# Kwame Nkrumah University of Science and Technology



# MATH 166: Introductory Probability and Statistics

Dr. Eric Nimako Aidoo

en.aidoo@yahoo.com 20-07-2021



# **Special Distributions**

- Discrete distributions
  - ✓ Bernoulli distribution
  - ✓ Binomial distribution
  - ✓ Poisson distribution

- > Continuous distributions
  - ✓ Uniform distribution
  - ✓ Exponential distribution
  - ✓ Normal distribution

# Discrete Distributions



# **Bernoulli Distribution**

### **Definition**

- A discrete random variable X is said to have a Bernoulli distribution with parameter p ( $0 \le p \le 1$ ), if it assumes the values, 0 and 1 for two outcomes.
- The probability distribution for the success in the trial, x is defined by

$$p(x) = p^{x}(1-p)^{1-x}, \quad x = 0, 1$$

### **Bernoulli Distribution**

# Properties of Bernoulli Distribution

$$\mu = E(x) = p$$

$$\sigma^2 = Var(x) = p(1-p)$$

### **Entention**

 The Bernoulli distribution is a special case of Binomial distribution



- The Binomial distribution is a discrete probability distribution that describes the probability of the number of successes x in a sequence of n independent and identical Bernoulli trials.
- The probability of a success, p, doesnot change from trial to trial.

 For a single trial, i.e., n = 1, the binomial distribution is a Bernoulli distribution.

### **Definition**

A discrete random variable X is said to have a
Binomial distribution with parameter n and p (0 ≤
p ≤ 1), and the probability distribution defined as:

$$P(x) = \binom{n}{x} p^{x} (1-p)^{n-x}, \quad x = 0,1,...,n$$

where:

p(x) = the probability of x successes in n trials

n =the number of trials

x =the number of successes

p = the probability of success on any one trial

Alternative formulation

$$P(x) = \frac{n!}{x!(n-x)!} p^{x} (1-p)^{n-x}, \quad x = 0, 1, ..., n$$

# Properties of Binomial Distribution

$$\mu = E(x) = np$$

$$\sigma^2 = Var(x) = np(1-p)$$

# Example 1:

In a computer manufacturing company, 10% of the memory chips produced are <u>defective</u>. If three (3) of the memory chips produced by the company are selected at random, what is the probability that 1 of the chips selected is defective

### Solution:

Let 
$$p=0.10$$
,  $n=3$ ,  $x=1$ 

$$P(x) = \frac{n!}{x!(n-x)!} p^{x} (1-p)^{n-x}, \quad x = 0, 1, ..., n$$

$$P(x=1) = \frac{3!}{1!(3-1)!}(0.1)^{1}(1-0.1)^{3-1} = 0.243$$

The probability that 1 of the chips selected is defective is 0.243

# **Example 2**

The number of serious accidents, in a manufacturing plant has approximately a Poisson distribution with a mean of 1.5 accidents per month. What is the probability that exactly three accidents will occur within a period of one month?

### Solution

First determine the value of  $\lambda$  which in this case is 1.5

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(x=3) = \frac{1.5^3 (2.7183)^{-1.5}}{3!} \approx 0.126$$

# Try it your self

A large retail shop purchases a certain brand of computers from a manufacturer. The manufacturer indicates that the defective rate of the product is 3% in a shipment. The inspector of the retail shop randomly picks 20 items of the product from a shipment. What is the probability that there will

- a) be 3 defective items?
- b) not be more than two (2) defective items?
- c) What is the average number of defective items

# Assignment

A multiple-choice test consists of 15 questions each with five possible answers of which only one is correct. Suppose one of the students taking the test answers the questions by guessing. What is the probability that he answers at most 3 questions correctly?

# Assignment

It is known that 25% of inhabitants of a community favour a political party A. A random sample of 20 inhabitants was selected from the community and each person was asked he/she will vote for party A in an impending election. What is the probability that:

- a) exactly two persons will vote for party A?
- b) at least three persons will vote for party A?



### **Definition**

• A discrete random variable X is said to have a Poisson distribution with parameter  $\lambda$  ( $\lambda > 0$ ), if the probability distribution is:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, \dots$$

The letter e is a constant equals 2.7183

# Properties of Poisson Distribution

$$\mu = E(x) = \lambda$$

$$\sigma^2 = Var(x) = \lambda$$

# Example

Suppose that the number of telephone calls received by a computer technician for IT support during a 30 minutes time interval is known to be a Poisson distribution with mean of 3. Find the probability that:

- a) no call will be received within the period.
- b) exactly 4 calls will be received
- c) at least one call will be received
- d) more than 4 calls will be received

# Solution

Let  $\lambda = 3$ 

a) no call will be received within the period

$$P(x=0) = \frac{3^0 e^{-3}}{0!} = 0.050$$

b) Exactly 4 calls will be received

$$P(x=4) = \frac{3^4 e^{-3}}{4!} = 0.168$$

c) at least one call

$$P(x \ge 1) = 1 - P(x = 0)$$

$$= 1 - \frac{3^{0} e^{-3}}{0!} = 1 - 0.050 = 0.950$$

d) more than 2 calls will be received

$$P(x > 2) = 1 - P(x \le 2)$$
$$= 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$$

$$=1 - \left[ \frac{3^{0}e^{-3}}{0!} + \frac{3^{1}e^{-3}}{1!} + \frac{3^{2}e^{-3}}{2!} \right] = 0.577$$

# Try it your self

The number of serious accidents, y in a manufacturing plant has approximately a Poisson distribution with a mean of 10 accidents per year. What is the probability that:

- a) at least 2 accidents will occur within a period of 3 months?
- b) at most one accident will occur within a period of 2 months?

# Assignment 2

- ➤ The number of telephone calls received by an office on averages is 4 per minute. Find the probability that:
  - ✓ No call will arrive in a given one-minute period.
  - ✓ At least two calls will arrive in a given one-minute period.
  - ✓ At least three calls will arrive in a given two-minute period.

# Further readings on Poisson approximation to binomial distribution

- ➤ The probability that a car will breakdown after falling into a pothole on a road is 0.00015. If 20,000 cars travel along the road, what is the expected number of break-downs and probability that at least one car will break down.
- ➤ A book has 300 pages and the probability of finding misprints, x, in a page is 0.015. Find the probability of detecting misprints in at most one page of the book.





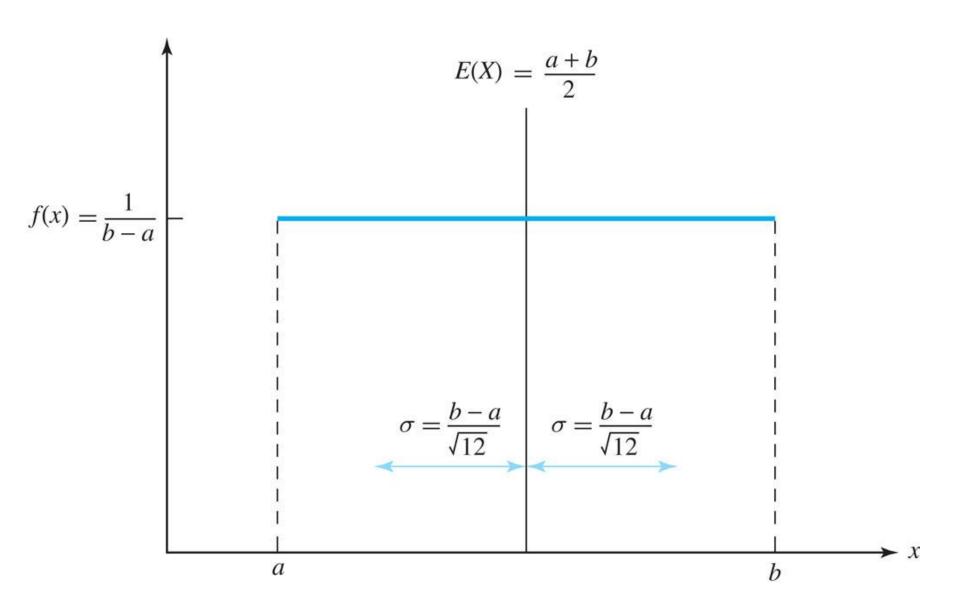
### **Definition**

 A continuous random variable X has the uniform distribution over the interval [a, b] if the probability distribution is:

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

# Properties of Uniform Distribution

$$\mu = E(x) = \frac{a+b}{2}$$
 $\sigma^2 = Var(x) = \frac{(b-a)^2}{12}$ 



# **Example**

Suppose a buses arrive at a bus stop every 6 minutes and that the waiting time for the next bus to arrive has a uniform probability distribution on the interval from 2 to 6 minutes.

