Optika vaje

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Kot v opombo, jaz ne verjamem v imaginarni lomni kolicnik. Je tocka, kjer mi domisljija ne omogoca, da bi dvomila v svoj dobro preizkusen vid.

1 Uvod in ponovitev geometrijske optike

1.1 Opticno vlakno z parabolicnim refrakcijskim koeficientom $n(x,y) = n_0\sqrt{1-\alpha^2(x^2-y^2)}$, $ax << 1$.

$$\nabla n = \frac{d}{ds}(n\frac{d\vec{r}}{ds}) \tag{1}$$

$$ds = \sqrt{dx^2+dz^2} = dz\sqrt{1+(\frac{dx}{dz})^2} \approx dz \tag{2}$$

$$\frac{dn}{dx} = \frac{d}{dz}(n\frac{dx}{dz}) = n\frac{d^2x}{dz^2} \tag{3}$$

$$\frac{d^2x}{dz^2} = \frac{1}{n}\frac{dn}{dx} = \frac{1}{n_0\sqrt{1-\alpha^2(x^2-y^2)}}n_0\frac{1}{\sqrt{1-\alpha^2(x^2-y^2)}}(-2\alpha^2x) = \frac{-2\alpha^2x}{(1-\alpha^2(x^2-y^2))} = \frac{-2\alpha^2xn_0^2}{n^2} \tag{4}$$

$$\frac{d^2x}{dz^2} = -\alpha^2\gamma => x = x_0sin(xz) \tag{5}$$

1.2 Curek svetlobe okrog Zemlje s polemerom R = 6400km

$$\vec{r} = (R+h)sin\theta\hat{e}_x + (R+h)cos\theta\hat{e}_y \tag{6}$$

$$ds = \sqrt{d\vec{r}d\vec{r}} = \sqrt{(R+h)^2(sin^2\theta+cos^2\theta)d\phi^2} = (R+h)d\theta \tag{7}$$

$$\nabla\vec{n} = \frac{d}{ds}(n\frac{d\vec{r}}{ds}) = n\frac{d^2\vec{r}}{ds^2} = \frac{n}{(R+h)^2}\frac{d^2\vec{r}}{ds^2} = -\frac{n}{R+h}(sin\theta\hat{e}_x+cos\theta\hat{e}_y) => \frac{dn}{dy} = -\frac{n}{} \tag{8}$$

$$4\cdot\Delta n = \frac{dn}{dy}\Delta y = -\frac{n}{R+h} \approx 1,56\cdot 10^{-6}; h << R \tag{9}$$

1.3 Izracunaj ABCD matriko za prehod zarka iz ene snovi v drugo $[y_2, \theta_2]^T = M[y_1, \theta_2]^2, M = \binom{AB}{CD}$

$$y_2 = 1 \cdot y_1 + 0 \cdot \theta_1 \tag{10}$$

Snellov zakon:

$$n_1 sin\theta_1 = n_2 sin\theta_2 =>_{\theta<<1} n_1\theta_1 = n_2\theta_2 \tag{11}$$

$$\Phi_1 = \theta_1 + \frac{y_1}{R}, \Phi_2 = \theta_2 + \frac{y_1}{R} \tag{12}$$

$$\Phi_1 = \Phi_2 => n_1(\theta_1 + \frac{y_1}{R}) = n_2(\theta_2 + \frac{y_1}{R}) \tag{13}$$

$$\theta_2 = (\frac{n_1 - n_2}{n_2 R})y_1 + \frac{n_1}{n_2}\theta_1; y_2 = 1y_1 + 0\theta_1 \tag{14}$$

$$n = \binom{A, B}{C, D} = \binom{1, 0}{\frac{n_1-n_2}{n_2R}, \frac{n_1}{n_2}} \tag{15}$$

1.4 Matrika za debelo leco (prehod med sredstvi).

$$\binom{y_2}{\theta_2} = M_3M_2M_1\binom{y_1}{\theta_2} \tag{16}$$

$$M_1 = \binom{1, 0}{\frac{n_1-n_2}{n_2R_1}, \frac{n_1}{n_2}} \tag{17}$$

$$M_2 = \binom{1, d}{0, 1} \tag{18}$$

$$M_3 = \binom{1, 0}{\frac{n_3-n_2}{n_3R_2}, \frac{n_2}{n_3}} \tag{19}$$

$$M = M_3M_2M_1 = \binom{A, B}{C, D} \tag{20}$$

1.5 Matrika za tanko leco (prek fokusa lec)

Fokus (tanke) lece: $\frac{1}{f_1} = \frac{n-1}{R_1}$ in $\frac{1}{f_2} = \frac{n-1}{R_2}$. Velja, da je f gorisce ali skupna locljivost dveh lec:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{n-1}{R_1} + \frac{n-1}{R_2} = \frac{(n+1)R_2 + (n-1)R_1}{R_1 - R_2} \tag{21}$$

Za elemente ABCD matrike za leco velja, da jih lahko prepisemo upostevajoc enacbi za fokus:

$$A = 1 + d\frac{1-n}{nR_1} = 1 - \frac{n-1}{nR_1} = 1 - \frac{d}{nf_1} \tag{22}$$

$$B = \frac{d}{n} \tag{23}$$

$$C=\frac{2-n}{R_2}(1+d\frac{1-n}{nR_1})+n\frac{1-n}{nR_1}=-\frac{1}{f_1}(1-\frac{d}{nf_1})-\frac{1}{f_1} \tag{24}$$

$$D=d\frac{1}{n_2}\frac{1-n}{R_2}+1=1-\frac{d}{nf_2} \tag{25}$$

$$M_{tanka-leca}=\begin{pmatrix} 1,0 \\ -\frac{2(n-1)}{R},1 \end{pmatrix} \tag{26}$$

1.6 Obravnavamo mikroskup iz dveh lec z fokusoma f_1, f_2 na razdalji d. Oddaljenost vzorca od 1. oznacuje f_1, oddaljenost detektorja od druge f_2.

$$povecava=\frac{y_2}{y_1} \tag{27}$$

$$M=M_5M_4M_3M_2M_1=\begin{pmatrix} 1,f_2 \\ 0,1 \end{pmatrix}\begin{pmatrix} 1,0 \\ -\frac{1}{f_2},1 \end{pmatrix}\begin{pmatrix} 1,d \\ 0,1 \end{pmatrix}\begin{pmatrix} 1,0 \\ -\frac{1}{f_1},1 \end{pmatrix}\begin{pmatrix} 1,f_1 \\ 0,1 \end{pmatrix} \tag{28}$$

V razmislek, ce bi dogajanje na lecah opisali kot kaj se zgodi pred in po prehodu bi veljalo: $Ay_1+B\theta_1=y_2$ in $Cy_1+D\theta_1=\theta_2 => \frac{y_2}{y_1}=A$, to se izkaze, da je res, ko 1. enacbo delimo z y_1 in je $\theta_1=0$ oz. ko je opticna os neodvisna od zacetnega kota.

$$M_{tot}=\begin{pmatrix} \frac{-f_2}{f_1},0 \\ -\frac{1}{f_2}(1-\frac{d}{f_1})-\frac{1}{f_3},\frac{-f_1}{f_2} \end{pmatrix} \tag{29}$$

1.7 Jonesove matrike za opticne elemente a) linearen polarizator vzporeden x in y osi, b) rotiran za 45° glede na x, c) cetrtinska retardacijska ploscica vzporedna x in y osi, kjer je y hitra os, d) dve identicni retardacijski ploscici, obe zarotirani za ± 45° glede na x in y os.

