MATH 166:

Introductory Probability and Statistics

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Special Distributions

Special Distributions

- Discrete distributions
  - ✓ Bernoulli distribution
  - ✓ Binomial distribution
  - ✓ Poisson distribution

- Continuous distributions
  - ✓ Uniform distribution
  - ✓ Exponential distribution
  - ✓ Normal distribution

Discrete Distributions

Bernoulli Distribution

Bernoulli Distribution

Definition

- A discrete random variable X is said to have a Bernoulli distribution with parameter p $(0 \leq p \leq 1)$, if it assumes the values, 0 and 1 for two outcomes.

- The probability distribution for the success in the trial, x is defined by

$$p(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$$

Bernoulli Distribution

Properties of Bernoulli Distribution

$$\mu = E(x) = p$$

$$\sigma^2 = Var(x) = p(1-p)$$

Entention

- The Bernoulli distribution is a special case of Binomial distribution

Binomial Distribution

Binomial Distribution

- The Binomial distribution is a discrete probability distribution that describes the probability of the number of successes x in a sequence of n independent and identical Bernoulli trials.

- The probability of a success, p, doesnot change from trial to trial.

- For a single trial, i.e., $n = 1$, the binomial distribution is a Bernoulli distribution.

Binomial Distribution

Definition

- A discrete random variable X is said to have a Binomial distribution with parameter *n* and *p* (0 ≤ *p* ≤ 1), and the probability distribution defined as:

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}, \qquad x = 0,1,\ldots,n$$

where:

$p(x)$ = the probability of x successes in n trials

n = the number of trials

x = the number of successes

p = the probability of success on any one trial

Binomial Distribution

- Alternative formulation

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \qquad x = 0, 1, \ldots, n$$

Properties of Binomial Distribution

$$\mu = E(x) = np$$

$$\sigma^2 = Var(x) = np(1-p)$$

Binomial Distribution

Example 1:

In a computer manufacturing company, 10% of the memory chips produced are defective. If three (3) of the memory chips produced by the company are selected at random, what is the probability that 1 of the chips selected is defective

Binomial Distribution

Solution:

Let p=0.10, n=3, x=1

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \qquad x = 0, 1, \ldots, n$$

$$P(x=1) = \frac{3!}{1!(3-1)!} (0.1)^1 (1-0.1)^{3-1} = 0.243$$

The probability that 1 of the chips selected is defective is 0.243

Example 2

The number of serious accidents, in a manufacturing plant has approximately a Poisson distribution with a mean of 1.5 accidents per month. What is the probability that exactly three accidents will occur within a period of one month?

Solution

First determine the value of λ which in this case is 1.5

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(x = 3) = \frac{1.5^3 (2.7183)^{-1.5}}{3!} \approx 0.126$$

Binomial Distribution

Try it your self

A large retail shop purchases a certain brand of computers from a manufacturer. The manufacturer indicates that the defective rate of the product is 3% in a shipment. The inspector of the retail shop randomly picks 20 items of the product from a shipment. What is the probability that there will

a) be 3 defective items?

b) not be more than two (2) defective items?

c) What is the average number of defective items

Binomial Distribution

Assignment

A multiple-choice test consists of 15 questions each with five possible answers of which only one is correct. Suppose one of the students taking the test answers the questions by guessing. What is the probability that he answers at most 3 questions correctly?

Binomial Distribution

Assignment

It is known that 25% of inhabitants of a community favour a political party A. A random sample of 20 inhabitants was selected from the community and each person was asked he/she will vote for party A in an impending election. What is the probability that:

a) exactly two persons will vote for party A?

b) at least three persons will vote for party A?

Poisson Distribution

Poisson Distribution

Definition

- A discrete random variable X is said to have a Poisson distribution with parameter λ ($\lambda>0$), if the probability distribution is:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, \ldots$$

The letter e is a constant equals 2.7183

Properties of Poisson Distribution

$$\mu = E(x) = \lambda$$

$$\sigma^2 = Var(x) = \lambda$$

Poisson Distribution

Example

Suppose that the number of telephone calls received by a computer technician for IT support during a 30 minutes time interval is known to be a Poisson distribution with mean of 3. Find the probability that:

a) no call will be received within the period.
b) exactly 4 calls will be received
c) at least one call will be received
d) more than 4 calls will be received

Poisson Distribution

Solution

Let λ=3

a) no call will be received within the period

$$P(x=0)=\frac{3^{0}e^{-3}}{0!}=0.050$$

b) Exactly 4 calls will be received

$$P(x=4)=\frac{3^{4}e^{-3}}{4!}=0.168$$

Poisson Distribution

c) at least one call

$$P(x \geq 1) = 1 - P(x = 0)$$

$$= 1 - \frac{3^0 e^{-3}}{0!} = 1 - 0.050 = 0.950$$

d) more than 2 calls will be received

$$P(x > 2) = 1 - P(x \leq 2)$$

$$= 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$$

$$= 1 - \left[\frac{3^0 e^{-3}}{0!} + \frac{3^1 e^{-3}}{1!} + \frac{3^2 e^{-3}}{2!} \right] = 0.577$$

Poisson Distribution

Try it your self

The number of serious accidents, y in a manufacturing plant has approximately a Poisson distribution with a mean of 10 accidents per year. What is the probability that:

a) at least 2 accidents will occur within a period of 3 months?

b) at most one accident will occur within a period of 2 months?

Poisson Distribution

Assignment 2

- The number of telephone calls received by an office on averages is 4 per minute. Find the probability that:
  - ✓ No call will arrive in a given one-minute period.
  - ✓ At least two calls will arrive in a given one-minute period.
  - ✓ At least three calls will arrive in a given two-minute period.

Poisson Distribution

Further readings on Poisson approximation to binomial distribution

- The probability that a car will breakdown after falling into a pothole on a road is 0.00015. If 20,000 cars travel along the road, what is the expected number of break-downs and probability that at least one car will break down.

- A book has 300 pages and the probability of finding misprints, x, in a page is 0.015. Find the probability of detecting misprints in at most one page of the book.

Continuous Probability Distributions

Uniform Distribution

Uniform Distribution

Definition

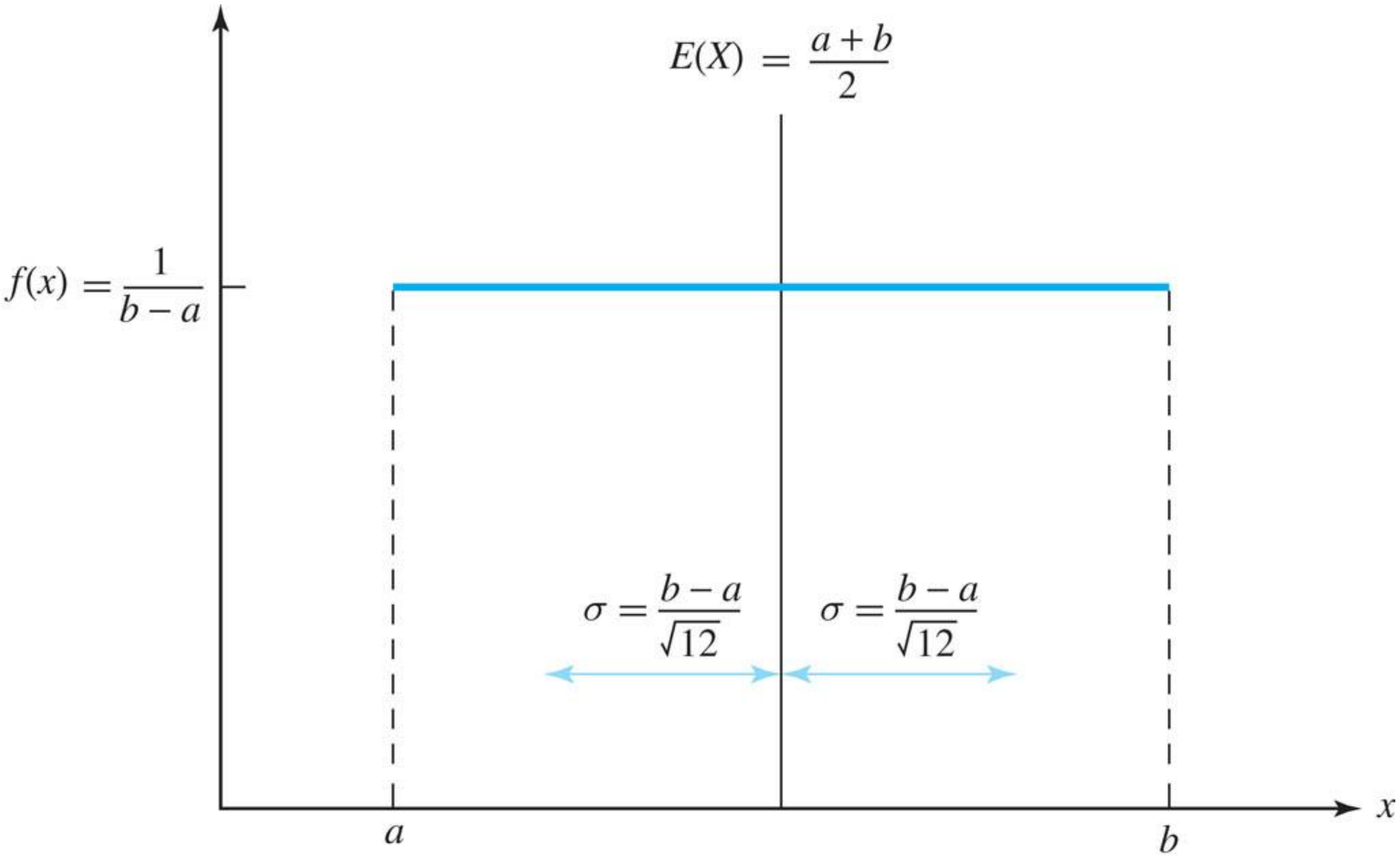
- A continuous random variable X has the uniform distribution over the interval [a, b] if the probability distribution is:

