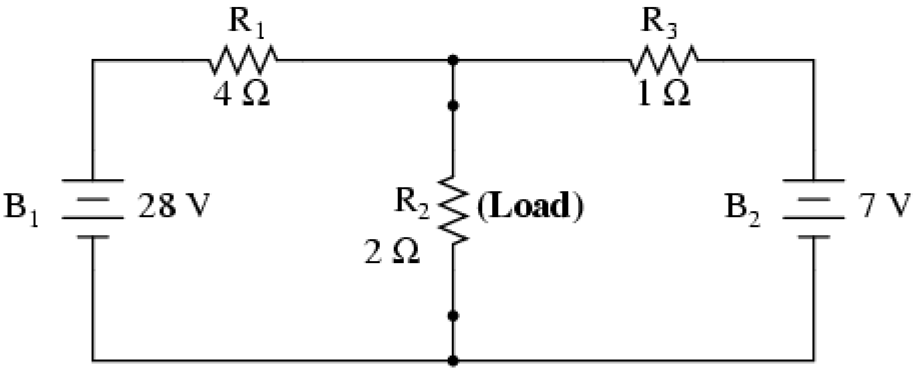
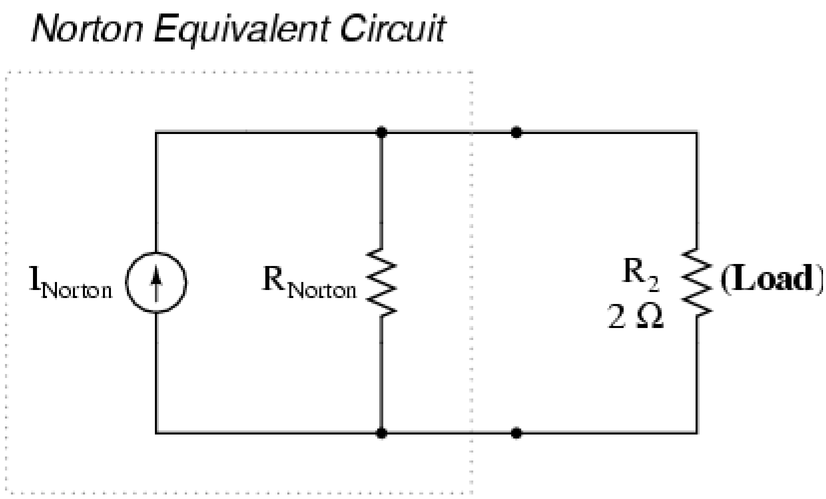
<u>Norton's Theorem</u> states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single current source and parallel resistance connected to a load. Just as with Thevenin's Theorem, the qualification of “linear” is identical to that found in the Superposition Theorem: all .(underlying equations must be linear (no exponents or roots

Contrasting our original example circuit against the Norton equivalent: it looks :something like this



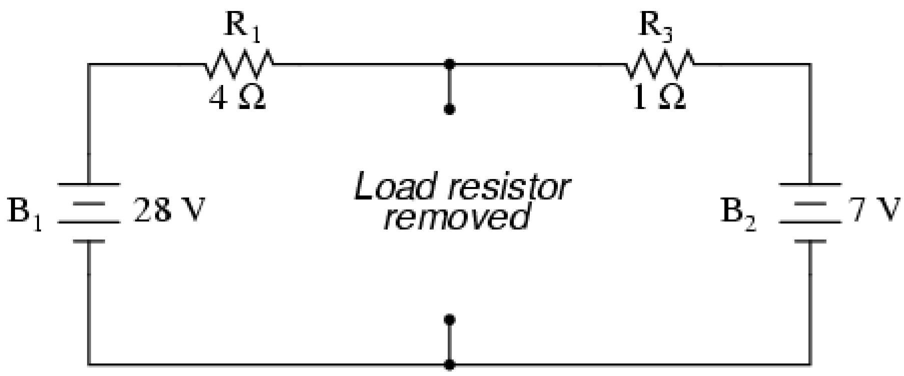
. . . after Norton conversion . . .



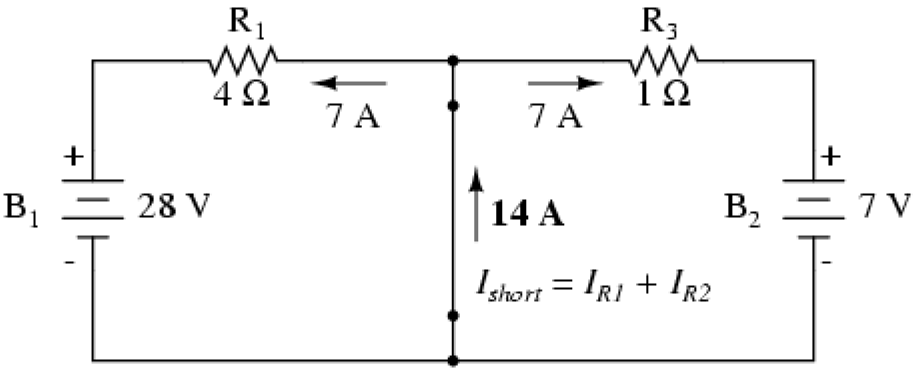
Remember that a *current source* is a component whose job is to provide a constant amount of current, outputting as much or as little voltage necessary to maintain that .constant current

As with Thevenin's Theorem, everything in the original circuit except the load resistance has been reduced to an equivalent circuit that is simpler to analyze. Also similar to Thevenin's Theorem are the steps used in Norton's Theorem to calculate the .(Norton source current (I_{Norton}) and Norton resistance (R_{Norton}

As before, the first step is to identify the load resistance and remove it from the :original circuit

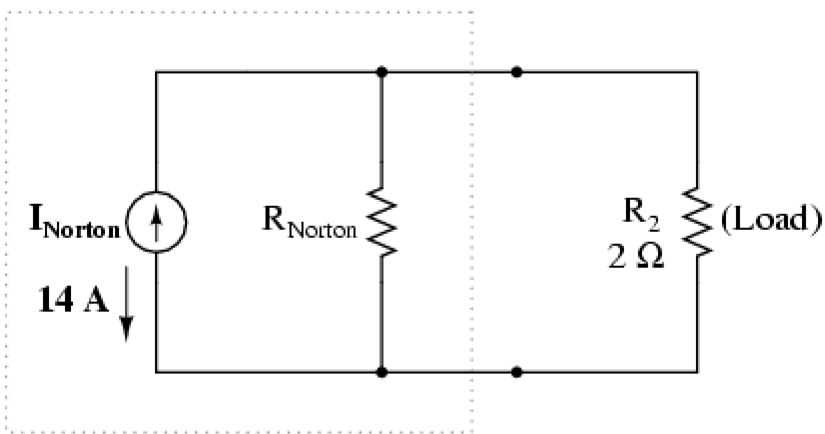


Then, to find the Norton current (for the current source in the Norton equivalent circuit), place a direct wire (short) connection between the load points and determine the resultant current. Note that this step is exactly opposite the respective step in :(Thevenin's Theorem, where we replaced the load resistor with a break (open circuit



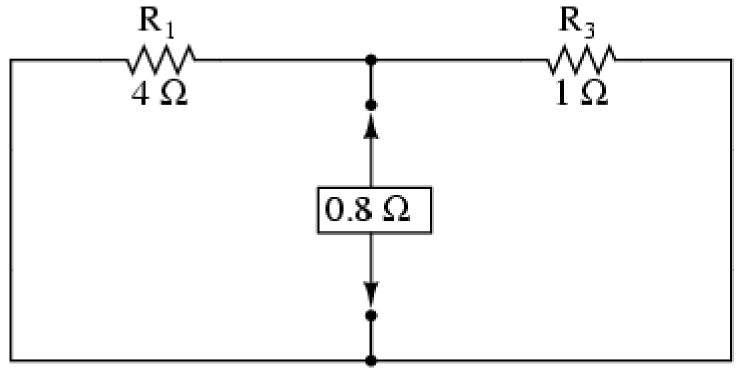
With zero voltage dropped between the load resistor connection points, the current through R_1 is strictly a function of B_1's voltage and R_1's resistance: 7 amps (I=E/R). Likewise, the current through R_3 is now strictly a function of B_2's voltage and R_3's resistance: 7 amps (I=E/R). The total current through the short between the load connection points is the sum of these two currents: 7 amps + 7 amps = 14 amps. This :figure of 14 amps becomes the Norton source current (I_{Norton}) in our equivalent circuit

