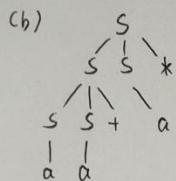


2.2.1

(a) $S \Rightarrow SS* \Rightarrow SS + S* \Rightarrow aS + S* \Rightarrow aa + S* \Rightarrow aa + a*$



(c) $L(G) = \{ \text{由 } +, * \text{ 和 } a \text{ 构成的后缀表达式} \}$

证明: 提供两种思路: ① 举例验证

如 $a + a * a$ 的后缀表达式为 $aaa*+$

$S \Rightarrow SS+ \Rightarrow aS+ \Rightarrow aSS*+ \Rightarrow aaS*+ \Rightarrow aaa*+$

② 用数学归纳法证明

2.2.2

(a) $L = \{ 0^n 1^n \mid n \geq 1 \}$

(b) $L = \{ \text{以 } a \text{ 为操作数, } +, - \text{ 为运算符的前缀表达式} \}$

(c) $L = \{ \text{成对括号的排列及嵌套及空串} \}$

(d) $L = \{ \text{由相同数量的 } a, b \text{ 组成的字符串} \}$

(e) $L = \{ \text{以 } a \text{ 为基本符号的正规表达式} \}$

2.2.4

(a) $S \rightarrow SS \text{ op } | a$

$\text{op} \rightarrow + | - | * | /$

(b) $\text{list} \rightarrow \text{list}, \text{id} | \text{id}$

(c) $\text{list} \rightarrow \text{id}, \text{list} | \text{id}$

(d) $\text{expr} \rightarrow \text{expr} + \text{expr} | \text{expr} - \text{expr} | \text{term}$

$\text{term} \rightarrow \text{term} * \text{factor} | \text{term} / \text{factor} | \text{factor}$

$\text{factor} \rightarrow \text{id} | \text{num} | (\text{expr})$

(e) $\text{expr} \rightarrow \text{expr} + \text{expr} | \text{expr} - \text{expr} | \text{term}$

$\text{term} \rightarrow \text{term} * \text{unary} | \text{term} / \text{unary} | \text{unary}$

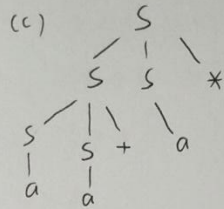
$\text{unary} \rightarrow + \text{factor} | - \text{factor} | \text{factor}$

$\text{factor} \rightarrow \text{id} | \text{num} | (\text{expr})$

4.2.1

(a) $S \Rightarrow SS^* \Rightarrow SS + S^* \Rightarrow aS + S^* \Rightarrow aa + S^* \Rightarrow aa + a^*$

(b) $S \Rightarrow SS^* \Rightarrow Sa^* \Rightarrow SS + a^* \Rightarrow Sa + a^* \Rightarrow aa + a^*$



4.2.3

(a) $S \rightarrow 1S \mid 0A \mid \epsilon$

$A \rightarrow 1S$

(b) $S \rightarrow 0S0 \mid 1S1 \mid 10 \mid 01 \mid \epsilon$

(c) $S \rightarrow 0S1S \mid 1S0S \mid \epsilon$

(d) $A: 11 \mid 10$

$B: 11 \mid 10$

$C: 11 \mid 10$

$A': 11 \mid 10$

$B': 11 \mid 10$

$S \rightarrow A \mid B$

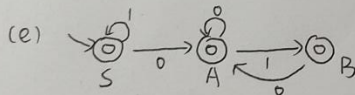
$A \rightarrow A'A \mid A'$

$B \rightarrow B'B \mid B'$

$A' \rightarrow 1A'A' \mid 0C$

$B' \rightarrow 0B'B' \mid 1C$

$C \rightarrow 1A' \mid 0B' \mid \epsilon$



$S \rightarrow 1S \mid 0A \mid \epsilon$

$A \rightarrow 0A \mid 1B \mid \epsilon$

$B \rightarrow 0A \mid \epsilon$

(f) $S \rightarrow AB \mid BA$

$A \rightarrow 0 \mid 0A0 \mid 0A1 \mid 1A0 \mid 1A1$

$B \rightarrow 1 \mid 0B0 \mid 0B1 \mid 1B0 \mid 1B1$

4.2.7

(a) 参考思路

- 1) 先检查非终结符是否与起始符 S 有关, 若无关, 则剔除该非终结符的相关式
 - 2) 再检查剩余的非终结符能否推出终结符, 若不能, 则剔除相关式
- 循环 (1) (2) 步, 直至结果不再变化

