

### 3: Nodal Analysis

- Aim of Nodal Analysis
- Nodal Analysis Stage 1:  
Label Nodes
- Nodal Analysis Stage 2:  
KCL Equations
- Current Sources
- Floating Voltage Sources
- Weighted Average Circuit
- Digital-to-Analog  
Converter
- Dependent Sources
- Dependent Voltage  
Sources
- Universal Nodal Analysis  
Algorithm
- Summary

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## Aim of Nodal Analysis

The aim of nodal analysis is to determine the voltage at each node relative to the reference node (or ground). Once you have done this you can easily work out anything else you need.

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- (1) **Nodal Analysis** - systematic; always works
- (2) **Circuit Manipulation** - ad hoc; but can be less work and clearer

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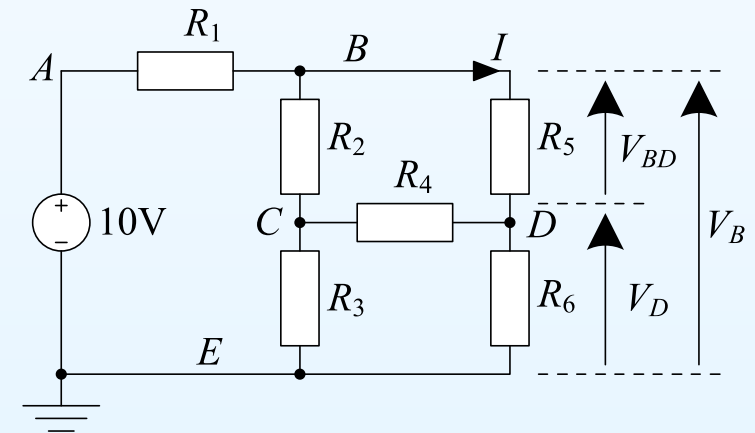
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### Reminders:

A node is all the points in a circuit that are directly interconnected.

We assume the interconnections have zero resistance so all points within a node have the same voltage. Five nodes:  $A, \dots, E$ .



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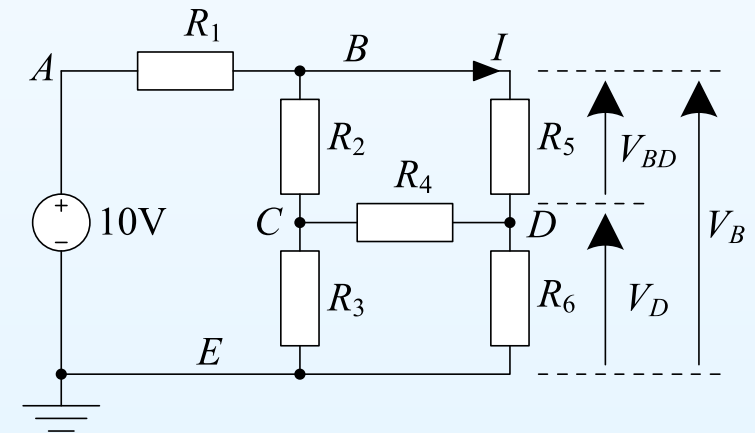
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Ohm's Law:  $V_{BD} = IR_5$



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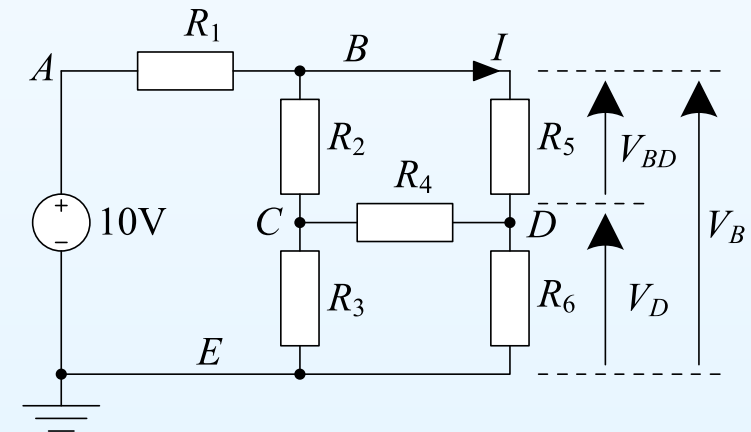
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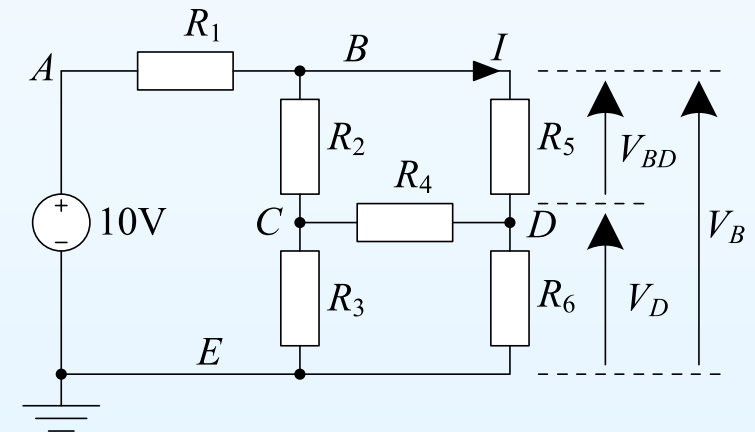
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**KVL:**  $V_{BD} = V_B - V_D$

**KCL:** Total current exiting any closed region is zero.

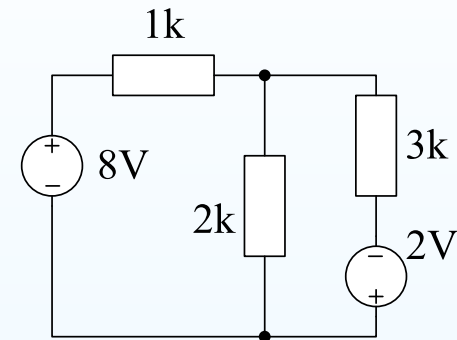


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## Nodal Analysis Stage 1: Label Nodes

To find the voltage at each node, the first step is to label each node with its voltage as follows



- (1) Pick any node as the voltage reference. Label its voltage as  $0\text{ V}$ .
- (2) If any fixed voltage sources are connected to a labelled node, label their other ends by adding the value of the source onto the voltage of the labelled end.
- (3) Pick an unlabelled node and label it with  $X$ ,  $Y$ ,  $\dots$ , then go back to step (2) until all nodes are labelled.

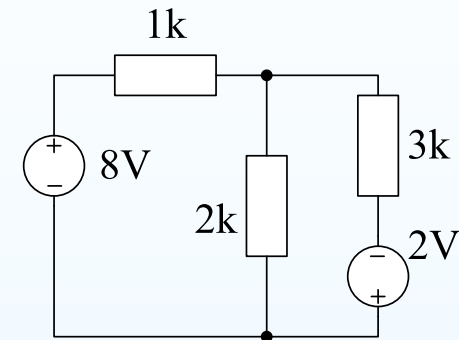


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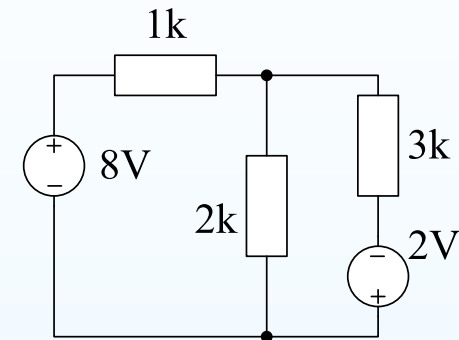
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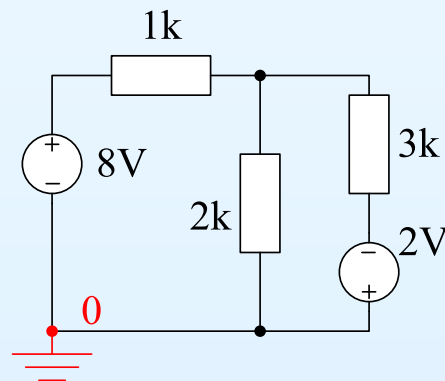
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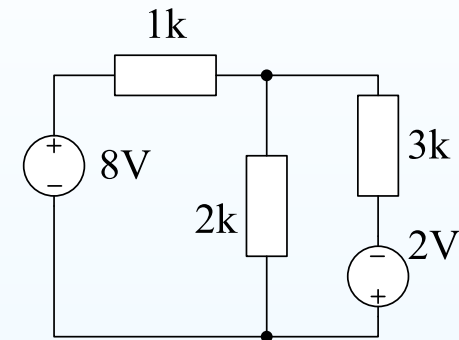
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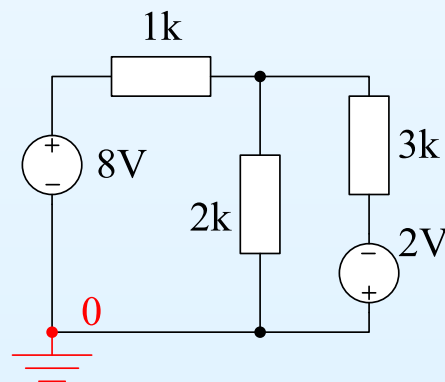
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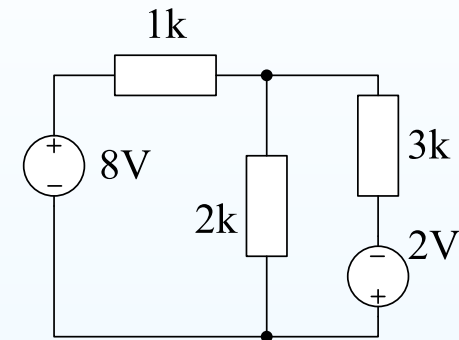


