In [1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
import datetime
from scipy.stats import shapiro
from statsmodels.tsa.stattools import adfuller
plt.style.use('fivethirtyeight')
```

配對交易所需三個時間序列的統計條件：

1. 相關性（Correlation）

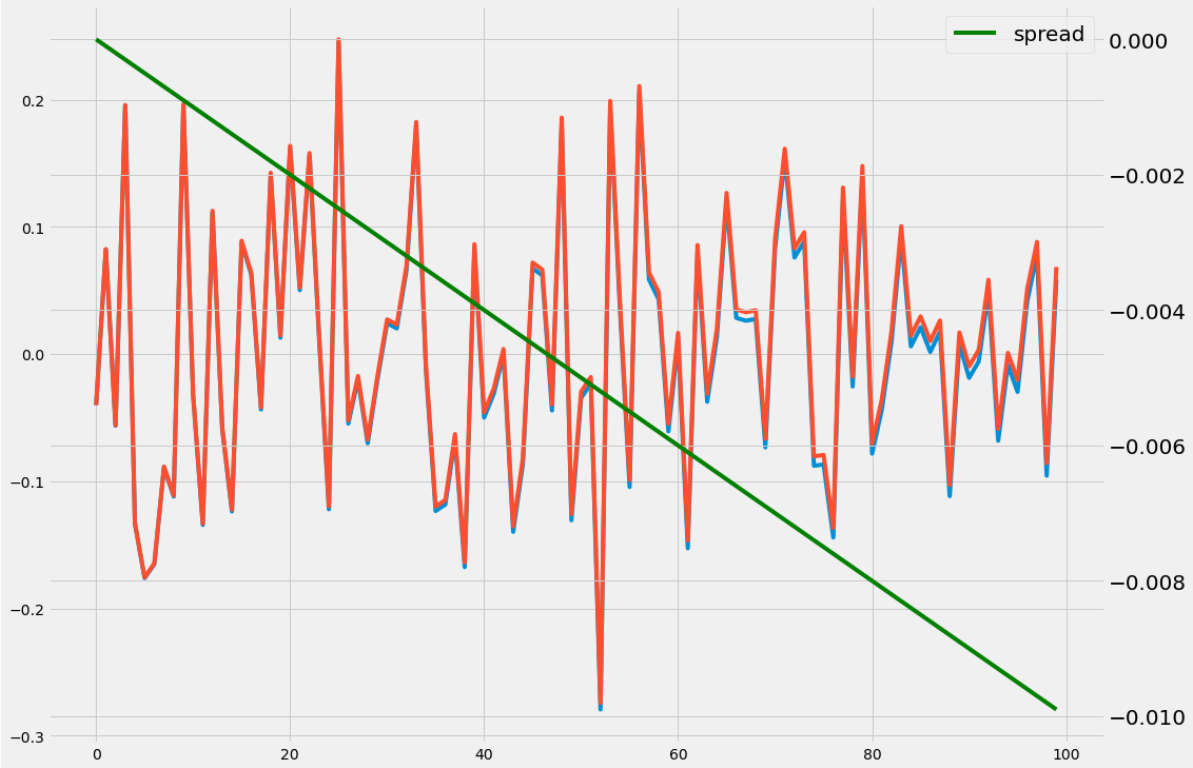
2. 共整合性（Cointegration）

3. 平穩性（Stationary）

In [87]:
```
mu, sigma = 0, 0.1
X=np.random.normal(mu,sigma,100)
Y=np.array([x+0.0001*t for t,x in enumerate(X)])
print(np.corrcoef(X,Y))
```

