CSM 153 Circuit Theory

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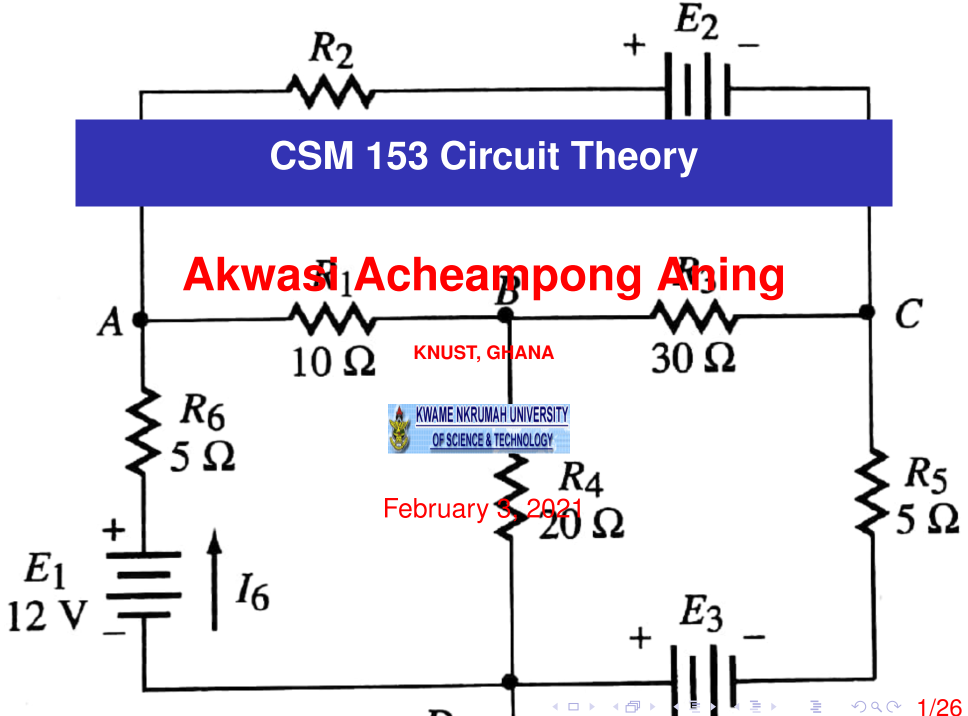


Outline I

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

1. **Unit Two**
   - Ohm's and Kirchoff's laws
   - Series and Parallel Circuits
   - Methods of Analysis

UNIT TWO

Direct Circuit Analysis

- Ohm's and Kirchoff's laws
- Series and Parallel Circuits
- Methods of Analysis

Ohm's Law

- Electric current flowing through a metallic conductor or wire is directly proportional to the potential difference applied, provided temperature and other physical factors remain constant. (i.e. $V \propto I$). Thus mathematical statement of the law is written as:

$$V = IR \tag{1}$$

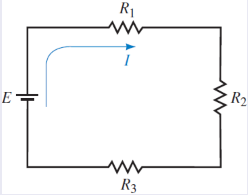
  where R is a constant defining resistance of the wire
- A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.
- Similarly, a conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

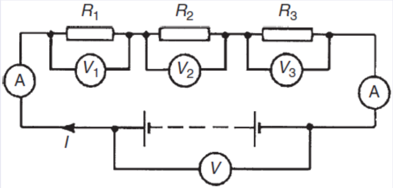
Ohm's Law

- Therefore, all homogeneous materials (conductors, pure semiconductors or impure semiconductors) obey Ohm's law within some range of values of the electric field.
- However, if the electric field is too strong there are departures from Ohm's law in all cases.
- A conductor whose function in a circuit is to provide a specific resistance is called a resistor.
- Thus, resistor is a conductor with a specified resistance, which remains the same no matter what the magnitude and direction (polarity) of the applied potential difference may be.
- This means the resistance R of the device is independent of the magnitude and polarity of the potential difference

Circuits: Series Circuit

- Two elements are said to be in series if they are connected at a single point and if there are no other current-carrying connections at this point
- The current I is the same in all parts of the circuit and hence the same reading is found on each of the two ammeters in the circuit
- The sum of the voltages V_1, V_2 and V_3 is equal to the total applied voltage V





Circuits: Series Circuit

- From Ohm's law, $V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$ and $V = IR$ where R is the total resistance

$$V = V_1 + V_2 + V_3 \tag{2}$$

then $IR = IR_1 + IR_2 + IR_3$
dividing through by I gives:

