Kwame Nkrumah University of Science & Technology

CSM 153
CIRCUIT THEORY

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COURSE OUTLINE

Unit 2: Direct Circuit Analysis

a) Ohm's law

b) Series and Parallel Circuits

c) Methods of Analysis

Ohm's Law

- **Electric current** flowing through a metallic conductor or wire is directly proportional to the **potential difference applied**, provided **temperature** and other **physical factors** remain constant. (i.e. $V \propto I$)
- Thus mathematical statement of the law is written as **V = IR**, where R is a constant defining resistance of the wire.
- This statement of Ohm's law is correct only in certain situations.

Ohm's Law

- The equation **defines resistance**, and it applies to all conducting devices, whether they obey Ohm's law or not.
- The importance of Ohm's law, however, is that a graph of V versus I is linear (straight line); that is R is independent of V.
- A **conducting material** obeys Ohm's law when the **resistivity** of the material is independent of the **magnitude** and **direction** of the applied electric field.

Ohm's Law

- Similarly, a **conducting device** obeys Ohm's law when the **resistance** of the device is independent of the **magnitude** and **polarity** of the applied potential difference.
- Therefore, all homogeneous materials (conductors, pure semiconductors or impure semiconductors) obey Ohm's law within some range of values of the electric field.
- However, if the electric field is too strong there are departures from Ohm's law in all cases.

Ohm's Law

- A conductor whose function in a circuit is to provide a **specific resistance** is called a **resistor**.
- Thus, resistor is a conductor with a specified resistance, which remains the same no matter what the magnitude and direction (polarity) of the applied potential difference may be.
- This means the resistance R of the device is independent of the magnitude and polarity of the potential difference.

Ohm's Law

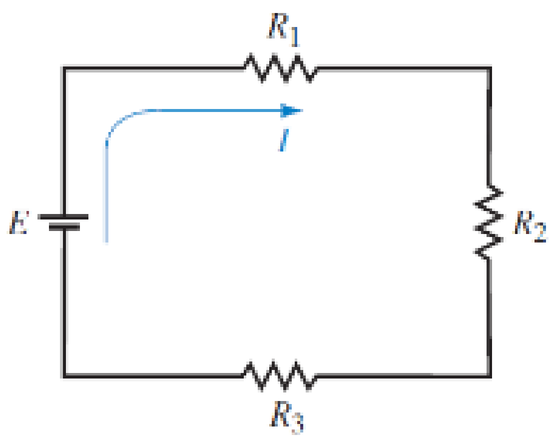
- When potential difference V is applied to the ends of a wire and the current I through it is measured, the ratio of the p.d. V to the current I defines resistance R given by

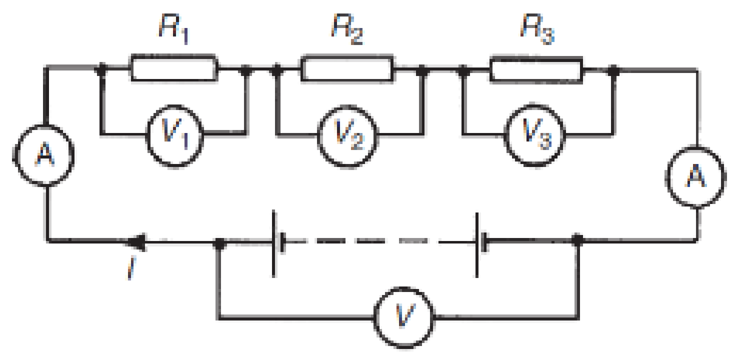
$$R = V/I \quad --- \quad (8).$$

- SI unit is the Volt per ampere or Ohm (Ω). 1 $\Omega = 1VA^{-1}$

Series and Parallel Circuits

- Two elements are said to be in series if they are connected at a single point and if there are no other current-carrying connections at this point





- The current I is the same in all parts of the circuit and hence the same reading is found on each of the two ammeters in the circuit
- The sum of the voltages V [redacted] voltage V

Series and Parallel Circuits

From Ohm's law:

$$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3 \text{ and } V = IR$$

where R is the total circuit resistance.

