

# The Numerical Utilities Reference Manual

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Numerical Utilities, version 1.0.0

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# Table of Contents

<b>Copying</b>	<b>1</b>
<b>1 Systems</b>	<b>3</b>
1.1 num-utils	3
<b>2 Files</b>	<b>5</b>
2.1 Lisp	5
2.1.1 num-utils/num-utils.asd	5
2.1.2 num-utils/packages.lisp	5
2.1.3 num-utils/utilities.lisp	5
2.1.4 num-utils/num=.lisp	6
2.1.5 num-utils/arithmetic.lisp	6
2.1.6 num-utils/elementwise.lisp	7
2.1.7 num-utils/extended-real.lisp	8
2.1.8 num-utils/interval.lisp	9
2.1.9 num-utils/print-matrix.lisp	10
2.1.10 num-utils/matrix.lisp	10
2.1.11 num-utils/matrix-shorthand.lisp	13
2.1.12 num-utils/statistics.lisp	14
2.1.13 num-utils/chebyshev.lisp	16
2.1.14 num-utils/polynomial.lisp	16
2.1.15 num-utils/rootfinding.lisp	16
2.1.16 num-utils/quadrature.lisp	17
2.1.17 num-utils/log-exp.lisp	19
2.1.18 num-utils/test-utilities.lisp	19
2.1.19 num-utils/pkgdcl.lisp	20
<b>3 Packages</b>	<b>21</b>
3.1 num-utils.quadrature	21
3.2 num-utils.print-matrix	23
3.3 num-utils.chebyshev	23
3.4 num-utils.interval	24
3.5 num-utils.matrix-shorthand	25
3.6 num-utils.log-exp	26
3.7 num-utils.matrix	26
3.8 num-utils.elementwise	28
3.9 num-utils.extended-real	29
3.10 num-utils.utilities	30
3.11 num-utils.statistics	31
3.12 num-utils.test-utilities	33
3.13 num-utils.arithmetic	34
3.14 num-utils.num=	35
3.15 num-utils.polynomial	35
3.16 num-utils	36
3.17 num-utils.rootfinding	36

<b>4</b>	<b>Definitions</b>	<b>39</b>
4.1	Public Interface	39
4.1.1	Special variables	39
4.1.2	Macros	39
4.1.3	Compiler macros	42
4.1.4	Ordinary functions	42
4.1.5	Generic functions	55
4.1.6	Standalone methods	72
4.1.7	Conditions	73
4.1.8	Structures	74
4.1.9	Classes	81
4.1.10	Types	82
4.2	Internals	83
4.2.1	Macros	83
4.2.2	Ordinary functions	85
4.2.3	Generic functions	97
4.2.4	Structures	99
4.2.5	Classes	102
4.2.6	Types	103
<b>Appendix A</b>	<b>Indexes</b>	<b>105</b>
A.1	Concepts	105
A.2	Functions	106
A.3	Variables	107
A.4	Data types	108

## Copying

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# 1 Systems

The main system appears first, followed by any subsystem dependency.

## 1.1 num-utils

Numerical utilities for Common Lisp

### Long Name

Numerical Utilities

**Author** Steven Nunez <steve@symbolics.tech>

### Source Control

(GIT <https://github.com/Lisp-Stat/numerical-utilities.git>)

### Bug Tracker

<https://github.com/Lisp-Stat/numerical-utilities/issues>

**License** MS-PL

### Long Description

This library implements simple numerical functions for Common Lisp, including  
 num=, a comparison operator for floats  
 simple arithmetic functions, like sum and l2norm  
 elementwise operations for arrays  
 intervals  
 special matrices and shorthand for their input  
 sample statistics  
 Chebyshev polynomials  
 univariate rootfinding

**Version** 1.0.0

### Dependencies

- anaphora (system).
- alexandria (system).
- array-operations (system).
- select (system).
- let-plus (system).

**Source** [num-utils.asd], page 5.

### Child Components

- [packages.lisp], page 5 (file).
- [utilities.lisp], page 5 (file).
- [num=.lisp], page 6 (file).
- [arithmetic.lisp], page 6 (file).
- [elementwise.lisp], page 7 (file).
- [extended-real.lisp], page 8 (file).
- [interval.lisp], page 9 (file).
- [print-matrix.lisp], page 10 (file).
- [matrix.lisp], page 10 (file).
- [matrix-shorthand.lisp], page 13 (file).

- `[statistics.lisp]`, page 14 (file).
- `[chebyshev.lisp]`, page 16 (file).
- `[polynomial.lisp]`, page 16 (file).
- `[rootfinding.lisp]`, page 16 (file).
- `[quadrature.lisp]`, page 17 (file).
- `[log-exp.lisp]`, page 19 (file).
- `[test-utilities.lisp]`, page 19 (file).
- `[pkgdcl.lisp]`, page 20 (file).



## 2 Files

Files are sorted by type and then listed depth-first from the systems components trees.

### 2.1 Lisp

#### 2.1.1 num-utils/num-utils.asd

**Source** [num-utils.asd], page 5.

**Parent Component**  
[num-utils], page 3 (system).

**ASDF Systems**  
[num-utils], page 3.

#### 2.1.2 num-utils/packages.lisp

**Source** [num-utils.asd], page 5.

**Parent Component**  
[num-utils], page 3 (system).

#### Packages

- [num-utils.quadrature], page 21.
- [num-utils.print-matrix], page 23.
- [num-utils.chebyshev], page 23.
- [num-utils.interval], page 24.
- [num-utils.matrix-shorthand], page 25.
- [num-utils.log-exp], page 26.
- [num-utils.matrix], page 26.
- [num-utils.elementwise], page 28.
- [num-utils.extended-real], page 29.
- [num-utils.utilities], page 30.
- [num-utils.statistics], page 31.
- [num-utils.test-utilities], page 33.
- [num-utils.arithmetic], page 34.
- [num-utils.num=], page 35.
- [num-utils.polynomial], page 35.
- [num-utils.rootfinding], page 36.

#### 2.1.3 num-utils/utilities.lisp

**Dependency**  
[packages.lisp], page 5 (file).

**Source** [num-utils.asd], page 5.

**Parent Component**  
[num-utils], page 3 (system).

**Public Interface**

- [as-alist], page 56 (generic function).

- [as-double-float], page 43 (function).
- [as-plist], page 56 (generic function).
- [as-simple-fixnum-vector], page 43 (function).
- [bic], page 43 (function).
- [binary-search], page 43 (function).
- [check-types], page 39 (macro).
- [curry\*], page 40 (macro).
- [define-with-multiple-bindings], page 40 (macro).
- [expanding], page 40 (macro).
- [fixnum?], page 47 (function).
- [generate-sequence], page 47 (function).
- [gethash\*], page 40 (macro).
- [make-vector], page 42 (compiler macro).
- [make-vector], page 50 (function).
- [sequencep], page 53 (function).
- [simple-double-float-vector], page 82 (type).
- [simple-fixnum-vector], page 83 (type).
- [simple-single-float-vector], page 83 (type).
- [splice-awhen], page 41 (macro).
- [splice-when], page 41 (macro).
- [with-double-floats], page 41 (macro).
- [within?], page 55 (function).

#### 2.1.4 num-utils/num=.lisp

##### Dependency

[utilities.lisp], page 5 (file).

##### Source

[num-utils.asd], page 5.

##### Parent Component

[num-utils], page 3 (system).

##### Public Interface

- [\*num=-tolerance\*], page 39 (special variable).
- [define-num=-with-accessors], page 40 (macro).
- [define-structure-num=], page 40 (macro).
- [num-delta], page 51 (function).
- [num=], page 68 (generic function).
- [num=-function], page 51 (function).

#### 2.1.5 num-utils/arithmetic.lisp

##### Dependency

[num=.lisp], page 6 (file).

##### Source

[num-utils.asd], page 5.

##### Parent Component

[num-utils], page 3 (system).

**Public Interface**

- [1c], page 42 (function).
- [abs-diff], page 42 (function).
- [absolute-square], page 42 (function).
- [as-integer], page 43 (function).
- [ceiling\*], page 43 (function).
- [cube], page 44 (function).
- [cumulative-product], page 44 (function).
- [cumulative-sum], page 44 (function).
- [divides?], page 45 (function).
- [floor\*], page 47 (function).
- [ivec], page 48 (function).
- [l2norm], page 49 (function).
- [l2norm-square], page 67 (generic function).
- [log10], page 49 (function).
- [log2], page 49 (function).
- [multf], page 41 (macro).
- [normalize-probabilities], page 51 (function).
- [numseq], page 51 (function).
- [product], page 69 (generic function).
- [round\*], page 52 (function).
- [same-sign-p], page 52 (function).
- [sequence-maximum], page 53 (function).
- [sequence-minimum], page 53 (function).
- [square], page 54 (function).
- [sum], page 71 (generic function).
- [truncate\*], page 54 (function).

**Internals**

- [define-rounding-with-offset], page 84 (macro).
- [ln], page 90 (function).
- [similar-element-type], page 94 (function).
- [similar-sequence-type], page 94 (function).

**2.1.6 num-utils/elementwise.lisp****Dependency**

[arithmetic.lisp], page 6 (file).

**Source**

[num-utils.asd], page 5.

**Parent Component**

[num-utils], page 3 (system).

**Public Interface**

- [e\*], page 45 (function).
- [e+], page 45 (function).
- [e-], page 45 (function).

- [e/], page 45 (function).
- [e1-], page 58 (generic function).
- [e1/], page 58 (generic function).
- [e1log], page 58 (generic function).
- [e2\*], page 59 (generic function).
- [e2+], page 60 (generic function).
- [e2-], page 61 (generic function).
- [e2/], page 62 (generic function).
- [e2log], page 63 (generic function).
- [e ceiling], page 63 (generic function).
- [e conjugate], page 63 (generic function).
- [ecos], page 63 (generic function).
- [eexp], page 64 (generic function).
- [eexpt], page 64 (generic function).
- [efloor], page 64 (generic function).
- [elementwise-float-contagion], page 45 (function).
- [elog], page 46 (function).
- [emax], page 46 (function).
- [emin], page 46 (function).
- [emod], page 65 (generic function).
- [erreduce], page 65 (generic function).
- [esin], page 65 (generic function).
- [esqrt], page 66 (generic function).

### Internals

- [define-e&], page 84 (macro).
- [define-e1], page 84 (macro).
- [define-e2], page 84 (macro).
- [define-elementwise-reduction], page 84 (macro).
- [esquare], page 97 (generic function).
- [mapping-array], page 85 (macro).

## 2.1.7 num-utils/extended-real.lisp

### Dependency

[elementwise.lisp], page 7 (file).

### Source

[num-utils.asd], page 5.

### Parent Component

[num-utils], page 3 (system).

### Public Interface

- [<], page 42 (function).
- [<=], page 42 (function).
- [=], page 42 (function).
- [>], page 42 (function).
- [>=], page 42 (function).

- `[extended-real]`, page 82 (type).
- `[infinite?]`, page 48 (function).
- `[lambda-template]`, page 41 (macro).
- `[with-template]`, page 42 (macro).

### Internals

- `[define-comparison]`, page 83 (macro).
- `[extend-pairwise-comparison]`, page 88 (function).
- `[infinite]`, page 103 (type).

## 2.1.8 num-utils/interval.lisp

### Dependency

`[extended-real.lisp]`, page 8 (file).

### Source

`[num-utils.asd]`, page 5.

### Parent Component

`[num-utils]`, page 3 (system).

### Public Interface

- `[&interval]`, page 39 (macro).
- `[extend-interval]`, page 66 (generic function).
- `[extendf-interval]`, page 40 (macro).
- `[finite-interval]`, page 81 (class).
- `[grid-in]`, page 47 (function).
- `[in-interval?]`, page 48 (function).
- `[initialize-instance]`, page 72 (method).
- `[interval]`, page 48 (function).
- `[interval]`, page 81 (class).
- `[interval-hull]`, page 48 (function).
- `[interval-length]`, page 48 (function).
- `[interval-midpoint]`, page 48 (function).
- `[left]`, page 67 (generic function).
- `[minusinf-interval]`, page 82 (class).
- `[num=]`, page 68 (method).
- `[num=]`, page 68 (method).
- `[open-left?]`, page 69 (generic function).
- `[open-right?]`, page 69 (generic function).
- `[plusinf-interval]`, page 82 (class).
- `[plusminus-interval]`, page 51 (function).
- `[print-object]`, page 72 (method).
- `[real-line]`, page 82 (class).
- `[relative]`, page 52 (function).
- `[relative]`, page 77 (structure).
- `[right]`, page 70 (generic function).
- `[shift-interval]`, page 70 (generic function).
- `[shrink-interval]`, page 53 (function).

- [spacer], page 53 (function).
- [spacer], page 78 (structure).
- [split-interval], page 53 (function).
- [subintervals-in], page 54 (function).

#### Internals

- [copy-relative], page 87 (function).
- [copy-spacer], page 87 (function).
- [interval/finite-left], page 102 (class).
- [interval/finite-right], page 102 (class).
- [interval/infinite-left], page 103 (class).
- [interval/infinite-right], page 103 (class).
- [print-left-endpoint], page 98 (generic function).
- [print-right-endpoint], page 98 (generic function).
- [relative-fraction], page 93 (reader).
- [relative-p], page 93 (function).
- [spacer-p], page 95 (function).
- [spacer-weight], page 95 (reader).

### 2.1.9 num-utils/print-matrix.lisp

#### Dependency

[interval.lisp], page 9 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

- [\*print-matrix-precision\*], page 39 (special variable).
- [print-length-truncate], page 51 (function).
- [print-matrix], page 51 (function).

**Internals** [print-matrix-formatter], page 92 (function).

### 2.1.10 num-utils/matrix.lisp

#### Dependency

[print-matrix.lisp], page 10 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

- [as-array], page 72 (method).
- [as-array], page 72 (method).
- [as-array], page 72 (method).
- [as-array], page 72 (method).
- [diagonal-matrix], page 45 (function).
- [diagonal-matrix], page 75 (structure).

- [diagonal-matrix-elements], page 45 (reader).
- [(setf diagonal-matrix-elements)], page 45 (writer).
- [diagonal-vector], page 57 (generic function).
- [(setf diagonal-vector)], page 57 (generic function).
- [dims], page 72 (method).
- [dims], page 72 (method).
- [e1-], page 58 (method).
- [e1-], page 58 (method).
- [e1-], page 58 (method).
- [e1-], page 58 (method).
- [e1/], page 58 (method).
- [e1/], page 58 (method).
- [e1/], page 58 (method).
- [e1/], page 58 (method).
- [e1log], page 59 (method).
- [e1log], page 59 (method).
- [e1log], page 59 (method).
- [e1log], page 59 (method).
- [e2\*], page 59 (method).
- [e2\*], page 59 (method).
- [e2\*], page 59 (method).
- [e2\*], page 59 (method).
- [e2\*], page 59 (method).
- [e2\*], page 59 (method).
- [e2\*], page 59 (method).
- [e2\*], page 59 (method).
- [e2\*], page 60 (method).
- [e2\*], page 60 (method).
- [e2\*], page 60 (method).
- [e2\*], page 60 (method).
- [e2\*], page 60 (method).
- [e2\*], page 60 (method).
- [e2\*], page 60 (method).
- [e2\*], page 60 (method).
- [e2+], page 60 (method).
- [e2+], page 60 (method).
- [e2+], page 61 (method).
- [e2+], page 61 (method).
- [e2+], page 61 (method).
- [e2+], page 61 (method).
- [e2-], page 61 (method).
- [e2-], page 61 (method).
- [e2-], page 61 (method).
- [e2-], page 61 (method).
- [e2-], page 61 (method).

- [e2-], page 62 (method).
- [e2/], page 62 (method).
- [e2/], page 62 (method).
- [e2/], page 62 (method).
- [e2/], page 62 (method).
- [e2/], page 62 (method).
- [e2/], page 62 (method).
- [e2/], page 62 (method).
- [e2/], page 62 (method).
- [eexp], page 64 (method).
- [eexp], page 64 (method).
- [eexp], page 64 (method).
- [eexp], page 64 (method).
- [element-type], page 72 (method).
- [element-type], page 72 (method).
- [esqrt], page 66 (method).
- [esqrt], page 66 (method).
- [esqrt], page 66 (method).
- [esqrt], page 66 (method).
- [hermitian-matrix], page 47 (function).
- [hermitian-matrix], page 76 (structure).
- [lower-triangular-matrix], page 50 (function).
- [lower-triangular-matrix], page 76 (structure).
- [map-array], page 67 (method).
- [num=], page 68 (method).
- [num=], page 68 (method).
- [print-object], page 72 (method).
- [print-object], page 73 (method).
- [print-object], page 73 (method).
- [select], page 73 (method).
- [select], page 73 (method).
- [select], page 73 (method).
- [transpose], page 71 (generic function).
- [triangular-matrix], page 83 (type).
- [upper-triangular-matrix], page 54 (function).
- [upper-triangular-matrix], page 80 (structure).
- [wrapped-matrix], page 80 (structure).
- [wrapped-matrix-elements], page 55 (reader).

## Internals

- [&diagonal-matrix], page 83 (macro).
- [&diagonal-matrix-r/o], page 83 (macro).
- [above-diagonal?], page 85 (function).



- [below-diagonal?], page 85 (function).
- [copy-diagonal-matrix], page 87 (function).
- [copy-hermitian-matrix], page 87 (function).
- [copy-lower-triangular-matrix], page 87 (function).
- [copy-upper-triangular-matrix], page 88 (function).
- [copy-wrapped-matrix], page 88 (function).
- [define-elementwise-as-array], page 84 (macro).
- [define-elementwise-same-class], page 84 (macro).
- [define-elementwise-univariate], page 84 (macro).
- [define-elementwise-with-constant], page 84 (macro).
- [define-wrapped-matrix], page 84 (macro).
- [diagonal-matrix-p], page 88 (function).
- [ensure-valid-elements], page 88 (function).
- [hermitian-matrix-elements], page 88 (function).
- [hermitian-matrix-p], page 88 (function).
- [lower-triangular-matrix-elements], page 90 (function).
- [lower-triangular-matrix-p], page 90 (function).
- [make-diagonal-matrix], page 90 (function).
- [make-hermitian-matrix], page 90 (function).
- [make-lower-triangular-matrix], page 90 (function).
- [make-upper-triangular-matrix], page 91 (function).
- [make-wrapped-matrix], page 91 (function).
- [upper-triangular-matrix-elements], page 96 (function).
- [upper-triangular-matrix-p], page 96 (function).
- [valid-sparse-type?], page 97 (function).
- [wrapped-matrix-p], page 97 (function).
- [zero-like], page 97 (function).

### 2.1.11 num-utils/matrix-shorthand.lisp

#### Dependency

[matrix.lisp], page 10 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

- [diagonal-mx], page 45 (function).
- [hermitian-mx], page 40 (macro).
- [lower-triangular-mx], page 41 (macro).
- [mx], page 41 (macro).
- [upper-triangular-mx], page 41 (macro).
- [vec], page 55 (function).

#### Internals

[pad-left-expansion], page 92 (function).

### 2.1.12 num-utils/statistics.lisp

#### Dependency

[matrix-shorthand.lisp], page 13 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

- [\*central-sample-moments-default-degree\*], page 39 (special variable).
- [add], page 55 (generic function).
- [as-alist], page 56 (method).
- [central-m2], page 56 (generic function).
- [central-m3], page 56 (generic function).
- [central-m4], page 57 (generic function).
- [central-sample-moments], page 57 (generic function).
- [central-sample-moments], page 74 (structure).
- [central-sample-moments-degree], page 43 (function).
- [cross-tabulate], page 44 (function).
- [empirical-quantile], page 46 (function).
- [empirical-quantile-probabilities], page 46 (function).
- [empty-accumulator], page 73 (condition).
- [ensure-sorted-reals], page 65 (generic function).
- [ensure-sorted-vector], page 46 (function).
- [information-not-collected-in-accumulator], page 73 (condition).
- [kurtosis], page 66 (generic function).
- [make-sparse-counter], page 50 (function).
- [mean], page 67 (generic function).
- [median], page 67 (generic function).
- [not-enough-elements-in-accumulator], page 73 (condition).
- [num=], page 68 (method).
- [pool], page 51 (function).
- [print-object], page 73 (method).
- [print-object], page 73 (method).
- [quantile], page 69 (generic function).
- [quantiles], page 70 (generic function).
- [sd], page 70 (generic function).
- [skewness], page 70 (generic function).
- [sorted-reals], page 77 (structure).
- [sorted-reals-elements], page 53 (function).
- [sparse-counter], page 78 (structure).
- [sparse-counter-count], page 53 (function).
- [sparse-counter-table], page 53 (reader).
- [tabulate], page 54 (function).

