STA 4020 HW 1

T1.

the skirk price $K = (S_t - d)\, e^{r(T-t)}$

if $K > (S_t - d)\, e^{r(T-t)}$

at time t:

1. sell K Unit of zero - coupon bond with face value \$1 and maturity T	$+K e^{-r(T-t)}$
2. Buy S_t stock	$-S_t$.
3. sell a forward contract	$+d$.

total at time t: $K e^{-r(T-t)} - S_t + d > 0$

at time T:

1. Pay K Unit zero - coupon bond.	$-K$
2. Sell S_T to forward contract	S_T
3. Sell the S_T at price K	$K - S_T$.

total at time $T = 0$

$\because K e^{-r(T-t)} - S_t + d > 0$ $\therefore$ Arbitrage.

$\therefore K \leq (S_t - d)\, e^{r(T-t)}$

if $K < (S_t - d)\, e^{r(T-t)}$

at time t:

1. Sell a stock S_t.	$+S_t$
2. Buy a forward contract	$-d$
3. Buy K Unit of zero-coupon matur at T.	$-Ke^{-r(T-t)}$

Total at time t: $S_t - d - Ke^{-r(T-t)} > 0$

at time T:

1. Buy a Stock at price K	$-S_T$
2. Use the forward contract Buy stock at price K	$S_T - K$
3. Get back the K Unit of zero-coupon bond	K

Total at time $T = 0$

$\therefore S_t - d - Ke^{-r(T-t)} > 0$ $\therefore$ Arbitrage

$\therefore K = (S_t - d)\, e^{r(T-t)}$

T2.

(a): the put-call parity:

$$Ke^{-r(T-t)} + C_t(T,K) = S_t + P_t(T,K)$$

$\because$ for zero-coupon: $1 \cdot e^{-r(T-t)} = 0.9948$

$$e^{-r \cdot 180} = 0.9948 \Rightarrow r = \frac{1}{180} \log \frac{1}{0.9948}$$

$$47.5 \cdot e^{-r \cdot 180} + 4.375 - 50 - 1.45$$

$$= 47.5 \cdot (0.9948) + 4.375 - 50 - 1.45$$

$$= 0.178 \neq 0$$

$\therefore$ this is NOT consistent with put-call parity.

(b):

$\because Ke^{-r(T-t)} + C_t(T,K) > S_t + P_t(T,K)$

at time t:

1. buy S_t stock	$-S_t$
2. buy a put option	$-P_t(T,K)$
3. Sell K unit zero-coupon bond	$+C_t(T,K)$
4. Sell a call option	$+Ke^{-r(T-t)}$

at time t. total:

$$Ke^{-r(T-t)} + C_t(T,K) - S_t - P_t(T,K) > 0.$$

at time T:

1. sell a S_T at call option at price K $\quad -(S_T-K)_+$

2. Stock price S_T $\quad +S_T$

3. exercise Put option $\quad (K-S_T)_+$

4. Pay for zero-coupon bond $\quad -K$

If $S_T > K \Rightarrow -S_T + K + S_T - K = 0.$

If $S_T \le K \Rightarrow S_T + K - S_T - K = 0.$

$\therefore$ the total at time $T = 0$.

$\therefore$ The above the total is:

$$Ke^{-r(T-t)} + C_t(T,K) - S_t - P_t(T,K) > 0.$$

$\therefore$ This is an Arbitrage.

T3.

if $C < \max(S - Ke^{-r(T-t)}, 0)$

at time t:

1. sell a call option	$(S - Ke^{-r(T-t)}, 0)_+$
2. Buy S stock	$-S$
3. Sell K unit zero-coupon bond	$Ke^{-r(T-t)}$

total at time t:

$$(S - Ke^{-r(T-t)}, 0)_+ - (S - Ke^{-r(T-t)}, 0) \geq 0$$

at time T:

1. sell the S stock at price K for the call option	$(K - S)_+$
2. the stock price	S
3. Get back money for K unit zero-coupon bond	$-K$

the total at time T:

$(K-S)_+ + S - K$

if $K-S > 0 \Rightarrow \quad K-S+S-K = 0$

if $K-S \leq 0 \Rightarrow \quad S-K \geq 0$

$\therefore \quad (K-S)_+ + S - K \geq 0$

$\therefore$ Payoff of total is:

$(S-Ke^{-r(T-t)}, 0)_+ - (S-Ke^{-r(T-t)}, 0)$

$+ (K-S)_+ + S - K \geq 0$

$\therefore$ this is Arbitrage.

$\therefore C \geq \max(S-Ke^{-r(T-t)}, 0)$