Vaje iz fizike jedra in osnovnih delcev

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1.

3 Vezana jedra

3.1 Vezavna jedra $.^{15}O$ in $.^{13}N$, ce sta njuni masi $m_{.^{15}O} = 15u$, $m_{.^{13}N}$ = 15u. Ista masa, drugaca sestava. V katerem prispevku se njuni vezavni energiji razlikujeta Kaksen je radij teh jeder v kapljicnem modelu

Velja $m_{.^{15}O=15u}$ z Z=8 in N=7 in $m_{15N} = 15u$ z Z = 7 in N = 8.

$$E_B = a_1 A - a_2 A^{\frac{2}{3}} - a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_4 \frac{(A-2Z)^2}{A} + \delta \tag{1}$$

Razlika v radijih je $-a_3\frac{Z(Z-1)}{A^{\frac{1}{3}}}$ za jedro, ki je homogena nabita krogla z nabojem:

$$\rho(r) = \frac{4\pi}{3}r^3\rho_0 \tag{2}$$

$$E = \frac{1}{4\pi\epsilon_0}\int\frac{1}{r}\rho d\rho = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(r)}{r}\rho_0 dV = \int_0^R \frac{1}{4\pi\epsilon_0}4\pi r^2\frac{\rho(r)}{r}\rho_0 dr = \tag{3}$$

$$= \frac{1}{4\pi\epsilon_0}(4\pi\int_0^R \frac{3e_0}{4\pi R^3}r^2 e_0(\frac{r}{R})^3\frac{1}{r}) = \frac{R_0^2}{4\pi\epsilon_0}\frac{3}{5}\frac{e_0^2}{R} \tag{4}$$

$$a_c = \frac{3}{5}\frac{e_0^2}{4\pi\epsilon_0 R_0} \tag{5}$$

$$R_N = Z_N^2 R_0 \tag{6}$$

Torej $R_0 = \frac{3}{5}\frac{e_0^2}{4\pi\epsilon_0 a_c}$

3.2 Masni defekt za $._{}^{21}F$ in $._{}^{238}U$ in kaj ta pojasni.

Iz SEMF ali tabele izberemo dva elementa, npr. $._{9}^{20}F$ in $._{92}^{238}U_{146}$ in izracunamo defekt:

$$\Delta = (Z\cdot m_p + N\cdot m_n - [Z\cdot m_p + N\cdot m_n - E_B]) = E_B \tag{7}$$

3.3 Iz masnih defektov za jedri $._{}^{24}Na$ in $._{}^{240}Pu$ izracunaj njuni masi

Podatka sta $\Delta_{.^{24}Na}$= -8,5MeV in $\Delta_{.^{240}Pu}$= 50,123MeV. Negativna vezavna energija pove, da je stanje nestabilno z razpadnim casom $\approx 15h.$, pozitivna (poz. masni defet), da je stabilno z $\tau \approx$ 6000let.

$$M_{.^{24}Na} = 11m_p + 13m_n - \Delta_{.^{24}Na} \doteq 24,2u \tag{8}$$

$$M_{.^{240}Pu} = ... \doteq 241,9u \tag{9}$$

3.4 Oblikovna funkcija za sipanje na jedru (točkasto porazdeljen naboj na jedru aka Rutherfordovo sipanje)

Za EM interakcijo velja:

$$< f|U|i >= \int \Psi_f(r)U(r)\Psi_i^*(r)dV \tag{10}$$

Kjer je U(r) potencial, na katerem se foton siplje $\Delta U(x) = -\frac{e}{\epsilon_0}\rho(x)$. Ψ je torej VF jedra, kjer je v jedru porazdeljen tako naboj, kot masa. Upostevamo razvoj VF po ravnih valovih $\Psi \to e^{i\vec{k}_{i,f}\vec{x}}$.

$$< f|V|i >= \int e^{i\vec{k}_{i,f}\vec{x}}V(r)e^{-i\vec{k}_{i,f}\vec{x}}dV = \int e^{i\vec{q}\vec{r}}V(r)dV|_{\vec{q}=\vec{k}_f-\vec{k}_i} \tag{11}$$

Z vmesnimi koraki, kjer gremo prek $\int f\Delta g dV = \int g\Delta f dV$ in $\int V\Delta e^{i\vec{q}\vec{x}}dV = \int e^{i\vec{q}\vec{x}}\Delta V dV$, $-q^2\int V e^{i\vec{q}\vec{x}}dV = \int e^{i\vec{q}\vec{x}}\Delta V dV$.

$$q^2 < f|V|i >= \int e^{i\vec{q}\vec{r}}\Delta V dV = \frac{e}{\epsilon_0}\int e^{i\vec{q}\vec{r}}\rho(r)dV \tag{12}$$

$$< f|V|i >= \frac{e}{\epsilon_0 q^2}\int e^{i\vec{q}\vec{r}}\rho(r)dV \tag{13}$$

Del pod integralom se imenuje oblikovni faktor F(q) $= \lambda\int e^{iq_x x+iq_y y+iq_z z}\delta(x-0)\delta(y-0)\delta(z-0)dxdydz$

3.5 Poenostavitev izraza F(q), ce je porazdelitev sferno simetricna

Za sferno simetricno porazdelitev velja $\rho(\vec{r}) = \rho(r)$.

$$F(q) = \int e^{i\vec{q}\vec{r}}\rho(\vec{r})dV = \int e^{i\vec{q}\vec{r}}\rho(\vec{r})sin\theta d\theta d\phi r^2 dr \tag{14}$$

$$= \int e^{iqrcos\theta}\rho(r)sin\theta d\theta d\phi r^2 dr \tag{15}$$

$$= 2\pi\int_0^R r^2\rho(r)dr\int_0^\pi sin\theta d\theta e^{iqrcos\theta} \tag{16}$$

$$= \frac{4\pi}{q}\int_0^\infty \rho(r)sin(qr)rdr \tag{17}$$

3.6 Izpelji izraz za radij jedra iz oblikovne funkcije F(q) in pokazi, kdaj je uporaben, $\rho(r) = (\lambda/4\pi)e^{-kr}/r$

$$F(q) = \frac{4\pi}{q}\int_0^\infty \rho(r)sin(qr)rdr \tag{18}$$

$$= \frac{4\pi}{q}\int_0^\infty \rho(r)[(qr) - \frac{1}{3!}(qr)^3 + ...]rdr \tag{19}$$

$$= \frac{4\pi}{q}\int_0^\infty \rho(r)qr^2dr - \frac{4\pi}{q}\int_0^\infty \rho(r)\frac{q^3r^4}{q3r}dr \tag{20}$$

$$= -\frac{2\pi q^2}{3}\int_0^\infty r^4 q(r)dr = -\frac{q^2}{6} < r^2 > \tag{21}$$

0. in 1. clen.

$$F(q) = \frac{4\pi}{q}\int_0^\infty \frac{\lambda^2}{4\pi}\frac{e^{/kr}}{r}sin(qr)rdr \tag{22}$$

Opomba, tu uporabimo $I = \frac{b}{a^2+b^2}$ in $\int_0^\infty e^{-ax}sin(bx)dx = *$