- a) What is the probability that the bus will arrive between 3 and 4 minute
- b) What is the probability that the bus will arrive before 5 minute
- c) Compute the average waiting time of a person.

# **Solution**

Let a=2 and b=6, the pdf is given by

$$f(x) = \frac{1}{6-2}, \quad 2 \le x \le 6$$

 a) probability that the bus will arrive between 3 and 4 minutes

$$P(3 \le x \le 4) = \int_3^4 \frac{1}{4} dx = \left[\frac{x}{4}\right]_3^4$$

$$= \left\lfloor \frac{4}{4} - \frac{3}{4} \right\rfloor = \frac{1}{4}$$

b) probability that the bus will arrive before 5 minutes

$$P(x < 5) = \int_{2}^{5} \frac{1}{4} dx = \left[\frac{x}{4}\right]_{2}^{5}$$

$$= \left\lfloor \frac{5}{4} - \frac{2}{4} \right\rfloor = \frac{3}{4}$$

c) the average waiting time of a person

$$\mu = E(x) = \frac{2+6}{2} = 4$$

# Example 2

- ➤ Let x denote the flight time of an airplane traveling from Chicago to New York. Assume that the minimum time is 2 hours and that the maximum time is 2 hours 20 minutes.
  - a) What is the probability that a flight will take between 135 and 140 minutes?
  - b) What is the probability that a flight will take between 124 and 136 minutes?
  - c) What is the average flying time?

### Solution

If X is the flight time, then X is uniformly distributed over [120, 140] and its probability distribution function is:

$$f(x) = \frac{1}{140 - 120} = \frac{1}{20}$$
 
$$f(x) = \begin{cases} \frac{1}{20}, & 120 \le x \le 140 \\ 0, & \text{otherwise} \end{cases}$$

a) The probability that a flight will take between 135 and 140 minutes is

$$P(135 \le x \le 140) = \int_{135}^{140} \frac{1}{20} dx$$
$$= \left[ \frac{1}{20} x \right]_{135}^{140} = \left[ \frac{140}{20} \right] - \left[ \frac{135}{20} \right] = 0.25$$

b) The probability that a flight will take between 124 and 136 minutes is

$$P(124 \le x \le 136) = \int_{124}^{136} \frac{1}{20} dx$$

$$= \left[\frac{1}{20}x\right]_{124}^{136} = \left[\frac{136}{20}\right] - \left[\frac{124}{20}\right] = 0.6$$

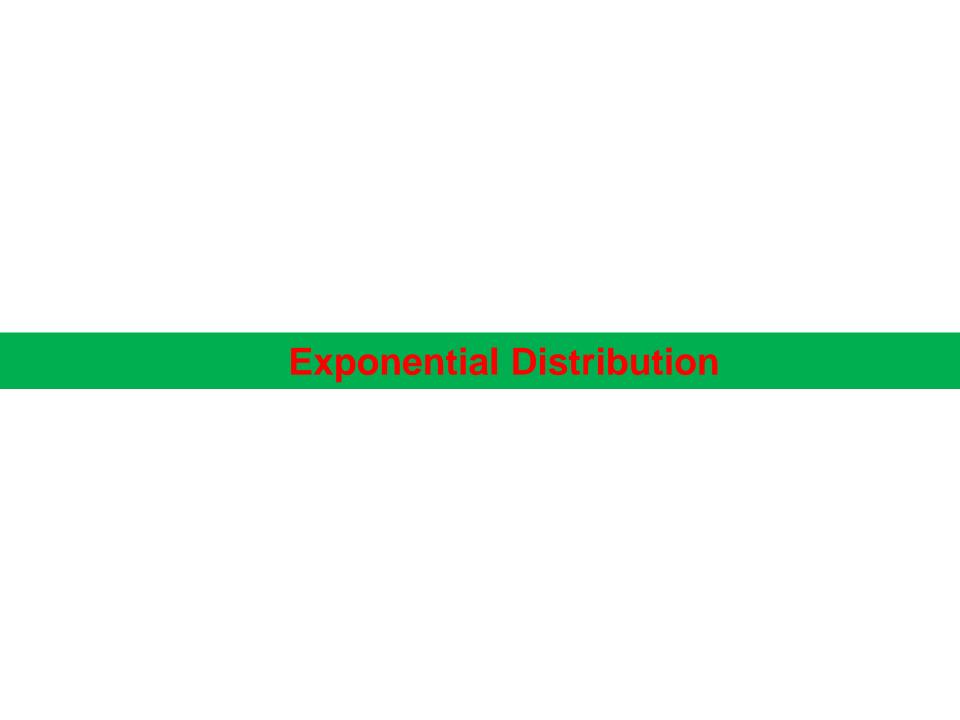
c) The average flying time is:

$$E(x) = \frac{140 + 120}{2} = \frac{260}{2} = 130 \text{ minutes}$$

# Try it your self

The probability density function of time, x required to complete an assembly operation is uniformly distributed for 30≤x≤40 seconds.

- a) Determine the proportion of assemblies that require less than 35 seconds to complete.
- b) What time is exceeded by 90% of the assemblies?



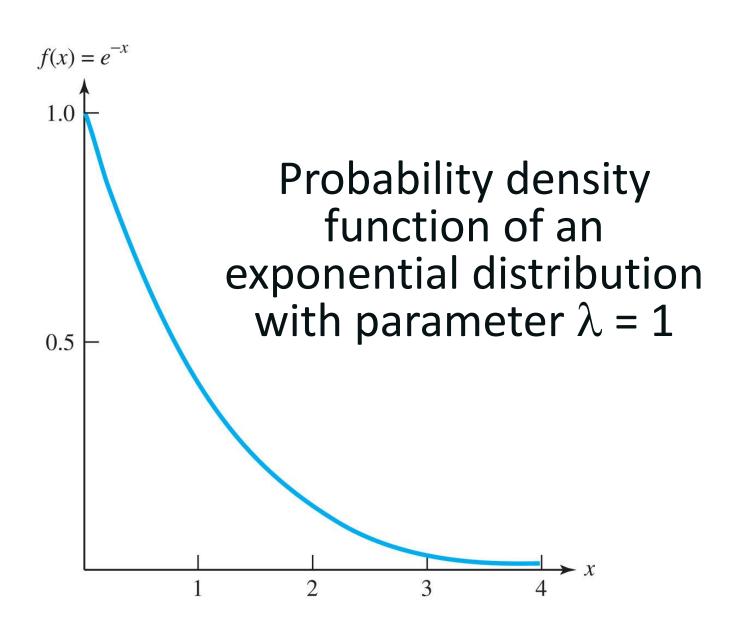
#### **Definition**

 A continuous random variable X has the exponential distribution with parameter θ if the probability distribution is:

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

#### Properties of Exponential Distribution

$$\mu = E(x) = \frac{1}{\theta} \qquad \qquad \sigma^2 = Var(x) = \frac{1}{\theta^2}$$



# Example 1

Suppose that the time until first failure of an electric component is an exponential random variables with a rate of  $\lambda$ =0.0625 per month. Calculate the probability that the device last:

- a) not more than 45 months
- b) longer than 40 months

#### Solution

a) not more than 45 months

$$P(x \le 45) = \int_0^{45} 0.0625e^{-0.0625x} dx = \left[ -e^{-0.0625x} \right]_0^{45}$$
$$= 1 - e^{-0.0625(45)} = 0.9399$$