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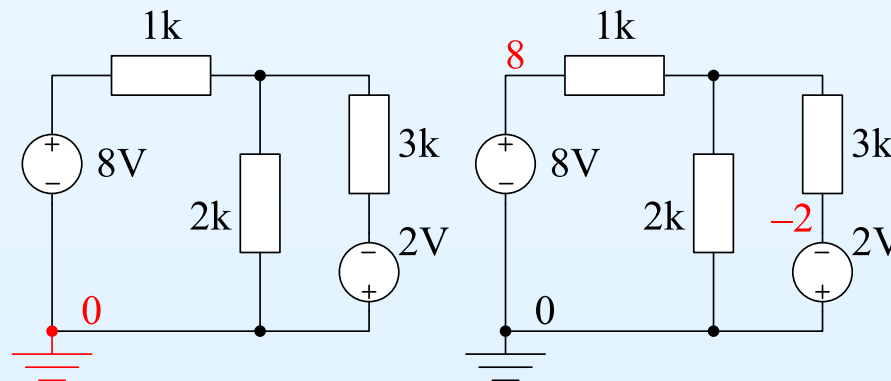
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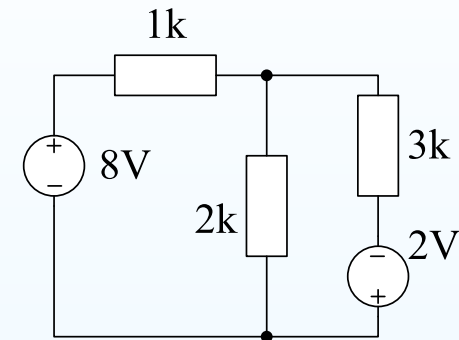


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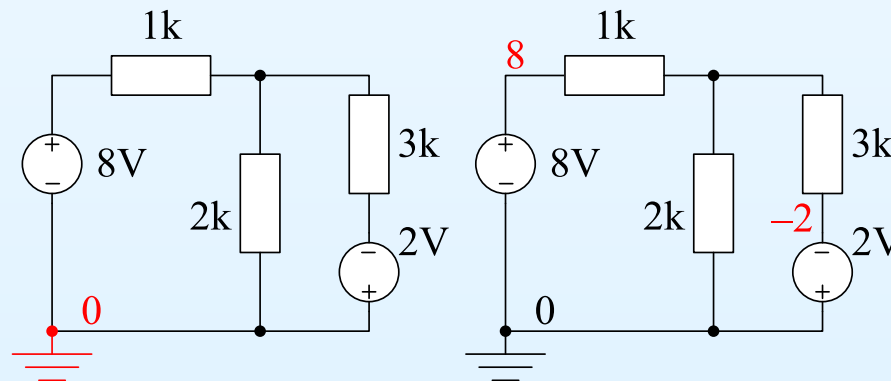
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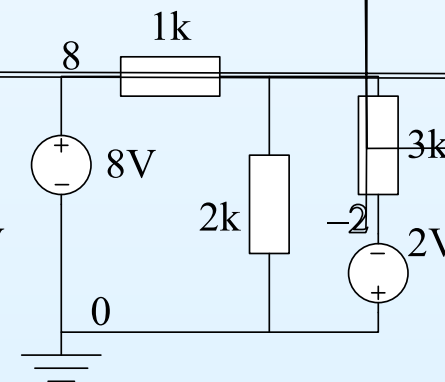
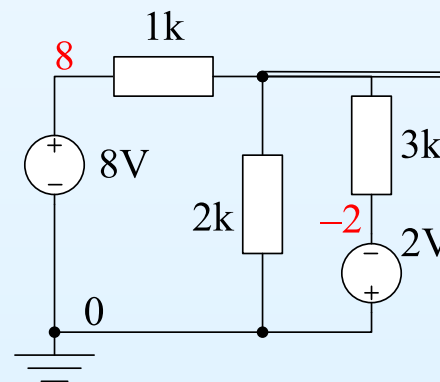
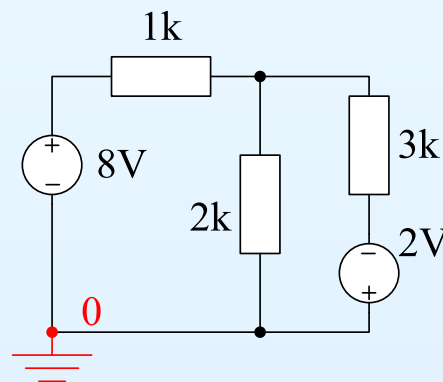
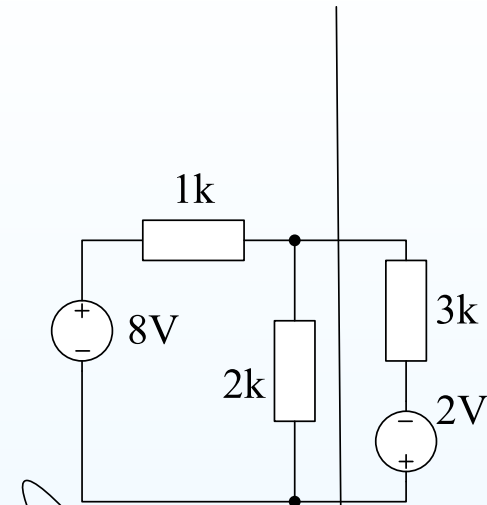
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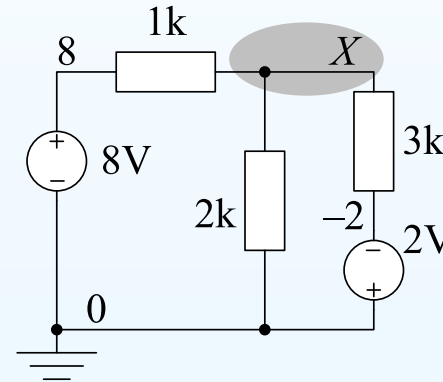
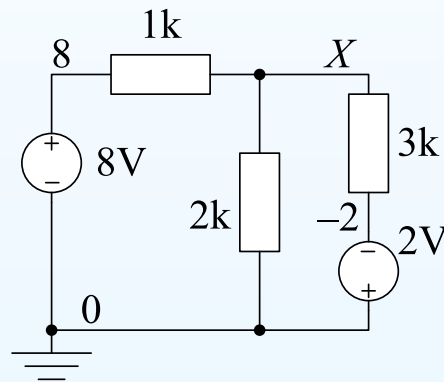


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The second step is to write down a KCL equation for each node labelled with a variable by setting the total current flowing out of the node to zero. For a circuit with  $N$  nodes and  $S$  voltage sources you will have  $N - S - 1$  simultaneous equations to solve.