Figure 1: 1

3.7 SEMF -¿ dolina stabilnih jeder, kjer je fiksen A in izrazi Z z maksimalno energijo.

$$M(A,Z) = Z \cdot m_p + (A-Z)m_n + ... \tag{23}$$

Minimum pri Z z fiksnim A:

$$\frac{\partial}{\partial z}M(A,Z) = \frac{\partial}{\partial z}(Zm_p+(A-Z)m_n+a_vA-a_sA^{\frac{2}{3}}-a_cA^{-\frac{2}{3}}Z(Z+1)-a_yA^{-1}(A-2Z)^2 \tag{24}$$

$$0 = m_p - m_n - 2a_cZA^{-\frac{1}{3}} - a_cA^{\frac{1}{3}} - a_y(-4A)A^{-1} - a_y(8Z)A^{-1} \tag{25}$$

$$0 = -a_c\frac{2Z-1}{A^{\frac{1}{3}}} - a_sy\frac{4(2Z-A)}{A} \tag{26}$$

$$Z = \frac{2a_sy}{\frac{2c}{A^{\frac{1}{3}}} + \frac{2a_{sy}}{A}} = \alpha\frac{A}{A^{\frac{2}{3}}+\alpha} \tag{27}$$

3.8 Pri katerem stevilu nukleonov so razpadi α dovoljeni: $._Z^AX \to ._{Z-2}^{A-4}Y + \alpha$

Na zacetku je $M_X(A_1,Z_1)$ in na koncu $M_Y(A_2,Z_2)+T_Y+M_\alpha+T_\alpha$. Iz enostavnosti predpostavimo, da smo v kineticni tocki, kjer je T_Y zanemarljiv.

$$M_X(A_1,Z_1) = M_Y(A_2,Z_2) + M_\alpha + T_\alpha \tag{28}$$

$$M_\alpha + T_\alpha = M_X - M_Y = z_1m_p + (A_1-Z_1)m_n + a_vA_1 - ... \tag{29}$$

$$T_\alpha = M_X - M_Y - 2m_p - 2m_n \tag{30}$$

$$= 4a_v - a_s(A_1^{\frac{2}{3}}-(A_1-4)^{\frac{2}{3}}) - a_c(\frac{Z_1(Z_1-1)}{A_1^{\frac{1}{3}}} - \frac{(Z_1-2)(Z_1-3)}{(A_1-4)^{\frac{2}{3}}}) + a_{sym}(A_1-2Z_1)^2(\frac{4}{A_1(A_1-4)}) \tag{31}$$

Ob upostevanju pogoja $T_\alpha >> 0$ in $Z_2 = Z_1 - 2$ ter $A_2 = A_1 - 4$ poenostavimo izraz. Za oceno vrednosti T_α pa na hitro vstavimo $A_1 = 2Z_1$.

3.9 Razpadi β: $._Z^AX_N \to ._{Z+1}^{A}Y_{N-1} + e^+ + \overline{\nu}_e$

Za te razpade velja, da se ohranja $Q = T_e + E_\nu$, je energijska balanca ¿ 0 ($Q = m_{zac} - m_{kon} - m_e$, poleg tega pa ni nujno, da se ohranja parnost, naboj se ohranja in stevilo vseh leptonov v razpadu se ohranja (za lepton L=1, za antilepton L=-1). Nekaj enacb v premislek:

- $\Gamma = \frac{|T_{fi}|^2}{60\pi^3}Q^5$: Sargentovo pravilo

- $K(T_e) = \frac{1}{p_e}\sqrt{\frac{d\Gamma}{dp_e}} = \frac{2|T_{fi}|}{2\pi^{\frac{3}{2}}}(Q - T_e)$: Curie plot
- $\vec{J}_i = \vec{J}_f + \vec{J}_{e\overline{\nu}}$, $\vec{J}_{e\overline{\nu}} = \vec{S}_{e\overline{\nu}} + \vec{l}$: ohranitev vrtilne kolicine
- Ohranja energijo, ko X delec miruje $m_x = E_y + E_e + E_\nu$
- Razpolovna sirina: $< \vec{p}|\vec{p} >= (2\pi)^3\delta^3(\vec{p} - \vec{p})$
- $T_{fi} =< i|B|f >$: ¡i— = ¡$J_i, M_i : L_i, S_i|$ in $< f| =< J_f, M_f : L_f, S_f|, |\beta| = \vec{J}_{e\nu}$
- Klasifikacija razpadov:
  1. Fermi razpad (l=0): $\vec{J}_i = \vec{J}_f + \vec{0}$, $P_i = P_f \cdot P_\beta = P_f(-1)^0$, ohranja parnost
  2. Fermi razpad (l≠l), prepovedan: $\vec{J}_i = \vec{J}_f + \vec{0} + \vec{l}, P_i = P_f \cdot (-1)^l$
  3. Gamow-Teler razpad: $\vec{J}_i = \vec{J}_f + \vec{1}, P_i = P_f \cdot P_\beta = P_f(-1)^0$
  4. se en razpad: $\vec{J}_i = \vec{J}_f + \vec{1} + \vec{l}, P_i = P_f \cdot (-1)^l$

3.10 Oceni radij jedra .^{58}Ni, ce pri sipanju z elektroni W_l = 450MeV opazimo 1.minimum pri kotu 25,3° in ga preimenuj z SEMF.

Pri elektricnem sipanju elektrona na jedru, bo to ostalo pri miru in bo veljala gibalna enacba: $|p_i| = |p_f| = p$.

$$\vec{q} = \vec{p}_i - \vec{p}_f \tag{32}$$

$$q^2 = (p_i - p_f)^2 = p_i^2 + p_f^2 - 2p_ip_fcos\theta = 2p^2(1 - cos\theta) \tag{33}$$

$$q = 2psin\frac{\theta}{2} \tag{34}$$

Vstavimo pogoj $2psin\frac{\theta}{2}R = \frac{3\pi}{2}$, za locitev fizicnih kotov $\frac{2psin\frac{\theta}{2}R}{\hbar c} = \frac{3\pi}{2}$. Radij je tako $R = \frac{3\pi\hbar c}{2psin\frac{\theta}{2}} = 4{,}71$fm.

SEMF formula je $R = R_0 \cdot A^{\frac{1}{3}}$.

3.11 Elektroni z E=180MeV se elasticno sipajo na tarci, kjer kotna distribucija pokaze minimume in maksimume. Imamo sfericno jedro, nabito homogeno kroglo in ocenjujemo stevilo minimumov, ki bi jih imel tak eksperiment.

$$\frac{d\theta}{d\Omega} = (\frac{e_0e_1m}{2\pi\epsilon_0\hbar^2})^2\frac{1}{sin^3\frac{\theta}{2}}|F(q)|^2 \tag{35}$$

Figure 2: 2

$$F(q) = \frac{4\pi}{q}\int_0^\infty \rho(r)sin(qr)rdr = \frac{1}{q}\int_0^R \frac{3}{R^3}sin(qr)rdr = \frac{3}{q^3R^3}\int_0^{qR} sinu\cdot udu \tag{36}$$

$$F(q) = \frac{3}{q^3R^3}(sin(qr) - (qr)cos(qr)) \tag{37}$$

Nicle isceno kot:

$$0 = \frac{3}{q^3R^3}(sin(qr) - (qr)cos(qr)) \tag{38}$$

$$0 = sin(qR) - (qR)cos(qR) = tan(qR) - (qR) \tag{39}$$

Pri relativnem delcu je masa zanemarljiva $E = pc$, kjer je $q_{max} = 2p_e = \frac{E}{c}$ oziroma $q_{max}R = \frac{2\cdot 180MeV\cdot 7fm}{\hbar c} = 22,8$ upostevajoc radij jedra prek SEMF.

3.11.1 Opazujemo lupinska jedra. Za primer a) imamo potencialno jamo $V_{pot} = -V_0 : r < a$ in 0 sicer, b) ima HO $V_{osc} = \frac{1}{2}m\omega^2 r^2$ in c) spin tir sklopitev $V = -2\eta\vec{L}\cdot\vec{S}$.