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

Properties of Uniform Distribution

$$\mu = E(x) = \frac{a+b}{2} \qquad \sigma^2 = Var(x) = \frac{(b-a)^2}{12}$$

Uniform Distribution



Uniform Distribution

Example

Suppose a buses arrive at a bus stop every 6 minutes and that the waiting time for the next bus to arrive has a uniform probability distribution on the interval from 2 to 6 minutes.

a) What is the probability that the bus will arrive between 3 and 4 minute
b) What is the probability that the bus will arrive before 5 minute
c) Compute the average waiting time of a person.

Solution

Let a=2 and b=6, the pdf is given by

$$f(x)=\frac{1}{6-2},\quad 2\le x\le 6$$

a) probability that the bus will arrive between 3 and 4 minutes

$$P(3\le x\le 4)=\int_3^4 \frac{1}{4}\,dx=\left[\frac{x}{4}\right]_3^4$$

$$=\left[\frac{4}{4}-\frac{3}{4}\right]=\frac{1}{4}$$

Uniform Distribution

b) probability that the bus will arrive before 5 minutes

$$P(x<5)=\int_2^5 \frac{1}{4}\,dx=\left[\frac{x}{4}\right]_2^5$$

$$=\left[\frac{5}{4}-\frac{2}{4}\right]=\frac{3}{4}$$

c) the average waiting time of a person

$$\mu=E(x)=\frac{2+6}{2}=4$$

Uniform Distribution

Example 2

- Let x denote the flight time of an airplane traveling from Chicago to New York. Assume that the minimum time is 2 hours and that the maximum time is 2 hours 20 minutes.
  - a) What is the probability that a flight will take between 135 and 140 minutes?
  - b) What is the probability that a flight will take between 124 and 136 minutes?
  - c) What is the average flying time?

Uniform Distribution

Solution

If X is the flight time, then X is uniformly distributed over [120, 140] and its probability distribution function is:

$$f(x) = \frac{1}{140-120} = \frac{1}{20} \qquad f(x) = \begin{cases} \frac{1}{20}, & 120 \le x \le 140 \\ 0, & \text{otherwise} \end{cases}$$

a) The probability that a flight will take between 135 and 140 minutes is

$$P(135 \le x \le 140) = \int_{135}^{140} \tfrac{1}{20}\, dx$$

$$= \left[\frac{1}{20}x\right]_{135}^{140} = \left[\frac{140}{20}\right] - \left[\frac{135}{20}\right] = 0.25$$

Uniform Distribution

b) The probability that a flight will take between 124 and 136 minutes is

$$P(124 \le x \le 136) = \int_{124}^{136} \tfrac{1}{20}\,dx$$

$$= \left[\frac{1}{20}x\right]_{124}^{136} = \left[\frac{136}{20}\right] - \left[\frac{124}{20}\right] = 0.6$$

c) The average flying time is:

$$E(x) = \frac{140 + 120}{2} = \frac{260}{2} = 130\,\text{minutes}$$

Uniform Distribution

Try it your self

The probability density function of time, x required to complete an assembly operation is uniformly distributed for $30 \leq x \leq 40$ seconds.

a) Determine the proportion of assemblies that require less than 35 seconds to complete.
b) What time is exceeded by 90% of the assemblies?

Exponential Distribution

Exponential Distribution

Definition

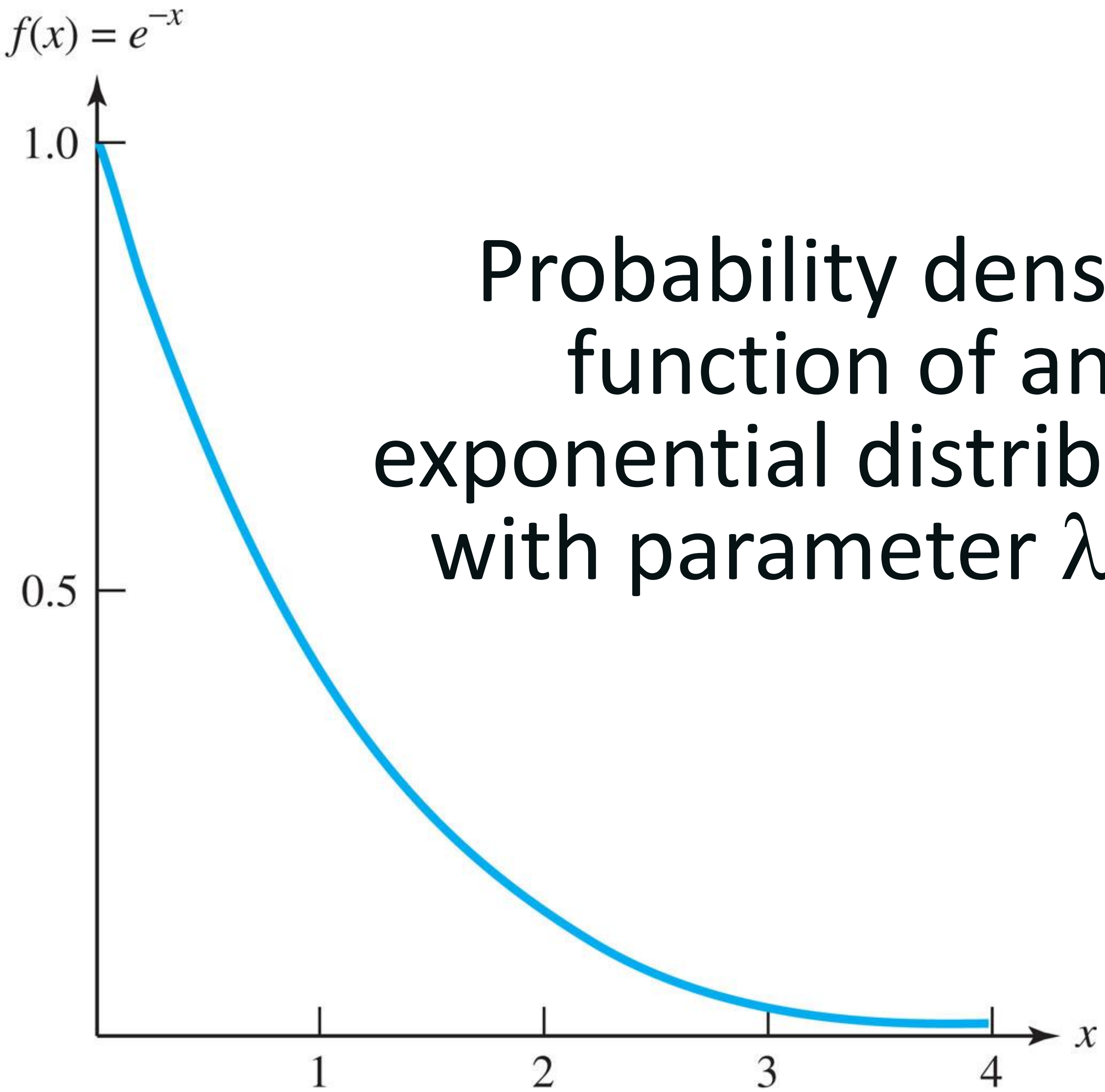
- A continuous random variable X has the exponential distribution with parameter θ if the probability distribution is:

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Properties of Exponential Distribution

$$\mu = E(x) = \frac{1}{\theta} \qquad \sigma^2 = Var(x) = \frac{1}{\theta^2}$$

Exponential Distribution



Probability density function of an exponential distribution with parameter $\lambda = 1$

Exponential Distribution

Example 1

Suppose that the time until first failure of an electric component is an exponential random variables with a rate of $\lambda=0.0625$ per month. Calculate the probability that the device last:

a) not more than 45 months

b) longer than 40 months

Exponential Distribution

Solution

a) not more than 45 months

$$P(x \le 45) = \int_0^{45} 0.0625e^{-0.0625x}dx = \left[-e^{-0.0625x}\right]_0^{45}$$

$$= 1 - e^{-0.0625(45)} = 0.9399$$

b) longer than 40 months

$$P(x > 40) = \int_{40}^{\infty} 0.0625e^{-0.0625x}dx = \left[-e^{-0.0625x}\right]_{40}^{\infty}$$

$$= e^{-0.0625(40)} = 0.0821$$

Exponential Distribution

Try it

Suppose the time in days between service calls on a photocopier machine follows an exponential distribution with a mean call of 2 days.

a) What is the probability that the time until the machine again requires service exceeds 60 days?

b) What is the probability that the time until the machine again requires service is less than 20 days?

Normal Distributions

Normal Distributions

Definition

- A continuous random variable X has the normal distribution with parameter *μ and σ^2* if the probability distribution is:

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, & -\infty < x < \infty \\ & \sigma > 0 \\ 0 & , elsewhere \end{cases}$$

Properties of Normal Distribution

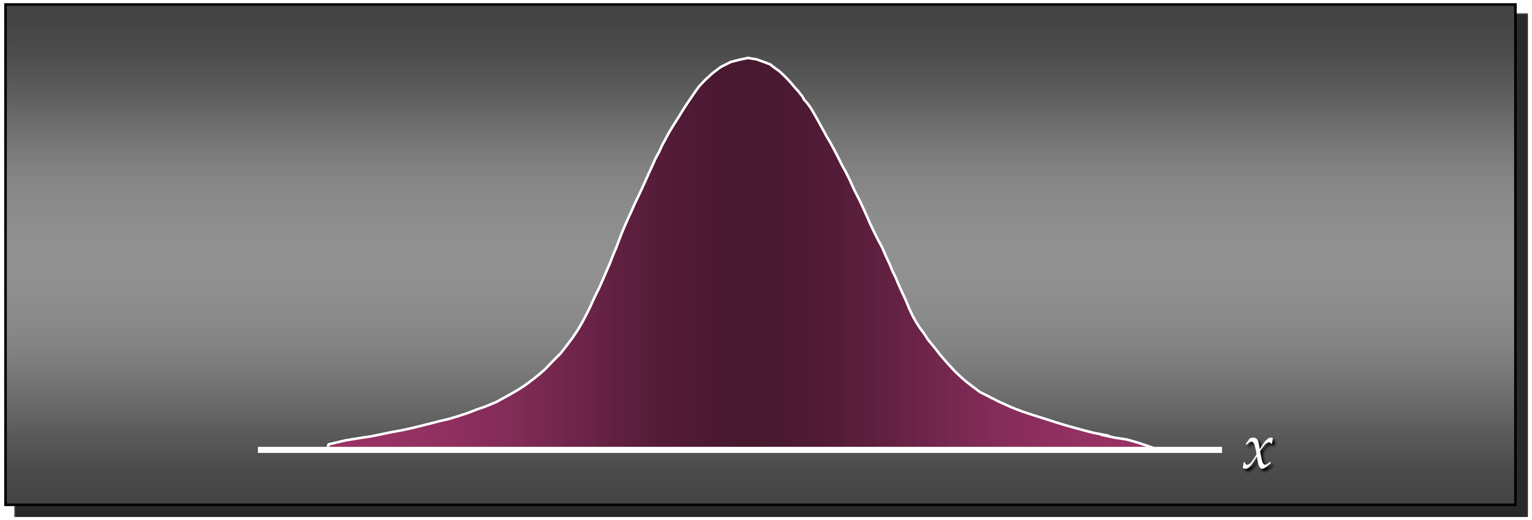
$$E(x) = \mu$$

$$Var(x) = \sigma^2$$

Normal Probability Distributions

- Characteristics

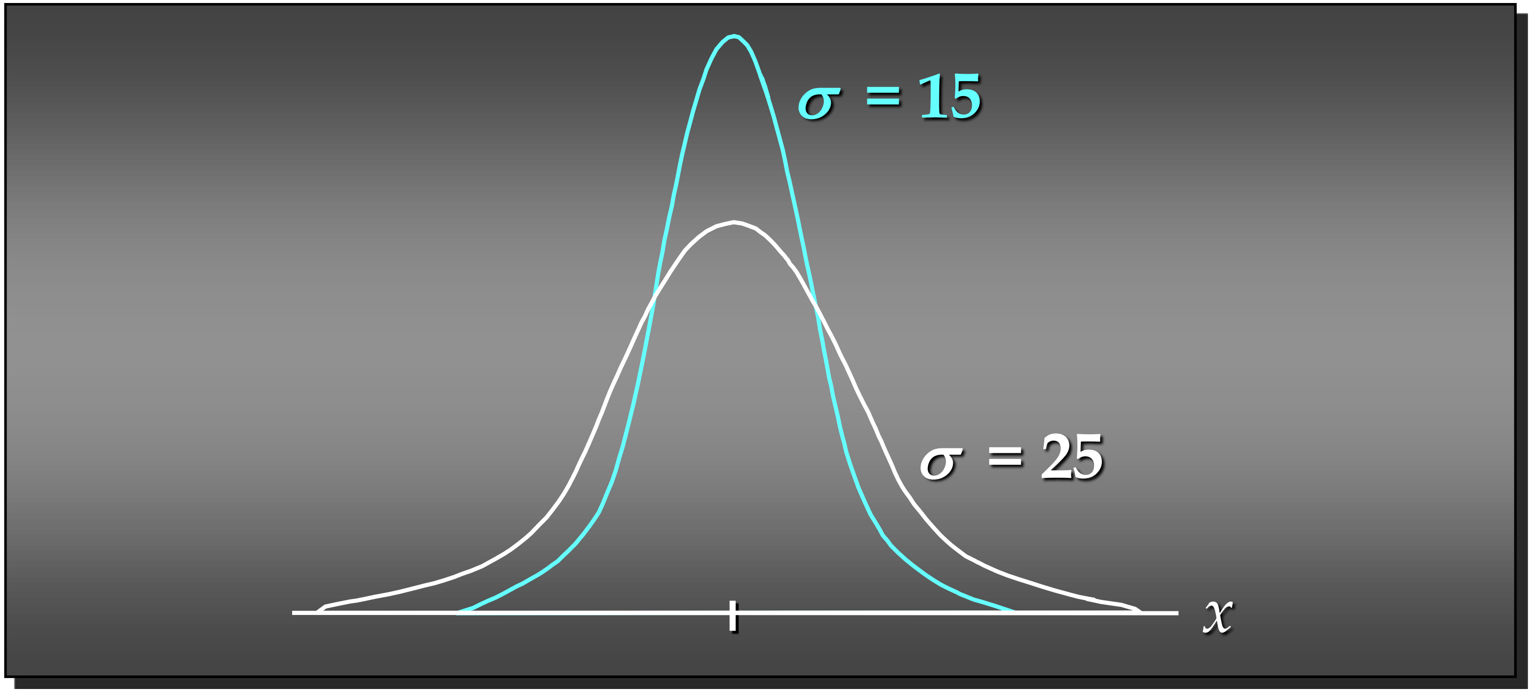
The distribution is <u>symmetric</u>; its skewness measure is zero.