```
[[1.        0.9995895]
 [0.9995895 1.       ]]
```

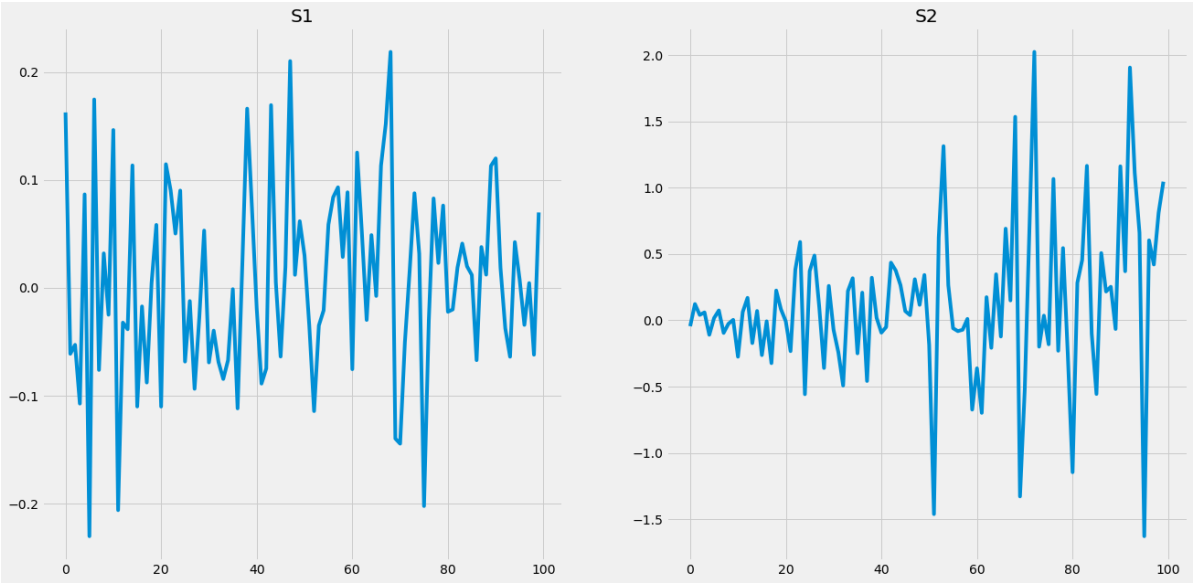
In [93]:
```
df=pd.DataFrame({'X':X,'Y':Y})
df['spread']=df['X']-df["Y"]
plt.figure(figsize=(16,12))
plt.plot(df['X'])
plt.plot(df['Y'])
plt.twinx()
plt.plot(df['spread'],color='g')
plt.legend(['spread'],fontsize=20)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
plt.show()
```



相關性 ≠ 共整合性 !!

共整合的主要概念為兩序列經過線性組合(加減乘除後)可以變成定態的時間序列(穩定的在均線軸上震盪)

In [94]:
```
np.random.seed(1)
S1=np.random.normal(0,0.1,100)
S2=np.array([])
for x in range(10):
    S_tem=np.random.normal(0,0.1*(x+1),10)
    S2=np.append(S2,S_tem)
fig,ax=plt.subplots(1,2,figsize=(20,10))
ax[0].plot(S1)
ax[0].set_title('S1')
ax[1].plot(S2)
ax[1].set_title('S2')
plt.show()
```



左邊是定態時間序列，右邊是非定態時間序列

Loading data

用台積電(2330)、聯發科（2454）做配對交易

In [2]:
```
TSMC=pd.read_csv('data/2330.TW.csv')[['Date','Close']]
MT=pd.read_csv('data/2454.TW.csv')[['Date','Close']]
TSMC.columns=['Date','TSMC']
MT.columns=['Date','MT']
prices=pd.merge(TSMC,MT,on='Date')
```

In [3]:
```
prices
```

Out[3]:

	Date	TSMC	MT
0	2017-12-06	227.0	286.5
1	2017-12-07	226.5	287.5
2	2017-12-08	227.0	296.0
3	2017-12-11	227.5	299.5
4	2017-12-12	227.5	296.0
...	...	...	...
1214	2022-11-29	487.0	713.0
1215	2022-11-30	490.0	733.0
1216	2022-12-01	498.5	741.0
1217	2022-12-02	492.5	739.0
1218	2022-12-05	489.0	739.0

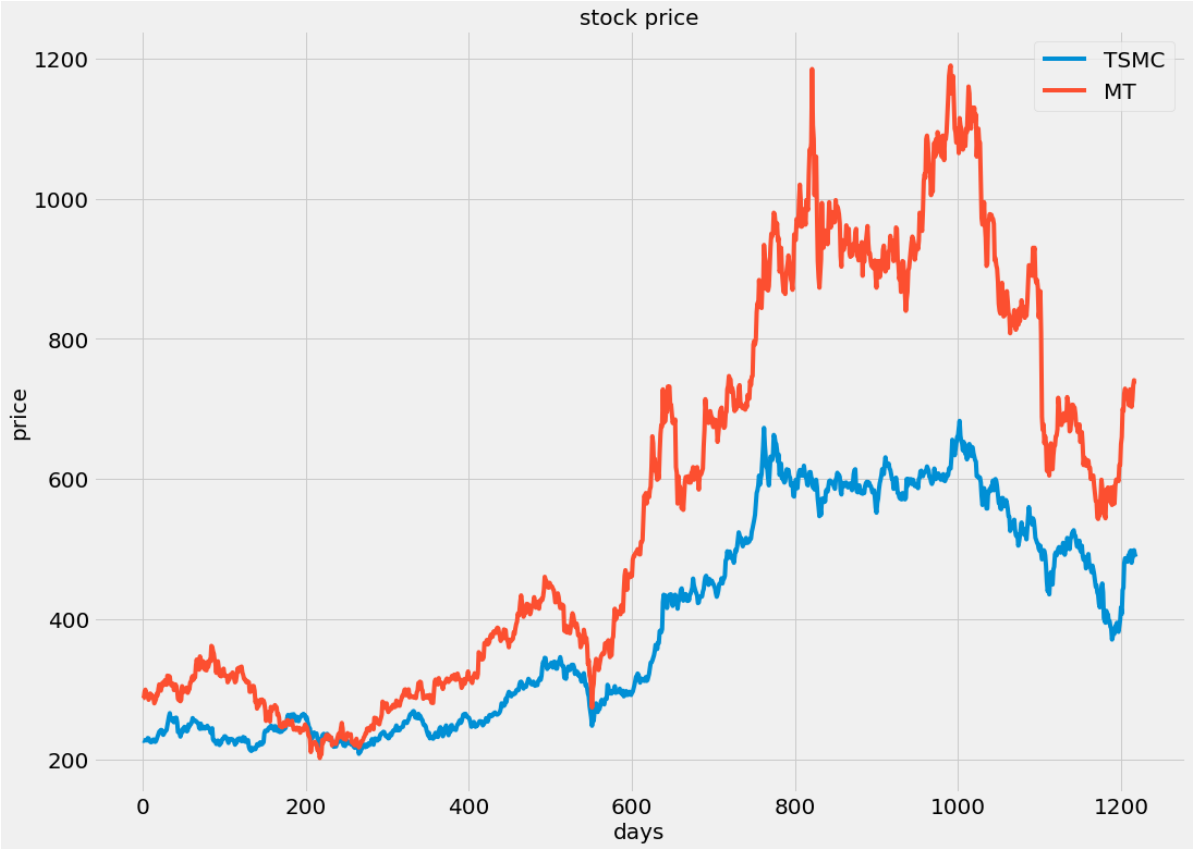
1219 rows × 3 columns

In [4]:
```
len(prices)
```

Out[4]:
```
1219
```

本次報告所回測的時間為 2017-12-06 ~ 2022-12-05，共1219個交易日。

In [5]:
```
plt.figure(figsize=(16,12))
plt.plot(TSMC[['TSMC']])
plt.plot(MT[['MT']])
plt.title('stock price',fontsize=20)
plt.legend(['TSMC','MT'], fontsize=20,title_fontsize=20)
plt.xlabel('days',fontsize=20)
plt.ylabel('price',fontsize=20)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
plt.show()
```



Variance-covariance matrix

In [6]:
```
# 計算日報酬率
prices_pct=prices[['TSMC','MT']].pct_change(1)
prices_pct.corr()
```

Out[6]:

	TSMC	MT
TSMC	1.000000	0.505825
MT	0.505825	1.000000

兩者股票的時間序列為中度相關性（Correlation）。

ADF Test

利用 ADF 檢定檢查每支股票是否為 nonstationary，並對其做一期差分，再檢查是否為 stationary 的資料，若一期差分為 stationary，則為共整合關係檢定的資料，適合進行配對交易。假設檢定為:

H_0：價差走勢為 nonstationary v.s. H_1：價差走勢為 stationary

In [7]:
```
prices_change_TSMC=prices[['TSMC']].diff()
prices_change_TSMC=prices_change_TSMC.dropna()
adfuller(prices_change_TSMC)
```

Out[7]:
```
(-21.006667236942402,
 0.0,
 2,
 1215,
 {'1%': -3.435743555099632,
  '5%': -2.8639217381867486,
  '10%': -2.568038075665972},
 8090.182647499236)
```

In [8]:
```
prices_change_MT=prices[['MT']].diff()
prices_change_MT=prices_change_MT.dropna()
adfuller(prices_change_MT)
```

Out[8]:
```
(-35.186250608297804,
 0.0,
 0,
 1217,
 {'1%': -3.4357346726088136,
  '5%': -2.863917819294165,
  '10%': -2.568035988607032},
 10079.057570547931)
```

兩支股票都通過檢定拒絕對立假設，差分序列為定態序列。

我們希望找到合適的比例 c，使得：

$$P_A \approx c \times P_B$$

$$\Rightarrow s = P_A - c \times P_B$$

$$\Rightarrow P_A = s + c \times P_B$$

用歷史資料做最小平方法(OLS)來找出合適投資組合的 c 與 s 。

In [9]:
```
TSMC_train=prices[["Date","TSMC"]].head(1000)
MT_train=prices[["Date","MT"]].head(1000)
S1=sm.add_constant(TSMC_train['TSMC'])
```