Linearni polarizator vzporeden z x osjo $M_{lin}^{(x)}=\binom{1,0}{0,0}\to\binom{1,0}{0,0}\binom{J_X}{J_Y}=\binom{J_X}{0}$, kjer vidimo, da prepusca svetlobo le v smeri x (enak postopek z y osjo). Primer, ko je rotiran linearni polarizator $M_{lin}^{45}=\frac{1}{2}\binom{1,\pm1}{\pm1,1}\to\frac{1}{2}\binom{1,\pm1}{\pm1,1}\binom{J_X}{J_Y}=\frac{1}{2}\binom{J_X\pm J_Y}{\pm J_X+J_Y}$. Retardacijske ploscice: velja, da bo y konponenta deformirana zaradi lomnega kolicnika n_y in x zaradi n_x. Faktor $\frac{\lambda}{4}$ oznacuje razliko v fazi med $\phi_y-\phi_x=\frac{\pi}{2}=\frac{2\pi}{\lambda_0}z\Delta n$, in izluscimo $z=\frac{1}{\Delta n}\frac{\lambda_0}{4}$.

$$M_{\lambda/4}=\begin{pmatrix} e^{2\pi i\frac{z}{\lambda_0}n_x},0 \\ 0,e^{2\pi i\frac{z}{\lambda_0}n_y} \end{pmatrix}=\begin{pmatrix} e^{i\phi_x},0 \\ 0,e^{i\phi_y} \end{pmatrix} \tag{30}$$

- Hitra os x: $n_y<n_x$,$c_y>c_x$ in $\phi_y<\phi_x => M_{qw}^{(y)}=e^{i\phi_x}\binom{1,0}{0,e^{i(\phi_y-\phi_x)}}=e^{i\phi_x}\binom{1,0}{0,+i}$

- Hitra os y: $M_{qw}^{(y)} = \binom{1,0}{0,-i}$

Retardacijske ploscice so lahko tudi zarotirane za poljuben kot α : $R(\alpha) = \binom{cos(\alpha),sin(\alpha)}{-sin(\alpha),cos(\alpha)}$, $R^T = R(-\alpha) = R^{-1}$.

$$M_{qw'} = R^T M_{qw} R => M_{qw} R \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = R \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} => R^T M_{qw} R \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \tag{31}$$

$$\alpha = \frac{\pi}{4} => \begin{pmatrix} cos^2(\alpha) + sin^2(\alpha), cos(\alpha)sin(\alpha) - isin(\alpha)cos(\alpha) \\ cos(\alpha)sin(\alpha) - isin(\alpha)cos(\alpha), sin^2(\alpha) + icos^2(\alpha) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1+i, 1-i \\ 1-i, 1+i \end{pmatrix} = M_{qr'} \tag{32}$$

$$M_{qw'} = \frac{\sqrt{2}}{2} \begin{pmatrix} e^{i\pi/4}, e^{-i\pi/4} \\ e^{-i\pi/4}, e^{i\pi/4} \end{pmatrix} = \frac{\sqrt{2}}{2} e^{i\pi/4} \begin{pmatrix} 1, -i \\ -i, 1 \end{pmatrix} \tag{33}$$

Delovanje te ploscice na vpadni snop je torej $M_{qw'} \binom{0}{1} = \frac{\sqrt{2}}{2} \binom{1,-1}{-i,1} \binom{0}{1} = \frac{1}{\sqrt{2}} \binom{-i}{1} = -\frac{i}{\sqrt{2}} \binom{1}{i}$.

Dve cetrtinski retardacijski ploscici skupaj data polovicno: $M_{hw'} = M_{qw'} \cdot M_{qw'} = \frac{1}{2} \binom{1,-i}{-i,1} \binom{1,-i}{-i,1} \xrightarrow{ideja:poenostavimo} => \binom{0,1}{1,0}$.

$$M_{hw'} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{34}$$

1.8 Podan je opticni filter za dolocitev polarizacije prepuscene svetlobe in je zarotiran za β, interpretiraj rezultat. β vpliva le na amplitudo prepuscene svetlobe.

$$T = \begin{pmatrix} cos^2(\theta), cos(\theta)sin(\theta) \\ cos(\theta)sin(\theta), sin^2(\theta) \end{pmatrix} \tag{35}$$

$$J_{in} = cos(\beta)\hat{e}_x + sin(\beta)\hat{e}_y = \begin{pmatrix} cos(\beta) \\ sin(\beta) \end{pmatrix} \tag{36}$$

$$J_{out} = TJ_{in} = \begin{pmatrix} cos^2(\theta), cos(\theta)sin(\theta) \\ cos(\theta)sin(\theta), sin^2(\theta) \end{pmatrix} \begin{pmatrix} cos(\beta) \\ sin(\beta) \end{pmatrix} = \begin{pmatrix} cos^2(\theta)cos(\beta) + cos(\theta)sin(\theta)sin(\beta) \\ cos(\theta)sin(\theta)cos(\beta) + sin^2(\theta)sin(\beta) \end{pmatrix} \tag{37}$$

$$= \begin{pmatrix} cos(\theta)cos(\theta - \beta) \\ sin(\theta)cos(\theta - \beta) \end{pmatrix} = cos(\theta - \beta) \begin{pmatrix} cos(\theta) \\ sin(\theta) \end{pmatrix} \tag{38}$$

Demonstracija rotacije opticnega elementa:

$$R^T M_{lin}^{(x)} R = \begin{pmatrix} c, -s \\ s, c \end{pmatrix} \begin{pmatrix} 1, 0 \\ 0, 0 \end{pmatrix} \begin{pmatrix} c, s \\ -s, c \end{pmatrix} = \begin{pmatrix} cos^2(\theta), cos(\theta)sin(\theta) \\ sin(\theta)cos(\theta), sin^2(\theta) \end{pmatrix} \tag{39}$$

1.9 Opticni izolator iz vec zaporednh elementov a) linearnega polarizatorja z osjo polarizacije v smeri y, b) $\lambda/4$ retardacijska ploscica z glavno osjo rotirano $\frac{\pi}{4}$ relativno na (x,y) in c) ogledalo

$$M_{lin}^{(y)} = \begin{pmatrix} 0, 0 \\ 0, 1 \end{pmatrix} \tag{40}$$

$$M_{qw}^{+} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1, -i \\ -i, 1 \end{pmatrix} \tag{41}$$

$$M_{qw}^{-} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1, i \\ i, 1 \end{pmatrix} \tag{42}$$

Ogledalo obrne opticno os $M_{mirror} = \binom{-1,0}{0,1}$ (torej iz $M_{qw}^{+} \rightarrow M_{qw}^{-}$. Opticni izolator je taka kombinacija opticnih elementov, ki blokira svetlobo na poti nazaj: $T_{tot} = \frac{1}{\sqrt{2}} M_{lin}^{(y)} M_{qw}^{-} M_{mirror} T_0 = ... = \frac{1}{2} \binom{0,0}{0,0}$

2 Jonesov kalkulus

2.1 Jonesovi vektorji elipticno polarizirane svetlobe, kjer je kot rotacije $\pi/4$.

Jonesov vektor v rotiranem sistemu S' prepisemo v sistem S, fazna razlika med sistemoma je kot rotacije.

$$J = R^T J' = \frac{1}{\sqrt{5}} \begin{pmatrix} cos(\theta), -sin(\theta) \\ sin(\theta), cos(\theta) \end{pmatrix} \begin{pmatrix} 2 \\ -i \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2cos(\theta + isin(\theta) \\ 2sin(\theta) - icos(\theta) \end{pmatrix} = \sqrt{\frac{2}{5}} \begin{pmatrix} 1 + \frac{i}{2} \\ 1 - \frac{i}{2} \end{pmatrix} \tag{43}$$

2.2 Elipticna polarizacija (desnosucna, kot $\frac{\pi}{6}$ z dolzinami osi $E_0, 2E_0$)Enkrat gre svetloba skozi vertikalno orientiran linearni polarizator in drugic skozi horizontalen. Posici razmerje intenzitet za prepusceno svetlobo, kjer je v prvem primeru rotacija polarizirane svetlobe Jonesovega vektorja taka, da je vzporeden ali x ali y osi in v drugem pa z rotacijo dveh linearnih polarizatorjev, da ta postaneta vzporedna elipticni polarizaciji sistema.