We only have one variable:

$$\frac{X-8}{1\text{ k}} + \frac{X-0}{2\text{ k}} + \frac{X-(-2)}{3\text{ k}} = 0$$

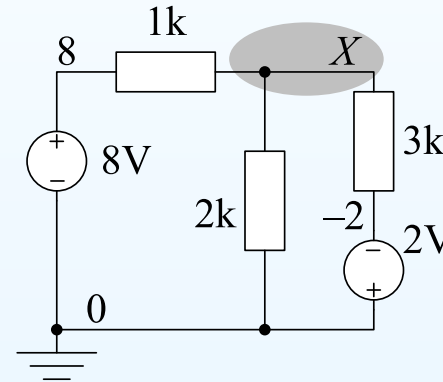
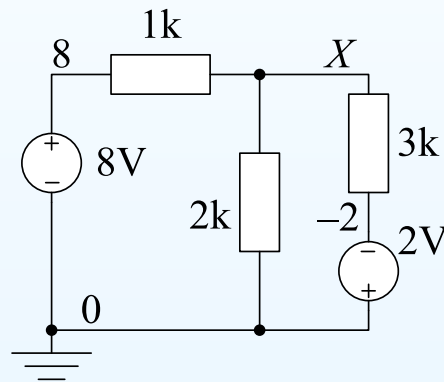


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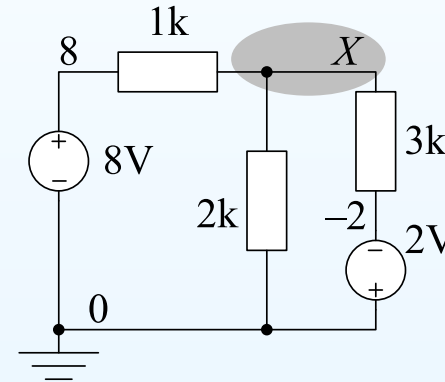
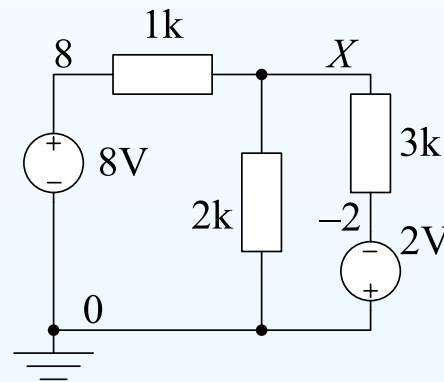
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$$\frac{X-8}{1\text{ k}} + \frac{X-0}{2\text{ k}} + \frac{X-(-2)}{3\text{ k}} = 0 \Rightarrow (6X - 48) + 3X + (2X + 4) = 0$$

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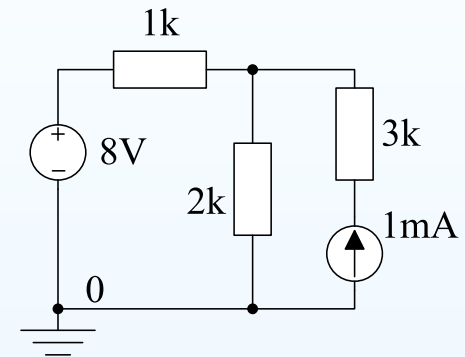


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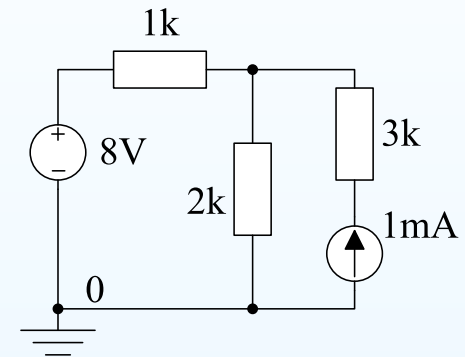
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(1) Pick reference node.



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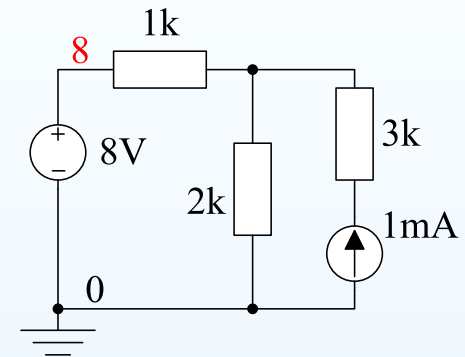
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(2) Label nodes: 8



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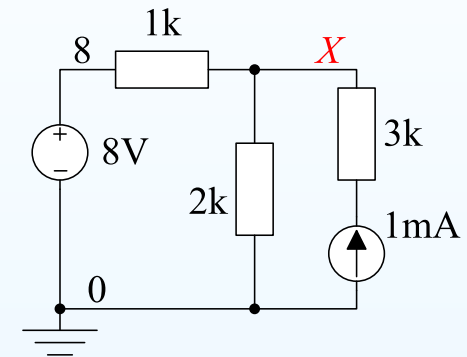
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(1) Pick reference node.

(2) Label nodes: 8,  $X$



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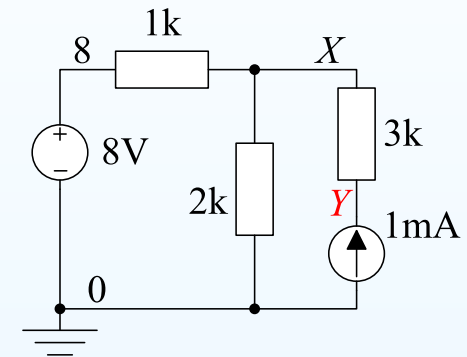
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(1) Pick reference node.

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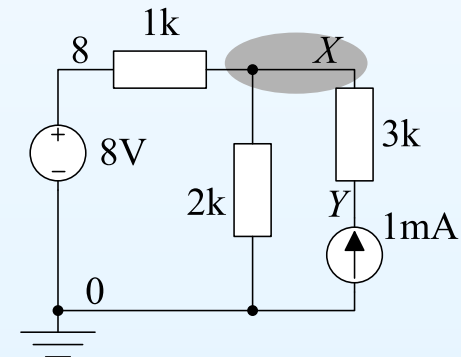
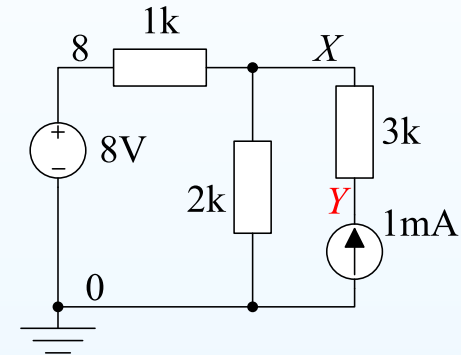
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(1) Pick reference node.

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(3) Write equations

$$\frac{X-8}{1} + \frac{X}{2} + \frac{X-Y}{3} = 0$$



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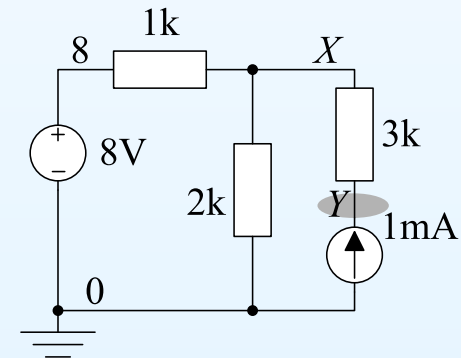
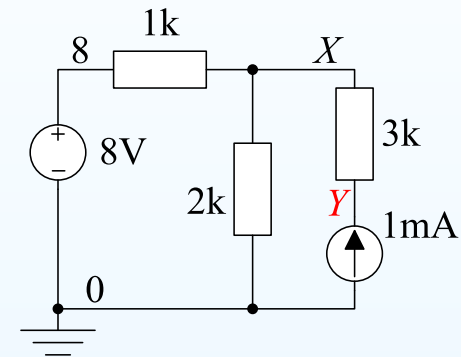
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$$\frac{Y-X}{3} + (-1) = 0$$



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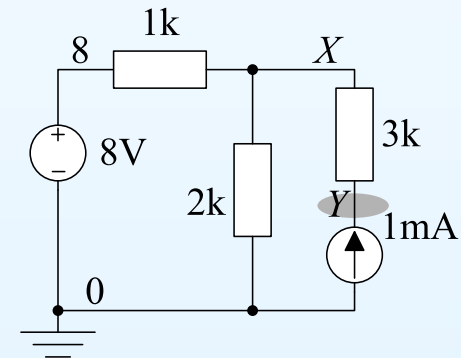
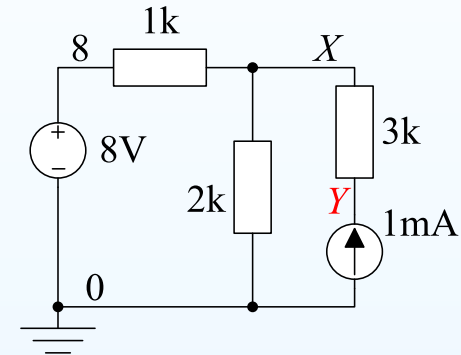
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(2) Label nodes: 8,  $X$  and  $Y$ .