Za razliko med $3d_{3/2}$ in $3d_{5/2}$ je 1,35MeV in $4f_{7/2}$ in $4d_{5/2}$ je 6,3MeV. Gledamo $H = \frac{p^2}{2m} + V(r)$ na potencialu za skoraj prost elektron $\Psi = R_N(r)Y_{lm}\Psi$, kjer je $|n, L, S >$ oziroma $|n, J, L, S >$ shematsko $._n^{2S+1}L_j$. Katere ferminone pa doprinesejo ti cleni v potencialu: jama-¿ vezana stanja, oscilator-¿ vezana stanja, LS-¿zaprtje orbital oz. nizje lezeca kvantna stevila. Zdruzen hamiltonjan je tako $H = \frac{p^2}{2V} - 2\eta\vec{L}\cdot\vec{S} + \frac{1}{2}m\omega^2r^2 - V_0$.

$$V_{LS} = -2\eta\vec{L}\cdot\vec{S} = -\eta(J^2 - L^2 - S^2) : J^2 = (\vec{L}+\vec{S})^2 = L^2 + S^2 + 2\vec{L}\cdot\vec{S} \tag{40}$$

Keoficient η dobimo prek energijske razlike $\Delta E = |E(3d_{3/2}) - E(3d_{5/2})|$ = 1,35MeV. Velja, da je $\Delta E = H|3, \frac{3}{2}, 2, \frac{1}{2} > -H|3, \frac{5}{2}, 2, \frac{1}{2} >= ... = 5\eta$. $\eta = 0,27MeV$. Sklopitev v oscilatorju (med $4f_{7/2}$ in $4d_{5/2}$) je $E_{osc}|m, J, L, S >= \hbar\omega(2n+L+\frac{3}{2})|n, J, L, S >$. $\Delta E = |H|4, \frac{7}{2}, 3, \frac{1}{2} > -H|4, \frac{5}{2}, 2, \frac{1}{2} > | = 6,3MeV = |V_{ls} + V_{ocs}|4, \frac{7}{2}, 3, \frac{1}{2} > -V_{LS} + V_{OSC}|4, \frac{5}{2}, 2, \frac{1}{2}|$.

3.12 Helijeve atome $._2^4He, ._3^4He$ trcimo ob fiksirano zlato tarco. Izbirnik hitrosti odfiltrira vse ione, ki nimajo hitrosti v=0,1c. Oba zarka imata enak pretok, kjer detektor delcev lociran ob kotu θ glede na smer zarka steje ione. Koliksno je masno stevilo, ce je razmerje detektiranih dogodkov $R = N_{He}/N_{Li}$

Za izracun uporabimo $p = mv$, upostevamo $v_1 = v_2 = 0,1c$ in upostevamo, da je prvi curek Helijevih delcev in drugi litijevih. Lahko upostevamo tudi

Figure 3: 3

relativno atomsko maso obeh delcev. Z predstavlja celoten naboj iona in ne stevila protonov.

$$R = \frac{N_1}{N_2} = \frac{(\frac{d\sigma}{d\Omega})_1}{(\frac{d\sigma}{d\Omega})_2} = \frac{(z_1 \frac{m_1}{p_1^2})^2 \cdot F_1(q)}{(z_2 \frac{m_2}{p_2^2})^2 \cdot F_2(q)} = (\frac{z_1 m_1 p_2^2}{z_2 m_2 p_1^2})^2 = (\frac{z_1 m_1 m_2^2 v_2^2}{z_2 m_2 m_1^2 v_1^2})^2 = (\frac{z_1 m_2}{z_2 m_1})^2 \tag{41}$$

3.13 Zarek α delcev z T= 0,1GeV trka ob tarco aluminija z gostoto 2,7g/cm^3 in debelino 1cm. Fluks zarka ob tarci je 10 odstotkov, scintilacijski detektor z aktivno povrsino $1cm^2 \times 1cm^2$ postavimo 1m stran od tarce, pri kotu $\theta = 30$ glede na smer zarka. Koliksno je stevilo dogodkov v detektorju

$$f = 1 : \frac{d\sigma}{d\Omega} = (\frac{Ze^2 m}{8\pi\epsilon_0 p^2})^2 \frac{1}{sin^4 \frac{\theta}{2}} = 2 \cdot 10^{-26} cm^2/s.d \tag{42}$$

Nas detektor zavzame kot 1cm x 1cm na sferi z radijem $1m^2$, torej je $\Delta\Omega \approx 0.001$. $\frac{dN}{dt} = 2\sigma = 1200Hz$.

4 Razpadi

4.1 Kateri izmed spodnjih razpadov so dovoljeni: a) $._6^{14}C \rightarrow ._4^{10}Be + \alpha$, b) $._{29}^{57}Cn \rightarrow ._{27}^{53}Co + \alpha$ in c)$._{92}^{235}U \rightarrow ._{90}^{231}Th + \alpha$

4.2 $\beta-$razpad. Ohranja energijo $Q = T_e + T_\nu$, $\Gamma = \frac{|T_{fi}|^2}{\epsilon_0 \pi^3} Q^5$, $K(T_e) = \frac{1}{p_e}\sqrt{\frac{d\Gamma}{dp_e}} = \frac{2|T_{fi}|}{(2\pi)^{\frac{3}{2}}}$. $\vec{J}_i = J_J + \vec{J}_{e\overline{\nu}}, \vec{J}_{e\nu} = \vec{S}_{e\nu} + \vec{R}$: prvi clen doloci vrsto razpada, singlet (0) da Fermijev razpad in triplet (1) da GamowTeller razpad. Velja, da se parnost lahko krsi, naboj se ohranja in leptonsko stevilo, ki je stevilo vseh leptonov, ki se ohranja. $._Z^A X \rightarrow ._{Z^1}^{Z^0} X + e\overline{\nu_e}$. Zanima nas, kako izpeljati izraz za razpadno sirino takega razpada.

$$._Z^A X_N \rightarrow ._{Z"}^{A"} Y_{N"} + e + \overline{\nu}_e; Z" = Z + 1, N" = N - 1 \tag{43}$$

$$n \rightarrow p + e^- + \overline{\nu}_e \tag{44}$$

V koncnem stanju so 3 delci z razlicnimi kineticnimi energijami te tvorijo spekter $m_x = E_y + E_e + E_\nu$). Podobno kot pri α razpadu bo nastalo jedro v stanjih pri miru, torej $E_Y = m_y$, $m_x - m_y - m_c = Q$.

4.3 Ocena energije delca α v jedru $._{84}^{210}Po$ v principu nedolocenosti

$$\Delta p \Delta x \approx \hbar \approx 1 \tag{45}$$

$$\Delta x \approx R = R_{206} + R_4 \tag{46}$$

$$\Delta p = m\alpha v \tag{47}$$

$$T_\alpha = \frac{1}{2} m_\alpha v^2 = ... = 0,07MeV. \tag{48}$$

4.4 $._{84}^{210}Po \rightarrow ._{82}^{206}Pb + \alpha$; Doloci delez energije, ki ga odnese delec α in presodi ali je ta relativisticen ali ne. Koliksna je vrednost Q razpada.

Na zacetku je $\vec{v} = 0, W_{tot} = W_{\mu_0}$ in $\vec{p} = 0$. Na koncu pa $W_{tot} = M_{Pb} + M_\alpha + T_{Pb} + T_\alpha$ in $\vec{p} = \vec{p}_{Pb} + \vec{p}_\alpha$. Ohranitev gibalne kolicine: $\vec{p}_z = -\vec{p}_\alpha$ in $M_{Pb}\vec{v}_{Pb} = -M_\alpha \vec{v}_\alpha$. Izrazena absolutna hitrost delca α je torej $|\vec{v}_\alpha| = -\frac{M_{Pb}}{M_\alpha}|\vec{v}_{Pb}|$. Ohranitev energije: $M_{Po} = M'_{Pb} + M'_\alpha + T_\alpha + T_{Pb}$. $\Delta M = M_{Pb} - M'_{Pb} - M'_\alpha = T_\alpha + T_{Pb} = \frac{1}{2} M_{Pb}|v_{Pb}|^2 + \frac{1}{2} M_\alpha |v_\alpha|^2 = \frac{1}{2} M_\alpha v^2 (1 + \frac{M_\alpha}{M_{Pb}})$. Vrednost Q: $Q = m_x - m'_y - m_\alpha = T'_x + T_\alpha = ... = 1MeV$.