2.2.1 a)

$$J = R^T(\theta) J' = \frac{1}{\sqrt{5}} \begin{pmatrix} 2cos(\theta) + isin(\theta) \\ 2sin(\theta) - icos(\theta) \end{pmatrix} \xrightarrow{\frac{\pi}{6}} J = \frac{1}{\sqrt{5}} \begin{pmatrix} \sqrt{3} + \frac{i}{2} \\ 1 - \frac{i\sqrt{3}}{2} \end{pmatrix} \tag{44}$$

Rotacija skozi horizontalni linearni polarizator $J_X = \frac{1}{\sqrt{5}}\binom{\sqrt{3}+\frac{i}{2}}{0}$ (horizontalni lin. pol.) in $J_Y = \frac{1}{\sqrt{5}}\binom{0}{1-\frac{i\sqrt{3}}{2}}$ (vertikalno skozi lin. pol v y smeri).

$$I \propto |J_X|^2 = J_X J_X^* = \frac{13}{20}; I_y \propto \frac{7}{20} => \frac{I_X}{I_Y} = 13/7 \tag{45}$$

2.2.2 b)

Jonesova matrika za zarotiran linearni polarizator iz laboratorijskega sistema v elipticni sistem $T' = \binom{cos^2(\theta), sin(\theta)cos(\theta)}{sin(\theta)cos(\theta), sin^2(\theta)}$. Ko rotiramo iz laboratorijskega sistema v elipticni sistem uporabimo pri kotu rotacije oznako -, torej v tem primeru bi rotacijo za $\frac{\pi}{6}$ oznacili za $\theta = -\frac{\pi}{6}$.

$$T'_X = \begin{pmatrix} \frac{3}{4}, \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4}, \frac{3}{4} \end{pmatrix} \tag{46}$$

Za zasuk opticne osi pristejemo kotu rotacije cetrtino celotnega kota, za primer y-osi $\theta = \frac{\pi}{2} + (-\frac{\pi}{6})$:

$$T'_y = \begin{pmatrix} \frac{1}{4}, \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4}, \frac{3}{4} \end{pmatrix} \tag{47}$$

J' v sistemu vzporednih osi ($\theta = 0$)=¿ $J' = \frac{1}{\sqrt{5}}(\binom{2}{-i})$. Koncni rezultat je torej $J'_X = T'_X J' = \frac{1}{4\sqrt{5}}\binom{6+i\sqrt{3}}{-2\sqrt{3}-i}, T'_Y J' = \frac{1}{4\sqrt{5}}\binom{2-i\sqrt{3}}{2\sqrt{3}-3i}$. $I_X = \frac{31}{90}, I_Y = \frac{14}{91} => \frac{I_X}{I_Y} = \frac{13}{7}$

3 EM valovanje v homogenih in prevodnih snoveh + EM valovanje v prevodnih snoveh

3.1 Orientacija elipticnega polarizatorja z fazno razliko $\delta = \frac{\pi}{4}, \frac{E_X}{E_Y} = 2$, a) θ je kot, za katerega je elipsa zarotirana glede na (x,y) in b) razmerje med elipsami dolge in kratke osi $\frac{x}{y}$ =?

Elipsa zarotirana za kot θ relativno na (x,y) prek $tan(2\theta) = \frac{2E_{0x}E_{oy}cos(\delta)}{E_{ox}^2+E_{oy}^2}$, vstavimo podatek o razmerju osi in $tan(2\theta) = \frac{2\sqrt{2}}{3}, \theta = \frac{1}{2}arctan(\frac{2\sqrt{2}}{3}) \approx 21,7$.

$$\frac{b}{a} = \frac{E_{0y}sin(\delta)cos(\theta)}{E_{0x}cos(\theta) + E_{0y}cos(\delta)sin(\theta)} = \frac{\frac{\sqrt{2}}{2}cos(\theta)}{2cos(\theta) + \frac{\sqrt{2}}{2}sin(\theta)} = \frac{1}{\frac{4}{\sqrt{2}} + tan(\theta)} \tag{48}$$

3.2 Globina pri koznem pojavu za elektrodo (ITD) specifikacij: $\lambda = 500nm, R = 0,02\Omega m, d = 50nm$.

Za prevodne materiale uporabimo enacbe: $k_0 = \frac{\omega}{c_0}, \sigma_E = \frac{1}{\xi}, d = \frac{1}{n_{im}k_0}$.

$$\mathcal{N}^2 = \epsilon_\mu + \frac{i\sigma_E^2}{\epsilon_0\omega} = n_{Re}^2 - n_{Im}^2 + 2in_{Re}n_{Im} \tag{49}$$

$$n_R^2 - n_{Im}^2 = \epsilon_\mu; \frac{\sigma_E}{\epsilon_0\omega} = 2n_R n_I; n_I = \frac{\sigma_E}{2n_R\epsilon_0\omega} \tag{50}$$

$$n_R^2 - \frac{\sigma_E^2}{4\epsilon_0\omega n_R^2} = \epsilon_E / \cdot n_R^2 \tag{51}$$

$$n_R^4 - \frac{\sigma_E^2}{4\epsilon_0\omega} - \epsilon_E n_R^2 = 0; t = n_R^2 \tag{52}$$

$$t^2 - \epsilon_E t - \alpha = 0; t_{1,2} = \frac{\epsilon_E \pm \sqrt{-4\alpha - \epsilon_E^2}}{2} = \frac{\epsilon_E \pm i\sqrt{4\alpha + \epsilon_E^2}}{2} \tag{53}$$

$$n_{Re}^2 = -\frac{1}{2}\epsilon_E + \frac{1}{2}\sqrt{\epsilon^2 + \frac{\sigma_n^2}{\epsilon_0^2\omega_2}} \tag{54}$$

$$n_{Im} = -\frac{\epsilon}{2} + \frac{1}{2}\sqrt{\epsilon^2 + \frac{\sigma_E^2}{\epsilon_0^2\omega}} = \sqrt{\frac{\sigma_E}{2\epsilon_0\omega}} \tag{55}$$

$$d = \frac{c_0}{\omega}\sqrt{\frac{2\epsilon_0\omega}{\sigma_E}} = \sqrt{\frac{2\xi}{\omega\mu_0}} \tag{56}$$

Za tocne vrednosti vstavimo podatke.

3.3 Odbojnost; s pomocjo tabele izracunaj odbojnost obeh svetlob na materialih (iz zraka na material).

Au	1,4	1,9
Ag	0,08	1,9
Al	0,4	4,5

Velja Snellov zakon $r = \frac{n_1 cos(\alpha) - n_2 cos(\beta)}{n_2 cos(\alpha) + n_2 cos(\beta)}$ in $n_1 = n_{air} = 1$.

$$r = \frac{1 - ncos(\beta)}{1 + ncos(\beta)} \tag{57}$$

$$R = |r|^2 = \frac{(1 - n_{Re})^2 + n_{Im}^2}{(1 + n_{Re})^2 + n_{Im}^2} \tag{58}$$

Iz tabele vzamemo podatke in jih "nasopamo" noter.

3.4 Odbojnost za dobre prevodnike, $\sigma_E >> \omega\epsilon_0, n >> 1$. Izpelji odvisnost odbojnosti za normalno svetlobo na dobrem prevodniku!

$$r_s = \frac{n_1 cos(\theta_i) - n_2 cos(\theta_t)}{n_1 cos(\theta_i) - n_2 cos(\theta_t)} \tag{59}$$

$$R = |r|^2 = \frac{(1-n_{Re})^2 + n_{Im}^2}{(1+n_{Re})^2 + n_{Im}^2} \tag{60}$$

$$n_{Re}^2 = \frac{1}{2}\epsilon + \frac{1}{2}\sqrt{\epsilon^2 + \frac{\sigma_E^2}{\epsilon_0^2\omega^2}}, n_{Im}^2 = -\frac{1}{2}\epsilon + \frac{1}{2}\sqrt{\epsilon^2 + \frac{\sigma_E^2}{\epsilon_0^2\omega^2}} \tag{61}$$

Za dobre prevodnike $n_{Re}^2 \approx n_{Im}^2 \approx \frac{1}{2}\frac{\sigma_E}{\epsilon_0\omega}$ sta dela priblizno enaka. Odvisnost $R(\lambda)$ za dobre prevodnike: $R \approx 1 - \frac{2}{n+1} \approx 1 - 2\sqrt{\sigma_E/2\epsilon_0\omega}$.