```
/opt/anaconda3/lib/python3.7/site-packages/statsmodels/tsa/tsatool
s.py:130: FutureWarning: In a future version of pandas all argumen
ts of concat except for the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

In [10]:
```
TSMC_test=prices[["Date","TSMC"]].iloc[1000:]
MT_test=prices[["Date","MT"]].iloc[1000:]
Spread=prices[['Date']]
```

In [11]:
```
result=sm.OLS(MT_train['MT'],S1).fit()
```

In [12]:
```
result.summary()
```

Out[12]:

OLS Regression Results

Dep. Variable:	MT	**R-squared:**	0.963
Model:	OLS	**Adj. R-squared:**	0.963
Method:	Least Squares	**F-statistic:**	2.606e+04
Date:	Thu, 08 Dec 2022	**Prob (F-statistic):**	0.00
Time:	21:40:32	**Log-Likelihood:**	-5410.6
No. Observations:	1000	**AIC:**	1.083e+04
Df Residuals:	998	**BIC:**	1.084e+04
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-151.6067	4.536	-33.426	0.000	-160.507	-142.706
TSMC	1.8394	0.011	161.426	0.000	1.817	1.862

Omnibus:	91.835	**Durbin-Watson:**	0.070
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	130.143
Skew:	0.703	**Prob(JB):**	5.49e-29
Kurtosis:	4.071	**Cond. No.**	1.05e+03