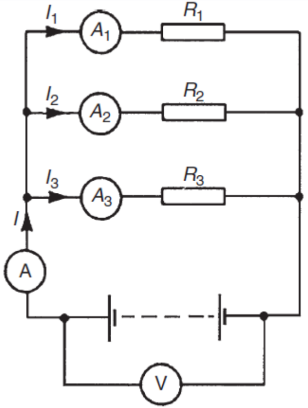
$$R = R_1 + R_2 + R_3 \tag{3}$$

- Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistances

Circuits: Parallel Circuit

- Elements or branches are said to be in a parallel connection when they have exactly two nodes in common. Additionally, these parallel elements or branches will have the same voltage across them.
- The sum of the currents I_1, I_2 and I_3 is equal to the current I
- The source potential difference (pd) is the same across each of the resistors

The Structure of the Atom



method

Circuits: Parallel Circuit

- From Ohm's law, $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, $I_3 = \frac{V}{R_3}$ and $I = \frac{V}{R}$ where R is the total resistance

$$I = I_1 + I_2 + I_3 \tag{4}$$

then $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

dividing through by V gives:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{5}$$

- This the total resistance is for a parallel circuit
For a special case of 2 resistors in parallel,

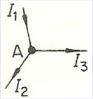
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 R_1}{R_1 + R_2} \tag{6}$$

Circuits: Kirchoff's Laws

- **Node**: A point at which two or more elements have a common connection
- **Branch**: A single path in a network composed of one simple element and the node at each end of that element
- **Loop**: A simple closed path in a circuit in which no circuit element or node is encountered more than once
- Electrical network is usually regarded to be a complicated (complex) system of electrical conductors.
- In dealing with such networks the Ohm's law was extended to the networks by a German physicist Gustav R. Kirchhoff (1847) in the form of two laws.
  However, it must be emphasized that:
- The laws enabled the current in any part of an electrical network to be calculated.
- All circuits can be solved by Kirchhoff's laws because they do not depend on series or parallel connection of resistors/conductors.

Circuits: Kirchoff's Laws

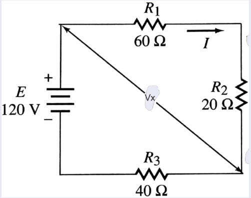
- The total current flowing into a junction in a circuit (electrical network) is equal to the total current flowing out of (leaving) the junction
- The algebraic sum of currents directed in and out at a junction of a circuit must be zero i.e. $I_1 + I_2 + I_3$
- The first law applies to any point or junction in an electrical network. If currents flowing in and out of the junction flows for a time t seconds, then we have $I_1 t = I_2 t + I_3 t \Rightarrow Q_1 = Q_2 + Q_3$
$$\Sigma Q_{in} = \Sigma Q_{out} \tag{7}$$
This is Kirchoff's 1st Law
- This means total charge flowing to the junction is equal to total charge flowing out of it.
- This implies there is neither a build up (accumulation or pile up) nor a depletion of charge at a junction.



- Therefore, the first law is a statement of the conservation of charge for a steady flow of charge or current.
- This is because charge is neither created nor destroyed, but can be transferred from one point to another.
- The law is often put in the form: The algebraic sum of currents directed in and out at a junction of a circuit must be zero, $\Sigma I = 0$
- Round a closed loop (path) the algebraic sum of the emfs is equal to the algebraic sum of the voltage (pd) drops, $\Sigma E = \Sigma IR$
This is Kirchoff's 2nd Law

Circuits: Voltage and Current Divider Theorems

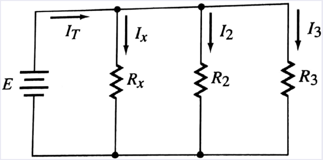
Voltage Divider Theorem



- The voltage across a part of a series circuit is equal to the resistance of the part multiplied by the total voltage and divided by the equivalent resistance
- Voltage division allows us to calculate what fraction of the total voltage across a series string of resistors is dropped across any one resistor (or group of resistors)