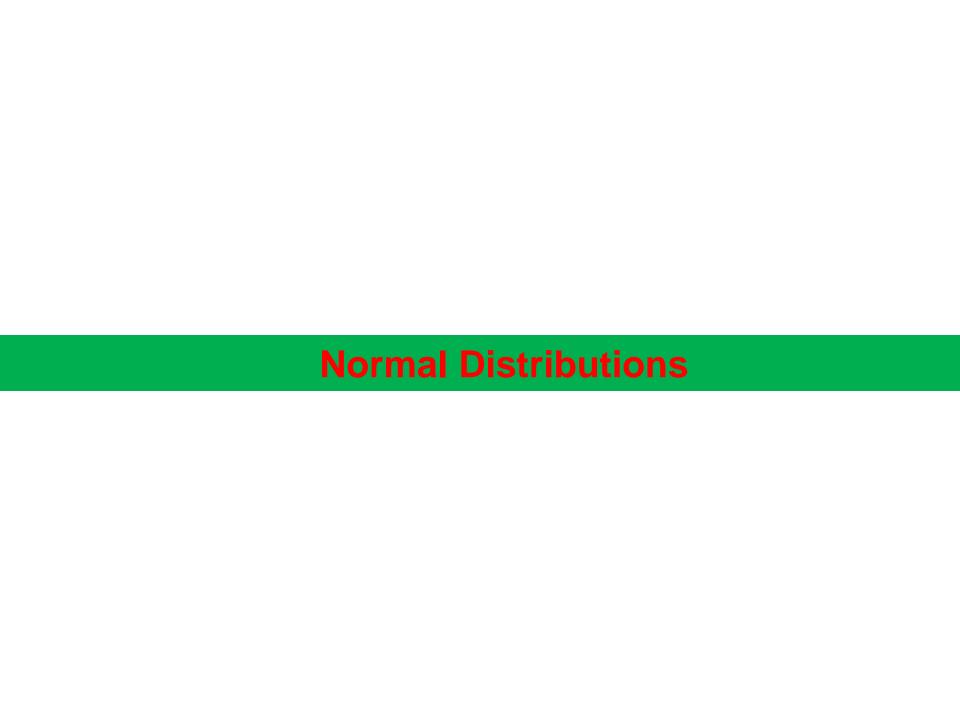
b) longer than 40 months

$$P(x > 40) = \int_{40}^{\infty} 0.0625e^{-0.0625x} dx = \left[ -e^{-0.0625x} \right]_{40}^{\infty}$$
$$= e^{-0.0625(40)} = 0.0821$$

# Try it

Suppose the time in days between service calls on a photocopier machine follows an exponential distribution with a mean call of 2 days.

- a) What is the probability that the time until the machine again requires service exceeds 60 days?
- b) What is the probability that the time until the machine again requires service is less than 20 days?



#### **Normal Distributions**

#### **Definition**

• A continuous random variable X has the normal distribution with parameter  $\mu$  and  $\sigma^2$  if the probability distribution is:

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}, & -\infty < x < \infty \\ 0, & elsewhere \end{cases}$$

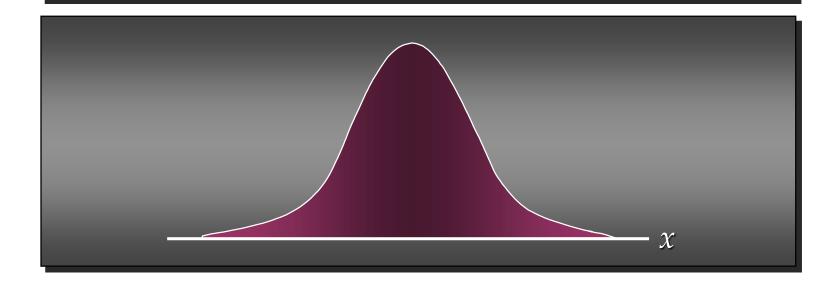
#### Properties of Normal Distribution

$$E(x) = \mu$$
$$Var(x) = \sigma^2$$

# **Normal Probability Distributions**

Characteristics

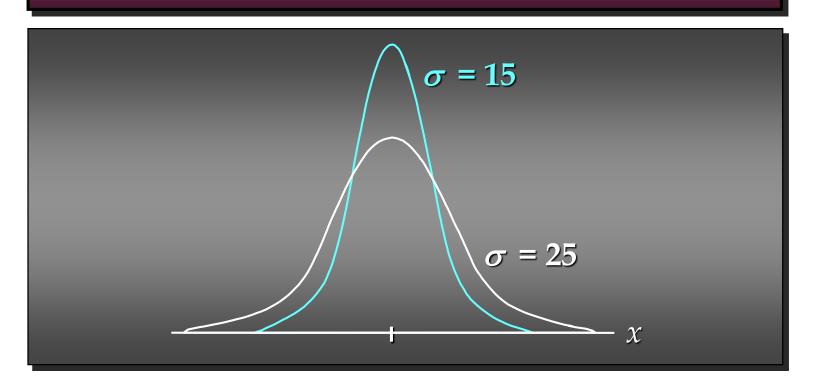
The distribution is <u>symmetric</u>; its skewness measure is zero.



#### **Normal Probability Distributions**

Characteristics

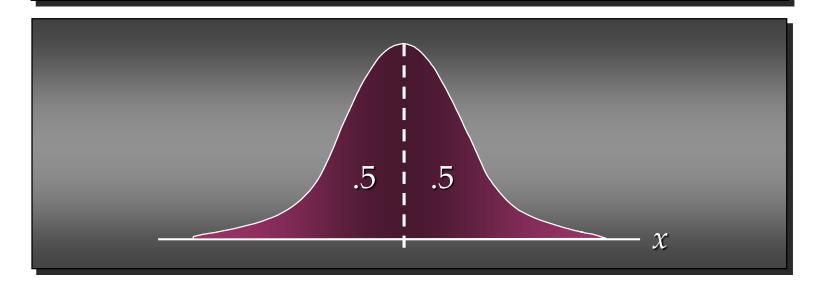
The standard deviation determines the width of the curve: larger values result in wider, flatter curves.



#### **Normal Probability Distributions**

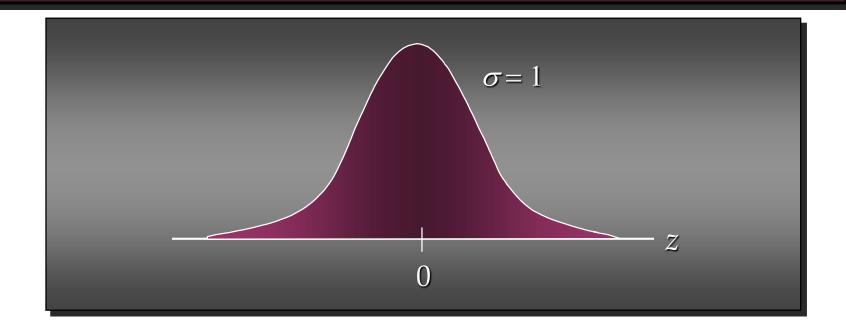
Characteristics

Probabilities for the normal random variable are given by <u>areas under the curve</u>. The total area under the curve is 1 (.5 to the left of the mean and .5 to the right).

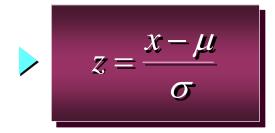


A random variable having a normal distribution with a mean of 0 and a standard deviation of 1 is said to have a standard normal probability distribution.

The letter *z* is used to designate the standard normal random variable.



Converting to the Standard Normal Distribution (z-score)



We can think of z as a measure of the number of standard deviations x is from  $\mu$ .

