warning buzzer using combinational logic. The criterion for the activation of the warning buzzer is as follows: The buzzer activates if the headlights are on *and* the driver's door is opened *or* if the key is in the ignition *and* the door is opened.

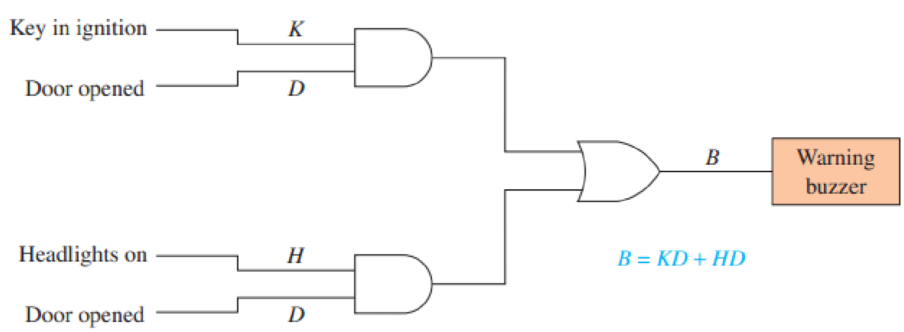
The logic function for the automobile warning buzzer is illustrated symbolically in Figure 5–1. The figure illustrates a *combination* of logic functions that can be written as a Boolean equation in the form

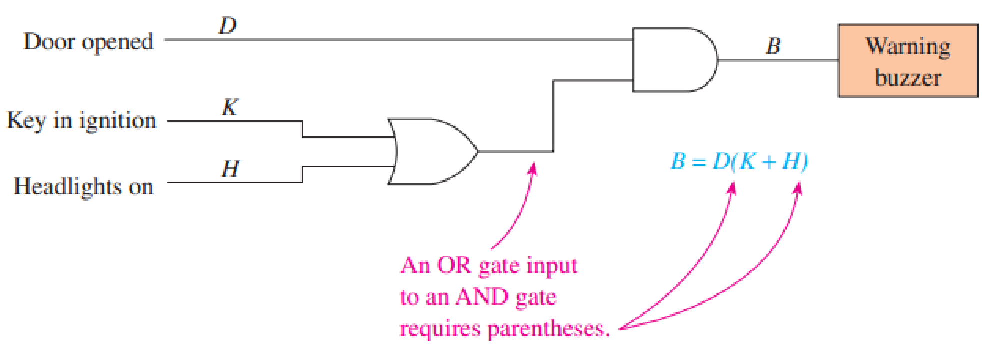
$$B = K \text{ and } D \quad \text{or} \quad H \text{ and } D$$

which is also written as

$$B = KD + HD$$

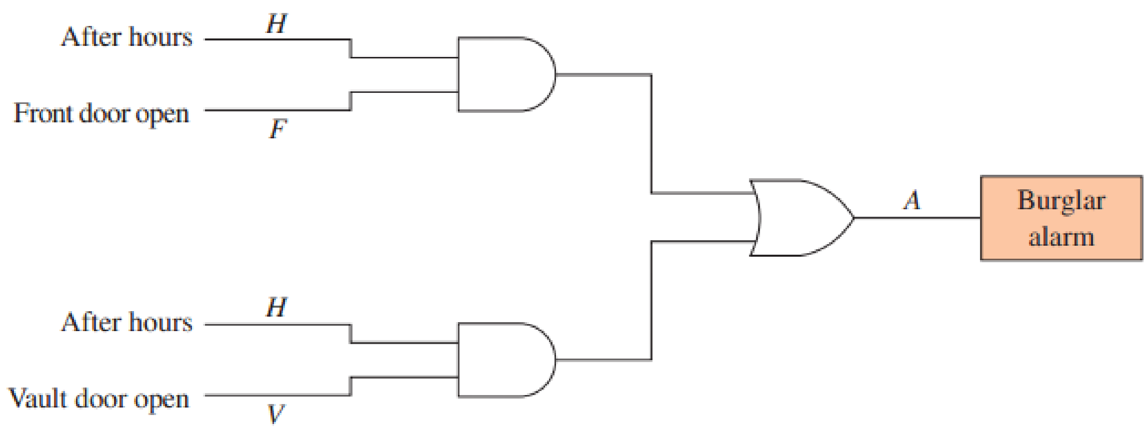
This equation can be stated as "B is HIGH if K *and* D are HIGH *or* if H *and* D are HIGH."





Write the Boolean logic equation, and draw the logic circuit and truth table that represents the following function: A bank burglar alarm (A) is to activate if it is after banking hours (H) *and* the front door (F) is opened *or* if it is after banking hours (H) and the vault door is opened (V).

Solution: $A = HF + HV$. The logic circuit and truth table are shown in Figure 5–4.

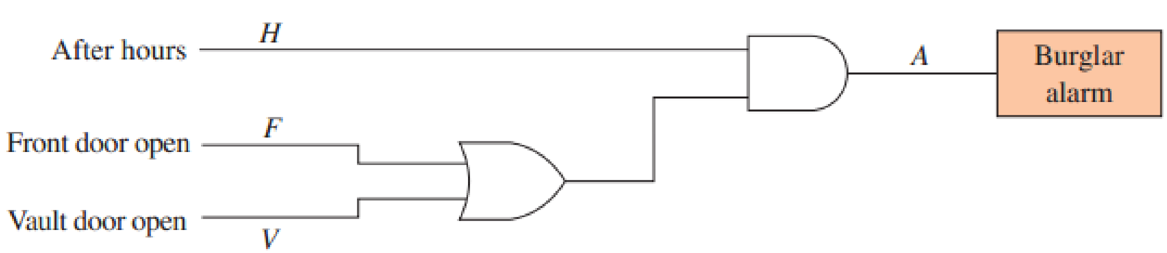


H	F	V	A
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Using common reasoning, reduce the logic function described in Example 5–1 to a simpler form.

Solution: The alarm is activated if it is after banking hours *and* if either the front door is opened *or* the vault door is opened (see Figure 5–5). The simplified equation is written as

$$A = H(F + V) \qquad \text{(Notice the use of parentheses.)}$$



Write the Boolean equation for the logic circuit shown in Figure 5–6.

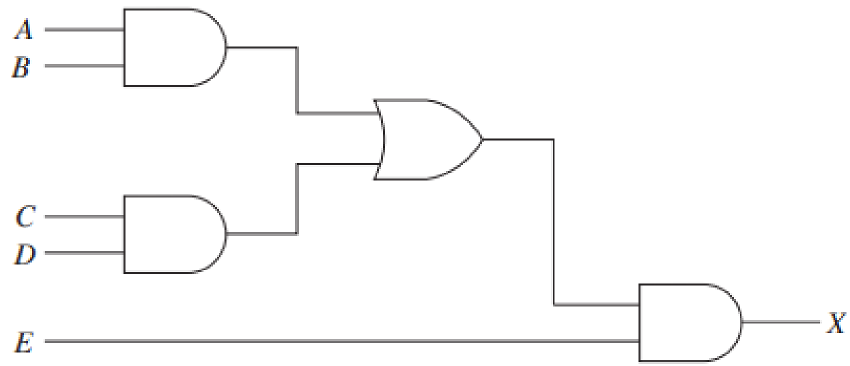


Figure 5–6 Combinational logic circuit for Example 5–3.

Solution: $X = (AB + CD)E$

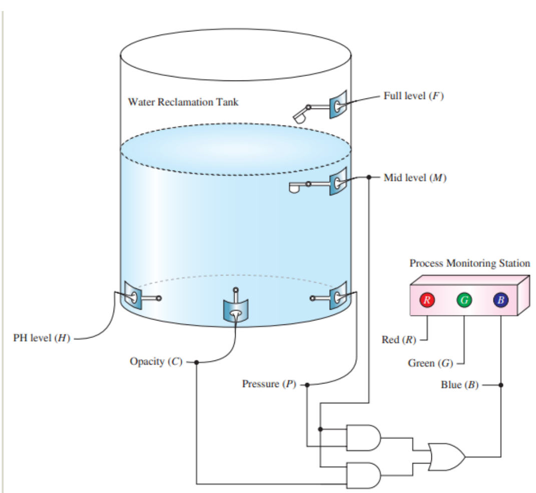
Figure 5–7 shows a gray water reclamation tank having five inputs and three outputs. The inputs are used to monitor HIGH/LOW levels on the quantities shown, and the outputs are used to illuminate the color lights in the Process Monitoring Station. The system is designed to capture gray water before it goes into a septic system. Gray water is the water drained in the bathroom sink or shower and water drained in a washing machine. This recycled water can then be used in the toilet or for landscape irrigation. In this example, logic gates are connected to the figure to turn on the blue light (B) if the water is at the mid level (M) and there is a HIGH pressure (P) or if the water is at the mid level (M) and there is a HIGH opacity (c). (Opacity is a measure of water clarity.)

(a) Reduce that Boolean equation to a simpler form.

(b) Write the Boolean equation for the new logic that would turn on the red light (R) if the PH level (H) or the Opacity (C) or the Pressure (P) are HIGH while the water is at the mid level (M). (The word *while* indicates an AND function).

(c) Write the Boolean equation for the new logic that would turn on the green light (G) if the PH level (H) or the Pressure (P) are HIGH while the water is at the mid level (M) or the full level (F).

(d) Write the Boolean equation for the new logic that would turn on the blue light (B) if the Opacity (C) and the pressure (P) are HIGH while the water is at the mid level (M) or the full level (F).



Solutions:

(a) $B = M(P + C)$ **(c)** $G = (H + P)(M + F)$

(b) $R = (H + C + P)M$ **(d)** $B = CP(M + F)$

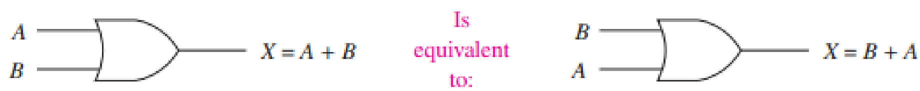


Figure 5–8 Using the commutative law of addition to rearrange an OR gate.

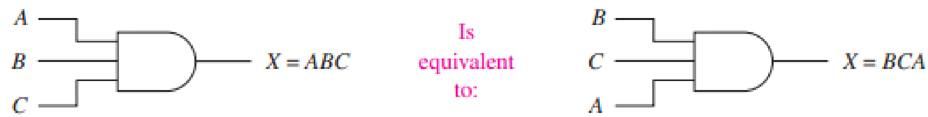


Figure 5–9 Using the commutative law of multiplication to rearrange an AND gate.

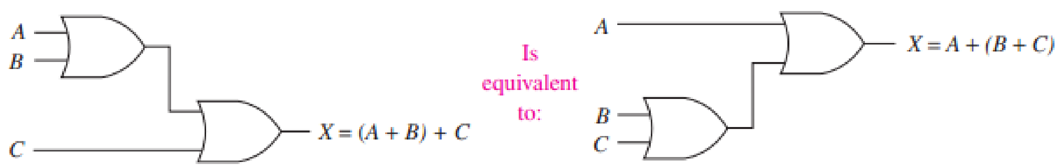


Figure 5–10 Using the associative law of addition to rearrange the grouping of OR gates.

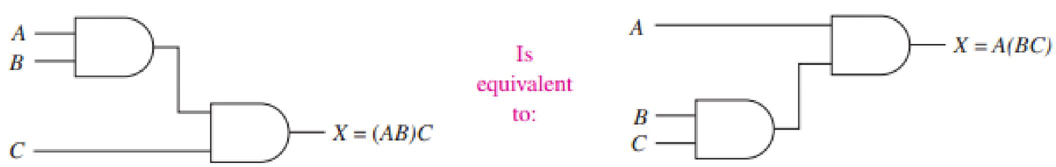


Figure 5–11 Using the associative law of multiplication to rearrange the grouping of AND gates.

