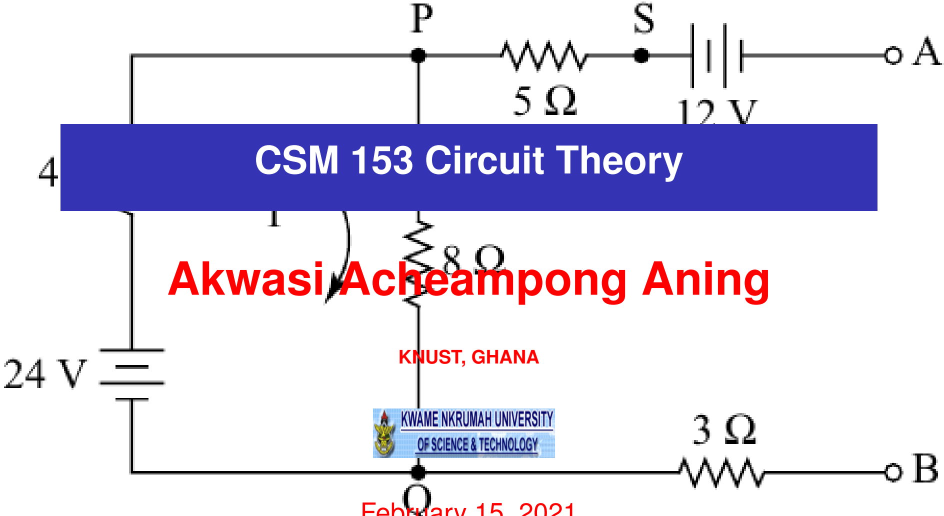

CSM 153 Circuit Theory

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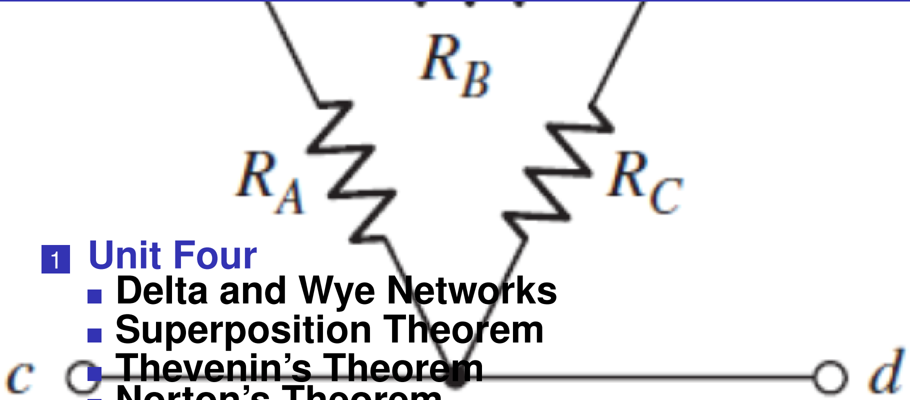
February 15, 2021

Outline I



1. **Unit Four**
   - Delta and Wye Networks
   - Superposition Theorem
   - Thevenin's Theorem
   - Norton's Theorem

UNIT FOUR

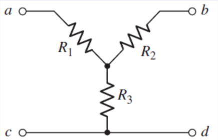
Network Theorems

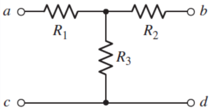
- Delta and Wye Networks
- Superposition Theorem
- Thevenin's Theorem
- Norton's Theorem

Delta and Wye Networks

Delta and Wye Networks

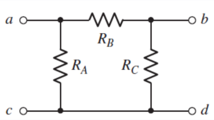
Wye(Y) - Circuits

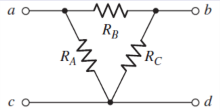




There are circuits that are neither series, parallel or series-parallel circuit.
Though these circuits can be analyzed using either the mesh or nodal analysis techniques, the number of linear equations are many.