#### **Standard Normal Probabilities**

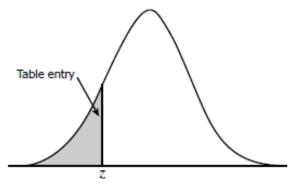


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367

#### **Standard Normal Probabilities**

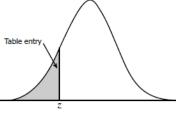
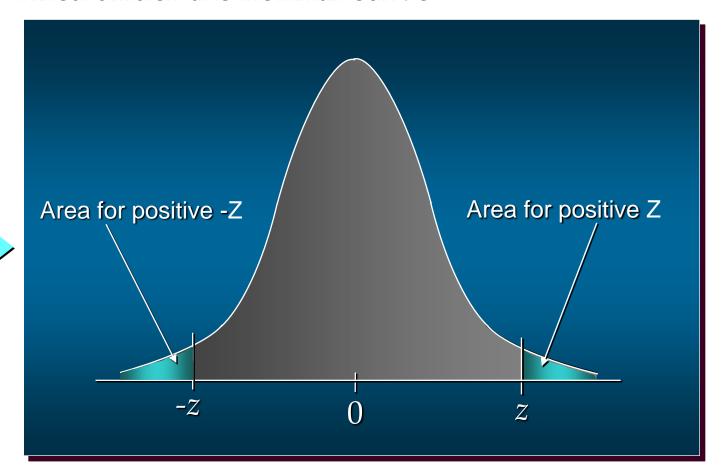


Table entry for z is the area under the standard normal curve to the left of z.

	z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Area under the normal curve



#### Example 1

➤ The store manager is concerned that sales are being lost due to stockouts while waiting for an order. It has been determined that demand during replenishment lead-time is normally distributed with a mean of 15 gallons and a standard deviation of 6 gallons.

The manager would like to know the probability of a stockout, P(x > 20).

#### Solution

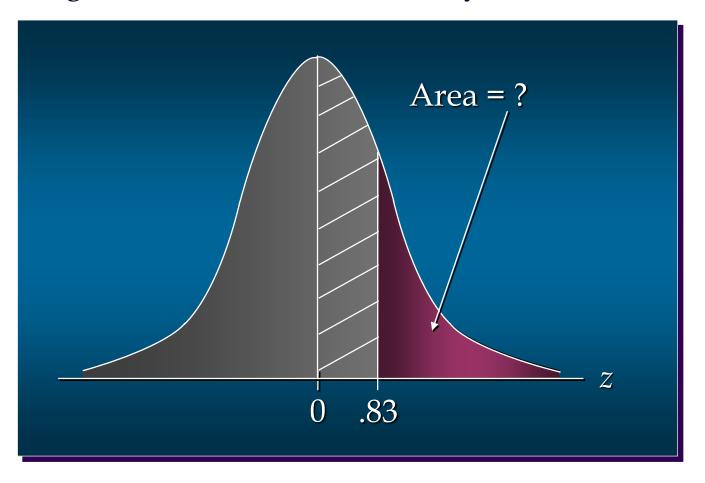
Let 
$$x=20$$
,  $\mu=15$ , and  $\sigma=6$ 

> Step 1: Convert x to the standard normal distribution.

$$z = (x - \mu)/\sigma$$
  
= (20 - 15)/6  
= 0.83

Step 2: Find the area under the standard normal curve to the left of z = 0.83.

Solving for the Stockout Probability



#### **Standard Normal Probabilities**

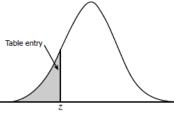
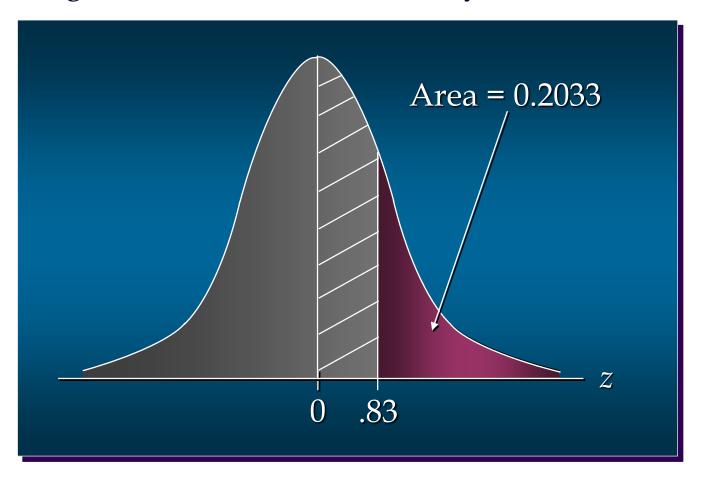


Table entry for z is the area under the standard normal curve to the left of z.

	~									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.00 03	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.00 04	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.00 06	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.00 09	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.00 17	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.00 23	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.00 32	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.00 43	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.00 57	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.00 75	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.00 99	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.01 29	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.01 56	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.02 58	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.03 36	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.04 18	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.05 16	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.06 30	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.07 54	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.09 18	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.10 93	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.12 92	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.15 15	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.17,72	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	2110	2000	2001	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Solving for the Stockout Probability

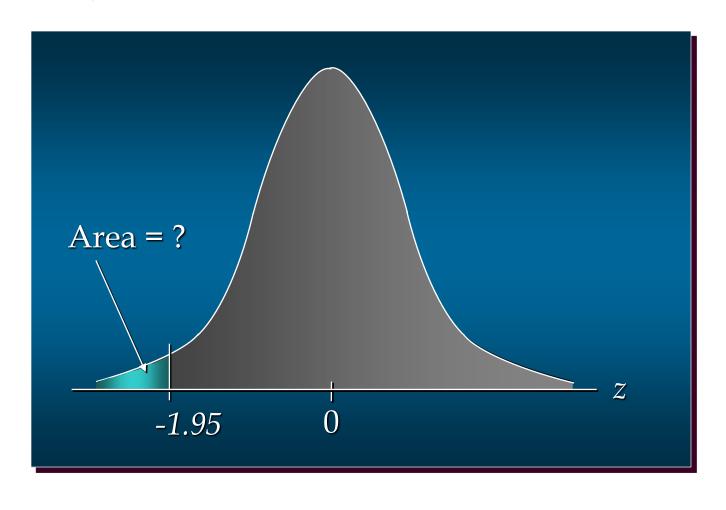


#### Example 2

Find the following probabilities using the normal table

- i.  $P(z \le -1.95)$
- ii  $P(-1.18 \le z \le 0.48)$

i).  $P(Z \le -1.95)$ 



#### **Standard Normal Probabilities**

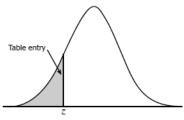
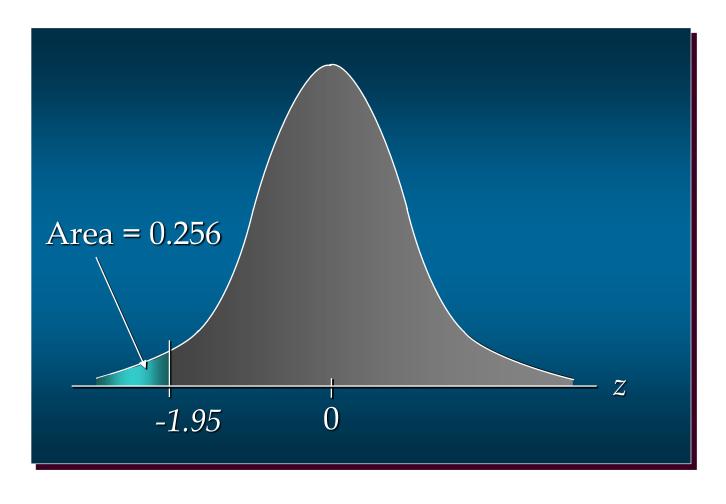


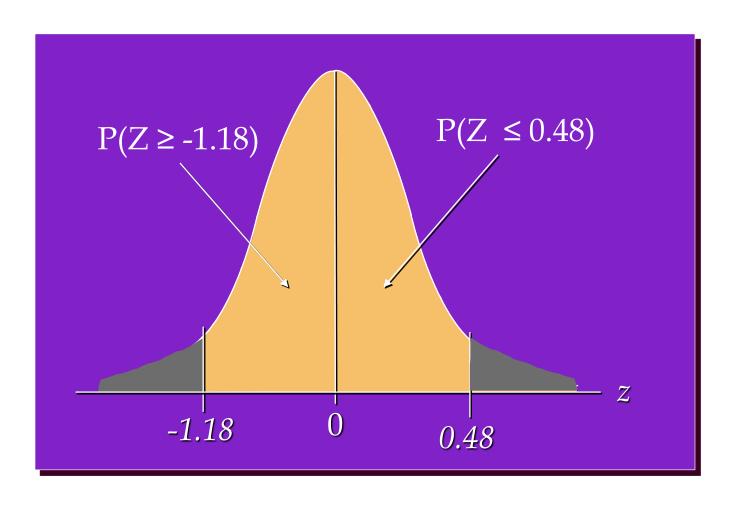
Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.00 03	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.00 04	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0080	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.00 10	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.00 94	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0 <mark>15</mark> 8	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	0207	0201	0274	0268	0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

i)  $P(Z \le -1.95) = 0.256$ 



(ii)  $P(-1.18 \le Z \le 0.48) = P(Z \ge -1.18) + P(Z \le 0.48)$ 



#### **Standard Normal Probabilities**

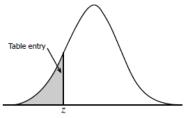


Table entry for z is the area under the standard normal curve to the left of z.