Norton Equivalent Circuit

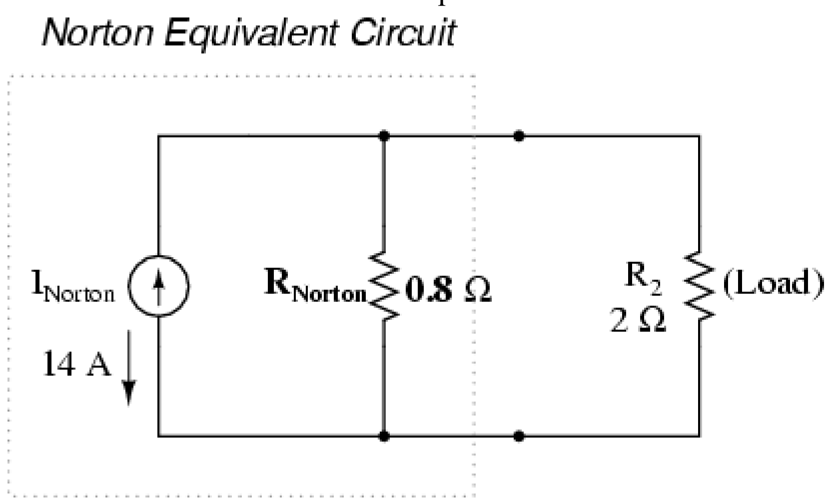


Remember, the arrow notation for a current source points in the direction *opposite* that of electron flow. Again, apologies for the confusion. For better or for worse, this !is standard electronic symbol notation. Blame Mr. Franklin again

To calculate the Norton resistance (R_{Norton}), we do the exact same thing as we did for calculating Thevenin resistance ($R_{Thevenin}$): take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure total resistance from one load connection point to :the other



:Now our Norton equivalent circuit looks like this



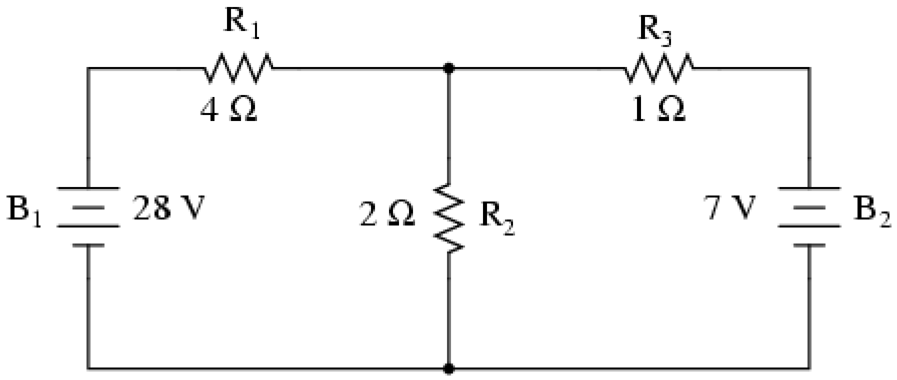
If we re-connect our original load resistance of 2 Ω, we can analyze the Norton circuit :as a simple parallel arrangement

	R_{Norton}	R_{Load}	Total	
E	8	8	8	Volts
I	10	4	14	Amps
R	0.8	2	571.43m	Ohms

Thevenin equivalent circuit, the only useful information from this analysis is the voltage and current values for R_2; the rest of the information is irrelevant to the original circuit. However, the same advantages seen with Thevenin's Theorem apply to Norton's as well: if we wish to analyze load resistor voltage and current over several different values of load resistance, we can use the Norton equivalent circuit again and again, applying nothing more complex than simple parallel circuit analysis .to determine what's happening with each trial

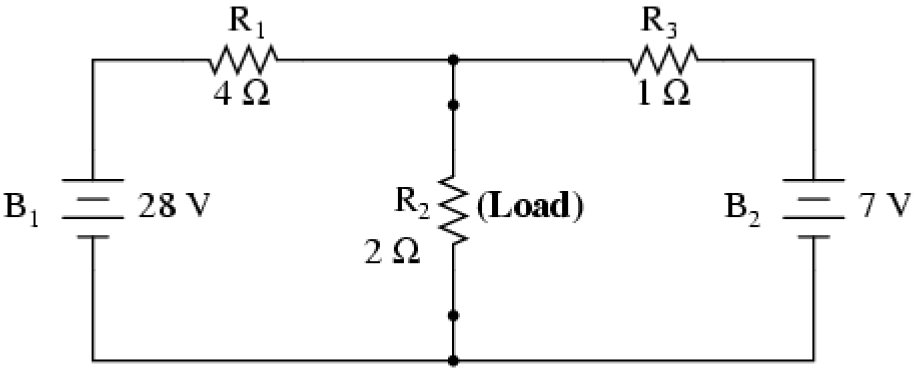
Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load. The qualification of “linear” is identical to that found in the Superposition Theorem, where all the underlying equations must be linear (no exponents or roots). If we're dealing with passive components (such as resistors, and later, inductors and capacitors), this is true. However, there are some components (especially certain gas-discharge and semiconductor components) which are nonlinear: that is, their opposition to current *changes* with voltage and/or current. As .such, we would call circuits containing these types of components, *nonlinear circuits*

Thevenin's Theorem is especially useful in analyzing power systems and other circuits where one particular resistor in the circuit (called the “load” resistor) is subject to change, and re-calculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it. Let's take another :look at our example circuit

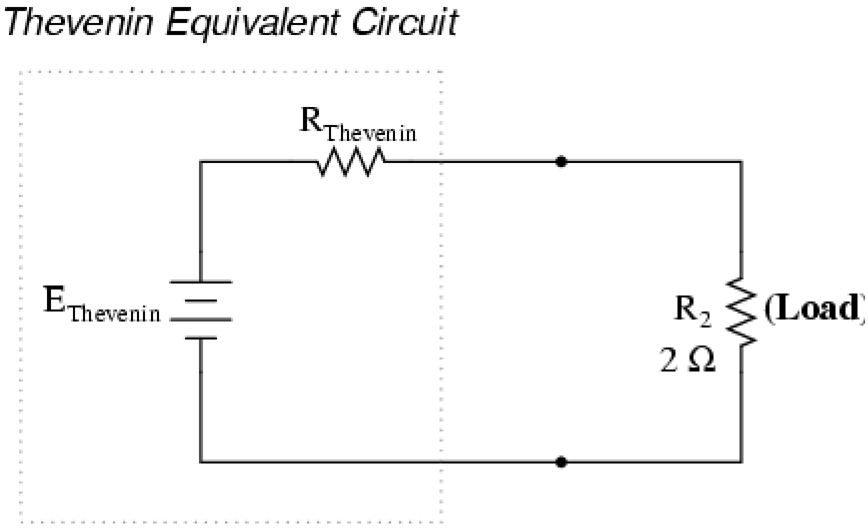


Let's suppose that we decide to designate R_2 as the “load” resistor in this circuit. We already have four methods of analysis at our disposal (Branch Current, Mesh Current, Millman's Theorem, and Superposition Theorem) to use in determining voltage across R_2 and current through R_2, but each of these methods are time-consuming. Imagine repeating any of these methods over and over again to find what would happen if the load resistance changed (changing load resistance is *very* common in power systems, as multiple loads get switched on and off as needed. the total resistance of their parallel connections changing depending on how many are connected at a time). This !could potentially involve a *lot* of work

Thevenin's Theorem makes this easy by temporarily removing the load resistance from the original circuit and reducing what's left to an equivalent circuit composed of a single voltage source and series resistance. The load resistance can then be re-connected to this “Thevenin equivalent circuit” and calculations carried out as if the :whole network were nothing but a simple series circuit



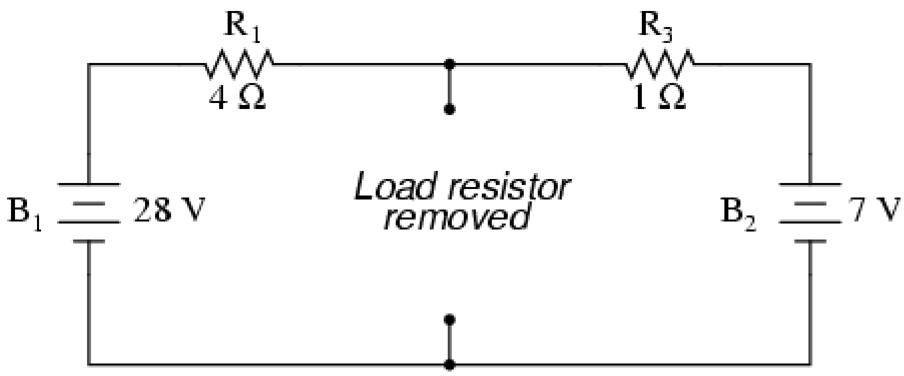
. . . after Thevenin conversion . . .