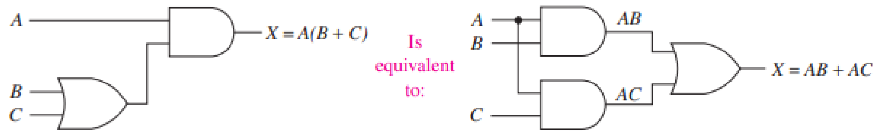
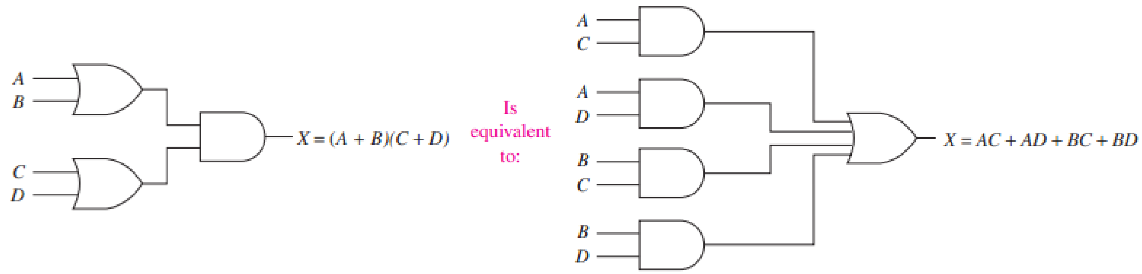


Figure 5–12 Using the distributive law to form an equivalent circiut.



A bank burglar alarm (B) will activate if it is after banking hours (A) and someone opens the front door (D). The logic level of the variable A is 1 after banking hours and 0 during banking hours. Also, the logic level of the variable D is 1 if the door sensing switch is opened and 0 if the door sensing switch is closed. The Boolean equation is, therefore, $B = AD$. The logic circuit to implement this function is shown in Figure 5–14(a).

Figure 5–14 (a) Logic circuit for a simple burglar alarm: (b) disabling the burglar alarm by making $D = 0$.

Later, a burglar comes along and puts tape on the door sensing switch, holding it closed so that it always puts out a 0 logic level. Now the Boolean equation ($B = AD$) becomes $B = A \cdot 0$ because the door sensing switch is always 0. The alarm will never sound in this condition because one input to the AND gate is always 0. The burglar must have studied the Boolean rules and realized that anything ANDed with a 0 will output a 0, as shown in Figure 5–14(b).

5–1. How many gates are required to implement the following Boolean equations?

(a) $X = (A + B)C$

(b) $Y = AC + BC$

(c) $Z = (ABC + CD)E$

5–2. Which Boolean law is used to transform each of the following equations?

(a) $B + (D + E) = (B + D) + E$

(b) $CAB = BCA$

(c) $(B + C)(A + D) = BA + BD + CA + CD$

5–3. The output of an AND gate with one of its inputs connected to 1 will always output a level equal to the level at the other input. True or false?

5–4. The output of an OR gate with one of its inputs connected to 1 will always output a level equal to the level at the other input. True or false?

5–5. If one input to an OR gate is connected to 0, the output will always be 0 regardless of the level on the other input. True or false?

5–6. Use one of the forms of Rule 10 to transform each of the following equations:

(a) $\overline{B} + AB = ?$

(b) $B + \overline{B}C = ?$

The logic circuit shown in Figure 5–23 is used to turn on a warning buzzer at X based on the input conditions at A, B, and C. A simplified equivalent circuit that will perform the same function can be formed by using Boolean algebra. Write the equation of the circuit in Figure 5–23, simplify the equation, and draw the logic circuit of the simplified equation.

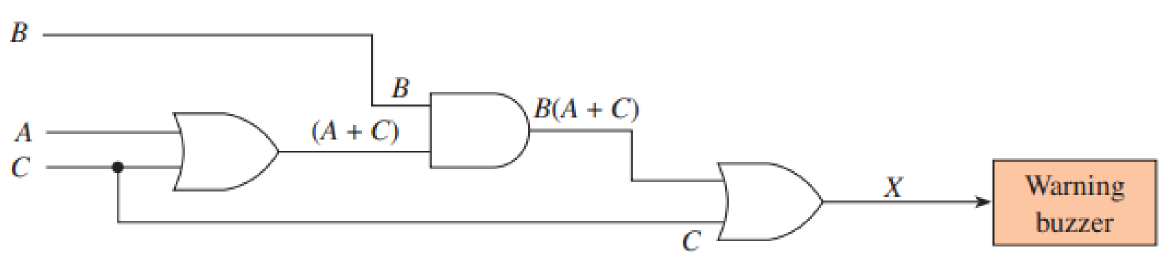


Figure 5–23 Logic circuit for Example 5–6.

Solution: The Boolean equation for X is

$$X = B(A + C) + C$$

To simplify, first apply *Law 3* $[B(A + C) = BA + BC]$:

$$X = BA + BC + C$$

Next, factor a C from terms 2 and 3:

$$X = BA + C(B + 1)$$

Apply *Rule 4* $(B + 1 = 1)$:

$$X = BA + C \cdot 1$$

Apply *Rule 2* $(C \cdot 1 = C)$:

$$X = BA + C$$

Apply *Law 1* $(BA = AB)$:

$$X = AB + C \quad \leftarrow \textit{simplified equation}$$

The logic circuit of the simplified equation is shown in Figure 5–24.

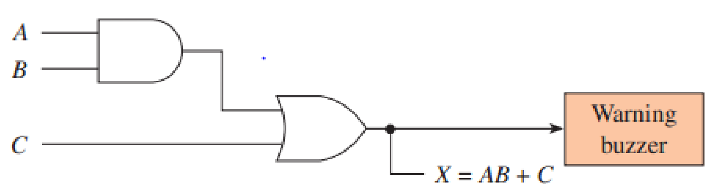


Figure 5–24 Simplified logic circuit for Example 5–6.

EXAMPLE 5–7

Repeat Example 5–6 for the logic circuit shown in Figure 5–25.

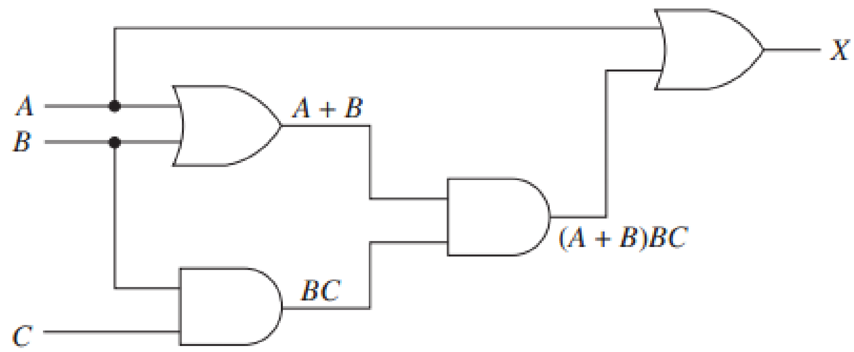


Figure 5–25 Logic circuit for Example 5–7.

Solution: The Boolean equation for X is

$$X = (A + B)BC + A$$

To simplify, first apply *Law 3* $[(A + B)BC = ABC + BBC]$:

$$X = ABC + BBC + A$$

Apply *Rule 5* $(B \cdot B = B)$:

$$X = ABC + BC + A$$

Factor a BC from terms 1 and 2:

$$X = BC(A + 1) + A$$

Apply *Rule 4* $(A + 1 = 1)$:

$$X = BC \cdot 1 + A$$

Apply *Rule 2* $(BC \cdot 1 = BC)$:

$$X = BC + A \quad \leftarrow \textit{simplified equation}$$

The logic circuit for the simplified equation is shown in Figure 5–26.

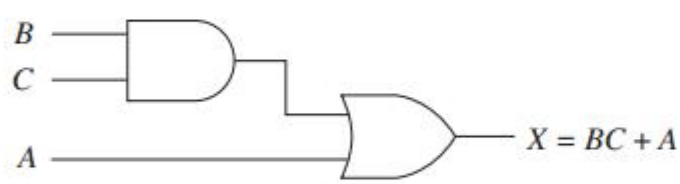
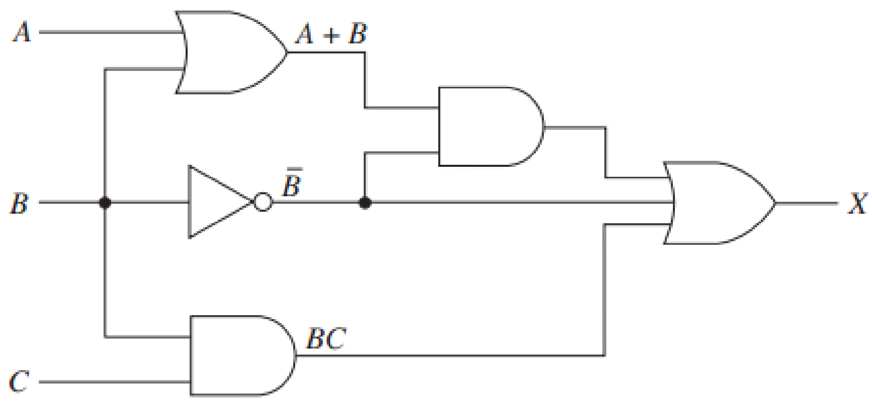


Figure 5–26 Simplified logic circuit for Example 5–7.

EXAMPLE 5–8

Repeat Example 5–6 for the logic circuit shown in Figure 5–27(a).



(a)

Figure 5–27 Logic circuit for Example 5–8: (a) Original circuit and (b) Simplified circuit.

$$X = (A + B)\overline{B} + \overline{B} + BC$$

To simplify, first apply *Law 3* $[(A + B)\overline{B} = A\overline{B} + B\overline{B}]$:

$$X = A\overline{B} + B\overline{B} + \overline{B} + BC$$

Apply *Rule 7* $(B\overline{B} = 0)$:

$$X = A\overline{B} + 0 + \overline{B} + BC$$

Apply *Rule 3* $(A\overline{B} + 0 = A\overline{B})$:

$$X = A\overline{B} + \overline{B} + BC$$

Factor a $\overline{B}$ from terms 1 and 2:

$$X = \overline{B}(A + 1) + BC$$

Apply *Rule 4* $(A + 1 = 1)$:

$$X = \overline{B} \cdot 1 + BC$$

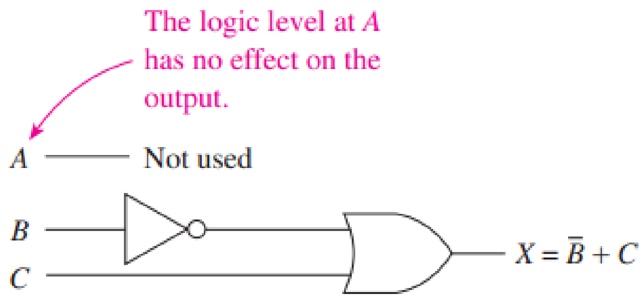
Apply *Rule 2* $(\overline{B} \cdot 1 = \overline{B})$:

$$X = \overline{B} + BC$$

Apply *Rule 10(b)* $(\overline{B} + BC = \overline{B} + C)$:

$$X = \overline{B} + C \quad \leftarrow \textit{simplified equation}$$

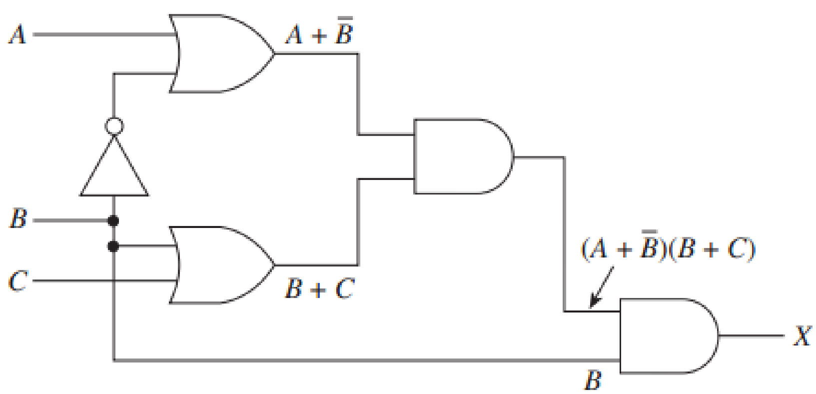
The logic circuit of the simplified equation is shown in Figure 5–27(b).