(3) Write equations

$$\frac{X-8}{1} + \frac{X}{2} + \frac{X-Y}{3} = 0$$

$$\frac{Y-X}{3} + (-1) = 0$$



Ohm's law works OK if **all resistors** are in  $k\Omega$  and **all currents** in  $mA$ .

# Current Sources

## 3: Nodal Analysis

- Aim of Nodal Analysis
- Nodal Analysis Stage 1: Label Nodes
- Nodal Analysis Stage 2: KCL Equations
- **Current Sources**
- Floating Voltage Sources
- Weighted Average Circuit
- Digital-to-Analog Converter
- Dependent Sources
- Dependent Voltage Sources
- Universal Nodal Analysis Algorithm
- Summary

Current sources cause no problems.

(1) Pick reference node.

(2) Label nodes: 8,  $X$  and  $Y$ .

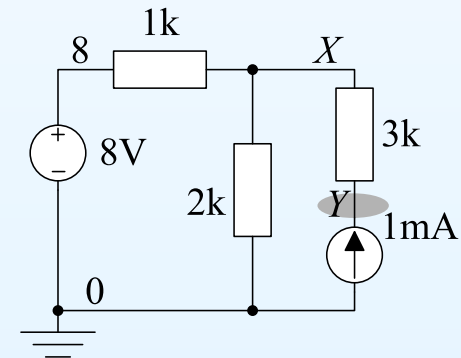
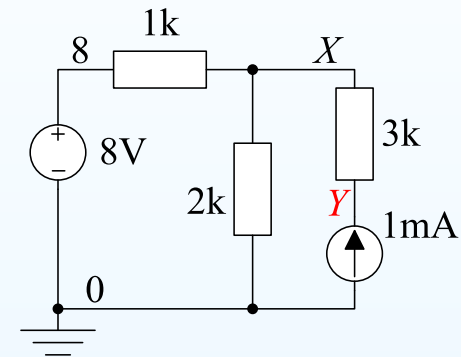
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$$\frac{X-8}{1} + \frac{X}{2} + \frac{X-Y}{3} = 0$$

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Ohm's law works OK if **all resistors** are in  $k\Omega$  and **all currents** in mA.

(4) Solve the equations:  $X = 6$ ,  $Y = 9$

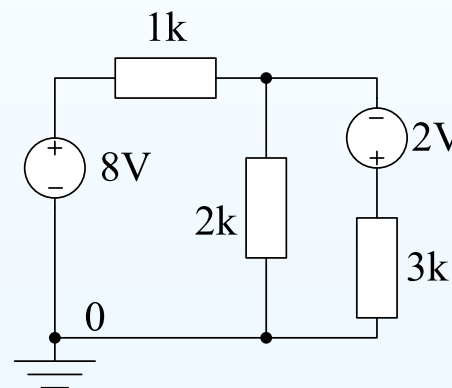


# Floating Voltage Sources

## 3: Nodal Analysis

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- Nodal Analysis Stage 2: KCL Equations
- Current Sources
- Floating Voltage Sources
- Weighted Average Circuit
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- Universal Nodal Analysis Algorithm
- Summary

*Floating voltage sources* have neither end connected to a known fixed voltage. We have to change how we form the KCL equations slightly.



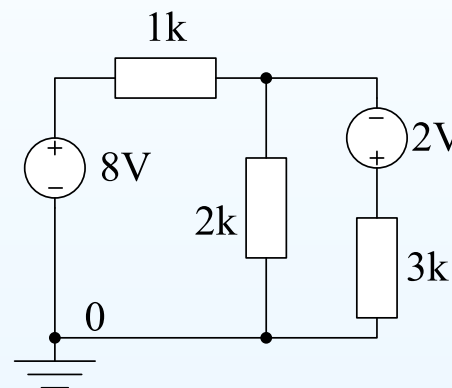
# Floating Voltage Sources

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(1) Pick reference node.



# Floating Voltage Sources

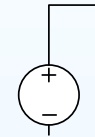
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(1) Pick reference node.

(2) Label nodes: 8









# Floating Voltage Sources

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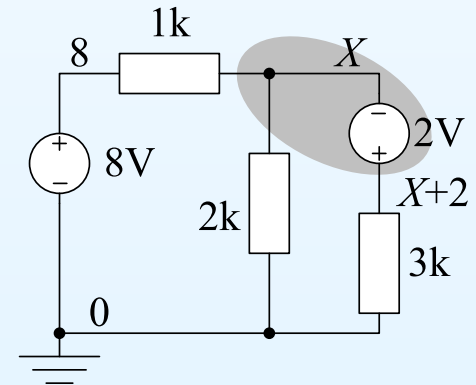
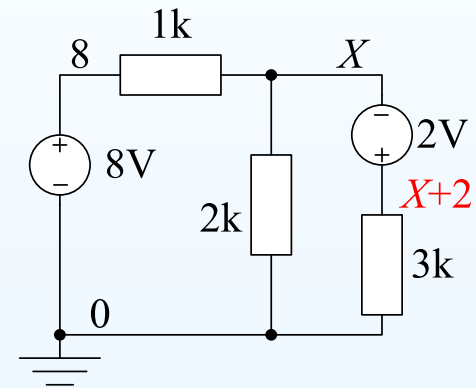
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(1) Pick reference node.

(2) Label nodes: 8,  $X$  and  $X + 2$  since it is joined to  $X$  via a voltage source.

(3) Write KCL equations but count all the nodes connected via floating voltage sources as a single “super-node” giving one equation

$$\frac{X-8}{1} + \frac{X}{2} + \frac{(X+2)-0}{3} = 0$$



# Floating Voltage Sources

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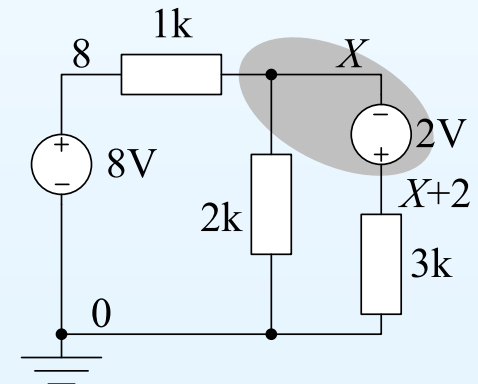
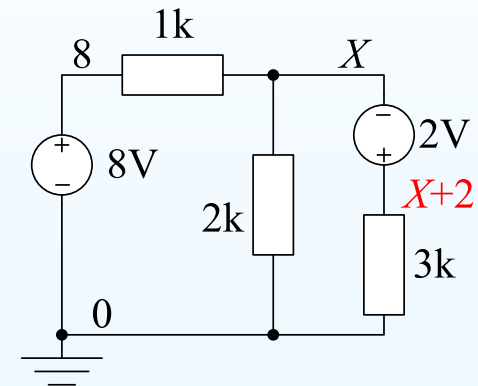
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Ohm's law always involves the difference between the voltages at **either end of a resistor**. (Obvious but easily forgotten)

# Floating Voltage Sources

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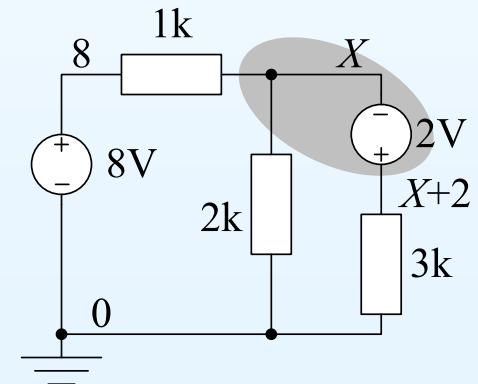
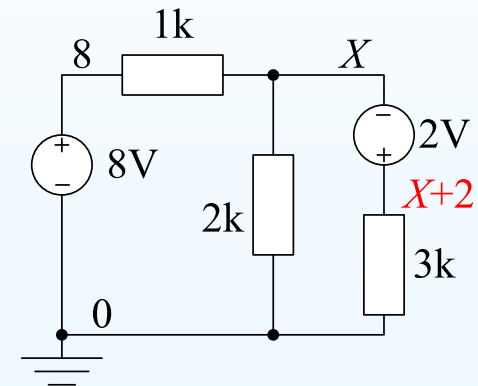
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$$\frac{X-8}{1} + \frac{X}{2} + \frac{(X+2)-0}{3} = 0$$