- [tally], page 71 (generic function).
- [variance], page 71 (generic function).
- [weighted-quantiles], page 55 (function).

## Internals

- [&sorted-reals], page 83 (macro).
- [&sorted-reals-r/o], page 83 (macro).
- [central-sample-moments-m], page 86 (reader).
- [(setf central-sample-moments-m)], page 86 (writer).
- [central-sample-moments-p], page 86 (function).
- [central-sample-moments-s2], page 86 (reader).
- [(setf central-sample-moments-s2)], page 86 (writer).
- [central-sample-moments-s3], page 86 (reader).
- [(setf central-sample-moments-s3)], page 86 (writer).
- [central-sample-moments-s4], page 86 (reader).
- [(setf central-sample-moments-s4)], page 86 (writer).
- [central-sample-moments-w], page 86 (function).
- [(setf central-sample-moments-w)], page 86 (function).
- [copy-central-sample-moments], page 87 (function).
- [copy-sorted-reals], page 87 (function).
- [copy-sparse-counter], page 87 (function).
- [copy-tally-mixin], page 88 (function).
- [define-central-sample-moment], page 83 (macro).
- [make-central-sample-moments], page 90 (function).
- [make-sorted-reals], page 90 (function).
- [make-sparse-counter%], page 90 (function).
- [make-tally-mixin], page 90 (function).
- [pool2], page 98 (generic function).
- [sort-reals], page 94 (function).
- [sorted-reals-ordered-elements], page 94 (reader).
- [(setf sorted-reals-ordered-elements)], page 94 (writer).
- [sorted-reals-p], page 94 (function).
- [sorted-reals-unordered-elements], page 94 (reader).
- [(setf sorted-reals-unordered-elements)], page 94 (writer).
- [sparse-counter-p], page 95 (function).
- [tally-mixin], page 101 (structure).
- [tally-mixin-p], page 95 (function).
- [tally-mixin-w], page 95 (reader).
- [(setf tally-mixin-w)], page 95 (writer).
- [weighted-empirical-quantile], page 97 (function).
- [weighted-quantile-p-table], page 97 (function).

### 2.1.13 num-utils/chebyshev.lisp

#### Dependency

[statistics.lisp], page 14 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

- [chebyshev-approximate], page 43 (function).
- [chebyshev-regression], page 44 (function).
- [chebyshev-root], page 44 (function).
- [chebyshev-roots], page 44 (function).
- [evaluate-chebyshev], page 46 (function).

#### Internals

- [ab-to-cd-intercept-slope], page 85 (function).
- [ab-to-cinf], page 85 (function).
- [chebyshev-approximate-implementation], page 97 (generic function).
- [chebyshev-recursion], page 86 (function).
- [cinf-to-ab], page 87 (function).

### 2.1.14 num-utils/polynomial.lisp

#### Dependency

[chebyshev.lisp], page 16 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

[evaluate-polynomial], page 46 (function).

### 2.1.15 num-utils/rootfinding.lisp

#### Dependency

[polynomial.lisp], page 16 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

- [\*rootfinding-delta-relative\*], page 39 (special variable).
- [\*rootfinding-epsilon\*], page 39 (special variable).
- [root-bisection], page 52 (function).

#### Internals

- [narrow-bracket?], page 92 (function).
- [near-root?], page 92 (function).
- [opposite-sign?], page 92 (function).
- [rootfinding-delta], page 94 (function).
- [univariate-rootfinder-loop%], page 85 (macro).

### 2.1.16 num-utils/quadrature.lisp

#### Dependency

[rootfinding.lisp], page 16 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

[romberg-quadrature], page 52 (function).

#### Internals

- [copy-iterative-quadrature], page 87 (function).
- [copy-midpoint-quadrature], page 87 (function).
- [copy-richardson-extrapolation], page 87 (function).
- [copy-trapezoidal-quadrature], page 88 (function).
- [iterative-quadrature], page 99 (structure).
- [iterative-quadrature-a], page 89 (reader).
- [(setf iterative-quadrature-a)], page 89 (writer).
- [iterative-quadrature-b], page 89 (reader).
- [(setf iterative-quadrature-b)], page 89 (writer).
- [iterative-quadrature-f], page 89 (reader).
- [(setf iterative-quadrature-f)], page 89 (writer).
- [iterative-quadrature-h], page 89 (reader).
- [(setf iterative-quadrature-h)], page 89 (writer).
- [iterative-quadrature-n], page 89 (reader).
- [(setf iterative-quadrature-n)], page 89 (writer).
- [iterative-quadrature-p], page 89 (function).
- [iterative-quadrature-sum], page 89 (reader).
- [(setf iterative-quadrature-sum)], page 89 (writer).
- [make-iterative-quadrature], page 90 (function).
- [midpoint-quadrature], page 91 (function).
- [midpoint-quadrature], page 100 (structure).
- [midpoint-quadrature%], page 91 (function).
- [midpoint-quadrature-a], page 91 (function).
- [(setf midpoint-quadrature-a)], page 91 (function).
- [midpoint-quadrature-b], page 91 (function).
- [(setf midpoint-quadrature-b)], page 91 (function).
- [midpoint-quadrature-f], page 91 (function).
- [(setf midpoint-quadrature-f)], page 91 (function).
- [midpoint-quadrature-h], page 91 (function).
- [(setf midpoint-quadrature-h)], page 91 (function).
- [midpoint-quadrature-n], page 92 (function).
- [(setf midpoint-quadrature-n)], page 92 (function).
- [midpoint-quadrature-p], page 92 (function).

- [midpoint-quadrature-sum], page 92 (function).
- [(setf midpoint-quadrature-sum)], page 92 (function).
- [refine-quadrature], page 98 (generic function).
- [richardson-coefficient], page 98 (generic function).
- [richardson-extrapolation], page 93 (function).
- [richardson-extrapolation], page 100 (structure).
- [richardson-extrapolation-coefficient], page 93 (reader).
- [(setf richardson-extrapolation-coefficient)], page 93 (writer).
- [richardson-extrapolation-diagonal], page 93 (reader).
- [(setf richardson-extrapolation-diagonal)], page 93 (writer).
- [richardson-extrapolation-n], page 93 (reader).
- [(setf richardson-extrapolation-n)], page 93 (writer).
- [richardson-extrapolation-p], page 93 (function).
- [richardson-iteration], page 93 (function).
- [romberg-quadrature%], page 94 (function).
- [transformed-quadrature], page 99 (generic function).
- [trapezoidal-quadrature], page 95 (function).
- [trapezoidal-quadrature], page 101 (structure).
- [trapezoidal-quadrature%], page 95 (function).
- [trapezoidal-quadrature-a], page 95 (function).
- [(setf trapezoidal-quadrature-a)], page 95 (function).
- [trapezoidal-quadrature-b], page 95 (function).
- [(setf trapezoidal-quadrature-b)], page 96 (function).
- [trapezoidal-quadrature-f], page 96 (function).
- [(setf trapezoidal-quadrature-f)], page 96 (function).
- [trapezoidal-quadrature-h], page 96 (function).
- [(setf trapezoidal-quadrature-h)], page 96 (function).
- [trapezoidal-quadrature-n], page 96 (function).
- [(setf trapezoidal-quadrature-n)], page 96 (function).
- [trapezoidal-quadrature-p], page 96 (function).
- [trapezoidal-quadrature-sum], page 96 (function).
- [(setf trapezoidal-quadrature-sum)], page 96 (function).

### 2.1.17 num-utils/log-exp.lisp

#### Dependency

[quadrature.lisp], page 17 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

- [exp-1], page 47 (function).
- [exp-1/x], page 47 (function).
- [expt-1], page 47 (function).

- [hypot], page 48 (function).
- [log1+], page 49 (function).
- [log1+/x], page 49 (function).
- [log1+exp], page 49 (function).
- [log1-], page 49 (function).
- [log1-exp], page 49 (function).
- [log1pmx], page 49 (function).
- [log2-exp], page 50 (function).
- [logexp-1], page 50 (function).

### 2.1.18 num-utils/test-utilities.lisp

#### Dependency

[log-exp.lisp], page 19 (file).

#### Source

[num-utils.asd], page 5.

#### Parent Component

[num-utils], page 3 (system).

#### Public Interface

- [compare-fns], page 44 (function).
- [compare-vectors], page 44 (function).
- [max-error], page 50 (reader).
- [(setf max-error)], page 50 (writer).
- [mean-error], page 50 (reader).
- [(setf mean-error)], page 50 (writer).
- [min-error], page 50 (reader).
- [(setf min-error)], page 50 (writer).
- [rms], page 52 (reader).
- [(setf rms)], page 52 (writer).
- [test-count], page 54 (reader).
- [(setf test-count)], page 54 (writer).
- [test-fn], page 54 (function).
- [test-results], page 78 (structure).
- [variance0], page 54 (reader).
- [(setf variance0)], page 54 (writer).
- [variance1], page 55 (reader).
- [(setf variance1)], page 55 (writer).
- [worst-case], page 55 (reader).
- [(setf worst-case)], page 55 (writer).

#### Internals

- [copy-test-results], page 88 (function).
- [make-test-results], page 90 (function).
- [test-results-p], page 95 (function).

### 2.1.19 num-utils/pkgdcl.lisp

**Dependency**

[test-utilities.lisp], page 19 (file).

**Source**

[num-utils.asd], page 5.

**Parent Component**

[num-utils], page 3 (system).

**Packages**

[num-utils], page 36.



## 3 Packages

Packages are listed by definition order.

### 3.1 num-utils.quadrature

**Source** [packages.lisp], page 5.

#### Use List

- alexandria.
- anaphora.
- common-lisp.
- let-plus.
- [num-utils.arithmetic], page 34.
- [num-utils.interval], page 24.
- [num-utils.utilities], page 30.

#### Used By List

- [num-utils], page 36.
- num-utils-tests.

#### Public Interface

[romberg-quadrature], page 52 (function).

#### Internals

- [copy-iterative-quadrature], page 87 (function).
- [copy-midpoint-quadrature], page 87 (function).
- [copy-richardson-extrapolation], page 87 (function).
- [copy-trapezoidal-quadrature], page 88 (function).
- [iterative-quadrature], page 99 (structure).
- [iterative-quadrature-a], page 89 (reader).
- [(setf iterative-quadrature-a)], page 89 (writer).
- [iterative-quadrature-b], page 89 (reader).
- [(setf iterative-quadrature-b)], page 89 (writer).
- [iterative-quadrature-f], page 89 (reader).
- [(setf iterative-quadrature-f)], page 89 (writer).
- [iterative-quadrature-h], page 89 (reader).
- [(setf iterative-quadrature-h)], page 89 (writer).
- [iterative-quadrature-n], page 89 (reader).
- [(setf iterative-quadrature-n)], page 89 (writer).
- [iterative-quadrature-p], page 89 (function).
- [iterative-quadrature-sum], page 89 (reader).
- [(setf iterative-quadrature-sum)], page 89 (writer).
- [make-iterative-quadrature], page 90 (function).
- [midpoint-quadrature], page 91 (function).
- [midpoint-quadrature], page 100 (structure).
- [midpoint-quadrature%], page 91 (function).

- [midpoint-quadrature-a], page 91 (function).
- [(setf midpoint-quadrature-a)], page 91 (function).
- [midpoint-quadrature-b], page 91 (function).
- [(setf midpoint-quadrature-b)], page 91 (function).
- [midpoint-quadrature-f], page 91 (function).
- [(setf midpoint-quadrature-f)], page 91 (function).
- [midpoint-quadrature-h], page 91 (function).
- [(setf midpoint-quadrature-h)], page 91 (function).
- [midpoint-quadrature-n], page 92 (function).
- [(setf midpoint-quadrature-n)], page 92 (function).
- [midpoint-quadrature-p], page 92 (function).
- [midpoint-quadrature-sum], page 92 (function).
- [(setf midpoint-quadrature-sum)], page 92 (function).
- [refine-quadrature], page 98 (generic function).
- [richardson-coefficient], page 98 (generic function).
- [richardson-extrapolation], page 93 (function).
- [richardson-extrapolation], page 100 (structure).
- [richardson-extrapolation-coefficient], page 93 (reader).
- [(setf richardson-extrapolation-coefficient)], page 93 (writer).
- [richardson-extrapolation-diagonal], page 93 (reader).
- [(setf richardson-extrapolation-diagonal)], page 93 (writer).
- [richardson-extrapolation-n], page 93 (reader).
- [(setf richardson-extrapolation-n)], page 93 (writer).
- [richardson-extrapolation-p], page 93 (function).
- [richardson-iteration], page 93 (function).
- [romberg-quadrature%], page 94 (function).
- [transformed-quadrature], page 99 (generic function).
- [trapezoidal-quadrature], page 95 (function).
- [trapezoidal-quadrature], page 101 (structure).
- [trapezoidal-quadrature%], page 95 (function).
- [trapezoidal-quadrature-a], page 95 (function).
- [(setf trapezoidal-quadrature-a)], page 95 (function).
- [trapezoidal-quadrature-b], page 95 (function).
- [(setf trapezoidal-quadrature-b)], page 96 (function).
- [trapezoidal-quadrature-f], page 96 (function).
- [(setf trapezoidal-quadrature-f)], page 96 (function).
- [trapezoidal-quadrature-h], page 96 (function).
- [(setf trapezoidal-quadrature-h)], page 96 (function).
- [trapezoidal-quadrature-n], page 96 (function).
- [(setf trapezoidal-quadrature-n)], page 96 (function).
- [trapezoidal-quadrature-p], page 96 (function).
- [trapezoidal-quadrature-sum], page 96 (function).
- [(setf trapezoidal-quadrature-sum)], page 96 (function).

## 3.2 num-utils.print-matrix

**Source** [packages.lisp], page 5.

**Use List**

- alexandria.
- anaphora.
- common-lisp.
- let-plus.

**Used By List**

- lisp-stat.
- [num-utils.matrix], page 26.

**Public Interface**

- [\*print-matrix-precision\*], page 39 (special variable).
- [print-length-truncate], page 51 (function).
- [print-matrix], page 51 (function).

**Internals** [print-matrix-formatter], page 92 (function).

## 3.3 num-utils.chebyshev

**Source** [packages.lisp], page 5.

**Use List**

- alexandria.
- anaphora.
- common-lisp.
- let-plus.
- [num-utils.interval], page 24.
- [num-utils.utilities], page 30.

**Used By List**

- [num-utils], page 36.
- num-utils-tests.

**Public Interface**

- [chebyshev-approximate], page 43 (function).
- [chebyshev-regression], page 44 (function).
- [chebyshev-root], page 44 (function).
- [chebyshev-roots], page 44 (function).
- [evaluate-chebyshev], page 46 (function).

**Internals**

- [ab-to-cd-intercept-slope], page 85 (function).
- [ab-to-cinf], page 85 (function).
- [chebyshev-approximate-implementation], page 97 (generic function).
- [chebyshev-recursion], page 86 (function).
- [cinf-to-ab], page 87 (function).

### 3.4 num-utils.interval

**Source** [packages.lisp], page 5.

**Use List**

- alexandria.
- anaphora.
- common-lisp.
- let-plus.
- [num-utils.num=], page 35.
- [num-utils.utilities], page 30.

**Used By List**

- [num-utils], page 36.
- num-utils-tests.
- [num-utils.chebyshev], page 23.
- [num-utils.quadrature], page 21.
- [num-utils.rootfinding], page 36.

**Public Interface**

- [&interval], page 39 (macro).
- [extend-interval], page 66 (generic function).
- [extendf-interval], page 40 (macro).
- [finite-interval], page 81 (class).
- [grid-in], page 47 (function).
- [in-interval?], page 48 (function).
- [interval], page 48 (function).
- [interval], page 81 (class).
- [interval-hull], page 48 (function).
- [interval-length], page 48 (function).
- [interval-midpoint], page 48 (function).
- [left], page 67 (generic function).
- [minusinf-interval], page 82 (class).
- [open-left?], page 69 (generic function).
- [open-right?], page 69 (generic function).
- [plusinf-interval], page 82 (class).
- [plusminus-interval], page 51 (function).
- [real-line], page 82 (class).
- [relative], page 52 (function).
- [relative], page 77 (structure).
- [right], page 70 (generic function).
- [shift-interval], page 70 (generic function).
- [shrink-interval], page 53 (function).
- [spacer], page 53 (function).
- [spacer], page 78 (structure).
- [split-interval], page 53 (function).

- [subintervals-in], page 54 (function).

#### Internals

- [copy-relative], page 87 (function).
- [copy-spacer], page 87 (function).
- [interval/finite-left], page 102 (class).
- [interval/finite-right], page 102 (class).
- [interval/infinite-left], page 103 (class).
- [interval/infinite-right], page 103 (class).
- [print-left-endpoint], page 98 (generic function).
- [print-right-endpoint], page 98 (generic function).
- [relative-fraction], page 93 (reader).
- [relative-p], page 93 (function).
- [spacer-p], page 95 (function).
- [spacer-weight], page 95 (reader).

### 3.5 num-utils.matrix-shorthand

**Source** [packages.lisp], page 5.

**Nickname** nu.mx

#### Use List

- alexandria.
- anaphora.
- common-lisp.
- let-plus.
- [num-utils.matrix], page 26.
- [num-utils.utilities], page 30.

#### Used By List

num-utils-tests.

#### Public Interface

- [diagonal-mx], page 45 (function).
- [hermitian-mx], page 40 (macro).
- [lower-triangular-mx], page 41 (macro).
- [mx], page 41 (macro).
- [upper-triangular-mx], page 41 (macro).
- [vec], page 55 (function).

**Internals** [pad-left-expansion], page 92 (function).

### 3.6 num-utils.log-exp

**Source** [packages.lisp], page 5.

#### Use List

- common-lisp.
- let-plus.

**Used By List**

- `[num-utils]`, page 36.
- `num-utils-tests`.

**Public Interface**

- `[exp-1]`, page 47 (function).
- `[exp-1/x]`, page 47 (function).
- `[expt-1]`, page 47 (function).
- `[hypot]`, page 48 (function).
- `[log1+]`, page 49 (function).
- `[log1+/x]`, page 49 (function).
- `[log1+exp]`, page 49 (function).
- `[log1-]`, page 49 (function).
- `[log1-exp]`, page 49 (function).
- `[log1pmx]`, page 49 (function).
- `[log2-exp]`, page 50 (function).
- `[logexp-1]`, page 50 (function).

**3.7 num-utils.matrix**

**Source**      `[packages.lisp]`, page 5.

**Use List**

- `alexandria`.
- `anaphora`.
- `common-lisp`.
- `let-plus`.
- `[num-utils.elementwise]`, page 28.
- `[num-utils.num=]`, page 35.
- `[num-utils.print-matrix]`, page 23.
- `[num-utils.utilities]`, page 30.
- `select`.

**Used By List**

- `[num-utils]`, page 36.
- `num-utils-tests`.
- `[num-utils.matrix-shorthand]`, page 25.

**Public Interface**

- `[diagonal-matrix]`, page 45 (function).
- `[diagonal-matrix]`, page 75 (structure).
- `[diagonal-matrix-elements]`, page 45 (reader).
- `[(setf diagonal-matrix-elements)]`, page 45 (writer).
- `[diagonal-vector]`, page 57 (generic function).
- `[(setf diagonal-vector)]`, page 57 (generic function).
- `[hermitian-matrix]`, page 47 (function).
- `[hermitian-matrix]`, page 76 (structure).