3.5 Odbojnost v prevodniku za dolocene kote - svetloba preide iz zraka na aluminij pod kotom $\theta = 30$. Velikost in smer kompleksnega valovnega vektorja aluminija: $n_{Re} = 0,4; n_{Im} = 4,5$

3.5.1 Velikost in smer kompleksnega valovnega vektorja $\mathcal{K} = k_{Re} + k_{Im} \cdot i$

$$e^{i\vec{\kappa}\vec{r}} = e^{-ik_{Re}\vec{r}} e^{-k_{Im}\vec{r}} \tag{62}$$

$$e^{ik\vec{r}} = e^{ik_{Re}\vec{r} - k_{Im}\vec{r}} = e^{ik_{Re}r - k_{Im}r} => ik_i r = ik_{Re}r - k_{Im}r \tag{63}$$

$$\mathcal{K}\mathcal{K} = k_0^2\mathcal{N}^2 = k_{Re}^2 + 2ik_{Im}k_{Re} - k_{Im}^2 \tag{64}$$

Upostevamo $k_0 = \frac{\omega}{c_0}$ in zvezo $cos^2(\beta) + sin^2(\beta) = 1$ in nadalje uporabimo $k_0 sin(\alpha) = k_{Re} sin(\beta)$:

$$k_0^2\mathcal{N}^2 = k_{Re}^2(cos^2(\beta)+sin^2(\beta))+2ik_{Im}k_{Re}cos(\beta)-k_{Im}^2 = k_{Re}^2cos^2(\beta)+2ik_{Im}k_{Re}cos(\beta)-k_{Im}^2+k_{Re}^2sin^2(\beta) \tag{65}$$

$$= (k_{Re}cos(\beta) + ik_{Im})^2 + k_{Re}^2 sin^2(\beta) \tag{66}$$

$$=> k_0^2(\mathcal{N} - sin^2(\alpha)) = (k_{Re}cos(\beta) + ik_{Im}) / \cdot \sqrt{}$$

(67)

$$k_{Re}sin(\beta) = k_0 sin(\alpha) => sin(\beta) = \frac{k_0}{k_{Re}} sin(\alpha) = ... => \beta \approx 51,7 \tag{68}$$

3.5.2 Odbojnost za TE polarizacijo na aluminiju pod kotom 30°, Rs(TE)=?, Aluminij je prevodnik zato je lahko θ_z kompleksen.

$$r_s = \frac{n_1 cos(\theta_i) - n_2 cos(\theta_t)}{n_1 cos(\theta_i) + n_2 cos(\theta_t)} = \frac{cos(\alpha) - \mathcal{N}\sqrt{1 - sin^2(\theta_t)}}{cos(\alpha) + \mathcal{N}\sqrt{1 - sin^2(\theta_t)}} \tag{69}$$

$$sin(\theta_t) = \frac{sin(\alpha)}{\mathcal{N}} \tag{70}$$

4 Fresnelove enacbe in Fraunhofferjev ter Fresnelov uklon

4.1 Globina pri koznem pojavu pri diamantu z $\lambda = 600nm, n = 2,417$in kote $\theta_i = [24,5,25,50]$

Poglejmo $sin\theta_c = \frac{n_2}{n_1} = \frac{1}{2{,}417}$ in $d = \frac{1}{k_0 n_2 \mathcal{K}} = \frac{\lambda}{2\pi n_2}(\frac{sin^2(\theta_i)}{sin^2(\theta_C)} - 1)^{-1/2} = 1,4\mu m$

4.2 Fraunhofferjev uklon na pravokotni zaslon ($R_0, a, b, \mathcal{K}_x = \frac{k_\xi}{R_0}, \mathcal{K}_y = \frac{k_\eta}{R_0}$)

Za izracun upostevamo definicijo $sin(cx) = \frac{sin(x)}{x}$ in vrednost funkcije f, ki je 1 na pravokotnem obmocju in 0 zunaj.

$$E(\mathcal{K}_\mathcal{X}, \mathcal{K}_\mathcal{Y}, R_0) = \frac{iE_0}{\lambda}\frac{e^{ikR_0}}{R_0}\iint f(x)e^{-i\kappa_x x}e^{-i\kappa_y y}dxdy \tag{71}$$

$$= \frac{iE_0}{\lambda}\frac{e^{ikR_0}}{R_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-i\kappa_x x}e^{-i\kappa_y y}dxdy \tag{72}$$

Resimo po delih, za x koordinato $I_X(\frac{a}{2}) = \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-i\kappa_x x}dx = -\frac{1}{i\kappa x}e^{-i\kappa_x x}|_{\frac{-a}{2}}^{\frac{a}{2}} = \frac{2}{\kappa_x}sinc(\frac{kappa_x a}{2})$ in $I_y(\frac{b}{2}) = b \cdot sinc(\frac{\kappa_y b}{2})$.

$$E = \frac{iE_0 e^{ikR_0}}{\lambda R_0} absinc(\frac{\kappa_x a}{2})sinc(\frac{\kappa_y b}{2}); I = |E|^2 \tag{73}$$

Naprej lahko racunamo $I(\theta_x, 0) = I_0 sinc(\frac{kasin\theta_x}{2})$, kar je difrakcija po kotu. MNinimum za sinc(x) je, ko je x = 0. Torej je x veckratnik π =¿ $\frac{kasin\theta_x}{2} = n\pi$ =¿ $\theta_x = sin^{-1}(\frac{2n\pi}{ka})$.

4.3 Fraunhofferjev uklon na difrakcijski resetki

$$E_0(\mathcal{K}_\mathcal{X}, \mathcal{K}_\mathcal{Y}) = \alpha\int f(x,y)e^{-i\kappa_x x}dx = e^{-\kappa_x x_0}E_0(\kappa_x, \kappa_y) \tag{74}$$

$$E = \sum_j^{\frac{N}{2}} E_0 e^{-i\kappa_x D_j}e^{-j\frac{\kappa_x D}{2}} + \sum_j^{\frac{N}{2}} E_0 e^{i\kappa_x D_j}e^{i\frac{\kappa_x D}{2}} \tag{75}$$

Uporabimo razvoj v vrsto $\sum_{j=0}^{N} x^j = \frac{1-x^N}{1-x}$,

$$E = E_0 e^{-i\frac{\kappa_x D}{2}} (\frac{1-e^{-i\frac{\kappa_x DN}{2}}}{1-e^{-i\kappa_x D}}) + E_0 e^{i\frac{\kappa_x D}{2}} (\frac{1-e^{i\frac{\kappa_x DN}{2}}}{1-e^{i\kappa_x D}}) = ... = 2E_0 \frac{sin(\frac{\kappa_x DN}{2})}{sin(\frac{\kappa_x D}{2})} \tag{76}$$

4.4 Resolucija difrakcijskih resetk

Podana sta $\lambda_1 = 589,0nm, \lambda_2 = 589,6nm$ in:

$$E \propto sinc(\frac{\kappa_x d}{2}) \cdot \frac{sin(\frac{\kappa_x DN}{2})}{sin(\frac{\kappa_x D}{2})} \propto \sigma(\theta) \cdot \Gamma(\theta) \tag{77}$$

$$I(\theta) = |E|^2 = I_0 sinc(\frac{\kappa_x d}{2}) \frac{sin^2(\frac{\kappa_x DN}{2})}{sin^2(\frac{\kappa_x D}{2})} \tag{78}$$

Za primer $N >> 1$ $I(\theta)$ zgleda tako, da pocasi oscilira zaradi $sinc^2\alpha$ in hitro zaradi $\frac{sin^2(\frac{\kappa_x DN}{2})}{sin^2(\frac{\kappa_x D}{2})}$, kjer zgornjemu clenu dolocimo minimum in spodnjemu maksimum za najhitrejse mozno pojemanje.