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.05e+03. This might indicate that there are strong multicollinearity or other numerical problems.

$$P_{MT} = -151.6067 + 1.8394 \times P_{TSMC}$$

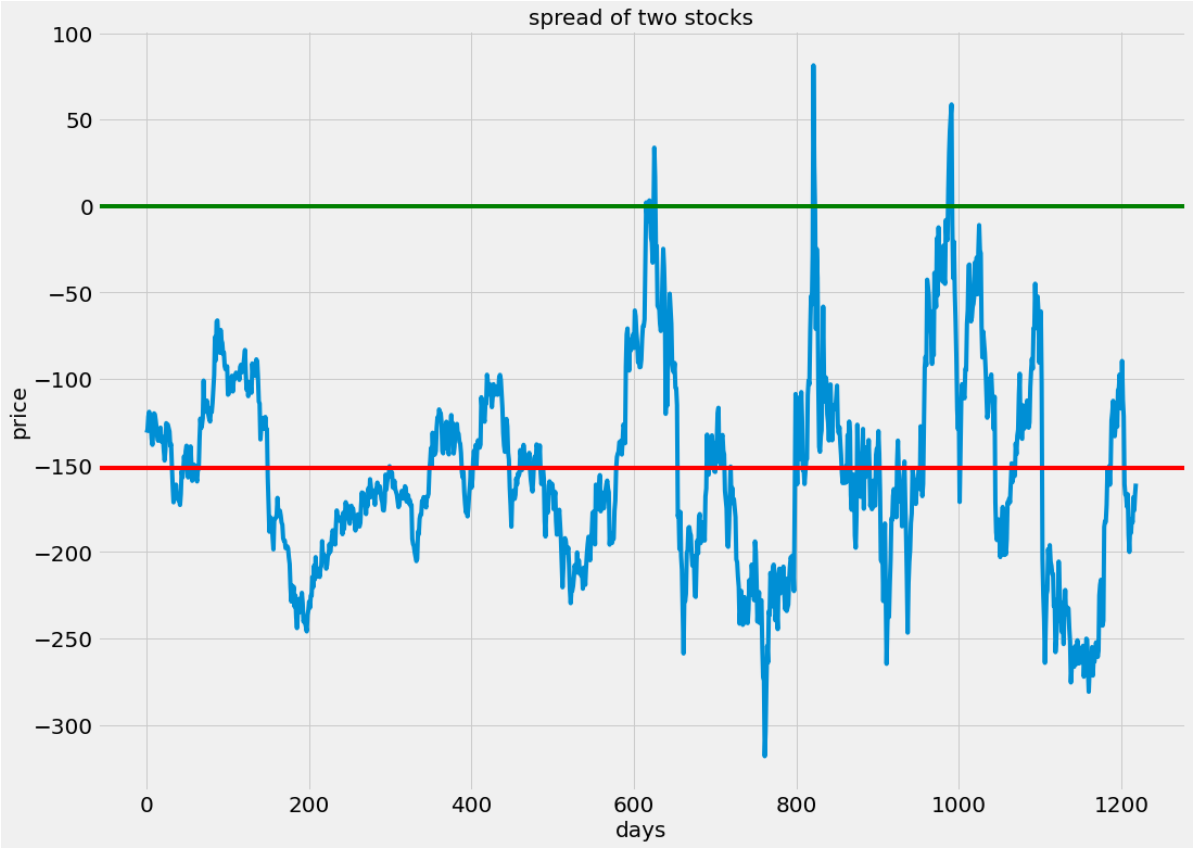
找尋回歸的誤差（$Y - \hat{Y}$）：偏誤的定價

我們期待偏誤的誤差會回復到0，而這段期間正是配對交易的時機。

In [13]:
```
Spread['spread']=prices['MT']-result.params['TSMC']*prices['TSMC']
```

```
/opt/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:1
: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pa
ndas-docs/stable/user_guide/indexing.html#returning-a-view-versus-
a-copy
  """Entry point for launching an IPython kernel.
```

In [14]:
```
plt.figure(figsize=(16,12))
plt.plot(Spread['spread'])
plt.title('spread of two stocks',fontsize=20)
plt.axhline(y=result.params[0],color="r",linestyle='-')
plt.axhline(y=0,color="g",linestyle='-')
plt.xlabel('days',fontsize=20)
plt.ylabel('price',fontsize=20)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
plt.show()
```