(b)

EXAMPLE 5–9

Repeat Example 5–6 for the logic circuit shown in Figure 5–28(a).



(a)

Figure 5–28 Logic circuit for Example 5–9: (a) Original circuit and (b) Simplified circuit.

Solution: The Boolean equation for X is

$$X = [(A + \overline{B})(B + C)]B$$

To simplify, first apply *Law 3*:

$$X = (AB + AC + \overline{B}B + \overline{B}C)B$$

The $\overline{B}B$ term can be eliminated using *Rule 7* and then *Rule 3*:

$$X = (AB + AC + \overline{B}C)\,B$$

Apply *Law 3* again:

$$X = ABB + ACB + \overline{B}CB$$

Apply *Law 1*:

$$X = ABB + ABC + \overline{B}BC$$

Apply *Rules 5* and *7*:

$$X = AB + ABC + 0 \cdot C$$

Apply *Rule 1*:

$$X = AB + ABC$$

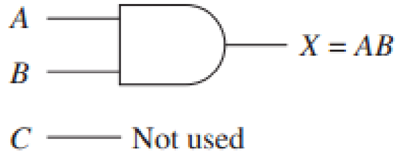
Factor an AB from both terms:

$$X = AB\,(1 + C)$$

Apply *Rule 4* and then *Rule 2*:

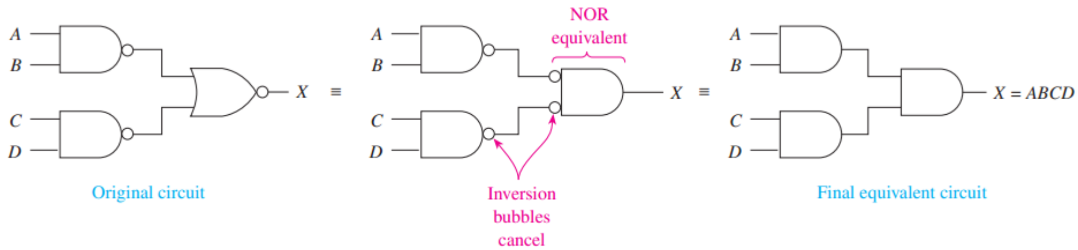
$$X = AB \quad \leftarrow \textit{simplified equation}$$

The logic circuit of the simplified equation is shown in Figure 5–28(b).

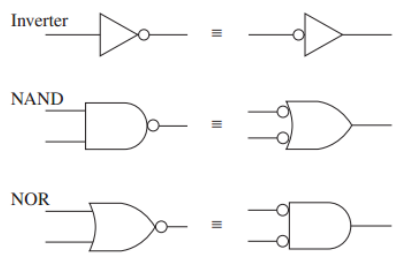


(b)

(a)


A
B
C
D
X
≡
NOR
equivalent
X
≡
X = ABCD
Original circuit
Inversion
bubbles
cancel
Final equivalent circuit

(b)


Inverter
≡
NAND
≡
NOR
≡

(c)

EXAMPLE 5–12

Write the Boolean equation for the circuit shown in Figure 5–42. Use De Morgan's theorem and then Boolean algebra rules to simplify the equation. Draw the simplified circuit.

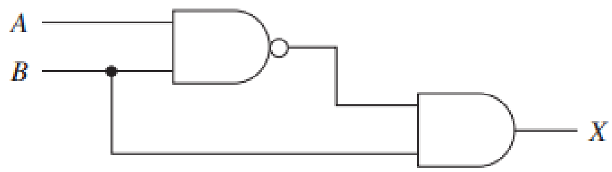


Figure 5–42

Solution: The Boolean equation at X is

$$X = \overline{AB} \cdot B$$

Applying De Morgan's theorem produces

$$X = (\overline{A} + \overline{B}) \cdot B$$

(Notice the use of parentheses to maintain proper grouping. *Rule:* Whenever you break the bar over a NAND you must use parentheses.) Using Boolean algebra rules produces

$$\begin{aligned} X &= \overline{A}B + \overline{B}B \\ &= \overline{A}B + 0 \\ &= \boldsymbol{\overline{A}B} \quad \leftarrow \textit{simplified equation} \end{aligned}$$

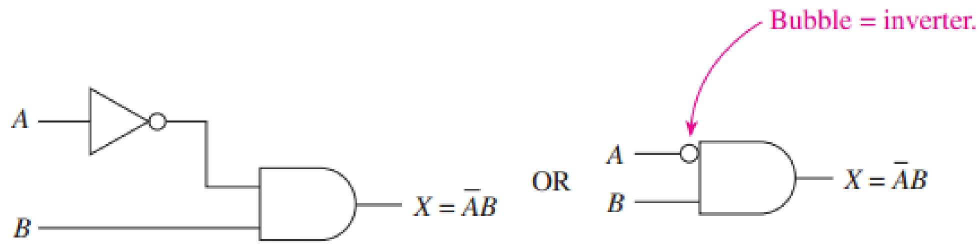


Figure 5–43 Simplified logic circuit for Example 5–12.

EXAMPLE 5–13

Repeat Example 5–12 for the circuit shown in Figure 5–44.

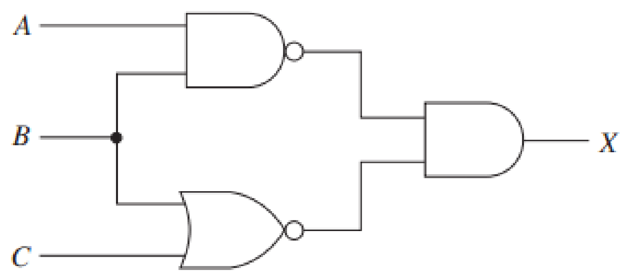


Figure 5–44

Solution: The Boolean equation at X is

$$X = \overline{AB} \cdot \overline{B + C}$$

Applying De Morgan's theorem produces

$$X = (\overline{A} + \overline{B}) \cdot \overline{B}\,\overline{C}$$

(Notice the use of parentheses to maintain proper grouping.) Using Boolean algebra rules produces

$$\begin{aligned} X &= \overline{A}\,\overline{B}\,\overline{C} + \overline{B}\,\overline{B}\,\overline{C} \\ &= \overline{A}\,\overline{B}\,\overline{C} + \overline{B}\,\overline{C} \\ &= \overline{B}\,\overline{C}(\overline{A} + 1) \\ &= \overline{B}\,\overline{C} \quad \leftarrow \textit{simplified equation} \end{aligned}$$

The simplified circuit is shown in Figure 5–45.

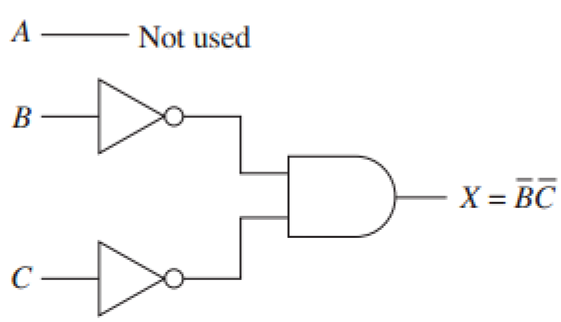


Figure 5–45 Simplified logic circuit for Example 5–13.

EXAMPLE 5–14

Repeat Example 5–12 for the circuit shown in Figure 5–47.

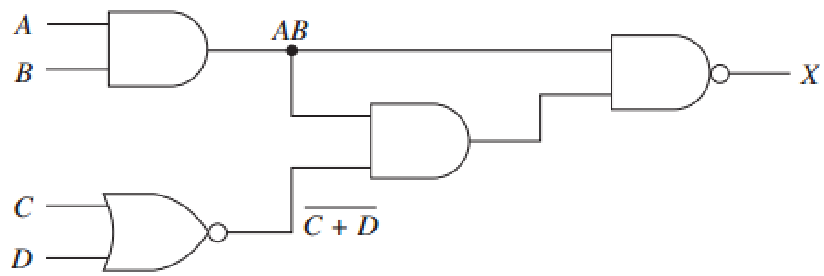


Figure 5–47

Solution:

$$
\begin{aligned}
X &= \overline{(AB \cdot \overline{C + D})\,AB} \\
&= \overline{AB \cdot \overline{C + D}} + \overline{AB} \\
&= \overline{AB} + \overline{\overline{C + D}} + \overline{AB} \\
&= \overline{A} + \overline{B} + C + D + \overline{A} + \overline{B} \\
&= \overline{A} + \overline{B} + C + D \quad \leftarrow \textit{simplified equation}
\end{aligned}
$$

The simplified circuit is shown in Figure 5–48.

The simplified circuit is shown in Figure 5–48.

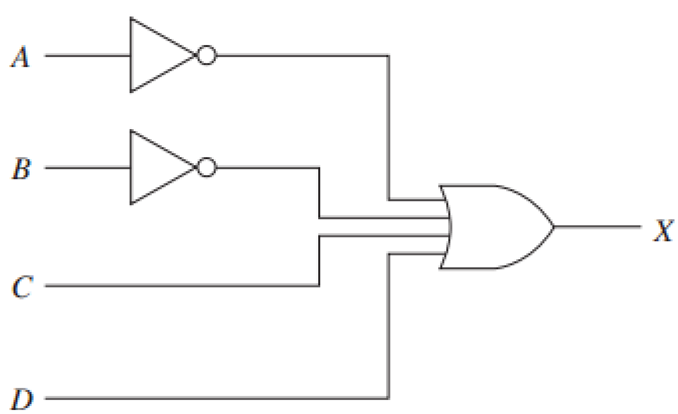


Figure 5–48 Simplified logic circuit for Example 5–14.

EXAMPLE 5–15

Use De Morgan's theorem and Boolean algebra on the circuit shown in Figure 5–49 to develop an equivalent circuit that has inversion bars covering only single variables.

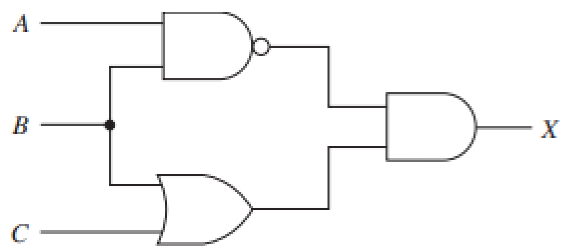


Figure 5–49

Solution: The Boolean equation at X is

$$X = \overline{AB} \cdot (B + C)$$

Applying De Morgan's theorem produces

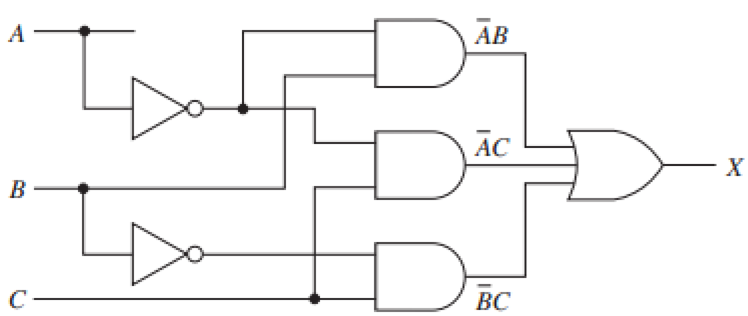
$$X = (\overline{A} + \overline{B}) \cdot (B + C)$$

(Notice the use of parentheses to maintain proper grouping.) Using Boolean algebra rules produces

$$X = \overline{A}B + \overline{A}C + \overline{B}B + \overline{B}C$$

$$= \overline{A}B + \overline{A}C + \overline{B}C \quad \leftarrow \textit{final equation (sum-of-products form)}$$

The equivalent circuit is shown in Figure 5–50.