(4) Solve the equations:  $X = 4$

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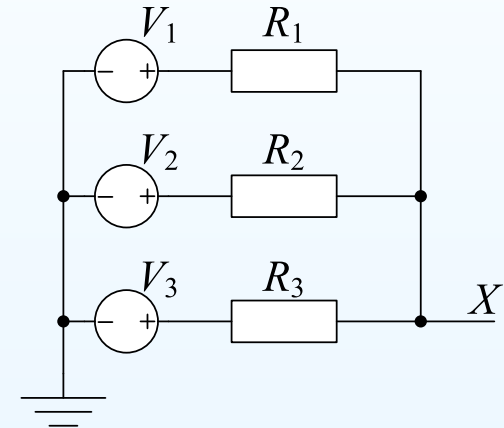


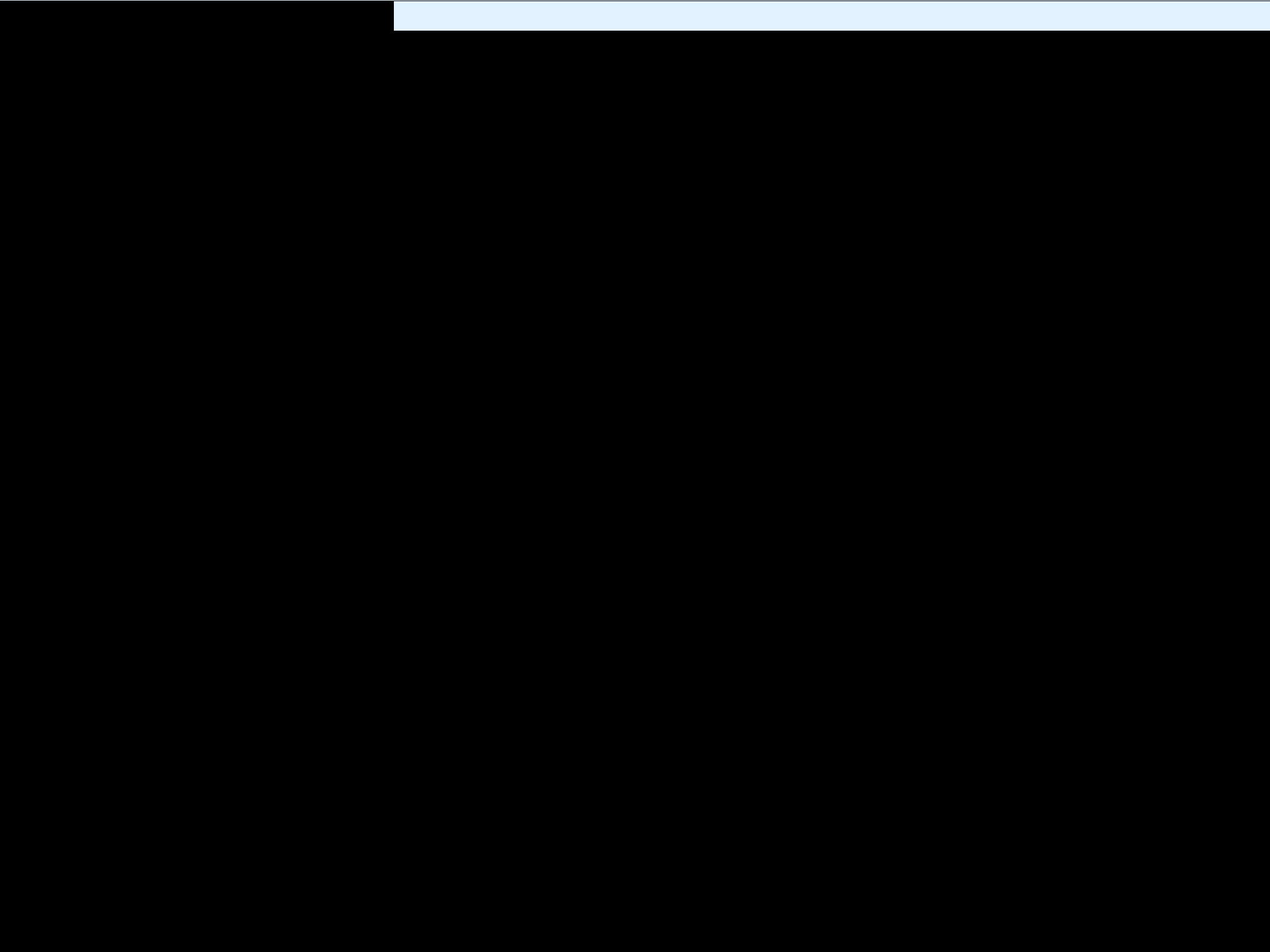
# Weighted Average Circuit

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- Dependent Sources
- Dependent Voltage Sources
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- Summary

A very useful sub-circuit that calculates the weighted average of any number of voltages.





# Weighted Average Circuit

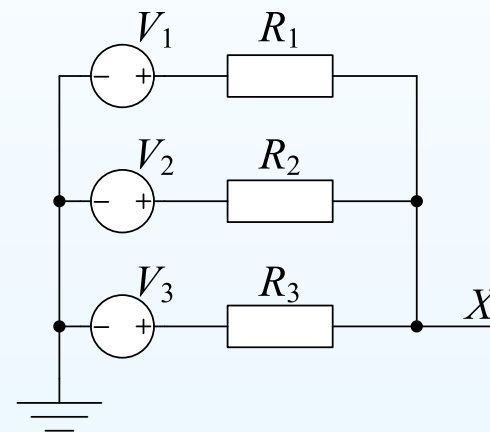
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KCL equation for node  $X$ :

$$\frac{X - V_1}{R_1} + \frac{X - V_2}{R_2} + \frac{X - V_3}{R_3} = 0$$



Or using conductances:

$$(X - V_1)G_1 + (X - V_2)G_2 + (X - V_3)G_3 = 0$$

# Weighted Average Circuit

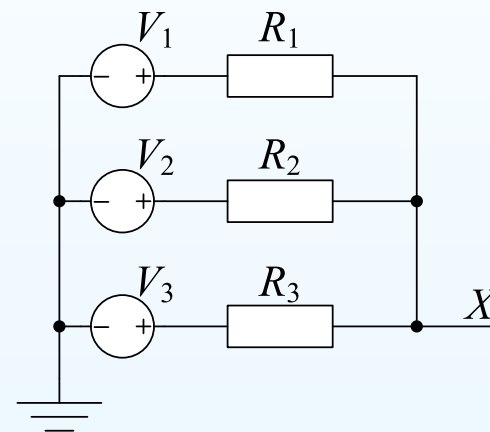
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$$X(G_1 + G_2 + G_3) = V_1G_1 + V_2G_2 + V_3G_3$$



# Weighted Average Circuit

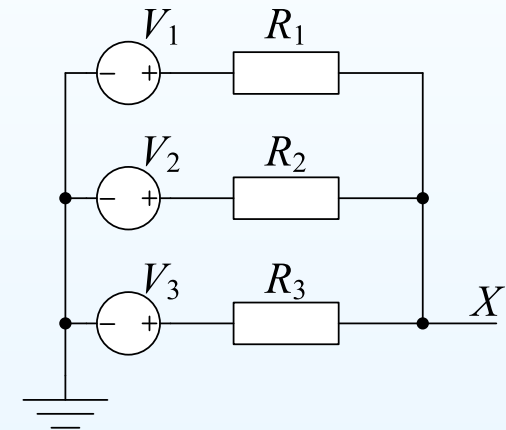
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## Weighted Average Circuit

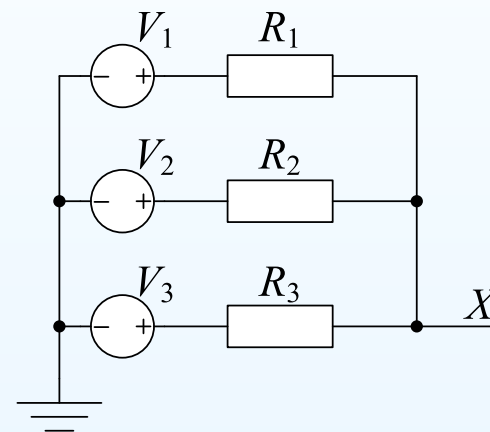
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Voltage  $X$  is the average of  $V_1$ ,  $V_2$ ,  $V_3$  weighted by the conductances.