The “Thevenin Equivalent Circuit” is the electrical equivalent of B_1, R_1, R_3, and B_2 as .seen from the two points where our load resistor (R_2) connects

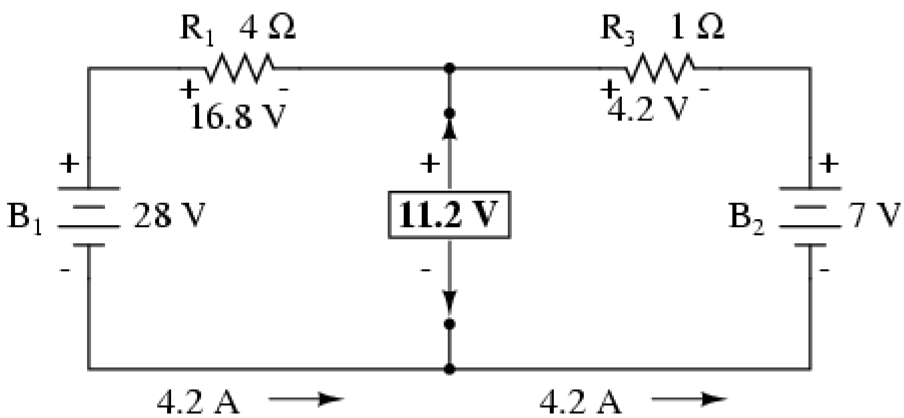
The Thevenin equivalent circuit, if correctly derived, will behave exactly the same as the original circuit formed by B_1, R_1, R_3, and B_2. In other words, the load resistor (R_2) voltage and current should be exactly the same for the same value of load resistance in the two circuits. The load resistor R_2 cannot “tell the difference” between the original network of B_1, R_1, R_3, and B_2, and the Thevenin equivalent circuit of $E_{Thevenin}$, and $R_{Thevenin}$, provided that the values for $E_{Thevenin}$ and $R_{Thevenin}$ have been calculated .correctly

The advantage in performing the “Thevenin conversion” to the simpler circuit, of course, is that it makes load voltage and load current so much easier to solve than in the original network. Calculating the equivalent Thevenin source voltage and series resistance is actually quite easy. First, the chosen load resistor is removed from the :(original circuit, replaced with a break (open circuit



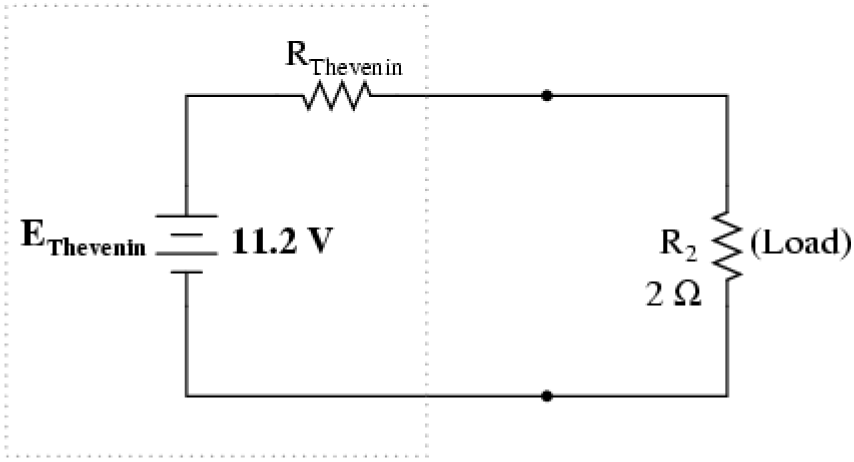
Next, the voltage between the two points where the load resistor used to be attached is determined. Use whatever analysis methods are at your disposal to do this. In this case, the original circuit with the load resistor removed is nothing more than a simple series circuit with opposing batteries, and so we can determine the voltage across the open load terminals by applying the rules of series circuits, Ohm's Law, and :Kirchhoff's Voltage Law

	R_1	R_3	Total	
E	16.8	4.2	21	Volts
I	4.2	4.2	4.2	Amps
R	4	1	5	Ohms

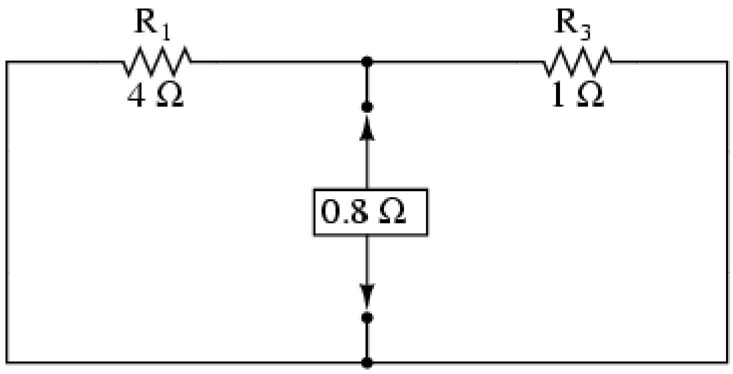


The voltage between the two load connection points can be figured from the one of the battery's voltage and one of the resistor's voltage drops, and comes out to 11.2 :volts. This is our “Thevenin voltage” ($E_{Thevenin}$) in the equivalent circuit

Thevenin Equivalent Circuit

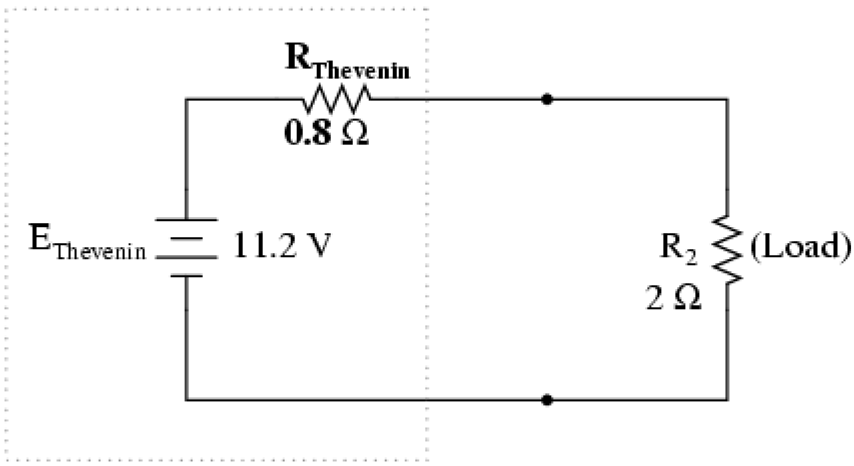


To find the Thevenin series resistance for our equivalent circuit, we need to take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure the resistance from one :load terminal to the other



With the removal of the two batteries, the total resistance measured at this location is equal to R_1 and R_3 in parallel: 0.8 Ω. This is our "Thevenin resistance" ($R_{Thevenin}$) for :the equivalent circuit

Thevenin Equivalent Circuit



With the load resistor (2 Ω) attached between the connection points, we can determine voltage across it and current through it as though the whole network were nothing :more than a simple series circuit

	$R_{Thevenin}$	R_{Load}	Total	
E	3.2	8	11.2	Volts
I	4	4	4	Amps
R	0.8	2	2.8	Ohms

Notice that the voltage and current figures for R_2 (8 volts, 4 amps) are identical to those found using other methods of analysis. Also notice that the voltage and current figures for the Thevenin series resistance and the Thevenin source (*total*) do not apply to any component in the original, complex circuit. Thevenin's Theorem is only useful .for determining what happens to a *single* resistor in a network: the load

The advantage, of course, is that you can quickly determine what would happen to that single resistor if it were of a value other than 2 Ω without having to go through a lot of analysis again. Just plug in that other value for the load resistor into the Thevenin equivalent circuit and a little bit of series circuit calculation will give you .the result