4.5 Razpadni cas za $._{93}^{243}Am$, pri tem sprosti Q=5,275MeV energije in $._{93}^{241}Am$, pri tem pa Q" = 5,485MeV in velja $t_{1/2} = 432$let.

Cilj GN zakona je pri fiksnem Z dolociti razpadni cas v odvisnosti od energije α delca.

$$lnt_{\frac{1}{2}} = A + \frac{B}{\sqrt{Q_{24}}}, t_{\frac{1}{2}}(Am) = 432let. \tag{49}$$

$$lnt_{1/2}^{243} = lnt_{1/2}^{2}41 + B(\frac{1}{\sqrt{Q_{243}}} - \frac{1}{\sqrt{Q_{241}}}) \tag{50}$$

$$B = \frac{1}{2}\sqrt{E_g}(1 - \frac{4}{\pi}\sqrt{\frac{R}{\overline{R}}}) = ... = ln243let + 6,0 \tag{51}$$

Razmerje izrazimo prek $6,0 = \frac{lnt_{1/2}^{243}}{lnt_{1/2}^{241}}$ in dobimo $t_{1/2} = 175000let$.

Izracun za razpad $.^{238}U \rightarrow .^{234}U + \alpha$.

$\Gamma = \frac{v_+}{R} e^{-2G}$; upostevamo $Z_\alpha = 4, Z' = 234, \mu = \frac{m_\alpha m_{Tb}}{m_\alpha + m_{Tb}}$. $T_I = Q + V_0 = \frac{1}{2}\mu v_I^2$, $Q = M(._{92}^{238}U) - M(._{90}^{234}U) - M(\alpha)$. Uporabimo SEMF z $Q = 92m_p + 146m_n - E_b(U) - 90m_p - 144m_n + E_Q(Tb) - 2m_p - 2m_n + E_B(He) = 4,3MeV$. Skupen radij izmerimo kot $R = R_{Tb} + R_\alpha = R_0(A_{Tb}^{1/3} + A_\alpha^{1/3}) = 9,3fm$. Velja, da je $Q = V_c = \frac{e_0^2 Z_\alpha Z'}{4\pi\overline{R}}$. Z izrazanjem iskanih kolicin dobimo rezultate za $\overline{R}, t_{1/2}, \Gamma$.

Figure 4: 4

4.6 Izpelji izraz za razpadno sirino $\alpha-$ delca v semiklasicnem priblizku oziroma v priblizku staticne Coulombske bariere.

Delca Th in He cutita dve interakciji:

- mocno, ki ju veze
- elektrostaticno, ki ju odvaja.

Potencial mocne interakcije med njimi je lahko veliko stvari, ampak za osnovo vzamemo potencialno jamo z $V = -V_0$, ko je r¡R in sicer 0. $V_C = \frac{e_0^2 2(Z=2)}{4\pi\epsilon_0 r}$. Ocena $R = R_{Th} + R_\alpha$, kjer je $\overline{R}$ ko V postane manjsi od Q. Tu je delec spet prost. Delec α lahko zapusti jamo, ker je Q¿0 ampak manjsi od Vc pri R. Skica poda informacijo o verjetnosti za tuneliranje cez to bariero $dP_T = e^{-2\kappa(r)dr}$.V takem sistemu bo stanje z neko verjetnostjo vezano, α bo ujet v potencialni jami in verjetnost za α pri $r > \overline{R}$ ni nicna. Verjetnost za prosto stanje je hkrati tudi verjetnost za tuneliranje $P_T = P_{VI} = \int_{\overline{R}}^{\infty} |\Psi_{III}|^2 dV$. Razpadna sirina $\Gamma = \frac{1}{\tau}$, kjer je τ zivljenski cas in $t_{1/2}$ razpadna doba. Ce zelimo dobiti razpadno sirino za tuneliranje moramo pomnoziti s frekvenco, s katero bi lahko .^{238}U nasli loceno.

$$\Gamma = fP_t \tag{52}$$

$$f \to \frac{v_F}{R} \tag{53}$$

Radialno odvisen potencial $K = K(r)$:

$$Q = T + V_C = \frac{k^2}{2\mu} + \frac{Z\alpha Z' e_0^2}{r} \tag{54}$$

$$\frac{\kappa^2}{2\mu} = \frac{Z_\alpha Z' e_0^2}{r} - Q \tag{55}$$

$$\Psi(r,\theta,\phi) = R_{n,l}(r)Y_{lm}(\theta,\phi) = N \cdot \frac{u(r)}{r} = \frac{e^{-\kappa r}}{r\sqrt{4\pi}} \tag{56}$$

Koeficient lahko izracunamo, ko poznamo $\Psi_L = \frac{e^{-\kappa r}}{r}, \Psi_D = \frac{e^{-\kappa(r+dr)}}{r+dr}$.

$$T = \frac{|\Psi_D|^2}{|\Psi_L|^2} = \frac{|e^{-\kappa|r+dr|} \cdot r|^2}{|(r+dr)e^{-\kappa r}|^2} \approx \frac{e^{-\kappa(r+dr)}}{r+dr}(\frac{r}{r+dr})^2 \approx e^{-2\kappa dr} \tag{57}$$

Naprej pogledamo verjetnost, da stuneliramo cez celotno bariero $P_T = \Pi_i dP_T, ln(P_T) = \sum_i ln(P_T)$.

$$ln(p_T) = \int_R^{\overline{R}} ln(P_T) = \int_R^{\overline{R}} (-2\kappa(r))dr \tag{58}$$

Figure 5: 5

$$P_T = e^{-2\int_R^{\overline{R}} \kappa(r)dr} = e^{-2G} \tag{59}$$

$$G = \int_R^{\overline{R}} \sqrt{2\mu(\frac{Z_\alpha Z' e_0^3}{r} - Q)}dr \tag{60}$$

$$G = Z_\alpha Z' e_0^2 \sqrt{\frac{2\mu}{Q}}[\sqrt{y - y'} - Arctan\sqrt{\frac{1}{y} - 1}]|_{R/\overline{R}}^1 \tag{61}$$

$$G = Z_\alpha Z' e_0^2 \sqrt{\frac{2\mu}{Q}}[Arctan(\sqrt{\frac{\sqrt{2} - R}{R}} - \sqrt{(\frac{R}{\overline{\overline{R}}}) - (\frac{R}{\overline{\overline{R}}})^2}] \tag{62}$$

V izrazu upostevamo $\overline{R} = \frac{Z_\alpha Z' e_0^2}{Q}$ in dobimo izraz za razpadno sirino $P = \frac{v_I}{R}e^{-2G}$. Gamwova energija in njena limita za $R << R'$:

$$E_G = \frac{\mu}{2}(8\pi\alpha^2 Z_\alpha Z')^2 \tag{63}$$

$$G = \frac{1}{2}\sqrt{E_G/Q}(1 - \frac{4}{\pi}\sqrt{\frac{R}{\overline{\overline{R}}}}) \tag{64}$$

Za velike vrednosti E_G dobimo majhne razpolovne dolzine in obratno. Geiger-Nuttalovo pravilo pravi $\Gamma = \frac{v_t}{R}e^{-2G} : G = G(r, Q, R, \overline{R})$. Za primer $G = G(a) = \frac{k}{\sqrt{Q}}$in $r \propto e^{\frac{2k}{\sqrt{Q}}}$