- `[lower-triangular-matrix]`, page 50 (function).
- `[lower-triangular-matrix]`, page 76 (structure).
- `[map-array]`, page 67 (generic function).
- `[transpose]`, page 71 (generic function).
- `[triangular-matrix]`, page 83 (type).
- `[upper-triangular-matrix]`, page 54 (function).
- `[upper-triangular-matrix]`, page 80 (structure).
- `[wrapped-matrix]`, page 80 (structure).
- `[wrapped-matrix-elements]`, page 55 (reader).

### Internals

- `[&diagonal-matrix]`, page 83 (macro).
- `[&diagonal-matrix-r/o]`, page 83 (macro).
- `[above-diagonal?]`, page 85 (function).
- `[below-diagonal?]`, page 85 (function).
- `[copy-diagonal-matrix]`, page 87 (function).
- `[copy-hermitian-matrix]`, page 87 (function).
- `[copy-lower-triangular-matrix]`, page 87 (function).
- `[copy-upper-triangular-matrix]`, page 88 (function).
- `[copy-wrapped-matrix]`, page 88 (function).
- `[define-elementwise-as-array]`, page 84 (macro).
- `[define-elementwise-same-class]`, page 84 (macro).
- `[define-elementwise-univariate]`, page 84 (macro).
- `[define-elementwise-with-constant]`, page 84 (macro).
- `[define-wrapped-matrix]`, page 84 (macro).
- `[diagonal-matrix-p]`, page 88 (function).
- `[ensure-valid-elements]`, page 88 (function).
- `[hermitian-matrix-elements]`, page 88 (function).
- `[hermitian-matrix-p]`, page 88 (function).
- `[lower-triangular-matrix-elements]`, page 90 (function).
- `[lower-triangular-matrix-p]`, page 90 (function).
- `[make-diagonal-matrix]`, page 90 (function).
- `[make-hermitian-matrix]`, page 90 (function).
- `[make-lower-triangular-matrix]`, page 90 (function).
- `[make-upper-triangular-matrix]`, page 91 (function).
- `[make-wrapped-matrix]`, page 91 (function).
- `[upper-triangular-matrix-elements]`, page 96 (function).
- `[upper-triangular-matrix-p]`, page 96 (function).
- `[valid-sparse-type?]`, page 97 (function).
- `[wrapped-matrix-p]`, page 97 (function).
- `[zero-like]`, page 97 (function).

### 3.8 num-utils.elementwise

**Source** [packages.lisp], page 5.

**Nickname** elmt

#### Use List

- alexandria.
- common-lisp.
- let-plus.
- [num-utils.arithmetic], page 34.
- [num-utils.utilities], page 30.

#### Used By List

- lisp-stat.
- [num-utils], page 36.
- num-utils-tests.
- [num-utils.matrix], page 26.

#### Public Interface

- [e\*], page 45 (function).
- [e+], page 45 (function).
- [e-], page 45 (function).
- [e/], page 45 (function).
- [e1-], page 58 (generic function).
- [e1/], page 58 (generic function).
- [e1log], page 58 (generic function).
- [e2\*], page 59 (generic function).
- [e2+], page 60 (generic function).
- [e2-], page 61 (generic function).
- [e2/], page 62 (generic function).
- [e2log], page 63 (generic function).
- [eceiling], page 63 (generic function).
- [econjugate], page 63 (generic function).
- [ecos], page 63 (generic function).
- [eexp], page 64 (generic function).
- [eexpt], page 64 (generic function).
- [efloor], page 64 (generic function).
- [elementwise-float-contagion], page 45 (function).
- [elog], page 46 (function).
- [emax], page 46 (function).
- [emin], page 46 (function).
- [emod], page 65 (generic function).
- [ereduce], page 65 (generic function).
- [esin], page 65 (generic function).
- [esqrt], page 66 (generic function).



**Internals**

- `[define-e&]`, page 84 (macro).
- `[define-e1]`, page 84 (macro).
- `[define-e2]`, page 84 (macro).
- `[define-elementwise-reduction]`, page 84 (macro).
- `[esquare]`, page 97 (generic function).
- `[mapping-array]`, page 85 (macro).

**3.9 num-utils.extended-real**

**Source** `[packages.lisp]`, page 5.

**Nickname** `xreal`

**Use List**

- `alexandria`.
- `common-lisp`.

**Public Interface**

- `[<]`, page 42 (function).
- `[<=]`, page 42 (function).
- `[=]`, page 42 (function).
- `[>]`, page 42 (function).
- `[>=]`, page 42 (function).
- `[extended-real]`, page 82 (type).
- `[infinite?]`, page 48 (function).
- `[lambda-template]`, page 41 (macro).
- `[with-template]`, page 42 (macro).

**Internals**

- `[define-comparison]`, page 83 (macro).
- `[extend-pairwise-comparison]`, page 88 (function).
- `[infinite]`, page 103 (type).

**3.10 num-utils.utilities**

A collection of utilities to work with floating point values. Optimised for double-float.

**Source** `[packages.lisp]`, page 5.

**Use List**

- `alexandria`.
- `anaphora`.
- `common-lisp`.
- `let-plus`.

**Used By List**

- `lisp-stat`.
- `[num-utils]`, page 36.
- `num-utils-tests`.

- `[num-utils.arithmetic]`, page 34.
- `[num-utils.chebyshev]`, page 23.
- `[num-utils.elementwise]`, page 28.
- `[num-utils.interval]`, page 24.
- `[num-utils.matrix]`, page 26.
- `[num-utils.matrix-shorthand]`, page 25.
- `[num-utils.polynomial]`, page 35.
- `[num-utils.quadrature]`, page 21.
- `[num-utils.rootfinding]`, page 36.
- `[num-utils.statistics]`, page 31.

### Public Interface

- `[as-alist]`, page 56 (generic function).
- `[as-double-float]`, page 43 (function).
- `[as-plist]`, page 56 (generic function).
- `[as-simple-fixnum-vector]`, page 43 (function).
- `[bic]`, page 43 (function).
- `[binary-search]`, page 43 (function).
- `[check-types]`, page 39 (macro).
- `[curry*]`, page 40 (macro).
- `[define-with-multiple-bindings]`, page 40 (macro).
- `[expanding]`, page 40 (macro).
- `[fixnum?]`, page 47 (function).
- `[generate-sequence]`, page 47 (function).
- `[gethash*]`, page 40 (macro).
- `[make-vector]`, page 42 (compiler macro).
- `[make-vector]`, page 50 (function).
- `[sequencep]`, page 53 (function).
- `[simple-double-float-vector]`, page 82 (type).
- `[simple-fixnum-vector]`, page 83 (type).
- `[simple-single-float-vector]`, page 83 (type).
- `[splice-when]`, page 41 (macro).
- `[splice-when]`, page 41 (macro).
- `[with-double-floats]`, page 41 (macro).
- `[within?]`, page 55 (function).

## 3.11 num-utils.statistics

**Source**     `[packages.lisp]`, page 5.

**Nickname**   `nu.stats`

### Use List

- `alexandria`.
- `anaphora`.
- `common-lisp`.

- `let-plus`.
- `[num-utils.arithmetic]`, page 34.
- `[num-utils.num=]`, page 35.
- `[num-utils.utilities]`, page 30.

#### Used By List

- `lisp-stat`.
- `[num-utils]`, page 36.
- `num-utils-tests`.

#### Public Interface

- `[*central-sample-moments-default-degree*]`, page 39 (special variable).
- `[add]`, page 55 (generic function).
- `[central-m2]`, page 56 (generic function).
- `[central-m3]`, page 56 (generic function).
- `[central-m4]`, page 57 (generic function).
- `[central-sample-moments]`, page 57 (generic function).
- `[central-sample-moments]`, page 74 (structure).
- `[central-sample-moments-degree]`, page 43 (function).
- `[cross-tabulate]`, page 44 (function).
- `[empirical-quantile]`, page 46 (function).
- `[empirical-quantile-probabilities]`, page 46 (function).
- `[empty-accumulator]`, page 73 (condition).
- `[ensure-sorted-reals]`, page 65 (generic function).
- `[ensure-sorted-vector]`, page 46 (function).
- `[information-not-collected-in-accumulator]`, page 73 (condition).
- `[kurtosis]`, page 66 (generic function).
- `[make-sparse-counter]`, page 50 (function).
- `[mean]`, page 67 (generic function).
- `[median]`, page 67 (generic function).
- `[not-enough-elements-in-accumulator]`, page 73 (condition).
- `[pool]`, page 51 (function).
- `[quantile]`, page 69 (generic function).
- `[quantiles]`, page 70 (generic function).
- `[sd]`, page 70 (generic function).
- `[skewness]`, page 70 (generic function).
- `[sorted-reals]`, page 77 (structure).
- `[sorted-reals-elements]`, page 53 (function).
- `[sparse-counter]`, page 78 (structure).
- `[sparse-counter-count]`, page 53 (function).
- `[sparse-counter-table]`, page 53 (reader).
- `[tabulate]`, page 54 (function).
- `[tally]`, page 71 (generic function).
- `[variance]`, page 71 (generic function).
- `[weighted-quantiles]`, page 55 (function).

## Internals

- [`&sorted-reals`], page 83 (macro).
- [`&sorted-reals-r/o`], page 83 (macro).
- [`central-sample-moments-m`], page 86 (reader).
- [`(setf central-sample-moments-m)`], page 86 (writer).
- [`central-sample-moments-p`], page 86 (function).
- [`central-sample-moments-s2`], page 86 (reader).
- [`(setf central-sample-moments-s2)`], page 86 (writer).
- [`central-sample-moments-s3`], page 86 (reader).
- [`(setf central-sample-moments-s3)`], page 86 (writer).
- [`central-sample-moments-s4`], page 86 (reader).
- [`(setf central-sample-moments-s4)`], page 86 (writer).
- [`central-sample-moments-w`], page 86 (function).
- [`(setf central-sample-moments-w)`], page 86 (function).
- [`copy-central-sample-moments`], page 87 (function).
- [`copy-sorted-reals`], page 87 (function).
- [`copy-sparse-counter`], page 87 (function).
- [`copy-tally-mixin`], page 88 (function).
- [`define-central-sample-moment`], page 83 (macro).
- [`make-central-sample-moments`], page 90 (function).
- [`make-sorted-reals`], page 90 (function).
- [`make-sparse-counter%`], page 90 (function).
- [`make-tally-mixin`], page 90 (function).
- [`pool2`], page 98 (generic function).
- [`sort-reals`], page 94 (function).
- [`sorted-reals-ordered-elements`], page 94 (reader).
- [`(setf sorted-reals-ordered-elements)`], page 94 (writer).
- [`sorted-reals-p`], page 94 (function).
- [`sorted-reals-unordered-elements`], page 94 (reader).
- [`(setf sorted-reals-unordered-elements)`], page 94 (writer).
- [`sparse-counter-p`], page 95 (function).
- [`tally-mixin`], page 101 (structure).
- [`tally-mixin-p`], page 95 (function).
- [`tally-mixin-w`], page 95 (reader).
- [`(setf tally-mixin-w)`], page 95 (writer).
- [`weighted-empirical-quantile`], page 97 (function).
- [`weighted-quantile-p-table`], page 97 (function).

## 3.12 num-utils.test-utilities

**Source**      [`packages.lisp`], page 5.

**Use List**    `common-lisp`.

**Used By List**

- [`num-utils`], page 36.

- `num-utils-tests`.

#### Public Interface

- `[compare-fns]`, page 44 (function).
- `[compare-vectors]`, page 44 (function).
- `[max-error]`, page 50 (reader).
- `[(setf max-error)]`, page 50 (writer).
- `[mean-error]`, page 50 (reader).
- `[(setf mean-error)]`, page 50 (writer).
- `[min-error]`, page 50 (reader).
- `[(setf min-error)]`, page 50 (writer).
- `[rms]`, page 52 (reader).
- `[(setf rms)]`, page 52 (writer).
- `[test-count]`, page 54 (reader).
- `[(setf test-count)]`, page 54 (writer).
- `[test-fn]`, page 54 (function).
- `[test-results]`, page 78 (structure).
- `[variance0]`, page 54 (reader).
- `[(setf variance0)]`, page 54 (writer).
- `[variance1]`, page 55 (reader).
- `[(setf variance1)]`, page 55 (writer).
- `[worst-case]`, page 55 (reader).
- `[(setf worst-case)]`, page 55 (writer).

#### Internals

- `[copy-test-results]`, page 88 (function).
- `[make-test-results]`, page 90 (function).
- `[test-results-p]`, page 95 (function).

### 3.13 `num-utils.arithmetic`

**Source**      `[packages.lisp]`, page 5.

#### Use List

- `alexandria-2`.
- `anaphora`.
- `common-lisp`.
- `let-plus`.
- `[num-utils.utilities]`, page 30.

#### Used By List

- `lisp-stat`.
- `[num-utils]`, page 36.
- `num-utils-tests`.
- `[num-utils.elementwise]`, page 28.
- `[num-utils.quadrature]`, page 21.
- `[num-utils.statistics]`, page 31.

- `special-functions`.

#### Public Interface

- `[1c]`, page 42 (function).
- `[abs-diff]`, page 42 (function).
- `[absolute-square]`, page 42 (function).
- `[as-integer]`, page 43 (function).
- `[ceiling*]`, page 43 (function).
- `[cube]`, page 44 (function).
- `[cumulative-product]`, page 44 (function).
- `[cumulative-sum]`, page 44 (function).
- `[divides?]`, page 45 (function).
- `[floor*]`, page 47 (function).
- `[ivec]`, page 48 (function).
- `[l2norm]`, page 49 (function).
- `[l2norm-square]`, page 67 (generic function).
- `[log10]`, page 49 (function).
- `[log2]`, page 49 (function).
- `[multf]`, page 41 (macro).
- `[normalize-probabilities]`, page 51 (function).
- `[numseq]`, page 51 (function).
- `[product]`, page 69 (generic function).
- `[round*]`, page 52 (function).
- `[same-sign-p]`, page 52 (function).
- `[sequence-maximum]`, page 53 (function).
- `[sequence-minimum]`, page 53 (function).
- `[square]`, page 54 (function).
- `[sum]`, page 100 (slot).
- `[sum]`, page 71 (generic function).
- `[truncate*]`, page 54 (function).

#### Internals

- `[define-rounding-with-offset]`, page 84 (macro).
- `[ln]`, page 90 (function).
- `[similar-element-type]`, page 94 (function).
- `[similar-sequence-type]`, page 94 (function).

### 3.14 `num-utils.num=`

**Source**      `[packages.lisp]`, page 5.

#### Use List

- `alexandria`.
- `anaphora`.
- `common-lisp`.
- `let-plus`.

**Used By List**

- [num-utils], page 36.
- num-utils-tests.
- [num-utils.interval], page 24.
- [num-utils.matrix], page 26.
- [num-utils.statistics], page 31.

**Public Interface**

- [\*num=-tolerance\*], page 39 (special variable).
- [define-num=-with-accessors], page 40 (macro).
- [define-structure-num=], page 40 (macro).
- [num-delta], page 51 (function).
- [num=], page 68 (generic function).
- [num=-function], page 51 (function).

**3.15 num-utils.polynomial**

Efficient evaluation of polynomial functions using Horner's method

**Source** [packages.lisp], page 5.

**Nickname** poly

**Use List**

- alexandria.
- common-lisp.
- [num-utils.utilities], page 30.

**Used By List**

- num-utils-tests.
- special-functions.

**Public Interface**

[evaluate-polynomial], page 46 (function).

**3.16 num-utils**

Numerical utilities for Lisp-Stat

**Source** [pkgdcl.lisp], page 20.

**Nickname** nu

**Use List**

- common-lisp.
- [num-utils.arithmetic], page 34.
- [num-utils.chebyshev], page 23.
- [num-utils.elementwise], page 28.
- [num-utils.interval], page 24.
- [num-utils.log-exp], page 26.
- [num-utils.matrix], page 26.
- [num-utils.num=], page 35.
- [num-utils.quadrature], page 21.

- `[num-utils.rootfinding]`, page 36.
- `[num-utils.statistics]`, page 31.
- `[num-utils.test-utilities]`, page 33.
- `[num-utils.utilities]`, page 30.

### 3.17 `num-utils.rootfinding`

**Source**      `[packages.lisp]`, page 5.

**Use List**

- `alexandria`.
- `common-lisp`.
- `let-plus`.
- `[num-utils.interval]`, page 24.
- `[num-utils.utilities]`, page 30.

**Used By List**

- `[num-utils]`, page 36.
- `num-utils-tests`.

**Public Interface**

- `[*rootfinding-delta-relative*]`, page 39 (special variable).
- `[*rootfinding-epsilon*]`, page 39 (special variable).
- `[root-bisection]`, page 52 (function).

**Internals**

- `[narrow-bracket?]`, page 92 (function).
- `[near-root?]`, page 92 (function).
- `[opposite-sign?]`, page 92 (function).
- `[rootfinding-delta]`, page 94 (function).
- `[univariate-rootfinder-loop%]`, page 85 (macro).



## 4 Definitions

Definitions are sorted by export status, category, package, and then by lexicographic order.

### 4.1 Public Interface

#### 4.1.1 Special variables

**\*central-sample-moments-default-degree\*** [Special Variable]

Default degree for (weighted) central sample moments.

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**\*num=-tolerance\*** [Special Variable]

Default tolerance for NUM=.

**Package** [num-utils.num=], page 35.

**Source** [num=.lisp], page 6.

**\*print-matrix-precision\*** [Special Variable]

Number of digits after the decimal point when printing numeric matrices.

**Package** [num-utils.print-matrix], page 23.

**Source** [print-matrix.lisp], page 10.

**\*rootfinding-delta-relative\*** [Special Variable]

Default relative interval width for rootfinding.

**Package** [num-utils.rootfinding], page 36.

**Source** [rootfinding.lisp], page 16.

**\*rootfinding-epsilon\*** [Special Variable]

Default maximum for the absolute value of the function, used for rootfinding.

**Package** [num-utils.rootfinding], page 36.

**Source** [rootfinding.lisp], page 16.

#### 4.1.2 Macros

**&interval** (*left right*) [Macro]

LET+ expansion for interval endpoints. If given a list of two values, the second value is an indicator for whether the endpoint is open.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**check-types** ((*&rest arguments*) *type*) [Macro]

CHECK-TYPE for multiple places of the same type. Each argument is either a place, or a list of a place and a type-string.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**curry\*** (*function &rest arguments*) [Macro]

Currying in all variables that are not \*. Note that this is a macro, so \* should not be quoted, and FUNCTION will be used as is, ie it can be a LAMBDA form.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**define-num=-with-accessors** (*class accessors*) [Macro]

Define a method for NUM=, specialized to the given class, comparing values obtained with accessors.

**Package** [num-utils.num=], page 35.

**Source** [num=.lisp], page 6.

**define-structure-num=** (*structure &rest slots*) [Macro]

Define a NUM= method for the given structure, comparing the given slots.

**Package** [num-utils.num=], page 35.

**Source** [num=.lisp], page 6.

**define-with-multiple-bindings** (*macro &key plural docstring*) [Macro]

Define a version of MACRO with multiple arguments, given as a list. Application of MACRO will be nested. The new name is the plural of the old one (generated using format by default).

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**expanding** (**&body** *body*) [Macro]

Expand BODY. Useful for generating code programmatically.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**extendf-interval** (*place object*) [Macro]

Apply EXTEND-INTERVAL on PLACE using OBJECT.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**gethash\*** (*key hash-table &optional datum &rest arguments*) [Macro]

Like GETHASH, but checking that KEY is present and raising the given error if not.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**hermitian-mx** (*element-type &body rows*) [Macro]

Macro for creating a lower triangular matrix. ROWS should be a list of lists, elements are evaluated. Masked elements (above the diagonal) are ignored at the expansion, rows which don't have enough elements are padded with zeros.

