$P_{173}.2$:

令 $\overline{X_n}=\frac{1}{n}\sum_{i=1}^{n}X_i$. $D(\sum_{i=1}^{n}X_i)<+\infty \Rightarrow D\overline{X_n}<+\infty$ 则 $E\overline{X_n}$ 存在（不考虑定义细节）

由切比雪夫不等式：

$$P(|\overline{X_n}-E\overline{X_n}|\geq\varepsilon)\leq\frac{D\overline{X_n}}{\varepsilon^2}=\frac{D(\sum_{i=1}^{n}X_i)}{\varepsilon^2 n^2}$$

当 $n\to+\infty$ 时 $0\leq\lim_{n\to\infty}P(|\overline{X_n}-E\overline{X_n}|\geq\varepsilon)=\frac{1}{\varepsilon^2}\lim_{n\to\infty}\frac{1}{n^2}D(\sum_{i=1}^{n}X_i)=0.$

$\therefore \overline{X_n}\xrightarrow{P}E\overline{X_n}$ $\therefore \{X_n\}$ 服从大数定律.

该定理也称为"马尔可夫"大数定律.

$P_{174}.4$:

设 X_i 表示每页印刷错误，$i=1,2,\cdots,400$.

$X_i\sim P(0.2)$，$\therefore EX_i=0.2$，$DX_i=0.2$

$$P(\sum_{i=1}^{400}X_i\leq90)=P\left(\frac{\sum_{i=1}^{400}X_i-E(\sum_{i=1}^{400}X_i)}{\sqrt{D(\sum_{i=1}^{400}X_i)}}\leq\frac{90-E(\sum_{i=1}^{400}X_i)}{\sqrt{D(\sum_{i=1}^{400}X_i)}}\right)$$

（$\overline{X_n}$ 或 $\sum_{i=1}^{n}X_i$ 的标准化随机变量. 在 $n\to\infty$ 时服从 $N(0,1)$.）

$$=P\left(\frac{\sum_{i=1}^{400}X_i-400\cdot0.2}{\sqrt{400\cdot0.2}}\leq\frac{10}{\sqrt{80}}\right)\simeq\Phi\left(\frac{10}{\sqrt{80}}\right)=\Phi(1.12)=0.86864$$

书后答案为超过90个的概率 $1-0.86864=0.13136$.

$P_{174}.8$:

$P(X_i=1)=\frac{1}{6}$，$P(X_i=2)=\frac{1}{3}$，$P(X_i=3)=\frac{1}{2}$，$i=1,2,\cdots,n$

$EX_i=\frac{1}{6}+2\cdot\frac{1}{3}+3\cdot\frac{1}{2}=\frac{7}{3}$

$DX_i=EX_i^2-(EX_i)^2=\frac{1}{6}+4\cdot\frac{1}{3}+9\cdot\frac{1}{2}-\left(\frac{7}{3}\right)^2=\frac{5}{9}$，$\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$

$E\overline{X}=\frac{7}{3}$

$D\overline{X}=\frac{5}{9n}$.

$$P(|\overline{X}-\tfrac{7}{3}|<0.1)=P\left(\left|\underbrace{\frac{\overline{X}-E\overline{X}}{\sqrt{D\overline{X}}}}_{\sim N(0,1)}\right|<\frac{0.1}{\sqrt{\frac{5}{9n}}}\right)=2\Phi\left(\frac{0.3\sqrt{n}}{\sqrt{5}}\right)-1\geq0.6826.$$

$\Rightarrow\Phi\left(\frac{0.3\sqrt{n}}{\sqrt{5}}\right)\geq0.8413=\Phi(1)\Rightarrow\frac{0.3\sqrt{n}}{\sqrt{5}}\geq1\Rightarrow n\geq55.5556.$ 取 $n\geq56$.

P174, 9:

1) 设X表示取到3号球的个数, $X \sim B(100, \frac{1}{2})$

$EX = 100 \cdot \frac{1}{2} = 50$, $DX = 100 \cdot \frac{1}{2} \cdot \frac{1}{2} = 25$

$$P\left(\frac{X}{100} \geqslant \frac{3}{5}\right) = P(X \geqslant 60) = P\left(\frac{X-50}{\sqrt{25}} \geqslant \frac{60-50}{\sqrt{25}}\right)$$

$= 1 - \Phi(2)$

$\simeq 0.02275$

2) $X \sim B(n, \frac{1}{2})$, $EX = \frac{1}{2}n$, $DX = \frac{1}{4}n$

$$P\left(\frac{X}{n} < 0.51\right) = P(X < 0.51n) = P\left(\frac{X-0.5n}{\sqrt{0.25n}} < \frac{0.51n - 0.5n}{\sqrt{0.25n}}\right)$$

$= \Phi(0.02\sqrt{n}) \geqslant 0.8413 \simeq \Phi(1)$

$\therefore 0.02\sqrt{n} \geqslant 1 \Rightarrow n \geqslant 2500.$

3) $X \sim B(225, \frac{1}{2})$

$$P\left(\left|\frac{X}{225} - \frac{1}{2}\right| < \varepsilon\right) \simeq 2\Phi\left(\varepsilon\sqrt{\frac{225}{0.25}}\right) - 1 = 2\Phi(30\varepsilon) - 1 = 0.95$$

$\Rightarrow \Phi(30\varepsilon) = 0.975 \simeq \Phi(1.96)$

$\Rightarrow \varepsilon \simeq \frac{1.96}{30} \simeq 0.065.$

P_{195}. 7:

1) $X_i \sim N(0, 0.5^2)$, $\frac{X_i}{0.5} \sim N(0,1)$

$\chi^2 = \sum_{i=1}^{10} \left(\frac{X_i}{0.5}\right)^2 = 4\sum_{i=1}^{10} X_i^2 \sim \chi^2(10)$

令 $Y = \sum_{i=1}^{10} X_i^2$，则 $\chi^2 = 4Y$

$F_Y(y) = P(Y \leq y) = P\left(\frac{\chi^2}{4} \leq y\right) = P(\chi^2 \leq 4y)$

$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} P(\chi^2 \leq 4y) = 4 f_{\chi^2}(4y)$

2) $P\left(\sum_{i=1}^{10} X_i^2 > 4\right) = P(Y > 4) = \int_4^{+\infty} f_Y(y)\,dy$

$= \int_4^{+\infty} 4 f_{\chi^2}(4y)\,dy = \int_{16}^{+\infty} f_{\chi^2}(z)\,dz$

→ 自由度为10

$= P(\chi^2 \geq 16) \simeq 0.1$

3) $P\left(\sum_{i=1}^{10} (X_i - \overline{X})^2 > 2.85\right)$，令 $\chi^2 = \sum_{i=1}^{10} \frac{(X_i - \overline{X})^2}{0.25} \sim \chi^2(9)$

P_{190}. 6.3.2

$\Rightarrow P\left(\sum_{i=1}^{10} (X_i - \overline{X})^2 = \frac{1}{4}\chi^2 > 2.85\right)$

$= P(\chi^2 > 11.4)$ → 自由度为9

$= \int_{11.4}^{+\infty} \frac{\chi^{\frac{9}{2}-1} e^{-\frac{\chi}{2}}}{2^{\frac{9}{2}} \Gamma(\frac{9}{2})}\,d\chi$

$\simeq 0.24928$.