- maksiumum: $sin^2\Gamma = 0 => \Gamma = \frac{kD}{2} sin\theta = m\pi => sin\theta_m = \frac{2\pi m}{kD} = \frac{m\lambda}{D} / \cdot \frac{d}{dt} => cos\theta\Delta\theta = \frac{m}{D}\Delta\lambda_C$
- minimum: $sin^2\Gamma N = 0 => \Gamma N = \frac{kDN}{2} sin\theta = n\pi => sin\theta_N = \frac{k\pi m}{kDN}$ 1.minimum se zgodi pri n+1, torej:

$$(n+1)\pi - n\pi = \frac{kDN}{2}(sin\theta_{min} - sin\theta_{max}) =] \approx \frac{kDN}{2}(\theta_{min} - \theta_{max}) \tag{79}$$

$$\pi = \frac{kD}{2} N\Delta(sin\theta) = \frac{kDN}{2} cos\theta d\theta \tag{80}$$

$$\Delta\theta = \frac{2\pi}{kDNcos\theta} = \frac{\lambda}{NDcos\theta_{max}} => \frac{\lambda}{\Delta\lambda} = mN \tag{81}$$

4.5 Fresnelove cone so okrogle cone radija a, nakopicene ena za drugo z drugacnimi polmeri. V primeru dveh je notranji radij $a_1 = \sqrt{\lambda L}$ in zunanji $a_2 = \sqrt{2\lambda L}$. Lahko racunamo za vsako cono posamicno ali skupaj z $\frac{1}{L} = \frac{1}{z_0'} + \frac{1}{z_0}$

Poenostavitev je sicer ta, da je vse na opticni osi $(x', y') = (\xi', \eta') = (0, 0), R_0 = z_0, R_0' = z_0'$:

$$E = \frac{E_0}{i\lambda} \frac{e^{ikz_0'}}{z_0'} \frac{e^{ikz_0}}{z_0} \iint f(x,y) e^{\frac{ik}{2z_0'}(x^2+y^2)} e^{\frac{ik}{2z_0}(x^2+y^2)} dxdy = \frac{c}{i\lambda} \iint f(x,y) e^{\frac{ik}{2L}(x^2+y^2)} dxdy \tag{82}$$

$$E = \frac{C}{i\lambda}\int_0^{2\pi} d\phi \int_0^{a_1} e^{\frac{ik}{2L}\rho^2}\rho d\rho = \frac{2\pi ic}{\lambda}\int_0^{a_1} e^{\frac{ik}{2L}\rho^2}\rho d\rho = \frac{2\pi c}{i\lambda}\frac{L}{k}\int_0^{\frac{ka_1^2}{2L}} e^{iu}du = \tag{83}$$

$$= \frac{CL}{i}\int_0^{\frac{ka_1^2}{2L}} e^{iu}du \xrightarrow{a_1,0} = -CL(e^{i\pi}-1) = 2CL \tag{84}$$

V tem primeru sta za podatka vstavljeni kolicini $(a_1, 0)$,naprej pa za $(a_2, 0)$ dobimo 0 in (a_1, a_2) $-2CL$.

4.6 Fresnelova leca, kjer je podan rekurzivni zapis za elektricno poljsko jakost $E_n = 2(-1)^n E_0$, $a_1 = \sqrt{n\pi L}$, $a_2 = \sqrt{(n+1)\lambda L}$

Velja, da sodo/liho stevilo takih odprtin deluje kot leca z fokusom $f = L = (a_2^2 - a_1^2)/\lambda$.

$$E_0 = \frac{\tilde{E_0}e^{ik(z_0'+z_0)}}{z_0'+z_0}; u = \frac{k\rho}{2L} \tag{85}$$

Za nadaljne clene upostevamo v mejah integrala vbistvu je $u(a_2) = \frac{k}{2L}[(n+1)\lambda L] = (n+1)\pi$ in $u(a_1) = n\pi$.

$$E_n = -CLe^{in}|_{n\pi}^{(n+1)\pi} = -CL(e^{in\pi}e^{i\pi} - e^{in\pi}) = 2CLe^{in\pi} = (-1)^n 2CL \tag{86}$$

$$= (-1)^n 2CL = (-1)^n \cdot 2 \cdot \frac{E_0 e^{ik(z_0'+z_0)}}{z_0+z_0'} => E_n = (-1)^n 2E_0 \tag{87}$$

5 Interferenca

5.1 Interferenca na tanki rezi z debelino odprtine $D = 0,54mm, \lambda = 600nm$. Zanima nas koliksna je oddaljenost od zalona, da bo razdalja med interferencnima vzorcema $\xi = 1mm$.

$$2\pi m = k_0 D sin(\theta) = k_0 D\theta => tan(\theta) = \frac{\xi}{z_0} => \xi = z_0 tan(\theta) \approx z_0\theta \tag{88}$$

Za m=1:$\theta = \frac{2\pi}{k_0 D} => z_0 = \frac{\xi}{\theta} = 0,9m$

5.2 Fresnelova bipiramida za $n = 1,5, \lambda = 633nm, \xi = 0,5nm$. Z α je oznacen prizmin kot, z β kot odboja.

Interferencni vzorec z $E = E_0 e^{i(\vec{k_1}\vec{r}-\omega t)} + E_0 e^{\vec{k_2}\vec{r}-\omega t}; k_1 = (-k_x, 0, k_y), k_z = (k_x, 0, k_y)$.

$$E(x,z) = E_0(e^{-ik_x x} + e^{ik_z z})e^{i(k_z z - \omega t)} \tag{89}$$

$$I \propto |E|^2 = |E_0|^2 cos^2(k_x x) \tag{90}$$

Maksimum intenzitete nastopi pri $cos(k_x x) = \pm 1 => \pi$. Snellov zakon je $n_{steklo} sin(\alpha) = n_{zrak} sin(\beta)$ postopkoma privede do $n\alpha \approx \beta$:

$$k_x = k_0 sin(\beta - \alpha) = \frac{2\pi}{\lambda} sin(\beta - \alpha) => \frac{2\pi x}{\lambda} sin(\beta - \alpha) = \pi => sin(\alpha(n-1)) \approx \frac{\lambda}{2x} \tag{91}$$

$$\alpha = \frac{\lambda}{2x(n-1)} \tag{92}$$

5.3 Fabry-Perrotov interferometer z $n = 1,5, R = 0,95$ in $\lambda_0 = 656,28nm, \Delta\lambda = 0,016nm$. Doloci visino h, da bo lahko interferometer locil neodvisni crti.

$$E_{out} = E_0 \frac{t_{12} t_{21}}{1 - r_{21}^2 e^{i\phi}}; \phi = 2k_0 ndcos(\alpha); R = |r_{21}|^2 \tag{93}$$

$$T = \frac{1}{1 + \frac{4R}{(1-R)^2} sin^2(\frac{\theta}{2})} = \frac{1}{1 + Fsin^2(\theta/2)} \tag{94}$$

Kriterij za dolocitev je, da sta dva $T(\lambda)$ vrha locena, ko sta se vedno locena, ce v $'\lambda'$ prostoru individualne vrh zadane svojo polovicno maksimalno vrednost.

$$T = \frac{1}{2} => Fsin^2 \frac{\theta}{2} = 1 => \frac{\theta}{2} = arcsin \frac{1}{\sqrt{F}} \tag{95}$$

$$d\lambda = -\frac{c}{\nu^2} d\nu => \frac{d\lambda}{\lambda} = -\frac{d\nu}{\nu} \tag{96}$$

$$\frac{\phi}{2} = arcsin(\frac{1}{\sqrt{F}}) = k_0 nd = \frac{\Delta\omega nd}{c} => \frac{1}{\sqrt{F}} = \frac{nd\omega\Delta\lambda}{c\lambda} => d = \frac{1}{2\pi n\sqrt{F}} \frac{\lambda^2}{\Delta\lambda} = 73\mu m \tag{97}$$

6 Vecplastni nanosi in sipanje

6.1 Popolna prepustnost skozi tanko plast (zrak-¿TP-¿steklo). Opazujemo svetlobo valovne dolzine λ=540nm, TP je debeline $d_2$in ima neznani lomni kolicnik n_2, steklo ima $n_3 = 1,54$. Zanima nas najmanjsa globina d_2 in n_2, da struktura prepusca svetlobo.

