The problem we study in this note is to maximize a function of the form

$$\mathcal{L}(\beta)=\sum_{i=1}^{n}\log\pi_i(\beta),$$

where

$$\pi_i(\beta)=\Phi(x_i'\beta)-\Phi(z_i'\beta).$$

Here $\Phi(\bullet)$ is a given cdf, and the x_i and z_i are known vectors of numbers. The elements of β are the unknown parameters that we have to estimate. Of course we have to restrict the β by requiring

$$(x_i-z_i)'\beta\geq 0$$

for all i, but as long as these inequalities are indeed solvable, it is clear that the fact that we are maximizing will keep us away from the boundary, and will keep us feasible.

Let us look at the derivatives. We find

$$\frac{\partial\mathcal{L}}{\partial\beta}=\sum_{i=1}^{n}\frac{1}{\pi_i}\{\phi(x_i'\beta)x_i-\phi(z_i'\beta)z_i\}.$$

Also

$$\begin{aligned}\frac{\partial^2\mathcal{L}}{\partial\beta^2}&=\sum_{i=1}^{n}\frac{1}{\pi_i}\{\phi'(x_i'\beta)x_ix_i'-\phi'(z_i'\beta)z_iz_i'\}\\&-\sum_{i=1}^{n}\frac{1}{\pi_i^2}\{\phi(x_i'\beta)x_i-\phi(z_i'\beta)z_i\}\{\phi(x_i'\beta)x_i-\phi(z_i'\beta)z_i\}'.\end{aligned}$$