Delta(Δ) - Circuits

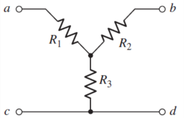


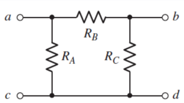


The Δ - Y and Y - Δ transformations are mathematical techniques use to simplify the analysis of an electrical network. The name derives from the shapes of the circuit diagrams, which look respectively like the letter Y and the Greek capital letter Δ

Delta and Wye Networks

Y - Δ Conversion





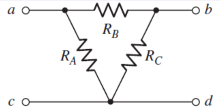
The relation to convert Y to Δ is

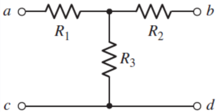
$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \quad (1)$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \quad (2)$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \quad (3)$$

Δ - Y Conversion





The relation to convert Δ to Y is

$$R_1 = \frac{R_AR_B}{R_A + R_B + R_C} \quad (4)$$

$$R_2 = \frac{R_BR_C}{R_A + R_B + R_C} \quad (5)$$

$$R_3 = \frac{R_CR_A}{R_A + R_B + R_C} \quad (6)$$

For an ideal situation $R_A = R_B = R_C$, and $R_1 = R_2 = R_3$ the equation is reduced to

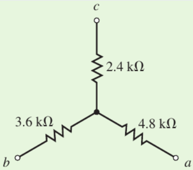
$$R_Y = \frac{1}{3}R_\Delta \quad (7)$$

and

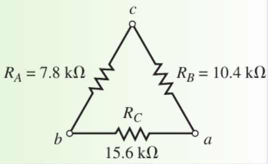
$$R_\Delta = 3R_Y \quad (8)$$

Delta and Wye Networks

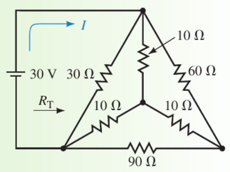
Question: Find the Δ network equivalent of the Y network below



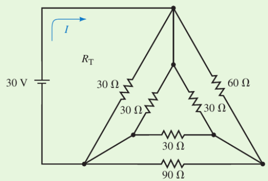
Solution:
Substitute the resistances in equations 1, 2 and 3, we obtain



Question: Find the total resistance, R_T , and the total current, I in the circuit below

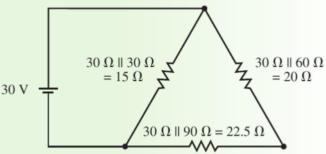


Solution:
Let's convert the Y into Δ equivalent. The equivalent Δ will have all resistors given as
$R_\Delta = 3(10)\Omega = 30\ \Omega$



Delta and Wye Networks

The sides of the resulting Δ are in parallel, which allows us to simplify the circuit even further. The equivalent circuit will be

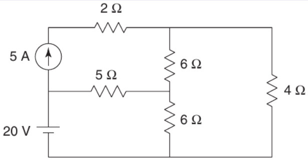


The total resistance of the circuit is

$R_T = 15\Omega \parallel (20\ \Omega + 22.5\ \Omega) = 11.09\ \Omega$

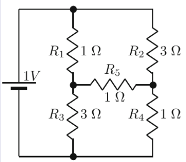
$I = \dfrac{30}{11.09} = 2.706\ A$

Question: Find the current through the 4 Ω resistor using the superposition Theorem



Answer $I_0 = 4.29\ mA \downarrow$

Question:Find the current I drawn by the circuit below



Hint: Transform Δ R_1, R_2 and R_5 to Y and solve

Answer I = 0.6 mA

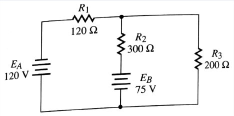
Superposition Theorem

Superposition Theorem

In any linear resistive network, the voltage across or the current through any resistor or source may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits

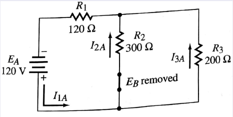
- Replace all sources except one by their internal resistances
- Calculate all the currents and voltages for that one source. Note the current directions and terminal polarities
- Repeat steps 1 and 2 for each source
- Determine the currents by algebraically adding the currents due to each source
- Determine the voltages by either algebraically adding the voltages for each source or using the total current and Ohm's law

Question: Use superposition theorem to find the currents and voltages in the circuit



Solution

Replace E_B with its internal resistance, we get



$I_{1A} = \dfrac{E_A}{R_{eqA}}$ and

$$R_{eqA} = R_1 + R_2 \| R_3 = 120\Omega + \frac{(200\Omega)(300\Omega)}{300\Omega + 200\Omega}$$
$= 240\Omega$

Superposition Theorem

$$I_{1A} = \frac{120V}{240\Omega} = 0.5A \downarrow$$

using the current divider theorem,

$$I_{2A} = \left(\frac{R_{eq}}{R_{300}}\right) I_{1A} = \left(\frac{120\Omega}{300\Omega}\right) 0.5A = 0.2A \uparrow$$