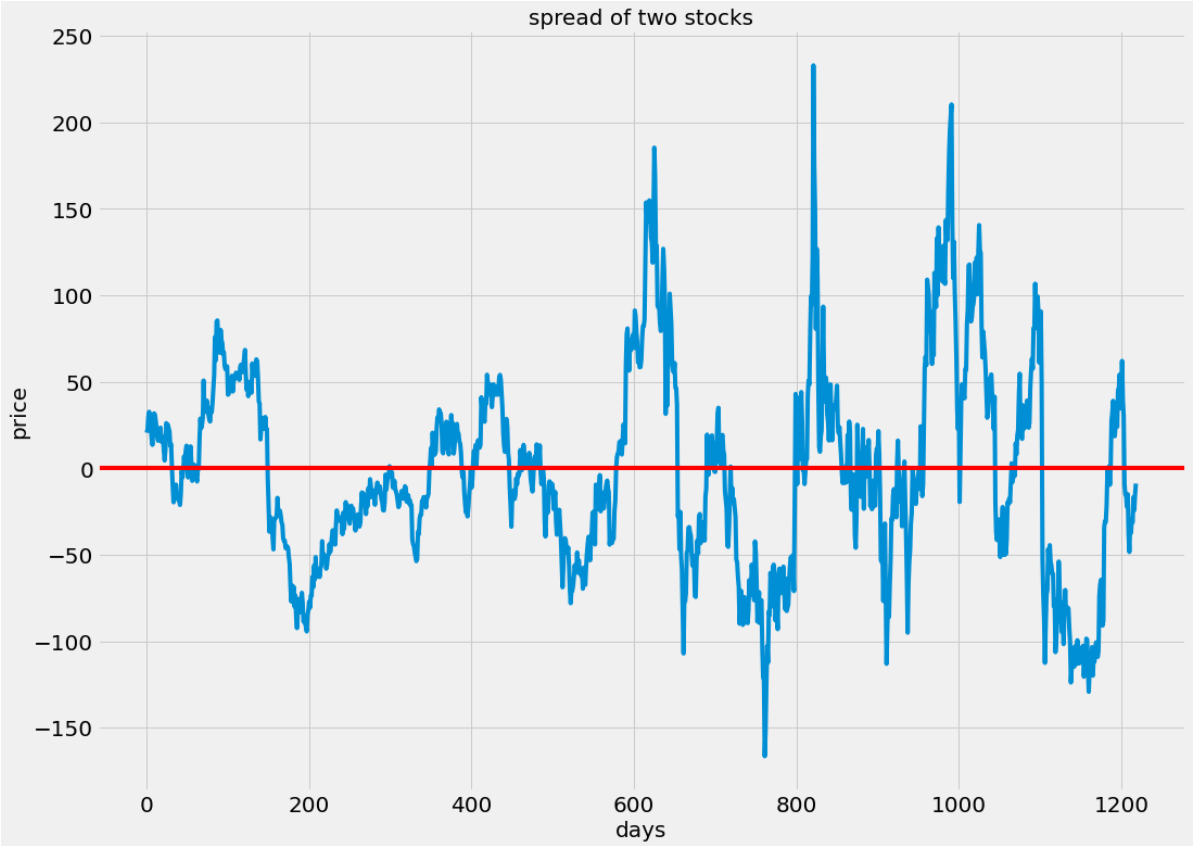


spread 的橫軸原點調整至 s=−151.6067，平移新的水平軸。

In [15]:
```
Spread['spread']=prices['MT']-result.params['TSMC']*prices['TSMC']-
result.params['const']
```

```
/opt/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:1
: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pa
ndas-docs/stable/user_guide/indexing.html#returning-a-view-versus-
a-copy
  """Entry point for launching an IPython kernel.
```

In [16]:
```
plt.figure(figsize=(16,12))
plt.plot(Spread['spread'])
plt.title('spread of two stocks',fontsize=20)
plt.axhline(y=0,color="r",linestyle='-')
plt.xlabel('days',fontsize=20)
plt.ylabel('price',fontsize=20)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
plt.show()
```



假設 A_t 與 B_t 之間為一度共整合，則存在 γ 使得

$$\Rightarrow u_t = A_t - \gamma B_t$$

為一定態的序列且 $\mathbb{E}(u_t) = 0$。

Shapiro-Wilk Test 常態性檢定

H_0 : sample is from the normal distributions. v.s. H_1 : sample isn't from the normal distributions.

In [17]:
```
# conduct the  Shapiro-Wilk Test
shapiro(Spread['spread'])
```

Out[17]:
```
(0.9870583415031433, 6.303703781185277e-09)
```

拒絕 H_0，資料不是常態分佈。

In [18]:
```
Spread['spread'].mean(),Spread['spread'].std()
```

Out[18]:
```
(-2.3235583740619945, 57.716430301159726)
```

配對交易（pair trade）演算法:

設定「做多一單位的聯發科做空c單位的台積電」為一個投資組合，投資組合的殘差為：

$$u_t = P^t_{MT} - c \times P^t_{TSMC} - s$$

1. 設定觸發交易門檻 s_0

2. 若 $u_t > s_0$ ，做空投資組合，也就是做空一單位的聯發科做多c單位的台積電 (Buy group)

3. 若 $u_t < -s_0$ ，做多投資組合，也就是做多一單位的聯發科做空c單位的台積電 (Short group)

4. 當 $u_t = 0$ ，結清部位。(Buy ⇒ Sell & Short ⇒ Cover)

設定交易訊號

In [48]:
```
threshold=93
signals=Spread[['Date']]
signals['buy']=Spread['spread']< -threshold
signals['short']=Spread['spread']> threshold
signals['sell']=(Spread['spread'].shift()<0) & (Spread['spread']>=0
)
signals['cover']=(Spread['spread'].shift()>0) & (Spread['spread']<=
0)
signals.head()
```