Normal Probability Distributions

- Characteristics

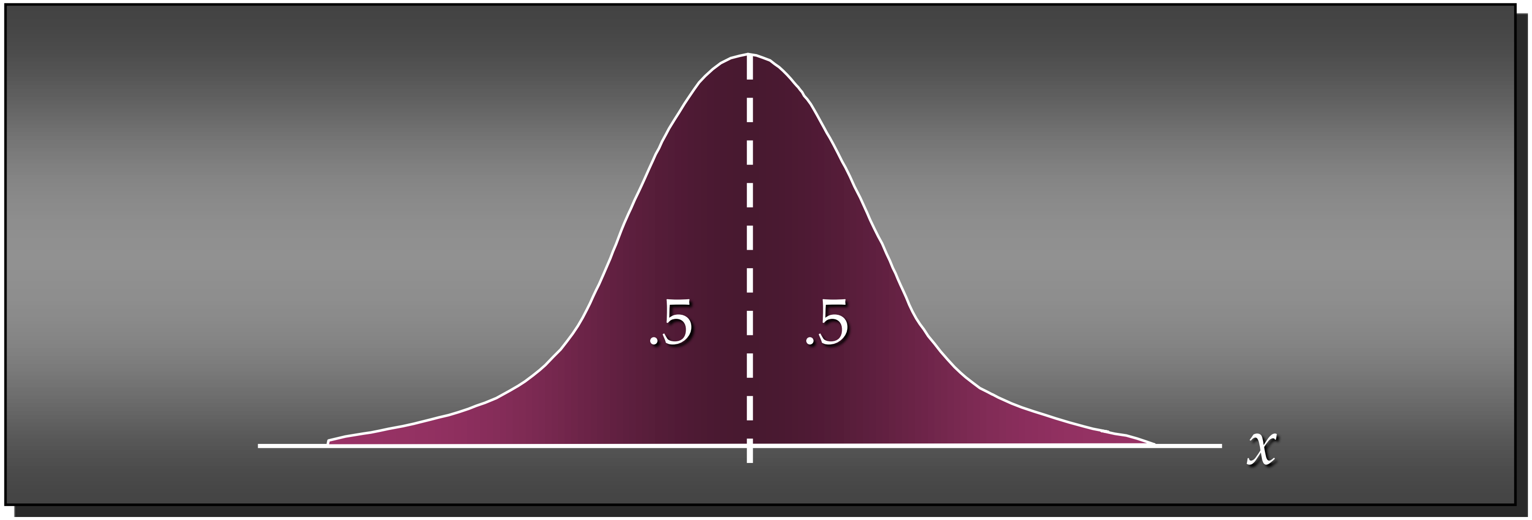
> The standard deviation determines the width of the curve: larger values result in wider, flatter curves.



Normal Probability Distributions

- Characteristics

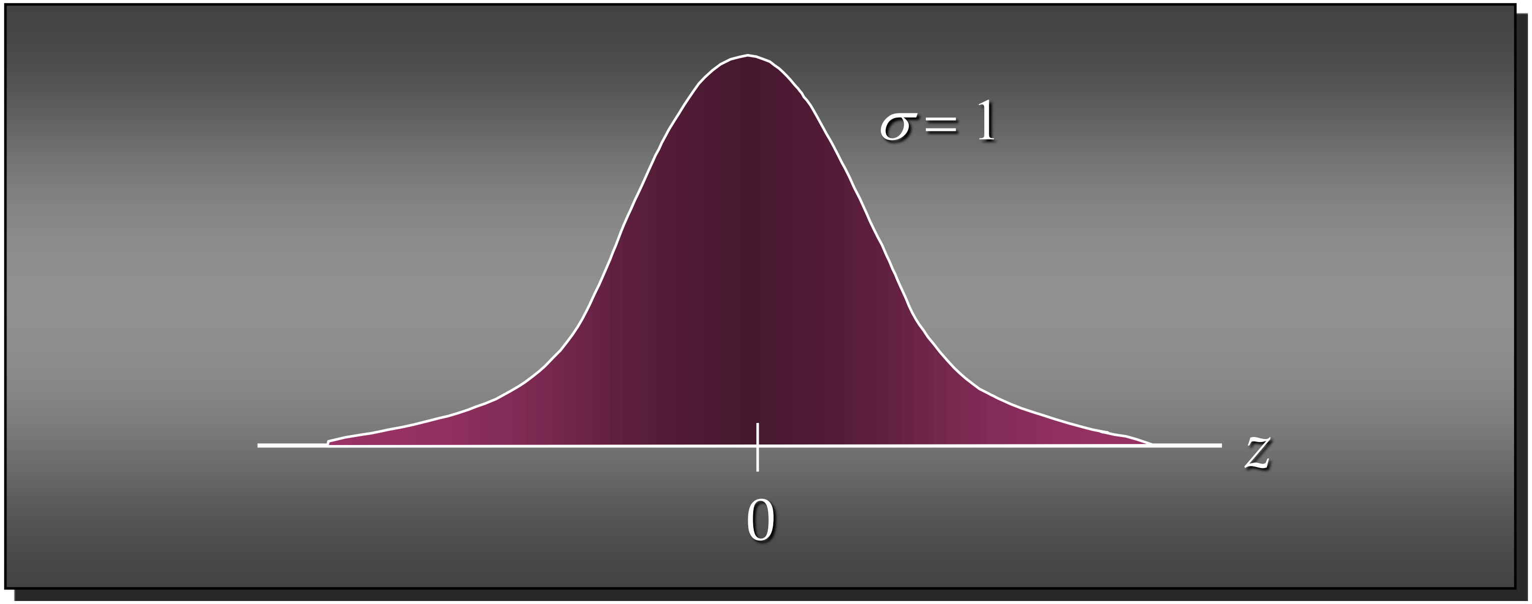
Probabilities for the normal random variable are given by <u>areas under the curve</u>. The total area under the curve is 1 (.5 to the left of the mean and .5 to the right).



Standard Normal Probability Distributions

- A random variable having a normal distribution with a mean of 0 and a standard deviation of 1 is said to have a standard normal probability distribution.

- The letter z is used to designate the standard normal random variable.



Standard Normal Probability Distributions

- Converting to the Standard Normal Distribution (z-score)

$$z = \frac{x - \mu}{\sigma}$$

We can think of z as a measure of the number of standard deviations x is from μ.

Standard Normal Probability Distributions

Standard Normal Probabilities

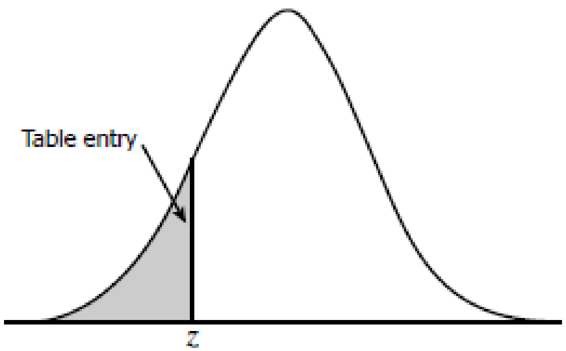


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367

Standard Normal Probability Distributions

Standard Normal Probabilities

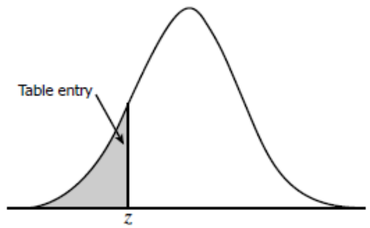


Table entry for z is the area under the standard normal curve to the left of z.