4.7 Opazujemo β^+ razpad, kjer gledamo $C'' \to B''$. Pokazi blizino spektra v blizini maksimalne energije pozitrona, podana podatka E_p = 1,983MeV in B'' = 11,005u. Koliksna je masa delca C''.

$$C'' \to B'' + e^+ + \nu_e \tag{65}$$

$$K(T_{e\nu}^{max}) = 0 \tag{66}$$

$$0 = \frac{2|T_{fi}|}{(2\pi)^{\frac{3}{2}}}(Q - T_e^{max}) \tag{67}$$

$$Q = T_e^{max} K(T_e) = \frac{1}{p_e}\sqrt{\frac{d\Gamma}{dp_e}} = \frac{2|T_{fi}|}{(2\pi)^{\frac{3}{2}}}(Q - T_c) == m_{C''} - m_{B''} - m_e \tag{68}$$

4.8 Presodi kateri izmed razpadov niso dovoljeni (i=initial, f=final)

$$.^{119}In(\frac{9}{2}^+) \rightarrow .^{115}Su(\frac{1}{2}^+) + e + \overline{\nu} \tag{69}$$

- $J_i^p = \frac{9}{2}^+$, $J_f^p = \frac{1}{2}^+$
- $\Delta J = J_i - J_f = 4; P_i = P_f \cdot P_\beta$
- $\Delta\vec{J} = \vec{l} + \vec{s}$: za s=0 velja $\Delta J = l + 0 = l$, resitev za l je ena, to je l = 4 $\rightarrow P_\beta = (-1)^4 = 1 =¿$ DOVOLJENO: za s=1 velja $\Delta J = l + 1$, zato sta resitvi dve l=3,5 in $P_\beta = -1 =¿$ NI DOVOLJENO.

$$.^{40}K(4^-) \rightarrow .^{40}Ar(0^+) + e^- + \overline{\nu} \tag{70}$$

$$.^{14}O(O^+) \rightarrow .^{14}N(O^+) + e^- + \overline{\nu} \tag{71}$$

$$.^{6}He(0^+) \rightarrow .^{6}Li(1^+) + e^- + \overline{\nu} \tag{72}$$

$$.^{32}_{15}P_{17} \rightarrow .^{32}_{16}S_{16} + e^- + \overline{\nu} \tag{73}$$

4.9 Opazujemo β razpad in iscemo spreosceno energijo E v odvisnosti od atomskih mas. Izpelji izraz za sirino razpada.

Razpadna sirina nerelativisticno je

$$< \vec{p}|\vec{p}' >= (2\pi)^3\delta^{(3)}(\vec{p} - \vec{p}') \tag{74}$$

$$d\delta = |T_{fi}|^2(2\pi)^4\delta^{(3)}(\vec{p_y} + \vec{p_e} + \vec{p_\nu})\delta(m_x - E_y - E_e - E_\nu)\frac{1}{(2\pi)^3} = \tag{75}$$

$$= \frac{|T_{fi}|^2}{(2\pi)^5}\delta(m_x - E_y - E_e - E_\nu)d\Omega_y p_y^2 dp_y d\Omega_{p_e} p_e^2 dp_e d\Omega p_\nu p_y^2 dp_y \tag{76}$$

Upostevamo naslednje:

- $\int \delta^3(\vec{p}_y + \vec{p}_e + \vec{p}_\nu)d\Omega_y p_y^2 dp_y = 1$
- $-\vec{p}_y = \vec{p}_e + \vec{p}_\nu$
- $E_y = \sqrt{(p_e + p_\nu)^2 + m_y^2}$; $m_\nu = 0, E_\nu = p_\nu$
- *naprej* $|T_{fi}|$ ni odvisen od $\vec{p}_e, \vec{p}_\nu$ zato se lahko znebimo ustreznih kotnih integralov

Figure 6: 6

$$= \frac{|T_{fi}|^2}{(2\pi)^5}\delta(m_x - \sqrt{|\vec{p}_e - \vec{p}_y| + m_y^2} - |\vec{p}_y|)p_e^2 dp_e d\Omega_e p_\nu^2 dp_\nu d\Omega_\nu \tag{77}$$

$$d\Gamma = \frac{4|T_{fi}|^2}{(2\pi)^3}\delta(m_x - ...) \tag{78}$$

Upostevamo $\delta(m_x - \sqrt{|\vec{p_e} + \vec{p_\nu}|^2 + m_y^2} - \sqrt{\vec{p_e}^2 + m_c^2} - |\vec{p}_\nu|)$, kjer lahko zanemarimo $|\vec{p}_e + \vec{p_\nu}|$ in se $< \vec{p}|\vec{p}' >= (2\pi)^3\delta^{(3)}(\vec{p} - \vec{p}')$.

$$d\Gamma = \frac{4|T_{fi}|^2}{(2\pi)^3}\delta(Q - T_e - p_\nu)p_\nu^2 dp_\nu p_e^2 dp_e \tag{79}$$

$$d\Gamma = \frac{4|T_{fi}|^2}{(2\pi)^3}\delta(Q - T_e)p_e^2 dp_e \tag{80}$$

To je Sargentovo pravilo in $K(T_e) = \frac{1}{p_e}\sqrt{\frac{d\Gamma}{dp_e}} = \frac{2|T_{fi}|}{(2\pi)^{\frac{3}{2}}}(Q - T_e)$

4.10 Kateri izmed spodnjih razpadov so dovoljeni in zakaj so in zakaj ne?

Zapis stanja $|\Psi >= |M, J, p, \lambda; B, Q, L, L_e, L_\mu, L_\tau >$, pojasni sucnost in $\mathbf{L} = L_e + L_\mu + L_\tau$. Gledamo delce: $(e^-, \nu_e), L_e = 1$; $(\mu^-, \nu_\mu), L_\mu = 1$; $(\tau^-, \nu_\tau), L_\tau = 1$ in antidelce, kjer je le obrnjen predznak npr. $L_{\tau^-} = -1$. Opomba -¿ pri elektrosibkem prehodu C parnost ni ohranjena.

$$\pi^- \to \mu^- + \overline{\nu}_e \tag{81}$$

$$\overline{\nu_\mu} + p^+ \to \mu^+ n \tag{82}$$

$$\overline{\nu}_\mu + n \to \mu^- + p^+ \tag{83}$$

$$\pi^0 \to \gamma\gamma\gamma \tag{84}$$

5 Klein-Gordonova enacba

5.1 Resi Klein-Gordonovo enacbo za prosti delec in interpretiraj resitev. Iz resitve za prost delec izracunaj casovno in prostorko komponento. Interpretiraj resitve in najdi nastavek, ki resi problem z negativnimi energijami.