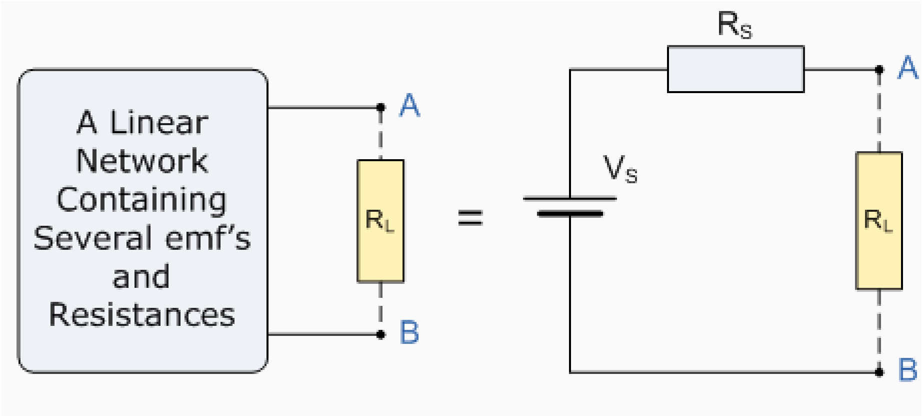
Tutorial: 7 of 10

Thevenins Theorem

In the previous 3 tutorials we have looked at solving complex electrical circuits using *Kirchoff's Circuit Laws*, *Mesh Analysis* and finally *Nodal Analysis* but there are many more "Circuit Analysis Theorems" available to calculate the currents and voltages at any point in a circuit. In this tutorial we will look at one of the more common circuit .analysis theorems (next to Kirchoff´s) that has been developed, **Thevenins Theorem**

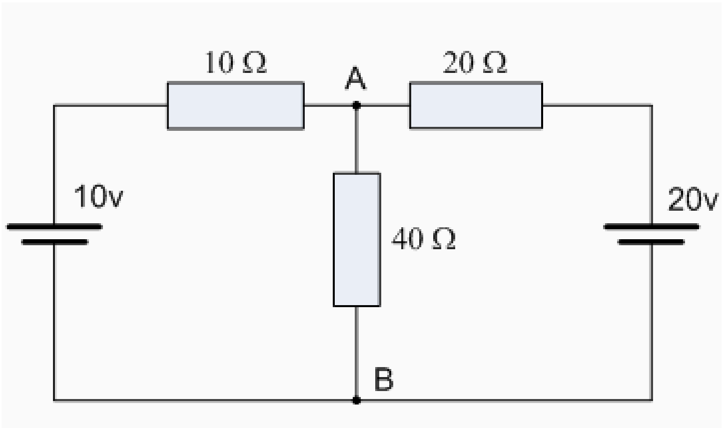
Thevenins Theorem states that "*Any linear circuit containing several voltages and resistances can be replaced by just a Single Voltage in series with a Single Resistor*". In other words, it is possible to simplify any "Linear" circuit, no matter how complex, to an equivalent circuit with just a single voltage source in series with a resistance connected to a load as shown below. **Thevenins Theorem** is especially useful in analyzing power or battery systems and other interconnected circuits where it will .have an effect on the adjoining part of the circuit

.Thevenins equivalent circuit

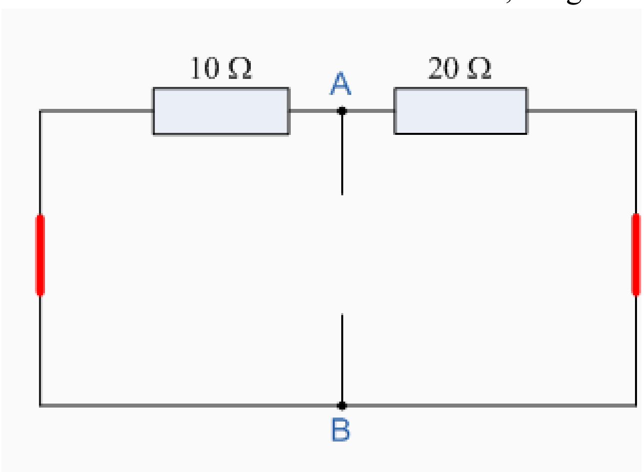


As far as the load resistor R_L is concerned, any "one-port" network consisting of resistive circuit elements and energy sources can be replaced by one single equivalent resistance Rs and equivalent voltage Vs, where Rs is the source resistance value .looking back into the circuit and Vs is the open circuit voltage at the terminals

.For example, consider the circuit from the previous section



Firstly, we have to remove the centre 40Ω resistor and short out (not physically as this would be dangerous) all the emf´s connected to the circuit, or open circuit any current sources. The value of resistor Rs is found by calculating the total resistance at the terminals A and B with all the emf´s removed, and the value of the voltage required Vs is the total voltage across terminals A and B with an open circuit and no load .resistor Rs connected. Then, we get the following circuit

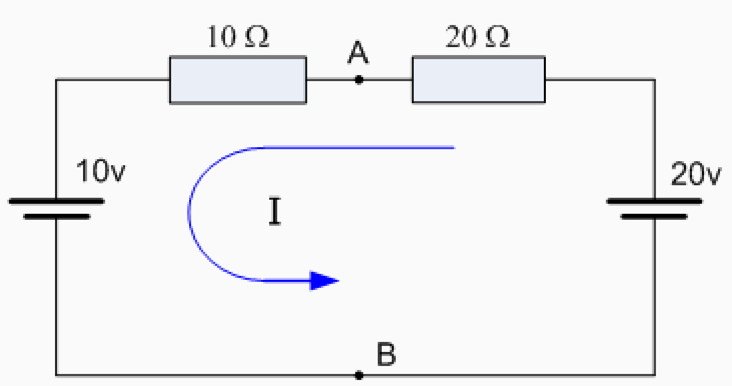


(Find the Equivalent Resistance (Rs

0Ω Resistor in Parallel with the 20Ω Resistor

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\Omega$$

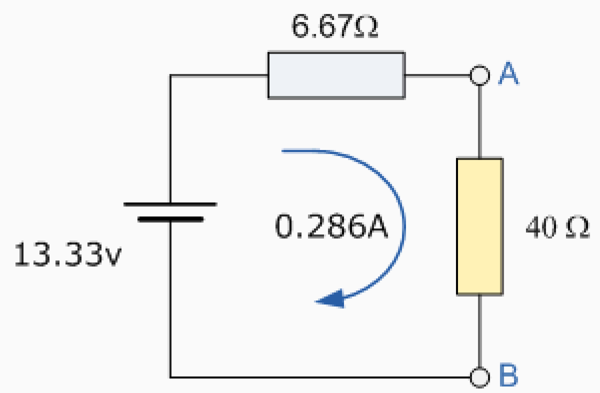
(Find the Equivalent Voltage (Vs



We now need to reconnect the two voltages back into the circuit, and as $V_S = V_{AB}$
:the current flowing around the loop is calculated as

$$I = \frac{20v - 10v}{20\Omega + 10\Omega} = 0.33\,amps$$

:so the voltage drop across the 20Ω resistor can be calculated as
.$V_{AB} = 20 - (20\Omega \times 0.33\text{amps}) = 13.33$ volts
Then the Thevenins Equivalent circuit is shown below with the 40Ω resistor
.connected



:and from this the current flowing in the circuit is given as

$$I = \frac{13.33v}{6.67\Omega + 40\Omega} = 0.286 amps$$

which again, is the same value of 0.286 amps, we found using *Kirchoff's* circuit law .in the previous tutorial

Thevenins theorem can be used as a circuit analysis method and is particularly useful if the load is to take a series of different values. It is not as powerful as *Mesh* or *Nodal* analysis in larger networks because the use of Mesh or Nodal analysis is usually necessary in any Thevenin exercise, so it might as well be used from the start. However, Thevenins equivalent circuits of **Transistors**, **Voltage Sources** such as .batteries etc, are very useful in circuit design

Thevenins Theorem Summary

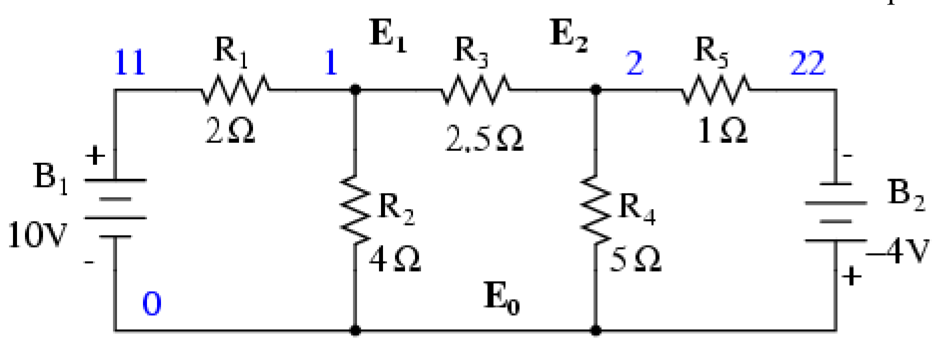
:The basic procedure for solving a circuit using **Thevenins Theorem** is as follows