## Weighted Average Circuit

### 3: Nodal Analysis

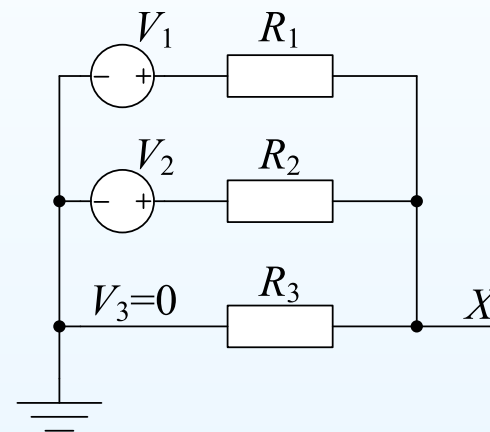
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KCL equation for node  $X$ :

$$\frac{X - V_1}{R_1} + \frac{X - V_2}{R_2} + \frac{X - V_3}{R_3} = 0$$

Still works if  $V_3 = 0$ .



Or using conductances:

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# Digital-to-Analog Converter

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A 3-bit binary number,  $b$ , has bit-weights of 4, 2 and 1. Thus 110 has a value 6 in decimal.

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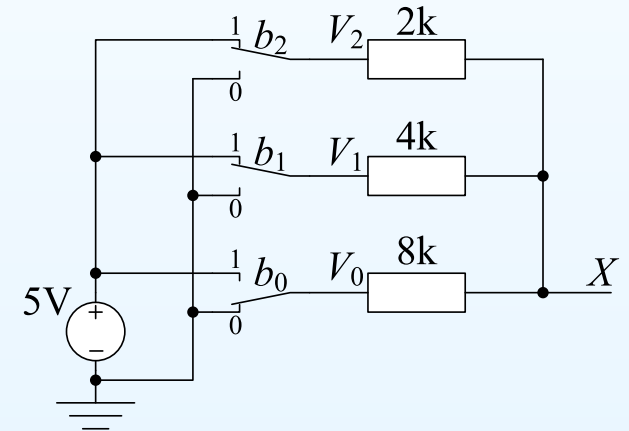
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We use  $b_2b_1b_0$  to control the switches which determine whether  $V_i = 5\text{ V}$  or  $V_i = 0\text{ V}$ . Thus  $V_i = 5b_i$ . Switches shown for  $b = 6$ .



# Digital-to-Analog Converter

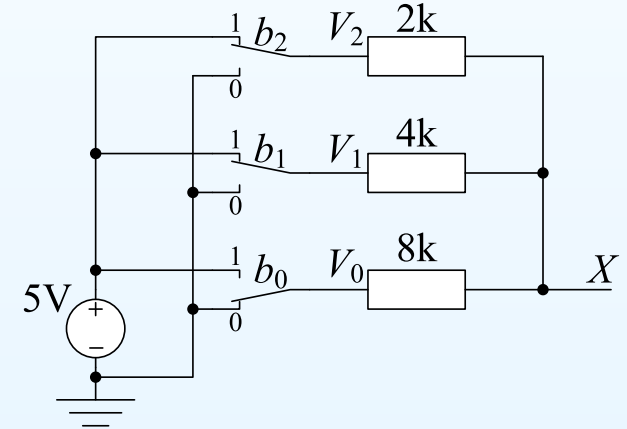
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$$X = \frac{\frac{1}{2}V_2 + \frac{1}{4}V_1 + \frac{1}{8}V_0}{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}}$$



$$G_2 = \frac{1}{R_2} = \frac{1}{2} \text{ mS}, \dots$$

# Digital-to-Analog Converter

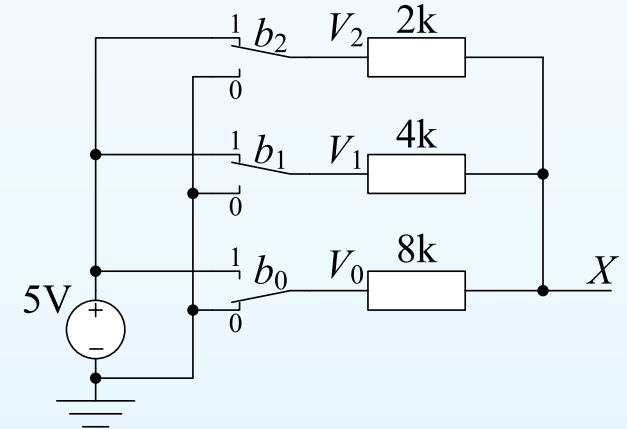
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# Digital-to-Analog Converter

## 3: Nodal Analysis

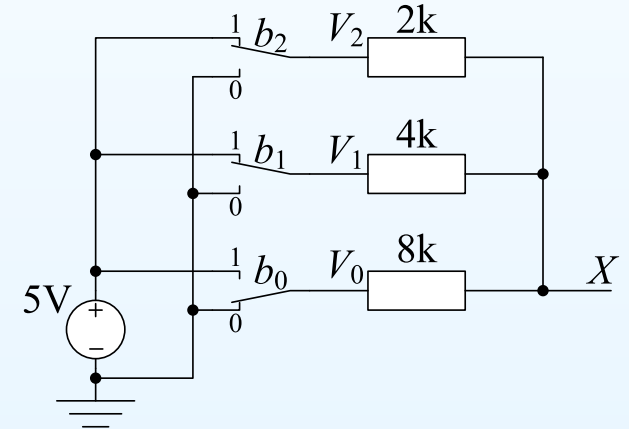
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but  $V_i = 5 \times b_i$  since it connects to either 0 V or 5 V



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# Digital-to-Analog Converter

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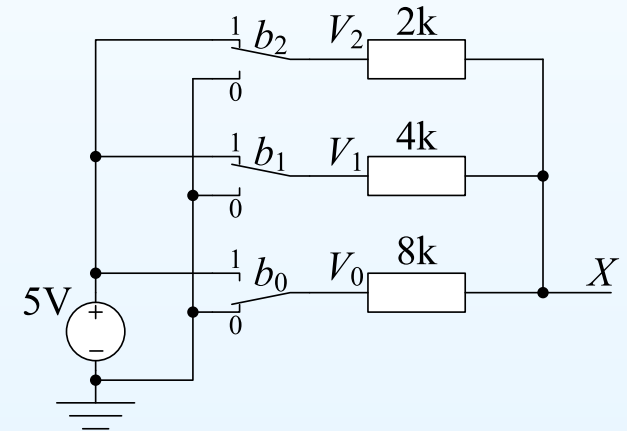
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$$= \frac{5}{7} (4b_2 + 2b_1 + b_0) = \frac{5}{7}b$$



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# Digital-to-Analog Converter

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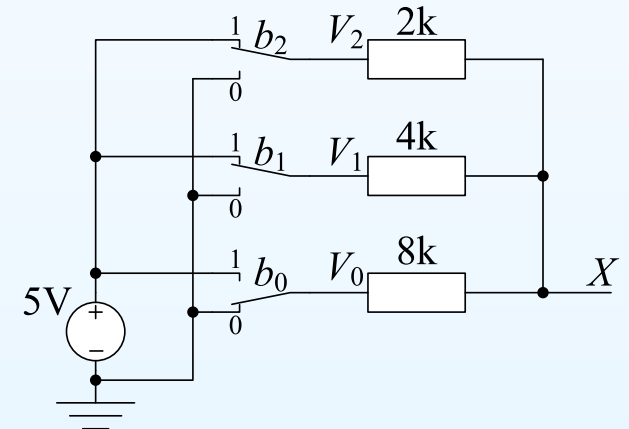
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$$= \frac{1}{7} (4V_2 + 2V_1 + V_0)$$

but  $V_i = 5 \times b_i$  since it connects to either 0 V or 5 V

$$= \frac{5}{7} (4b_2 + 2b_1 + b_0) = \frac{5}{7}b$$

So we have made a circuit in which  $X$  is proportional to a binary number  $b$ .



$$G_2 = \frac{1}{R_2} = \frac{1}{2} \text{ mS}, \dots$$

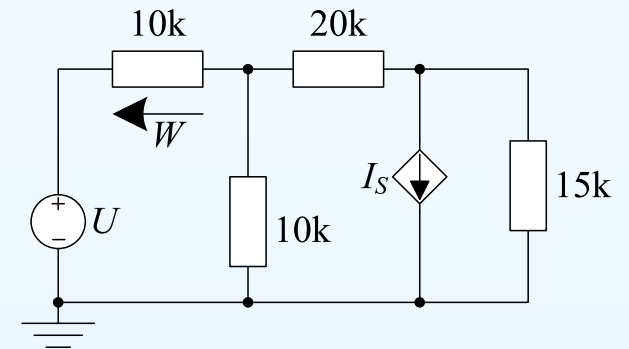
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In this circuit:  $I_S = 0.2W$  mA where  $W$  is in volts.



