Quadratic Voting

How Mechanism Design Can Radicalize Democracy

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Vickrey (1961) launched mechanism design as a project to implement Lerner (1944)'s "socialist" dreams of welfare optimality absent a benevolent and omniscient planner. When Vickrey became a Nobel Laureate, he announced that he planned to bring this vision to the broader public using the "bully pulpit" the prize afforded him, but he sadly died days later. Furthermore, while Vickrey's ambitions gained some early adherents (Tideman and Tullock, 1976), experience (Ausubel and Milgrom, 2005; Rothkopf, 2007) and Vickrey's own warnings suggested his exact scheme was usually impractical.

Thus, while mechanism design transformed economic theory, the novel mechanisms it suggests have largely been treated as a series of insightful theoretical curiosities of little practical relevance or applied in quite narrow settings, for example to spectrum and internet advertising auctions (Milgrom, 2004; Edelman et al., 2007). In a book that will come out as this article does (Posner and Weyl, 2018) one of us aims to revive the more radical ambitions of mechanism design and suggest they may offer a solution to the economic, political and social crisis of our times.

Perhaps the most glaring element of that crisis has been the struggles of democracies to resolve majority-minority conflicts and the resulting ascendancy of "populist" tyrannies of the majority or plurality. This oldest and most persistent problem of democracy seems particularly to lend itself to Vickrey's approach both because of its sweeping scope and because the basic problem seems to be precisely the lack of markets.

One-person-one-vote (1p1v) rations a single unit of influence on any collective decision to each voter, preventing potential Pareto-improving trade. When we complain about the failure of standard democracy to account for intensity of preference or knowledge, we mean that it blocks trades where some citizens would be willing to give up their influence on some issues to gain more influence on others. Furthermore, given that the standard market system of exchange may be derived as a limit of Vickrey's procedure in a large society (Tideman and Plassmann, Forthcoming), it seems natural that applying a similar logic to public goods could yield a solution.

Yet, to our knowledge, prior to our work on the subject no simple and flexible pricing system for collective decisions that could apply to common settings such as binary decisions had been proposed.[1] Groves and Ledyard (1977) and Hylland and Zeckhauser (1980) proposed closely related market systems for choosing continuous public goods, but groups of voters rarely directly choose continuous public goods. Here we propose a simple version of Hylland and Zeckhauser's scheme for the common setting of communities making repeated binary collective decisions. As with the market system for private goods, it is the large population limit (this time for public goods) of Vickrey's scheme and we will show that it is the unique way to price votes that achieves optimality in a "price-taking" model analogous to the conditions under which markets achieve optimality.

I. Price-Theoretic Model

Consider a society of N voters $i = 1, \ldots N$ where many binary collective decisions (e.g. referenda or choice of leaders) arise. To create the opportunity for market trade, each voter is endowed with a large stock of "voice credits" that they may spend influencing the outcome of

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[1] Slightly after the first draft of this paper circulated, Goeree and Zhang (2017) proposed a closely related mechanism, though one applicable only in a narrower scope.

these decisions. As is common in the analysis of markets for private goods (Willig, 1976), we assume there are enough issues and that each is sufficiently inconsequential that every voter has a quasi-linear "continuation value" for retaining voice credits for future votes. As in the theory of fair resource allocation (Varian, 1974), we assume that voice credits have been distributed in a manner (such as equal division) considered fair by the relevant society in the sense that maximizing total equivalent continuation value in units of voice credits defines social optimality.

Consider some particular decision. Call the value that voters would receive, in units of voice credits, for seeing alternative A prevail over alternative B $2u_i$, with negative values indicating a preference for B over A. The community votes to determine which alternative is implemented, with each voter choosing a continuous number of votes v_i either positive or negative depending on which alternative she favors. A is implemented just when $\sum_i v_i \geq 0$. Each voter pays a cost $c(v_i)$ voice credits for her votes where c is differentiable, convex, even and strictly increasing in $|v_i|$ to a central clearing house. We describe c as a *vote pricing rule*.

A game-theoretic analysis would specify beliefs of each voter about the distribution of values and derive an equilibrium strategy. We conduct such an analysis, which involves narrow assumptions about beliefs and behavior and statistical subtleties, elsewhere (Lalley and Weyl, 2017). We focus here instead on a price-taking approximation (Weyl, Forthcoming) that appears to be valid more broadly (Weyl, 2017) and shows simply the core logic.