Since $V = V_1 + V_2 + V_3$

then $IR = IR_1 + IR_2 + IR_3$

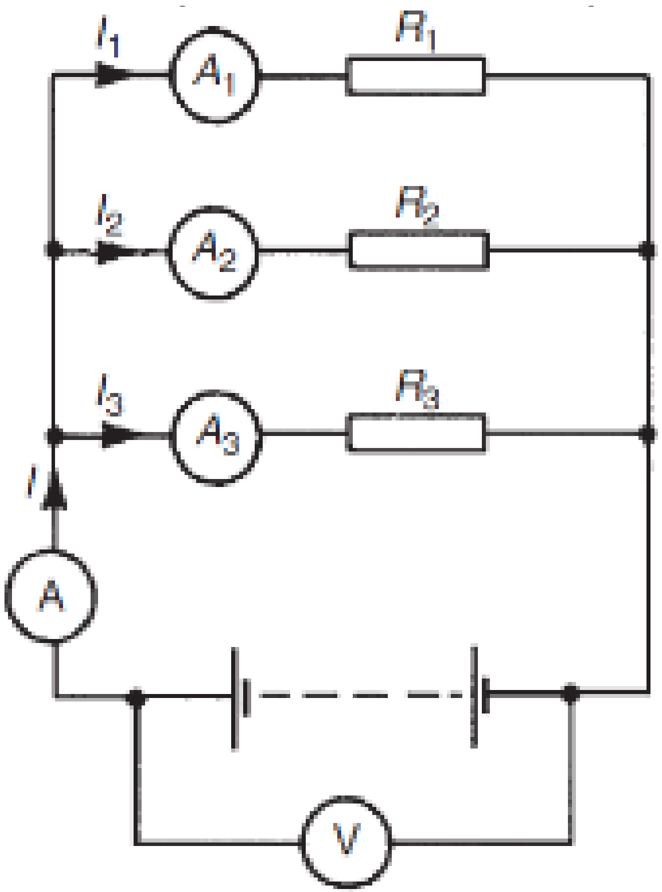
Dividing throughout by I gives

$$R = R_1 + R_2 + R_3$$

Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistances.

Series and Parallel Circuits

- Elements or branches are said to be in a parallel connection when they have exactly two nodes in common. Additionally, these parallel elements or branches will have the same m.



- The sum of the currents I [redacted] is equal to the current I
- The source potential difference (pd) is the same across each of the resistors

Series and Parallel Circuits

From Ohm's law:

$$I_1 = V/_{R_1}, I_2 = V/_{R_2}, I_3 = V/_{R_3} \; and \; I = V/_{R}$$

where R is the total resistance of the circuit

Since $I = I_1 + I_2 + I_3$

then $V/_{R} = V/_{R_1} + V/_{R_2} + V/_{R_3}$

Dividing throughout by V gives

$$1/_{R} = 1/_{R_1} + 1/_{R_2} + 1/_{R_3}$$

Series and Parallel Circuits

The total resistance R for a parallel circuit is:

$$1/_{R} = 1/_{R_1} + 1/_{R_2} + 1/_{R_3}$$

For a special case of 2 resistors in parallel

$$1/_{R} = 1/_{R_1} + 1/_{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

Hence $\mathrm{R} = \frac{R_2 R_1}{R_1 + R_2}$

Series and Parallel Circuits

Node: A point at which two or more elements have a common connection called a node

Branch: A single path in a network composed of one simple element and the node at each end of that element

Loop: A simple closed path in a circuit in which no circuit element or node is encountered more than once

Series and Parallel Circuits

- Electrical network is usually regarded to be a complicated (complex) system of electrical conductors.
- In dealing with such networks the Ohm's law was extended to the networks by a German physicist Gustav R. Kirchhoff (1847) in the form of two laws.

However, it must be emphasized that:

1) The laws enabled the **current in any part** of an electrical network to be calculated.
2) All circuits can be solved by Kirchhoff's laws because they **do not depend on series or parallel** connection of resistors/ conductors.

Series and Parallel Circuits

Kirchoff's Law 1

- The total current flowing into a junction in a circuit (electrical network) is equal to the total current flowing out of (leaving) the junction.
- The algebraic sum of currents directed in and out at a junction of a circuit must be zero.

i.e. I

Series and Parallel Circuits

Kirchoff's Law 1

The first law applies to any point or junction in an electrical network.

If currents flowing in and out of the junction flows for a time t seconds, then we have

$$I \; [illegible]$$

$$\Rightarrow Q \; [illegible]$$

$$\therefore \sum Q \; [illegible]$$

This means total charge flowing to the junction is equal to total charge flowing out of it.