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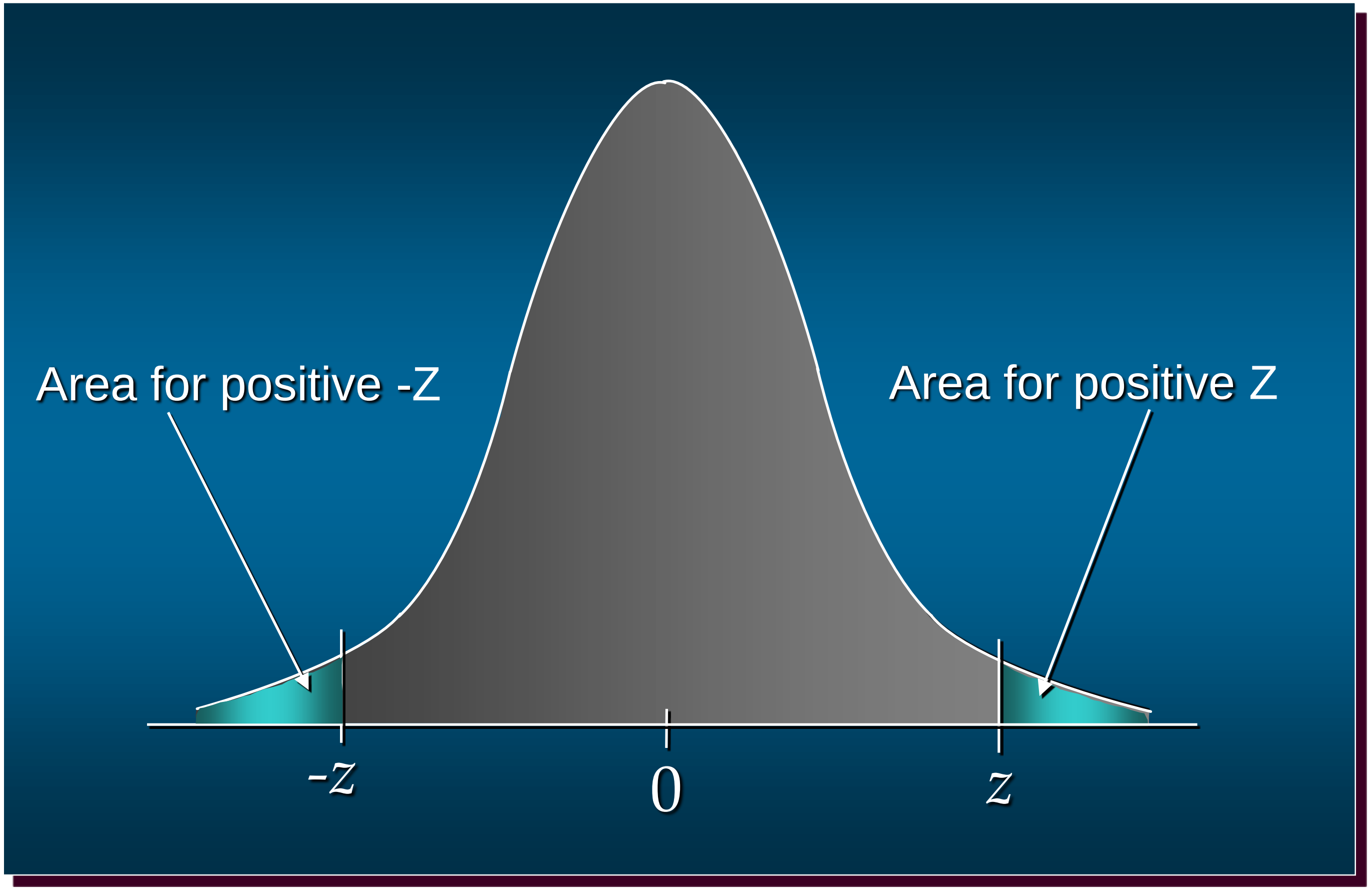
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probability Distributions

- Area under the normal curve

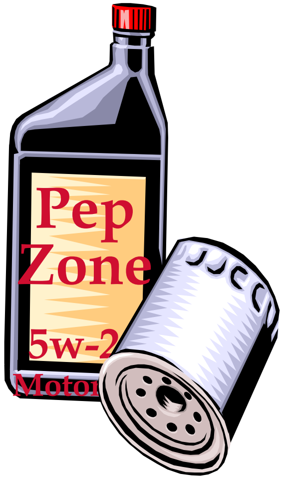


Standard Normal Probability Distributions

Example 1

- The store manager is concerned that sales are being lost due to stockouts while waiting for an order. It has been determined that demand during replenishment lead-time is normally distributed with a mean of 15 gallons and a standard deviation of 6 gallons.

The manager would like to know the probability of a stockout, $P(x > 20)$.



Standard Normal Probability Distributions

Solution

Let $x=20$, $\mu=15$, and $\sigma= 6$

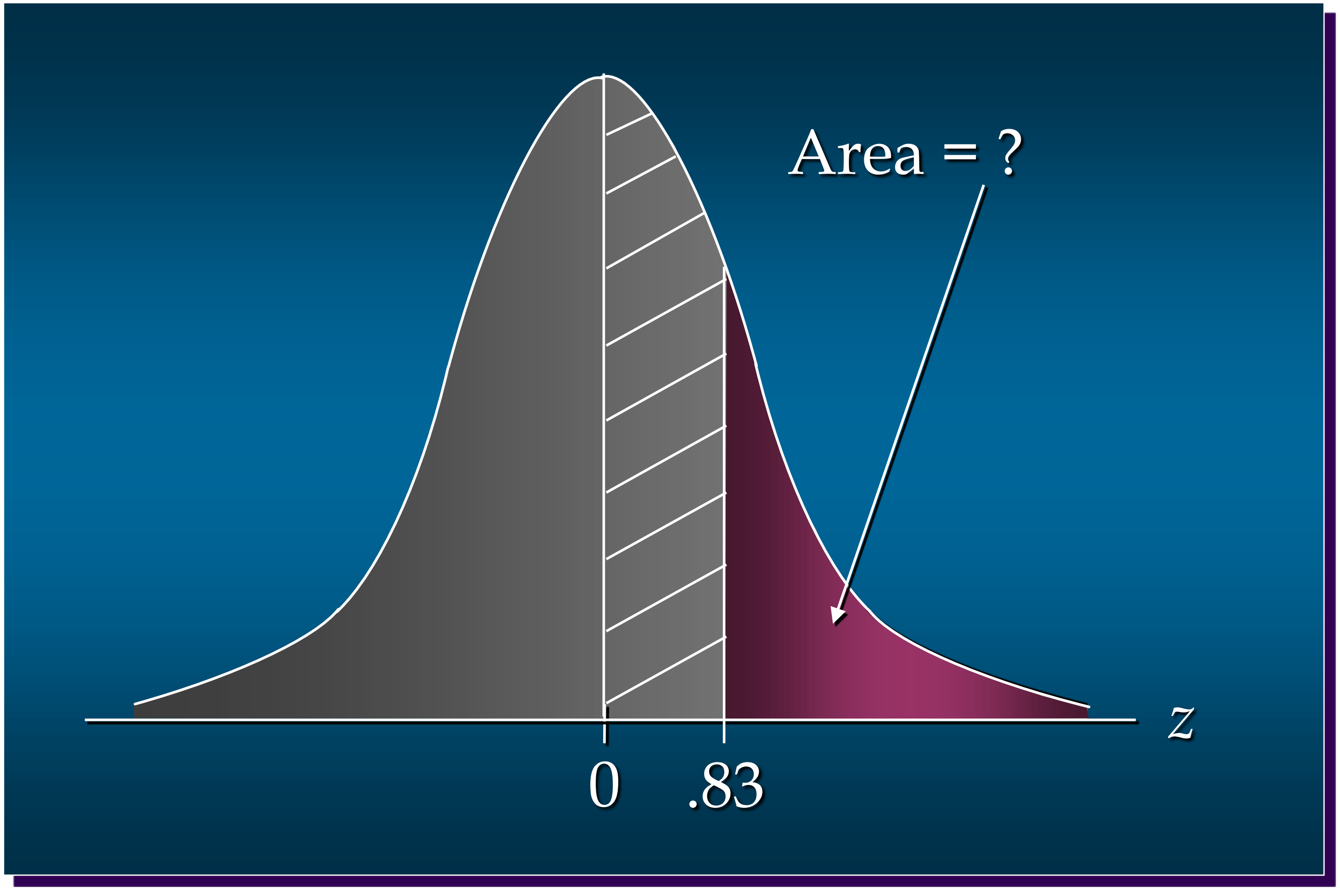
- Step 1: Convert x to the standard normal distribution.

$$
\begin{aligned}
z &= (x - \mu)/\sigma \\
&= (20 - 15)/6 \\
&= 0.83
\end{aligned}
$$

- Step 2: Find the area under the standard normal curve to the left of $z = 0.83$.