**Package** [num-utils.matrix-shorthand], page 25.

**Source** [matrix-shorthand.lisp], page 13.

**lambda-template** (*((prefix &rest variables) &body body)*) [Macro]  
 LAMBDA with WITH-TEMPLATE in its BODY.

**Package** [num-utils.extended-real], page 29.

**Source** [extended-real.lisp], page 8.

**lower-triangular-mx** (*element-type &body rows*) [Macro]

Macro for creating a lower triangular matrix. ROWS should be a list of lists, elements are evaluated. Masked elements (above the diagonal) are ignored at the expansion, rows which don't have enough elements are padded with zeros.

**Package** [num-utils.matrix-shorthand], page 25.

**Source** [matrix-shorthand.lisp], page 13.

**multf** (*place coefficient*) [Macro]

Multiply place by COEFFICIENT.

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**mx** (*element-type &body rows*) [Macro]

Macro for creating a (dense) matrix (ie a rank 2 array). ROWS should be a list of lists (or atoms, which are treated as lists), elements are evaluated.

**Package** [num-utils.matrix-shorthand], page 25.

**Source** [matrix-shorthand.lisp], page 13.

**splice-when** (*test &body forms*) [Macro]

Similar to splice-when, but binds IT to test.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**splice-when** (*test &body forms*) [Macro]

Similar to when, but wraps the result in list.

Example: '(,foo ,@(splice-when add-bar? bar))

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**upper-triangular-mx** (*element-type &body rows*) [Macro]

Macro for creating an upper triangular matrix. ROWS should be a list of lists, elements are evaluated. Masked elements (below the diagonal) are ignored at the expansion.

**Package** [num-utils.matrix-shorthand], page 25.

**Source** [matrix-shorthand.lisp], page 13.

**with-double-floats** (*bindings &body body*) [Macro]

For each binding = (variable value), coerce VALUE to DOUBLE-FLOAT and bind it to VARIABLE for BODY. When VALUE is omitted, VARIABLE is used instead. When BINDING is an atom, it is used for both the value and the variable.

Example:

```
(with-double-floats (a
  (b)
  (c 1))
...)
```

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**with-template** ((*prefix* &rest *variables*) &body *body*) [Macro]

Define the function (PREFIX &rest VARIABLES) which can be used to match variables using :PLUSINF, :MINUSINF, REAL, or T.

**Package** [num-utils.extended-real], page 29.

**Source** [extended-real.lisp], page 8.

### 4.1.3 Compiler macros

**make-vector** (*element-type* &rest *initial-contents*) [Compiler Macro]

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

### 4.1.4 Ordinary functions

**1c** (*number*) [Function]

Return 1-number. The mnemonic is "1 complement", 1- is already a CL library function.

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**<** (*number* &rest *more-numbers*) [Function]

**Package** [num-utils.extended-real], page 29.

**Source** [extended-real.lisp], page 8.

**<=** (*number* &rest *more-numbers*) [Function]

**Package** [num-utils.extended-real], page 29.

**Source** [extended-real.lisp], page 8.

**=** (*number* &rest *more-numbers*) [Function]

**Package** [num-utils.extended-real], page 29.

**Source** [extended-real.lisp], page 8.

**>** (*number* &rest *more-numbers*) [Function]

**Package** [num-utils.extended-real], page 29.

**Source** [extended-real.lisp], page 8.

**>=** (*number* &rest *more-numbers*) [Function]

**Package** [num-utils.extended-real], page 29.

**Source** [extended-real.lisp], page 8.

**abs-diff** (*a b*) [Function]

Absolute difference of A and B.

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**absolute-square** (*number*) [Function]

Number multiplied by its complex conjugate.

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**as-double-float** (*v*) [Function]

Convert argument to DOUBLE-FLOAT.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**as-integer** (*number*) [Function]

If NUMBER represents an integer (as an integer, complex, or float, etc), return it as an integer, otherwise signal an error. Floats are converted with RATIONALIZE.

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**as-simple-fixnum-vector** (*sequence &optional copy?*) [Function]

Convert SEQUENCE to a SIMPLE-FIXNUM-VECTOR. When COPY?, make sure that the they don't share structure.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**bic** (*a b*) [Function]

Biconditional. Returns  $A \iff B$ .

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**binary-search** (*sorted-reals value*) [Function]

Return INDEX such that

(WITHIN? (AREF SORTED-REALS INDEX) VALUE (AREF SORTED-REALS (1+ INDEX))).

SORTED-REALS is assumed to be reals sorted in ascending order (not checked, if this does not hold the result may be nonsensical, though the algorithm will terminate).

If value is below (or above) the first (last) break, NIL (T) is returned.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**ceiling\*** (*number &optional divisor offset*) [Function]

Find the lowest  $A = I * \text{DIVISOR} + \text{OFFSET} \geq \text{NUMBER}$ , return (values A (- A NUMBER)).

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**central-sample-moments-degree** (*central-sample-moments*) [Function]

Return the degree of CENTRAL-SAMPLE-MOMENTS.

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**chebyshev-approximate** (*f interval n-polynomials &key n-points*) [Function]

Return a closure approximating F on the given INTERVAL (may be infinite on either end) using the given number of Chebyshev polynomials.

**Package** [num-utils.chebyshev], page 23.

**Source** [chebyshev.lisp], page 16.

- chebyshev-regression** (*f n-polynomials &optional n-points*) [Function]  
 Chebyshev polynomial regression using the given number of polynomials and points (zeroes of the corresponding Chebyshev polynomial).  
**Package** [num-utils.chebyshev], page 23.  
**Source** [chebyshev.lisp], page 16.
- chebyshev-root** (*m i*) [Function]  
 Return the iTH root of the Mth Chebyshev polynomial as double-float.  
**Package** [num-utils.chebyshev], page 23.  
**Source** [chebyshev.lisp], page 16.
- chebyshev-roots** (*m*) [Function]  
 Return the roots of the Mth Chebyshev polynomial as a vector of double-floats.  
**Package** [num-utils.chebyshev], page 23.  
**Source** [chebyshev.lisp], page 16.
- compare-fns** (*fn-1 fn-2 &rest fn-params*) [Function]  
 Compare the values returned by two functions  
**Package** [num-utils.test-utilities], page 33.  
**Source** [test-utilities.lisp], page 19.
- compare-vectors** (*reference-values computed-values*) [Function]  
 Compare two vectors containing the results of previous computations  
**Package** [num-utils.test-utilities], page 33.  
**Source** [test-utilities.lisp], page 19.
- cross-tabulate** (*sequence1 sequence2 &key test*) [Function]  
 Cross-tabulate two sequences (using a SPARSE-COUNTER with the given TEST). TEST is used to compare conses.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- cube** (*number*) [Function]  
 Cube of number.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- cumulative-product** (*sequence &key result-type*) [Function]  
 Cumulative product of sequence. Return a sequence of the same kind and length; last element is the total product. The latter is also returned as the second value.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- cumulative-sum** (*sequence &key result-type*) [Function]  
 Cumulative sum of sequence. Return a sequence of the same kind and length; last element is the total. The latter is returned as the second value.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.

- diagonal-matrix** (*elements*) [Function]  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.
- diagonal-matrix-elements** (*instance*) [Reader]  
 (setf **diagonal-matrix-elements**) (*instance*) [Writer]  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.  
**Target Slot**  
 [elements], page 75.
- diagonal-mx** (*element-type &rest elements*) [Function]  
 Return a DIAGONAL-MATRIX with elements coerced to ELEMENT-TYPE.  
**Package** [num-utils.matrix-shorthand], page 25.  
**Source** [matrix-shorthand.lisp], page 13.
- divides?** (*number divisor*) [Function]  
 Test if DIVISOR divides NUMBER without remainder, and if so, return the quotient. Works generally, but makes most sense for rationals.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- e\*** (*argument &rest more-arguments*) [Function]  
 Elementwise \*.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- e+** (*argument &rest more-arguments*) [Function]  
 Elementwise +.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- e-** (*argument &rest more-arguments*) [Function]  
 Elementwise -.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- e/** (*argument &rest more-arguments*) [Function]  
 Elementwise /.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- elementwise-float-contagion** (*&rest objects*) [Function]  
 Return the resulting float type when objects (or their elements) are combined using arithmetic operations.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.

- elog** (*a* &**optional** *base*) [Function]  
 Elementwise logarithm.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- emax** (*object*) [Function]  
 Elementwise MAX.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- emin** (*object*) [Function]  
 Elementwise MIN.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- empirical-quantile** (*sorted-vector q*) [Function]  
 Return the empirical quantile of a vector of real numbers, sorted in ascending order (not checked). Uses a 0.5 correction.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- empirical-quantile-probabilities** (*n*) [Function]  
 Probabilities that correspond to the empirical quantiles of a vector of length N. That is to say,  
 (== (quantiles sample (empirical-quantile-probabilities (length sample)))  
 sample)  
 for any vector SAMPLE.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- ensure-sorted-vector** (*object*) [Function]  
 Return the elements of OBJECT as a vector (or reals) sorted in ascending order.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- evaluate-chebyshev** (*coefficients x*) [Function]  
 Return the sum of Chebyshev polynomials, weighted by COEFFICIENTS, at X.  
**Package** [num-utils.chebyshev], page 23.  
**Source** [chebyshev.lisp], page 16.
- evaluate-polynomial** (*coefficients x*) [Function]  
 Return the sum of polynomials, weighted by COEFFICIENTS, at X. COEFFICIENTS are ordered from the highest degree down to the constant term. X must be of the same type as COEFFICIENTS.  
**Package** [num-utils.polynomial], page 35.  
**Source** [polynomial.lisp], page 16.



**exp-1** (*x*) [Function]

Compute  $(- (\exp x) 1)$  stably even when  $X$  is near 0

**Package** [num-utils.log-exp], page 26.

**Source** [log-exp.lisp], page 19.

**exp-1/x** (*x*) [Function]

Compute  $(/ (- (\exp x) 1) x)$  stably even when  $X$  is near zero.

**Package** [num-utils.log-exp], page 26.

**Source** [log-exp.lisp], page 19.

**expt-1** (*a z*) [Function]

Compute  $(a^z)-1$  stably even when  $A$  is close to 1 or  $Z$  is close to zero.

**Package** [num-utils.log-exp], page 26.

**Source** [log-exp.lisp], page 19.

**fixnum?** (*object*) [Function]

Check if type of OBJECT is fixnum.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**floor\*** (*number &optional divisor offset*) [Function]

Find the highest  $A = I * \text{DIVISOR} + \text{OFFSET} \leq \text{NUMBER}$ , return (values A (- A NUMBER)).

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**generate-sequence** (*result-type size function*) [Function]

Like MAKE-SEQUENCE, but using a function to fill the result.

Example to create a sequence of random numbers between 0-1 from the uniform distribution:  
(generate-sequence '(vector double-float) 100 (lambda () (random 1.0))).

Essentially the initial values are ignored when using this function.

See also: aops:generate

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**grid-in** (*interval size &optional sequence-type*) [Function]

Return an arithmetic sequence of the given size (length) between the endpoints of the interval.

The endpoints of the sequence coincide with the respective endpoint of the interval iff it is closed. RESULT-TYPE determines the result type (eg list), if not given it is a simple-array (of rank 1), narrowing to the appropriate float type or fixnum if possible.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**hermitian-matrix** (*elements*) [Function]

Create a lower-triangular-matrix.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

- hypot** (*x y*) [Function]  
 Compute the hypotenuse of X and Y without danger of floating-point overflow or underflow.  
**Package** [num-utils.log-exp], page 26.  
**Source** [log-exp.lisp], page 19.
- in-interval?** (*interval number*) [Function]  
 Test if NUMBER is in INTERVAL (which can be NIL, designating the empty set).  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- infinite?** (*object*) [Function]  
 Test if an object represents positive or negative infinity.  
**Package** [num-utils.extended-real], page 29.  
**Source** [extended-real.lisp], page 8.
- interval** (*left right &key open-left? open-right?*) [Function]  
 Create an INTERVAL.  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- interval-hull** (*object*) [Function]  
 Return the smallest connected interval that contains (elements in) OBJECT.  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- interval-length** (*interval*) [Function]  
 Difference between left and right.  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- interval-midpoint** (*interval &optional alpha*) [Function]  
 Convex combination of left and right, with alpha (defaults to 0.5) weight on right.  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- ivec** (*end-or-start &optional end by strict-direction?*) [Function]  
 Return a vector of fixnums.
- (ivec end) => #(0 ... end-1) (or #(0 ... end+1) when end is negative).
- (ivec start end) => #(start ... end-1) or to end+1 when end is negative.
- When BY is given it determines the increment, adjusted to match the direction unless STRICT-DIRECTION, in which case an error is signalled.
- Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.

- l2norm** (*object*) [Function]  
 $\$L_2\$$  norm of OBJECT.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- log1+** (*x*) [Function]  
 Compute  $(\log(1+x))$  stably even when  $X$  is near 0.  
**Package** [num-utils.log-exp], page 26.  
**Source** [log-exp.lisp], page 19.
- log1+/x** (*x*) [Function]  
 Compute  $(/ (\log(1+x)) x)$  stably even when  $X$  is near zero.  
**Package** [num-utils.log-exp], page 26.  
**Source** [log-exp.lisp], page 19.
- log1+exp** (*a*) [Function]  
 Accurately compute  $\log(1+\exp(x))$  even when  $A$  is near zero.  
**Package** [num-utils.log-exp], page 26.  
**Source** [log-exp.lisp], page 19.
- log1-** (*x*) [Function]  
 Compute  $(\log(-1+x))$  stably even when  $X$  is near zero.  
**Package** [num-utils.log-exp], page 26.  
**Source** [log-exp.lisp], page 19.
- log1-exp** (*a*) [Function]  
 Compute  $\log(1-\exp(x))$  stably even when  $A$  is near zero.  
 This is sometimes known as the E<sub>3</sub>, the third Einstein function.  
 See Mächler 2008 for notes on accurate calculation. <https://cran.r-project.org/web/packages/Rmpfr/vignettes/log1mexp-note.pdf>  
**Package** [num-utils.log-exp], page 26.  
**Source** [log-exp.lisp], page 19.
- log10** (*number*) [Function]  
 Abbreviation for decimal logarithm.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- log1pmx** (*x*) [Function]  
 Compute  $(- (\log(1+x)) x)$   
 Accuracy within  $\sim 2$ ulps for  $-0.227 < x < 0.315$   
**Package** [num-utils.log-exp], page 26.  
**Source** [log-exp.lisp], page 19.
- log2** (*number*) [Function]  
 Abbreviation for binary logarithm.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.

- log2-exp** (*x*) [Function]  
 Compute  $\log(2\text{-exp}(x))$  stably even when  $X$  is near zero.  
**Package** [num-utils.log-exp], page 26.  
**Source** [log-exp.lisp], page 19.
- logexp-1** (*a*) [Function]  
 Compute  $\log(\exp(a)-1)$  stably even when  $A$  is small.  
**Package** [num-utils.log-exp], page 26.  
**Source** [log-exp.lisp], page 19.
- lower-triangular-matrix** (*elements*) [Function]  
 Create a lower-triangular-matrix.  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.
- make-sparse-counter** (**&key** *test*) [Function]  
 Create a sparse counter. Elements are compared with **TEST** (should be accepted by **HASH-TABLE**).  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- make-vector** (*element-type* **&rest** *initial-contents*) [Function]  
**Package** [num-utils.utilities], page 30.  
**Source** [utilities.lisp], page 5.
- max-error** (*instance*) [Reader]  
**(setf max-error)** (*instance*) [Writer]  
**Package** [num-utils.test-utilities], page 33.  
**Source** [test-utilities.lisp], page 19.  
**Target Slot**  
 [max-error], page 79.
- mean-error** (*instance*) [Reader]  
**(setf mean-error)** (*instance*) [Writer]  
**Package** [num-utils.test-utilities], page 33.  
**Source** [test-utilities.lisp], page 19.  
**Target Slot**  
 [mean-error], page 79.
- min-error** (*instance*) [Reader]  
**(setf min-error)** (*instance*) [Writer]  
**Package** [num-utils.test-utilities], page 33.  
**Source** [test-utilities.lisp], page 19.  
**Target Slot**  
 [min-error], page 79.

- normalize-probabilities** (*vector &key element-type result*) [Function]  
 Verify that each element of VECTOR is nonnegative and return a vector multiplied so that they sum to 1. ELEMENT-TYPE can be used to specify the element-type of the result. When RESULT is given, the result is placed there. When RESULT is NIL, VECTOR is modified instead.
- Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- num-delta** (*a b*) [Function]  
 $|a-b|/\max(1,|a|,|b|)$ . Useful for comparing numbers.
- Package** [num-utils.num=], page 35.  
**Source** [num=.lisp], page 6.
- num=-function** (*tolerance*) [Function]  
 Curried version of num=, with given tolerance.
- Package** [num-utils.num=], page 35.  
**Source** [num=.lisp], page 6.
- numseq** (*from to &key length by type*) [Function]  
 Return a sequence between FROM and TO, progressing by BY, of the given LENGTH. Only 3 of these a parameters should be given, the missing one (NIL) should be inferred automatically. The sign of BY is adjusted if necessary. If TYPE is LIST, the result is a list, otherwise it determines the element type of the resulting simple array. If TYPE is nil, it as autodetected from the arguments (as a FIXNUM, a RATIONAL, or some subtype of FLOAT). Note that the implementation may upgrade the element type.
- Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- plusminus-interval** (*center half-width &key open-left? open-right?*) [Function]  
 A symmetric interval around CENTER.
- Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- pool** (*&rest accumulators*) [Function]  
 Pool ACCUMULATORS.
- Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- print-length-truncate** (*dimension*) [Function]  
 Return values (min dimension \*print-length\*) and whether the constraint is binding.
- Package** [num-utils.print-matrix], page 23.  
**Source** [print-matrix.lisp], page 10.
- print-matrix** (*matrix stream &key formatter masked-fn aligned? padding indent*) [Function]  
 Format and print the elements of MATRIX (a 2d array) to STREAM, using PADDING between columns.

MASKED-FN is called on row and column indices. If it returns nil, the corresponding element is formatted using FORMATTER and printed. Otherwise, it should return a string, which is printed as is. INDENT is printed before each row.

If ALIGNED?, columns will be right-aligned. At most \*PRINT-LENGTH\* rows and columns are printed, more is indicated with ellipses (...).

**Package** [num-utils.print-matrix], page 23.

**Source** [print-matrix.lisp], page 10.

**relative** (*fraction*) [Function]

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**rms** (*instance*) [Reader]

(setf rms) (*instance*) [Writer]

**Package** [num-utils.test-utilities], page 33.

**Source** [test-utilities.lisp], page 19.

**Target Slot**  
[rms], page 80.

**romberg-quadrature** (*f interval &key epsilon min-iter max-iter transformation*) [Function]

Romberg quadrature of F on the interval. The iteration stops if the relative change is below EPSILON, but only after MIN-ITER refinements (to avoid spurious premature convergence). An error occurs when MAX-ITER iterations are reached without convergence.

**Package** [num-utils.quadrature], page 21.

**Source** [quadrature.lisp], page 17.

**root-bisection** (*f bracket &key delta epsilon*) [Function]

Find the root of f bracketed between a and b using bisection.

The algorithm stops when either the root is bracketed in an interval of length TOLERANCE (relative to the initial |a-b|), or root is found such that abs(f(root)) <= epsilon.

Return five values: the root, the value of the function at the root, and a boolean which is true iff abs(f(root)) <= epsilon. If the third value is true, the fourth and fifth values are the endpoints of the bracketing interval, otherwise they are undefined.

**Package** [num-utils.rootfinding], page 36.

**Source** [rootfinding.lisp], page 16.

**round\*** (*number &optional divisor offset*) [Function]

Find A=I\*DIVISOR+OFFSET that minimizes |A-NUMBER|, return (values A (- A NUMBER)). When NUMBER is exactly in between two possible A's, the rounding rule of ROUND is used on NUMBER-OFFSET.

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**same-sign-p** (*&rest arguments*) [Function]

Test whether all arguments have the same sign (ie all are positive, negative, or zero).