Series and Parallel Circuits

Kirchoff's Law 1

- This implies there is neither a build up (accumulation or pile up) nor a depletion of charge at a junction.
- Therefore, the first law is a statement of the conservation of charge for a steady flow of charge or current.
- This is because charge is neither created nor destroyed, but can be transferred from one point to another.
- The law is often put in the form: *The algebraic sum of currents directed in and out at a junction of a circuit must be zero* .

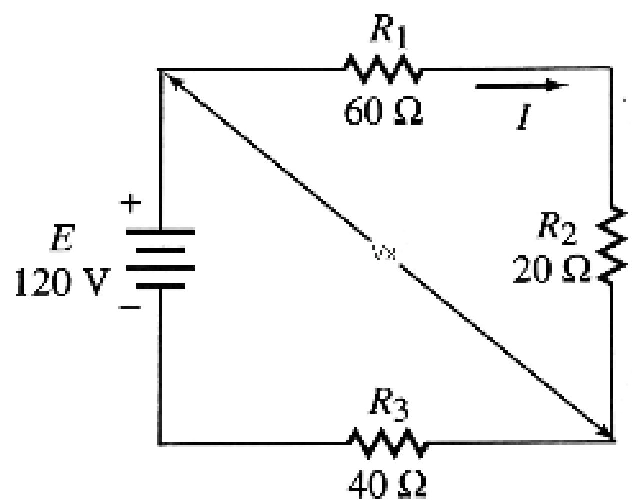
$$\sum I = 0, \qquad \text{i.e. } I$$

Series and Parallel Circuits

Kirchoff's Law 2

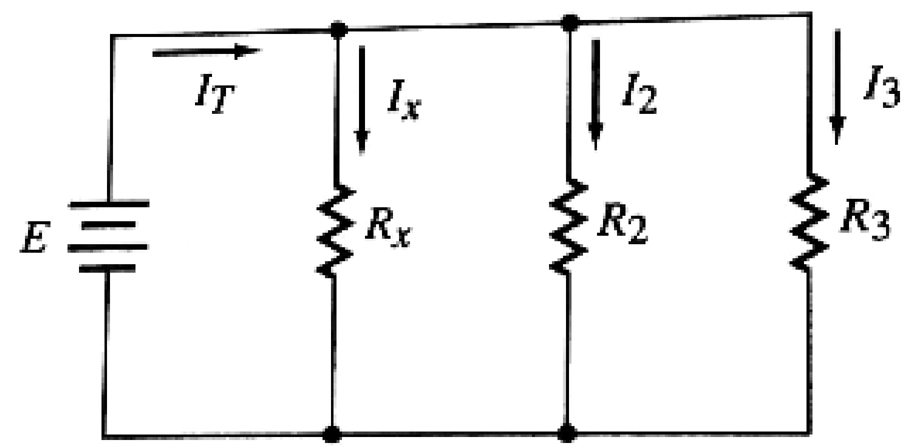
- Round a closed loop (path) the algebraic sum of the emfs is equal to the algebraic sum of the voltage (pd) drops. i.e. $\sum E = \sum IR$

- **Voltage Divider Rule**: The voltage across a part of a series circuit is equal to the resistance of the part multiplied by the total voltage and divided by the equivalent resistance
- Voltage division allows us to calculate what fraction of the total voltage across a series string of resistors is dropped across any one resistor (or group of resistors)
- $$V_x = \frac{R_x}{R_{eq}} V$$