- .Remove the load resistor R_L or component concerned **.1**
- Find R_S by shorting all voltage sources or by open circuiting all the current **.2** .sources
- .Find V_S by the usual circuit analysis methods **.3**
- .Find the current flowing through the load resistor R_L **.4**

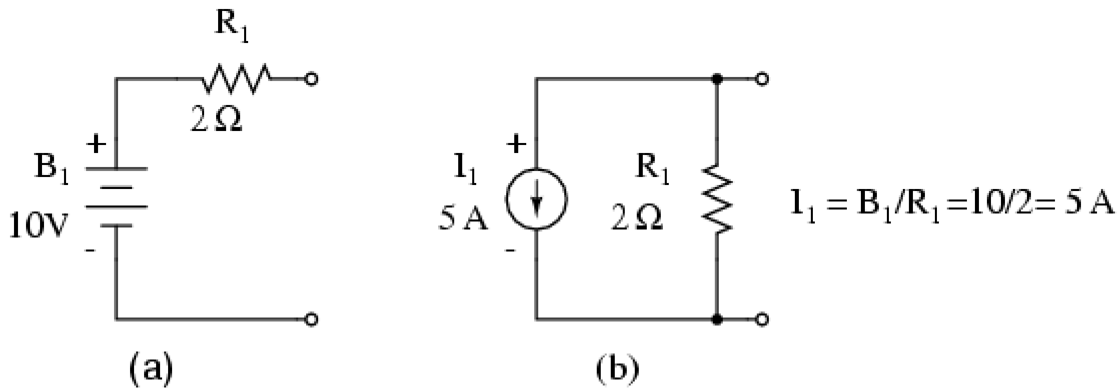
In the next tutorial we will look at *Nortons Theorem* which allows a network consisting of linear resistors and sources to be represented by an equivalent circuit .with a single current source in parallel with a single source resistance

The node voltage method of analysis solves for unknown voltages at circuit nodes in terms of a system of KCL equations. This analysis looks strange because it involves replacing voltage sources with equivalent current sources. Also, resistor values in ohms are replaced by equivalent conductances in siemens, G = 1/R. The siemens (S) is the unit of conductance, having replaced the mho unit. In any event S = .(Ω^{-1}. And S = mho (obsolete

We start with a circuit having conventional voltage sources. A common node E_0 is chosen as a reference point. The node voltages E_1 and E_2 are calculated with respect to .this point

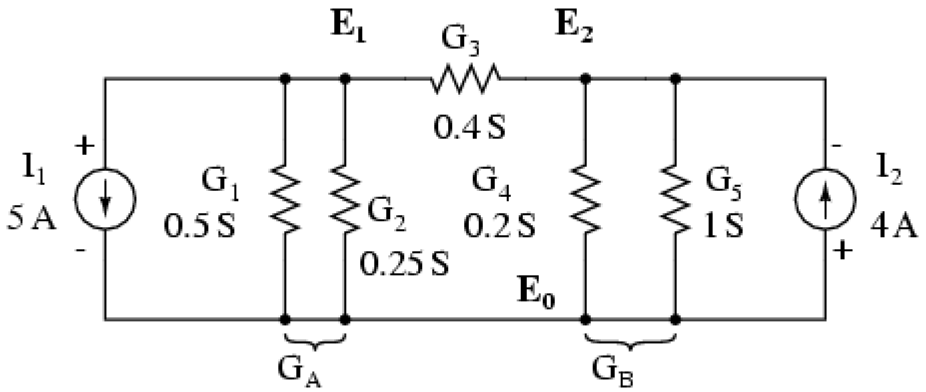


A voltage source in series with a resistance must be replaced by an equivalent current source in parallel with the resistance. We will write KCL equations for each node. The .right hand side of the equation is the value of the current source feeding the node



Replacing voltage sources and associated series resistors with equivalent current sources and parallel resistors yields the modified circuit. Substitute resistor .conductances in siemens for resistance in ohms

$$I_1 = E_1/R_1 = 10/2 = 5\ A$$
$$I_2 = E_2/R_5 = 4/1 = 4\ A$$
$$G_1 = 1/R_1 = 1/2\ \Omega = 0.5\ S$$
$$G_2 = 1/R_2 = 1/4\ \Omega = 0.25\ S$$
$$G_3 = 1/R_3 = 1/2.5\ \Omega = 0.4\ S$$
$$G_4 = 1/R_4 = 1/5\ \Omega = 0.2\ S$$
$$G_5 = 1/R_5 = 1/1\ \Omega = 1.0\ S$$



The Parallel conductances (resistors) may be combined by addition of the conductances. Though, we will not redraw the circuit. The circuit is ready for .application of the node voltage method

$$G_A = G_1 + G_2 = 0.5\ S + 0.25\ S = 0.75\ S$$
$$G_B = G_4 + G_5 = 0.2\ S + 1\ S = 1.2\ S$$

Deriving a general node voltage method, we write a pair of KCL equations in terms of unknown node voltages V_1 and V_2 this one time. We do this to illustrate a pattern for .writing equations by inspection

$$(G_AE_1 + G_3(E_1 - E_2) = I_1 \qquad (1$$
$$(G_BE_2 - G_3(E_1 - E_2) = I_2 \qquad (2$$

$$(G_A + G_3\ )E_1 \qquad -G_3E_2 = I_1 \qquad (1)$$
$$(G_3E_1 + (G_B + G_3)E_2 = I_2 \qquad (2-$$

The coefficients of the last pair of equations above have been rearranged to show a pattern. The sum of conductances connected to the first node is the positive coefficient of the first voltage in equation (1). The sum of conductances connected to the second node is the positive coefficient of the second voltage in equation (2). The other coefficients are negative, representing conductances between nodes. For both equations, the right hand side is equal to the respective current source connected to the node. This pattern allows us to quickly write the equations by inspection. This .leads to a set of rules for the node voltage method of analysis

- **:Node voltage rules**
- Convert voltage sources in series with a resistor to an equivalent current .source with the resistor in parallel
- .Change resistor values to conductances
- (Select a reference node(E_0
- .Assign unknown voltages (E_1)(E_2) ... (E_N)to remaining nodes
- Write a KCL equation for each node 1,2, ... N. The positive coefficient of the first voltage in the first equation is the sum of conductances connected to the node. The coefficient for the second voltage in the second equation is the sum of conductances connected to that node. Repeat for coefficient of third voltage, .third equation, and other equations. These coefficients fall on a diagonal
- All other coefficients for all equations are negative, representing conductances between nodes. The first equation, second coefficient is the conductance from node 1 to node 2, the third coefficient is the conductance from node 1 to node .3. Fill in negative coefficients for other equations
- The right hand side of the equations is the current source connected to the .respective nodes
- .Solve system of equations for unknown node voltages

Example: Set up the equations and solve for the node voltages using the numerical .values in the above figure

:Solution

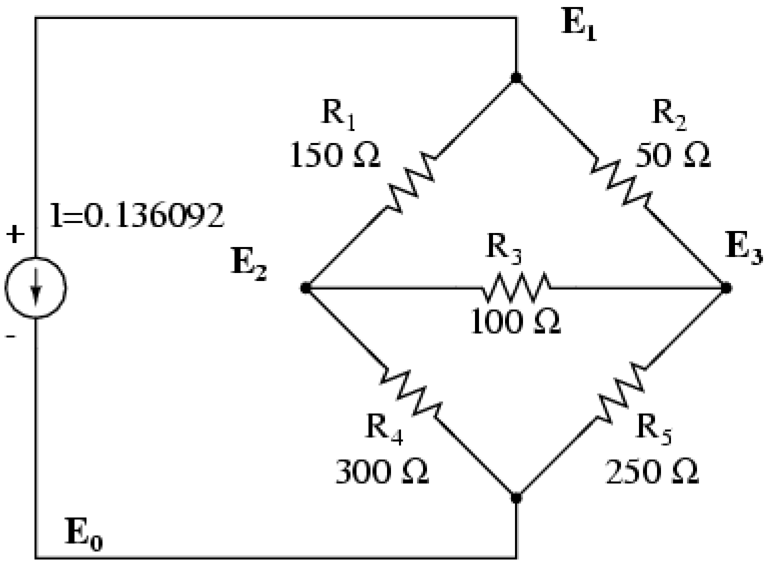
$$E_1 \;-(0.4)E_2= \;\;5(0.5+0.25+0.4)$$
$$E_1 \;+(0.4+0.2+1.0)E_2 \;=\; -4(0.4)-$$
$$E_1 \;-(0.4)E_2= \;\;5(1.15)$$
$$E_1 \;+(1.6)E_2 \;\;=\; -4(0.4)-$$
$$E_1 \;=\;\; 3.8095$$
$$E_2 \;=\; -1.5476$$