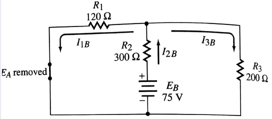
$$I_{3A} = \left(\frac{R_{eq}}{R_{200}}\right) I_{1A} = \left(\frac{120\Omega}{200\Omega}\right) 0.5A = 0.3A \uparrow$$

$$V_{1A} = I_{1A}R_1 = (0.5A)(120\Omega) = 60 \text{ V}$$

$$V_{2A} = I_{2A}R_2 = (0.2A)(300\Omega) = 60 \text{ V}$$

$$V_{3A} = I_{3A}R_3 = (0.3A)(200\Omega) = 60 \text{ V}$$

Replace E_A with its internal resistance, we get



$$I_{2B} = \frac{E_B}{R_{eqB}} \text{ and}$$

$$R_{eqB} = R_2 + R_1 \| R_3 = 300\Omega + \frac{(120\Omega)(200\Omega)}{120\Omega + 200\Omega}$$
$$= 375\Omega$$

$$I_{2B} = \frac{75V}{375\Omega} = 0.2A \uparrow$$

using the current divider theorem,

$$I_{1B} = \left(\frac{R_{eq}}{R_{120}}\right) I_{2B} = \left(\frac{75\Omega}{120\Omega}\right) 0.2A = 0.125 \text{ A} \downarrow$$

$$I_{3B} = \left(\frac{R_{eq}}{R_{200}}\right) I_{2B} = \left(\frac{75\Omega}{200\Omega}\right) 0.2A = 0.075 \text{ A} \downarrow$$

$$V_{1B} = I_{1B}R_1 = (0.125A)(120\Omega) = 15 \text{ V}$$

$$V_{2B} = I_{2B}R_2 = (0.2A)(300\Omega) = 60 \text{ V}$$

$$V_{3B} = I_{3B}R_3 = (0.075A)(200\Omega) = 15 \text{ V}$$

Superposition Theorem

By superposition theorem, I_{1A} and I_{1B} are in the same direction

$I_1 = I_{1A} + I_{1B} = 0.5 + 0.125 = 0.625\ A$ in the direction of I_{1A}

I_{2A} and I_{2B} are in the same direction

$I_2 = I_{2A} + I_{2B} = 0.2 + 0.2 = 0.4\ A$ in the direction of I_{2A}

I_{3A} and I_{3B} are in opposite direction, so

$I_3 = I_{3A} - I_{3B} = 0.3 - 0.075 = 0.225\ A$ in the direction of I_{3A}

By superposition, the voltages are:

For V_1, the polarity is the same for all sources

$V_1 = V_{1A} + V_{1B} = 60 + 15 = 75\ V$

The left terminal is negative

For V_2, the polarity is the same for all sources

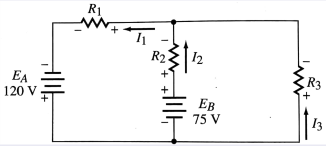
$V_2 = V_{2A} + V_{2B} = 60 + 60 = 120\ V$

The bottom terminal is negative

For V_3, the polarities are different for all sources

$V_3 = V_{3A} - V_{3B} = 60 - 15 = 45\ V$

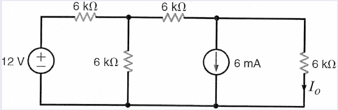
The bottom terminal is positive



method

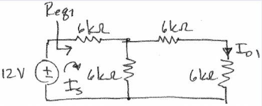
Superposition Theorem

Question: Find I_0 using superposition



Solution

Replace 6 mA source with its internal resistance, we get



By superposition theorem,

$$R_{eq1} = 6000 + [6000||12000]$$

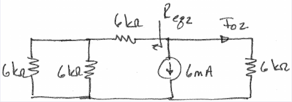
$$= 6000 + \frac{(6000)(12000)}{6000 + 12000} = 10\ k\Omega$$

$$I_s = \frac{V_{12}}{R_{eq1}} = \frac{12V}{10k\Omega} = 1.2\ mA$$

using the current divider theorem,

$$I_{01} = \left(\frac{R_{eq}}{R_{1200}}\right) I_s = \left(\frac{4000}{12000}\right) 1.2\ mA = 0.4\ mA$$