-3.3 .0005 .0005 .0005 .0004 .0004 .0004 .0004 .0004 .00 p4 -3.2 .0007 .0007 .0006 .0006 .0006 .0006 .0006 .0006 .0005 .00 p5 -3.1 .0010 .0009 .0009 .0009 .0008 .0008 .0008 .0008 .00 p7 -3.0 .0013 .0013 .0013 .0012 .0012 .0011 .0011 .0011 .0011 .0011 -2.9 .0019 .0018 .0018 .0017 .0016 .0016 .0015 .0015 .00 p1 -2.8 .0026 .0025 .0024 .0023 .0023 .0022 .0021 .0021 .0021 .00 p1 -2.7 .0035 .0034 .0033 .0032 .0031 .0030 .0029 .0028 .00 p7 -2.6 .0047 .0045 .0044 .0043 .0041 .0040 .0039 .0038 .00 p7 -2.5 .0062 .0060 .0059 .0057 .0055 .0054 .0052 .0051 .00 p4 -2.4 .0082 .0080 .0078 .0075 .0073 .0071 .0069 .0068 .00 p6 -2.3 .0107 .0104 .0102 .0099 .0096 .0094 .0091 .0089 .00 p7 -2.2 .0139 .0136 .0132 .0129 .0125 .0122 .0119 .0116 .01 p1 -2.1 .0179 .0174 .0170 .0166 .0162 .0158 .0154 .0150 .01 p4 -2.0 .0228 .0222 .0217 .0212 .0207 .0202 .0197 .0192 .01 p8 -1.9 .0287 .0281 .0274 .0268 .0262 .0256 .0250 .0244 .02 p9 -1.8 .0359 .0351 .0344 .0336 .0329 .0322 .0314 .0307 .05 p1 -1.7 .0446 .0436 .0427 .0418 .0409 .0401 .0392 .0384 .03 p7 -1.6 .0548 .0537 .0526 .0516 .0505 .0495 .0485 .0475 .04 p5 -1.5 .0668 .0655 .0643 .0630 .0618 .0606 .0594 .0582 .05 p1 -1.1 .1131 .1112 .1093 .1075 .1056 .1038 .1020 .1103 -1.1 .1557 .1355 .1311 .122 .1093 .1075 .1056 .1038 .1020 .1103 -1.1 .1557 .1355 .1311 .122 .1093 .1075 .1056 .1038 .1020 .1103 -1.0 .1587 .1562 .1539 .1515 .1492 .1469 .1446 .1423 .1401 .008 .2119 .0090 .2389 .2358 .2327 .2296 .2266 .2236 .2206 .2217 .2420 .2389 .2358 .2327 .2296 .2266 .2236 .2206 .2217 .2420 .2389 .2358 .2327 .2296 .2266 .2236 .2266 .2214 .2483 .0055 .0055 .3085 .3050 .3015 .2981 .2946 .2912 .2877 .2843 .2810	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00	z
-3.2 .0007 .0007 .0006 .0006 .0006 .0006 .0006 .0005 .00 5 -3.1 .0010 .0009 .0009 .0009 .0008 .0008 .0008 .0008 .0008 .0007 -3.0 .0013 .0013 .0013 .0012 .0012 .0011 .0011 .0011 .0010 -2.9 .0019 .0018 .0018 .0017 .0016 .0016 .0015 .0015 .0014 -2.8 .0026 .0025 .0024 .0023 .0023 .0022 .0021 .0021 .0021 -2.7 .0035 .0034 .0033 .0032 .0031 .0030 .0029 .0028 .00 27 -2.6 .0047 .0045 .0044 .0043 .0041 .0040 .0039 .0038 .00 37 -2.5 .0062 .0060 .0059 .0057 .0055 .0054 .0052 .0051 .00 49 -2.4 .0082 .0080 .0078 .0075 .0073 .0071 .0069 .0068 .00 66 -2.3 .0107 .0104 .0102 .0099 .0096 .0094 .0091 .0089 .00 87 -2.2 .0139 .0136 .0132 .0129 .0125 .0122 .0119 .0116 .01 13 -2.1 .0179 .0174 .0170 .0166 .0162 .0158 .0154 .0150 .01 46 -2.0 .0228 .0222 .0217 .0212 .0207 .0202 .0197 .0192 .01 88 -1.9 .0287 .0281 .0274 .0268 .0262 .0256 .0250 .0244 .07 39 -1.8 .0359 .0351 .0344 .0336 .0329 .0322 .0314 .0307 .03 11 -1.7 .0446 .0436 .0427 .0418 .0409 .0401 .0392 .0384 .03 75 -1.5 .0668 .0655 .0643 .0630 .0618 .0606 .0594 .0582 .017 -1.1 .131 .1112 .1093 .1075 .1056 .1038 .1020 .1193 -1.1 .1597 .1595 .1511 .1202 .1271 .1251 .1251 .1202 .1199 -1.10 .1587 .1562 .1539 .1515 .1492 .1469 .1446 .1423 .1401 -0.9 .1841 .1814 .1788 .1762 .1736 .1741 .1685 .1600 .1635 .008 -0.1 .1079 .1841 .1814 .1788 .1762 .1736 .1741 .1685 .1660 .1635 .008 -0.1 .1079 .1841 .1814 .1788 .1762 .1736 .1741 .1685 .1660 .1635 .008 -0.1 .1079 .2389 .2358 .2327 .2296 .2266 .2236 .2206 .2216 .2217 -0.6 .2743 .2709 .2676 .2643 .2611 .2578 .2546 .2514 .2483 .0055	.0002	.00 03	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	-3.4
-3.1 .0010 .0009 .0009 .0009 .0008 .0008 .0008 .0008 .0007 .0010 .0013 .0013 .0013 .0012 .0012 .0011 .0011 .0011 .0011 .0010 .010 .	.0003	.00 04	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005	-3.3
-3.0	.0005	.00 05	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007	-3.2
-2.9 .0019 .0018 .0018 .0017 .0016 .0016 .0015 .0015 .0014 .02.8 .0026 .0025 .0024 .0023 .0023 .0022 .0021 .0021 .0020 .020 .027 .0035 .0034 .0033 .0032 .0031 .0030 .0029 .0028 .0027 .026 .0047 .0045 .0044 .0043 .0041 .0040 .0039 .0038 .0037 .025 .0062 .0060 .0059 .0057 .0055 .0054 .0052 .0051 .0049 .026 .0060 .0059 .0057 .0055 .0054 .0052 .0051 .0049 .024 .0082 .0080 .0078 .0075 .0073 .0071 .0069 .0068 .0066 .023 .0107 .0104 .0102 .0099 .0096 .0094 .0091 .0089 .0037 .022 .0139 .0136 .0132 .0129 .0125 .0122 .0119 .0116 .0133 .021 .0179 .0174 .0170 .0166 .0162 .0158 .0154 .0150 .0146 .020 .0228 .0222 .0217 .0212 .0207 .0202 .0197 .0192 .0138 .199 .0287 .0281 .0274 .0268 .0262 .0256 .0250 .0244 .0239 .18 .0359 .0351 .0344 .0336 .0329 .0322 .0314 .0307 .031 .17 .0446 .0436 .0427 .0418 .0409 .0401 .0392 .0384 .0375 .15 .0668 .0655 .0643 .0630 .0618 .0606 .0594 .0582 .0171 .14 .0808 .0793 .0778 .0764 .0749 .0735 .0721 .0708 .0694 .13 .1151 .1131 .1112 .1093 .1075 .1056 .1038 .1020 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1103 .1075 .1056 .1038 .1020 .1103 .0184 .0190 .0191 .0192 .0192 .0194 .0192 .0194 .0192 .0194 .0192 .0194	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010	-3.1
-2.8         .0026         .0025         .0024         .0023         .0023         .0022         .0021         .002         .002           -2.7         .0035         .0034         .0033         .0032         .0031         .0030         .0029         .0028         .0027           -2.6         .0047         .0045         .0044         .0043         .0041         .0040         .0039         .0038         .0037           -2.5         .0062         .0060         .0059         .0057         .0055         .0054         .0052         .0051         .0049           -2.4         .0082         .0080         .0078         .0075         .0073         .0071         .0069         .0068         .0056           -2.3         .0107         .0104         .0102         .0099         .0096         .0094         .0091         .0089         .0087           -2.2         .0139         .0136         .0132         .0129         .0125         .0122         .0119         .0116         .013           -2.1         .0179         .0174         .0170         .0166         .0162         .0158         .0154         .0150         .0146           -2.0         .0228         <	.0010		.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013	-3.0
-2.7         .0035         .0034         .0033         .0032         .0031         .0030         .0029         .0028         .0027           -2.6         .0047         .0045         .0044         .0043         .0041         .0040         .0039         .0038         .0037           -2.5         .0062         .0060         .0059         .0057         .0055         .0054         .0052         .0051         .0049           -2.4         .0082         .0080         .0078         .0075         .0073         .0071         .0069         .0068         .0056           -2.3         .0107         .0104         .0102         .0099         .0096         .0094         .0091         .0089         .0067           -2.2         .0139         .0136         .0132         .0129         .0125         .0122         .0119         .0116         .013           -2.1         .0179         .0174         .0170         .0166         .0162         .0158         .0154         .0150         .0146           -2.0         .0228         .0222         .0217         .0212         .0207         .0202         .0197         .0192         .0138           -1.9         .0287	.0014	.0(14	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019	-2.9
-2.6         .0047         .0045         .0044         .0043         .0041         .0040         .0039         .0038         .0037           -2.5         .0062         .0060         .0059         .0057         .0055         .0054         .0052         .0051         .0049           -2.4         .0082         .0080         .0078         .0075         .0073         .0071         .0069         .0068         .0056           -2.3         .0107         .0104         .0102         .0099         .0096         .0094         .0091         .0089         .0037           -2.2         .0139         .0136         .0132         .0129         .0125         .0122         .0119         .0116         .013           -2.1         .0179         .0174         .0170         .0166         .0162         .0158         .0154         .0150         .016           -2.0         .0228         .0222         .0217         .0212         .0207         .0202         .0197         .0192         .018           -1.9         .0287         .0281         .0274         .0268         .0262         .0256         .0250         .0244         .0239           -1.8         .0359         <	.0019		.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026	-2.8
-2.5         .0062         .0060         .0059         .0057         .0055         .0054         .0052         .0051         .00         49           -2.4         .0082         .0080         .0078         .0075         .0073         .0071         .0069         .0068         .0         56           -2.3         .0107         .0104         .0102         .0099         .0096         .0094         .0091         .0089         .0         37           -2.2         .0139         .0136         .0132         .0129         .0125         .0122         .0119         .0116         .0         13           -2.1         .0179         .0174         .0170         .0166         .0162         .0158         .0154         .0150         .0         46           -2.0         .0228         .0222         .0217         .0212         .0207         .0202         .0197         .0192         .0         38           -1.9         .0287         .0281         .0274         .0268         .0262         .0256         .0250         .0244         .0         39           -1.8         .0359         .0351         .0344         .0336         .0329         .0322         .0314 </td <td>.0026</td> <td>.0027</td> <td>.0028</td> <td>.0029</td> <td>.0030</td> <td>.0031</td> <td>.0032</td> <td>.0033</td> <td>.0034</td> <td>.0035</td> <td>-2.7</td>	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035	-2.7