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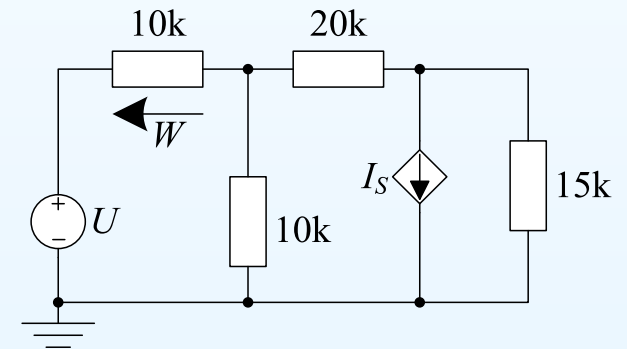
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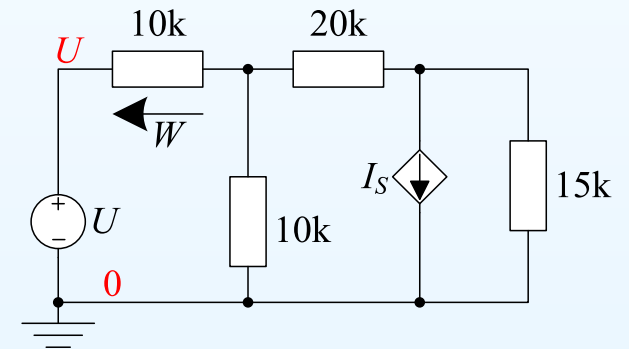
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# Dependent Sources

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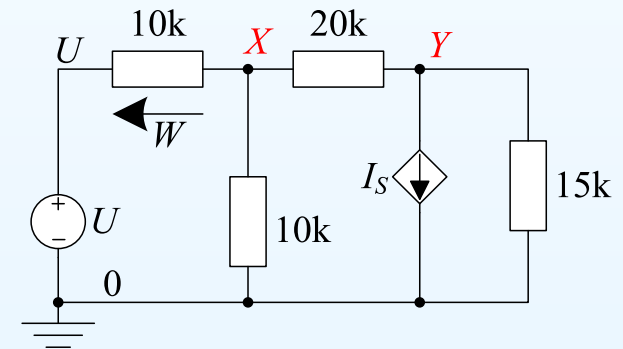
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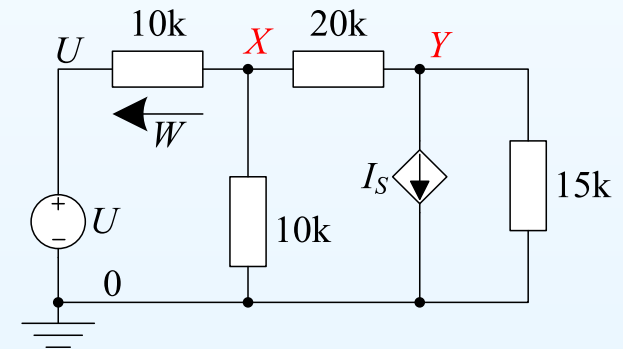
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$$I_S = 0.2(U - X)$$





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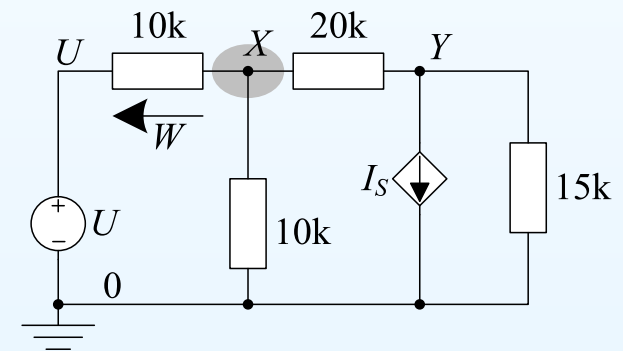
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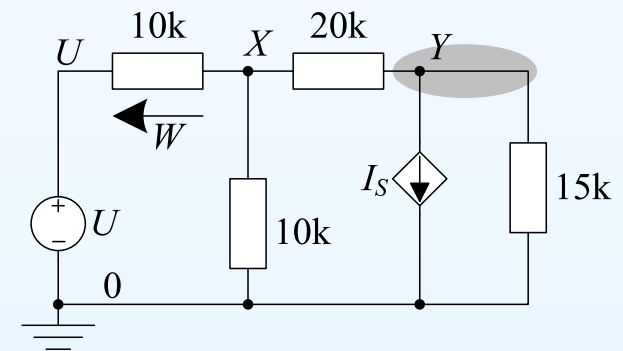
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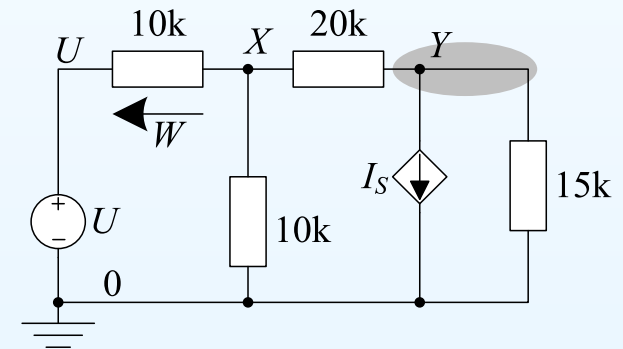
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(5) Solve all three equations to find  $X$ ,  $Y$  and  $I_S$  in terms of  $U$ :



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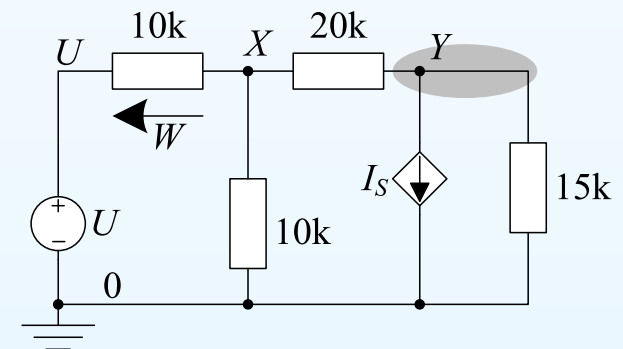
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$$X = 0.1U, Y = -1.5U, I_S = 0.18U$$



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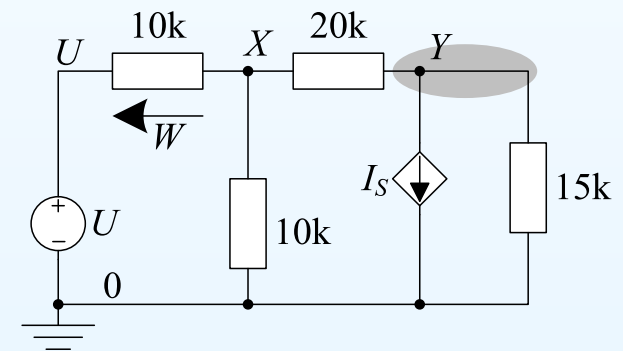
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$$X = 0.1U, Y = -1.5U, I_S = 0.18U$$

Note that the value of  $U$  is assumed to be known.