$$M = \begin{pmatrix} M_{11}, M_{12} \\ M_{21}, M_{22} \end{pmatrix} = M_{12} P M_{23} = M_{12} \begin{pmatrix} e^{-i\delta}, 0 \\ 0, e^{i\delta} \end{pmatrix} M_{23} = \frac{1}{t_{12}} \begin{pmatrix} 1, r_{12} \\ r_{12}, 1 \end{pmatrix} \begin{pmatrix} e^{-i\delta}, 0 \\ 0, e^{i\delta} \end{pmatrix} \frac{1}{t_{23}} \begin{pmatrix} 1, r_{23} \\ r_{23}, 1 \end{pmatrix} \tag{98}$$

$$= \frac{1}{t_{12} t_{23}} \begin{pmatrix} r_{12} + r_{23} e^{i\delta}, r_{23} e^{-i\delta} + r_{12} e^{i\delta} \\ r_{12} e^{-i\delta} + r_{23} e^{i\delta}, r_{12} r_{23} e^{-i\delta} + e^{i\delta} \end{pmatrix} \tag{99}$$

$$T = \frac{n_3}{n_1} \frac{|t_{12} t_{23}|^2}{1 + |r_{12} r_{23}|^2 + 2r_{12} r_{23} cos(2\delta)}; T_{max} => cos(2\delta_{min}) => 2\delta = (2m-1)\pi \tag{100}$$

$$M_{11} = \frac{1}{t_{12}t_{23}}(e^{-i\delta} + r_{12}r_{23}e^{i\delta}), M_{21} = ... \tag{101}$$

$$t = \frac{1}{M_{11}} = \frac{t_{12}t_{23}}{e^{-i\delta} + r_{12}r_{23}e^{i\delta}} \tag{102}$$

$$r = \frac{M_{21}}{M_{11}} = \frac{r_{12}e^{-i\delta} + r_{23}e^{i\delta}}{e^{-i\delta} + r_{12}r_{23}e^{i\delta}} \tag{103}$$

Maksimalna prepustnost je, ko je prepustnost T=1 in odbojnost R=0. Pri teh pogojih je $r_{12} = r_{23}$ in $n_2 = 1,24$.

6.2 Vecplastno dielektricno steklo z 17 plastmi z alternirajocimi lomnimi kolicnikomi $n_a = 1,38$ in $n_b = 2,32$ na steklu z $n_3 = 1,52$.Koliksna je odbojnost?

$$a = \frac{\lambda}{4na}, b = \frac{\lambda}{4nb} \tag{104}$$

$$M_{tot} = M_{1a}P_aM_{ab}P_b...P_aM_{a3} = M_{1a} \cdot M_N \cdot P_aM_{a3} \tag{105}$$

$$M_{1a} = \frac{1}{t_{1a}}\begin{pmatrix}1, r_{1a}\\ r_{1a}, 1\end{pmatrix}; r_{1a} = \frac{1-na}{1+na} \tag{106}$$

$$M_{a3} = \frac{1}{t_{a3}}\begin{pmatrix}1, r_{a3}\\ r_{a3}, 1\end{pmatrix} \tag{107}$$

$$P_{ab} = \begin{pmatrix}e^{-i\delta_{ab}}, 0\\ 0, e^{i\delta_{ab}}\end{pmatrix} = \begin{pmatrix}-i, 0\\ 0, i\end{pmatrix}; \delta_{ab} = n_{a,b}k_0d_{a,b} = \frac{\pi}{2} \tag{108}$$

7 Koherenca svetlobe

7.1 Mikelsonov interferometer z $\lambda_1 = 588,995nm$ in $\lambda_2 = 589,592nm$. Kako dalec od ogledala moramo opazovati, da ju locimo?

$$\Delta\omega = |\omega_1 - \omega_2| \tag{109}$$

$$S(\omega) = \frac{I}{2}[\delta(\omega-(\omega_0-\frac{\Delta\omega}{2}))+\delta(\omega-(\omega_0+\frac{\Delta\omega}{2}))] = \frac{I_0}{2}|\delta(\omega-\omega_0)+\delta(\omega-(\omega+\Delta\omega))| \tag{110}$$

$$G^{(1)}(\tau) = \int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau}d\omega = \frac{I_0}{2}[e^{i(\omega_0-\frac{\Delta\omega}{2})\tau} + e^{i(\omega_0+\frac{\Delta\omega}{2})\tau}] \tag{111}$$

$$I_{det} = 2I_0+2Re(G^{(1)}) = 2I_0+I_0(cos((\omega_0-\frac{\Delta\omega}{2})\tau)) = 2I_0(1+cos(\omega_0\tau)cos(\frac{\Delta\tau}{2})) \tag{112}$$

Da poiscemo minimum, upostevamo, da bo to tam kjer je oscilajoc clen $cos(\frac{\Delta\omega\tau}{2}) = 0 => \Delta\omega = \pi => \pi = \frac{2\Delta\omega x}{c_0}$:

$$\Delta x = \frac{\pi c_0}{2\Delta\omega} = \frac{\pi c_)}{2}\frac{1}{2\pi c_0(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})} \tag{113}$$

7.2 Spekter in avtokorelacija Gavsovega paketa $E(t) = E_0 e^{-\pi(\frac{t}{t_c})^2} e^{-i\omega_0\tau}$, doloci $S(\omega) =?$

Zacetni nastavek je avtokorelacijska funkcija $G^{(1)}(\tau) = lim_{T->\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} E(t)E^*(t+\tau)dt$ in njeno normalizirano vrednostjo $g(\tau)$.

$$g(\tau) = \frac{G^{(1)}(\tau)}{G^{(1)}(0)} = lim_{T->\infty} \frac{\int_{-\frac{T}{2}}^{\frac{T}{2}} E(t)E^*(t+\tau)dt}{\int_{-\frac{T}{2}}^{\frac{T}{2}} |E(t)|^2 dt} = \frac{1}{C} lim_{T->\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} E(t)E^*(t+\tau)dt \tag{114}$$

Nato izracunamo normalizacijsko konstanto $C = lim_{T->\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} |E(t)|^2 dt = ... = |E_0|^2 \frac{t_c}{\sqrt{2}}$:

$$= \frac{1}{C} lim_{T->0} \int_{-\frac{T}{2}}^{\frac{T}{2}} E_0 e^{-\pi(\frac{t}{t_c})^2} e^{-i\omega_0 t} E_0^* e^{i\omega_0(t+\tau)} e^{-\pi(\frac{t+\tau}{t_c})^2} dt \tag{115}$$

$$= \frac{|E_0|^2}{c} lim_{T->\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-\pi(\frac{t}{t_c})^2} e^{-\pi(\frac{t+\tau}{t_c})^2} e^{i\omega_0\tau} dt \tag{116}$$

$$= \frac{\sqrt{2}}{t_c} lim_{T->\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-\frac{2\pi}{t_c^2}(t^2+t\tau+\frac{\tau}{2})} e^{i\omega_0\tau} dt \tag{117}$$

$$= \frac{\sqrt{2}}{t_c} lim_{T->0} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-\frac{2\pi}{t_c^2}(t^2+t\tau+\frac{\tau}{2})} e^{-\frac{2\pi\tau^2}{4t_c^2}} e^{i\omega_0\tau} dt \tag{118}$$

$$= \frac{\sqrt{2}}{t_c} e^{-\frac{2\pi\tau^2}{4t_c^2}} e^{i\omega_0\tau} lim_{T->\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-\frac{2\pi}{t_c^2}(t+\frac{\tau}{2})^2} dt \tag{119}$$

$$=_{NS}= \frac{\sqrt{2}}{t_c} e^{-\frac{2\pi\tau^2}{4t_c^2}} e^{i\omega_0\tau} \int_{-\infty}^{\infty} e^{-\frac{2\pi}{t_c^2}(x)^2} dx = e^{-\frac{2\pi\tau^2}{2t_c^2}} e^{i\omega_0\tau} \tag{120}$$

Po Fourierovi transformaciji dobimo:

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G^{(1)}(\tau) E^{-i\omega\tau} d\tau = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\tau) e^{-i\omega\tau} d\tau \tag{121}$$

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7.3 Spekter in avtokorelacija eksponentnega pulza $E = E_0 e^{-t/t_c} e^{-i\omega_0 t} : t > 0$ in 0 sicer.