$$V_x = V\frac{R_x}{R_{eq}} \tag{8}$$

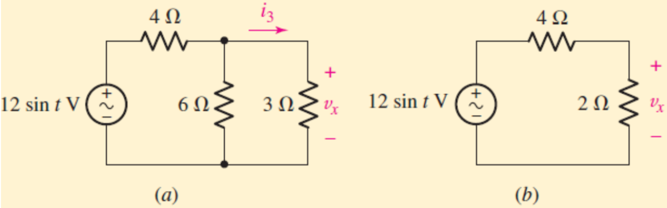
Current Divider Theorem



- The current in a branch of a parallel circuit is equal to the current entering the circuit multiplied by the equivalent resistance of the branches divided by the resistance in the branch
- Current division allows us to calculate what fraction of the total current into a parallel string of resistors flows through any one of the resistors

$$I_x = I_T\frac{R_{eq}}{R_x} \tag{9}$$

Circuits: Circuits: Voltage and Current Divider Theorems



Calculate the voltage v_x and the current through the 3Ω resistor

Solution:

$$v_x = 12\sin t\frac{2}{4+2} = 4\sin t \text{ V (voltage divider theorem)}$$

The total current in the circuit is:

$$i_t(t) = \frac{12\sin t}{4+3||6} = \frac{12\sin t}{4+2} = 2sint \text{ A}$$

$$i_3(t) = 2\sin t\frac{2}{3} = \frac{4}{3}\sin t \text{ A (current divider theorem)}$$

Circuits: Nodal and Mesh Analysis

Mesh Analysis

In mesh analysis, the values of the independent current variables are determined. Steps in establishing equilibrium equations for the mesh analysis of a circuit and finding the solution are as follows:

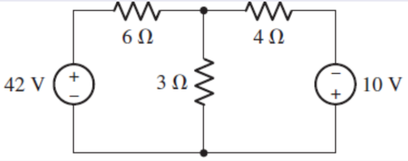
- Select an appropriate number of independent current variables and the directions of current flow
- Express the dependent current variables, by applying KCL at nodes, in terms of independent current variables
- Apply KVL around the selected loops to set up a set of simultaneous equations
- Solve for the independent currents and find the currents in all the branches

Nodal Analysis

In nodal analysis, the values of the independent voltage variables are determined. The steps in nodal analysis are as follows:

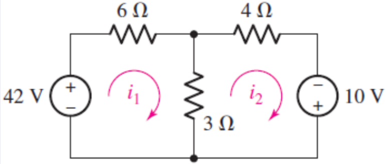
- Select an appropriate number of independent voltage variables
- Express the dependent voltage variables, by applying KVL around the loops, in terms of independent voltage variables
- Apply KCL at the selected nodes to set up a set of simultaneous equations
- Solve for the independent voltages and find the voltages at all the nodes
- Select the node connected to the maximum number of elements and sources as the ground node. A ground node acts as a reference for voltage levels at various points in the circuit. The voltage at the ground node is assumed to be zero.

Mesh Analysis



1. Find the current in each resistor
2. Find the voltage across each resistor

Solution

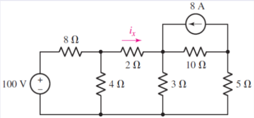


Mesh Analysis

For mesh 1:
$-42 + 6i + 3(i_1 - i_2) = 0$
For mesh 2:
$-10 + 3(i_2 - i_1) + 4i_2 = 0$
$i_1 = 6$ A; $i_2 = 4$ A;
current through 3Ω is $i_1 - i_2 = 2$ A;

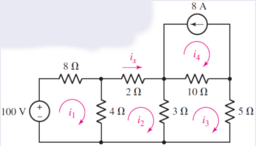
Circuits: Nodal and Mesh Analysis

Mesh Analysis



1. Determine i_x

Solution



▸ method

Mesh Analysis

For mesh 1:

$-100 + 8i + 4(i_1 - i_2) = 0 \Rightarrow$

$12i_1 - 4i_2 = 100$ .... (1)

For mesh 2:

$4(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0 \Rightarrow$

$-4i_1 + 9i_2 - 3i_3 = 0$ .... (2)

For mesh 3:

$3(i_3 - i_2) + 10(i_3 + 8) + 5i_3 = 0 \Rightarrow$

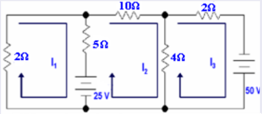
$-3i_2 + 18i_3 = -80$ .... (3)

solving eqns 1, 2 and 3 simultaneously we have

$i_x = i_2 = 2.79$ A

Circuits: Nodal and Mesh Analysis

Mesh Analysis



1. Find the mesh currents I_1, I_2 and I_3

Solution For mesh 1:
$2I_1 + 5(I_1 - I_2) = -25 \Rightarrow 7I_1 - 5I_2 = -25$ .... (1)
For mesh 2:
$10I_2 + 4(I_2 - I_3) + 5(I_2 - I_1) = 25 \Rightarrow$
$-5I_1 + 19I_2 - 4I_3 = 25$ .... (2)
For mesh 3:
$2I_3 + 4(I_3 - I_2) = 50 \Rightarrow -4I_2 + 6I_3 = 50$ .... (3)