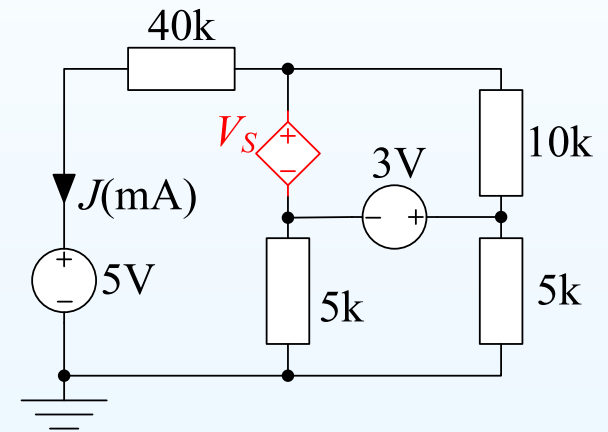


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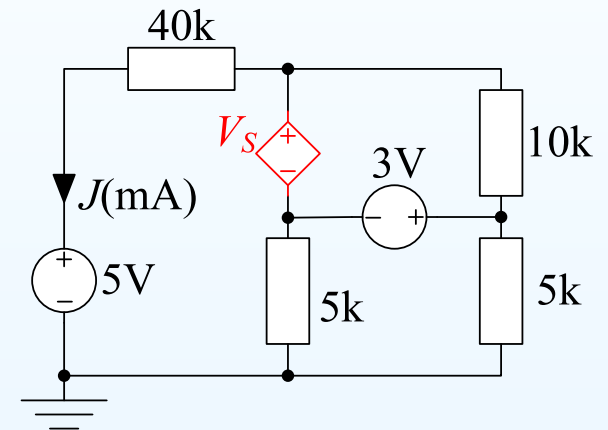
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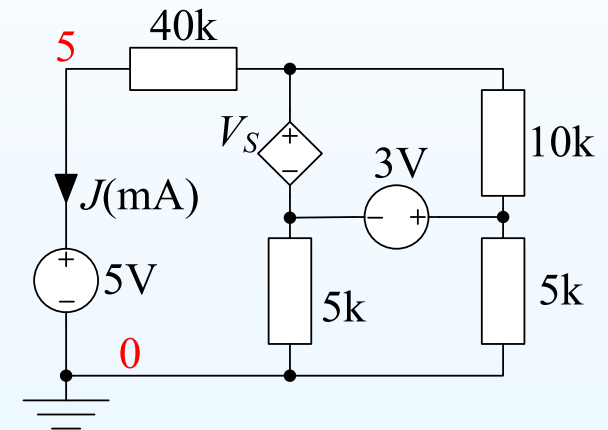
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(2) Label nodes: 0, 5





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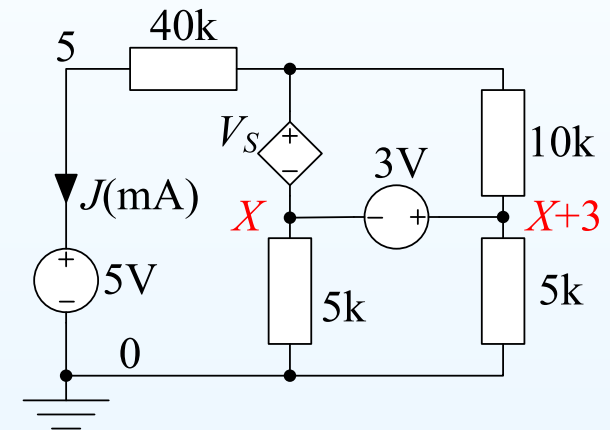
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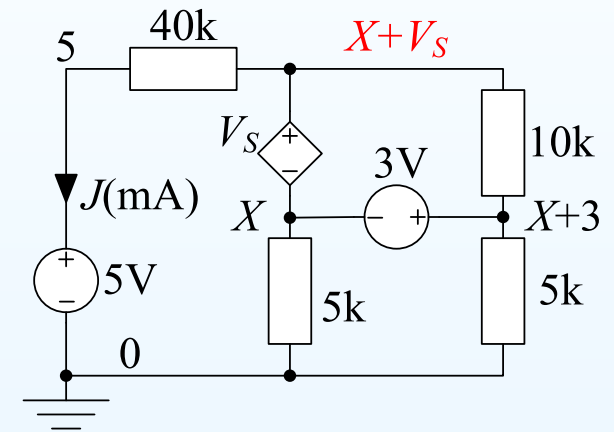
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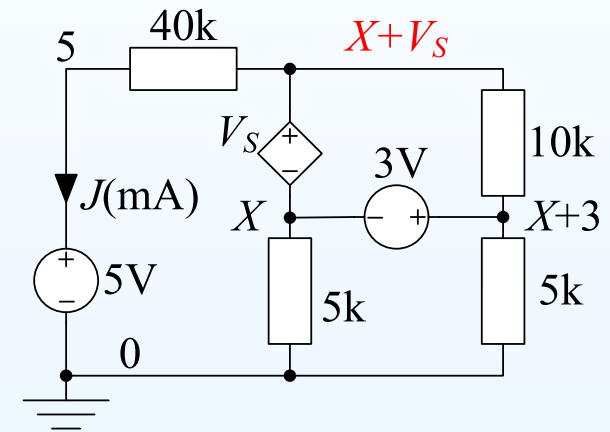
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$$V_S = 10J = 10 \times \frac{X + V_S - 5}{40} \Rightarrow 3V_S = X - 5$$



# Dependent Voltage Sources

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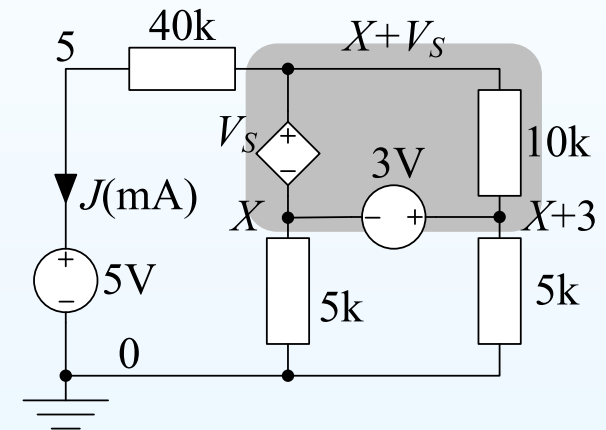
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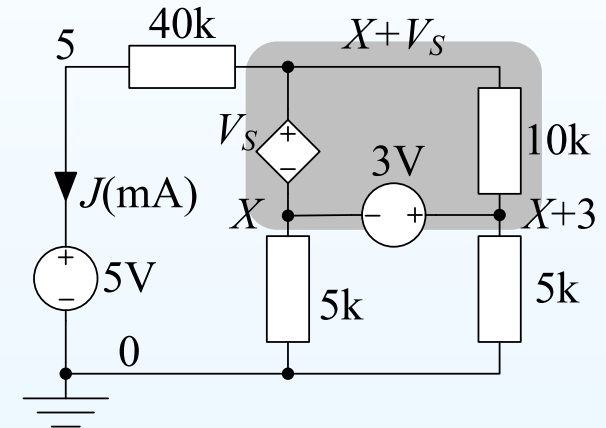
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(5) Solve the two equations:  $X = -1$  and  $V_S = -2$

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(3) Pick an unlabelled node and label it with  $X$ ,  $Y$ , ..., then loop back to step (2) until all nodes are labelled.

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(5) Write down a KCL equation for each “normal” node (i.e. one that is not connected to a floating voltage source).

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- (1) Pick any node as the voltage reference. Label its voltage as  $0\text{ V}$ . Label any dependent sources with  $V_S, I_S, \dots$
- (2) If any voltage sources are connected to a labelled node, label their other ends by adding the value of the source onto the voltage of the labelled end. Repeat as many times as possible.
- (3) Pick an unlabelled node and label it with  $X, Y, \dots$ , then loop back to step (2) until all nodes are labelled.
- (4) For each **dependent source**, write down an equation that expresses its value in terms of other node voltages.
- (5) Write down a KCL equation for each “normal” node (i.e. one that is not connected to a floating voltage source).
- (6) Write down a KCL equation for each “super-node”. A super-node consists of a set of nodes that are joined by floating voltage sources and includes any other components joining these nodes.
- (7) Solve the set of simultaneous equations that you have written down.

### 3: Nodal Analysis

- Aim of Nodal Analysis
- Nodal Analysis Stage 1: Label Nodes
- Nodal Analysis Stage 2: KCL Equations
- Current Sources
- Floating Voltage Sources
- Weighted Average Circuit
- Digital-to-Analog Converter
- Dependent Sources
- Dependent Voltage Sources
- Universal Nodal Analysis Algorithm
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- Nodal Analysis
  - Simple Circuits (no floating or dependent voltage sources)

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  - ▷ use supernodes: all the nodes connected by floating voltage sources (independent or dependent)

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  - Simple Circuits (no floating or dependent voltage sources)
  - Floating Voltage Sources
    - ▷ use supernodes: all the nodes connected by floating voltage sources (independent or dependent)
  - Dependent Voltage and Current Sources
    - ▷ Label each source with a variable
    - ▷ Write extra equations expressing the source values in terms of node voltages
    - ▷ Write down the KCL equations as before

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  - Alternative to nodal analysis but doesn't work for all circuits
  - No significant benefits  $\Rightarrow$  ignore it



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For further details see Hayt Ch 4 or Irwin Ch 3.