Standard Normal Probability Distributions

- Solving for the Stockout Probability



Standard Normal Probability Distributions

Standard Normal Probabilities

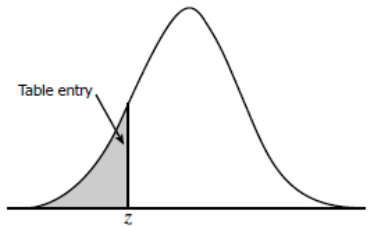


Table entry for z is the area under the standard normal curve to the left of z.

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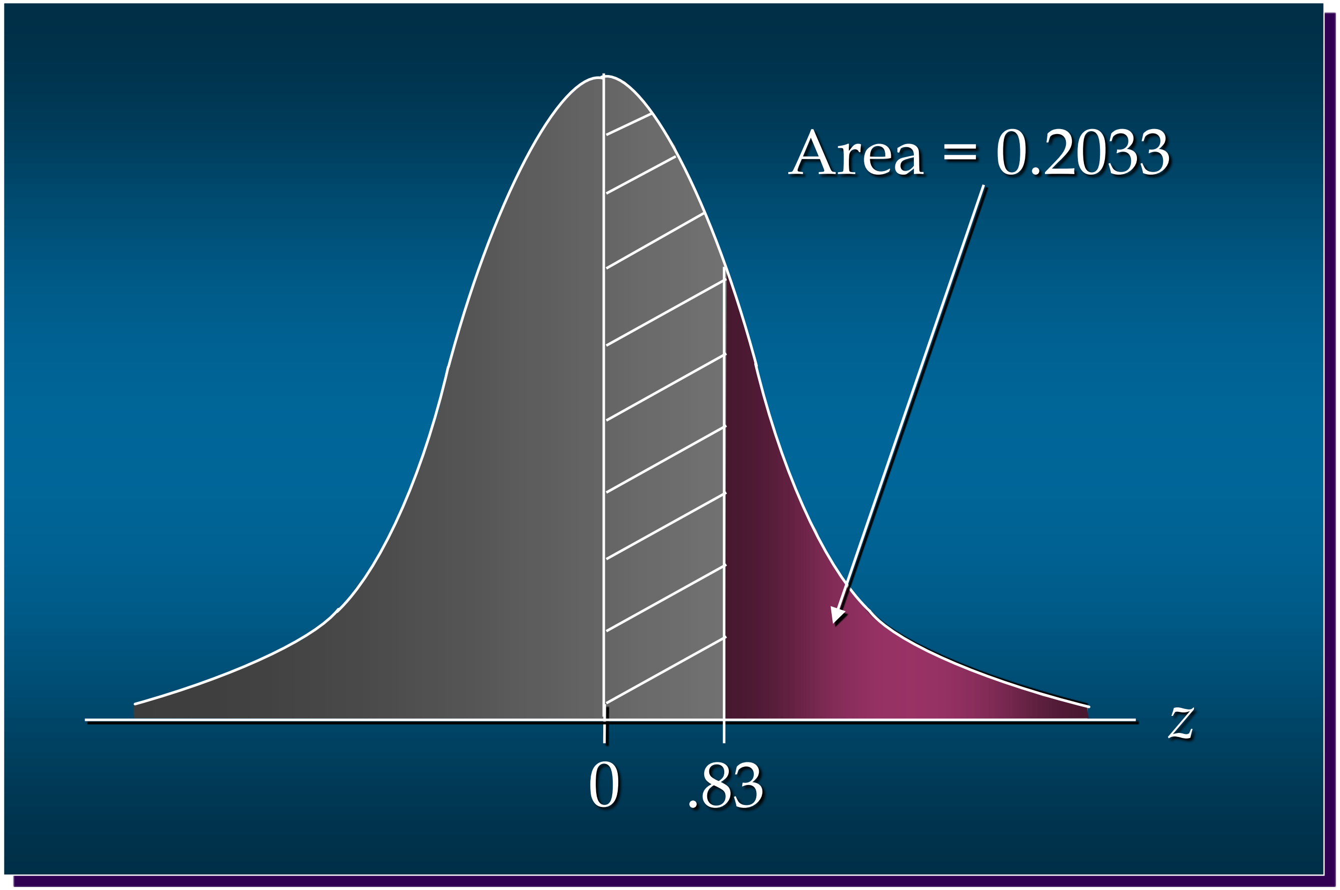
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probability Distributions

- Solving for the Stockout Probability



Standard Normal Probability Distributions

Example 2

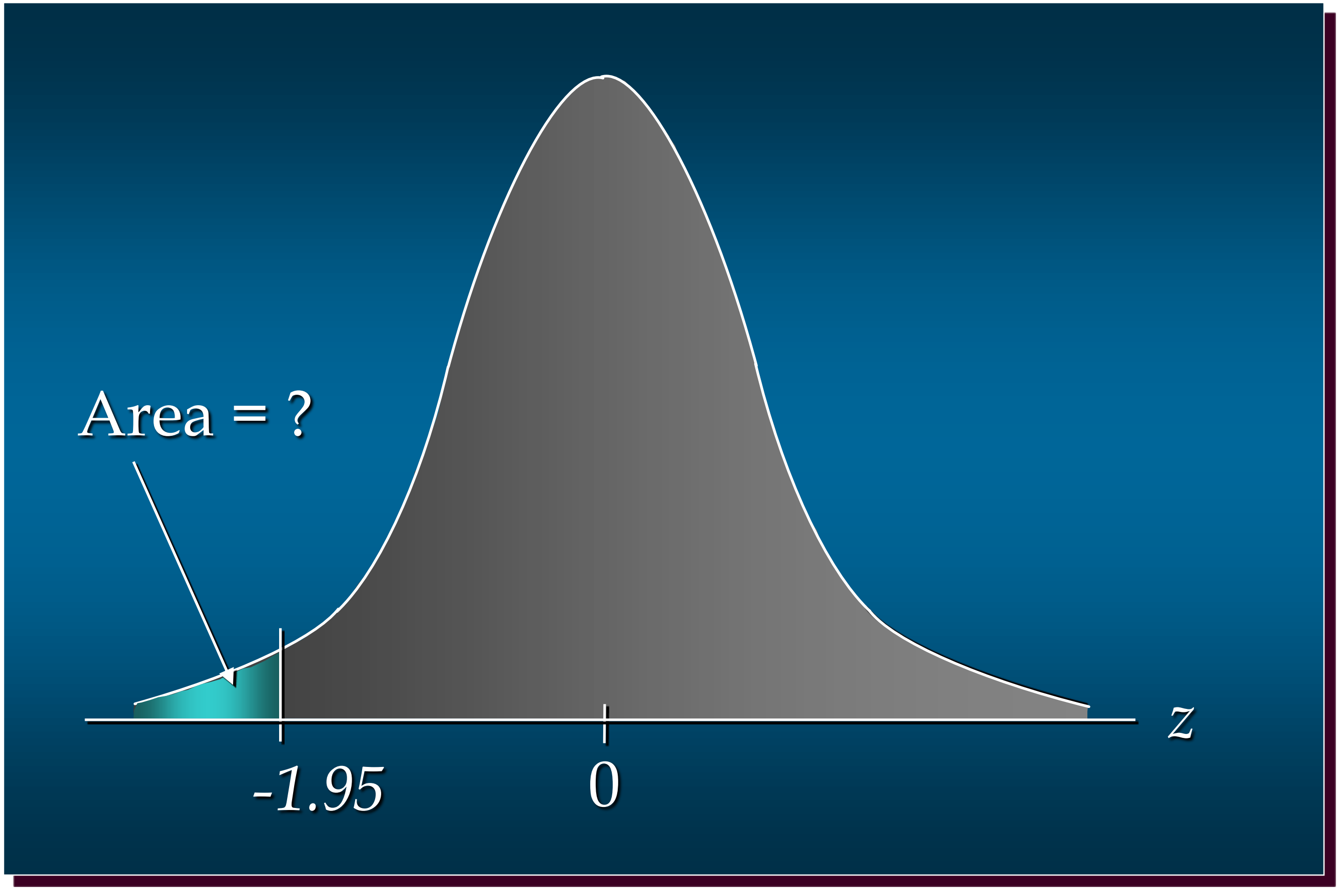
Find the following probabilities using the normal table