The solution of two equations can be performed with a calculator, or with octave (not shown).[octav] The solution is verified with SPICE based on the original schematic diagram with voltage sources. [spi] Though, the circuit with the current sources could .have been simulated

```
V1 11 0 DC 10
V2 22 0 DC -4
r1 11 1 2
r2 1 0 4
r3 1 2 2.5
r4 2 0 5
r5 2 22 1
DC V1 10 10 1 V2 -4 -4 1.
(print DC V(1) V(2.
end.

(v(1)           v(2
3.809524e+00    -1.547619e+00
```

One more example. This one has three nodes. We do not list the conductances on the .schematic diagram. However, $G_1 = 1/R_1$, etc



There are three nodes to write equations for by inspection. Note that the coefficients are positive for equation (1) E_1, equation (2) E_2, and equation (3) E_3. These are the sums of all conductances connected to the nodes. All other coefficients are negative, representing a conductance between nodes. The right hand side of the equations is the associated current source, 0.136092 A for the only current source at node 1. The other equations are zero on the right hand side for lack of current sources. We are too lazy to calculate the conductances for the resistors on the diagram. Thus, the subscripted .G's are the coefficients

G_1 + G_2)E_1 $-G_1E_2$ $-G_2E_3$ =) 0.136092

G_1E_1 +(G_1 + G_3 + G_4)E_2 $-G_3E_3$ = 0-

G_2E_1 $-G_3E_2$ +(G_2 + G_3 + G_5)E_3 = 0-

We are so lazy that we enter reciprocal resistances and sums of reciprocal resistances into the octave "A" matrix, letting octave compute the matrix of conductances after "A=".[octav] The initial entry line was so long that it was split into three rows. This is different than previous examples. The entered "A" matrix is delineated by starting and ending square brackets. Column elements are space separated. Rows are "new line" separated. Commas and semicolons are not need as separators. Though, the current .vector at "b" is semicolon separated to yield a column vector of currents

```
octave:12> A = [1/150+1/50 -1/150 -1/50
1/100- 1/150+1/100+1/300 1/150- <
[1/50+1/100+1/250 1/100- 1/50- <
= A
0.0200000-   0.0066667-   0.0266667
0.0100000-   0.0200000    0.0066667-
0.0340000    0.0100000-   0.0200000-

[octave:13> b = [0.136092;0;0
= b
0.13609
0.00000
0.00000

octave:14> x=A\b
= x
24.000
17.655
```

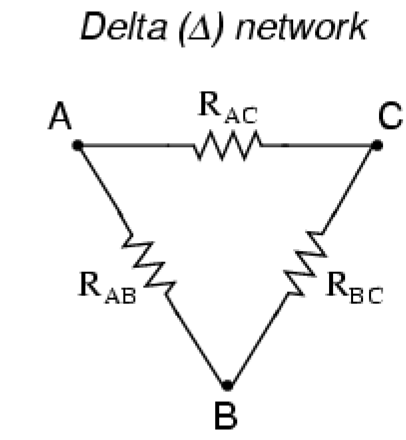
19.310

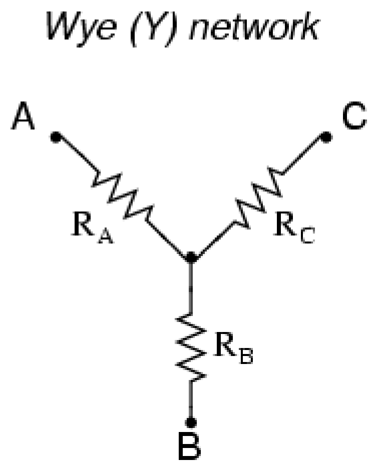
Note that the “A” matrix diagonal coefficients are positive, That all other coefficients .are negative

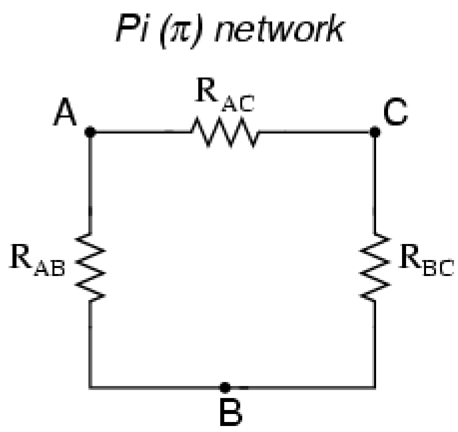
The solution as a voltage vector is at “x”. E_1 = 24.000 V, E_2 = 17.655 V, E_3 = 19.310 V. These three voltages compare to the previous mesh current and SPICE solutions to the unbalanced bridge problem. This is no coincidence, for the 0.13609 A current source was purposely chosen to yield the 24 V used as a voltage source in that .problem

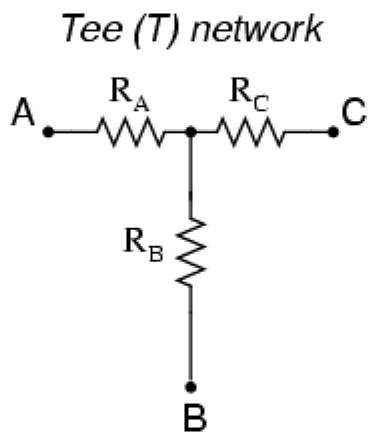
- Summary
- Given a network of conductances and current sources, the node voltage method of circuit analysis solves for unknown node voltages from KCL .equations
- .See rules above for details in writing the equations by inspection
- The unit of conductance G is the siemens S. Conductance is the reciprocal of resistance: G = 1/R

In many circuit applications, we encounter components connected together in one of two ways to form a three-terminal network: the “Delta,” or Δ (also known as the “Pi,” .or π) configuration, and the “Y” (also known as the “T”) configuration









It is possible to calculate the proper values of resistors necessary to form one kind of network (Δ or Y) that behaves identically to the other kind, as analyzed from the terminal connections alone. That is, if we had two separate resistor networks, one Δ and one Y, each with its resistors hidden from view, with nothing but the three terminals (A, B, and C) exposed for testing, the resistors could be sized for the two networks so that there would be no way to electrically determine one network apart .from the other. In other words, equivalent Δ and Y networks behave identically

:There are several equations used to convert one network to the other

To convert a Delta (Δ) to a Wye (Y)

$$R_A = \frac{R_{AB}\,R_{AC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_B = \frac{R_{AB}\,R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_C = \frac{R_{AC}\,R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

To convert a Wye (Y) to a Delta (Δ)

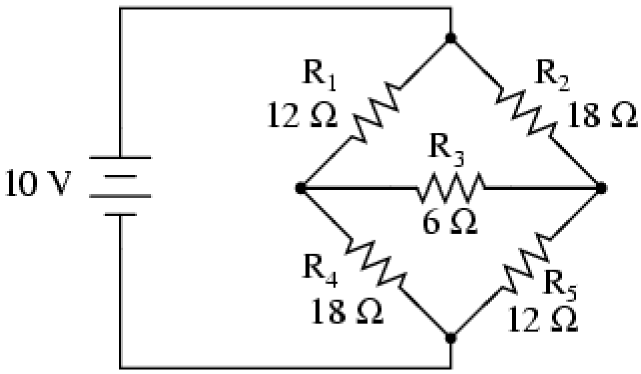
$$R_{AB} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_C}$$

$$R_{BC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_A}$$

$$R_{AC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_B}$$

Δ and Y networks are seen frequently in 3-phase AC power systems (a topic covered in volume II of this book series), but even then they're usually balanced networks (all resistors equal in value) and conversion from one to the other need not involve such complex calculations. When would the average technician ever need to use these ?equations