EXAMPLE 5–16

Using De Morgan's theorem and Boolean algebra, prove that the two circuits shown in Figure 5–51 are equivalent.

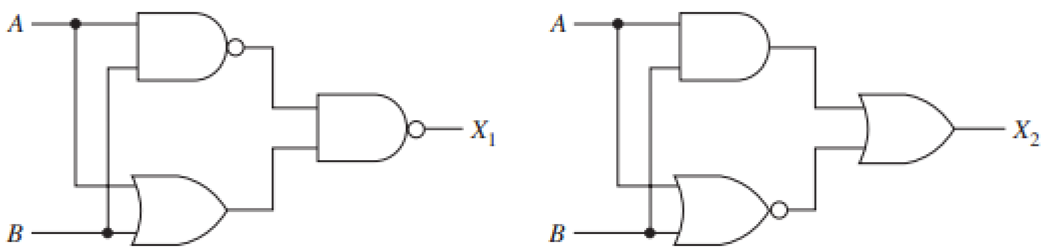


Figure 5–51

Solution: They can be proved to be equivalent if their simplified equations match.

$$\begin{aligned} X_1 &= \overline{\overline{AB} \cdot (A + B)} \\ &= \overline{\overline{AB}} + \overline{A + B} \\ &= AB + \overline{A}\,\overline{B} \end{aligned} \qquad\qquad \begin{aligned} X_2 &= AB + \overline{A + B} \\ &= AB + \overline{A}\,\overline{B} \end{aligned}$$

Equivalent

Draw the logic circuit for the following equation, simplify the equation, and construct a truth table for the simplified equation

$$X = \overline{A \cdot \overline{B}} + \overline{A \cdot (\overline{A} + C)}$$

Solution: To draw the circuit, we have to reverse our thinking from the previous examples. When we study the equation, we see that we need two NANDs feeding into an OR gate, as shown in Figure 5–54. Then we have to provide the inputs to the NAND gates, as shown in Figure 5–55.

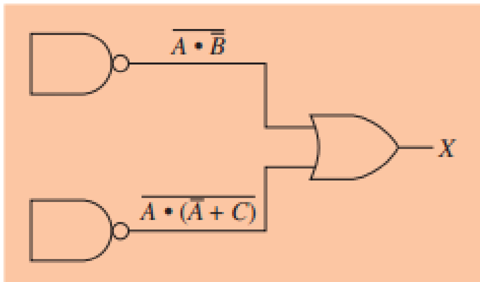


Figure 5–54 Partial solution to Example 5–18.

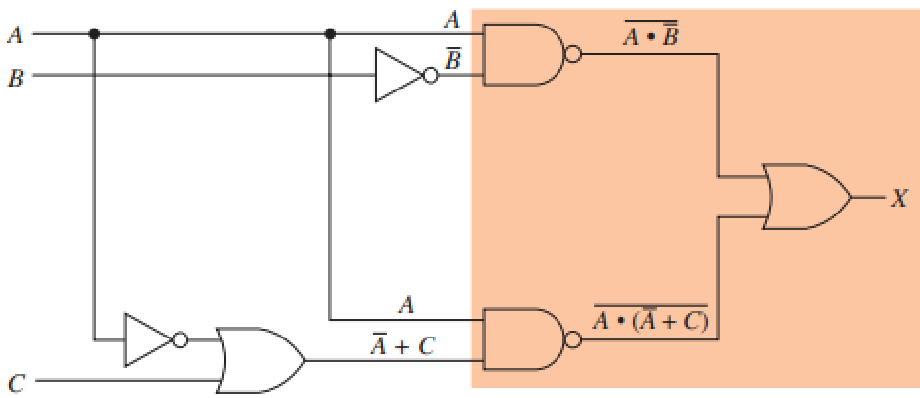


Figure 5–55 Logic circuit of the equation for Example 5–18.

Next, we use De Morgan's theorem and Boolean algebra to simplify the equation:

$$
\begin{aligned}
X &= \overline{A \cdot \overline{B}} + \overline{A \cdot (\overline{A} + C)} \\
&= (\overline{A} + \overline{\overline{B}}) + (\overline{A} + \overline{\overline{A} + C}) \\
&= \overline{A} + B + \overline{A} + \overline{\overline{A}} \cdot \overline{C} \\
&= \overline{A} + \overline{A} + A\overline{C} + B \\
&= \overline{A} + A\overline{C} + B
\end{aligned}
$$

Apply *Rule 10:*

$$X = \overline{A} + \overline{C} + B \quad \leftarrow \textit{simplified equation}$$

This equation can be interpreted as: X is HIGH if A is LOW or C is LOW or B is HIGH. Now, to construct a truth table (Table 5–3), we need three input columns (A, B, C) and eight entries ($2^3 = 8$), and we fill in a 1 for X when $A = 0$, $C = 0$, or $B = 1$.

TABLE 5–3 Truth Table for Example 5–18

A	B	C	$X = \overline{A} + \overline{C} + B$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

EXAMPLE 5–19

Repeat Example 5–18 for the following equation:

$$X = \overline{A\overline{B} \cdot (A + C)} + \overline{A}B \cdot \overline{A + \overline{B} + \overline{C}}$$

Solution: The required logic circuit is shown in Figure 5–56. The Boolean equation simplification is

$$\begin{aligned}
X &= \overline{A\overline{B} \cdot (A + C)} + \overline{A}B \cdot \overline{A + \overline{B} + \overline{C}} \\
&= \overline{A}\,\overline{\overline{B}} + \overline{A + C} + \overline{A}B \cdot (\overline{A} \cdot \overline{\overline{B}} \cdot \overline{\overline{C}}) \\
&= (\overline{A} + \overline{\overline{B}}) + \overline{A} \cdot \overline{C} + \overline{A}\,\overline{A}BBC \\
&= \overline{A} + B + \overline{A}\,\overline{C} + \overline{A}BC \\
&= \overline{A}(1 + \overline{C}) + B + \overline{A}BC \\
&= \overline{A} + B + \overline{A}BC \\
&= \overline{A} + B(1 + \overline{A}C) \\
&= \overline{A} + B \quad \leftarrow \textit{simplified equation}
\end{aligned}$$

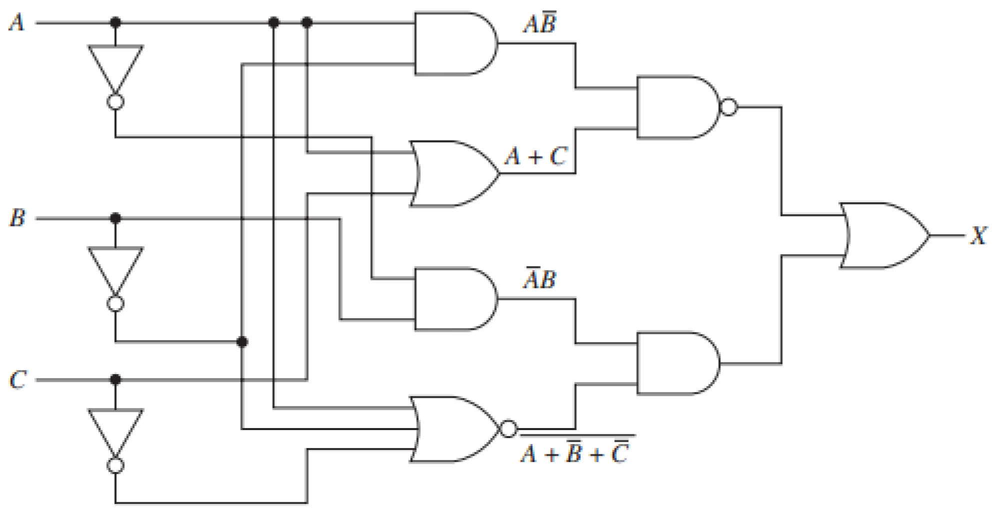
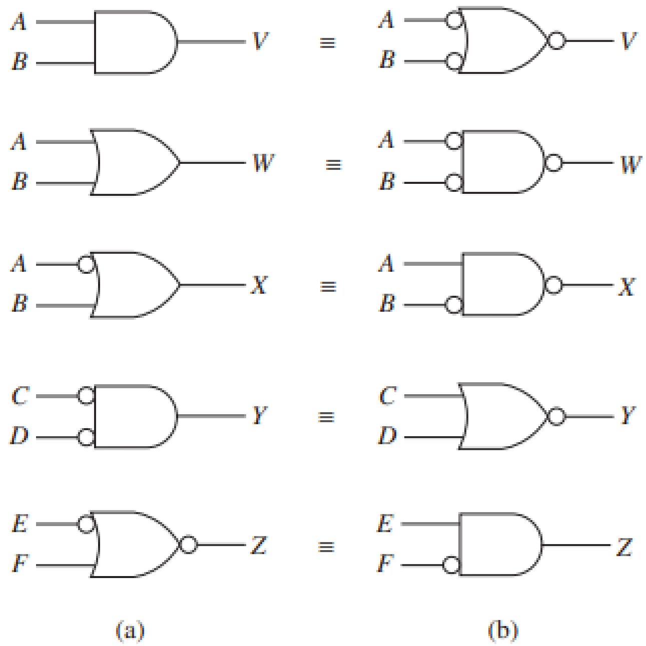


Figure 5–56 Logic circuit for the equation of Example 5–19.

TABLE 5–4 Truth Table for Example 5–19

A	*B*	*C*	$X = \overline{A} + B$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$X = 1$ if $A = 0$ or $B = 1$



(a) Original logic circuits; (b) equivalent logic circuits.

Review Questions

5–7. Why is De Morgan's theorem important in the simplification of Boolean equations?

5–8. Using De Morgan's theorem, you can prove that a NOR gate is equivalent to an ___________ (OR, AND) gate with inverted inputs.

5–9. Using the bubble-pushing technique, an AND gate with one of its inputs inverted is equivalent to a ___________ (NAND, NOR) gate with its other input inverted.

5–10. Using bubble pushing to convert an inverted-input OR gate will yield a(n) ___________ (AND, NAND) gate.

Problems

Section 5–1

5–1. Write the Boolean equation for each of the logic circuits shown in Figure P5–1.

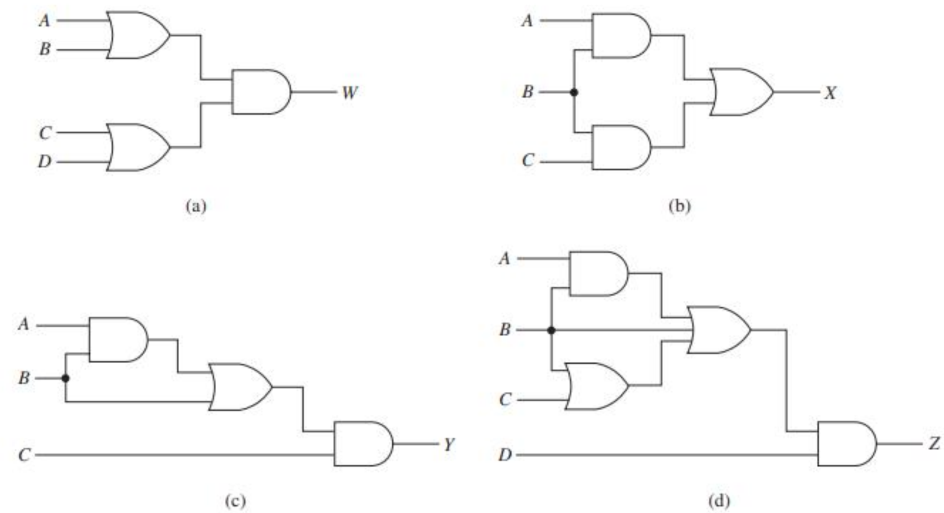


Figure P5–1

5–2. Refer to the gray water reclamation tank in Figure 5–7 (Example 5–4). Write the Boolean equation and draw the logic circuit to implement the following functions:

(a) Turn on the red light (R) if there is a HIGH opacity (C) and pressure (P) when the level is full (F).