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

- sequence-maximum** (*x*) [Function]  
 Return the maximum value in the sequence *X*  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- sequence-minimum** (*x*) [Function]  
 Return the minimum value in the sequence *X*  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- sequencep** (*x*) [Function]  
 Return T if *X* is type SEQUENCE.  
**Package** [num-utils.utilities], page 30.  
**Source** [utilities.lisp], page 5.
- shrink-interval** (*interval left &optional right check-flip?*) [Function]  
 Shrink interval by given magnitudes (which may be REAL or RELATIVE). When *check-flip?*, the result is checked for endpoints being in a different order than the original. Negative LEFT and RIGHT extend the interval.  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- sorted-reals-elements** (*sorted-reals*) [Function]  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- spacer** (*&optional weight*) [Function]  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- sparse-counter-count** (*sparse-counter object*) [Function]  
 Return the count for OBJECT.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- sparse-counter-table** (*instance*) [Reader]  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.  
**Target Slot**  
 [table], page 78.
- split-interval** (*interval divisions*) [Function]  
 Return a vector of subintervals (same length as DIVISIONS), splitting the interval using the sequence DIVISIONS, which can be nonnegative real numbers (or RELATIVE specifications) and SPACERS which divide the leftover proportionally. If there are no spacers and the divisions don't fill up the interval, and error is signalled.  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.

- square** (*number*) [Function]  
 Square of number.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- subintervals-in** (*interval count &optional mid-open-right?*) [Function]  
 Return INTERVAL evenly divided into COUNT subintervals as a vector. When MID-OPEN-RIGHT?, subintervals in the middle are open on the right and closed on the left, otherwise the opposite; openness of endpoints on the edge follows INTERVAL.  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- tabulate** (*sequence &key test*) [Function]  
 Tabulate a sequence (using a SPARSE-COUNTER with the given TEST).  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- test-count** (*instance*) [Reader]  
 (setf test-count) (*instance*) [Writer]  
**Package** [num-utils.test-utilities], page 33.  
**Source** [test-utilities.lisp], page 19.  
**Target Slot**  
 [test-count], page 79.
- test-fn** (*expected-column fn &rest fn-param-columns*) [Function]  
 Test the differences between expected values and the given function  
**Package** [num-utils.test-utilities], page 33.  
**Source** [test-utilities.lisp], page 19.
- truncate\*** (*number &optional divisor offset*) [Function]  
 Find  $A = I * DIVISOR + OFFSET$  that maximizes  $|A| \leq |NUMBER|$  with the same sign, return (values A (- A NUMBER)).  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- upper-triangular-matrix** (*elements*) [Function]  
 Create a lower-triangular-matrix.  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.
- variance0** (*instance*) [Reader]  
 (setf variance0) (*instance*) [Writer]  
**Package** [num-utils.test-utilities], page 33.  
**Source** [test-utilities.lisp], page 19.  
**Target Slot**  
 [variance0], page 79.



<code>variance1</code> ( <i>instance</i> )	[Reader]
<code>(setf variance1)</code> ( <i>instance</i> )	[Writer]
<b>Package</b> [ <code>num-utils.test-utilities</code> ], page 33.	
<b>Source</b> [ <code>test-utilities.lisp</code> ], page 19.	
<b>Target Slot</b>	
[ <code>variance1</code> ], page 79.	
<code>vec</code> ( <i>element-type</i> & <b>rest</b> <i>elements</i> )	[Function]
Return a vector with elements coerced to ELEMENT-TYPE.	
<b>Package</b> [ <code>num-utils.matrix-shorthand</code> ], page 25.	
<b>Source</b> [ <code>matrix-shorthand.lisp</code> ], page 13.	
<code>weighted-quantiles</code> ( <i>values weights qs</i> )	[Function]
Calculate quantiles QS of weighted observations. Uses a 0.5 correction.	
<b>Package</b> [ <code>num-utils.statistics</code> ], page 31.	
<b>Source</b> [ <code>statistics.lisp</code> ], page 14.	
<code>within?</code> ( <i>left value right</i> )	[Function]
Return non-nil iff value is in [left,right).	
<b>Package</b> [ <code>num-utils.utilities</code> ], page 30.	
<b>Source</b> [ <code>utilities.lisp</code> ], page 5.	
<code>worst-case</code> ( <i>instance</i> )	[Reader]
<code>(setf worst-case)</code> ( <i>instance</i> )	[Writer]
<b>Package</b> [ <code>num-utils.test-utilities</code> ], page 33.	
<b>Source</b> [ <code>test-utilities.lisp</code> ], page 19.	
<b>Target Slot</b>	
[ <code>worst-case</code> ], page 79.	
<code>wrapped-matrix-elements</code> ( <i>instance</i> )	[Reader]
<b>Package</b> [ <code>num-utils.matrix</code> ], page 26.	
<b>Source</b> [ <code>matrix.lisp</code> ], page 10.	
<b>Target Slot</b>	
[ <code>elements</code> ], page 81.	

#### 4.1.5 Generic functions

<code>add</code> ( <i>accumulator object</i> & <b>key</b> <i>weight</i> )	[Generic Function]
Add OBJECT to ACCUMULATOR. Return OBJECT. NILs are ignored by the accumulator, unless a specialized method decides otherwise. Keywords may be used to specify additional information (eg weight).	
<b>Package</b> [ <code>num-utils.statistics</code> ], page 31.	
<b>Source</b> [ <code>statistics.lisp</code> ], page 14.	
<b>Methods</b>	

- add** ((*accumulator* [*sparse-counter*], page 78) *object* &**key** *weight*) [Method]  
 Increments the count of OBJECT in SPARSE-COUNTER, optionally with a weight
- add** ((*accumulator* [*sorted-reals*], page 77) *object* &**key**) [Method]
- add** ((*moments* [*central-sample-moments*], page 74) (*y* *real*) &**key** *weight*) [Method]
- add** (*accumulator* (*object* null) &**key**) [Method]
- as-alist** (*object*) [Generic Function]  
 Return OBJECT as an ALIST. Semantics depends on OBJECT.
- Package** [num-utils.utilities], page 30.
- Source** [utilities.lisp], page 5.
- Methods**
- as-alist** ((*object* [*sparse-counter*], page 78)) [Method]  
 Return (OBJECT . COUNT) pairs as an alist.
- Source** [statistics.lisp], page 14.
- as-plist** (*object*) [Generic Function]  
 Return OBJECT as a PLIST. Semantics depends on OBJECT. The default method uses AS-ALIST.
- Package** [num-utils.utilities], page 30.
- Source** [utilities.lisp], page 5.
- Methods**
- as-plist** (*object*) [Method]
- central-m2** (*object* &**key** *weights*) [Generic Function]  
 Second central moment. For samples, normalized by the total weight (and thus not the unbiased estimator, see VARIANCE).
- Package** [num-utils.statistics], page 31.
- Source** [statistics.lisp], page 14.
- Methods**
- central-m2** (*object* &**key** *weights*) [Method]
- central-m2** ((*object* [*central-sample-moments*], page 74) &**key** *weights*) [Method]
- central-m3** (*object* &**key** *weights*) [Generic Function]  
 Third central moment.
- Package** [num-utils.statistics], page 31.
- Source** [statistics.lisp], page 14.
- Methods**
- central-m3** (*object* &**key** *weights*) [Method]
- central-m3** ((*object* [*central-sample-moments*], page 74) &**key** *weights*) [Method]

**central-m4** (*object &key weights*) [Generic Function]  
 Fourth central moment.

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Methods**

central-m4 (*object &key weights*) [Method]

central-m4 ((*object [central-sample-moments]*, page 74) *&key weights*) [Method]

**central-sample-moments** (*object &key degree weights*) [Generic Function]

Return a CENTRAL-SAMPLE-MOMENTS object that allows the calculation of the central sample moments of OBJECT up to the given DEGREE.

When WEIGHTS are given, they need to be a sequence of matching length.

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Methods**

central-sample-moments ((*object null*) *&key degree weights*) [Method]

central-sample-moments ((*moments [central-sample-moments]*, page 74) *&key degree weights*) [Method]

central-sample-moments ((*sequence sequence*) *&key degree weights*) [Method]

**diagonal-vector** (*matrix*) [Generic Function]

Return the diagonal elements of MATRIX as a vector.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**Methods**

diagonal-vector ((*matrix array*)) [Method]

diagonal-vector (*matrix*) [Method]

**(setf diagonal-vector)** (*matrix*) [Generic Function]

Set the diagonal elements of MATRIX using VECTOR.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**Methods**

(setf diagonal-vector) ((*matrix array*)) [Method]

**e1-** (*a*) [Generic Function]

Univariate elementwise -.

**Package** [num-utils.elementwise], page 28.

**Source** [elementwise.lisp], page 7.

**Methods**

e1-	((a [diagonal-matrix], page 75))	[Method]
Source	[matrix.lisp], page 10.	
e1-	((a [hermitian-matrix], page 76))	[Method]
Source	[matrix.lisp], page 10.	
e1-	((a [upper-triangular-matrix], page 80))	[Method]
Source	[matrix.lisp], page 10.	
e1-	((a [lower-triangular-matrix], page 76))	[Method]
Source	[matrix.lisp], page 10.	
e1-	((a number))	[Method]
e1-	((a array))	[Method]
e1/ (a)		[Generic Function]
Univariate elementwise /.		
Package	[num-utils.elementwise], page 28.	
Source	[elementwise.lisp], page 7.	
Methods		
e1/ ((a [diagonal-matrix], page 75))		[Method]
Source	[matrix.lisp], page 10.	
e1/ ((a [hermitian-matrix], page 76))		[Method]
Source	[matrix.lisp], page 10.	
e1/ ((a [upper-triangular-matrix], page 80))		[Method]
Source	[matrix.lisp], page 10.	
e1/ ((a [lower-triangular-matrix], page 76))		[Method]
Source	[matrix.lisp], page 10.	
e1/ ((a number))		[Method]
e1/ ((a array))		[Method]
e1log (a)		[Generic Function]
Univariate elementwise LOG.		
Package	[num-utils.elementwise], page 28.	
Source	[elementwise.lisp], page 7.	
Methods		
e1log ((a [diagonal-matrix], page 75))		[Method]
Source	[matrix.lisp], page 10.	
e1log ((a [hermitian-matrix], page 76))		[Method]
Source	[matrix.lisp], page 10.	
e1log ((a [upper-triangular-matrix], page 80))		[Method]
Source	[matrix.lisp], page 10.	

<code>e1log ((a [lower-triangular-matrix], page 76))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e1log ((a number))</code>	[Method]
<code>e1log ((a array))</code>	[Method]
<b>e2* (a b)</b>	[Generic Function]
Bivariate elementwise *.	
<b>Package</b> [num-utils.elementwise], page 28.	
<b>Source</b> [elementwise.lisp], page 7.	
<b>Methods</b>	
<code>e2* ((a [diagonal-matrix], page 75) (b [diagonal-matrix], page 75))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a [hermitian-matrix], page 76) (b [hermitian-matrix], page 76))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a [upper-triangular-matrix], page 80) (b [upper-triangular-matrix], page 80))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a [lower-triangular-matrix], page 76) (b [lower-triangular-matrix], page 76))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a number) (b [diagonal-matrix], page 75))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a [diagonal-matrix], page 75) (b number))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a number) (b [hermitian-matrix], page 76))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a [hermitian-matrix], page 76) (b number))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a number) (b [upper-triangular-matrix], page 80))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a [upper-triangular-matrix], page 80) (b number))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a number) (b [lower-triangular-matrix], page 76))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a [lower-triangular-matrix], page 76) (b number))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	

<code>e2* (a (b [wrapped-matrix], page 80))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a [wrapped-matrix], page 80) b)</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2* ((a number) (b number))</code>	[Method]
<code>e2* ((a vector) (b number))</code>	[Method]
<code>e2* ((a number) (b vector))</code>	[Method]
<code>e2* ((a vector) (b vector))</code>	[Method]
<code>e2* ((a array) (b number))</code>	[Method]
<code>e2* ((a number) (b array))</code>	[Method]
<code>e2* ((a array) (b array))</code>	[Method]
<code>e2+ (a b)</code>	[Generic Function]
Bivariate elementwise +.	
<b>Package</b> [num-utils.elementwise], page 28.	
<b>Source</b> [elementwise.lisp], page 7.	
<b>Methods</b>	
<code>e2+ ((a [diagonal-matrix], page 75) (b [diagonal-matrix], page 75))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2+ ((a [hermitian-matrix], page 76) (b [hermitian-matrix], page 76))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2+ ((a [upper-triangular-matrix], page 80) (b [upper-triangular-matrix], page 80))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2+ ((a [lower-triangular-matrix], page 76) (b [lower-triangular-matrix], page 76))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2+ (a (b [wrapped-matrix], page 80))</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2+ ((a [wrapped-matrix], page 80) b)</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>e2+ ((a number) (b number))</code>	[Method]
<code>e2+ ((a vector) (b number))</code>	[Method]
<code>e2+ ((a number) (b vector))</code>	[Method]
<code>e2+ ((a vector) (b vector))</code>	[Method]
<code>e2+ ((a array) (b number))</code>	[Method]
<code>e2+ ((a number) (b array))</code>	[Method]

	<code>e2+ ((a array) (b array))</code>	[Method]
<code>e2- (a b)</code>	Bivariate elementwise -.	[Generic Function]
<b>Package</b>	[num-utils.elementwise], page 28.	
<b>Source</b>	[elementwise.lisp], page 7.	
<b>Methods</b>		
	<code>e2- ((a [diagonal-matrix], page 75) (b [diagonal-matrix], page 75))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>e2- ((a [hermitian-matrix], page 76) (b [hermitian-matrix], page 76))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>e2- ((a [upper-triangular-matrix], page 80) (b [upper-triangular-matrix], page 80))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>e2- ((a [lower-triangular-matrix], page 76) (b [lower-triangular-matrix], page 76))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>e2- (a (b [wrapped-matrix], page 80))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>e2- ((a [wrapped-matrix], page 80) b)</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>e2- ((a number) (b number))</code>	[Method]
	<code>e2- ((a vector) (b number))</code>	[Method]
	<code>e2- ((a number) (b vector))</code>	[Method]
	<code>e2- ((a vector) (b vector))</code>	[Method]
	<code>e2- ((a array) (b number))</code>	[Method]
	<code>e2- ((a number) (b array))</code>	[Method]
	<code>e2- ((a array) (b array))</code>	[Method]
<code>e2/ (a b)</code>	Bivariate elementwise /.	[Generic Function]
<b>Package</b>	[num-utils.elementwise], page 28.	
<b>Source</b>	[elementwise.lisp], page 7.	
<b>Methods</b>		
	<code>e2/ ((a number) (b [diagonal-matrix], page 75))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>e2/ ((a [diagonal-matrix], page 75) (b number))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	

e2/ ((a number) (b <i>[hermitian-matrix]</i> , page 76))	[Method]
Source [matrix.lisp], page 10.	
e2/ ((a <i>[hermitian-matrix]</i> , page 76) (b number))	[Method]
Source [matrix.lisp], page 10.	
e2/ ((a number) (b <i>[upper-triangular-matrix]</i> , page 80))	[Method]
Source [matrix.lisp], page 10.	
e2/ ((a <i>[upper-triangular-matrix]</i> , page 80) (b number))	[Method]
Source [matrix.lisp], page 10.	
e2/ ((a number) (b <i>[lower-triangular-matrix]</i> , page 76))	[Method]
Source [matrix.lisp], page 10.	
e2/ ((a <i>[lower-triangular-matrix]</i> , page 76) (b number))	[Method]
Source [matrix.lisp], page 10.	
e2/ ((a number) (b number))	[Method]
e2/ ((a vector) (b number))	[Method]
e2/ ((a number) (b vector))	[Method]
e2/ ((a vector) (b vector))	[Method]
e2/ ((a array) (b number))	[Method]
e2/ ((a number) (b array))	[Method]
e2/ ((a array) (b array))	[Method]
e2log (a b)	[Generic Function]
Bivariate elementwise LOG.	
Package [num-utils.elementwise], page 28.	
Source [elementwise.lisp], page 7.	
Methods	
e2log ((a number) (b number))	[Method]
e2log ((a vector) (b number))	[Method]
e2log ((a number) (b vector))	[Method]
e2log ((a vector) (b vector))	[Method]
e2log ((a array) (b number))	[Method]
e2log ((a number) (b array))	[Method]
e2log ((a array) (b array))	[Method]
eceiling (a)	[Generic Function]
Univariate elementwise CEILING.	
Package [num-utils.elementwise], page 28.	
Source [elementwise.lisp], page 7.	
Methods	
eceiling ((a number))	[Method]



	<code>eceiling ((a array))</code>	[Method]
<code>econjugate (a)</code>	Univariate elementwise CONJUGATE.	[Generic Function]
<b>Package</b>	[num-utils.elementwise], page 28.	
<b>Source</b>	[elementwise.lisp], page 7.	
<b>Methods</b>		
	<code>econjugate ((a number))</code>	[Method]
	<code>econjugate ((a array))</code>	[Method]
<code>ecos (a)</code>	Univariate elementwise COS.	[Generic Function]
<b>Package</b>	[num-utils.elementwise], page 28.	
<b>Source</b>	[elementwise.lisp], page 7.	
<b>Methods</b>		
	<code>ecos ((a number))</code>	[Method]
	<code>ecos ((a array))</code>	[Method]
<code>eexp (a)</code>	Univariate elementwise EXP.	[Generic Function]
<b>Package</b>	[num-utils.elementwise], page 28.	
<b>Source</b>	[elementwise.lisp], page 7.	
<b>Methods</b>		
	<code>eexp ((a [diagonal-matrix], page 75))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>eexp ((a [hermitian-matrix], page 76))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>eexp ((a [upper-triangular-matrix], page 80))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>eexp ((a [lower-triangular-matrix], page 76))</code>	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>eexp ((a number))</code>	[Method]
	<code>eexp ((a array))</code>	[Method]
<code>eexpt (a b)</code>	Bivariate elementwise EXPT.	[Generic Function]
<b>Package</b>	[num-utils.elementwise], page 28.	
<b>Source</b>	[elementwise.lisp], page 7.	
<b>Methods</b>		
	<code>eexpt ((a number) (b number))</code>	[Method]
	<code>eexpt ((a vector) (b number))</code>	[Method]

- `eexpt ((a number) (b vector))` [Method]  
`eexpt ((a vector) (b vector))` [Method]  
`eexpt ((a array) (b number))` [Method]  
`eexpt ((a number) (b array))` [Method]  
`eexpt ((a array) (b array))` [Method]
- efloor** (*a*) [Generic Function]  
 Univariate elementwise FLOOR.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.  
**Methods**
- `efloor ((a number))` [Method]  
`efloor ((a array))` [Method]
- emod** (*a b*) [Generic Function]  
 Bivariate elementwise MOD.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.  
**Methods**
- `emod ((a number) (b number))` [Method]  
`emod ((a vector) (b number))` [Method]  
`emod ((a number) (b vector))` [Method]  
`emod ((a vector) (b vector))` [Method]  
`emod ((a array) (b number))` [Method]  
`emod ((a number) (b array))` [Method]  
`emod ((a array) (b array))` [Method]
- ensure-sorted-reals** (*object*) [Generic Function]  
 Return the contents of OBJECT as a SORTED-REALS.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.  
**Methods**
- `ensure-sorted-reals ((sorted-reals [sorted-reals],  
                     page 77))` [Method]  
`ensure-sorted-reals ((array array))` [Method]  
`ensure-sorted-reals ((list list))` [Method]
- erreduce** (*function object &key key*) [Generic Function]  
 Elementwise reduce, traversing in row-major order.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.  
**Methods**

	<code>erreduce</code> ( <i>function</i> ( <i>array</i> <i>array</i> ) <b>&amp;key</b> <i>key</i> )	[Method]
	<code>erreduce</code> ( <i>function</i> ( <i>sequence</i> <i>sequence</i> ) <b>&amp;key</b> <i>key</i> )	[Method]
	<code>erreduce</code> ( <i>function</i> <i>object</i> <b>&amp;key</b> <i>key</i> )	[Method]
<code>esin</code> ( <i>a</i> )		[Generic Function]
	Univariate elementwise SIN.	
<b>Package</b>	[num-utils.elementwise], page 28.	
<b>Source</b>	[elementwise.lisp], page 7.	
<b>Methods</b>		
	<code>esin</code> (( <i>a</i> <i>number</i> ))	[Method]
	<code>esin</code> (( <i>a</i> <i>array</i> ))	[Method]
<code>esqrt</code> ( <i>a</i> )		[Generic Function]
	Univariate elementwise Sqrt.	
<b>Package</b>	[num-utils.elementwise], page 28.	
<b>Source</b>	[elementwise.lisp], page 7.	
<b>Methods</b>		
	<code>esqrt</code> (( <i>a</i> [ <i>diagonal-matrix</i> ], page 75))	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>esqrt</code> (( <i>a</i> [ <i>hermitian-matrix</i> ], page 76))	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>esqrt</code> (( <i>a</i> [ <i>upper-triangular-matrix</i> ], page 80))	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>esqrt</code> (( <i>a</i> [ <i>lower-triangular-matrix</i> ], page 76))	[Method]
	<b>Source</b> [matrix.lisp], page 10.	
	<code>esqrt</code> (( <i>a</i> <i>number</i> ))	[Method]
	<code>esqrt</code> (( <i>a</i> <i>array</i> ))	[Method]
<code>extend-interval</code> ( <i>interval</i> <i>object</i> )		[Generic Function]
	Return an interval that includes INTERVAL and OBJECT. NIL stands for the empty set.	
<b>Package</b>	[num-utils.interval], page 24.	
<b>Source</b>	[interval.lisp], page 9.	
<b>Methods</b>		
	<code>extend-interval</code> (( <i>interval</i> <i>null</i> ) ( <i>object</i> <i>null</i> ))	[Method]
	<code>extend-interval</code> (( <i>interval</i> <i>null</i> ) ( <i>number</i> <i>real</i> ))	[Method]
	<code>extend-interval</code> (( <i>interval</i> [ <i>interval</i> ], page 81) ( <i>number</i> <i>real</i> ))	[Method]
	<code>extend-interval</code> ( <i>interval</i> ( <i>object</i> [ <i>interval</i> ], page 81))	[Method]
	<code>extend-interval</code> ( <i>interval</i> ( <i>list</i> <i>list</i> ))	[Method]
	<code>extend-interval</code> ( <i>interval</i> ( <i>array</i> <i>array</i> ))	[Method]

**kurtosis** (*object* **&key** *weights*) [Generic Function]

Kurtosis FIXME talk about bias, maybe implement unbiased?