Replace 12 V source with its internal resistance, we get

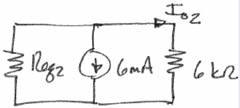


By superposition theorem,

$$R_{eq2} = 6000 + [6000||6000]$$

$$= 6000 + \frac{(6000)(6000)}{6000 + 6000} = 9\ k\Omega$$

Superposition Theorem

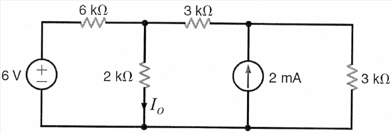


$$R_{eq} = [R_{eq2} || R_{6000}] = \frac{(9000)(6000)}{9000+6000} = 3.6\ k\Omega$$

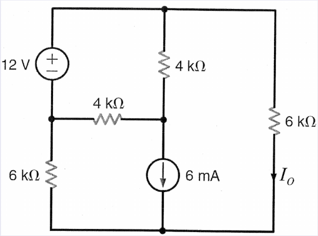
$$I_{02} = \left(\frac{R_{eq}}{R_{6000}}\right) I_6 = \left(\frac{3600}{6000}\right)(-6)\ mA = \text{-}\ 3.6\ mA$$

$$\therefore I_0 = I_{01} + I_{02} = -3.2\ mA$$

Find I_0 in the following circuits using superposition theorem



Answer I_0 = 1.2 mA



Answer I_0 = -2 mA

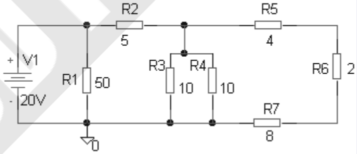
Thevenin's Theorem

Thevenin's Theorem

Thévenin's theorem for linear electrical networks states that any combination of voltage sources, current sources, and resistors with two terminals is electrically equivalent to a single voltage source V and a single series resistor R. For single frequency AC systems the theorem can also be applied to general impedances, not just resistors

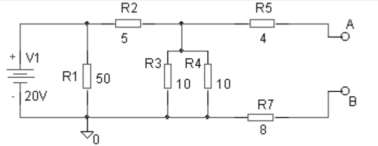
- Identify the circuit that is to be Thévenized and the load that is connected to it
- Disconnect the load from the circuit that is to be Thévenized
- Use circuit concepts to find the voltage across the open circuited two terminals. This is E_{TH}
- Find the resistance looking into the two terminals with the sources replaced by their internal resistances. This is R_{TH}
- Voltage source is replaced with a short circuit and current source replaced with an open circuit
- Reconnect the load to the Thévenin equivalent and make any required analysis of the load condition

Question: Find the power in the 2 Ω resistor $R6$ using Thevenin's theorem



Solution

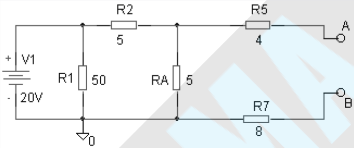
Remove $R6$ from the circuit



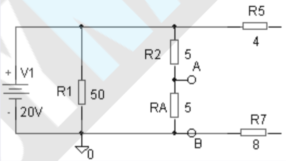
Redraw the circuit after finding the resistance, RA of the parallel branch $R3||R4$

$$RA = \frac{R3R4}{R3+R4} = \frac{(10)(10)}{10+10} = 5\ \Omega$$

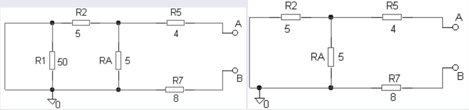
Thevenin's Theorem



Redraw the circuit and calculate the voltage across the break AB which is the voltage across $RA = V_{TH}$. Remember that 20 V is dropped across $R1$ and also across $R2 + RA$. Since $R2 = RA = 5\Omega$ half of the voltage is dropped across each resistor $V_{TH} = 10\ V$

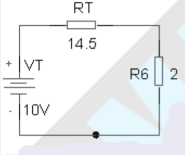


Remove $V1$ and replace it with a short circuit



This action also short circuits $R1$ out of the circuit

$$R_{TH} = \frac{(RA)(R2)}{RA + R2} + R5 + R7 = 14.5\ \Omega$$



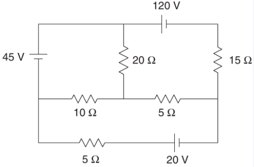
$$I = \frac{V_{TH}}{R_{TH} + R6} = \frac{10}{16.5} = 0.606\ A$$