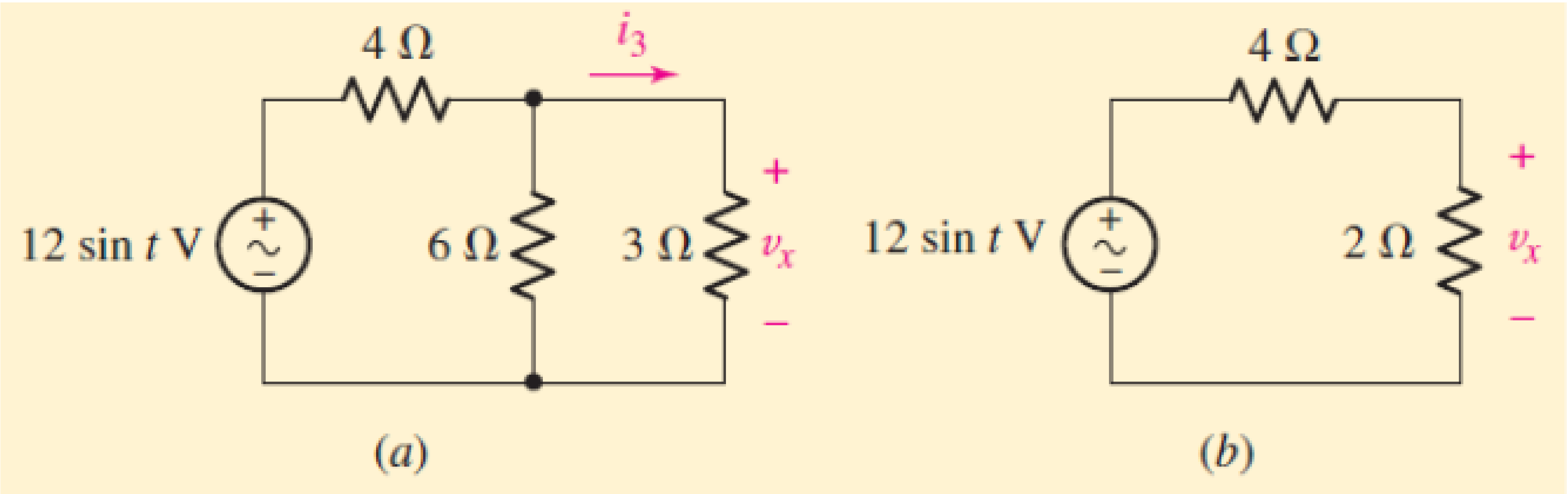


Current and Voltage Divider Rules

- **Current Divider Rule**: The current in a branch of a parallel circuit is equal to the current entering the circuit multiplied by the equivalent resistance of the branches divided by the resistance in the branch
- Current division allows us to calculate what fraction of the total current into a parallel string of resistors flows through any one of the resistors
- $I_x = \frac{R_{eq}}{R_x} I_T$



Direct Circuit Analysis
Series and Parallel Circuits
Current and Voltage Divider Rules

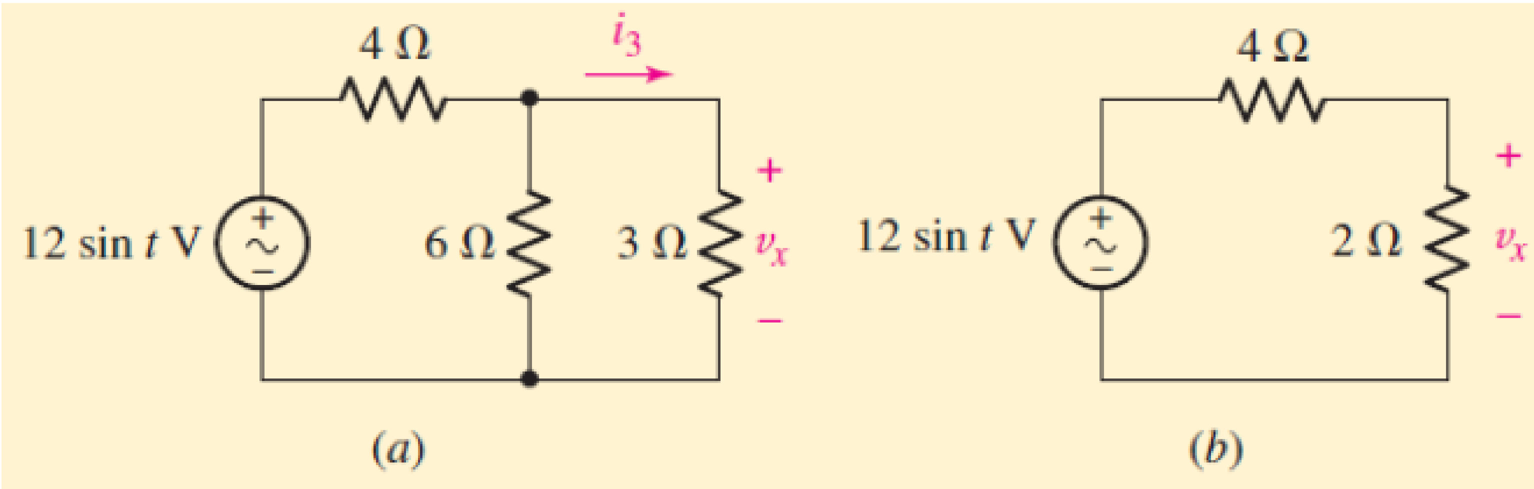


- Calculate the voltage V_x.