Klein-Gordonova enacba:

$$(-\partial_t^2 + \partial_x^2 - m^2)\phi(x) = (\partial^2 + m^2) = 0 \tag{85}$$

$$\partial^2 = \frac{\partial}{\partial t^2} - \nabla^2 \tag{86}$$

Uporabimo nastavek za prosti delec oz. ravni val $\phi(x) = NT(t)e^{i\vec{k}\vec{x}}$.

$$(\partial_t^2 - \partial_x^2 + m^2)NT(t)e^{i\vec{k}\vec{x}} = 0 \tag{87}$$

$$\partial_t^2(NT(t)e^{i\vec{k}\vec{x}}) = Ne^{i\vec{k}\vec{x}} \cdot \partial_t^2(T(t)) \tag{88}$$

$$\partial_x^2(NT(t)e^{i\vec{k}\vec{x}}) = -k^2NT(t)e^{i\vec{k}\vec{x}} \tag{89}$$

$$(\partial_t^2T(t) + k^2T(t) + m^2T(t))Ne^{i\vec{k}\vec{x}} = 0 \tag{90}$$

In sicer bo to drzalo, ko bo izraz v oklepaju enak 0. Test z nastavkom $T(t) = e^{-i\sqrt{k^2+m^2}t}$ in sicer $\partial_tT(t) = -i\sqrt{k^2+m^2}e^{-i\sqrt{k^2+m^2}t}$

$$\partial_tT(t) + (k^2 + m^2)T(t) = 0 \tag{91}$$

$$\phi(x) = Ne^{-i\sqrt{k^2+m^2}t}e^{i\vec{k}\vec{x}} = Ne^{-iEt}e^{i\vec{k}\vec{x}} = Ne^{ik^\mu x_\mu} \tag{92}$$

$$(\partial^2 + m^2)\phi(x) = 0 = ((iE)^2 - (i\vec{k})^2 + m^2)\phi(x) \tag{93}$$

Ponovno mora biti izraz v oklepaju 0, in to bo ko $-E^2 + k^2 + m^2 = 0$, resitev je torej energija E= $\pm\sqrt{k^2+m^2}$. Ideja energij je ta, da recemo delcem, ki se premikajo naprej v casu, da imajo pozitivno in delcem, ki se premikajo nazaj negativno energijo oz. to je antidelec, ki se premika naprej.

Gostota tokov (+ ponovitev):

$$\vec{j}(x) = -i[\phi^*(x)\vec{\nabla}\phi(x) - \phi(x)\vec{\nabla}\phi^*(x)] \tag{94}$$

$$\rho(x) = i[\phi^*(x)\partial_t\phi(x) - \phi(x)\partial_t\phi^*(x)] \tag{95}$$

Uporabimo delovanje operatorjev na funkcije za $\vec{\nabla}\phi(x) = i\vec{k}\phi(x), \partial_t\phi(x) = Ei\phi(x)$ in $\vec{\nabla}\phi^*(x) = i\vec{k}\phi(x)\partial_t\phi^*(x) = iE\phi$

$$\phi(x) = Ne^{-ikx} = Ne^{-\partial Et}e^{e^{i\vec{k}\vec{x}}} \tag{96}$$

Sledi, da je tok:

$$j(x) = -i[N^*e^{-ikx}(i\vec{k})Ne^{ikx} - Ne^{ikx}(-ik)N^*e^{-ikx}] = |N|^2[\vec{k} + \vec{k}] = 2|N|^2\vec{k} \tag{97}$$

$$\rho(x) = i[N^*e^{-ikx}(-iE)Ne^{ikx} - Ne^{ikx}(iE)N^*e^{-ikx}] = 2|N|^2E \tag{98}$$

Figure 7: 7

6 Izospin in simetrija

6.1 Zapisi izospinska stanja Δ barionske resonance. Diskutiraj izospinski dekuplet, v katerem se nahaja Δ. Skunstruiraj izospinsko reprezentacijo Δ resonance iz sipanja protona in π. Kaksno je razmerje razpadnih sirin Δ v $\pi^- p$ in $\pi^+ p$?

6.1.1 Izospinska stanja Δ resonance

- IZOSPIN =¿ dodatna simetrija narave $[N, I] = 0$ =¿ nukleon $|n >= (p, n)^T = (\frac{1}{2}, -\frac{1}{2})^T => (|\frac{1}{2}, \frac{1}{2} >, |\frac{1}{2}, -\frac{1}{2} >)^T$
- p in n spadata v reprezentacijo $I = \frac{1}{2}$, p ima $I_Z = \frac{1}{2}$ =¿ $|p >= |\frac{1}{2}, \frac{1}{2} >$ $, m_p|p >= H|p >$ in p ima $I_Z = -\frac{1}{2}$ =¿ $|n >= |\frac{1}{2}, -\frac{1}{2} >, m_n|n >= H|n >$.
- Δpade v izospinsko reprezentacijo $I = \frac{3}{2}$, izospin je lastnost mocne interakcije , zato imajo vse konponente znotraj reprezentacije iste lastnosti $m_{\Delta^{++}} = m_{\Delta^+} = m_{\Delta^0} = m_{\Delta^-}$

  $$|s >= (\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)^T = (|\frac{3}{2}, \frac{3}{2} >, |\frac{3}{2}, \frac{3}{2} >, |\frac{3}{2}, \frac{3}{2} >, |\frac{3}{2}, \frac{3}{2} >)^T \quad (99)$$

  Imamo 4 Δ resonance z nabojem 2,1,0,-1, kar skupno imenujemo Δ -¿ 1 resonanca z 4 I_Z konponentami.

6.1.2 Izospinski dekuplet v katerem se nahaja Δ

Taka stanja znamo opisati z kvarki $\Omega^-(x) = s(x)s(x)s(x)$ in $\Delta^0(x) = u(x)d(x)d(x)$.

6.1.3 Skonstruiraj Δ resonanco iz sipanja N in π

- barioni iz kvarkov $\rightarrow^{disociirajo}$ drugi barioni, mezoni: $\Delta^+ \approx uud \approx udd\overline{u}u \approx uud\overline{d}d$, $\Delta^+ \approx p\pi^0, n\pi^+$
- sipanje $p, \pi \rightarrow \Delta^0$, z enacbo $\mathcal{U}_{\Delta \rightarrow p\pi} =< i|T|f >$. V izospinski bazi, to je $< i|m|f >, < i| =< p| < \pi|, < i| =< p| < \pi^-|, < \pi^-| =< 1, -1|$ in torej: $< p| < \pi^-| =< \frac{1}{2}, \frac{1}{2}| < 1, -1| = \frac{1}{\sqrt{3}} < \frac{3}{2}, -\frac{1}{2}| - \sqrt{\frac{2}{3}} < \frac{1}{2}, -\frac{1}{2}|$

6.1.4 Razmerje sipalnih presekov $K^+\pi^0$ in $\overline{K}^0\pi^-$

Drugi zapis za $|K^+\pi^0 >= |K^+ > |\pi^0 >$ in posamezen delec je $|K^+ >= |\frac{1}{2}, \frac{1}{2} >$ in izospinska stevila naj bodo ista kot za K^{*+}. K^+ je psevdoskalarni mezon

$(J^P = 0^-)$ in $K^{*+}(J^P = 1^-)$ je vektorski mezon. Imata ista izospinska KS, vendar razlicne prostorske lastnosti. Upostevamo se $|\pi^0>=|1,0>$ in dobimo:

$$|K^+\pi^0>=|\frac{1}{2},\frac{1}{2}>|1,0>\to^{razpis-baze-M_1xM_2}_{v-M_3-prek-CG-koef.}|K^+\pi^0>=\sqrt{\frac{2}{3}}|\frac{3}{2},\frac{1}{2}>-\sqrt{\frac{1}{3}}|\frac{1}{2},\frac{1}{2}> \tag{100}$$

Poglejmo sipalni presek $\sigma \propto |\mathcal{U}_{K^+\pi^0\to K^+\pi^0}|^2$, kjer je $\mathcal{U} =< K^+\pi^0|T|K^+\pi^0 >$ sipalna amplituda in T je operator sipanja.

