P_{125}, 22:

(1) $P(Z=k) = P(X>k, Y=k) + P(X=k, Y>k) + P(X=k, Y=k)$

$= P(X>k)P(Y=k) + P(X=k)P(Y>k) + P(X=k)P(Y=k)$

$= 2\cdot\left(\sum_{i=k+1}^{+\infty} pq^{i-1}\right)pq^{k-1} + p^2q^{2k-2}$.

$= 2p\dfrac{q^k}{1-q}pq^{k-1} + p^2q^{2k-2}$

$= 2pq^{2k-1} + p^2q^{2k-2}$

$= pq^{2k-2}(1+q)$, $k=1,2,\cdots$

(2) $P(Z=k) = P(X+Y=k) = \sum_{i=1}^{k-1} P(X=i, Y=k-i) = \sum_{i=1}^{k-1} P(X=i)P(Y=k-i)$

$= \sum_{i=1}^{k-1} pq^{i-1}\cdot pq^{k-i-1} = \sum_{i=1}^{k-1} p^2q^{k-2}$

$= (k-1)p^2q^{k-2}$, $k=2,3,\cdots$

P_{125}, 23:

$Y\sim U(-h,h)$，Y的概率密度为：$f_Y(y) = \begin{cases} \dfrac{1}{2h}, & -h<y<h \\ 0, & 其他 \end{cases}$

由卷积公式：

$f_Z(z) = \int_{-\infty}^{+\infty} f_X(z-y)f_Y(y)\,dy = \dfrac{1}{2h}\int_{-h}^{h} f(z-y)\,dy$

$= \dfrac{1}{2h}\int_{-\infty}^{h} f(z-y)\,dy - \dfrac{1}{2h}\int_{-\infty}^{-h} f(z-y)\,dy$

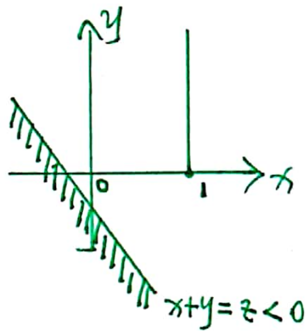
$= \dfrac{1}{2h}\left(F(z+h) - F(z-h)\right)$

P_{126}, 25:

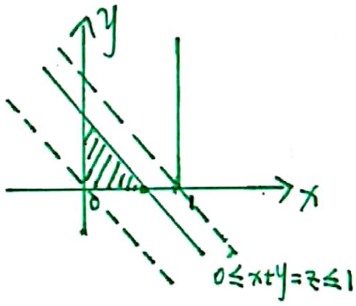
X与Y相互独立，$\therefore f(x,y)=f_X(x)\cdot f_Y(y)=\begin{cases} e^{-y}, & 0<x<1,\ y>0 \\ 0, & 其他 \end{cases}$

法①：$F_Z(z)=P(Z\le z)=P(X+Y\le z)$

$$=\iint\limits_{x+y\le z} f(x,y)\,dx\,dy$$

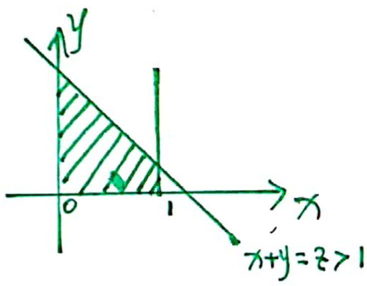


(1) 当 $z<0$ 时，$F_Z(z)=0$



(2) 当 $0\le z\le 1$ 时，$F_Z(z)=\int_0^z\int_0^{z-x} e^{-y}\,dy\,dx$

$$=z-1+e^{-z}$$



(3) 当 $z>1$ 时，$F_Z(z)=\int_0^1\int_0^{z-x} e^{-y}\,dy\,dx$

$$=1-e^{1-z}+e^{-z}$$

$$\therefore F_Z(z)=\begin{cases} 0, & z<0 \\ z-1+e^{-z}, & 0\le z\le 1 \\ 1-e^{1-z}+e^{-z}, & z>1 \end{cases}$$

$$\therefore f_Z(z)=F_Z'(z)=\begin{cases} 0, & z<0 \\ 1-e^{-z}, & 0\le z\le 1 \\ e^{1-z}-e^{-z}, & z>1. \end{cases}$$

法②：$f_Z(z)=\int_{-\infty}^{+\infty} f_X(x)f_Y(z-x)\,dx$，卷积公式

$$=\int_0^1 f_Y(z-x)\,dx$$

注意只有当y的范围确定后才能将f_Y的具体形式代入.

