

Homework Problem 7

Let a and b be positive integers such that $a \mid n$ and $b \mid n$.

Show that $\text{lcm}(a, b) \mid n$.

Justify your answer.

Due Thu, Sept 19, in class.

Homework Problem 8

Let a and b be positive integers such that a^3 divides b^2 .

Does it follow that a divides b ?

Justify your answer.

Due Thu, Sept 19, in class.

Homework Problem 9

From your textbook:

Klain: Essential Number Theory, Exercise 8.4 on page 41.

Due Thu, Sept 19, in class.

Homework Problem 10

In Smurfland, there are 5-dollar and 12-dollar bills in circulation. There are no other types of paper bills and no coins either.

Show that Smurfs can make cash payments of n dollars, for any integer amount n that is 44 or larger.

PS: Assume that Smurfs can print as many \$5 and \$12 bills as necessary, but *only* \$5 and \$12 bills.

Due Thu, Sept 19, in class.

Homework Problem 11* (optional)

Find the *smallest* pair of integers on which the Euclidean algorithm takes exactly 10 steps.

PS: “smallest pair” means that if the inputs are n, m with $n \geq m$, then n should be as small as possible.

* Due (optionally) Thu, Sept 26, in class.

Homework Problem 12

(a) What is $\gcd(F_n, F_{n+1})$?

(b) What is $\gcd(F_n, F_{n+2})$?

Justify your answer.

Due Thu, Sept 19, in class.

Homework Problem 13

Set $\phi = \frac{1+\sqrt{5}}{2}$.

(a) Show that

$$\lim_{n \rightarrow \infty} \left(F_n - \frac{1}{\sqrt{5}} \phi^n \right) = 0$$

(b) Show that for any $n \geq 0$, F_n is the integer closest to the value of $\frac{1}{\sqrt{5}} \phi^n$.

Due Thu, Sept 19, in class.