i. $P(z \leq -1.95)$

ii $P(-1.18 \leq z \leq 0.48)$

Standard Normal Probability Distributions

i). $P(Z \leq -1.95)$



Standard Normal Probability Distributions

Standard Normal Probabilities

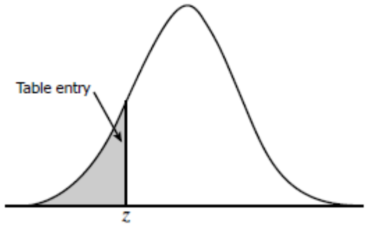
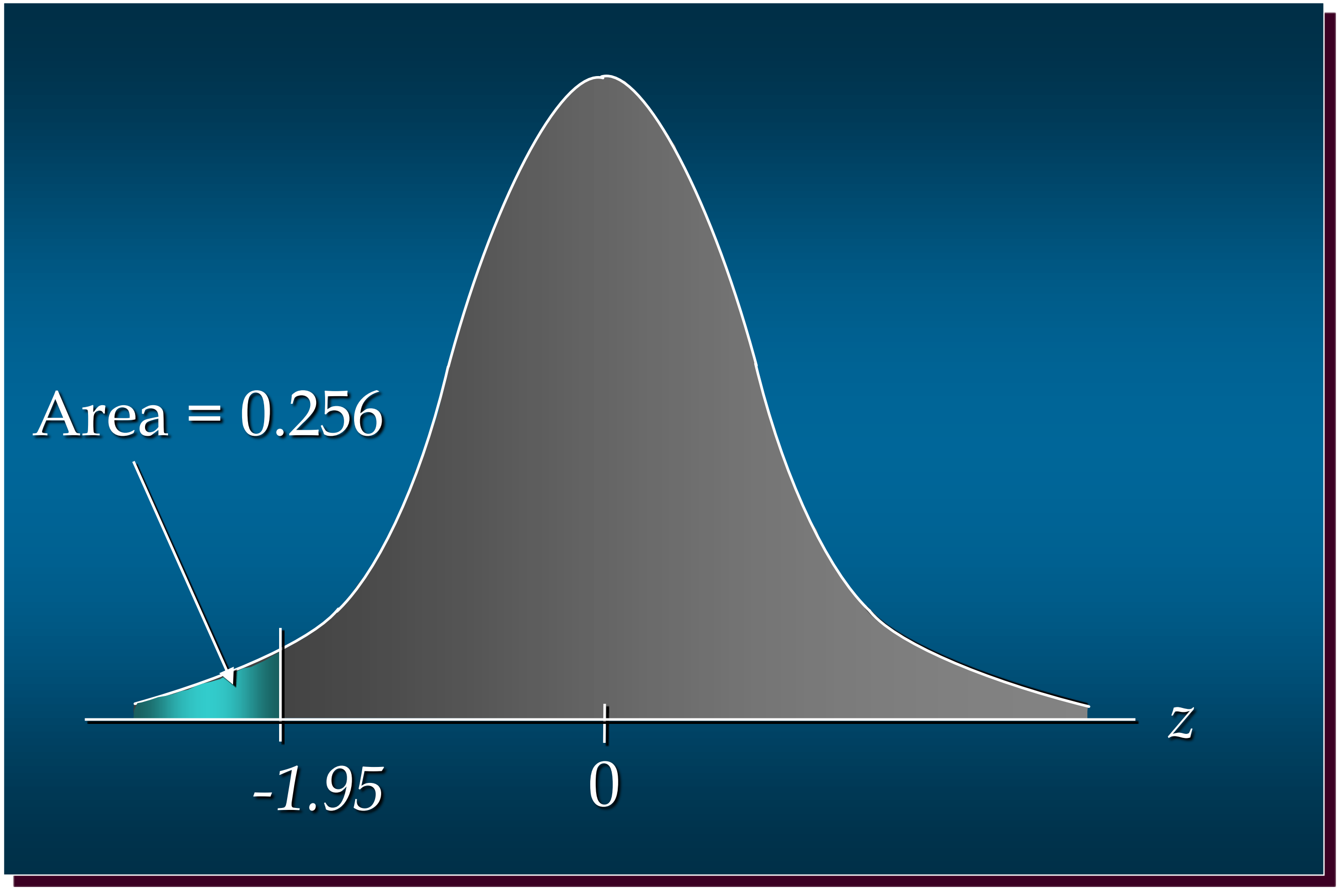


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

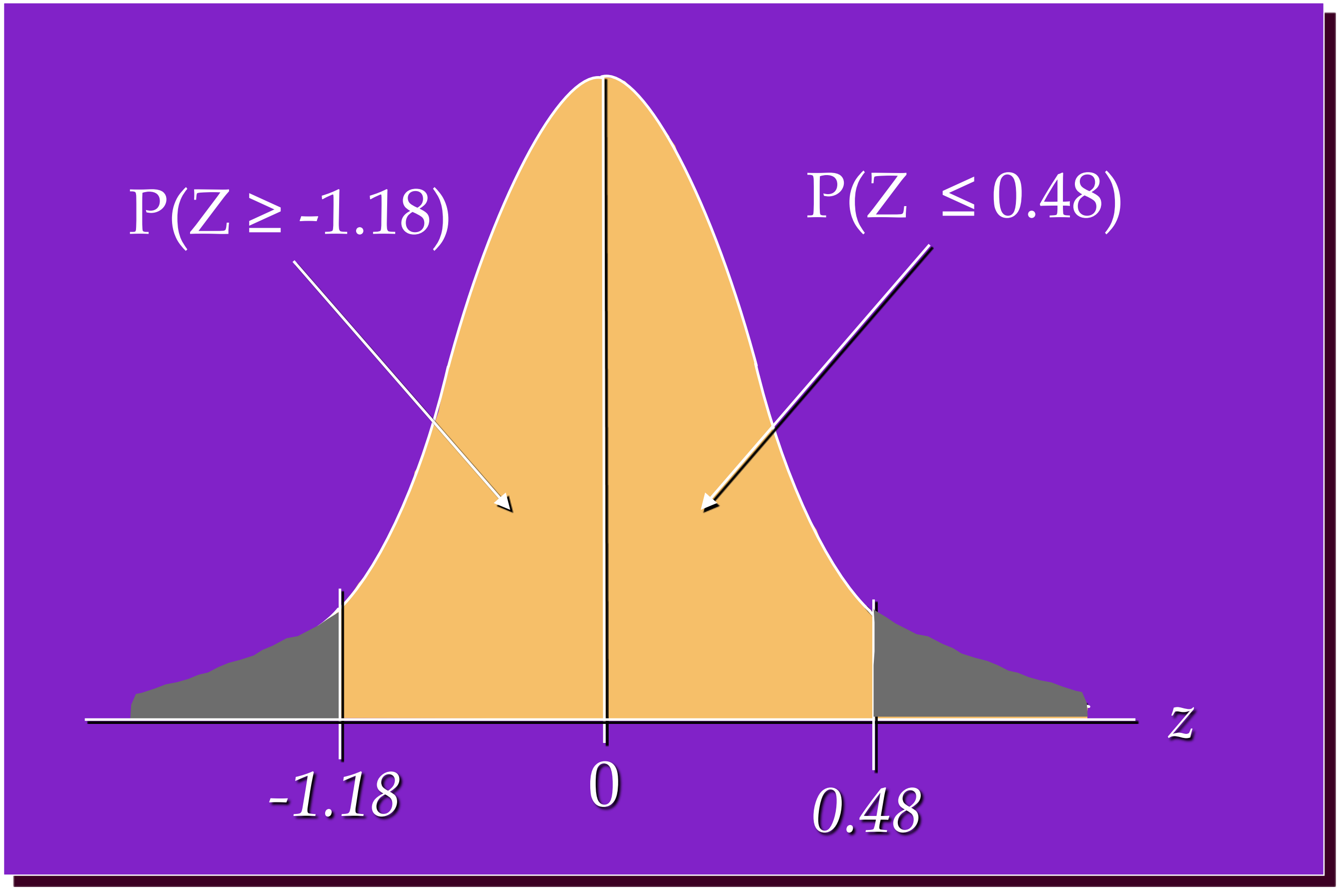
Standard Normal Probability Distributions