(b) $S \rightarrow \emptyset$

4.4.6

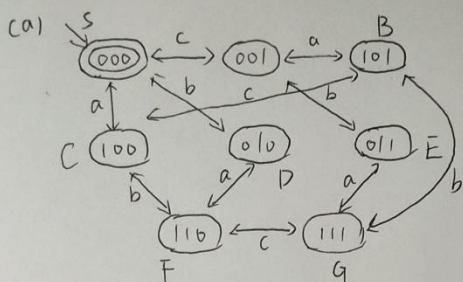
- (a)
- ① 找出所有产生 ϵ 的非终结符, 记为 V
 - ② 改写相关产生式, 对 $\forall V \in V$, 将其分别替换成 ϵ 和 V
 - ③ 删去除 $S \rightarrow \epsilon$ 以外的产生式
 - ④ 若存在 $S \rightarrow \epsilon$, 改写成 $S' \rightarrow \epsilon | S$

(b) $S' \rightarrow \epsilon | S$

$S \rightarrow ab | abs | asb | asbs |$

$S \rightarrow ba | bas | bsa | bsas$

补充练习



P: $S \rightarrow aC | bD | cA | c$
 $A \rightarrow aB | bE | cS$
 $B \rightarrow aA | bG | cC$
 $C \rightarrow aS | bF | cB$
 $D \rightarrow aF | bS | cE$
 $E \rightarrow aG | bA | cD$
 $F \rightarrow aD | bC | cG$
 $G \rightarrow aE | bB | cF$

$V_0 = \{S\}$

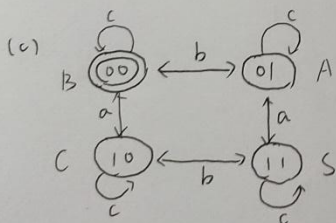
P: $S' \rightarrow S | \epsilon$
 $S \rightarrow aC | bD | cA$
 $A \rightarrow aB | bE | cS | c$
 $B \rightarrow aA | bG | cC$
 $C \rightarrow a | aS | bF | cE$
 $E \rightarrow aG | bA | cD$
 $F \rightarrow aD | bC | cG$
 $G \rightarrow aE | bB | cF$

(b) $a^i b^j = a^{i-2j} a^{2j} b^j = a^m a^{2j} b^j \quad (m \geq 1, j \geq 0)$

P: $S \rightarrow AB$
 $A \rightarrow aA | a$
 $B \rightarrow aaBb | \epsilon$

$V_0 = \{B\}$

$S \rightarrow AB | A$
 $A \rightarrow aA | a$
 $B \rightarrow aaBb | aab$



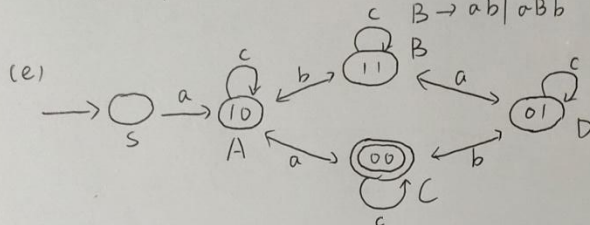
P: $S \rightarrow aA | bC | cS$
 $A \rightarrow aS | bB | cA$
 $B \rightarrow aC | bA | cB | \epsilon$
 $C \rightarrow aB | bS | cC$

$V_0 = \{B\}$

P: $S \rightarrow aA | bC | cS$
 $A \rightarrow aS | bB | cA | b$
 $B \rightarrow aC | bA | cB | c$
 $C \rightarrow aB | bS | cC | a$

$$(d) a^i b^j = a^{i-j} a^j b^j \\ = a^m a^j b^j \quad (m \geq 1, j \geq 0)$$

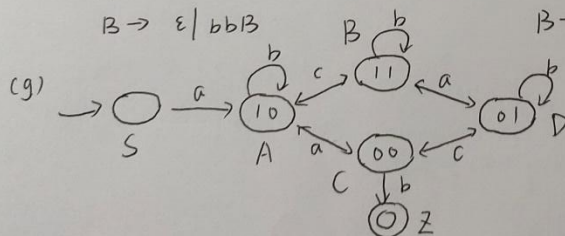
P: $S \rightarrow AB$ $V_0 = \{B\}$
 $A \rightarrow a | aA$ $P': S \rightarrow A | AB$
 $B \rightarrow \epsilon | aBb$ $A \rightarrow a | aA$
 $B \rightarrow ab | aBb$