-2.4         .0082         .0080         .0078         .0075         .0073         .0071         .0069         .0068         .00 56           -2.3         .0107         .0104         .0102         .0099         .0096         .0094         .0091         .0089         .0037           -2.2         .0139         .0136         .0132         .0129         .0125         .0122         .0119         .0116         .013           -2.1         .0179         .0174         .0170         .0166         .0162         .0158         .0154         .0150         .0166           -2.0         .0228         .0222         .0217         .0212         .0207         .0202         .0197         .0192         .0188           -1.9         .0287         .0281         .0274         .0268         .0262         .0256         .0250         .0244         .0299           -1.8         .0359         .0351         .0344         .0336         .0329         .0322         .0314         .0307         .011           -1.7         .0446         .0436         .0427         .0418         .0409         .0401         .0392         .0384         .0375           -1.5         .0668	.0036		.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047	-2.6
-2.3         .0107         .0104         .0102         .0099         .0096         .0094         .0091         .0089         .0037           -2.2         .0139         .0136         .0132         .0129         .0125         .0122         .0119         .0116         .013           -2.1         .0179         .0174         .0170         .0166         .0162         .0158         .0154         .0150         .0146           -2.0         .0228         .0222         .0217         .0212         .0207         .0202         .0197         .0192         .0138           -1.9         .0287         .0281         .0274         .0268         .0262         .0256         .0250         .0244         .0239           -1.8         .0359         .0351         .0344         .0336         .0329         .0322         .0314         .0307         .011           -1.7         .0446         .0436         .0427         .0418         .0409         .0401         .0392         .0384         .0375           -1.5         .0668         .0655         .0643         .0630         .0618         .0606         .0594         .0582         .051           -1.4         .0808         <	.0048		.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062	-2.5
-2.2         .0139         .0136         .0132         .0129         .0125         .0122         .0119         .0116         .013           -2.1         .0179         .0174         .0170         .0166         .0162         .0158         .0154         .0150         .0146           -2.0         .0228         .0222         .0217         .0212         .0207         .0202         .0197         .0192         .0138           -1.9         .0287         .0281         .0274         .0268         .0262         .0256         .0250         .0244         .0239           -1.8         .0359         .0351         .0344         .0336         .0329         .0322         .0314         .0307         .011           -1.7         .0446         .0436         .0427         .0418         .0409         .0401         .0392         .0384         .0275           -1.6         .0548         .0537         .0526         .0516         .0505         .0495         .0485         .0475         .055           -1.5         .0668         .0655         .0643         .0630         .0618         .0606         .0594         .0582         .051           -1.4         .0808 <t< td=""><td>.0064</td><td></td><td>.0068</td><td>.0069</td><td>.0071</td><td>.0073</td><td>.0075</td><td>.0078</td><td>.0080</td><td>.0082</td><td>-2.4</td></t<>	.0064		.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082	-2.4
-2.1         .0179         .0174         .0170         .0166         .0162         .0158         .0154         .0150         .0146           -2.0         .0228         .0222         .0217         .0212         .0207         .0202         .0197         .0192         .0138           -1.9         .0287         .0281         .0274         .0268         .0262         .0256         .0250         .0244         .0239           -1.8         .0359         .0351         .0344         .0336         .0329         .0322         .0314         .0307         .011           -1.7         .0446         .0436         .0427         .0418         .0409         .0401         .0392         .0384         .0375           -1.6         .0548         .0537         .0526         .0516         .0505         .0495         .0485         .0475         .055           -1.5         .0668         .0655         .0643         .0630         .0618         .0606         .0594         .0582         .051           -1.4         .0808         .0793         .0778         .0764         .0749         .0735         .0721         .0708         .0694           -1.3         .0968         <	.0084		.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107	-2.3
-2.0         .0228         .0222         .0217         .0212         .0207         .0202         .0197         .0192         .0188           -1.9         .0287         .0281         .0274         .0268         .0262         .0256         .0250         .0244         .0239           -1.8         .0359         .0351         .0344         .0336         .0329         .0322         .0314         .0307         .031           -1.7         .0446         .0436         .0427         .0418         .0409         .0401         .0392         .0384         .0375           -1.6         .0548         .0537         .0526         .0516         .0505         .0495         .0485         .0475         .055           -1.5         .0668         .0655         .0643         .0630         .0618         .0606         .0594         .0582         .051           -1.4         .0808         .0793         .0778         .0764         .0749         .0735         .0721         .0708         .064           -1.3         .0968         .0951         .0934         .0918         .0901         .0885         .0869         .0853         .088           -1.2         .1151 <td< td=""><td>.0110</td><td>.0113</td><td>.0116</td><td>.0119</td><td>.0122</td><td>.0125</td><td>.0129</td><td>.0132</td><td>.0136</td><td>.0139</td><td></td></td<>	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139	
-1.9         .0287         .0281         .0274         .0268         .0262         .0256         .0250         .0244         .039           -1.8         .0359         .0351         .0344         .0336         .0329         .0322         .0314         .0307         .031           -1.7         .0446         .0436         .0427         .0418         .0409         .0401         .0392         .0384         .0375           -1.6         .0548         .0537         .0526         .0516         .0505         .0495         .0485         .0475         .0455           -1.5         .0668         .0655         .0643         .0630         .0618         .0606         .0594         .0582         .0171           -1.4         .0808         .0793         .0778         .0764         .0749         .0735         .0721         .0708         .064           -1.3         .0968         .0951         .0934         .0918         .0901         .0885         .0869         .0853         .088           -1.2         .1151         .1131         .1112         .1093         .1075         .1056         .1038         .1020         .123           -1.1         .1557 <td< td=""><td>.0143</td><td>.01<mark>4</mark>6</td><td>.0150</td><td>.0154</td><td>.0158</td><td>.0162</td><td>.0166</td><td>.0170</td><td>.0174</td><td>.0179</td><td>-2.1</td></td<>	.0143	.01 <mark>4</mark> 6	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179	-2.1
-1.8       .0359       .0351       .0344       .0336       .0329       .0322       .0314       .0307       .031         -1.7       .0446       .0436       .0427       .0418       .0409       .0401       .0392       .0384       .0375         -1.6       .0548       .0537       .0526       .0516       .0505       .0495       .0485       .0475       .0 55         -1.5       .0668       .0655       .0643       .0630       .0618       .0606       .0594       .0582       .0 71         -1.4       .0808       .0793       .0778       .0764       .0749       .0735       .0721       .0708       .0 94         -1.3       .0968       .0951       .0934       .0918       .0901       .0885       .0869       .0853       .0 88         -1.2       .1151       .1131       .1112       .1093       .1075       .1056       .1038       .1020       .1 33         -1.1       .1357       .1335       .1311       .1292       .1271       .1251       .1230       .1190         -1.0       .1587       .1562       .1539       .1515       .1492       .1469       .1446       .1423       .1401	.0183		.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228	-2.0
-1.7       .0446       .0436       .0427       .0418       .0409       .0401       .0392       .0384       .0375         -1.6       .0548       .0537       .0526       .0516       .0505       .0495       .0485       .0475       .0455         -1.5       .0668       .0655       .0643       .0630       .0618       .0606       .0594       .0582       .0171         -1.4       .0808       .0793       .0778       .0764       .0749       .0735       .0721       .0708       .0694         -1.3       .0968       .0951       .0934       .0918       .0901       .0885       .0869       .0853       .088         -1.2       .1151       .1131       .1112       .1093       .1075       .1056       .1038       .1020       .123         -1.1       .1557       .1335       .1511       .1202       .1271       .1251       .1230       .1190         -1.0       .1587       .1562       .1539       .1515       .1492       .1469       .1446       .1423       .1401         -0.9       .1841       .1814       .1788       .1762       .1736       .1711       .1685       .1660       .1635      <	.0233	.02 39	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287	-1.9