(1) 当 $z<0$ 时，$f_Z(z)=\int_0^1 0\,dx=0$，$(z-x<0,\ 0<x<1)$

(2) 当 $0\le z\le 1$ 时，$f_Z(z)=\int_0^z e^{-(z-x)}dx+\int_z^1 0\,dx$

$$=1-e^{-z},$$

(3) 当 $z>1$ 时，$f_Z(z)=\int_0^1 e^{-(z-x)}dx$

$$=e^{1-z}-e^{-z}.$$

$$\therefore f_Z(z)=\begin{cases}0, & z<0\\ 1-e^{-z}, & 0\le z\le 1\\ e^{1-z}-e^{-z}, & z>1\end{cases}$$

P_{126}，29：

参考课上例题，如下：

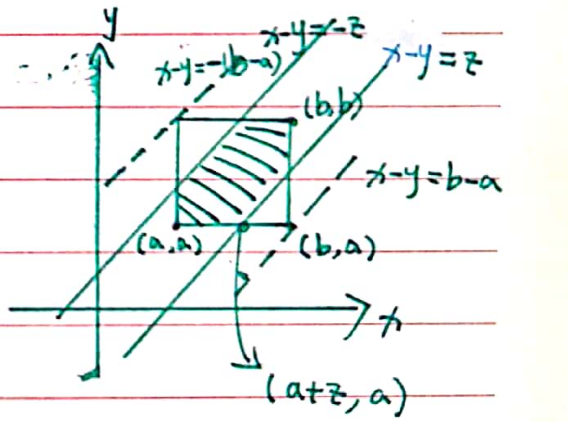
例：往区间 $[a,b]$ 上随机投两点，求两点之间的距离满足的分布.

解：令 X, Y 分别表示两点的坐标，则

$$f_X(x)=\begin{cases}\dfrac{1}{b-a}, & a<x<b\\ 0, & \text{其他}\end{cases},\quad f_Y(y)=\begin{cases}\dfrac{1}{b-a}, & a<y<b\\ 0, & \text{其他}\end{cases}$$

$$f(x,y)=\begin{cases}\dfrac{1}{(b-a)^2}, & a<x,y<b\\ 0, & \text{其他}\end{cases}$$

（X, Y 相互独立，$f(x,y)=f_X(x)\cdot f_Y(y)$）



令 $Z=|X-Y|$，表示两点之间的距离，则分布函数：

$$F_Z(z)=P(Z\le z)=P(|X-Y|\le z)$$

① 当 $z\le 0$ 时，$F_Z(z)=0$

② 当 $z\ge b-a$ 时，$F_Z(z)=1$

③ 当 $0<z<b-a$ 时：

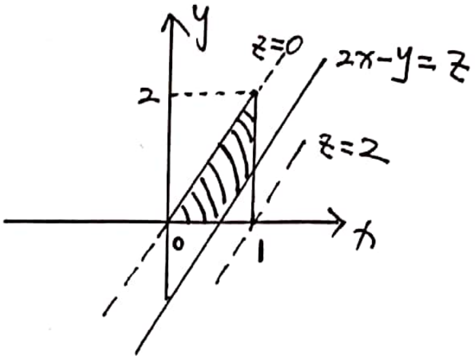
$$F_Z(z)=P(-z\le X-Y\le z)=\frac{\text{阴影部分面积}}{(b-a)^2}=\frac{(b-a)^2-2\times\text{三角面积}}{(b-a)^2}$$

$$=\frac{(b-a)^2-(b-a-z)^2}{(b-a)^2}.$$

$$\therefore F_Z(z)=\begin{cases}0, & z\le 0\\ \dfrac{(b-a)^2-(b-a-z)^2}{(b-a)^2}, & 0<z<b-a\\ 1, & z\ge b-a\end{cases},\quad f_Z(z)=\begin{cases}\dfrac{2(b-a-z)}{(b-a)^2}, & 0<z\le b-a\\ 0, & \text{其他}\end{cases}$$

P_{126} 28:

方法①: $F_Z(z)=P(2X-Y\leq z)=\iint\limits_{2x-y\leq z} f(x,y)\,dx\,dy$



$$=\begin{cases} 0, & z<0 \\ \text{阴影面积}/1, & 0\leq z\leq 2 \\ 1, & z>2 \end{cases}$$

$$=\begin{cases} 0, & z<0 \\ 1-\frac{1}{2}(1-\frac{z}{2})(2-z), & 0\leq z\leq 2 \\ 1, & z>2 \end{cases} \Rightarrow f_Z(z)=\begin{cases} 1-\frac{z}{2}, & 0\leq z\leq 2 \\ 0, & \text{其他} \end{cases}$$

方法②: $f_Z(z)=\int_{-\infty}^{+\infty}\frac{1}{|b|}f(x,\frac{1}{b}(z-ax))\,dx$, $a=2, b=-1$

$$=\int_{-\infty}^{+\infty} f(x,2x-z)\,dx$$

$$=\int_0^1 f(x,2x-z)\,dx$$

(1) 当 $z<0$ 时, $2x-z>2x$, $f(x,2x-z)=0$, $\therefore f_Z(z)=0$

(2) 当 $z>2$ 时, $2x-z<0$, $f(x,2x-z)=0$, $\therefore f_Z(z)=0$

(3) 当 $0\leq z\leq 2$ 时, 要使 $f(x,y)=1$, 则 $0\leq 2x-z\leq 2x$ 且 $0\leq x\leq 1$

$$\Rightarrow \frac{z}{2}\leq x\leq 1$$

$$\therefore f_Z(z)=\begin{cases} \int_{\frac{z}{2}}^{1} 1\,dx = 1-\frac{z}{2}, & 0\leq z\leq 2 \\ 0, & \text{其他} \end{cases}$$