i) $P(Z \leq -1.95) = 0.256$



Standard Normal Probability Distributions

(ii) $P(-1.18 \le Z \le 0.48) = P(Z \ge -1.18) + P(Z \le 0.48)$



Standard Normal Probability Distributions

Standard Normal Probabilities

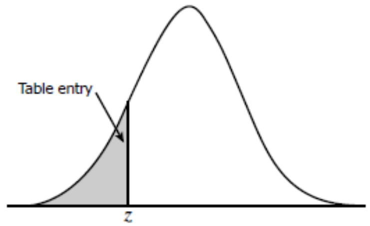


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	$\boxed{.1190}$	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probability Distributions

Standard Normal Probabilities

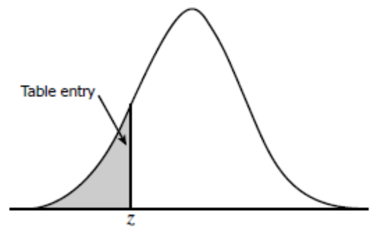
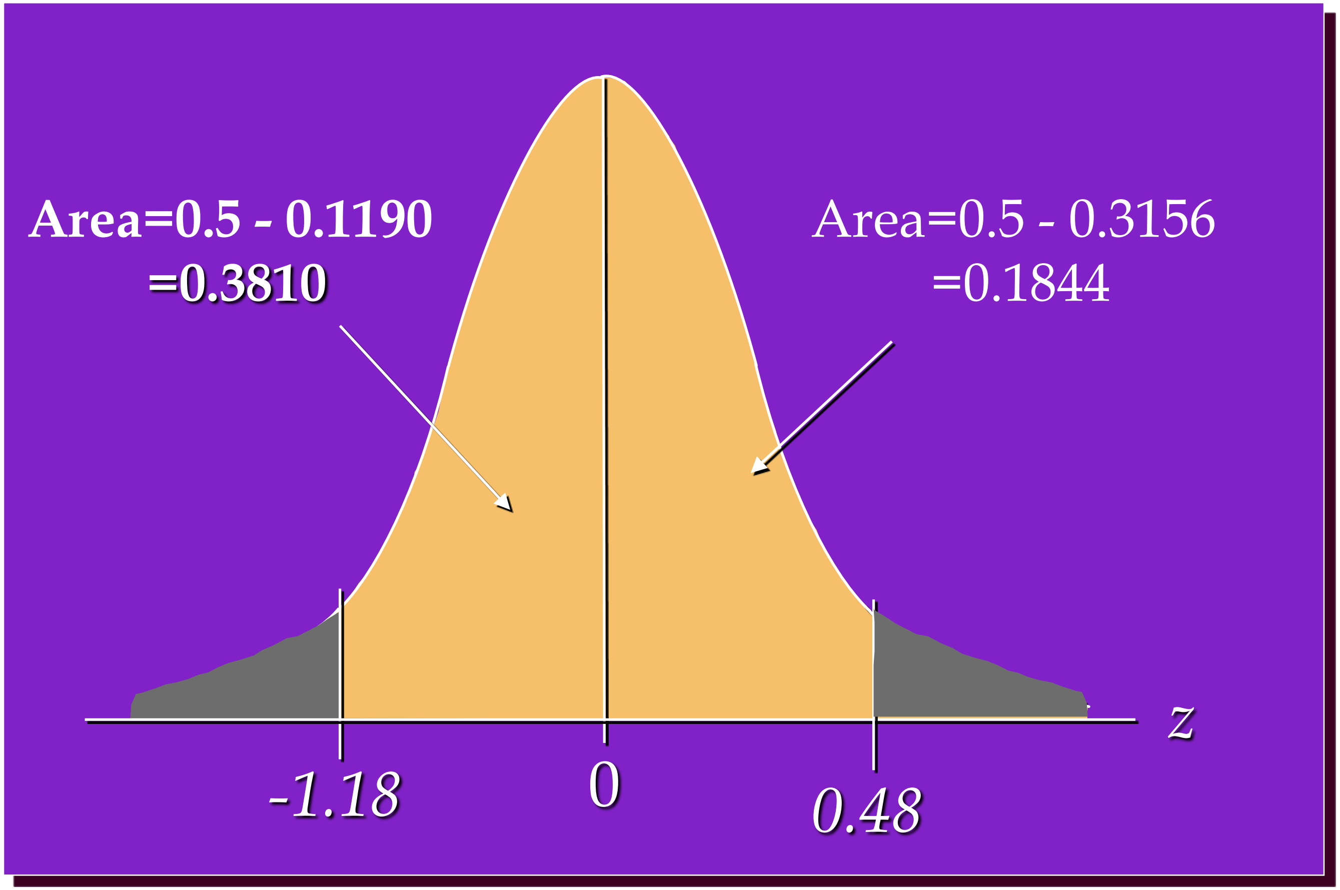


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	$\boxed{.3156}$	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probability Distributions

(ii) $P(-1.18 \le Z \le 0.48) = P(Z \ge -1.18) + P(Z \le 0.48)$



$P(-1.18 \le Z \le 0.48) = 0.3810 + 0.1844 = 0.5654$

Standard Normal Probability Distributions

Try 1

(a) Find the following probabilities using the normal table

- iii $P(0 \leq z \leq 2.58)$
- iv $P(z > 2.63)$
- v $P(-2.35 \leq z \leq 2.35)$

b Suppose that $y \sim N(6, 4)$. What percentage will y fall between 5 and 10?

Standard Normal Probability Distributions

Try 2

The nicotine content of a brand of cigarettes is normally distributed with a mean of $2.0mg$ and a standard deviation of $0.25mg$. What is the probability that a cigarette will have nicotine content

i of $1.65mg$ or less?

ii between $1.50mg$ and $2.25mg$?

iii of $2.18mg$ or more?

Standard Normal Probability Distributions

Try 3

The weekly amount spent for maintenance and repairs in a certain company was observed, over a long period of time, to be approximately normally distributed with a mean of \$400 and a standard deviation of \$20.

- If \$450 is budgeted for the week, what is the probability that the actual costs will exceed the budgeted amount?
- How much should be budgeted for weekly repairs and maintenance in order for the budgeted amount is exceeded with a probability of 0.1?

Standard Normal Probability Distributions

b. The weekly amount spent for maintenance and repairs in a certain company was observed, over a long period of time, to be approximately normally distributed with a mean of \$400 and a standard deviation of \$20.

- If \$450 is budgeted for the week, what is the probability that the actual costs will exceed the budgeted amount?
- How much should be budgeted for weekly repairs and maintenance in order for the budgeted amount is exceeded with a probability of 0.1?

Standard Normal Probability Distributions

	Second decimal place in z									
z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
-3.9										0.0000*
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.6	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
-3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
-3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
-3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
-1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
-1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
-1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
-1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
-1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
-1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
-1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
-0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
-0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
-0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
-0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
-0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
-0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
-0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
-0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
-0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
-0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

*For $z \le -3.90$, the areas are 0.0000 to four decimal places