(b) Turn on the green light (G) if there is a HIGH opacity (C) and pressure (P) when the level is mid (M) or full (F).

(c) Turn on the blue light (B) when the tank level is full and any of the sensors for PH (H), opacity (C), or pressure (P) are HIGH.

Section 5–2

5–3. Draw the logic circuit that would be used to implement the following Boolean equations. Also, construct a truth table for each of the equations. (*Hint:* Where applicable, apply Law 3 to the equation first. *Do not* simplify the equation for this problem.)

(a) $M = (AB) + (C + D)$

(b) $N = (A + B + C)D$

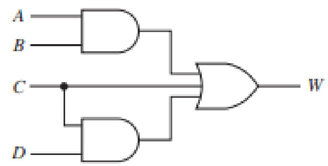
(c) $P = (AC + BC)(A + C)$

(d) $Q = (A + B)BCD$

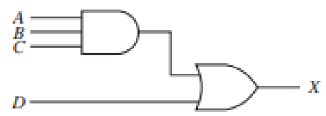
(e) $R = BC + D + AD$

(f) $S = B(A + C) + AC + D$

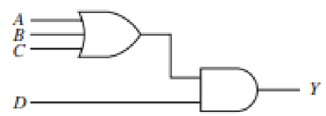
5–4. Write the Boolean equation and then complete the timing diagram at W, X, Y, and Z for the logic circuits shown in Figure P5–4.



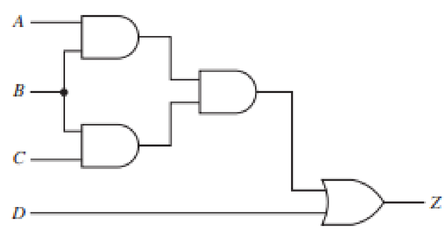
(a)

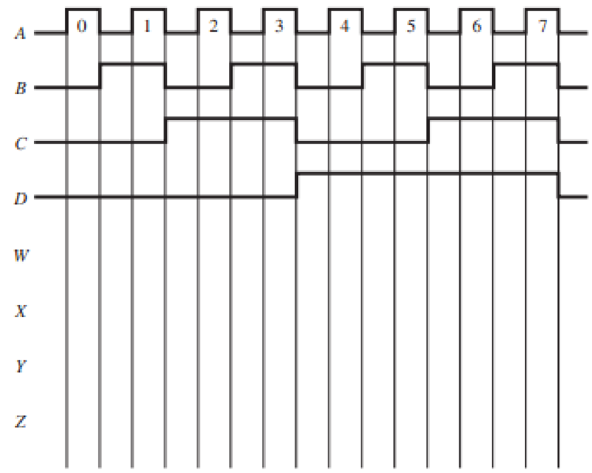


(b)

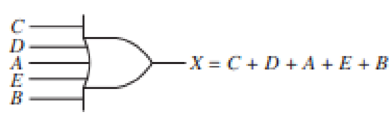


(c)

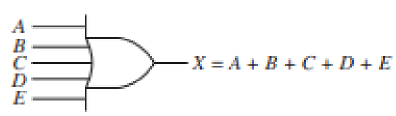




5–5. State the Boolean law that makes each of the equivalent circuits shown in Figure P5–5 valid.



Original circuit



Equivalent circuit

(a)

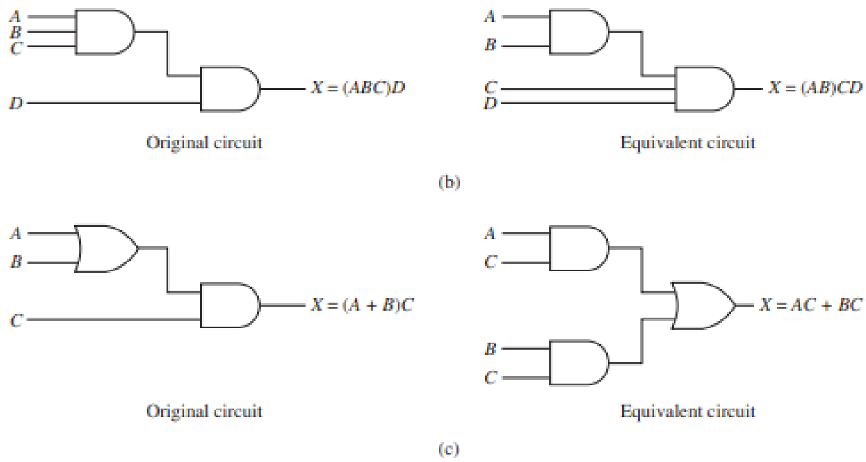
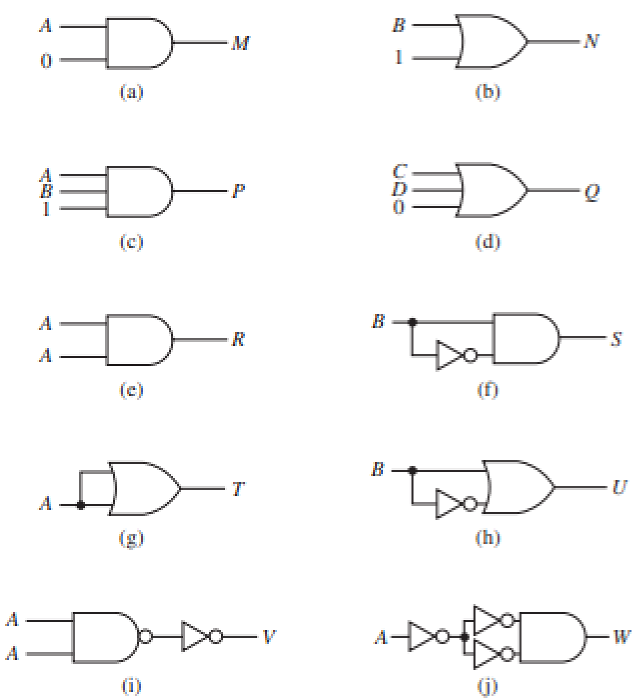


Figure P5–5 *Continued*

5–6. Using the 10 Boolean rules presented in Table 5–2, determine the outputs of the logic circuits shown in Figure P5–6.



Section 5–3

5–7. Write the Boolean equation for the circuits of Figure P5–7. Simplify the equations, and draw the simplified logic circuit.

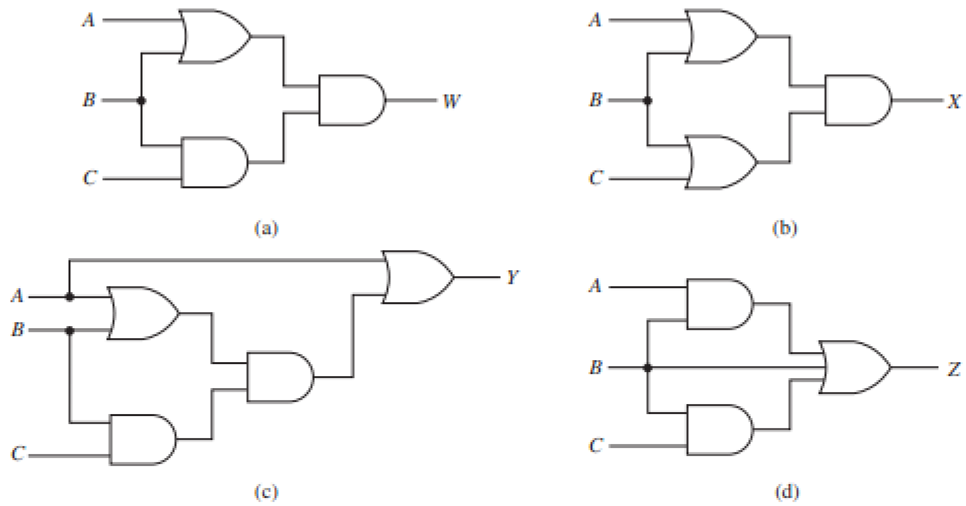
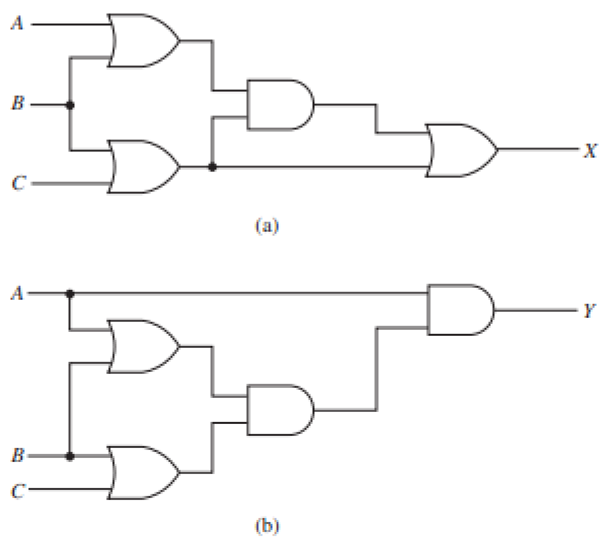
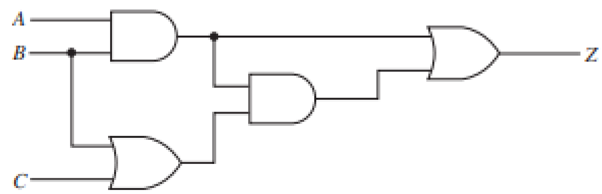


Figure P5–7

5–8. Repeat Problem 5–7 for the circuits shown in Figure P5–8.





(c)

Figure P5–8

5–9. Draw the logic circuit for the following equations. Simplify the equations, and draw the simplified logic circuit.

(a) $V = AC + ACD + CD$

(b) $W = (BCD + C)CD$

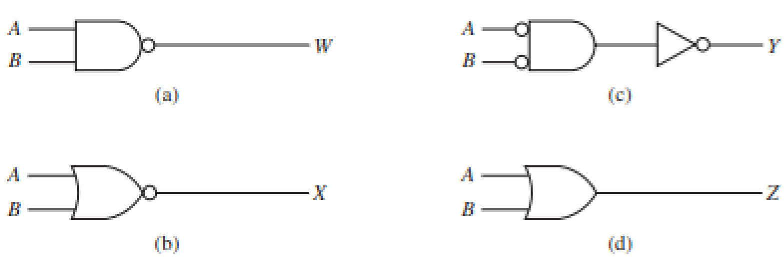
(c) $X = (B + D)(A + C) + ABD$

(d) $Y = AB + BC + ABC$

(e) $Z = ABC + CD + CDE$

5–10. Construct a truth table for each of the simplified equations of Problem 5–9.

5–15. Which two circuits in Figure P5–15 produce equivalent output equations?



5–17. Draw the logic circuit for the following equations. Apply De Morgan's theorem and Boolean algebra rules to reduce them to equations having inversion bars over single variables only. Draw the simplified circuit.