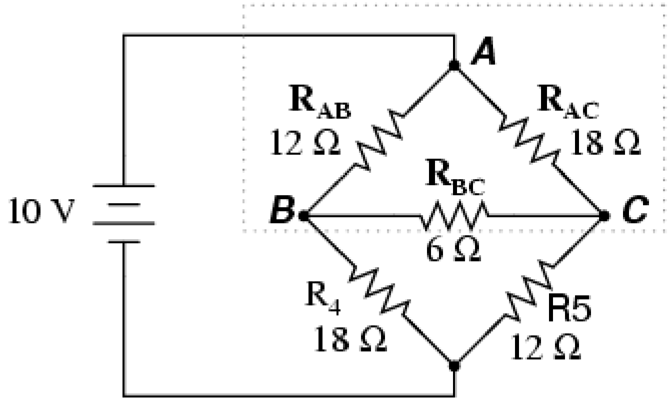
A prime application for Δ-Y conversion is in the solution of unbalanced bridge :circuits, such as the one below



Solution of this circuit with Branch Current or Mesh Current analysis is fairly involved, and neither the Millman nor Superposition Theorems are of any help, since there's only one source of power. We could use Thevenin's or Norton's Theorem, ?treating R_3 as our load, but what fun would that be

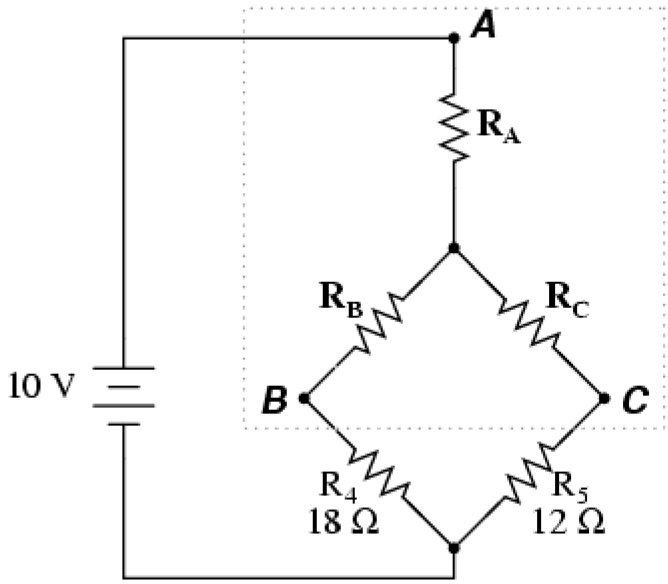
If we were to treat resistors R_1, R_2, and R_3 as being connected in a Δ configuration (R_{ab}, R_{ac}, and R_{bc}, respectively) and generate an equivalent Y network to replace them, :we could turn this bridge circuit into a (simpler) series/parallel combination circuit

Selecting Delta (Δ) network to convert:



. . . After the Δ-Y conversion

Δ converted to a Y

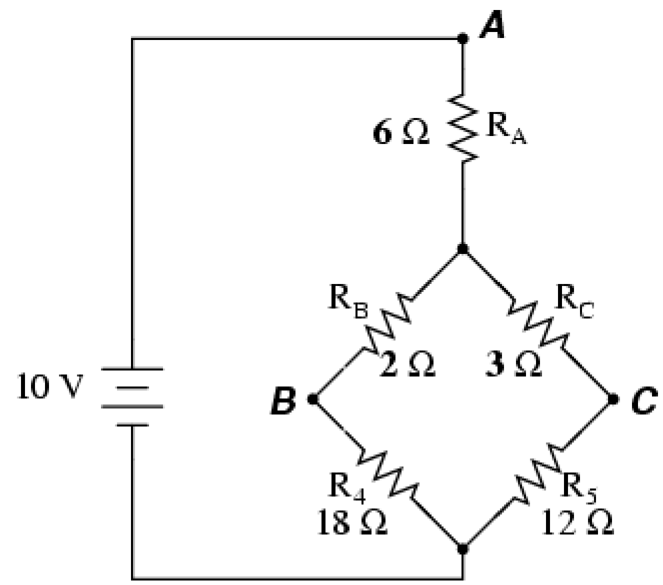


If we perform our calculations correctly, the voltages between points A, B, and C will be the same in the converted circuit as in the original circuit, and we can transfer .those values back to the original bridge configuration

$$R_A = \frac{(12\ \Omega)(18\ \Omega)}{(12\ \Omega) + (18\ \Omega) + (6\ \Omega)} = \frac{216}{36} = \mathbf{6\ \Omega}$$

$$R_B = \frac{(12\ \Omega)(6\ \Omega)}{(12\ \Omega) + (18\ \Omega) + (6\ \Omega)} = \frac{72}{36} = \mathbf{2\ \Omega}$$

$$R_C = \frac{(18\ \Omega)(6\ \Omega)}{(12\ \Omega) + (18\ \Omega) + (6\ \Omega)} = \frac{108}{36} = \mathbf{3\ \Omega}$$

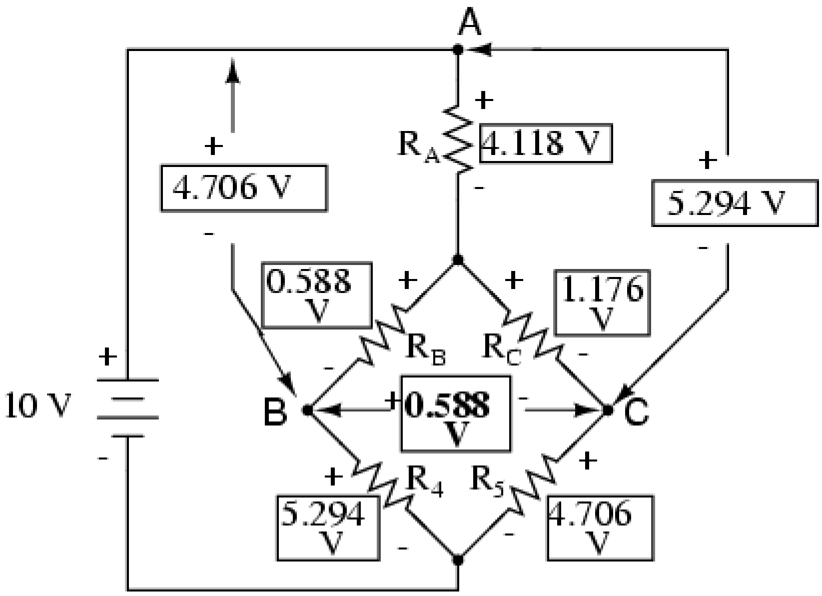


Resistors R_4 and R_5, of course, remain the same at 18 Ω and 12 Ω, respectively. Analyzing the circuit now as a series/parallel combination, we arrive at the following :figures

	R_A	R_B	R_C	R_4	R_5	
E	4.118	588.24m	1.176	5.294	4.706	Volts
I	686.27m	294.12m	392.16m	294.12m	392.16m	Amps
R	6	2	3	18	12	Ohms

	$R_B + R_4$	$R_C + R_5$	$R_B + R_4$ // $R_C + R_5$	Total	
E	5.882	5.882	5.882	10	Volts
I	294.12m	392.16m	686.27m	686.27m	Amps
R	20	15	8.571	14.571	Ohms

We must use the voltage drops figures from the table above to determine the voltages between points A, B, and C, seeing how the add up (or subtract, as is the case with :(voltage between points B and C

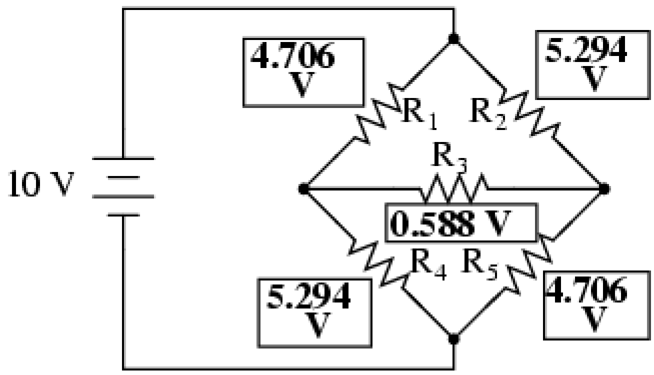


$$E_{A\text{-}B} = 4.706 \text{ V}$$

$$E_{A\text{-}C} = 5.294 \text{ V}$$

$$E_{B\text{-}C} = 588.24 \text{ mV}$$

Now that we know these voltages, we can transfer them to the same points A, B, and :C in the original bridge circuit