We thus assume that each voter, in deciding how many votes to purchase, weighs the marginal cost of an additional vote against the perceived chance the vote will be *pivotal* in swinging the election. The price-taking assumption we adopt, originally proposed by Mueller (1973) and Laine (1977), is that all voters agree on this *marginal pivotality* of votes p. Under this assumption, a rational voter will choose v_i to maximize $2pv_iu_i - c(v_i)$. By our assumption that vote credits are distributed fairly, society wishes to implement A exactly when $\sum_i u_i \geq 0$. We say a vote pricing rule is *robustly optimal* if, for every $p > 0$, N and vector u, each price-taking voter i chooses votes $v_i^\star$ so that $\sum_i v_i^\star$ has the same sign as $\sum_i u_i$.

II. Uniqueness

Our main result is that the set of robustly optimal vote pricing rules are precisely the set of quadratic rules.

THEOREM 1: *A vote pricing rule is robustly optimal if and only if it is quadratic.*

PROOF:

See our online Appendix.

The central idea of the omitted formal proof is that quadratic functions are the only ones with linear derivatives and thus the only functions where a voter buying votes equates her marginal benefit and cost at a number of votes proportional to her value. Consider the class of vote pricing rules $c(x) = x^a$ for $a > 1$. The first-order condition for voter optimization is, by differentiation,

$$2pu_i = a(v_i)^{a-1} \implies$$

$$v_i = \text{sign}(u_i)\left(\frac{2p}{a}\right)^{\frac{1}{a-1}} |u_i|^{\frac{1}{a-1}}$$

If $a = 2$ this leads to $v_i^\star$ proportional to u_i and thus robust optimality. For any other a, $v_i^\star$ is not proportional to u_i and thus the costly voting rule will be suboptimal for some arrangements of values and p.

In constrast, consider the extremes of $a \to 1$ (as voting cost approaches linear) and $a \to \infty$ (as voting cost becomes very convex). In the former case, the power on u_i determining $v_i^\star$ becomes infinite and thus voters with only slightly greater values will vote infinitely more. Linear voting thus leads to dictatorship of the most intense voter, as in Casella et al. (2012). This reflects the common intuition against vote trading: allowing it will lead to capture by the most intense special interests.

In the case of $a \to \infty$, the power on u_i approaches 0 so all voters buy exactly one vote in the direction of their preference as in 1p1v. In this sense, QV is an optimal intermediate point between the extremes of dictatorship and majority rule. It is the one vote pricing rule under which voters who intend only their own gain are led, as if by an invisible hand, to advance the interests of society. We refer to voting based on a quadratic pricing rule as Quadratic Voting (QV) for obvious reasons.

III. Practical Promise

While price-taking is a useful heuristic, it is unlikely to literally hold in game theoretic models of voter behavior. In other work, we have considered a variety of these models. In Lalley and Weyl (2017) we consider the most canonical case from the game theoretic literature, in which voters values are drawn independently and identically from a known value distribution and act as rational, risk-neutral expected utility maximizers. We show that under appropriate conditions and in all symmetric Bayes-Nash equilibria in large populations the price-taking assumption approximately holds for almost all voters and thus that the welfare losses from QV decay, generically at a rate $\frac{1}{N}$ as the population grows. In another parallel to markets, this is the same rate of convergence of private goods markets towards efficiency as competition grows.

The structure of game theoretic equilibrium is quite subtle and thus our rigorous mathematical proof of this result is long and technical. However, taking the approximations that we rigorously prove hold in this case to hold, appropriately adjusted, in other cases and using numerical methods, allows analysis of QV for other environments. In Chandar and Weyl (2017) one of us, with another co-author, considers the baseline model in smaller populations for a range of distributions numerically and finds that the welfare lost from QV relative to the optimum is very small (rarely more than a few percentage points) while that lost under 1p1v may easily be near one hundred percent.

In Weyl (2017) one of us uses these approximations to allow for collusion, uncertainty about the value distribution and the possibility that voters are not perfectly rational and consequentialist. In some of these settings QV performs better than in our baseline and it rarely has more than a very small welfare loss, again in contrast to 1p1v. Furthermore, laboratory experiments on QV (Goeree and Zhang, 2017; Cárdenas et al., 2014) are largely consistent with these robustness conclusions. While participants do not play precisely as game theoretic equilibrium would predict, votes are generally linearly proportional to values with some noise, leading to outcome much closer to optimal than 1p1v.