Mesh Analysis

Let's write eqns 1, 2 and 3 in matrix form

$$A = \begin{pmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -25 \\ 25 \\ 50 \end{pmatrix}$$

The determinant of the coefficient matrix is

$$\det A = \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix} = 536$$

From Cramer's rule,

$$\det I_1 = \begin{vmatrix} -25 & -5 & 0 \\ 25 & 19 & -4 \\ 50 & -4 & 6 \end{vmatrix} \div \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}$$

$I_1 = -700 \div 536 = -1.31$ A

method

Circuits: Circuits: Nodal and Mesh Analysis

Solution Cont:

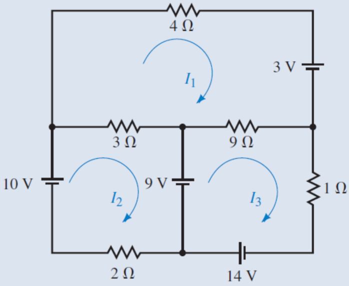
$$\det I_2 = \begin{vmatrix} 7 & -25 & 0 \\ -5 & 25 & -4 \\ 0 & 50 & 6 \end{vmatrix} \div \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}$$

$I_2 = 1700 \div 536 = 3.17$ A

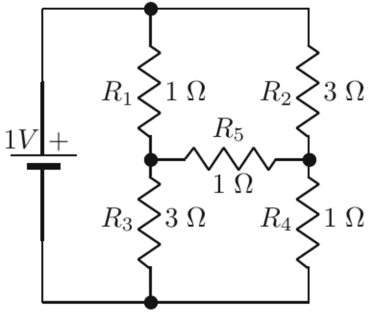
$$\det I_1 = \begin{vmatrix} 7 & -5 & -25 \\ -5 & 19 & 25 \\ 0 & -4 & 50 \end{vmatrix} \div \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}$$

$I_3 = 5600 \div 536 = 10.45$ A

Circuits: Nodal and Mesh Analysis



- Use the mesh analysis to find the loop currents



- Use the mesh analysis to find the currents in R_1, R_3 and R_5

method

Circuits: Circuits: Cramer's Rule

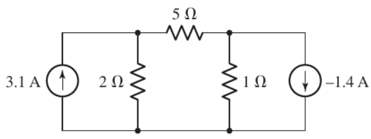
The solution of the system of equations

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

is given by $x = \dfrac{D_x}{D}$, $y = \dfrac{D_y}{D}$, and $z = \dfrac{D_z}{D}$, where

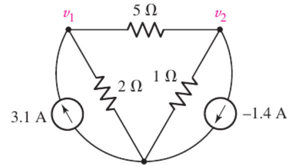
$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},\ D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix},\ D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}, \text{ and } D \neq 0.$$

Nodal Analysis



1. Determine the voltage across the 5 Ω resistor

Solution



Nodal Analysis

Applying KCL to nodes 1 and 2, the total current leaving the node through the several resistors is equal to the total source current entering the node

For node 1:

$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1 \Rightarrow$$

$$0.7v_1 - 0.2v_2 = 3.1 \quad \text{..................}(i)$$

For node 2:

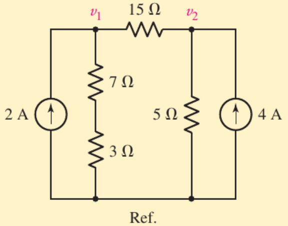
$$\frac{v_2}{1} + \frac{v_2 - v_1}{5} = -(-1.4) \Rightarrow$$

$$-0.2v_1 + 1.2v_2 = 1.4 \quad \text{..............}(ii)$$

solving (i) and (ii) gives v_1 = 5 V and v_2 = 2 V

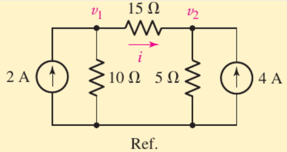
and the voltage across the 5 Ω resistor is 3 V