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Methods**

kurtosis (*object* **&key** *weights*) [Method]

kurtosis ((*object* [*central-sample-moments*], page 74) **&key** *weights*) [Method]

**l2norm-square** (*object*) [Generic Function]

Square of the  $L_2$  norm of OBJECT.

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**Methods**

l2norm-square ((*sequence* *sequence*)) [Method]

**left** (*interval*) [Generic Function]

Left endpoint of interval.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Methods**

left ((*interval* [*interval/infinite-left*], page 103)) [Method]

left ((*interval/finite-left* [*interval/finite-left*], page 102)) [Reader Method]

automatically generated reader method

**Target Slot**

[left], page 102.

**map-array** (*array function* **&optional** *retval*) [Generic Function]

**Package** [num-utils.matrix], page 26.

**Methods**

map-array (*array function* **&optional** *retval*) [Method]

Apply FUNCTION to each element of ARRAY

Return a new array, or write into the optional 3rd argument.

**Source** [matrix.lisp], page 10.

**mean** (*object* **&key** *weights*) [Generic Function]

The mean of elements in OBJECT.

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Methods**

mean (*object* **&key** *weights*) [Method]

`mean ((object [central-sample-moments], page 74) &key weights)` [Method]

`median (object)` [Generic Function]  
Median of OBJECT.

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Methods**

`median ((sample sequence))` [Method]  
Returns median of SAMPLE. SAMPLE must be a sequence of real numbers.

`median (object)` [Method]

`num= (a b &optional tolerance)` [Generic Function]  
Compare A and B for approximate equality, checking corresponding elements when applicable (using TOLERANCE).

Two numbers A and B are NUM= iff  $|a-b|/\max(1,|a|,|b|) \leq \text{tolerance}$ .

Unless a method is defined for them, two objects are compared with EQUALP.

Generally, methods should be defined so that two objects are NUM= if they the same class, same dimensions, and all their elements are NUM=.

**Package** [num-utils.num=], page 35.

**Source** [num=.lisp], page 6.

**Methods**

`num= ((a [central-sample-moments], page 74) (b [central-sample-moments], page 74) &optional tolerance)` [Method]  
**Source** [statistics.lisp], page 14.

`num= ((a [diagonal-matrix], page 75) (b [diagonal-matrix], page 75) &optional tolerance)` [Method]  
**Source** [matrix.lisp], page 10.

`num= ((a [wrapped-matrix], page 80) (b [wrapped-matrix], page 80) &optional tolerance)` [Method]  
**Source** [matrix.lisp], page 10.

`num= ((a [finite-interval], page 81) (b [finite-interval], page 81) &optional tolerance)` [Method]  
**Source** [interval.lisp], page 9.

`num= ((a [real-line], page 82) (b [real-line], page 82) &optional tolerance)` [Method]  
**Source** [interval.lisp], page 9.

`num= (a b &optional tolerance)` [Method]

`num= ((a number) (b number) &optional tolerance)` [Method]

- `num= ((a array) (b array) &optional tolerance)` [Method]  
`num= ((a cons) (b cons) &optional tolerance)` [Method]  
`num= ((a null) (b null) &optional tolerance)` [Method]
- open-left?** (*interval*) [Generic Function]  
 True iff the left endpoint of the interval is open.  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.  
**Methods**
- `open-left? ((interval [interval/infinite-left], page 103))` [Method]  
`open-left? ((interval/finite-left [interval/finite-left], page 102))` [Reader Method]  
 automatically generated reader method  
**Target Slot**  
 [open-left?], page 102.
- open-right?** (*interval*) [Generic Function]  
 True iff the right endpoint of the interval is open.  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.  
**Methods**
- `open-right? ((interval [interval/infinite-right], page 103))` [Method]  
`open-right? ((interval/finite-right [interval/finite-right], page 102))` [Reader Method]  
 automatically generated reader method  
**Target Slot**  
 [open-right?], page 103.
- product** (*object*) [Generic Function]  
 Product of elements in object.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.  
**Methods**
- `product ((sequence sequence))` [Method]  
`product ((array array))` [Method]
- quantile** (*object q*) [Generic Function]  
 Return an element at quantile Q. May be an interpolation or an approximation, depending on OBJECT and Q. NOTE: Extensions should define methods for QUANTILES, not QUANTILE.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.  
**Methods**

- `quantile` ((*object sequence*) *q*) [Method]
- `quantile` (*object* *q*) [Method]
- `quantiles` (*object* *qs*) [Generic Function]  
 Multiple quantiles (see QUANTILE). NOTE: Extensions should define methods for QUANTILES, not QUANTILE.
- Package** [num-utils.statistics], page 31.
- Source** [statistics.lisp], page 14.
- Methods**
- `quantiles` ((*object sequence*) *qs*) [Method]
- `quantiles` ((*accumulator* [*sorted-reals*], page 77) *q*) [Method]
- `right` (*interval*) [Generic Function]  
 Right endpoint of interval.
- Package** [num-utils.interval], page 24.
- Source** [interval.lisp], page 9.
- Methods**
- `right` ((*interval* [*interval/infinite-right*], page 103)) [Method]
- `right` ((*interval/finite-right* [*interval/finite-right*], page 102)) [Reader Method]  
 automatically generated reader method
- Target Slot**  
 [right], page 103.
- `sd` (*object* &key *weights*) [Generic Function]  
 Standard deviation. For samples, the square root of the unbiased estimator (see VARIANCE).
- Package** [num-utils.statistics], page 31.
- Source** [statistics.lisp], page 14.
- Methods**
- `sd` (*object* &key *weights*) [Method]
- `shift-interval` (*interval* *offset*) [Generic Function]
- Package** [num-utils.interval], page 24.
- Source** [interval.lisp], page 9.
- Methods**
- `shift-interval` ((*interval* [*finite-interval*], page 81) (*offset* *real*)) [Method]
- `skewness` (*object* &key *weights*) [Generic Function]  
 Skewness FIXME talk about bias, maybe implement unbiased?
- Package** [num-utils.statistics], page 31.
- Source** [statistics.lisp], page 14.
- Methods**

- skewness** (*object* &**key** *weights*) [Method]
- skewness** ((*object* [*central-sample-moments*], *page 74*) &**key** *weights*) [Method]
- sum** (*object* &**key** *key*) [Generic Function]  
Sum of elements in object. KEY is applied to each element.
- Package** [num-utils.arithmetic], page 34.
- Source** [arithmetic.lisp], page 6.
- Methods**
- sum** ((*sequence* *sequence*) &**key** *key*) [Method]
- sum** ((*array* *array*) &**key** *key*) [Method]
- tally** (*accumulator*) [Generic Function]  
The total weight of elements in ACCUMULATOR.
- Package** [num-utils.statistics], page 31.
- Source** [statistics.lisp], page 14.
- Methods**
- tally** ((*accumulator* [*sparse-counter*], *page 78*)) [Method]  
Return the total 'weight' of the accumulator
- tally** ((*accumulator* [*tally-mixin*], *page 101*)) [Method]
- transpose** (*array*) [Generic Function]  
Transpose.
- Package** [num-utils.matrix], page 26.
- Source** [matrix.lisp], page 10.
- Methods**
- transpose** ((*array* *array*)) [Method]
- transpose** ((*matrix* [*lower-triangular-matrix*], *page 76*)) [Method]
- transpose** ((*matrix* [*upper-triangular-matrix*], *page 80*)) [Method]
- transpose** ((*matrix* [*hermitian-matrix*], *page 76*)) [Method]
- transpose** ((*diagonal* [*diagonal-matrix*], *page 75*)) [Method]
- variance** (*object* &**key** *weights*) [Generic Function]  
Variance of OBJECT. For samples, normalized by the weight-1 (and thus unbiased if certain assumptions hold, eg weights that count frequencies).
- Package** [num-utils.statistics], page 31.
- Source** [statistics.lisp], page 14.
- Methods**
- variance** (*object* &**key** *weights*) [Method]
- variance** ((*object* [*central-sample-moments*], *page 74*) &**key** *weights*) [Method]



### 4.1.6 Standalone methods

<code>as-array ((<i>matrix0</i> [<i>upper-triangular-matrix</i>], page 80))</code>	[Method]
<b>Package</b> array-operations/generic.	
<b>Source</b> [matrix.lisp], page 10.	
<code>as-array ((<i>matrix0</i> [<i>hermitian-matrix</i>], page 76))</code>	[Method]
<b>Package</b> array-operations/generic.	
<b>Source</b> [matrix.lisp], page 10.	
<code>as-array ((<i>matrix0</i> [<i>lower-triangular-matrix</i>], page 76))</code>	[Method]
<b>Package</b> array-operations/generic.	
<b>Source</b> [matrix.lisp], page 10.	
<code>as-array ((<i>diagonal-matrix</i> [<i>diagonal-matrix</i>], page 75))</code>	[Method]
<b>Package</b> array-operations/generic.	
<b>Source</b> [matrix.lisp], page 10.	
<code>dims ((<i>wrapped-matrix</i> [<i>wrapped-matrix</i>], page 80))</code>	[Method]
<b>Package</b> array-operations/generic.	
<b>Source</b> [matrix.lisp], page 10.	
<code>dims ((<i>diagonal-matrix</i> [<i>diagonal-matrix</i>], page 75))</code>	[Method]
<b>Package</b> array-operations/generic.	
<b>Source</b> [matrix.lisp], page 10.	
<code>element-type ((<i>wrapped-matrix</i> [<i>wrapped-matrix</i>], page 80))</code>	[Method]
<b>Package</b> array-operations/generic.	
<b>Source</b> [matrix.lisp], page 10.	
<code>element-type ((<i>diagonal-matrix</i> [<i>diagonal-matrix</i>], page 75))</code>	[Method]
<b>Package</b> array-operations/generic.	
<b>Source</b> [matrix.lisp], page 10.	
<code>initialize-instance :after ((<i>interval</i> [<i>finite-interval</i>], page 81) &amp;key                               &amp;allow-other-keys)</code>	[Method]
<b>Source</b> [interval.lisp], page 9.	
<code>print-object ((<i>interval</i> [<i>interval</i>], page 81) <i>stream</i>)</code>	[Method]
<b>Source</b> [interval.lisp], page 9.	
<code>print-object ((<i>matrix0</i> [<i>upper-triangular-matrix</i>], page 80) <i>stream1</i>)</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>print-object ((<i>matrix0</i> [<i>hermitian-matrix</i>], page 76) <i>stream1</i>)</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	
<code>print-object ((<i>matrix0</i> [<i>lower-triangular-matrix</i>], page 76) <i>stream1</i>)</code>	[Method]
<b>Source</b> [matrix.lisp], page 10.	

- `print-object` ((*acc* [*sorted-reals*], page 77) *stream*) [Method]  
**Source** [statistics.lisp], page 14.
- `print-object` ((*sparse-counter* [*sparse-counter*], page 78) *stream*) [Method]  
**Source** [statistics.lisp], page 14.
- `select` ((*matrix0* [*upper-triangular-matrix*], page 80) &rest *slices*) [Method]  
**Package** select.  
**Source** [matrix.lisp], page 10.
- `select` ((*matrix0* [*hermitian-matrix*], page 76) &rest *slices*) [Method]  
**Package** select.  
**Source** [matrix.lisp], page 10.
- `select` ((*matrix0* [*lower-triangular-matrix*], page 76) &rest *slices*) [Method]  
**Package** select.  
**Source** [matrix.lisp], page 10.

#### 4.1.7 Conditions

- `empty-accumulator` [Condition]  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.  
**Direct superclasses**  
 error.
- `information-not-collected-in-accumulator` [Condition]  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.  
**Direct superclasses**  
 error.
- `not-enough-elements-in-accumulator` [Condition]  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.  
**Direct superclasses**  
 error.

#### 4.1.8 Structures

- `central-sample-moments` [Structure]  
 Central sample moments calculated on-line/single-pass.
- M weighted mean  
 S2 weighted sum of squared deviations from the mean, not calculated when NIL  
 S3 weighted sum of cubed deviations from the mean, not calculated when NIL  
 S4 weighted sum of 4th power deviations from the mean, not calculated when NIL  
 Allows on-line, numerically stable calculation of moments. See cite{bennett2009numerically} and cite{pebay2008formulas} for the description of the algorithm. M\_2, ..., M\_4 in the paper are s2, ..., s4 in the code.
- Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

### Direct superclasses

[tally-mixin], page 101.

### Direct methods

- [add], page 56.
- [central-m2], page 56.
- [central-m3], page 57.
- [central-m4], page 57.
- [central-sample-moments], page 57.
- [kurtosis], page 66.
- [mean], page 67.
- [num=], page 68.
- [pool2], page 98.
- [skewness], page 71.
- [variance], page 71.

### Direct slots

<b>m</b>		[Slot]
<b>Type</b>	real	
<b>Initform</b>	0.0d0	
<b>Readers</b>	[central-sample-moments-m], page 86.	
<b>Writers</b>	[(setf central-sample-moments-m)], page 86.	
<b>s2</b>		[Slot]
<b>Type</b>	(or (real 0) null)	
<b>Initform</b>	0.0d0	
<b>Readers</b>	[central-sample-moments-s2], page 86.	
<b>Writers</b>	[(setf central-sample-moments-s2)], page 86.	
<b>s3</b>		[Slot]
<b>Type</b>	(or real null)	
<b>Initform</b>	0.0d0	
<b>Readers</b>	[central-sample-moments-s3], page 86.	
<b>Writers</b>	[(setf central-sample-moments-s3)], page 86.	
<b>s4</b>		[Slot]
<b>Type</b>	(or (real 0) null)	
<b>Initform</b>	0.0d0	
<b>Readers</b>	[central-sample-moments-s4], page 86.	
<b>Writers</b>	[(setf central-sample-moments-s4)], page 86.	

**diagonal-matrix** [Structure]

Diagonal matrix. The elements in the diagonal are stored in a vector.

**Package** `[num-utils.matrix]`, page 26.

**Source** `[matrix.lisp]`, page 10.

**Direct superclasses**

`structure-object`.

**Direct methods**

- `[as-array]`, page 72.
- `[dims]`, page 72.
- `[e1-]`, page 58.
- `[e1/]`, page 58.
- `[e1log]`, page 59.
- `[e2*]`, page 59.
- `[e2*]`, page 59.
- `[e2*]`, page 59.
- `[e2+]`, page 60.
- `[e2-]`, page 61.
- `[e2/]`, page 62.
- `[e2/]`, page 62.
- `[eexp]`, page 64.
- `[element-type]`, page 72.
- `[esqrt]`, page 66.
- `[num=]`, page 68.
- `[transpose]`, page 71.

**Direct slots**

**elements** [Slot]

**Type** `vector`

**Readers** `[diagonal-matrix-elements]`, page 45.

**Writers** `[(setf diagonal-matrix-elements)]`, page 45.

**hermitian-matrix** [Structure]

Hermitian/symmetric matrix, with elements stored in the `_lower_` triangle.

Implements `_both_` real symmetric and complex Hermitian matrices — as technically, real symmetric matrices are also Hermitian. Complex symmetric matrices are `_not_` implemented as a special matrix type, as they don't have any special properties (eg real eigenvalues, etc).

**Package** `[num-utils.matrix]`, page 26.

**Source** `[matrix.lisp]`, page 10.

**Direct superclasses**

`[wrapped-matrix]`, page 80.

**Direct methods**

- `[as-array]`, page 72.
- `[e1-]`, page 58.

- [e1/], page 58.
- [e1log], page 59.
- [e2\*], page 59.
- [e2\*], page 59.
- [e2\*], page 60.
- [e2+], page 60.
- [e2-], page 61.
- [e2/], page 62.
- [e2/], page 62.
- [eexp], page 64.
- [esqrt], page 66.
- [print-object], page 73.
- [select], page 73.
- [transpose], page 71.

**lower-triangular-matrix** [Structure]

Lower triangular matrix. ELEMENTS in the upper triangle are treated as zero.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**Direct superclasses**

[wrapped-matrix], page 80.

**Direct methods**

- [as-array], page 72.
- [e1-], page 58.
- [e1/], page 58.
- [e1log], page 59.
- [e2\*], page 59.
- [e2\*], page 60.
- [e2\*], page 60.
- [e2+], page 61.
- [e2-], page 61.
- [e2/], page 62.
- [e2/], page 62.
- [eexp], page 64.
- [esqrt], page 66.
- [print-object], page 73.
- [select], page 73.
- [transpose], page 71.

**relative** [Structure]

Relative sizes are in terms of width.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

<b>Direct superclasses</b>	
structure-object.	
<b>Direct slots</b>	
fraction [Slot]	
<b>Type</b>	(real 0)
<b>Readers</b>	[relative-fraction], page 93.
<b>Writers</b>	<i>This slot is read-only.</i>
sorted-reals [Structure]	
Accumulator which sorts elements. ELEMENTS return the sorted elements.	
<b>Package</b>	[num-utils.statistics], page 31.
<b>Source</b>	[statistics.lisp], page 14.
<b>Direct superclasses</b>	
structure-object.	
<b>Direct methods</b>	
<ul style="list-style-type: none"> <li>• [add], page 56.</li> <li>• [ensure-sorted-reals], page 65.</li> <li>• [print-object], page 73.</li> <li>• [quantiles], page 70.</li> </ul>	
<b>Direct slots</b>	
ordered-elements [Slot]	
<b>Type</b>	vector
<b>Initform</b>	#()
<b>Readers</b>	[sorted-reals-ordered-elements], page 94.
<b>Writers</b>	[(setf sorted-reals-ordered-elements)], page 94.
unordered-elements [Slot]	
<b>Type</b>	list
<b>Readers</b>	[sorted-reals-unordered-elements], page 94.
<b>Writers</b>	[(setf sorted-reals-unordered-elements)], page 94.
spacer [Structure]	
Spacers divide the leftover portion of an interval.	
<b>Package</b>	[num-utils.interval], page 24.
<b>Source</b>	[interval.lisp], page 9.
<b>Direct superclasses</b>	
structure-object.	
<b>Direct slots</b>	
weight [Slot]	
<b>Type</b>	(real 0)
<b>Initform</b>	1
<b>Readers</b>	[spacer-weight], page 95.
<b>Writers</b>	<i>This slot is read-only.</i>

**sparse-counter** [Structure]

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Direct superclasses**  
structure-object.

