$p^{n\bar{x}}(1-p)^{n-n\bar{x}}$

$\frac{1}{2}$, $\frac{3}{10}$

P_{195}, 5:

(1) $X \sim N(\mu, \sigma^2)$, $\{X_i, i=1,\dots,n\}$ 是样本. $\therefore X_i \sim N(\mu, \sigma^2)$ 且相互独立

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N\left(\frac{1}{n}\sum_{i=1}^{n} EX_i, \frac{1}{n^2}\sum_{i=1}^{n} DX_i\right) = N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\therefore \frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}} = \frac{\bar{X}-\mu}{\sigma}\sqrt{n} \sim N(0,1).$$

$n=16$, $\sigma=5$

$$p(|\bar{X}-\mu|<2) = p\left(\left|\frac{\bar{X}-\mu}{5}\sqrt{16}\right| < \frac{2}{5}\sqrt{16}\right) = p\left(-\frac{8}{5} < \left|\frac{\bar{X}-\mu}{5}4\right| < \frac{8}{5}\right)$$

$$= 2\Phi(1.6)-1 = 2\cdot 0.9452-1 = 0.8904.$$

(2). $\frac{\bar{X}-\mu}{S}\sqrt{n} \sim t(n-1)$, $n=16$. $S=\sqrt{20.8}$, (用观察值代替样本值,且体计算中)

$$\frac{\bar{X}-\mu}{\sqrt{20.8}}\sqrt{16} = T_{15} \sim t(16-1)$$

$$p(|\bar{X}-\mu|<2) = p\left(|T_{15}| < \frac{2}{\sqrt{20.8}}\cdot\sqrt{16}\right) = p(|T_{15}| < 1.754)$$

$$= p(-1.754 < T_{15} < 1.754) = 1-\alpha.$$

$\Rightarrow t_{\frac{\alpha}{2}}(15) = 1.754 \underset{P_{373}}{\Rightarrow} \alpha/2 = 0.05 \Rightarrow 1-\alpha = 0.9.$

P_{196}, 9:

$X_{n+1} \sim N(\mu, \sigma^2)$, $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$, X_{n+1}, $\bar{X}$ 相互独立

$$X_{n+1}-\bar{X} \sim N\left(0, \sigma^2+\frac{\sigma^2}{n}\right) = N\left(0, \frac{n+1}{n}\sigma^2\right)$$

$$\frac{X_{n+1}-\bar{X}}{\sqrt{\frac{n+1}{n}\sigma^2}} \sim N(0,1). \text{ 又} \because \frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2(n-1), \therefore \frac{\frac{X_{n+1}-\bar{X}}{\sqrt{\frac{n+1}{n}\sigma^2}}}{\sqrt{\frac{(n-1)S_n^2}{\sigma^2}/(n-1)}} = \frac{X_{n+1}-\bar{X}}{S_n}\sqrt{\frac{n}{n+1}} \sim t(n-1)$$

相互独立

S_n^2 与 $\bar{X}$ 相互独立

P_{196}, 10:

令 $X \sim N(0,1)$, $Y \sim \chi^2(n)$, 且 X, Y 相互独立.

则 $T = \frac{X}{\sqrt{Y/n}} \sim t(n)$, $T^2 = \frac{X^2/1}{Y/n} = F(1, n)$

($\because X^2 \sim \chi^2(1)$, 且 X^2, Y 相互独立
$Y \sim \chi^2(n)$)

P_{226}, 1,

$f(x,\theta) = \theta^{2-x}(1-\theta)^{x-1}$

1): $\alpha_1 = EX = 1\cdot P(X=1) + 2\cdot P(X=2) = \theta + 2\cdot(1-\theta) = 2-\theta == A_1 = \overline{X}$

$\Rightarrow \hat{\theta} = 2-\overline{X}$

2): $L(\theta) = \prod_{i=1}^{n} f(x_i,\theta) = \prod_{i=1}^{n} \theta^{2-x_i}(1-\theta)^{x_i-1} = \theta^{2n-n\overline{X}}(1-\theta)^{n\overline{X}-n}$

$\ln L(\theta) = (2n-n\overline{X})\ln\theta + (n\overline{X}-n)\ln(1-\theta)$

$\frac{\partial}{\partial\theta}\ln L(\theta) = \frac{2n-n\overline{X}}{\theta} - \frac{n\overline{X}-n}{1-\theta} = 0$

$\Rightarrow \hat{\theta} = 2-\overline{X}$

P_{227}, 6,

1): $\alpha_1 = EX = \int_0^{+\infty} \frac{x^2}{\theta} e^{-\frac{x^2}{2\theta}} dx = \sqrt{2\theta}\int_0^{+\infty}\left(\frac{x^2}{2\theta}\right)^{\frac{1}{2}} e^{-\frac{x^2}{2\theta}} d\left(\frac{x^2}{2\theta}\right)$

$= \sqrt{2\theta}\,\Gamma\left(\frac{3}{2}\right) = \sqrt{\frac{\pi\theta}{2}} == A_1 = \overline{X}$

$\Rightarrow \hat{\theta} = \frac{2\overline{X}^2}{\pi}$

2): $L(\theta) = \prod_{i=1}^{n} f(x_i,\theta) = \prod_{i=1}^{n} \frac{x_i}{\theta} e^{-\frac{x_i^2}{2\theta}} = \frac{\prod_{i=1}^{n} x_i}{\theta^n} e^{-\sum_{i=1}^{n}\frac{x_i^2}{2\theta}}$

$\ln L(\theta) = \ln\prod_{i=1}^{n} x_i - n\ln\theta - \frac{\sum_{i=1}^{n} x_i^2}{2\theta}$