P: $S \rightarrow aA$ $V_0 = \{c\}$
 $A \rightarrow aC | bB | cA$ $P': S \rightarrow aA$
 $B \rightarrow aD | bA | cB$ $A \rightarrow a | aC | bB | cA$
 $C \rightarrow aA | bD | cC | \epsilon$ $B \rightarrow aD | bA | cB$
 $D \rightarrow aB | bC | b | cD$ $C \rightarrow aA | bD | cC | c$
 $D \rightarrow aB | bC | b | cD$

$$(f) a^{2i} b^{2j} = a^{2i} b^{2i} b^{2j-2i} \\ = a^{2i} b^{2i} b^{2m} \quad (i \geq 1, m \geq 0)$$

P: $S \rightarrow AB$ $V_0 = \{B\}$
 $A \rightarrow aa bb | aa A bb$ $P': S \rightarrow A | AB$
 $B \rightarrow \epsilon | bb B$ $A \rightarrow aabb | aa A bb$
 $B \rightarrow bb | bb B$

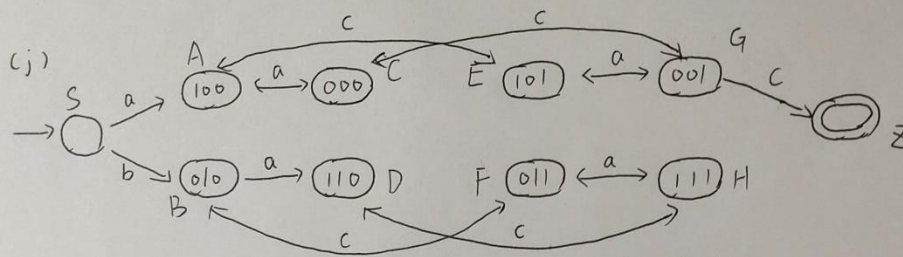


P: $S \rightarrow aA$ $V_0 = \{z\}$
 $A \rightarrow aC | bA | cB$ $P': S \rightarrow aA$
 $B \rightarrow aD | bB | cA$ $A \rightarrow aC | bA | cB$
 $C \rightarrow aA | bC | cD | bz$ $B \rightarrow aD | bB | cA$
 $D \rightarrow aB | bD | cC$ $C \rightarrow aA | bC | cD | b$
 $Z \rightarrow \epsilon$ $D \rightarrow aB | bD | cC$

(b) $a^i b^j c^k = a^i b^i b^{j-i-k} b^k c^k$
 $= a^i b^i b^m b^k c^k \quad (i \geq 0, m \geq 1, k \geq 1)$

P: $S \rightarrow ABC$
 $A \rightarrow \epsilon | aAb$
 $B \rightarrow b | bB$
 $C \rightarrow bc | bCc$

P': $V_0 = \{A\}$
 $S \rightarrow ABC | BC$
 $A \rightarrow ab | aAb$
 $B \rightarrow b | bB$
 $C \rightarrow bc | bCc$



P: $S \rightarrow aA | bB$
 $A \rightarrow aC | bD | cE$
 $B \rightarrow aD | bC | cF$
 $C \rightarrow aA | bB | cG$
 $D \rightarrow aB | bA | cH$
 $E \rightarrow aG | bH | cA$
 $F \rightarrow aH | bG | cB$
 $G \rightarrow aE | bF | cC | cZ$
 $H \rightarrow aF | bE | cD$
 $Z \rightarrow \epsilon$

$V_0 = \{Z\}$

P': $S \rightarrow aA | bB$
 $A \rightarrow aC | bD | cE$
 $B \rightarrow aD | bC | cF$
 $C \rightarrow aA | bB | cG$
 $D \rightarrow aB | bA | cH$
 $E \rightarrow aG | bH | cA$
 $F \rightarrow aH | bG | cB$
 $G \rightarrow aE | bF | cC | c$
 $H \rightarrow aF | bE | cD$

(k) $a^{2i-1} b^{2j-1} c^{2k-1} = a^{2i-1} b^{2i-1} b^{2(j-i-k)+1} b^{2k-1} c^{2k-1}$
 $= a^{2i-1} b^{2i-1} b^{2m+1} b^{2k-1} c^{2k-1} \quad (i \geq 1, m \geq 0, k \geq 1)$

P: $S \rightarrow ABC$
 $A \rightarrow aA | bB | a^2 b^2$
 $B \rightarrow bB | b$
 $C \rightarrow bC | bC$