Ustvarjena baza z celotnim izospinom obeh delcev:

$$< K^+\pi^0|T|K^+\pi^0>=(\sqrt{\frac{2}{3}}<\frac{3}{2},\frac{1}{2}|-\sqrt{\frac{1}{2}}<\frac{1}{2},\frac{1}{2}|)\cdot T\cdot(\sqrt{\frac{2}{3}}|\frac{3}{2}>,\frac{1}{2}>-\sqrt{\frac{1}{3}}|\frac{1}{2},\frac{1}{2}>) \tag{101}$$

$$=\frac{2}{3}<\frac{3}{2},\frac{1}{2}|T|\frac{3}{2},\frac{1}{2}>-\frac{\sqrt{2}}{3}<\frac{3}{2},\frac{1}{2}|T|\frac{1}{2},\frac{1}{2}>-\frac{\sqrt{2}}{3}<\frac{1}{2},\frac{1}{2}|T|\frac{3}{2},\frac{1}{2}>+\frac{1}{3}<\frac{1}{2},\frac{1}{2}|T|\frac{1}{2},\frac{1}{2}> \tag{102}$$

$$=\frac{2}{3}<\frac{3}{2},\frac{1}{2}|T|\frac{3}{2},\frac{1}{2}>+\frac{1}{3}<\frac{1}{2},\frac{1}{2}|T|\frac{1}{2},\frac{1}{2}>=\frac{2}{3}T_{\frac{3}{2}}+\frac{1}{3}T_{\frac{1}{2}} \tag{103}$$

Vmes smo upostevali $<\frac{3}{2},\frac{1}{2}|T|\frac{1}{2},\frac{1}{2}>=<\frac{1}{2},\frac{1}{2}|T|\frac{3}{2},\frac{1}{2}>^*$:: Pri dolocenih vrstah interakcij sta ta dva matricna elementa nenicelna, ce recimo $[T,I]\neq 0$ ali mocno sipanje $[T,I]=0$. T ne more mesati baz =¿ $T|\frac{1}{2},\frac{1}{2}>=t|\frac{1}{2},\frac{1}{2}>$. Na primeru je to sicer: $<\frac{3}{2},\frac{1}{2}|T|\frac{1}{2},\frac{1}{2}>=t<\frac{3}{2},\frac{1}{2}|\frac{1}{2},\frac{1}{2}>=0$.

Enak postopek na $\overline{K}^0\pi^-$:

$$|\overline{K}^0\pi^->= \tag{104}$$

$$<\overline{K}^0\pi^-|T|\overline{K}^0\pi^->=(\sqrt{\frac{1}{3}}<\frac{3}{2},-\frac{1}{2}-\sqrt{\frac{2}{3}}<\frac{1}{2},-\frac{1}{2}|)\cdot T\cdot(\sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}>-\sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}>)= \tag{105}$$

$$=\frac{1}{2}<\frac{3}{2},-\frac{1}{2}|T|\frac{3}{2},-\frac{1}{2}>-\frac{\sqrt{2}}{3}<\frac{1}{2},-\frac{1}{2}|T|\frac{3}{2},-\frac{1}{2}>-\frac{\sqrt{2}}{3}<\frac{3}{2},-\frac{1}{2}|T|\frac{1}{2},-\frac{1}{2}>+\frac{2}{3}<\frac{1}{2},-\frac{1}{2}|T|\frac{1}{2},-\frac{1}{2}> \tag{106}$$

$$|I_1,I_{1Z}>\cdot|I_2,I_{2Z}>=\sum_{I,I_Z}C^{I,I_Z}_{I_1,I_{1Z},I_2,I_{2Z}}|I,I_Z> \tag{107}$$

Torej, ce vas zanima sipanje v Δ^0 bo: $<\Delta^0|T|p\pi^->=\sqrt{\frac{1}{3}}<\frac{3}{2},-\frac{1}{2}|T|\frac{3}{2},-\frac{1}{2}>$,
$\Gamma \propto |<\Delta^0|T|p\pi^->|^2$

6.1.5 Razmerje razpadnih sirin $\frac{\Gamma_{\Delta\to\pi^-p}}{\Gamma_{\Delta\to\pi^+p}}$

Skonstruirajmo tip $\pi^+p<i|=<p|<\pi^+|=-<\frac{1}{2},\frac{1}{2}|<1,1|=<\frac{3}{2},\frac{3}{2}|=<\Delta^{++}|$, kjer je $\Delta^{++}=<\frac{3}{2},\frac{3}{2}|$ in torej $<\Delta^{++}|T|p\pi^+>=<\frac{3}{2},\frac{3}{2}|T|\frac{3}{2},\frac{3}{2}>=_{WE}=<\frac{3}{2}|T|\frac{3}{2}>$. Velja, da imamo Δ^{++} resonanco pri π^+p in Δ^0 pri π^-p. Ti dve sta zaradi izospinske simetrije enaki, zato dobimo:

$$R=\frac{\Gamma_{\Delta\to\pi^-p}}{\Gamma_{\Delta\to\pi^+p}}=\frac{\frac{1}{3}|<\frac{3}{2}|T|\frac{3}{2}>|^2}{|<\frac{3}{2}|T|\frac{3}{2}>|^2}=\frac{1}{3} \tag{108}$$

Figure 8: 8

6.2 Razlozi izospinska stanja K^* resonance, diskutiraj izospinski nonet v katerem nastopa K^* in skonstruiraj K^* resonanco iz sipanja π in K. Koliksno je razmerje sipalnih presekov $K^+M\pi^0$ in $\overline{K}^0\pi^-$

Izospinska stanja K^* resonance:

$$|\pi >= (\pi^+, \pi^0, \pi^-)^T = (1, 0, -1)^T => (|1, 1 >, |1, 0 >, |1, -1 >)^T \tag{109}$$

, kjer π^+, π^0, π^- spadajo v reprezentacijo I=1, dm=3 in imajo vsi isto maso.

$$|K^* >= (K^{*+}, K^{*0})^T = (|\frac{1}{2}, \frac{1}{2} >, |\frac{1}{2}, -\frac{1}{2} >)^T; S = 1 \tag{110}$$

$$|\overline{K}^* >= (\overline{K}^{*0}, K^{*-})^T = (|\frac{1}{2}, \frac{1}{2} >, |\frac{1}{2}, -\frac{1}{2} >)^T; S = -1 \tag{111}$$

Zapomniti velja, da je delec $K^* \approx s\overline{u}$ v reprezentaciji $\frac{1}{2}$. Izospin je dobro kvantno stevilo in delci so z S=1 in S=-1 povezani prek delec-antidelec simetrije: $m_{K^{*0}} = m_{K^{*+}} = m_{\overline{K}^{*-}} = m_{\overline{K^{*0}}}$, $\Gamma_{K^{*0}} = \Gamma_{K^{*+}} = \Gamma_{\overline{K}^{*-}} = \Gamma_{\overline{K}^{*0}}$

6.2.1 Izospinski nonent: Simetrija je bolj zlomljena (usd) in delci K^*, ρ, ω nimajo istih mas =¿ "priblizna simetrija"

6.3 Izracunaj magnetni dipolni moment p,n in Ω

Najprej poglejmo VF hadronov, kjer je η = barvni del (antisimetricen), l,m = vrtilna kolicina, s = spin, f = okus in n = radialne ekstincije (n=0).

$$|n >= \eta|l, m > |s > |f > |n > \tag{112}$$

$$|p \uparrow>= Y_{00}\eta\frac{1}{\sqrt{18}}[|udu > (2|\uparrow\downarrow\uparrow> -|\downarrow\uparrow\uparrow> -|\uparrow\uparrow\downarrow>)+|duu > (2|\downarrow\uparrow\uparrow> -|\uparrow\downarrow\uparrow> -|\uparrow\uparrow\downarrow>)+ \tag{113}$$

$$...|uud|(2|\uparrow\uparrow\downarrow> -|\uparrow\downarrow\uparrow> -|\downarrow\uparrow\uparrow>)] \tag{114}$$