-1.6       .0548       .0537       .0526       .0516       .0505       .0495       .0485       .0475       .0 55         -1.5       .0668       .0655       .0643       .0630       .0618       .0606       .0594       .0582       .0171         -1.4       .0808       .0793       .0778       .0764       .0749       .0735       .0721       .0708       .0694         -1.3       .0968       .0951       .0934       .0918       .0901       .0885       .0869       .0853       .088         -1.2       .1151       .1131       .1112       .1093       .1075       .1056       .1038       .1020       .1193         -1.1       .1357       .1335       .1311       .1292       .1271       .1251       .1239       .1190         -1.0       .1587       .1562       .1539       .1515       .1492       .1469       .1446       .1423       .1401         -0.9       .1841       .1814       .1788       .1762       .1736       .1711       .1685       .1660       .1635         -0.8       .2119       .2090       .2061       .2033       .2005       .1977       .1949       .1922       .1894	.0294	.03 01	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359	
-1.5       .0668       .0655       .0643       .0630       .0618       .0606       .0594       .0582       .0571         -1.4       .0808       .0793       .0778       .0764       .0749       .0735       .0721       .0708       .0694         -1.3       .0968       .0951       .0934       .0918       .0901       .0885       .0869       .0853       .088         -1.2       .1151       .1131       .1112       .1093       .1075       .1056       .1038       .1020       .123         -1.1       .1557       .1335       .1511       .1202       .1271       .1251       .1230       .1190         -1.0       .1587       .1562       .1539       .1515       .1492       .1469       .1446       .1423       .1401         -0.9       .1841       .1814       .1788       .1762       .1736       .1711       .1685       .1660       .1635         -0.8       .2119       .2090       .2061       .2033       .2005       .1977       .1949       .1922       .1894         -0.7       .2420       .2389       .2358       .2327       .2296       .2266       .2236       .2206       .2177      <	.0367	.03 75	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446	-1.7
-1.4       .0808       .0793       .0778       .0764       .0749       .0735       .0721       .0708       .0094         -1.3       .0968       .0951       .0934       .0918       .0901       .0885       .0869       .0853       .088         -1.2       .1151       .1131       .1112       .1093       .1075       .1056       .1038       .1020       .123         -1.1       .1357       .1335       .1311       .1232       .1272       .1251       .1239       .1190         -1.0       .1587       .1562       .1539       .1515       .1492       .1469       .1446       .1423       .1401         -0.9       .1841       .1814       .1788       .1762       .1736       .1711       .1685       .1660       .1635         -0.8       .2119       .2090       .2061       .2033       .2005       .1977       .1949       .1922       .1894         -0.7       .2420       .2389       .2358       .2327       .2296       .2266       .2236       .2206       .2177         -0.6       .2743       .2709       .2676       .2643       .2611       .2578       .2546       .2514       .2483      <	.0455	.04 <mark>65</mark>	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548	-1.6
-1.3       .0968       .0951       .0934       .0918       .0901       .0885       .0869       .0853       .08 38         -1.2       .1151       .1131       .1112       .1093       .1075       .1056       .1038       .1020       .11 23         -1.1       .1357       .1335       .1311       .1202       .1271       .1251       .1230       .1210       .1190         -1.0       .1587       .1562       .1539       .1515       .1492       .1469       .1446       .1423       .1401         -0.9       .1841       .1814       .1788       .1762       .1736       .1711       .1685       .1660       .1635         -0.8       .2119       .2090       .2061       .2033       .2005       .1977       .1949       .1922       .1894         -0.7       .2420       .2389       .2358       .2327       .2296       .2266       .2236       .2206       .2177         -0.6       .2743       .2709       .2676       .2643       .2611       .2578       .2546       .2514       .2483         -0.5       .3085       .3050       .3015       .2981       .2946       .2912       .2877       .2843       .	.0559	.05 71	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668	-1.5
-1.2       .1151       .1131       .1112       .1093       .1075       .1056       .1038       .1020       .1033         -1.1       .1957       .1335       .1511       .1292       .1271       .1251       .1239       .1210       .1190         -1.0       .1587       .1562       .1539       .1515       .1492       .1469       .1446       .1423       .1401         -0.9       .1841       .1814       .1788       .1762       .1736       .1711       .1685       .1660       .1635         -0.8       .2119       .2090       .2061       .2033       .2005       .1977       .1949       .1922       .1894         -0.7       .2420       .2389       .2358       .2327       .2296       .2266       .2236       .2206       .2177         -0.6       .2743       .2709       .2676       .2643       .2611       .2578       .2546       .2514       .2483         -0.5       .3085       .3050       .3015       .2981       .2946       .2912       .2877       .2843       .2810	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808	-1.4
-1.1     .1957     .1935     .1911     .1292     .1271     .1251     .1239     .1210     .1190       -1.0     .1587     .1562     .1539     .1515     .1492     .1469     .1446     .1423     .1401       -0.9     .1841     .1814     .1788     .1762     .1736     .1711     .1685     .1660     .1635       -0.8     .2119     .2090     .2061     .2033     .2005     .1977     .1949     .1922     .1894       -0.7     .2420     .2389     .2358     .2327     .2296     .2266     .2236     .2206     .2177       -0.6     .2743     .2709     .2676     .2643     .2611     .2578     .2546     .2514     .2483       -0.5     .3085     .3050     .3015     .2981     .2946     .2912     .2877     .2843     .2810	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968	
-1.1     .1537     .1535     .1517     .1252     .1272     .1232     .1243     .1190       -1.0     .1587     .1562     .1539     .1515     .1492     .1469     .1446     .1423     .1401       -0.9     .1841     .1814     .1788     .1762     .1736     .1711     .1685     .1660     .1635       -0.8     .2119     .2090     .2061     .2033     .2005     .1977     .1949     .1922     .1894       -0.7     .2420     .2389     .2358     .2327     .2296     .2266     .2236     .2206     .2177       -0.6     .2743     .2709     .2676     .2643     .2611     .2578     .2546     .2514     .2483       -0.5     .3085     .3050     .3015     .2981     .2946     .2912     .2877     .2843     .2810	.0985						.1093	.1112	.1131	.1151	-1.2
-0.9       .1841       .1814       .1788       .1762       .1736       .1711       .1685       .1660       .1635         -0.8       .2119       .2090       .2061       .2033       .2005       .1977       .1949       .1922       .1894         -0.7       .2420       .2389       .2358       .2327       .2296       .2266       .2236       .2206       .2177         -0.6       .2743       .2709       .2676       .2643       .2611       .2578       .2546       .2514       .2483         -0.5       .3085       .3050       .3015       .2981       .2946       .2912       .2877       .2843       .2810	.1170		.1210	.1230	.1251	11271	11202	.1311	.1335	.1357	-1.1
-0.8       .2119       .2090       .2061       .2033       .2005       .1977       .1949       .1922       .1894         -0.7       .2420       .2389       .2358       .2327       .2296       .2266       .2236       .2206       .2177         -0.6       .2743       .2709       .2676       .2643       .2611       .2578       .2546       .2514       .2483         -0.5       .3085       .3050       .3015       .2981       .2946       .2912       .2877       .2843       .2810	.1379								.1562		
-0.7       .2420       .2389       .2358       .2327       .2296       .2266       .2236       .2206       .2177         -0.6       .2743       .2709       .2676       .2643       .2611       .2578       .2546       .2514       .2483         -0.5       .3085       .3050       .3015       .2981       .2946       .2912       .2877       .2843       .2810	.1611										
-0.6       .2743       .2709       .2676       .2643       .2611       .2578       .2546       .2514       .2483         -0.5       .3085       .3050       .3015       .2981       .2946       .2912       .2877       .2843       .2810	.1867										
-0.5 .3085 .3050 .3015 .2981 .2946 .2912 .2877 .2843 .2810	.2148						.2327				
	.2451										
_0 4 3446 3409 3372 3336 3300 3264 3228 3192 3156	.2776										
	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446	-0.4
	.3483										
	.3859										
	.4247										
-0.0 .5000 .4960 .4920 .4880 .4840 .4801 .4761 .4721 .4681	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000	-0.0