Out[48]:

	Date	buy	short	sell	cover
0	2017-12-06	False	False	False	False
1	2017-12-07	False	False	False	False
2	2017-12-08	False	False	False	False
3	2017-12-11	False	False	False	False
4	2017-12-12	False	False	False	False

In [49]:
```
def day(x,y):
    return int(str(pd.to_datetime(x)-pd.to_datetime(y)).split(' ')[
0])
```

交易成本計算

- 融券買入時：交易手續費0.1425%(未折扣)
- 融券賣出時：交易手續費0.1425%(未折扣)、證券交易稅0.3%、借券費0.08%、融券利息0.2%/年
- 交易手續費算法:
  - 做多: 交割金額*0.1425%
  - 做多獲利了結: 交割金額*(0.001425+0.003)
  - 放空:交割金額*0.1425%
  - 保證金:交割金額*0.9，若(首次買進交割金額+(保證金+未實現損益))/目前交割金額>130%, 則直接以當前市價斷頭平倉該次交易結束
  - 放空獲利了結:交割金額*(0.001425+0.003+0.0008+0.002*(持有天數/360))

buy-sell group

In [50]:
```
bs=pd.merge(prices,signals[['Date','buy','sell']],on='Date')
bs=pd.merge(bs,Spread,on='Date')
bs
```

Out[50]:

	Date	TSMC	MT	buy	sell	spread
0	2017-12-06	227.0	286.5	False	False	20.557478
1	2017-12-07	226.5	287.5	False	False	22.477190
2	2017-12-08	227.0	296.0	False	False	30.057478
3	2017-12-11	227.5	299.5	False	False	32.637766
4	2017-12-12	227.5	296.0	False	False	29.137766
...	...	...	...	...	...	...
1214	2022-11-29	487.0	713.0	False	False	-31.192789
1215	2022-11-30	490.0	733.0	False	False	-16.711061
1216	2022-12-01	498.5	741.0	False	False	-24.346166
1217	2022-12-02	492.5	739.0	False	False	-15.309622
1218	2022-12-05	489.0	739.0	False	False	-8.871637

1219 rows × 6 columns

In [51]:
```
tem_spread=0
earn_bs=0
day_buy=[]
day_sell=[]
for i in range(1,bs.shape[0]):
    if tem_spread==0:
        if (bs['buy'][i-1]==False) & (bs['buy'][i]==True):
            tem_spread=bs['MT'][i]*(1+0.001425)-result.params['TSMC
']*bs['TSMC'][i]*(1-0.001425)
            day_buy.append(i)
            date_buy=bs["Date"][i]
    else:
        if (bs['sell'][i-1]==False) & (bs['sell'][i]==True):
            sell_spread=bs['MT'][i]*(1-0.004425)-result.params['TSM
C']*bs['TSMC'][i]*(1+(0.005225+0.002*day(bs["Date"][i],date_buy)/36
0))
            earn_bs+=(sell_spread-tem_spread)
            print('buy:',tem_spread,'sell:',sell_spread,'accumulate
d earn:',earn_bs)
            tem_spread=0
            day_sell.append(i)
day_buy=day_buy[0:len(day_sell)]
earn_bs
```

```
buy: -244.90863742165536 sell: -154.2958123315688 accumulated earn
: 90.61282509008655
buy: -256.74534835930183 sell: -139.9318280022751 accumulated earn
: 207.42634544711328
buy: -252.7073074638297 sell: -118.79994407934305 accumulated earn
: 341.3337088315999
buy: -261.7458451908095 sell: -158.4918907881331 accumulated earn:
444.5876632342763
buy: -243.8605295395971 sell: -137.6032713433059 accumulated earn:
550.8449214305675
buy: -261.88177980574915 sell: -157.3033083406035 accumulated earn
: 655.4233928957132
```

Out[51]: 655.4233928957132

買進（Buy）在相對低點，賣在（Sell）在相對高點。

signal of buy and sell

In [52]:
```
trade_bs=np.zeros(prices.shape[0])
for i in range(len(day_buy)):
    trade_bs[day_buy[i]:day_sell[i]+1]=np.ones(day_sell[i]+1-day_bu
y[i])
df_trade_bs=pd.DataFrame({'trade_bs':trade_bs})
```

short-cover group

In [53]:
```
sc=pd.merge(prices,signals[['Date','short','cover']],on='Date')
sc=pd.merge(sc,Spread,on='Date')
sc
```

Out[53]:

	Date	TSMC	MT	short	cover	spread
0	2017-12-06	227.0	286.5	False	False	20.557478
1	2017-12-07	226.5	287.5	False	False	22.477190
2	2017-12-08	227.0	296.0	False	False	30.057478
3	2017-12-11	227.5	299.5	False	False	32.637766
4	2017-12-12	227.5	296.0	False	False	29.137766
...	...	...	...	...	...	...
1214	2022-11-29	487.0	713.0	False	False	-31.192789
1215	2022-11-30	490.0	733.0	False	False	-16.711061
1216	2022-12-01	498.5	741.0	False	False	-24.346166
1217	2022-12-02	492.5	739.0	False	False	-15.309622
1218	2022-12-05	489.0	739.0	False	False	-8.871637