Voltage drops across R_4 and R_5, of course, are exactly the same as they were in the .converted circuit

At this point, we could take these voltages and determine resistor currents through the :(repeated use of Ohm's Law (I=E/R

$$I_{R1} = \frac{4.706 \text{ V}}{12\ \Omega} = 392.16 \text{ mA}$$

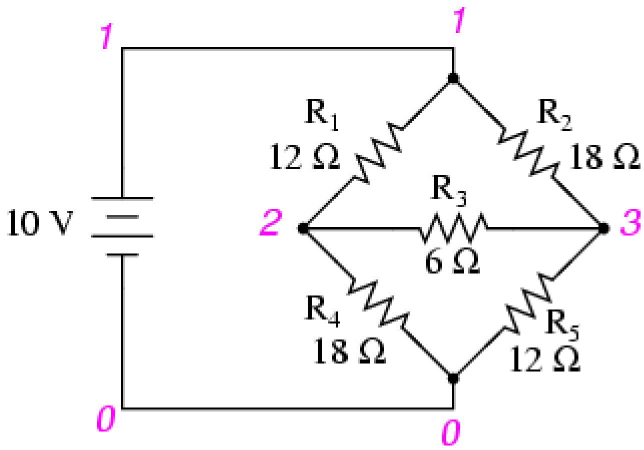
$$I_{R2} = \frac{5.294 \text{ V}}{18\ \Omega} = 294.12 \text{ mA}$$

$$I_{R3} = \frac{588.24 \text{ mV}}{6\ \Omega} = 98.04 \text{ mA}$$

$$I_{R4} = \frac{5.294 \text{ V}}{18\ \Omega} = 294.12 \text{ mA}$$

$$I_{R5} = \frac{4.706 \text{ V}}{12\ \Omega} = 392.16 \text{ mA}$$

[A quick simulation with SPICE will serve to verify our work:[spi




```
unbalanced bridge circuit
v1 1 0
r1 1 2 12
r2 1 3 18
r3 2 3 6
r4 2 0 18
r5 3 0 12
dc v1 10 10 1.
(print dc v(1,2) v(1,3) v(2,3) v(2,0) v(3,0.
end.
```

```
(v1            v(1,2)      v(1,3)      v(2,3)      v(2)        v(3
      1.000E+01     4.706E+00   5.294E+00   5.882E-01   5.294E+00
                                                        4.706E+00
```

The voltage figures, as read from left to right, represent voltage drops across the five respective resistors, R_1 through R_5. I could have shown currents as well, but since that would have required insertion of “dummy” voltage sources in the SPICE netlist, and since we're primarily interested in validating the Δ-Y conversion equations and not .Ohm's Law, this will suffice

Nodal Analysis of Electric Circuits

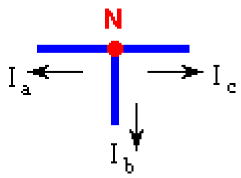
In this method, we set up and solve a system of equations in which the unknowns are the **voltages at the principal nodes of the circuit**. From these nodal voltages the .currents in the various branches of the circuit are easily determined

:The steps in the nodal analysis method are

- Count the number of principal nodes or junctions in the circuit. Call this number *n*. (A **principal node or junction** is a point where 3 or more branches join. We will indicate them in a circuit diagram with a red dot. Note that if a branch contains no voltage sources or loads then that entire branch (.can be considered to be one node
- Number the nodes N_1, N_2, . . . , N_n and draw them on the circuit diagram. Call .the voltages at these nodes V_1, V_2, . . . , V_n, respectively
- Choose one of the nodes to be the reference node or ground and assign it a .voltage of zero
- For each node except the reference node write down Kirchoff's Current Law in the form *"the **algebraic sum** of the currents flowing out of a node equals zero"*. (By algebraic sum we mean that a current flowing into a node is to be (.considered a negative current flowing out of the node

For example, for the node to the right KCL yields the :equation

$I_a + I_b + I_c = 0$



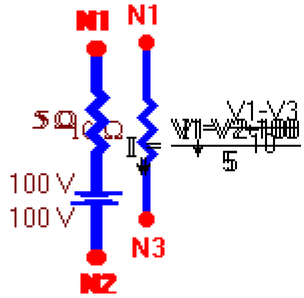
Express the current in each branch in terms of the nodal voltages at each end of the branch using Ohm's Law ($I = V / R$). Here are :some examples

The current downward out of node 1 depends on the voltage difference V1 - .V3 and the resistance in the branch

In this case the voltage difference across the resistance is V1 - V2 **minus the voltage across the voltage source**. Thus the downward current is as .shown

In this case the voltage difference across the resistance must be 100 volts .greater than the difference V1 - V2. Thus the downward current is as shown

The result, after simplification, is a system of *m* linear equations in the *m* unknown nodal voltages (where *m* is one less than the number of nodes; *m* = :*n* - 1). The equations are of this form



$$\begin{cases} G_{11}\cdot V_1 + G_{12}\cdot V_2 + \cdots + G_{1m}\cdot V_m = I_1 \\ G_{21}\cdot V_1 + G_{22}\cdot V_2 + \cdots + G_{2m}\cdot V_m = I_2 \\ \vdots \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ G_{m1}\cdot V_1 + G_{m2}\cdot V_2 + \cdots + G_{mm}\cdot V_m = I_m \end{cases}$$

.where G_{11}, G_{12}, . . . , G_{mm} and I_1, I_2, . . . , I_m are constants

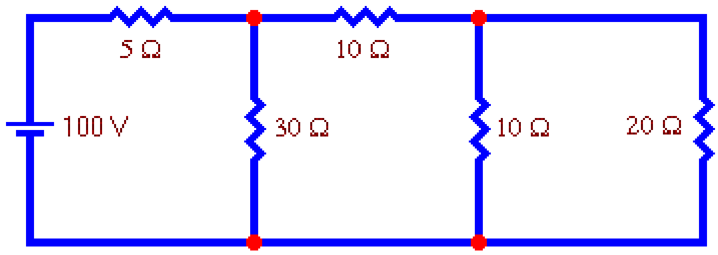
Alternatively, the system of equations can be gotten (already in simplified .form) by using [the inspection method]

- Solve the system of equations for the *m* node voltages V_1, V_2, . . . , V_m using .[Gaussian elimination] or some other method

.**Example 1:** Use nodal analysis to find the voltage at each node of this circuit

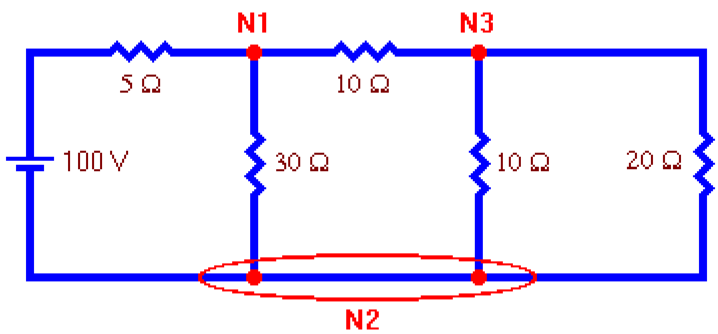
:**Solution**

- Note that the "pair of nodes" at the bottom is actually 1 extended node. Thus the number .of nodes is 3



- We will number the nodes as shown to .the right

- We will choose node 2 as the reference node and assign it a .voltage of zero



- Write down Kirchoff's Current Law for each node. Call V_1 the voltage at node 1, V_3 the voltage at node 3, and remember that $V_2 = 0$. The result is the following system of equations:

$$\frac{V1}{30} + \frac{V1-100}{5} + \frac{V1-V3}{10} = 0$$

$$\frac{V3-V1}{10} + \frac{V3}{10} + \frac{V3}{20} = 0$$

The first equation results from KCL applied at node 1 and the second equation results from KCL applied at node 3. Collecting terms this becomes:

$$\left(\frac{1}{30} + \frac{1}{5} + \frac{1}{10}\right)V1 - \left(\frac{1}{10}\right)V3 = \frac{100}{5}$$

$$-\left(\frac{1}{10}\right)V1 + \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{20}\right)V3 = 0$$

This form for the system of equations could have been gotten immediately by using the inspection method.

- Solving the system of equations using Gaussian elimination or some other method gives the following voltages:
  V_1=68.2 volts and V_3=27.3 volts