Soln

- $V_x = 12 \sin t \frac{2}{4+2} = 4 \sin t \; V$

Direct Circuit Analysis
Series and Parallel Circuits
Current and Voltage Divider Rules



- Calculate the voltage the current through the 3Ω.

Soln

$$i(t) = \frac{12 \sin t}{4+3||6} = \frac{12 \sin t}{4+2} = 2 \sin t \text{ A} \quad \text{... (total current in cct)}$$

$$i_3(t) = 2 \sin t \frac{6}{6+3} = \frac{4}{3} \sin t \, A$$

Series and Parallel Circuits

Nodal analysis

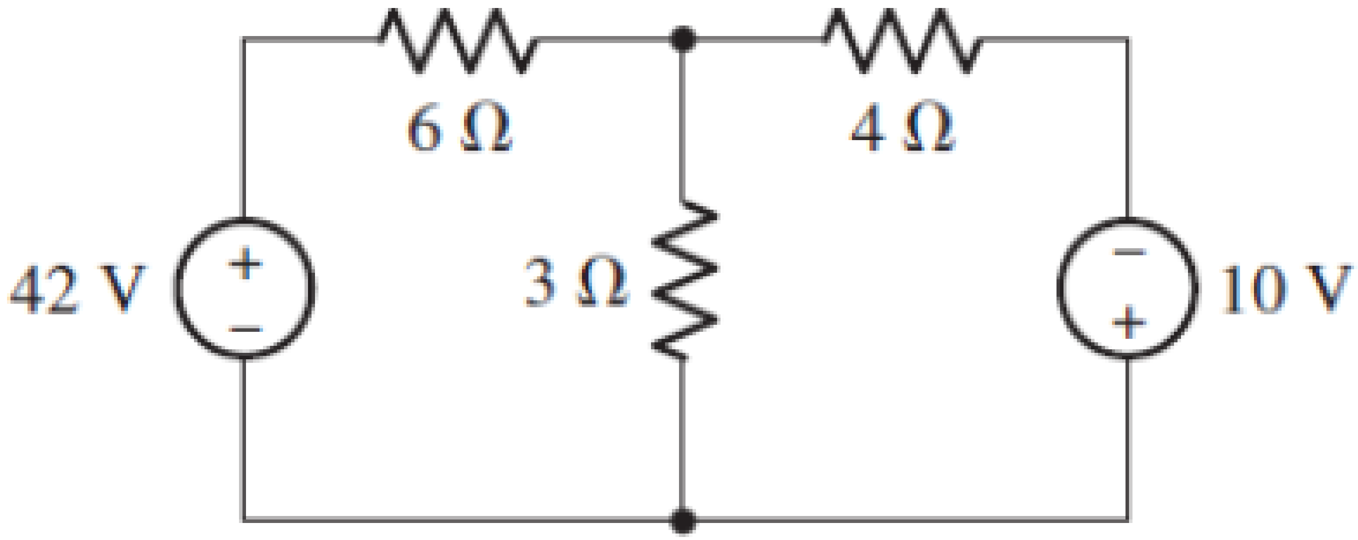
- Choose one node as the reference node. Then label the node voltages v
  measured with respect to the reference node
- If the circuit contains only current sources, apply KCL at each nonreference node

Mesh analysis

- First make certain that the network is a planar network.
- Assign a clockwise mesh current in each mesh: i
- If the circuit contains only voltage sources, apply KVL around each mesh