(a) $W = \overline{AB} + \overline{A + C}$

(b) $X = A\overline{B + C} + \overline{BC}$

(c) $Y = \overline{(AB) + C} + B\overline{C}$

(d) $Z = \overline{AB + (\overline{A} + C)}$

5–18. Write the Boolean equation for the circuits of Figure P5–18. Use De Morgan's theorem and Boolean algebra rules to simplify the equation. Draw the simplified circuit.

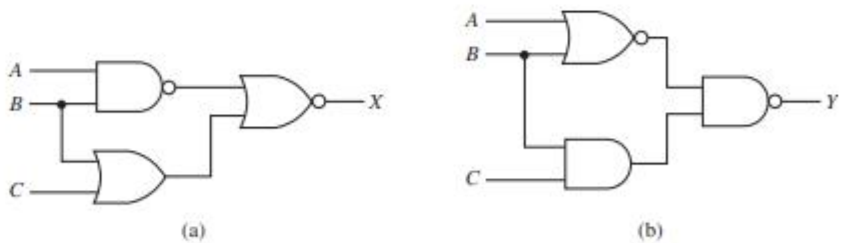


Figure P5–18

C **5–19.** Repeat Problem 5–17 for the following equations.

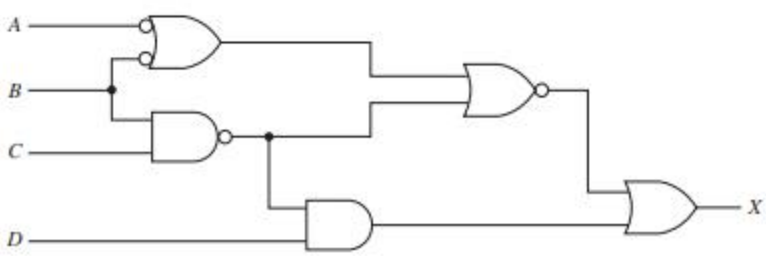
(a) $W = \overline{\overline{A}B + CD} + \overline{AC\overline{D}}$

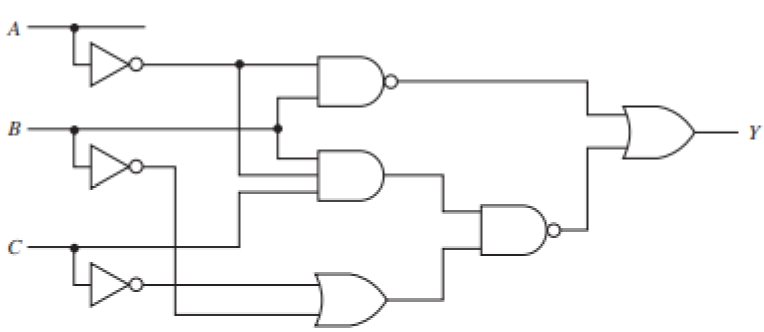
(b) $X = \overline{\overline{\overline{A} + B} \cdot BC} + \overline{B}C$

(c) $Y = \overline{AB\overline{C} + D} + \overline{A\overline{B} + B\overline{C}}$

(d) $Z = \overline{(C + D)\overline{A\overline{C}D}(\overline{A}C + \overline{D})}$

C **5–20.** Repeat Problem 5–18 for the circuits of Figure P5–20.





(b)

D* **5–21.** Design a logic circuit that will output a 1 (HIGH) only if *A* and *B* are both 1 while either *C* or *D* is 1.

D **5–22.** Design a logic circuit that will output a 0 only if *A* or *B* is 0.

D **5–23.** Design a logic circuit that will output a LOW only if *A* is HIGH or *B* is HIGH while *C* is LOW or *D* is LOW.

C D **5–24.** Design a logic circuit that will output a HIGH if only one of the inputs *A*, *B*, or *C* is LOW.

Section 5–7

5–31. Draw the connections required to convert

(a) A NAND gate into an inverter

(b) A NOR gate into an inverter

5–32. Draw the connections required to construct

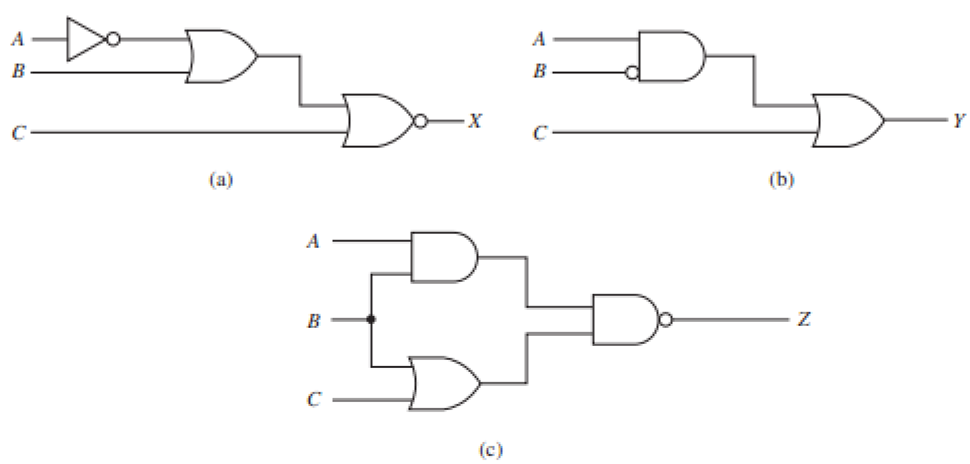
(a) An OR gate from two NOR gates

(b) An AND gate from two NAND gates

(c) An AND gate from several NOR gates

(d) A NOR gate from several NAND gates

5–33. Redraw the logic circuits of Figure P5–33 to their equivalents *using only* NOR gates.



C **5–34.** Convert the circuits of Figure P5–34 to their equivalents *using only* NAND gates. Next, make the external connections to a 7400 quad NAND to implement the new circuit. (Each new equivalent circuit is limited to *four* NAND gates.)

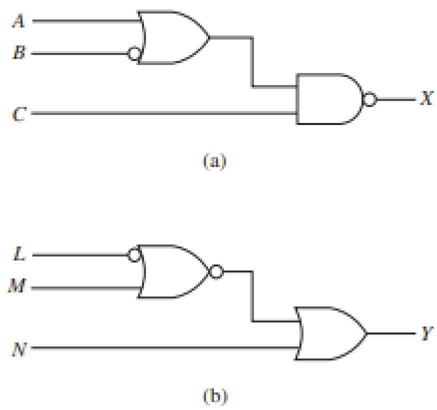


Figure P5–34

Section 5–8

5–35. Identify each of the following Boolean equations as a POS expression, a SOP expression, or both.

(a) $U = A\overline{B}C + BC + \overline{A}C$

(b) $V = (A + C)(\overline{B} + \overline{C})$

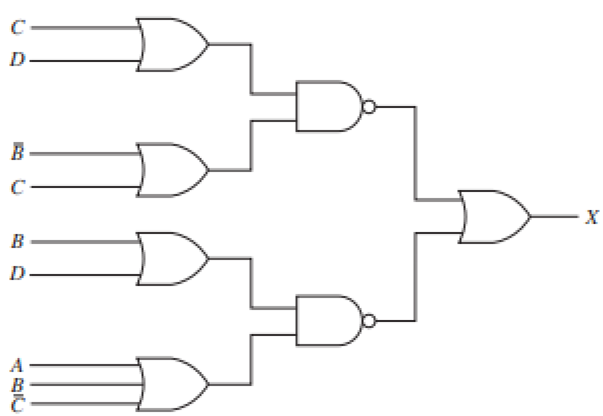
(c) $W = A\overline{C}(\overline{B} + C)$

(d) $X = AB + \overline{C} + BD$

(e) $Y = (A\overline{B} + D)(A + \overline{C}D)$

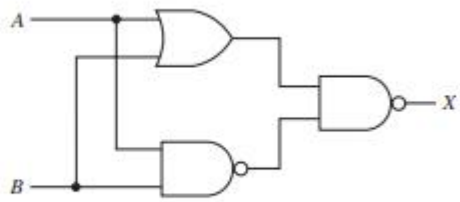
(f) $Z = (A + \overline{B})(BC + A) + \overline{A}B + CD$

5–36. Simplify the circuit of Figure P5–36 down to its SOP form, then draw the logic circuit of the simplified from implemented using a 74LS54 AOI gate.

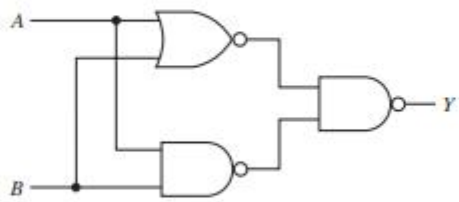


EXAMPLE 6–1

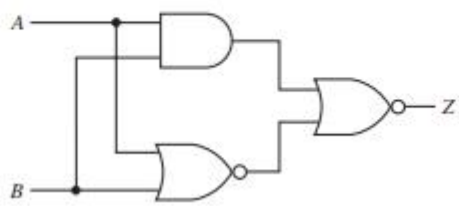
Determine for each circuit shown in Figure 6–5 if its output provides the Ex-OR function, the Ex-NOR function, or neither.



(a)



(b)



(c)

Solution:

(a) $X = \overline{(A + B)\overline{AB}}$

$= \overline{A + B} + \overline{\overline{AB}}$

$= \overline{A}\,\overline{B} + AB$ ← Ex-NOR

(b) $Y = \overline{\overline{A + B}\,\overline{AB}}$

$= \overline{\overline{A + B}} + \overline{\overline{AB}}$

$= A + B + AB$

$= A + B(1 + A)$

$= A + B$ ← neither (OR function)

(c) $Z = \overline{AB + \overline{A + B}}$

$= \overline{AB}\,\overline{\overline{A + B}}$

$= (\overline{A} + \overline{B})(A + B)$

$= \overline{A}B + \overline{A}A + \overline{B}A + \overline{B}B$

$= \overline{A}B + A\overline{B}$ ← Ex-OR

EXAMPLE 6–3

Write the Boolean equation for the circuit shown in Figure 6–7 and simplify.

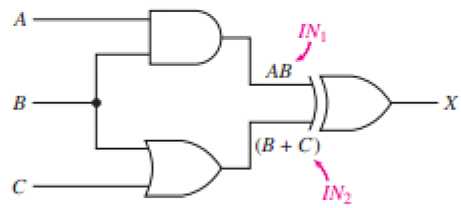


Figure 6–7

Solution:

$$
\begin{aligned}
X &= \overline{AB}(B + C) + AB(\overline{B + C}) \\
&= (\bar{A} + \bar{B})(B + C) + AB\bar{B}\,\bar{C} \\
&= \bar{A}B + \bar{A}C + \bar{B}B + \bar{B}C \\
&= \bar{A}B + \bar{A}C + \bar{B}C
\end{aligned}
$$

Hint:

$$X = \overline{IN_1}IN_2 + IN_1\overline{IN_2}$$

EXAMPLE 6–2

Write the Boolean equation for the circuit shown in Figure 6–6 and simplify.

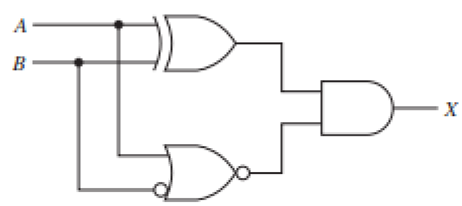


Figure 6–6

Solution:

$$
\begin{aligned}
X &= (\bar{A}B + A\bar{B})\overline{A + \bar{B}} \\
&= (\bar{A}B + A\bar{B})\bar{A}\,\bar{\bar{B}} \\
&= \bar{A}B\bar{A}B + A\bar{B}\,\bar{A}B \\
&= \bar{A}B
\end{aligned}
$$

Review Questions

6–1. The exclusive-OR gate is the complement (or inverse) of the OR gate. True or false?

6–2. The exclusive-OR gate is the complement of the exclusive-NOR gate. True or false?