Power dissipated by

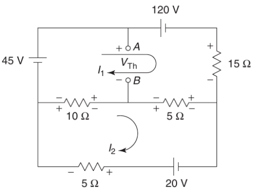
$R6 = I^2 R = 0.73\ W$

Thevenin's Theorem

Question: Find the current through the 20 Ω resistor in the fig below



Solution



Calculation of V_{TH}

Applying KVL to mesh 1

$15(I_1) + 5(I_1 - I_2) + 10(I_1 - I_2)$ = -75

$\therefore 30I_1 - 15I_2$ = -75.......(i)

Applying KVL to mesh 2

$5(I_2) + 5(I_2 - I_1) + 10(I_2 - I_1)$ = 20

$\therefore -15I_1 + 20I_2$ = 20.......(ii)

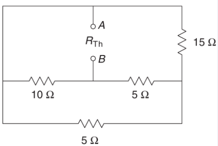
From equations (i) and (ii) we've

I_1 = -3.2 A and I_2 = -1.4 A

45 - V_{TH} - 10$(I_1 - I_2)$ = 0

V_{TH} = 45 - 10 $(I_1 - I_2)$

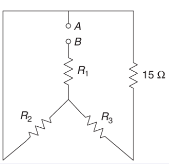
= 45 - 10[-3.2 - (-1.4)] = 63 V

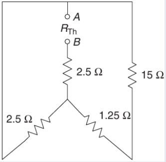


Calculation of R_{TH}

Converting the delta formed by resistors of 10 Ω, 5 Ω and 5 Ω into an equivalent star network

Thevenin's Theorem

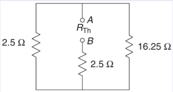


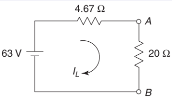


using

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{10x5}{20} = 2.5\ \Omega$$

$$R_2 = \frac{10x5}{20} = 2.5\ \Omega, \quad R_3 = \frac{5x5}{20} = 1.25\ \Omega$$

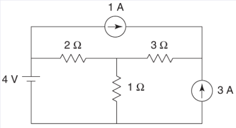




$$R_{TH} = (16.25 \| 2.5) + 2.5 = 4.67\ \Omega$$

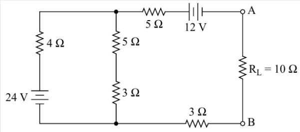
$$I_L = \frac{63}{4.67 + 20} = 2.55\ A$$

Question Find the current through the 1 Ω resistor in circuit below using Thevenin's Theorem



Answer $R_{TH} = 2\ \Omega$, $I_L = 4\ A$

Question Using Thevenin's theorem calculate the current flowing through the load resistance R_L



Answer $R_{TH} = 10.67\ \Omega$, $I_L = 0.193\ A$

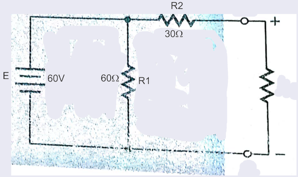
Norton's Theorem

Norton's Theorem

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current and a parallel resistor

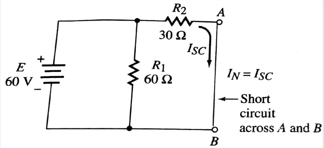
- Identify the circuit that is to be Nortonized and the load that is connected to it
- Disconnect the load from the circuit that is to be Nortonized
- Short circuit the terminals and use circuit concepts to find the short circuit current. This is I_N
- Open the terminals, replace the sources by their internal resistance and find the resistance looking into the terminals. This is R_N
- Reconnect the load and make any required analysis

Question: Find the Norton equivalent of the circuit below



Solution

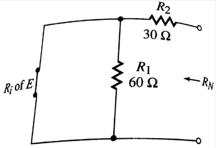
Remove R_L from the circuit



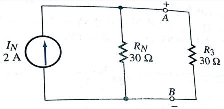
$$I_{SC} = \frac{E}{R_2} = \frac{60V}{30\Omega} = 2\ A$$

Norton's Theorem

Replacing E by its R_i a short circuit also eliminates R_1

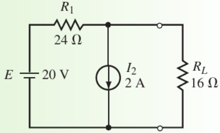


So $R_N = R_2 = 30\ \Omega$



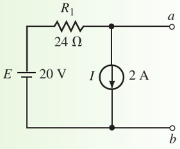
This is the Norton's equivalent circuit

Question: Determine the Norton equivalent of the circuit below. Use the Norton equivalent circuit to calculate the current through R_L .