$\frac{\partial}{\partial\theta}\ln L(\theta) = -\frac{n}{\theta} + \frac{\sum_{i=1}^{n} x_i^2}{2\theta^2} = 0$

$\Rightarrow \hat{\theta} = \frac{\sum_{i=1}^{n} x_i^2}{2n}$

P_{228}, 12,

令 $S_1^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \frac{1}{n}\sum_{j=1}^{n} x_j\right)^2$ 为 $x_1, \cdots, x_n$ 的样本方差

$S_2^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_{n+i} - \frac{1}{n}\sum_{j=1}^{n} X_{n+j}\right)^2$ 为 $X_{n+1}, \cdots, X_{2n}$ 的样本方差

$E\left(\sum_{i=1}^{n}(X_i + X_{n+i} - 2\overline{X})^2\right) = E\left(\sum_{i=1}^{n}\left(x_i - \frac{1}{n}(x_1+\cdots+x_n) + X_{n+i} - \frac{1}{n}(X_{n+1}+\cdots+X_{2n})\right)^2\right)$

$= E\left((n-1)S_1^2 + (n-1)S_2^2\right) + \sum_{i=1}^{n} \underbrace{E\left(X_i - \frac{1}{n}\sum_{j=1}^{n} x_j\right)}_{=0} \underbrace{E\left(X_{n+i} - \frac{1}{n}\sum_{j=1}^{n} X_{n+j}\right)}_{=0}$

$= (n-1)\left(ES_1^2 + ES_2^2\right) = 2(n-1)\sigma^2$

$\therefore E\hat{\sigma}^2 = C\cdot 2(n-1)\sigma^2 = \sigma^2 \Rightarrow C = \frac{1}{2(n-1)}$

P_{228}，13，

1) $E\hat{\theta}_1 = E\overline{X} = EX = \int_0^{+\infty} \frac{x}{\theta} e^{-\frac{x}{\theta}} dx = \theta \int_0^{+\infty} t e^{-t} dt$

$= \theta \Gamma(2) = \theta$

$\therefore \hat{\theta}_1$ 是 θ 的无偏估计

令 $T = \min\{X_1, \cdots, X_n\}$

$f_T(t) = n(1-F(t))^{n-1} f(t)$

$$= \begin{cases} n(e^{-\frac{t}{\theta}})^{n-1} \frac{1}{\theta} e^{-\frac{t}{\theta}} = \frac{n}{\theta} e^{-\frac{nt}{\theta}}, & t>0 \\ 0, & t \le 0 \end{cases}$$

$\int_0^x \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = -\int_0^x e^{-\frac{x}{\theta}} d(-\frac{x}{\theta})$

$= -e^{-\frac{x}{\theta}} \Big|_0^x = 1 - e^{-\frac{x}{\theta}}$

$$F(x) = \begin{cases} \int_0^x f(x)dx = 1 - e^{-\frac{x}{\theta}}, & x>0 \\ 0, & x<0 \end{cases}$$

$ET = \int_0^{+\infty} t f_T(t) dt = \int_0^{+\infty} \frac{nt}{\theta} e^{-\frac{nt}{\theta}} dt \overset{r=\frac{nt}{\theta}}{=\!=} \frac{\theta}{n} \int_0^{+\infty} r e^{-r} dr = \frac{\theta}{n}\Gamma(2) = \frac{\theta}{n}$

$\therefore E\hat{\theta}_2 = nET = \theta$

$\therefore \hat{\theta}_2$ 是 θ 的无偏估计

2) $D\hat{\theta}_1 = D\overline{X} = \frac{n}{n^2} DX = \frac{1}{n}(EX^2 - (EX)^2)$

$EX^2 = \int_0^{+\infty} \frac{x^2}{\theta} e^{-\frac{x}{\theta}} dx = \theta^2 \int_0^{+\infty} (\frac{x}{\theta})^2 e^{-\frac{x}{\theta}} d(\frac{x}{\theta}) = \theta^2 \Gamma(3) = 2\theta^2$

$\therefore D\hat{\theta}_1 = \frac{1}{n}\theta^2$

$ET^2 = \int_0^{+\infty} t^2 f_T(t) dt = \int_0^{+\infty} \frac{nt^2}{\theta} e^{-\frac{nt}{\theta}} dt \overset{r=\frac{nt}{\theta}}{=\!=} (\frac{\theta}{n})^2 \int_0^{+\infty} r^2 e^{-r} dr$

$= (\frac{\theta}{n})^2 \Gamma(3)$

$= 2\frac{\theta^2}{n^2}$

$\therefore D\hat{\theta}_2 = n^2 DT = n^2(ET^2 - (ET)^2) = n^2(\frac{\theta^2}{n^2}) = \theta^2$

$\therefore D\hat{\theta}_2 > D\hat{\theta}_1$

$\therefore \hat{\theta}_1$ 比 $\hat{\theta}_2$ 有效.

P229. 19.

$\overline{X}=5899$, $S^2=1.72137\times10^6$, $S=1312$

1) $P(\overline{X}-\frac{S}{\sqrt{n}}t_{0.025}(n-1)<\mu<\overline{X}+\frac{S}{\sqrt{n}}t_{0.025}(n-1))=1-0.05$

$n=10 \Rightarrow 0.95$置信区间 of μ 为 $(4960.52, 6837.48)$

2) $P(\frac{(n-1)S^2}{\chi^2_{0.025}(n-1)}<\sigma^2<\frac{(n-1)S^2}{\chi^2_{0.975}(n-1)})=1-0.05$

$n=10 \Rightarrow 0.95$置信区间 of σ^2 为 $(814400, 5.7379\times10^6)$

(单位: mg)