#### **Standard Normal Probabilities**

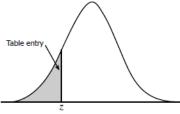
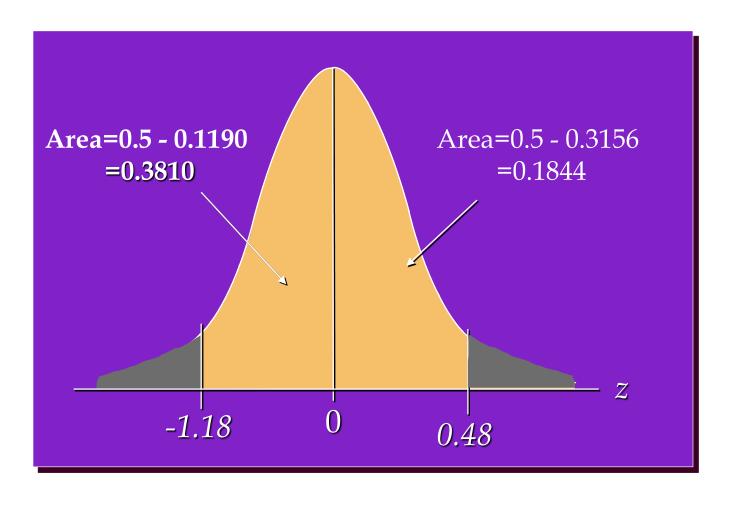


Table entry for z is the area under the standard normal curve to the left of z.

	~									
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.00 03	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.00 05	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.00 07	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.00 49	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.00 66	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.00 37	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.01 <mark>4</mark> 6	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.02 39	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.03 01	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.03 75	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.04 65	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.05 71	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.06 94	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.08 38	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.10 03	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.14 01	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.18 94	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.21 77	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.24 83	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.21/10	.2776
-0.4	.3116	<del>13 100</del>	10072	13336	.3300	.3264	.3220	.3103	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(ii)  $P(-1.18 \le Z \le 0.48) = P(Z \ge -1.18) + P(Z \le 0.48)$ 



$$P(-1.18 \le Z \le 0.48) = 0.3810 + 0.1844 = 0.5654$$

#### Try 1

(a) Find the following probabilities using the normal table

```
iii P(0 \le z \le 2.58)
iv P(z > 2.63)
v P(-2.35 \le z \le 2.35)
```

b Suppose that  $y \sim N(6,4)$ . What percentage will y fall between 5 and 10?

#### Try 2

The nicotine content of a brand of cigarettes is normally distributed with a mean of 2.0mg and a standard deviation of 0.25mg. What is the probability that a cigarette will have nicotine content

```
i of 1.65mg or less?
```

ii between 1.50mg and 2.25mg?

iii of 2.18mg or more?

#### Try 3

The weekly amount spent for maintenance and repairs in a certain company was observed, over a long period of time, to be approximately normally distributed with a mean of \$400 and a standard deviation of \$20.

- If \$450 is budgeted for the week, what is the probability that the actual costs will exceed the budgeted amount?
- How much should be budgeted for weekly repairs and maintenance in order for the budgeted amount is exceeded with a probability of 0.1?

- b. The weekly amount spent for maintenance and repairs in a certain company was observed, over a long period of time, to be approximately normally distributed with a mean of \$400 and a standard deviation of \$20.
  - If \$450 is budgeted for the week, what is the probability that the actual costs will exceed the budgeted amount?
  - How much should be budgeted for weekly repairs and maintenance in order for the budgeted amount is exceeded with a probability of 0.1?

	i i			Sec	cond decin	nal place in	n z			
Z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
-3.9										0.0000*
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.6	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
-3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
-3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
-3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
-1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
-1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
-1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
-1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
-1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
-1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
-1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
-0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
-0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
-0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
-0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
-0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
-0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
-0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
-0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
-0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
-0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

\*For  $z \le -3.90$ , the areas are 0.0000 to four decimal places