8 Lomni kolicnik

8.1 Koeficienti v Cauchyjevi formuli za $\lambda >> \lambda_0$, doloci formuli za koeficienta C_1, C_2 prek Sellmarjevih koeficientov A in G za material z eno resonanco λ_0 v zunanjem magnetnem polju. + Z podatki A=1, G =1,17, $\lambda_0 = 500nm$ in $\lambda = 800nm$ lahko vrednosti izracunas.

$$\mathcal{N} = C_1 + \frac{C_2}{\lambda^2} /\cdot^2 => \mathcal{N}^2 = A + \frac{C\lambda^2}{\lambda^2 - \lambda_0^2} = A + \frac{G}{1 - (\frac{\lambda_0}{\lambda})^2} = C_1^2 + 2\frac{C_1 C_2}{\lambda^2} + \frac{C_2^2}{\lambda^4} \tag{122}$$

$$C_1^2 + 2\frac{C_1 C_2}{\lambda^2} + \frac{C_2^2}{\lambda^4} = A + \frac{G}{1 - (\frac{\lambda_0}{\lambda})^2} =_{\lambda >> \lambda_0} = A + G + G(\frac{\lambda_0^2}{\lambda^2}) \tag{123}$$

Z enacenjem koeficientov pri enakih vrednostih λ dobimo, da je $C_1^2 = A + G$ in $2C_1C_2 = G\lambda_0^2$ ter izrazimo iskane vrednosti.

$$\mathcal{N} = \sqrt{A+G} + \frac{G}{2\sqrt{A+G}} \frac{\lambda_0^2}{\lambda^2} \tag{124}$$

9 Opticno anizotropne snovi

9.1 Opticno aktivni materiali in cirkularni dihroizem; obravnavaj Jonesovo matriko, matriko, ki opise opticni fenomen cirkularnega dihroizma in matriko, ki opise prehod skozi material, ki je hkrati opticno aktiven in 'ima' cirkularni dihroizem.

9.1.1 Jonesova matrika

Linearno polarizirano svetlobo razdelimo na levo in desno sucno polarizacijo ($\tilde{n} = \frac{n_{LCP} + n_{RCP}}{2}, \Delta n = n_{RCP} - n_{LCP}$), ki je pred in po prehodu skozi opticno aktiven material:

$$J_{in} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{125}$$

$$J_{out} = \frac{1}{2}\begin{pmatrix} 1 \\ i \end{pmatrix} e^{in_{LCP}k_0 d} + \frac{1}{2}\begin{pmatrix} 1 \\ -i \end{pmatrix} e^{in_{RCP}k_0 d} = \frac{1}{2}\begin{pmatrix} e^{ik_0\tilde{n}d} e^{-ik_0\frac{\Delta n}{2}d} + e^{ik_0\tilde{n}d} e^{ik_0\frac{\Delta n}{2}d} \\ ie^{ik_0\tilde{n}d} e^{-ik_0\frac{\Delta n}{2}d} - ie^{-ik_0\tilde{n}d} e^{ik_0\frac{\Delta n}{2}d} \end{pmatrix} = \tag{126}$$

$$= \frac{e^{ik_0\tilde{n}d}}{2}\begin{pmatrix} e^{-ik_0\frac{\Delta n}{2}d} + e^{ik_0\frac{\Delta n}{2}d} \\ ie^{-ik_0\frac{\Delta n}{2}d} - ie^{ik_0\frac{\Delta n}{2}d} \end{pmatrix} = e^{ik_0\tilde{n}d}\begin{pmatrix} cos(k_0\frac{\Delta n}{2}d) \\ sin(k_0\frac{\Delta n}{2}d) \end{pmatrix} = e^{ik_0\tilde{n}d}\begin{pmatrix} cos(\theta), -sin(\theta) \\ sin(\theta), cos(\theta) \end{pmatrix} = \tag{127}$$

$$J_{out} = R_{0A} J_{in} \tag{128}$$

9.1.2 Cirkularni dihroizem

- $\kappa_{LCP} = n_{Im}^{LCP} \cdot k$, $\kappa_{RCP} = n_{Im}^{RCP} \cdot k$
- $\overline{\kappa} = \frac{\kappa_{LCP}+\kappa_{RCP}}{2}, \Delta\kappa = \kappa_{RCP} - \kappa_{LCP}$
- $\kappa_{RCP} = \overline{\kappa} + \frac{\Delta\kappa}{2}, \kappa_{LCP} = \overline{\kappa} - \frac{\Delta\kappa}{2}$

$$J_{in} = \begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\i\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-i\end{pmatrix} \tag{129}$$

$$J_{out} = \frac{1}{2}\begin{pmatrix}1\\i\end{pmatrix}e^{-\kappa_{LCP}d} + \frac{1}{2}\begin{pmatrix}1\\-i\end{pmatrix}e^{-\kappa_{LCP}d} = \frac{e^{-\frac{\overline{\kappa}d}{2}}}{2}\begin{pmatrix}e^{\frac{\Delta\kappa d}{2}} + e^{\frac{\Delta\kappa d}{2}}\\ ie^{\frac{\Delta\kappa d}{2}} - ie^{\frac{\Delta\kappa d}{2}}\end{pmatrix} = \tag{130}$$

$$= e^{-\overline{\kappa}d}\begin{pmatrix}cosh(\frac{\Delta\kappa d}{2})\\ isinh(\frac{\Delta\kappa d}{2})\end{pmatrix} = e^{-\overline{\kappa}d}\begin{pmatrix}cos(\frac{i\Delta\kappa d}{2})\\ sin(\frac{i\Delta\kappa d}{2})\end{pmatrix} \tag{131}$$

$$R_{C.D.} = e^{-\overline{\kappa}d}\begin{pmatrix}cos(\theta), sin(\theta)\\ sin(\theta), cos(\theta)\end{pmatrix} : \theta = i\frac{\Delta\kappa d}{2} \tag{132}$$

9.1.3 Opticno aktiven material z cirkularnim dihroizmom (matriki zmnozimo)

- $n_{Re}^{LCP} \neq n_{Re}^{RCP}, n_{Im}^{LCP} \neq n_{Im}^{RCP}$

$$R_{both} = R_{0A}R_{CD} - e^{ik_0\overline{n}d}e^{-\overline{\kappa}d}\begin{pmatrix}cos(\theta), -sin(\theta)\\ sin(\theta), cos(\theta)\end{pmatrix}; \theta = \frac{\Delta nRek_0d}{2} + i\frac{\Delta\kappa d}{2} \tag{133}$$

9.2 Rotacija polarizacije v magneto-opticnem pojavu z $\theta = \frac{k_0\Delta nd}{2}, \Delta n = n_{RCP} - n_{LCP}$

Resujemo enacbi za magneto opticni efekt za $\Omega << \omega$ oz. majhna magnetna polja:

$$n_{RCP} = 1+\frac{1}{2}\frac{\omega_p^2}{\omega_0^2 - \omega^2 + \omega\Omega} = 1+\frac{1}{2}\frac{\omega_p^2}{\omega_0^2 - \omega^2}\frac{1}{1+\frac{\omega\Omega}{\omega_0^2-\omega^2}} = 1+\frac{1}{2}\frac{\omega_p^2}{\omega_0^2 - \omega^2}(1-\frac{\omega\Omega}{\omega_0^2 - \omega^2}) \tag{134}$$

$$n_{LCP} = 1+\frac{1}{2}\frac{\omega_p^2}{\omega_0^2 - \omega^2 - \omega\Omega} = 1+\frac{1}{2}\frac{\omega_p^2}{\omega_0^2 - \omega^2}\frac{1}{1-\frac{\omega\Omega}{\omega_0^2-\omega^2}} = 1+\frac{1}{2}\frac{\omega_p^2}{\omega_0^2 - \omega^2}(1+\frac{\omega\Omega}{\omega_0^2 - \omega^2}) \tag{135}$$

$$\Omega << \omega => \frac{1}{1+\frac{\omega\Omega}{\omega_0^2-\omega^2}} = 1 + \frac{\omega\Omega}{\omega_0^2 - \omega^2} \tag{136}$$

$$\Delta n = n_{RCP} - n_{LCP} = -\frac{\omega_p^2\Omega\omega}{(\omega_0^2 - \omega^2)^2}; \Omega = \frac{e_0B_0}{me}, \omega_p^2 = \frac{\rho e_0^2}{m\epsilon_0}, k = \frac{\omega}{c} \tag{137}$$

$$\theta = \frac{k_0\Delta nd}{2} = -\frac{1}{2}k_0d\frac{\omega_p^2\omega\Omega}{(\omega_0^2 - \omega^2)^2} \tag{138}$$

10 Indeks elipsoida, sipanje svetlobe in laserji

10.1 Izracunaj in skiciraj indeks elipsoida in valovni vektor za prehod svetlobe skozi biaksialen kristal z lomnimi kolicniki n_{xx}, n_{yy}, n_{zz} v xz,xy in yz ravninah