Nodal Analysis



1. Determine the voltage across the 5 Ω resistor

Solution



Nodal Analysis

Applying KCL to nodes 1 and 2, the total current leaving the node through the several resistors is equal to the total source current entering the node

For node 1:

$$\frac{v_1}{10} + \frac{v_1 - v_2}{15} = 2$$

$$\Rightarrow 5v_1 - 2v_2 = 60 \quad ....(i)$$

For node 2:

$$\frac{v_2}{5} + \frac{v_2 - v_1}{15} = 4 \Rightarrow -v_1 + 4v_2 = 60$$

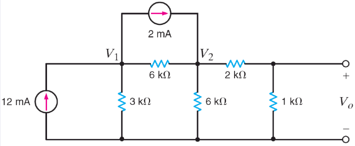
...............(ii) solving (i) and (ii) gives v_1 = 20 V and v_2 = 20 V

and the voltage across the 15 Ω resistor is $-v_1 - v_2 = 0$

No current flows through the 15 Ω resistor

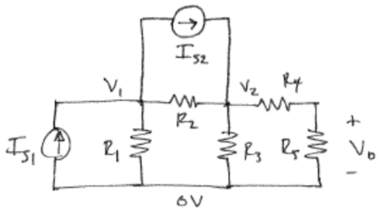
Circuits: Nodal and Mesh Analysis

Nodal Analysis



1. Use nodal analysis to find V_1 and V_o

Solution



Nodal Analysis

$R_1 = 3\ k\Omega$; $R_2 = R_3 = 6\ k\Omega$; $R_4 = 2\ k\Omega$; $R_5 = 1\ k\Omega$; $I_{s1} = 12$ mA; $I_{s2} = 2$ mA

For V_1:

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + I_{s2} = I_{s1} \quad ......(i)$$

For V_2:

$$\frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2} + \frac{V_2 - V_0}{R_4} = I_{s2} \quad ......(ii)$$

For V_0:

$$\frac{V_0 - V_2}{R_4} + \frac{V_0}{R_5} = 0$$

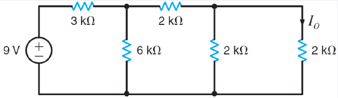
$$\frac{V_2 - V_0}{R_4} = \frac{V_0}{R_5} \quad ......(iii)$$

solving (i), (ii) and (iii) gives $V_0 = 2.91$ V and $V_1 = 22.90$ V

method

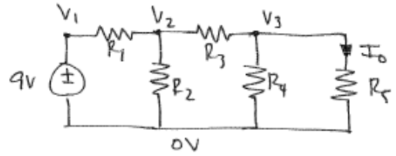
Circuits: Nodal and Mesh Analysis

Nodal Analysis



1. Use nodal analysis to find I_0

Solution



▸ method

Nodal Analysis

$R_1 = 3\ k\Omega$; $R_2 = 6\ k\Omega$; $R_3 = R_4 = R_5 = 2\ k\Omega$

For V_1:

$V_1 = 9$ V ......(i)

For V_2:

$$\frac{V_2}{R_2} + \frac{V_2 - V_1}{R_1} + \frac{V_2 - V_3}{R_3} = 0 \quad ......(ii)$$

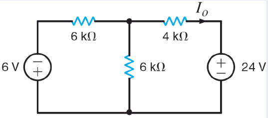
For V_0:

$$\frac{V_3}{R_4} + \frac{V_3 - V_2}{R_3} + \frac{V_3}{R_5} = 0 \quad ......(iii)$$

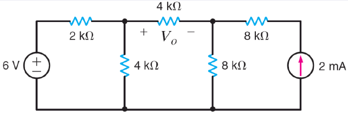
solving (i), (ii) and (iii) gives $V_3 = 1.2$ V and

$$I_0 = \frac{V_3}{R_5} = 0.6 \text{ mA}$$

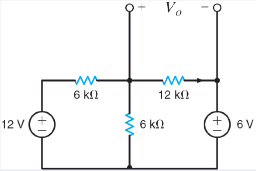
Circuits: Nodal and Mesh Analysis



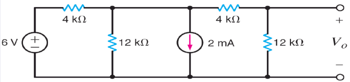
- Use the nodal analysis to find I_0



- Use the nodal analysis to find V_0



- Use the nodal analysis to find V_0



- Use the nodal analysis to find V_0

method