Operator za magnetni moment, maksimalna projekcija spina:

$$\mu_i = g_s\frac{Q_iS_i}{2m_i} \tag{115}$$

$$\mu_i^{max} = g_s\frac{Q_iS_i^{max=3}}{2m_i} \tag{116}$$

$$\mu_p =< p\uparrow | \sum_{i=1,u} \mu_i|p\uparrow> \tag{117}$$

$$= \frac{g_{sl}}{2m_u}[2 < u_1 d_2 u_3| <\uparrow_1\downarrow_2\uparrow_3 | - < u_1 d_2 u_3| <\downarrow\uparrow\uparrow | - < u_1 d_2 u_3| <\uparrow\uparrow\downarrow + 2(Q_u S_u + Q_d S_d)\cdot(?)\cdot(\frac{1}{\sqrt{18}}\frac{1}{\sqrt{18}}) \quad (118)$$

$$\sum_{i=u_1,u_2} Q_i S_i |u_1 d_2 u_3 > | \uparrow\downarrow\uparrow >= \sum Q_i |u_i > |d_2 > |u_3 > \sum S_i | \uparrow\downarrow\uparrow >= \quad (119)$$

$$= \frac{2}{3}|u_1 > |d_2 > |u_3 > \frac{1}{2}| \uparrow\downarrow\uparrow > +0 + \frac{2}{3}|u_1 > |d_2 > |u_3 > \frac{1}{2}| \uparrow\downarrow\uparrow > \quad (120)$$

$$\mu_p = \frac{g_{sl}}{2m\mu_n}\frac{1}{2} \quad (121)$$

Nevtronska valovna funkcija:

$$|n \uparrow >= Y_{00}\eta\frac{1}{\sqrt{18}}[|d_1 u_2 d_3 > (2| \uparrow\downarrow\uparrow > -| \downarrow\uparrow\uparrow > -| \uparrow\uparrow\downarrow >) + |u_1 d_2 d_3 > (2| \downarrow\uparrow\uparrow > -| \uparrow\downarrow\uparrow > -| \uparrow\uparrow\downarrow >) + \quad (122)$$

$$...|d_2 d_2 u_3 > (2| \uparrow\uparrow\downarrow > -| \uparrow\downarrow\uparrow > -| \downarrow\uparrow\uparrow >)] \quad (123)$$

$$\mu_N = \frac{g_{sl}}{2m_N}(-\frac{1}{3}) \quad (124)$$

Zdaj pogledamo se VF Ω bariona (fermion=¿VF simetricna), η =barvni del=¿antisimetricen in l,m del je simetricen —n¿ $= \eta|l, m > |s > |f >$. Okusni del Ωbariona je sss in je v celoti simetricen, spinski $|s >= \frac{1}{\sqrt{3}}[| \uparrow\uparrow\downarrow > +| \uparrow\downarrow\uparrow > +| \downarrow\uparrow\uparrow >]$ mora biti v celoti simetricen.

$$|\Omega > \frac{1}{\sqrt{3}}\eta Y_{00}|S_1 S_2 S_3 > [<\uparrow\uparrow\downarrow | + | <\uparrow\downarrow\uparrow | + <\downarrow\uparrow\uparrow |] \quad (125)$$

$$< \Omega|\mu_s|\Omega >= \frac{1}{3} < s_1 s_2 s_3|[<\uparrow\uparrow\downarrow | + <\uparrow\downarrow\uparrow | + <\downarrow\uparrow\uparrow |]\cdot\frac{g_{ls}}{2m_s}[Q_{s1}S_{s1} + Q_{s2}S_{s2} + Q_{s3}S_{s3}]\cdot \quad (126)$$

$$...[| \uparrow\uparrow\downarrow > +| \uparrow\downarrow\uparrow > +| \downarrow\uparrow\uparrow >]|s_1 s_2 s_3 >= \quad (127)$$

$$=< s_1 s_2 s_3| <\uparrow\uparrow\downarrow |[Q_{s1}S_{s1} + Q_{s2}S_{s2} + Q_{s3}S_{s3}]| \uparrow\uparrow\downarrow > |s_1 s_2 s_3 > \quad (128)$$

Nadalje upostevamo $Q_i S_i |f_1 f_2 f_3 > |s_1 s_2 s_3 >= l_1 s_1 |f_1 f_2 f_3 > |s_1 s_2 s_3 >$ in $< f_i|f_j >= \delta_{ij}$ ter $< s_a|s_b >= \delta_{ab}$.

$$= \frac{g_{ls}}{2m_s}([\frac{1}{2} + \frac{1}{2} - \frac{1}{2}] + [\frac{1}{2} - \frac{1}{2} + \frac{1}{2}] + ...vsi - cleni...) \quad (129)$$

$$= -\frac{g_{ls}}{4m_y} \quad (130)$$

6.4 Za psevdoskalarne mezone iz noneta zapisi mozna mesanja in kako mesanje vpliva na mase? Doloci mesalni kot med η_8 in η_0 za η in η' mezona, ce ves $m_Y = 550MeV, m_{Y'} = 950MeV, m_B = 525MeV, m_i = 975MeV$.

Clenom okteta vcasih pravimo goldsteinovi bozoni(psevdo), mesajo se lahko stvari z istim KS =¿ 3 koordinate η, η', π^0. Prva dva se fenomenolosko mesata, π^0 se ne.

- $\eta_8 = \frac{1}{\sqrt{6}}(\overline{u}u + \overline{d}d - 2s\overline{s})$
- $\eta_1 = \frac{1}{\sqrt{3}}(\overline{u}u + \overline{d}d + \overline{s}s)$
- $\pi^0 = \frac{1}{\sqrt{2}}(\overline{u}u - \overline{d}d)$
- H—f¿ $= m_f|f>$
- $H|\eta_8>= m_8|\eta_8>$
- $H|\eta_1>= m_1|\eta_1>$
- $H|\eta>= m_y|\eta>$
- $H|\eta'>= m_{\eta'}|\eta'>$

Izpeljava za masno formulo:

$$|\eta>= cos\alpha|\eta_8> -sin\alpha|\eta_i> \tag{131}$$

$$|\eta'>= sin\alpha|\eta_8> +cos\alpha|\eta_i> \tag{132}$$

$$H|\eta>= m_\eta|\eta>= \hbar(cos\alpha|\eta_8> -sin\alpha|\eta_1>) = m_8 cos\alpha|\eta_8> -m_1 sin\alpha|\eta_1> \tag{133}$$

$$H|\eta'>= m_{\eta'}|\eta'>= H(sin\alpha|\eta_8> +cos\alpha|\eta_1>) = m_8 sin\alpha|\eta_8> +m_1 cos\alpha|\eta_1> \tag{134}$$

$$<\eta|H|\eta>= m_\eta = [<\eta_8|cos\alpha- <\eta_1|cos\alpha] \cdot [m_8 cos\alpha|\eta_8> -m_1 sin\alpha|\eta_l>] \tag{135}$$

$$<\eta'|H|\eta'>= m_{\eta'} = [<\eta_8|sin\alpha+ <\eta_1|cos\alpha] \cdot [m_8 sin\alpha|\eta_8> +m_1 cos\alpha|\eta_l>] \tag{136}$$

$$m_\eta = m_8 cos^2\alpha + m_1 sin^2\alpha \tag{137}$$

$$m_{\eta'} = m_8 sin^2\alpha + m_1 cos^2\alpha \tag{138}$$

To sta dva sta enacb, ki ju sestejemo in odstejemo da dobimo:

$$+|m_{\eta'} + m_\eta = m_8 + m_1 \tag{139}$$

$$-|m_{\eta'} - m_\eta = xm_8 - m_1 x = x(m_8 - m_1) \tag{140}$$

Tu izracunamo vrednost x in prek zveze $sin^2\alpha - cos^2\alpha = x$ se α, vrednosti sta x=-0,8829 in α=14°.

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