**Direct methods**

- [add], page 56.
- [as-alist], page 56.
- [print-object], page 73.
- [tally], page 71.

**Direct slots**

<b>table</b>	[Slot]
<b>Type</b>	hash-table
<b>Readers</b>	[sparse-counter-table], page 53.
<b>Writers</b>	<i>This slot is read-only.</i>

**test-results** [Structure]

Differences between reference values and computed values

**Package** [num-utils.test-utilities], page 33.

**Source** [test-utilities.lisp], page 19.

**Direct superclasses**  
structure-object.

**Direct slots**

<b>worst-case</b>	[Slot]
<b>Type</b>	integer
<b>Initform</b>	0
<b>Readers</b>	[worst-case], page 55.
<b>Writers</b>	[(setf worst-case)], page 55.

<b>min-error</b>	[Slot]
<b>Type</b>	double-float
<b>Initform</b>	0.0d0
<b>Readers</b>	[min-error], page 50.
<b>Writers</b>	[(setf min-error)], page 50.

<b>max-error</b>	[Slot]
<b>Type</b>	double-float
<b>Initform</b>	0.0d0
<b>Readers</b>	[max-error], page 50.
<b>Writers</b>	[(setf max-error)], page 50.

<b>mean-error</b>	[Slot]
<b>Type</b>	double-float
<b>Initform</b>	0.0d0
<b>Readers</b>	[mean-error], page 50.
<b>Writers</b>	[(setf mean-error)], page 50.
<b>test-count</b>	[Slot]
<b>Type</b>	integer
<b>Initform</b>	0
<b>Readers</b>	[test-count], page 54.
<b>Writers</b>	[(setf test-count)], page 54.
<b>variance0</b>	[Slot]
<b>Type</b>	double-float
<b>Initform</b>	0.0d0
<b>Readers</b>	[variance0], page 54.
<b>Writers</b>	[(setf variance0)], page 54.
<b>variance1</b>	[Slot]
<b>Type</b>	double-float
<b>Initform</b>	0.0d0
<b>Readers</b>	[variance1], page 55.
<b>Writers</b>	[(setf variance1)], page 55.
<b>rms</b>	[Slot]
<b>Type</b>	double-float
<b>Initform</b>	0.0d0
<b>Readers</b>	[rms], page 52.
<b>Writers</b>	[(setf rms)], page 52.
<b>upper-triangular-matrix</b>	[Structure]
Upper triangular matrix. ELEMENTS in the lower triangle are treated as zero.	
<b>Package</b>	[num-utils.matrix], page 26.
<b>Source</b>	[matrix.lisp], page 10.
<b>Direct superclasses</b>	[wrapped-matrix], page 80.
<b>Direct methods</b>	
•	[as-array], page 72.
•	[e1-], page 58.
•	[e1/], page 58.
•	[e1log], page 59.
•	[e2*], page 59.
•	[e2*], page 60.



- [e2\*], page 60.
- [e2+], page 61.
- [e2-], page 61.
- [e2/], page 62.
- [e2/], page 62.
- [eexp], page 64.
- [esqrt], page 66.
- [print-object], page 72.
- [select], page 73.
- [transpose], page 71.

**wrapped-matrix** [Structure]

A matrix that has some special structure (eg triangular, symmetric/hermitian). **ELEMENTS** is always a matrix. Not used directly, not exported.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**Direct superclasses**

structure-object.

**Direct subclasses**

- [hermitian-matrix], page 76.
- [lower-triangular-matrix], page 76.
- [upper-triangular-matrix], page 80.

**Direct methods**

- [dims], page 72.
- [e2\*], page 60.
- [e2\*], page 60.
- [e2+], page 61.
- [e2+], page 61.
- [e2-], page 61.
- [e2-], page 62.
- [element-type], page 72.
- [num=], page 68.

**Direct slots**

**elements** [Slot]

**Type** (array \* (\* \*))

**Readers** [wrapped-matrix-elements], page 55.

**Writers** *This slot is read-only.*

#### 4.1.9 Classes

**finite-interval** [Class]

Interval with finite endpoints.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Direct superclasses**

- [interval], page 81.
- [interval/finite-left], page 102.
- [interval/finite-right], page 102.

**Direct methods**

- [chebyshev-approximate-implementation], page 97.
- [initialize-instance], page 72.
- [num=], page 68.
- [shift-interval], page 70.
- [transformed-quadrature], page 99.

**interval** [Class]

Abstract superclass for all intervals.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Direct subclasses**

- [finite-interval], page 81.
- [minusinf-interval], page 82.
- [plusinf-interval], page 82.
- [real-line], page 82.

**Direct methods**

- [extend-interval], page 66.
- [extend-interval], page 66.
- [print-object], page 72.

**minusinf-interval** [Class]

Interval from  $-\infty$  to RIGHT.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Direct superclasses**

- [interval], page 81.
- [interval/finite-right], page 102.
- [interval/infinite-left], page 103.

**plusinf-interval** [Class]

Interval from LEFT to  $\infty$ .

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Direct superclasses**

- [interval], page 81.
- [interval/finite-left], page 102.
- [interval/infinite-right], page 103.

**Direct methods**

- [chebyshev-approximate-implementation], page 97.
- [transformed-quadrature], page 99.

**real-line** [Class]

Representing the real line  $(-\infty, \infty)$ .

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Direct superclasses**

- [interval], page 81.
- [interval/infinite-left], page 103.
- [interval/infinite-right], page 103.

**Direct methods**

[num=], page 68.

#### 4.1.10 Types

**extended-real** (*&optional base*) [Type]

Extended real number.

**Package** [num-utils.extended-real], page 29.

**Source** [extended-real.lisp], page 8.

**simple-double-float-vector** (*&optional length*) [Type]

Simple vector of double-float elements.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**simple-fixnum-vector** () [Type]

Simple vector of fixnum elements.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**simple-single-float-vector** (*&optional length*) [Type]

Simple vector of single-float elements.

**Package** [num-utils.utilities], page 30.

**Source** [utilities.lisp], page 5.

**triangular-matrix** () [Type]

Triangular matrix (either lower or upper).

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

## 4.2 Internals

### 4.2.1 Macros

**&diagonal-matrix** (*elements*) [Macro]

LET+ form for slots of the structure DIAGONAL-MATRIX.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

- &diagonal-matrix-r/o** (*elements*) [Macro]  
 LET+ form for slots of the structure DIAGONAL-MATRIX. Read-only.  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.
- &sorted-reals** (*ordered-elements unordered-elements*) [Macro]  
 LET+ form for slots of the structure SORTED-REALS.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- &sorted-reals-r/o** (*ordered-elements unordered-elements*) [Macro]  
 LET+ form for slots of the structure SORTED-REALS. Read-only.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- define-central-sample-moment** (*function (variable degree) &body body*) [Macro]  
 FIXME documentation, factor out general part  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- define-comparison** (*name test*) [Macro]  
 Define a comparison, extendeding a pairwise comparison to an arbitrary number of arguments.  
**Package** [num-utils.extended-real], page 29.  
**Source** [extended-real.lisp], page 8.
- define-e&** (*operation &key function bivariate univariate docstring*) [Macro]  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- define-e1** (*operation &key function docstring*) [Macro]  
 Define an univariate elementwise operation.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- define-e2** (*operation &key function docstring*) [Macro]  
 Define a bivariate elementwise operation.  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.
- define-elementwise-as-array** (*type &key functions*) [Macro]  
 Define binary elementwise operations for FUNCTION, implemented by converting them to arrays.  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.
- define-elementwise-reduction** (*name function &optional docstring*) [Macro]  
**Package** [num-utils.elementwise], page 28.  
**Source** [elementwise.lisp], page 7.

**define-elementwise-same-class** (*type &key functions elements-accessor*) [Macro]  
 Define binary elementwise operations for FUNCTION for two arguments of the same class.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**define-elementwise-univariate** (*type &key functions elements-accessor*) [Macro]  
 Define unary elementwise operations for FUNCTION for all subclasses of wrapped-elements.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**define-elementwise-with-constant** (*type &key functions elements-accessor*) [Macro]  
 Define binary elementwise operations for FUNCTION for all subclasses of wrapped-elements.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**define-rounding-with-offset** (*name function docstring*) [Macro]

**Package** [num-utils.arithmetic], page 34.

**Source** [arithmetic.lisp], page 6.

**define-wrapped-matrix** (*type elements struct-docstring (masked-test masked-string) check-and-convert-elements regularize-elements*) [Macro]

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**mapping-array** ((*ref array &rest other*) *form*) [Macro]

**Package** [num-utils.elementwise], page 28.

**Source** [elementwise.lisp], page 7.

**univariate-rootfinder-loop%** (((*f a b fa fb*) (*f-tested test-bracket delta epsilon*)) **&body** *body*) [Macro]  
 Common parts for univariate rootfinder functions.

Sets up the following:

- function OPPOSITE-SIGN-P for checking that two numbers are on the opposite side of 0
- function EVALUATE-AND-RETURN-IF-WITHIN-EPSILON which checks that  $|f(x)| \leq \text{EPSILON}$ , if so, returns from the block with (VALUES X FX T), otherwise simply returns the value
- function RETURN-IF-WITHIN-TOLERANCE checks if the interval [A,B] bracketing X is small enough (smaller than TOLERANCE) and if so, returns (X FX NIL (INTERVAL A B))
- variables FA and FB to hold function values at A and B

Initially, it checks for either  $f(a)$  or  $f(b)$  being a root, and establishes  $a \leq b$  by exchanging  $a, f(a)$  and  $b, f(b)$  if necessary. Also checks that  $f(a)$  and  $f(b)$  are of opposite sign. Checks that both tolerance and epsilon are nonnegative.

**Package** [num-utils.rootfinding], page 36.

**Source** [rootfinding.lisp], page 16.

### 4.2.2 Ordinary functions

**ab-to-cd-intercept-slope** (*a b c d*) [Function]  
 Return (values INTERCEPT SLOPE) for linear mapping  $x \mapsto \text{intercept} + \text{slope} * x$  from  $[a, b]$  to  $[c, d]$ .

**Package** [num-utils.chebyshev], page 23.

**Source** [chebyshev.lisp], page 16.

**ab-to-cinf** (*z a b c*) [Function]  
 Inverse of cinf-to-ab.

**Package** [num-utils.chebyshev], page 23.

**Source** [chebyshev.lisp], page 16.

**above-diagonal?** (*row col*) [Function]  
 Test if element with indexes row and col is (strictly) above the diagonal.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**below-diagonal?** (*row col*) [Function]  
 Test if element with indexes row and col is (strictly) below the diagonal.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**central-sample-moments-m** (*instance*) [Reader]  
**(setf central-sample-moments-m)** (*instance*) [Writer]

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Target Slot**  
 [m], page 74.

**central-sample-moments-p** (*object*) [Function]

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**central-sample-moments-s2** (*instance*) [Reader]  
**(setf central-sample-moments-s2)** (*instance*) [Writer]

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Target Slot**  
 [s2], page 74.

**central-sample-moments-s3** (*instance*) [Reader]  
**(setf central-sample-moments-s3)** (*instance*) [Writer]

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Target Slot**  
 [s3], page 75.

<code>central-sample-moments-s4</code> ( <i>instance</i> )	[Reader]
<code>(setf central-sample-moments-s4)</code> ( <i>instance</i> )	[Writer]
<b>Package</b> [ <code>num-utils.statistics</code> ], page 31.	
<b>Source</b> [ <code>statistics.lisp</code> ], page 14.	
<b>Target Slot</b>	
[s4], page 75.	
<code>central-sample-moments-w</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.statistics</code> ], page 31.	
<b>Source</b> [ <code>statistics.lisp</code> ], page 14.	
<code>(setf central-sample-moments-w)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.statistics</code> ], page 31.	
<b>Source</b> [ <code>statistics.lisp</code> ], page 14.	
<code>chebyshev-recursion</code> ( <i>x value previous-value</i> )	[Function]
Chebyshev polynomial recursion formula.	
<b>Package</b> [ <code>num-utils.chebyshev</code> ], page 23.	
<b>Source</b> [ <code>chebyshev.lisp</code> ], page 16.	
<code>cinf-to-ab</code> ( <i>x a b c</i> )	[Function]
Map <i>x</i> in [ <i>c</i> ,plus-infinity) to <i>z</i> in [ <i>a</i> , <i>b</i> ] using $x \rightarrow (x-c)/(1+x-c)+(b-a)+a$ .	
<b>Package</b> [ <code>num-utils.chebyshev</code> ], page 23.	
<b>Source</b> [ <code>chebyshev.lisp</code> ], page 16.	
<code>copy-central-sample-moments</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.statistics</code> ], page 31.	
<b>Source</b> [ <code>statistics.lisp</code> ], page 14.	
<code>copy-diagonal-matrix</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.matrix</code> ], page 26.	
<b>Source</b> [ <code>matrix.lisp</code> ], page 10.	
<code>copy-hermitian-matrix</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.matrix</code> ], page 26.	
<b>Source</b> [ <code>matrix.lisp</code> ], page 10.	
<code>copy-iterative-quadrature</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>copy-lower-triangular-matrix</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.matrix</code> ], page 26.	
<b>Source</b> [ <code>matrix.lisp</code> ], page 10.	
<code>copy-midpoint-quadrature</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	

<code>copy-relative</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.interval</code> ], page 24.	
<b>Source</b> [ <code>interval.lisp</code> ], page 9.	
<code>copy-richardson-extrapolation</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>copy-sorted-reals</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.statistics</code> ], page 31.	
<b>Source</b> [ <code>statistics.lisp</code> ], page 14.	
<code>copy-spacer</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.interval</code> ], page 24.	
<b>Source</b> [ <code>interval.lisp</code> ], page 9.	
<code>copy-sparse-counter</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.statistics</code> ], page 31.	
<b>Source</b> [ <code>statistics.lisp</code> ], page 14.	
<code>copy-tally-mixin</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.statistics</code> ], page 31.	
<b>Source</b> [ <code>statistics.lisp</code> ], page 14.	
<code>copy-test-results</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.test-utilities</code> ], page 33.	
<b>Source</b> [ <code>test-utilities.lisp</code> ], page 19.	
<code>copy-trapezoidal-quadrature</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>copy-upper-triangular-matrix</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.matrix</code> ], page 26.	
<b>Source</b> [ <code>matrix.lisp</code> ], page 10.	
<code>copy-wrapped-matrix</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.matrix</code> ], page 26.	
<b>Source</b> [ <code>matrix.lisp</code> ], page 10.	
<code>diagonal-matrix-p</code> ( <i>object</i> )	[Function]
<b>Package</b> [ <code>num-utils.matrix</code> ], page 26.	
<b>Source</b> [ <code>matrix.lisp</code> ], page 10.	



**ensure-valid-elements** (*array rank &rest predicates*) [Function]

Convert OBJECT to an array, check that it

1. has the required rank,
2. has a valid sparse element type, and
3. that it satisfies PREDICATES.

Return the array.

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**extend-pairwise-comparison** (*test first rest*) [Function]

Extend TEST (a pairwise comparison) to an arbitrary number of arguments (but at least one, FIRST).

**Package** [num-utils.extended-real], page 29.

**Source** [extended-real.lisp], page 8.

**hermitian-matrix-elements** (*instance*) [Function]

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**hermitian-matrix-p** (*object*) [Function]

**Package** [num-utils.matrix], page 26.

**Source** [matrix.lisp], page 10.

**iterative-quadrature-a** (*instance*) [Reader]

(setf iterative-quadrature-a) (*instance*) [Writer]

**Package** [num-utils.quadrature], page 21.

**Source** [quadrature.lisp], page 17.

**Target Slot**  
[a], page 99.

**iterative-quadrature-b** (*instance*) [Reader]

(setf iterative-quadrature-b) (*instance*) [Writer]

**Package** [num-utils.quadrature], page 21.

**Source** [quadrature.lisp], page 17.

**Target Slot**  
[b], page 100.

**iterative-quadrature-f** (*instance*) [Reader]

(setf iterative-quadrature-f) (*instance*) [Writer]

**Package** [num-utils.quadrature], page 21.

**Source** [quadrature.lisp], page 17.

**Target Slot**  
[f], page 99.

<code>iterative-quadrature-h</code> ( <i>instance</i> )	[Reader]
<code>(setf iterative-quadrature-h)</code> ( <i>instance</i> )	[Writer]
Package [num-utils.quadrature], page 21.	
Source [quadrature.lisp], page 17.	
Target Slot [h], page 100.	
<code>iterative-quadrature-n</code> ( <i>instance</i> )	[Reader]
<code>(setf iterative-quadrature-n)</code> ( <i>instance</i> )	[Writer]
Package [num-utils.quadrature], page 21.	
Source [quadrature.lisp], page 17.	
Target Slot [n], page 100.	
<code>iterative-quadrature-p</code> ( <i>object</i> )	[Function]
Package [num-utils.quadrature], page 21.	
Source [quadrature.lisp], page 17.	
<code>iterative-quadrature-sum</code> ( <i>instance</i> )	[Reader]
<code>(setf iterative-quadrature-sum)</code> ( <i>instance</i> )	[Writer]
Package [num-utils.quadrature], page 21.	
Source [quadrature.lisp], page 17.	
Target Slot [sum], page 100.	
<code>ln</code> ( <i>n</i> )	[Function]
Natural logarithm.	
Package [num-utils.arithmetic], page 34.	
Source [arithmetic.lisp], page 6.	
<code>lower-triangular-matrix-elements</code> ( <i>instance</i> )	[Function]
Package [num-utils.matrix], page 26.	
Source [matrix.lisp], page 10.	
<code>lower-triangular-matrix-p</code> ( <i>object</i> )	[Function]
Package [num-utils.matrix], page 26.	
Source [matrix.lisp], page 10.	
<code>make-central-sample-moments</code> ( <b>&amp;key</b> <i>w m s2 s3 s4</i> )	[Function]
Package [num-utils.statistics], page 31.	
Source [statistics.lisp], page 14.	
<code>make-diagonal-matrix</code> ( <b>&amp;key</b> <i>elements</i> )	[Function]
Package [num-utils.matrix], page 26.	
Source [matrix.lisp], page 10.	