Each of these results has important limitations on its own; some are in stylized, or specific experimental, settings, while others are approximate or numerical. Together, however, we believe they suggest significant promise as they illustrate the robustness of this simple yet (approximately) optimal voting rule. This contrasts with other voting rules which are either optimal, but widely seen as too fragile to apply in practice, or which do not aim at optimality.

In the first category, the most famous proposal was Vickrey's original scheme as refined by Clarke (1971) and Groves (1973), the first fully optimal mechanism. However, this approach is extremely sensitive to collusion by even a very small group (Ausubel and Milgrom, 2005). It also requires large and highly uncertain expenditures of money that make it heavily dependent on using real money rather than voice credits (Greenberg et al., 1977), a possibility that may be unjust given the inequality of wealth in most societies (Laurence and Sher, 2017; Ober, 2017). As noted in our introduction, to address these problems, Groves and Ledyard (1977) and Hylland and Zeckhauser (1980) proposed mechanism closely related to QV, but only applicable to continuous public goods problems, which rarely arise in practical applications. Other optimal mechanisms proposed in the literature are even less robust (Weyl, 2017). Given this, practical attention has primarily focused on mechanisms that do not aspire to even approximate optimality, such as ones that do not allow expression of preference intensity (Brams and Fishburn, 1978) or are like QV but with a linear costly voting function (Casella, 2005; Hortala-Vallve, 2012) and thus induce dictatorship of the most intense voters.

QV aims to combine the simplicity and practicality of this latter group of designs with an approximation to the optimality of the first. This has allowed applications to polling and survey research (Quarfoot et al., 2017; Holland, 2017), where evidence from initial studies shows significant improvements over existing methods of preference elicitation. Higher-stakes applications to politics (Posner and Weyl, 2015) or corporate governance (Posner and Weyl, 2014) remain far more speculative and are not advisable without further experimentation at smaller scales.

IV. Radical Democracy, Radical Markets

However, cautious but increasingly ambitious experimentation seem warranted given the potential of QV to create a truly radical democracy in several senses. First, and most mechanically, QV may also be viewed as individuals receiving votes equal to the square root or radical of the voice credits they spend. Second, QV gets to the roots of what the framers of modern democratic institutions, such as Alexander Hamilton, the Marquis de Condorcet and Jeremy Bentham, viewed as the aim of democracy: to ensure the state serves the general happiness of the people maximally. Third, QV radically expands the rights of citizens to fully and freely express their political views, liberating them from the straight jacket of 1p1v influence rationing. In this sense, it shows how mechanism design can suggest fundamental and yet also practical reforms of social institutions.

Yet Vickrey's vision has nearly as much to give to markets for private as public goods. A follower of Henry George, Vickrey was skeptical of private property and the central argument of his 1961 classic was that auctioning assets ensured their efficient allocation, which Myerson and Satterthwaite (1981) show this is impossible beginning from private ownership. One can imagine many assets (other than personal property) being continuously auctioned for rental according to Vickrey's favored English auction, much as occurs with internet advertising slots. However, rather than the revenue being appropriated by a web platform, it could be returned to the public in equal shares as a social dividend, much as envisioned by George. Such a *Vickrey Commons* would on the one hand take the logic of the free market to its furthest extreme, while on the other hand abolishing private property and most inequality. It is the natural private goods markets analog of the radical market logic of QV.

As with the direct application of Vickrey's schemes to collective decisions, a Vickrey Commons would face a host of practical challenges and variations on it, such as those proposed by Weyl and Zhang (2017), are of greater practical relevance. Yet as a thought experiment it, along with QV, illustrates the potential to break the deadlocked ideologies of left and right that plague our society. For mechanism designers to realize this potential, they will have to broaden their horizons beyond both narrow applications and impractical theory, to learn to be at once idealistic and pragmatic, to deploy rigor flexibly and to account for sociological, psychological, legal and other factors that they have traditionally shied away from given their analytical intractability. Here we tried to illustrate briefly the challenges and promise of such an approach.

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