6–3. Write the Boolean equation for an exclusive-NOR gate.

6–4. Write the Boolean equations for the circuits in Figure P6–4. Simplify the equations and determine if they function as an Ex-OR, Ex-NOR, or neither.

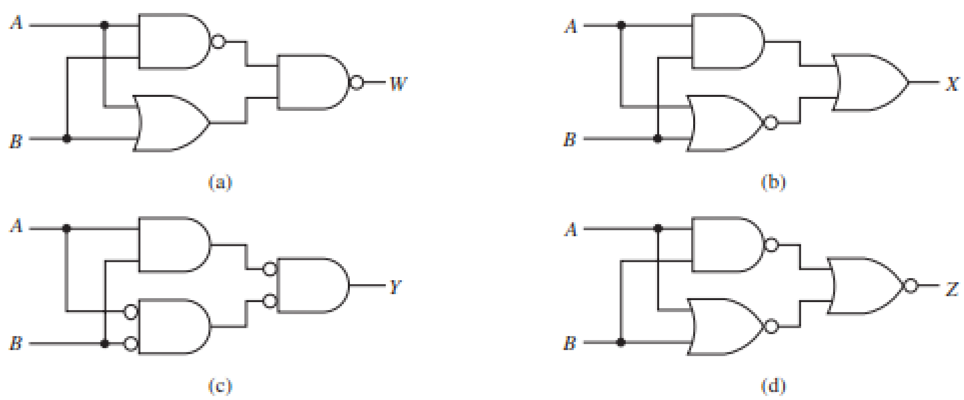


Figure P6–4

EXAMPLE 7–1

Perform the following decimal additions. Convert the original decima numbers to binary and add them. Compare answers. **(a)** 5 + 2; **(b)** 8 + 3 **(c)** 18 + 2; **(d)** 147 + 75; **(e)** 31 + 7.

Solution:

	Decimal	Binary
(a)	5	0000 010
	+ 2	+ 0000 0010
	7	0000 0111 = 7_{10} ✓
(b)	8	0000 1000
	+ 3	+ 0000 0011
	11	0000 1011 = 11_{10} ✓
(c)	18	0001 0010
	+ 2	+ 0000 0010
	20	0001 0100 = 20_{10} ✓
(d)	147	1001 0011
	+ 75	+ 0100 1011
	222	1101 1110 = 222_{10} ✓
(e)	31	0001 1111
	+ 7	+ 0000 0111
	38	0010 0110 = 38_{10} ✓

EXAMPLE 7–2

Perform the following decimal subtractions. Convert the original deci numbers to binary and subtract them. Compare answers. **(a)** 27 − **(b)** 9 − 4; **(c)** 172 − 42; **(d)** 154 − 54; **(e)** 192 − 3.

Solution:

	Decimal	Binary
(a)	27	0001 1011
	− 10	− 0000 1010
	17	0001 0001 = 17_{10} ✓
(b)	9	0000 1001
	− 4	− 0000 0100
	5	0000 0101 = 5_{10} ✓
(c)	172	1010 1100
	− 42	− 0010 1010
	130	1000 0010 = 130_{10} ✓
(d)	154	1001 1010
	− 54	− 0011 0110
	100	0110 0100 = 100_{10} ✓
(e)	192	1100 0000
	− 3	− 0000 0011
	189	1011 1101 = 189_{10} ✓

EXAMPLE 7–3

Perform the following decimal multiplications. Convert the original decimal numbers to binary and multiply them. Compare answers. **(a)** 5 × 3; **(b)** 45 × 3; **(c)** 15 × 15; **(d)** 23 × 9.

Solution:

	Decimal	Binary
(a)	5	0000 0101
	× 3	× 0000 0011
	15	0000 0101
		+ 00000 101
		00000 1111 = 0000 1111 = 15_{10} ✓
(b)	45	0010 1101
	× 3	× 0000 0011
	135	0010 1101
		+ 00101 101
		01000 0111 = 1000 0111 = 135_{10} ✓
(c)	15	0000 1111
	× 15	× 0000 1111
	75	0000 1111
	+ 15	00001 111
	225	000011 11
		+ 0000111 1
		0001110 0001 = 1110 0001 = 225_{10} ✓
(d)	23	0001 0111
	× 9	× 0000 1001
	207	0001 0111
		00000 000
		000000 00
		0001011 1
		0001100 1111 = 1100 1111 = 207_{10} ✓

EXAMPLE 7–4

Perform the following decimal divisions. Convert the original decimal numbers to binary and divide them. Compare answers. **(a)** 9 ÷ 3; **(b)** 35 ÷ 5; **(c)** 135 ÷ 15; **(d)** 221 ÷ 17.

Solution:

	Decimal	**Binary**

(a)

$$\begin{array}{r} 3 \\ 3\overline{)\ 9} \\ -\ \underline{9} \\ 0 \end{array} \qquad\qquad \begin{array}{r} 11 \\ 0000\ 0011\overline{)0000\ 1001} \\ -\ \underline{11\ \ } \\ 11 \\ -\ \underline{11} \\ 0 \end{array} = 3_{10}\checkmark$$

(b)

$$\begin{array}{r} 7 \\ 5\overline{)\ 35} \\ -\ \underline{35} \\ 0 \end{array} \qquad\qquad \begin{array}{r} 111 \\ 0000\ 0101\overline{)0010\ 0011} \\ -\ \underline{1\ 01\ \ } \\ 111\ \\ -\ \underline{101\ } \\ 101 \\ -\ \underline{101} \\ 0 \end{array} = 7_{10}\checkmark$$

(c)

$$\begin{array}{r} 9 \\ 15\overline{)\ 135} \\ -\ \underline{135} \\ 0 \end{array} \qquad\qquad \begin{array}{r} 1001 \\ 0000\ 1111\overline{)1000\ 0111} \\ -\ \underline{111\ 1\ \ \ } \\ 1111 \\ -\ \underline{1111} \\ 0 \end{array} = 9_{10}\checkmark$$

(d)

$$\begin{array}{r} 13 \\ 17\overline{)\ 221} \\ -\ \underline{17\ } \\ 51 \\ \underline{51} \\ 0 \end{array} \qquad\qquad \begin{array}{r} 1101 \\ 0001\ 0001\overline{)1101\ 1101} \\ -\ \underline{1000\ 1\ \ } \\ 101\ 01\ \ \\ -\ \underline{100\ 01\ \ } \\ 1\ 0001 \\ -\ \underline{1\ 0001} \\ 0 \end{array} = 13_{10}\checkmark$$

Review Questions

7–1. Binary addition in the least significant column deals with how many inputs and how many outputs?

7–2. In binary subtraction, the borrow-out of the least significant column becomes the borrow-in of the next-more-significant column. True or false?

7–3. Binary multiplication and division are performed by a series of additions and subtractions. True or false?

Steps for Two's-Complement-to-Decimal Conversion

1. If the two's-complement number is positive (sign bit = 0), do a regular binary-to-decimal conversion.
2. If the two's-complement number is negative (sign bit = 1), the decimal sign will be −, and the decimal number is found by
   - **(a)** Complementing the entire two's-complement number, bit by bit.
   - **(b)** Adding 1 to arrive at the true binary equivalent.
   - **(c)** Doing a regular binary-to-decimal conversion to get the decimal numeric value.

The following examples illustrate the conversion process.

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EXAMPLE 7–5

Convert $+35_{10}$ to two's complement.

Solution:

$$\text{True binary} = 0010\ 0011$$
$$\text{Two's complement} = 0010\ 0011 \quad \textit{Answer}$$

EXAMPLE 7–7

Convert 1101 1101 two's complement back to decimal.

Solution: The sign bit is 1, so the decimal result will be negative.

$$
\begin{aligned}
\text{Two's complement} &= 1101\ 1101 \\
\text{Complement} &= 0010\ 0010 \\
\text{Add } 1 &= +1 \\
\text{True binary} &= 0010\ 0011 \\
\text{Decimal complement} &= -35 \quad \textit{Answer}
\end{aligned}
$$

EXAMPLE 7–8

Convert -98_{10} to two's complement.

Solution:

$$
\begin{aligned}
\text{True binary} &= 0110\ 0010 \\
\text{One's complement} &= 1001\ 1101 \\
\text{Add } 1 &= +1 \\
\text{Two's complement} &= 1001\ 1110 \quad \textit{Answer}
\end{aligned}
$$

EXAMPLE 7–9

Convert 1011 0010 two's complement to decimal.

Solution: The sign bit is 1, so the decimal result will be negative.

$$
\begin{aligned}
\text{Two's complement} &= 1011\ 0010 \\
\text{Complement} &= 0100\ 1101 \\
\text{Add } 1 &= +1 \\
\text{True binary} &= 0100\ 1110 \\
\text{Decimal complement} &= -78 \quad \textit{Answer}
\end{aligned}
$$

EXAMPLE 7–10

Add 19 + 27 using 8-bit two's-complement arithmetic.

Solution:

$$\begin{aligned} 19 &= 0001\ 0011 \\ 27 &= \underline{0001\ 1011} \\ \text{Sum} &= 0010\ 1110 = 46_{10} \end{aligned}$$

EXAMPLE 7–11

Perform the following subtractions using 8-bit two's-complement arithmetic.

(a) 18 − 7;

(b) 21 − 13;

(c) 118 − 54;

(d) 59 − 96.

Solution:

(a) 18 − 7 is the same as 18 + (−7), so just add 18 to negative 7.

$$\begin{aligned} +18 &= 0001\ 0010 \\ -7 &= \underline{1111\ 1001} \\ \text{Sum} &= 0000\ 1011 = 11_{10} \end{aligned}$$

Note: The carry-out of the MSB is ignored. (It will always occur for positive sums.) The 8-bit answer is 0000 1011.

(b)
$$\begin{aligned} +21 &= 0001\ 0101 \\ -13 &= \underline{1111\ 0011} \\ \text{Sum} &= 0000\ 1000 = 8_{10} \end{aligned}$$

(c)
$$\begin{aligned} +118 &= 0111\ 0110 \\ -54 &= \underline{1100\ 1010} \\ \text{Sum} &= 0100\ 0000 = 64_{10} \end{aligned}$$

(d)
$$\begin{aligned} +59 &= 0011\ 1011 \\ -96 &= \underline{1010\ 0000} \\ \text{Sum} &= 1101\ 1011 = -37_{10} \end{aligned}$$

Review Questions

7–6. Which of the following decimal numbers cannot be converted to 8-bit two's-complement notation?

(a) 89

(b) 135

(c) −107

(d) −144

7–7. The procedure for subtracting numbers in two's-complement notation is exactly the same as for adding numbers. True or false?

7–8. When subtracting a smaller number from a larger number in two's complement, there will always be a carry-out of the MSB, which will be ignored. True or false?

Hexadecimal Addition

Remember, hexadecimal is a base 16 numbering system, meaning that it has 16 different digits (as shown in Table 7–4). Adding 3 + 6 in hex equals 9, and 5 + 7 equals C. But, adding 9 + 8 in hex equals a sum greater than F, which will create a carry. The sum of 9 + 8 is 17_{10}, which is 1 larger than 16, making the answer 11_{16}.

TABLE 7–4 Hexadecimal Digits with Their Equivalent Binary and Decimal Values

Hexadecimal	Binary	Decimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

EXAMPLE 7–12

Add 9 + C in hex.

Solution: C is equivalent to decimal 12.

$$12 + 9 = 21$$

Because 21 is greater than 16: **(a)** subtract $21 - 16 = 5$, and **(b)** carry 1 to the next-more-significant column. Therefore,

$$9 + C = 15_{16} \quad \textit{Answer}$$

EXAMPLE 7–13

Add 4F + 2D in hex.