<code>make-hermitian-matrix</code> ( <b>&amp;key</b> <i>elements</i> )	[Function]
<b>Package</b> [num-utils.matrix], page 26.	
<b>Source</b> [matrix.lisp], page 10.	
<code>make-iterative-quadrature</code> ( <b>&amp;key</b> <i>f a b h n sum</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>make-lower-triangular-matrix</code> ( <b>&amp;key</b> <i>elements</i> )	[Function]
<b>Package</b> [num-utils.matrix], page 26.	
<b>Source</b> [matrix.lisp], page 10.	
<code>make-sorted-reals</code> ( <b>&amp;key</b> <i>ordered-elements unordered-elements</i> )	[Function]
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<code>make-sparse-counter%</code> ( <b>&amp;key</b> <i>table</i> )	[Function]
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<code>make-tally-mixin</code> ( <b>&amp;key</b> <i>w</i> )	[Function]
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<code>make-test-results</code> ( <b>&amp;key</b> <i>worst-case min-error max-error mean-error test-count variance0 variance1 rms</i> )	[Function]
<b>Package</b> [num-utils.test-utilities], page 33.	
<b>Source</b> [test-utilities.lisp], page 19.	
<code>make-upper-triangular-matrix</code> ( <b>&amp;key</b> <i>elements</i> )	[Function]
<b>Package</b> [num-utils.matrix], page 26.	
<b>Source</b> [matrix.lisp], page 10.	
<code>make-wrapped-matrix</code> ( <b>&amp;key</b> <i>elements</i> )	[Function]
<b>Package</b> [num-utils.matrix], page 26.	
<b>Source</b> [matrix.lisp], page 10.	
<code>midpoint-quadrature</code> ( <i>f a b</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>midpoint-quadrature%</code> ( <b>&amp;key</b> <i>f a b h n sum</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>midpoint-quadrature-a</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	

<code>(setf midpoint-quadrature-a)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>midpoint-quadrature-b</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>(setf midpoint-quadrature-b)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>midpoint-quadrature-f</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>(setf midpoint-quadrature-f)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>midpoint-quadrature-h</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>(setf midpoint-quadrature-h)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>midpoint-quadrature-n</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>(setf midpoint-quadrature-n)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>midpoint-quadrature-p</code> ( <i>object</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>midpoint-quadrature-sum</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	
<code>(setf midpoint-quadrature-sum)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [ <code>num-utils.quadrature</code> ], page 21.	
<b>Source</b> [ <code>quadrature.lisp</code> ], page 17.	

- narrow-bracket?** (*a b delta*) [Function]  
 Return true iff  $|a-b| < \delta$ .
- Package** [num-utils.rootfinding], page 36.  
**Source** [rootfinding.lisp], page 16.
- near-root?** (*f epsilon*) [Function]  
 Return true iff  $|f| < \epsilon$ .
- Package** [num-utils.rootfinding], page 36.  
**Source** [rootfinding.lisp], page 16.
- opposite-sign?** (*a b*) [Function]  
 Return true iff A and B are on opposite sides of 0.
- Package** [num-utils.rootfinding], page 36.  
**Source** [rootfinding.lisp], page 16.
- pad-left-expansion** (*rows ncol*) [Function]  
 Pad ragged-right rows. Used internally to implement ragged right matrix specifications.
- Package** [num-utils.matrix-shorthand], page 25.  
**Source** [matrix-shorthand.lisp], page 13.
- print-matrix-formatter** (*x*) [Function]  
 Standard formatter for matrix printing. Respects `*print-precision*`, and formats complex numbers as `a+bi`, eg `0.0+1.0i`.
- Package** [num-utils.print-matrix], page 23.  
**Source** [print-matrix.lisp], page 10.
- relative-fraction** (*instance*) [Reader]  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.  
**Target Slot** [fraction], page 77.
- relative-p** (*object*) [Function]  
**Package** [num-utils.interval], page 24.  
**Source** [interval.lisp], page 9.
- richardson-extrapolation** (*coefficient iterations*) [Function]  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.
- richardson-extrapolation-coefficient** (*instance*) [Reader]  
 (setf richardson-extrapolation-coefficient) (*instance*) [Writer]  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.  
**Target Slot** [coefficient], page 101.

- richardson-extrapolation-diagonal** (*instance*) [Reader]  
 (setf richardson-extrapolation-diagonal) (*instance*) [Writer]  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.  
**Target Slot**  
 [diagonal], page 101.
- richardson-extrapolation-n** (*instance*) [Reader]  
 (setf richardson-extrapolation-n) (*instance*) [Writer]  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.  
**Target Slot**  
 [n], page 101.
- richardson-extrapolation-p** (*object*) [Function]  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.
- richardson-iteration** (*extrapolation step*) [Function]  
 Add STEP (=  $A(h q^{-k})$ ) to an existing Richardson EXTRAPOLATION. See the documentation of RICHARDSON-EXTRAPOLATION for details.  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.
- romberg-quadrature%** (*quadrature epsilon min-iter max-iter*) [Function]  
 Internal function implementing Romberg quadrature. Requires an iterative quadrature instance, a relative EPSILON and MIN-ITER for the stopping criterion, and the maximum number of iterations allowed. Works on finite intervals.  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.
- rootfinding-delta** (*interval &optional delta-relative*) [Function]  
 Default DELTA for rootfinding methods, uses bracket width.  
**Package** [num-utils.rootfinding], page 36.  
**Source** [rootfinding.lisp], page 16.
- similar-element-type** (*element-type*) [Function]  
 Return a type that is a supertype of ELEMENT-TYPE and is closed under arithmetic operations. May not be the narrowest.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.
- similar-sequence-type** (*sequence*) [Function]  
 Return type that sequence can be mapped to using arithmetic operations.  
**Package** [num-utils.arithmetic], page 34.  
**Source** [arithmetic.lisp], page 6.

<b>sort-reals</b> ( <i>sequence</i> )	[Function]
Return a SORTED-REALS structure.	
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<b>sorted-reals-ordered-elements</b> ( <i>instance</i> )	[Reader]
(setf sorted-reals-ordered-elements) ( <i>instance</i> )	[Writer]
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<b>Target Slot</b>	
[ordered-elements], page 77.	
<b>sorted-reals-p</b> ( <i>object</i> )	[Function]
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<b>sorted-reals-unordered-elements</b> ( <i>instance</i> )	[Reader]
(setf sorted-reals-unordered-elements) ( <i>instance</i> )	[Writer]
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<b>Target Slot</b>	
[unordered-elements], page 78.	
<b>spacer-p</b> ( <i>object</i> )	[Function]
<b>Package</b> [num-utils.interval], page 24.	
<b>Source</b> [interval.lisp], page 9.	
<b>spacer-weight</b> ( <i>instance</i> )	[Reader]
<b>Package</b> [num-utils.interval], page 24.	
<b>Source</b> [interval.lisp], page 9.	
<b>Target Slot</b>	
[weight], page 78.	
<b>sparse-counter-p</b> ( <i>object</i> )	[Function]
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<b>tally-mixin-p</b> ( <i>object</i> )	[Function]
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<b>tally-mixin-w</b> ( <i>instance</i> )	[Reader]
(setf tally-mixin-w) ( <i>instance</i> )	[Writer]
<b>Package</b> [num-utils.statistics], page 31.	
<b>Source</b> [statistics.lisp], page 14.	
<b>Target Slot</b>	
[w], page 101.	

<code>test-results-p</code> ( <i>object</i> )	[Function]
<b>Package</b> [num-utils.test-utilities], page 33.	
<b>Source</b> [test-utilities.lisp], page 19.	
<code>trapezoidal-quadrature</code> ( <i>f a b</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>trapezoidal-quadrature%</code> ( <b>&amp;key</b> <i>f a b h n sum</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>trapezoidal-quadrature-a</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>(setf trapezoidal-quadrature-a)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>trapezoidal-quadrature-b</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>(setf trapezoidal-quadrature-b)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>trapezoidal-quadrature-f</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>(setf trapezoidal-quadrature-f)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>trapezoidal-quadrature-h</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>(setf trapezoidal-quadrature-h)</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	
<code>trapezoidal-quadrature-n</code> ( <i>instance</i> )	[Function]
<b>Package</b> [num-utils.quadrature], page 21.	
<b>Source</b> [quadrature.lisp], page 17.	



- (setf trapezoidal-quadrature-n) (*instance*) [Function]  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.
- trapezoidal-quadrature-p (*object*) [Function]  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.
- trapezoidal-quadrature-sum (*instance*) [Function]  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.
- (setf trapezoidal-quadrature-sum) (*instance*) [Function]  
**Package** [num-utils.quadrature], page 21.  
**Source** [quadrature.lisp], page 17.
- upper-triangular-matrix-elements (*instance*) [Function]  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.
- upper-triangular-matrix-p (*object*) [Function]  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.
- valid-sparse-type? (*type*) [Function]  
 Check if TYPE is a valid type for sparse matrices. Only supertypes and subtypes of NUMBER are allowed.  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.
- weighted-empirical-quantile (*sorted-reals p-table q*) [Function]  
 Return the empirical quantile of a vector of real numbers, sorted in ascending order (not checked). Uses a 0.5 correction.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- weighted-quantile-p-table (*weights*) [Function]  
 Return table of probability brackets for weighted quantile calculations., built from the weights (which should be positive reals, not checked). Uses a 0.5 correction.  
**Package** [num-utils.statistics], page 31.  
**Source** [statistics.lisp], page 14.
- wrapped-matrix-p (*object*) [Function]  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.
- zero-like (*array*) [Function]  
 Return 0 coerced to the element type of ARRAY. It is assumed that the latter satisfies VALID-SPARSE-TYPE?.  
**Package** [num-utils.matrix], page 26.  
**Source** [matrix.lisp], page 10.

### 4.2.3 Generic functions

**chebyshev-approximate-implementation** (*f interval* [Generic Function]  
*n-polynomials n-points*)

Implementation of CHEBYSHEV-APPROXIMATE.

**Package** [num-utils.chebyshev], page 23.

**Source** [chebyshev.lisp], page 16.

**Methods**

**chebyshev-approximate-implementation** (*f (interval* [Method]  
*[plusinf-interval], page 82) n-polynomials n-points*)

**chebyshev-approximate-implementation** (*f (interval* [Method]  
*[finite-interval], page 81) n-polynomials n-points*)

**esquare** (*a*) [Generic Function]

Univariate elementwise SQUARE.

**Package** [num-utils.elementwise], page 28.

**Source** [elementwise.lisp], page 7.

**Methods**

**esquare** ((*a number*)) [Method]

**esquare** ((*a array*)) [Method]

**pool2** (*accumulator1 accumulator2*) [Generic Function]

Pool two accumulators. When they are of a different type, the resulting accumulator will be downgraded to the level afforded by the information available in the accumulators.

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Methods**

**pool2** ((*moments-a [central-sample-moments], page 74* [Method]  
*(moments-b [central-sample-moments], page 74)*)

**print-left-endpoint** (*interval stream*) [Generic Function]

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Methods**

**print-left-endpoint** ((*interval [interval/finite-left],* [Method]  
*page 102) stream*)

**print-left-endpoint** ((*interval* [Method]  
*[interval/infinite-left], page 103) stream*)

**print-right-endpoint** (*interval stream*) [Generic Function]

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Methods**

- `print-right-endpoint` ((*interval* [interval/finite-right], page 102) *stream*) [Method]
- `print-right-endpoint` ((*interval* [interval/infinite-right], page 103) *stream*) [Method]
- `refine-quadrature` (*quadrature*) [Generic Function]  
 Refine quadrature with more points. Return the sum for those points.
- Package** [num-utils.quadrature], page 21.
- Source** [quadrature.lisp], page 17.
- Methods**
- `refine-quadrature` ((*quadrature* [midpoint-quadrature], page 100)) [Method]
- `refine-quadrature` ((*quadrature* [trapezoidal-quadrature], page 101)) [Method]
- `richardson-coefficient` (*quadrature*) [Generic Function]  
 Return the coefficient  $q$  for Richardson approximation.
- Package** [num-utils.quadrature], page 21.
- Source** [quadrature.lisp], page 17.
- Methods**
- `richardson-coefficient` ((*quadrature* [midpoint-quadrature], page 100)) [Method]
- `richardson-coefficient` ((*quadrature* [trapezoidal-quadrature], page 101)) [Method]
- `transformed-quadrature` (*function interval transformation*) [Generic Function]  
 Return a quadrature for integrating FUNCTION on INTERVAL, which may be infinite, in which case FUNCTION will be transformed. TRANSFORMATION can be used to select the transformation when applicable, otherwise it is NIL.
- Package** [num-utils.quadrature], page 21.
- Source** [quadrature.lisp], page 17.
- Methods**
- `transformed-quadrature` (*function (interval [finite-interval], page 81) (transformation null)*) [Method]
- `transformed-quadrature` (*function (interval [plusinf-interval], page 82) (transformation null)*) [Method]

#### 4.2.4 Structures

- `iterative-quadrature` [Structure]  
 Quadrature building block.  
 F is the function.
- A and B are the endpoints.  
 H is the stepsize.
- Package** [num-utils.quadrature], page 21.

**Source** [quadrature.lisp], page 17.

**Direct superclasses**

structure-object.

**Direct subclasses**

- [midpoint-quadrature], page 100.
- [trapezoidal-quadrature], page 101.

**Direct slots**

f		[Slot]
<b>Type</b>	(function (double-float) double-float)	
<b>Readers</b>	[iterative-quadrature-f], page 89.	
<b>Writers</b>	[(setf iterative-quadrature-f)], page 89.	
a		[Slot]
<b>Type</b>	double-float	
<b>Readers</b>	[iterative-quadrature-a], page 89.	
<b>Writers</b>	[(setf iterative-quadrature-a)], page 89.	
b		[Slot]
<b>Type</b>	double-float	
<b>Readers</b>	[iterative-quadrature-b], page 89.	
<b>Writers</b>	[(setf iterative-quadrature-b)], page 89.	
h		[Slot]
<b>Type</b>	double-float	
<b>Readers</b>	[iterative-quadrature-h], page 89.	
<b>Writers</b>	[(setf iterative-quadrature-h)], page 89.	
n		[Slot]
<b>Type</b>	fixnum	
<b>Initform</b>	0	
<b>Readers</b>	[iterative-quadrature-n], page 89.	
<b>Writers</b>	[(setf iterative-quadrature-n)], page 89.	
sum		[Slot]
<b>Package</b>	[num-utils.arithmetic], page 34.	
<b>Type</b>	double-float	
<b>Initform</b>	0.0d0	
<b>Readers</b>	[iterative-quadrature-sum], page 89.	
<b>Writers</b>	[(setf iterative-quadrature-sum)], page 89.	

midpoint-quadrature [Structure]

**Package** [num-utils.quadrature], page 21.

**Source** [quadrature.lisp], page 17.

**Direct superclasses**

[iterative-quadrature], page 99.

**Direct methods**

- [refine-quadrature], page 98.
- [richardson-coefficient], page 99.

**richardson-extrapolation**

[Structure]

Given  $A(h) = A_0 + \sum_{k=1}^{\infty} a_k h^{\{kp\}}$ , calculate approximations for  $A$  given  $A(h q^{\{-k\}})$ , where the latter can be incorporated using RICHARDSON-ITERATION with consecutive values for  $k=1, \dots, \text{max\_iter}$ , which returns the latest  $A(0)$  as the first and the largest relative change, which can be used to test termination.

The algorithm uses Richardson extrapolation, the required coefficient is  $q^k$ .

**Package** [num-utils.quadrature], page 21.

**Source** [quadrature.lisp], page 17.

**Direct superclasses**

structure-object.

**Direct slots****coefficient**

[Slot]

**Type** double-float

**Readers** [richardson-extrapolation-coefficient], page 93.

**Writers** [(setf richardson-extrapolation-coefficient)], page 93.

**n**

[Slot]

**Type** fixnum

**Initform** 0

**Readers** [richardson-extrapolation-n], page 93.

**Writers** [(setf richardson-extrapolation-n)], page 93.

**diagonal**

[Slot]

**Type** (array double-float (\*))

**Readers** [richardson-extrapolation-diagonal], page 93.

**Writers** [(setf richardson-extrapolation-diagonal)], page 93.

**tally-mixin**

[Structure]

Mixin structure that contains a tally. Not exported.  $W$  is the total weight.

**Package** [num-utils.statistics], page 31.

**Source** [statistics.lisp], page 14.

**Direct superclasses**

structure-object.

**Direct subclasses**

[central-sample-moments], page 74.

**Direct methods**

[tally], page 71.

**Direct slots**

<b>w</b>		[Slot]
<b>Type</b>	(real 0)	
<b>Initform</b>	0	
<b>Readers</b>	[tally-mixin-w], page 95.	
<b>Writers</b>	[(setf tally-mixin-w)], page 95.	

trapezoidal-quadrature [Structure]

**Package** [num-utils.quadrature], page 21.

**Source** [quadrature.lisp], page 17.

**Direct superclasses**

[iterative-quadrature], page 99.

**Direct methods**

- [refine-quadrature], page 98.
- [richardson-coefficient], page 99.

**4.2.5 Classes**

interval/finite-left [Class]

Interval with left endpoint.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Direct subclasses**

- [finite-interval], page 81.
- [plusinf-interval], page 82.

**Direct methods**

- [left], page 67.
- [open-left?], page 69.
- [print-left-endpoint], page 98.

**Direct slots**

<b>left</b>		[Slot]
<b>Type</b>	real	
<b>Initargs</b>	:left	
<b>Readers</b>	[left], page 67.	
<b>Writers</b>	<i>This slot is read-only.</i>	
<b>open-left?</b>		[Slot]
<b>Type</b>	boolean	
<b>Initargs</b>	:open-left?	
<b>Readers</b>	[open-left?], page 69.	
<b>Writers</b>	<i>This slot is read-only.</i>	

`interval/finite-right` [Class]

Interval with right endpoint.

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Direct subclasses**

- [finite-interval], page 81.
- [minusinf-interval], page 82.

**Direct methods**

- [open-right?], page 69.
- [print-right-endpoint], page 98.
- [right], page 70.

**Direct slots**

`right` [Slot]

**Type** real

**Initargs** :right

**Readers** [right], page 70.

**Writers** *This slot is read-only.*

`open-right?` [Slot]

**Type** boolean

**Initargs** :open-right?

**Readers** [open-right?], page 69.

**Writers** *This slot is read-only.*

`interval/infinite-left` [Class]

Left endpoint is  $-\infty$ .

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Direct subclasses**

- [minusinf-interval], page 82.
- [real-line], page 82.

**Direct methods**

- [left], page 67.
- [open-left?], page 69.
- [print-left-endpoint], page 98.

`interval/infinite-right` [Class]

Right endpoint is  $\infty$ .

**Package** [num-utils.interval], page 24.

**Source** [interval.lisp], page 9.

**Direct subclasses**

- [plusinf-interval], page 82.
- [real-line], page 82.

**Direct methods**

- `[open-right?]`, page 69.
- `[print-right-endpoint]`, page 98.
- `[right]`, page 70.

**4.2.6 Types**

`infinite ()` [Type]

Representing infinity (extending the real line).

**Package**     `[num-utils.extended-real]`, page 29.

**Source**     `[extended-real.lisp]`, page 8.



## Appendix A Indexes

### A.1 Concepts

(Index is nonexistent)

## A.2 Functions

(	(setf rms).....	52
(setf central-sample-moments-m).....	(setf sorted-reals-ordered-elements).....	94
(setf central-sample-moments-s2).....	(setf sorted-reals-unordered-elements).....	94
(setf central-sample-moments-s3).....	(setf tally-mixin-w).....	95
(setf central-sample-moments-s4).....	(setf test-count).....	54
(setf central-sample-moments-w).....	(setf trapezoidal-quadrature-a).....	95
(setf diagonal-matrix-elements).....	(setf trapezoidal-quadrature-b).....	96
(setf diagonal-vector).....	(setf trapezoidal-quadrature-f).....	96
(setf iterative-quadrature-a).....	(setf trapezoidal-quadrature-h).....	96
(setf iterative-quadrature-b).....	(setf trapezoidal-quadrature-n).....	96
(setf iterative-quadrature-f).....	(setf trapezoidal-quadrature-sum).....	96
(setf iterative-quadrature-h).....	(setf variance0).....	54
(setf iterative-quadrature-n).....	(setf variance1).....	55
(setf iterative-quadrature-sum).....	(setf worst-case).....	55
(setf max-error).....		
(setf mean-error).....	<	
(setf midpoint-quadrature-a).....	<.....	42
(setf midpoint-quadrature-b).....	<=.....	42
(setf midpoint-quadrature-f).....		
(setf midpoint-quadrature-h).....	=	
(setf midpoint-quadrature-n).....	=.....	42
(setf midpoint-quadrature-sum).....		
(setf min-error).....	>	
(setf richardson-extrapolation-coefficient).....	>.....	42
(setf richardson-extrapolation-diagonal).....	>=.....	42
(setf richardson-extrapolation-n).....		

### A.3 Variables

(Index is nonexistent)

## A.4 Data types

(Index is nonexistent)