1219 rows × 6 columns

In [54]:
```
tem_spread=0
earn_sc=0
day_short=[]
day_cover=[]
for i in range(1,bs.shape[0]):
    if tem_spread==0:
        if (sc['short'][i-1]==False) & (sc['short'][i]==True):
            tem_spread=sc['MT'][i]*(1-0.001425)-result.params['TSMC
']*sc['TSMC'][i]*(1+0.001425)
            day_short.append(i)
            date_short=sc["Date"][i]
    else:
        if (sc['cover'][i-1]==False) & (sc['cover'][i]==True):
            cover_spread=sc['MT'][i]*(1+(0.005225+0.002*day(bs["Dat
e"][i],date_short)/360))-result.params['TSMC']*sc['TSMC'][i]*(1-0.0
04425)
            earn_sc+=(-cover_spread+tem_spread)
            print('short:',tem_spread,'cover:',cover_spread,'accumu
lated earn:',earn_sc)
            tem_spread=0
            day_cover.append(i)
day_short=day_short[0:len(day_cover)]
earn_sc
```

```
short: -34.10129148081364 cover: -172.51676261552518 accumulated e
arn: 138.41547113471154
short: -55.17237258599812 cover: -144.55680207047965 accumulated e
arn: 227.79990061919307
short: -45.71988343478165 cover: -159.72141109153108 accumulated e
arn: 341.8014282759425
short: -37.62491409333893 cover: -161.6182599820114 accumulated ea
rn: 465.794774164615
short: -47.609249951768675 cover: -196.43891239698144 accumulated
earn: 614.6244366098277
```

Out[54]:
```
614.6244366098277
```

賣空（Short）在相對高點，回補（Cover）在相對低點。

signal of short and cover

In [55]:
```
trade_sc=np.zeros(prices.shape[0])
for i in range(len(day_short)):
    trade_sc[day_short[i]:day_cover[i]+1]=np.ones(day_cover[i]+1-da
y_short[i])
df_trade_sc=pd.DataFrame({'trade_sc':trade_sc})
```