Solution:

$$\begin{array}{r} 4\,F \\ +\ \underline{2\,D} \\ 7\,C \end{array} \quad \textit{Answer}$$

Explanation: F + D = 15 + 13 = 28, which is 12 with a carry (28 − 16 = 12). The 12 is written down as C; 4 + 2 + carry = 7.

EXAMPLE 7–14

Add A7C5 + 2DA8 in hex.

Solution:

$$\begin{array}{r} A\,7\,C\,5 \\ +\ \underline{2\,D\,A\,8} \\ D\,5\,6\,D \end{array} \quad \textit{Answer}$$

Explanation: 5 + 8 = 13, which is D, C + A = 22, which is 6 with a carry. 7 + D + carry = 21, which is 5 with a carry. A + 2 + carry = 13, which is D.

EXAMPLE 7–15

Subtract D7 − A8 in hex.

Solution:

$$\begin{array}{r} \text{D}\,7 \\ -\ \underline{\text{A}\,8} \\ 2\,\text{F} \end{array} \quad \textit{Answer}$$

Explanation: 7 borrows from the D, which increases its value to 23 (7 + 16 = 23), and 23 − 8 = 15, which is an F. D becomes a C, and C − A = 12 − 10 = 2.

EXAMPLE 7–16

Subtract A05C − 24CA in hex.

Solution:

$$\begin{array}{r} \text{A}\,0\,5\,\text{C} \\ -\ \underline{2\,4\,\text{C}\,\text{A}} \\ 7\,\text{B}\,9\,2 \end{array} \quad \textit{Answer}$$

Explanation: C − A = 12 − 10 = 2. The 5 borrows from the 0, which borrows from the A (5 + 16 = 21); 21 − C = 21 − 12 = 9. The 0 borrowed from the A, but it was also borrowed from, so it is now a 15; 15 − 4 = 11, which is a B. The A was borrowed from, so it is now a 9; 9 − 2 = 7.

BCD Addition

Addition is the most important operation because subtraction, multiplication, and division can all be done by a series of additions or two's-complement additions.

The procedure for BCD addition is as follows:

1. Add the BCD numbers as regular true binary numbers.
2. If the sum is 9 (1001) or less, it is a valid BCD answer; leave it as is.
3. If the sum is greater than 9 or there is a carry-out of the MSB, it is an invalid BCD number; do step 4.
4. If it is invalid, add 6 (0110) to the result to make it valid. Any carry-out of the MSB is added to the next-more-significant BCD number.
5. Repeat steps 1 to 4 for each group of BCD bits.

Use this procedure for the following example.

EXAMPLE 7–17

Convert the following decimal numbers to BCD and add them. Convert the result back to decimal to check your answer.

(a) 8 + 7;

(b) 9 + 9;

(c) 52 + 63;

(d) 78 + 69.

Solution:

(a)

$$\begin{array}{rl} 8 = & 1000 \\ +\ 7 = & \underline{0111} \\ \text{Sum} = & 1111 \text{ (invalid BCD, so add six)} \\ \text{Add 6} = & \underline{0110} \\ 1 & 0101 = 0001\ \ 0101_{BCD} = 15_{10}\ \checkmark \end{array}$$

(b)

$$\begin{array}{rl} 9 = & 1001 \\ +\ 9 = & \underline{1001} \\ \text{Sum} = & 1\ \ 0010 \text{ (invalid because of carry)} \\ & \nwarrow \text{cy} \\ \text{Add 6} = & \underline{0110} \\ & 1\ \ 1000 = 0001\ \ 1000_{BCD} = 18_{10}\ \checkmark \end{array}$$

(c)

$$\begin{array}{rl} 52 = & 0101\ \ 0010 \\ +\ 63 = & \underline{0110\ \ 0011} \\ \text{Sum} = & 1011\ \ 0101 \\ \text{Add 6} = & \underline{0110} \text{ invalid} \\ & 1\ 0001\ \ 0101 = 0001\ \ 0001\ \ 0101 = 115_{10}\ \checkmark \end{array}$$

(d)

$$\begin{array}{rl} 78 = & 0111\ \ 1000 \\ +\ 69 = & \underline{0110\ \ 1001} \\ \text{Sum} = & 1110\ \ 0001 \quad \text{(both groups of 4} \\ & \quad\ \nwarrow \text{cy} \quad \text{BCD bits are invalid)} \\ \text{Add 6} = & \underline{0110} \\ & 1110\ \ 0111 \\ \text{Add 6} = & \underline{0110} \\ & 1\ \ 0100\ \ 0111 = 0001\ \ 0100\ \ 0111 = 147_{10}\ \checkmark \end{array}$$

Review Questions

7–12. When adding two BCD digits, the sum is invalid and needs correction if it is ___________ or if ___________.

7–13. What procedure is used to correct the result of a BCD addition if the sum is greater than 9?

EXAMPLE 7–18

Apply the following input bits to the full-adder of Figure 7–10 to demonstrate its operation ($A_1 = 0$, $B_1 = 1$, $C_{in} = 1$).

Solution: The full-adder operation is shown in Figure 7–11.

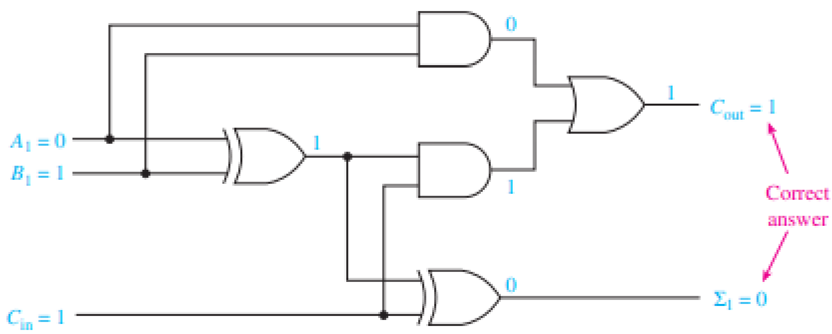

Figure 7–11 Full-adder operation for Example 7–18.

Section 7–1

7–1. Perform the following decimal additions, convert the original decimal numbers to binary, and add them. Compare answers.

(a) $6 + 3$	**(b)** $8 + 7$	**(c)** $22 + 6$	**(d)** $29 + 37$
(e) $134 + 66$	**(f)** $254 + 36$	**(g)** $208 + 127$	**(h)** $196 + 156$

7–2. Repeat Problem 7–1 for the following subtractions.

(a) $15 - 4$	**(b)** $22 - 11$	**(c)** $84 - 36$	**(d)** $66 - 31$
(e) $126 - 64$	**(f)** $113 - 88$	**(g)** $109 - 60$	**(h)** $111 - 104$

7–3. Repeat Problem 7–1 for the following multiplications.

(a) 7×3	**(b)** 6×7	**(c)** 12×5	**(d)** 39×7
(e) 63×125	**(f)** 127×15	**(g)** 31×13	**(h)** 255×127

7–4. Repeat Problem 7–1 for the following divisions.

(a) $4\overline{)12}$	**(b)** $3\overline{)15}$	**(c)** $12\overline{)48}$	**(d)** $5\overline{)25}$
(e) $5\overline{)125}$	**(f)** $14\overline{)294}$	**(g)** $15\overline{)195}$	**(h)** $12\overline{)228}$

Section 7–2

7–5. Produce a table of 8-bit two's-complement numbers from +15 to −15.

7–6. Convert the following decimal numbers to 8-bit two's-complement notation.

(a) 7	**(b)** −7	**(c)** 14	**(d)** 36	**(e)** −36
(f) 66	**(g)** −48	**(h)** 112	**(i)** −112	**(j)** −125

7–7. Convert the following two's-complement numbers to decimal.

(a) 0001 0110	**(b)** 0000 1111
(c) 0101 1100	**(d)** 1000 0110
(e) 1110 1110	**(f)** 1000 0001
(g) 0111 1111	**(h)** 1111 1111

Section 7–3

7–8. What is the maximum positive-to-negative range of a two's-complement number in each of the following?

(a) An 8-bit system **(b)** A 16-bit system

7–9. Convert the following decimal numbers to two's-complement form and perform the operation indicated.

(a)	$\begin{array}{r} 5 \\ + \underline{7} \end{array}$	**(b)**	$\begin{array}{r} 12 \\ - \underline{\ 6} \end{array}$
(c)	$\begin{array}{r} 32 \\ + \underline{18} \end{array}$	**(d)**	$\begin{array}{r} 32 \\ - \underline{18} \end{array}$
(e)	$\begin{array}{r} -28 \\ + \underline{38} \end{array}$	**(f)**	$\begin{array}{r} 125 \\ - \underline{\ 66} \end{array}$
(g)	$\begin{array}{r} 36 \\ - \underline{48} \end{array}$	**(h)**	$\begin{array}{r} -36 \\ - \underline{48} \end{array}$

Section 7–4

7–10. Build a table similar to Table 7–4 for hex digits 0C to 22.

7–11. Add the following hexadecimal numbers.

(a)	$\begin{array}{r} \mathrm{A} \\ + \underline{4} \end{array}$	**(b)**	$\begin{array}{r} 7 \\ + \underline{6} \end{array}$
(c)	$\begin{array}{r} \mathrm{0B} \\ + \underline{16} \end{array}$	**(d)**	$\begin{array}{r} 23 \\ + \underline{\mathrm{A7}} \end{array}$
(e)	$\begin{array}{r} \mathrm{8A} \\ + \underline{82} \end{array}$	**(f)**	$\begin{array}{r} \mathrm{A7} \\ + \underline{\mathrm{BB}} \end{array}$
(g)	$\begin{array}{r} \mathrm{A049} \\ + \underline{\mathrm{0AFC}} \end{array}$	**(h)**	$\begin{array}{r} \mathrm{0FFF} \\ + \underline{9001} \end{array}$

7–12. Subtract the following hexadecimal numbers.

(a)	$\begin{array}{r} \mathrm{A} \\ - \underline{4} \end{array}$	**(b)**	$\begin{array}{r} 8 \\ - \underline{2} \end{array}$
(c)	$\begin{array}{r} \mathrm{1B} \\ - \underline{06} \end{array}$	**(d)**	$\begin{array}{r} \mathrm{A7} \\ - \underline{18} \end{array}$
(e)	$\begin{array}{r} \mathrm{2A} \\ - \underline{07} \end{array}$	**(f)**	$\begin{array}{r} \mathrm{A7} \\ - \underline{\mathrm{1D}} \end{array}$
(g)	$\begin{array}{r} \mathrm{4A2D} \\ - \underline{\mathrm{1A2F}} \end{array}$	**(h)**	$\begin{array}{r} \mathrm{8BB0} \\ - \underline{\mathrm{4AC8}} \end{array}$

7–16. Convert the following decimal numbers to BCD and add them. Convert the result back to decimal to check your answer.

(a) $\begin{array}{r} 8 \\ +\ \underline{3} \end{array}$ **(b)** $\begin{array}{r} 12 \\ +\ \underline{16} \end{array}$

(c) $\begin{array}{r} 43 \\ +\ \underline{72} \end{array}$ **(d)** $\begin{array}{r} 47 \\ +\ \underline{38} \end{array}$

(e) $\begin{array}{r} 12 \\ +\ \underline{89} \end{array}$ **(f)** $\begin{array}{r} 36 \\ +\ \underline{22} \end{array}$

(g) $\begin{array}{r} 99 \\ +\ \underline{11} \end{array}$ **(h)** $\begin{array}{r} 80 \\ +\ \underline{23} \end{array}$

Section 7–6

7–17. Under what circumstances would you use a half-adder instead of a full-adder?

7–18. Reconstruct the half-adder circuit of Figure 7–7 using only NOR gates.

C **7–19.** The circuit in Figure P7–19 is an attempt to build a half-adder. Will the C_{out} and Σ_0 function properly? (*Hint:* Write the Boolean equation at C_{out} and Σ_0.)