Solution

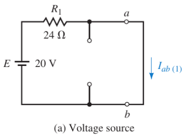
Remove R_L from the circuit

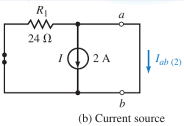


$\therefore R_N = 24\ \Omega$

The short-circuit current is determined by finding the current through the short due to each source

Norton's Theorem





The current due to the voltage source: E

$$I_{ab1} = \frac{20V}{24\Omega} = 0.833\ A$$

The current due to the current source: I

$$I_{ab1} = -\ 2.00\ A$$

Note that the current is indicated as negative because the actual current is opposite to the assumed reference direction

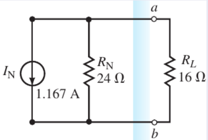
method

From the superposition theorem, we find the Norton current as

$$I_N = I_{ab1} + I_{ab2} = 0.833 - 2.00 = -1.167\ A$$

We can calculate the current through R_L by using the current divider rule

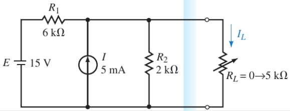
$$I_L = \frac{24\Omega}{24\Omega + 16\Omega}(1.167)A = 0.700\ A \uparrow$$



This is the Norton's equivalent circuit

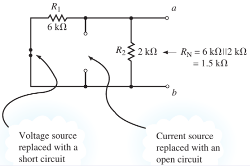
Norton's Theorem

Question: Find the Norton equivalent of the circuit external to resistor R_L in the circuit below. Use the equivalent circuit to determine the load current I_L when R_L is 0, 2 Ω, and 5 Ω.



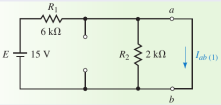
Solution

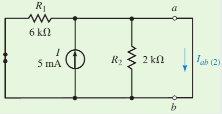
Remove R_L from the circuit



The Norton resistance of the circuit is found as

$$R_N = 6\text{ k}\Omega \,||\, 2\text{ k}\Omega = 1.5\text{ k}\Omega$$





The current due to the voltage source: E

$$I_{ab1} = \frac{15V}{6k\Omega} = 2.50\text{ m}A$$

the short circuit between terminals a and b eliminates resistor R_2

The current due to the current source: I

$$I_{ab2} = 5.00\text{ m}A$$

the short circuit between terminals a and b eliminates both resistors R_1 and R_2

From the superposition theorem, we find the Norton current as

$$I_N = I_{ab1} + I_{ab2} = 2.50 + 5.00 = 7.50\text{ m}A$$

Norton's Theorem

Let $R_L = 0$

The current I_L must equal the source current $\therefore\ I_L = 7.50\ \text{m}A$ Let $R_L = 2\ \text{k}\Omega$

The current I_L is calculated using the current divider rule

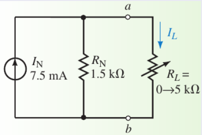
$$\therefore\ I_L = \frac{1.5k\Omega}{1.5k\Omega + 2.0k\Omega}(7.50mA) = 3.21\ \text{m}A$$

Let $R_L = 5\ \text{k}\Omega$

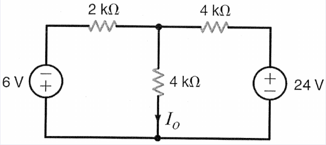
The current I_L is calculated using the current divider rule

$$\therefore\ I_L = \frac{1.5k\Omega}{1.5k\Omega + 5.0k\Omega}(7.50mA) = 1.73\ \text{m}A$$

This is the Norton equivalent circuit

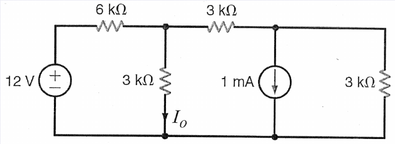


Question Find the current I_0 in the circuit using Norton's theorem



Answer $R_N = \frac{4}{3}\ \text{k}\Omega,\ I_L = 0.75\ \text{m}A$

Question Find the current I_0 in the circuit using Norton's theorem



Answer $R_N = 3\ \text{k}\Omega,\ I_L = 0.75\ \text{m}A$

THANK YOU FOR YOUR ATTENTION