$P_{122}.1:$

$$\left.\begin{array}{l} F(+\infty,+\infty)=1 \Rightarrow A(B+\frac{\pi}{2})(\frac{\pi}{2}+\frac{\pi}{2})=1 \\ F(-\infty,y)=0 \Rightarrow A(B-\frac{\pi}{2})(\arctan\frac{y}{2}+\frac{\pi}{2})=0 \end{array}\right\} \Rightarrow A=\frac{1}{\pi^2},\ B=\frac{\pi}{2}$$

$P_{123}, 2:$

令 $A:\{X\le x\}$, $B:\{Y\le y\}$

$$F(x,y)=P(AB)=1-P(\overline{AB})=1-P(\overline{A}\cup\overline{B})=1-P(\overline{A})-P(\overline{B})+P(\overline{A}\overline{B})$$

$$\ge 1-P(\overline{A})-P(\overline{B})=1-P(X>x)-P(Y>y)$$

$$=1-(1-P(X\le x))-(1-P(Y\le y))$$

$$=1-(1-F_X(x))-(1-F_Y(y)).$$

$P_{123}, 4:$

$$P(X=i,Y=j)=\frac{C_2^i C_2^j C_3^{3-i-j}}{C_7^3},\quad i,j=0,1,2.\quad i+j\le 3$$

$P_{123}, 8:$

(1) $1=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x,y)\,dxdy=\int_0^1\int_0^1 kxy\,dxdy=k/4 \Rightarrow k=4$

(2) $F(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}f(x,y)\,dxdy$

① 当 $x<0$ 或 $y<0$ 时:

$F(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}0\,dxdy=0$

情形①的反面:

$x\ge 0$ 且 $y\ge 0$

②③④⑤再对其细分.

② 当 $0\le x<1$ 且 $0\le y<1$ 时:

$F(x,y)=\int_0^x\int_0^y 4xy\,dxdy=x^2\cdot y^2$

③ 当 $0\le x<1$ 且 $y\ge 1$ 时:

$F(x,y)=\int_0^x\int_0^1 4xy\,dxdy=x^2$

④ 当 $x\ge 1$ 且 $0\le y<1$ 时:

$F(x,y)=\int_0^1\int_0^y 4xy\,dxdy=y^2$

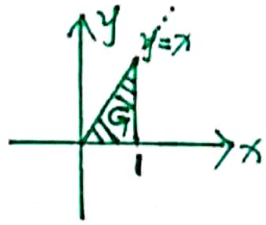
⑤ 当 $x\ge 1$ 且 $y\ge 1$ 时:

$F(x,y)=\int_0^1\int_0^1 4xy\,dxdy=1$

$$\therefore F(x,y)=\begin{cases}0, & x<0 \text{或} y<0\\ x^2y^2, & 0\le x<1,\ 0\le y<1\\ x^2, & 0\le x<1,\ y\ge 1\\ y^2, & x\ge 1,\ 0\le y<1\\ 1, & x\ge 1,\ y\ge 1\end{cases}$$

(3) $$P(Y\le x)=\iint\limits_{G} f(x,y)\,dx\,dy=\int_0^1\left(\int_0^x 4xy\,dy\right)dx$$

$$=\int_0^1 2x^3\,dx=\frac{1}{2}$$



P_{123}, 10:

X \ Y	1	3	$P_{\cdot j}$
0	0	$C_3^0\cdot\frac{1}{8}$	$\frac{1}{8}$
1	$C_3^1\cdot\frac{1}{8}$	0	$\frac{3}{8}$
2	$C_3^2\cdot\frac{1}{8}$	0	$\frac{3}{8}$
3	0	$C_3^3\cdot\frac{1}{8}$	$\frac{1}{8}$
$P_{i\cdot}$	$\frac{3}{4}$	$\frac{1}{4}$	

P_{123}, 11:

X的边缘分布律:

(1) $$P(X=i)=\sum_{j=0}^{n}P(X=i,Y=j)=\sum_{j=0}^{n-i}\frac{n!}{i!\,j!\,(n-i-j)!}P_1^iP_2^jP_3^{n-i-j}$$

$$=\frac{n!}{i!\,(n-i)!}P_1^i\sum_{j=0}^{n-i}\frac{(n-i)!}{j!\,(n-i-j)!}P_2^jP_3^{n-i-j}$$

$$=\frac{n!}{i!\,(n-i)!}P_1^i\,(P_2+P_3)^{n-i}$$

$$=C_n^i P_1^i(1-P_1)^{n-i},\quad i=0,1,\cdots,n,\quad X\sim B(n,P_1)$$

同理对i从0到n-j求和得:

$$P(Y=j)=C_n^jP_2^j(1-P_2)^{n-j},\quad j=0,1,\cdots,n,\quad Y\sim B(n,P_2).$$

(2) $$P(X=i\mid Y=j)=\frac{P(X=i,Y=j)}{P(Y=j)}=\frac{\frac{n!}{i!\,j!\,(n-i-j)!}P_1^iP_2^jP_3^{n-i-j}}{\frac{n!}{j!\,(n-j)!}P_2^j(1-P_2)^{n-j}}$$

$$=\frac{(n-j)!}{i!\,(n-i-j)!}\cdot\frac{P_1^i}{(1-P_2)^i}\cdot\frac{P_3^{n-i-j}}{(1-P_2)^{n-i-j}}$$

$$=C_{n-j}^i\left(\frac{P_1}{P_1+P_3}\right)^i\left(\frac{P_3}{P_1+P_3}\right)^{n-i-j}$$

令 $P_1=\frac{\#A_1}{\#\Omega}$

$P_2=\frac{\#A_2}{\#\Omega}$

$P_3=\frac{\#A_3}{\#\Omega}$

$\frac{P_1}{P_1+P_3}=\frac{\#A_1}{\#A_1+\#A_3}\to$ 仅限空间大小.

P_{124}, 13:

(1)

$$f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy=\begin{cases}\int_0^2 \left(x^2+\frac{xy}{3}\right)dy = 2x^2+\frac{2}{3}x, & 0\leq x\leq 1\\ 0, & \text{其他}\end{cases}$$

$$f_Y(y)=\int_{-\infty}^{+\infty} f(x,y)\,dx=\begin{cases}\int_0^1 \left(x^2+\frac{xy}{3}\right)dx = \frac{1}{3}+\frac{y}{6}, & 0\leq y\leq 2\\ 0, & \text{其他}\end{cases}$$

(2)

$$\begin{cases}\text{当}\ 0\leq y\leq 2\ \text{时},\ f_{X|Y}(x|y)=\dfrac{f(x,y)}{f_Y(y)}=\begin{cases}\dfrac{6x^2+2xy}{2+y}, & 0\leq x\leq 1\\ 0, & \text{其他}\end{cases}\\ \text{当}\ y<0\ \text{或}\ y>2\ \text{时},\ f_{X|Y}(x|y)\ \text{无定义}.\end{cases}$$

$$\begin{cases}\text{当}\ 0<x\leq 1\ \text{时},\ f_{Y|X}(y|x)=\dfrac{f(x,y)}{f_X(x)}=\begin{cases}\dfrac{3x+y}{6x+2}, & 0\leq y\leq 2\\ 0, & \text{其他}\end{cases}\\ \text{当}\ x\leq 0\ \text{或}\ x>1\ \text{时},\ f_{Y|X}(y|x)\ \text{无定义}.\end{cases}$$

(3)

$$P\left(Y\leq\frac{1}{2}\,\middle|\,X\leq\frac{1}{2}\right)=\frac{\int_{-\infty}^{\frac{1}{2}}\int_{-\infty}^{\frac{1}{2}} f(x,y)\,dx\,dy}{\int_{-\infty}^{\frac{1}{2}} f_X(x)\,dx}=\frac{\int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}}\left(x^2+\frac{xy}{3}\right)dx\,dy}{\int_0^{\frac{1}{2}}\left(2x^2+\frac{2}{3}x\right)dx}$$

$$=\frac{5}{32}.$$

P124, 16:

(1) 已知：$f_X(x)=\begin{cases}1, & 0<x<1\\ 0, & 其他\end{cases}$

当 $0<x<1$ 时，$f_{Y|X}(y|x)=\begin{cases}\frac{1}{x}, & 0<y<x\\ 0, & 其他\end{cases}$

$\therefore f(x,y)=f_X(x)\cdot f_Y(y|x)=\begin{cases}\frac{1}{x}, & 0<y<x<1\\ 0, & 其他\end{cases}$

验证：$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x,y)dxdy=\int_0^1\left(\int_0^x\frac{1}{x}dy\right)dx=1$

(2)

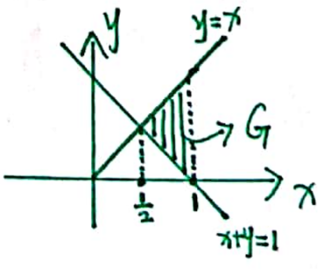
$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)dx=\begin{cases}\int_y^1\frac{1}{x}dx=-\ln y, & 0<y<1\\ 0, & 其他\end{cases}$

验证：$\int_{-\infty}^{+\infty}f_Y(y)dy=\int_0^1 -\ln y\,dy=\int_{-\infty}^0 -z\,de^z$ (令 $\ln y=z$)

$=-ze^z\Big|_{-\infty}^0+\int_{-\infty}^0 e^z dz$

$=1$

(3)



$P(X+Y>1)=\iint_G f(x,y)\,dxdy=\int_{\frac{1}{2}}^1\left(\int_{1-x}^x\frac{1}{x}dy\right)dx$

$=\int_{\frac{1}{2}}^1\frac{1}{x}(2x-1)dx=1-\ln 2$

P_{124}, 17:

$X \backslash Y$	y_1	y_2	$P_{i\cdot}$
x_1	a	$\frac{1}{8}$	$\frac{1}{6}$
x_2	$\frac{1}{8}$	b	$b+\frac{1}{8}$
x_3	c	d	$c+d$
$P_{\cdot j}$	$a+c+\frac{1}{8}$	$b+d+\frac{1}{8}$	

$a=\frac{1}{6}-\frac{1}{8}=\frac{1}{24}$

X与Y相互独立

$a=\frac{1}{6}(a+c+\frac{1}{8}) \Rightarrow c=\frac{1}{12}$

$c=(c+d)(a+c+\frac{1}{8}) \Rightarrow d=\frac{1}{4}$

$\frac{1}{8}=\frac{1}{6}(b+d+\frac{1}{8}) \Rightarrow b=\frac{3}{8}$

$\therefore$

$X \backslash Y$	y_1	y_2	$P_{i\cdot}$
x_1	$\frac{1}{24}$	$\frac{1}{8}$	$\frac{1}{6}$
x_2	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
x_3	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{3}$
$P_{\cdot j}$	$\frac{1}{4}$	$\frac{3}{4}$	验证1

P_{125}, 20:

$$f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy=\begin{cases}\int_{-1}^{1}\frac{1}{4}(1+x^3y-xy^3)\,dy=\frac{1}{2}, & |x|\le 1\\ 0, & \text{其他}\end{cases}$$

$$f_Y(y)=\int_{-\infty}^{+\infty} f(x,y)\,dx=\begin{cases}\int_{-1}^{1}\frac{1}{4}(1+x^3y-xy^3)\,dx=\frac{1}{2}, & |y|\le 1\\ 0, & \text{其他}\end{cases}$$

$f(x,y)\neq f_X(x)\cdot f_Y(y)$，X与Y不相互独立.