一單位的投資組合下所賺取的金額

In [56]:
```
earn_bs+earn_sc
```

Out[56]: 1270.047829505541

Find optimal threshold

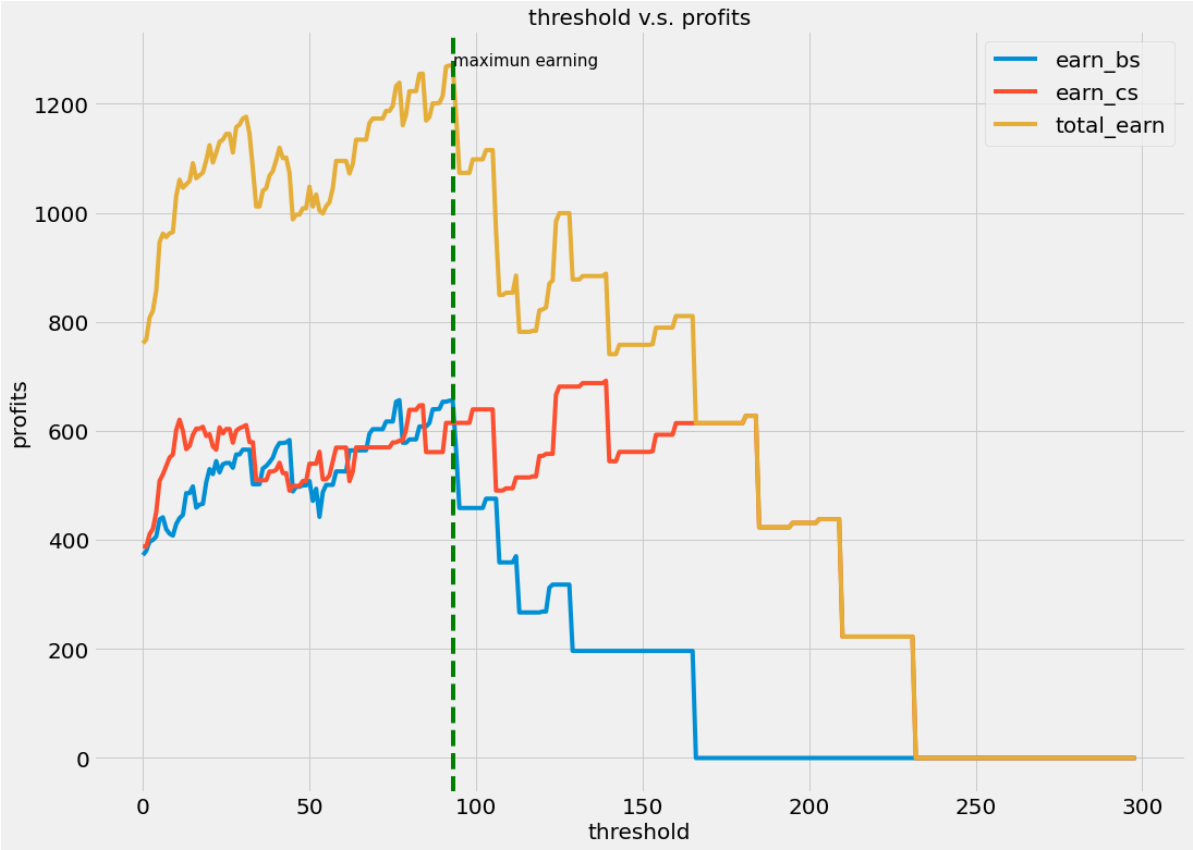
In [45]:
```
threshold_set=[i for i in range(1,300)]
earn_bs_set=[]
earn_sc_set=[]
total_earn_set=[]

for i in range(len(threshold_set)):
    threshold=threshold_set[i]
    signals=Spread[['Date']]
    signals['buy']=Spread['spread']< -threshold
    signals['short']=Spread['spread']> threshold
    signals['sell']=(Spread['spread'].shift()<0) & (Spread['spread'
]>=0)
    signals['cover']=(Spread['spread'].shift()>0) & (Spread['spread
']<=0)
    signals.head()


    # buy-sell group

    bs=pd.merge(prices,signals[['Date','buy','sell']],on='Date')
    bs=pd.merge(bs,Spread,on='Date')

    tem_spread=0
    earn_bs=0
    for i in range(1,bs.shape[0]):
        if tem_spread==0:
            if (bs['buy'][i-1]==False) & (bs['buy'][i]==True):
                tem_spread=bs['MT'][i]*(1+0.001425)-result.params['
TSMC']*bs['TSMC'][i]*(1-0.001425)
                day_buy.append(i)
                date_buy=bs["Date"][i]
        else:
            if (bs['sell'][i-1]==False) & (bs['sell'][i]==True):
                sell_spread=bs['MT'][i]*(1-0.004425)-result.params[
'TSMC']*bs['TSMC'][i]*(1+(0.005225+0.002*day(bs["Date"][i],date_buy
)/360))
                earn_bs+=(sell_spread-tem_spread)
                tem_spread=0
                day_sell.append(i)
    earn_bs_set.append(earn_bs)
```

```
    # short-cover group
    sc=pd.merge(prices,signals[['Date','short','cover']],on='Date')
    sc=pd.merge(sc,Spread,on='Date')

    tem_spread=0
    earn_sc=0
    for i in range(1,sc.shape[0]):
        if tem_spread==0:
            if (sc['short'][i-1]==False) & (sc['short'][i]==True):
                tem_spread=sc['MT'][i]*(1-0.001425)-result.params['
TSMC']*sc['TSMC'][i]*(1+0.001425)
                day_short.append(i)
                date_short=sc["Date"][i]
        else:
            if (sc['cover'][i-1]==False) & (sc['cover'][i]==True):
                cover_spread=sc['MT'][i]*(1+(0.005225+0.002*day(bs[
"Date"][i],date_short)/360))-result.params['TSMC']*sc['TSMC'][i]*(1
-0.004425)
                earn_sc+=(-cover_spread+tem_spread)
                tem_spread=0
                day_cover.append(i)
    earn_sc_set.append(earn_sc)

    total_earn_set.append(earn_bs+earn_sc)
```

In [46]:
```
index=total_earn_set.index(max(total_earn_set))
print('threshold:',threshold_set[index],'  ','profit:',max(total_ea
rn_set))
```

```
threshold: 93    profit: 1270.047829505541
```

In [76]:
```
df_earn=pd.DataFrame({'earn_bs':earn_bs_set,'earn_sc':earn_sc_set,'
total_earn':total_earn_set})
plt.figure(figsize=(16,12))
plt.plot(df_earn)
plt.title('threshold v.s. profits',fontsize=20)
plt.legend(['earn_bs','earn_cs','total_earn'], fontsize=20,title_fo
ntsize=20)
plt.axvline(x=threshold_set[index],color="g",linestyle='--')
plt.xlabel('threshold',fontsize=20)
plt.ylabel('profits',fontsize=20)
plt.annotate('maximun earning',(threshold_set[index],max(total_earn
_set)),fontsize=15)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
plt.show()
```



In [69]:
```
fig=plt.figure(figsize=(20,16))
ax1=fig.add_subplot(311)
ax1.plot(Spread[['spread']])
plt.axhline(y=-threshold,color="r",linestyle='--')
plt.axhline(y=threshold,color="r",linestyle='--')
plt.legend(['spread','threshold'])
plt.title('Spread of portfolio')
plt.ylabel('price',fontsize=20)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
ax1=fig.add_subplot(312)
ax1.plot(df_trade_bs,color='orange')
plt.title('signal of buy and sell')
plt.ylabel('signal',fontsize=20)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
ax1=fig.add_subplot(313)
ax1.plot(df_trade_sc,color='g')
plt.title('signal of short and cover')
plt.ylabel('signal',fontsize=20)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
plt.xlabel('days',fontsize=20)
plt.show()
```