Navadno je $\vec{k} \cdot \vec{E} = 0$ oziroma nista vzporedna v izotropnih materialih.

$$(k^2 l - k_0^2 \epsilon)\vec{E} = (\vec{k}\vec{E})\vec{k} \tag{139}$$

V lastnem sistemu je matrika taka in sicer pogosto velja $\epsilon_{xx} < \epsilon_{yy} < \epsilon_{zz}$:

$$\epsilon = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \tag{140}$$

$$\begin{bmatrix} k_y^2 + k_z^2 - k_0^2\epsilon_{xx} & -k_xk_y & -k_xk_z \\ -k_xk_y & k_x^2 + k_y^2 - k_0^2\epsilon_{yy} & -k_yk_z \\ -k_zk_x & -k_yk_z & k_x^2 + k_y^2 - k_0^2\epsilon_{zz} \end{bmatrix} \vec{E} = \underline{\underline{M}}\vec{E} = 0 \tag{141}$$

$$det\underline{\underline{M}} = \begin{bmatrix} k_y^2 + k_z^2 - k_0^2\epsilon_{xx} & -k_xk_y & -k_xk_z \\ -k_xk_y & k_x^2 + k_y^2 - k_0^2\epsilon_{yy} & -k_yk_z \\ -k_zk_x & -k_yk_z & k_x^2 + k_y^2 - k_0^2\epsilon_{zz} \end{bmatrix} = 0 \tag{142}$$

Za lazji izracun gremo v podane ravnine, za primer bo izracun v xy ravnini.

10.1.1 xy-ravnina $\vec{k} = (k_x, k_y, 0)$

$$\underline{\underline{\vec{M}}} = \begin{bmatrix} k_y^2 - k_0^2k_{xx} & -k_xk_y & 0 \\ -k_xk_y & k_x^2 - k_0^2\epsilon_{yy} & 0 \\ 0 & 0 & k_x^2 + k_y^2 - k_0^2\epsilon_{zz} \end{bmatrix} \tag{143}$$

$$det\underline{\underline{M}} = (k_x^2 + k_y^2 - k_0^2\epsilon_{zz})[k_0^4\epsilon_{xx}\epsilon_{yy} - k_0^2k_x^2\epsilon_{xx} - k_0^2k_y^2\epsilon_{yy}] \tag{144}$$

1. $k_x^2 + k_y^2 = k_0^2\epsilon_{zz} => \kappa_x^2 + \kappa_y^2 = \epsilon_{zz}$ =>graficno kroznica

2. $k_0^2k_x^2\epsilon_{xx} + k_0^2k_y^2\epsilon_{yy} = k_0^4\epsilon_{xx}\epsilon_{zz} => \frac{\kappa_x^2}{\epsilon_{yy}} + \frac{\kappa_y^2}{\epsilon_{xx}} = 1$ =>graficno elipsa

V yz ravnini naredimo enak korak in ta da pride enak par resitev. Naprej se nekaj o polarizaciji na primeru xy ravnine:

- Vsaka resitev pripada dolocenemu kolicniku n in svoji polarizaciji
- Ena polarizacija je vzporedna na elipso, druga pravokotna "ven"
- Ena polarizacija kaze v κ_z smer: $k_x^2 + k_y^2 - k_0^2\epsilon_{zz} = 0$; $\vec{E} = (0, 0, E_0)$

10.2 Smer $\vec{E},\vec{k}$ na povrsini indeksnega elipsoida.

10.2.1 xy ravnina

$$\tilde{M} = \frac{M}{k_0^2} = \begin{bmatrix} \kappa_y^2 - \epsilon_{xx} & -\kappa_x\kappa_y & 0 \\ -\kappa_x\kappa_y & \kappa_x^2 - \epsilon_{yy} & 0 \\ 0 & 0 & \kappa_x^2 + \kappa_y^2 - \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

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$$(\kappa_y^2 - \epsilon_{xx})E_X - \kappa_x\kappa_y E_y = 0 => \frac{E_y}{E_x} = \frac{\kappa_y^2 - \epsilon_{xx}}{\kappa_x\kappa_y} \tag{146}$$

$$\frac{\kappa_y^2}{\epsilon_{xx}} + \frac{\kappa_x^2}{\epsilon_{yy}} = 1 => \kappa_y^2 = \epsilon_{xx} - \kappa_x^2\frac{\epsilon_{xx}}{\epsilon_{yy}} \tag{147}$$

Enacbi zdruzimo v $\frac{E_y}{E_x} = \frac{\kappa_y^2-\epsilon_{xx}}{\kappa_x\kappa_y} = -\frac{\epsilon_{xx}\kappa_x}{\epsilon_{yy}\kappa_y}$ in tangentno linijo za resitev dobimo prek odvoda $\frac{d\kappa_y}{d\kappa_x}$:

$$0 = \frac{2\kappa_x}{\epsilon_{yy}} + \frac{2\kappa_y}{\epsilon_{xx}}\frac{d\kappa_y}{d\kappa_x} => \frac{d\kappa_y}{d\kappa_x} = -\frac{\epsilon_{xx}}{\epsilon_{yy}}\frac{\kappa_x}{\kappa_y} \tag{148}$$

10.2.2 xz ravnina

$$\tilde{M} = \frac{M}{k_0^2} = \begin{bmatrix} \kappa_z^2 - \epsilon_{xx} & 0 & -\kappa_x\kappa_z \\ 0 & \kappa_x^2 + \kappa_z^2 - \epsilon_{yy} & 0 \\ -\kappa_z\kappa_x & 0 & \kappa_x^2 - \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

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$$(\kappa_z^2 - \epsilon_{xx})E_x - \kappa_x\kappa_z E_z = 0 => \frac{E_z}{E_x} = \frac{\kappa_z^2 - \epsilon_{xx}}{\kappa_x\kappa_z} \tag{150}$$

10.3 Glavna polarizacija v biaksialnem materialu z $n_{xx} = 1,619; n_{yy} = 1,620; n_{zz} = 1,627$. Material osvetimo tako, da svetloba lezi v xz ravnini pod kotom 30° glede na os z. Analiziraj polarizacijo svetlobe, izracunaj n, smeri E in D polj ter kot med njima in poyntingov vektor $\mathcal{S}$ ter doloci kot med njima.

Ce vektor lezi v xz ravnini uporabimo ze dobljene resitve $\kappa_x^2 + \kappa_y^2 = \epsilon_{yy}$ in $\frac{\kappa_x^2}{\epsilon_{zz}} + \frac{\kappa_z^2}{\epsilon_{xx}} = 1$. Pripravimo podatke za izracun elipticne resitve:

- $\epsilon_{xx} = n_{xx}^2 = 2,62$ enako za $\epsilon_{yy}, \epsilon_{zz}$
- $\epsilon_{yy} = \kappa_x^2 + \kappa_y^2 => k_x^2 + k_z^2 = \epsilon_{yy}k_0^2$

10.3.1 Elipticna resitev

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10.3.2 Resitev za elektricno polje in Poytingov vektor

10.3.3 Opticne osi

Cilj je najti kot med $(\vec{k}, \vec{z})$, da sta n_1, n_2 enaka.

$$n_1 = n_{yy} \tag{152}$$

$$n_2 = \sqrt{\frac{\epsilon_{xx}\epsilon_{zz}}{\epsilon_{xx} sin^2(\theta) + \epsilon_{yy} cos^2(\theta)}} \tag{153}$$

$$n_1 = n_2 => n_{yy} = \sqrt{\frac{\epsilon_{xx}\epsilon_{zz}}{\epsilon_{xx} sin^2(\theta) + \epsilon_{yy} cos^2(\theta)}} = \sqrt{\frac{\epsilon_{xx}\epsilon_{zz}}{\epsilon_{xx}(1 - cos^2(\theta)) + \epsilon_{yy} cos^2(\theta)}} \tag{154}$$

$$sin^2(\theta) = \frac{1}{\epsilon_{zz} - \epsilon_{xx}}(\epsilon_{zz} - \frac{\epsilon_{xx}\epsilon_{zz}}{\epsilon_{yy}}) \approx 0,128 \tag{155}$$