Series and Parallel Circuits

Circuit Analysis



Find

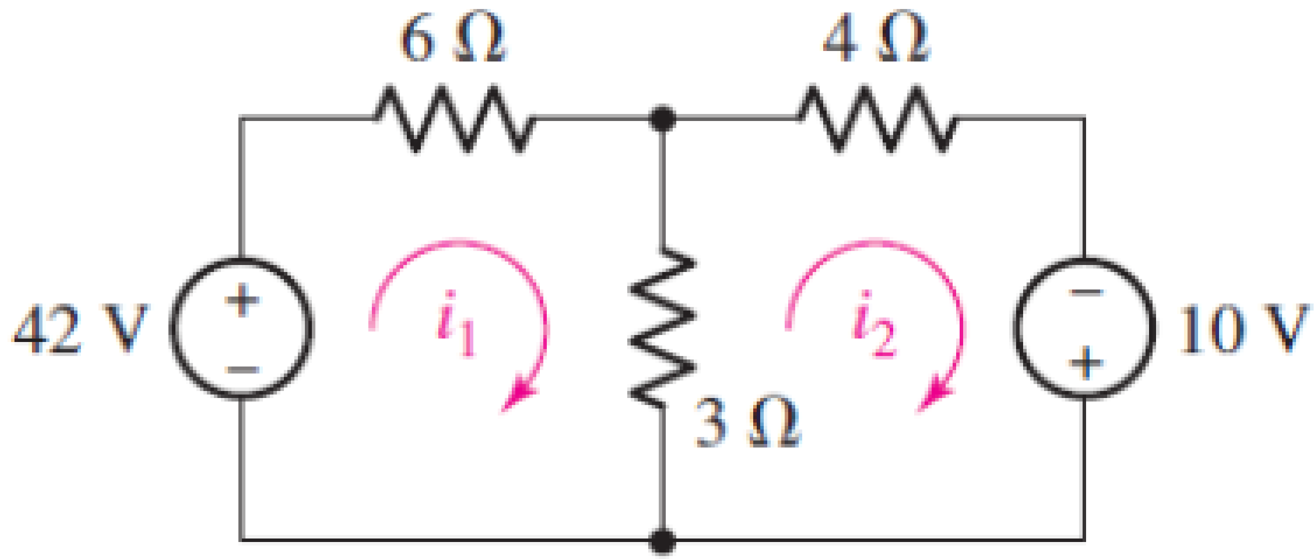
i. the current in each resistor

ii. the voltage across each resistor

Direct Circuit Analysis

Series and Parallel Circuits

Circuit Analysis



For mesh 1: $-42 + 6i + 3(i_1 - i_2) = 0$

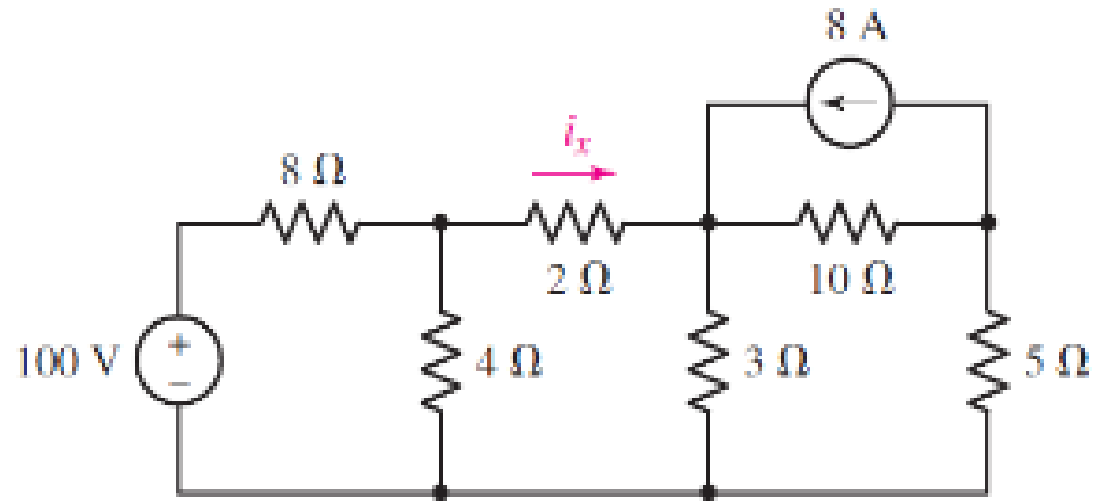
For mesh 2: $-10 + 3(i_2 - i_1) + 4i_2 = 0$

$i_1 = 6\ A,$ $\qquad i_2 = 4\ A\ and\ current\ the\ 3\Omega\ is$

$$i_1 - i_2 = 2\ A$$

Direct Circuit Analysis
Series and Parallel Circuits
Circuit Analysis

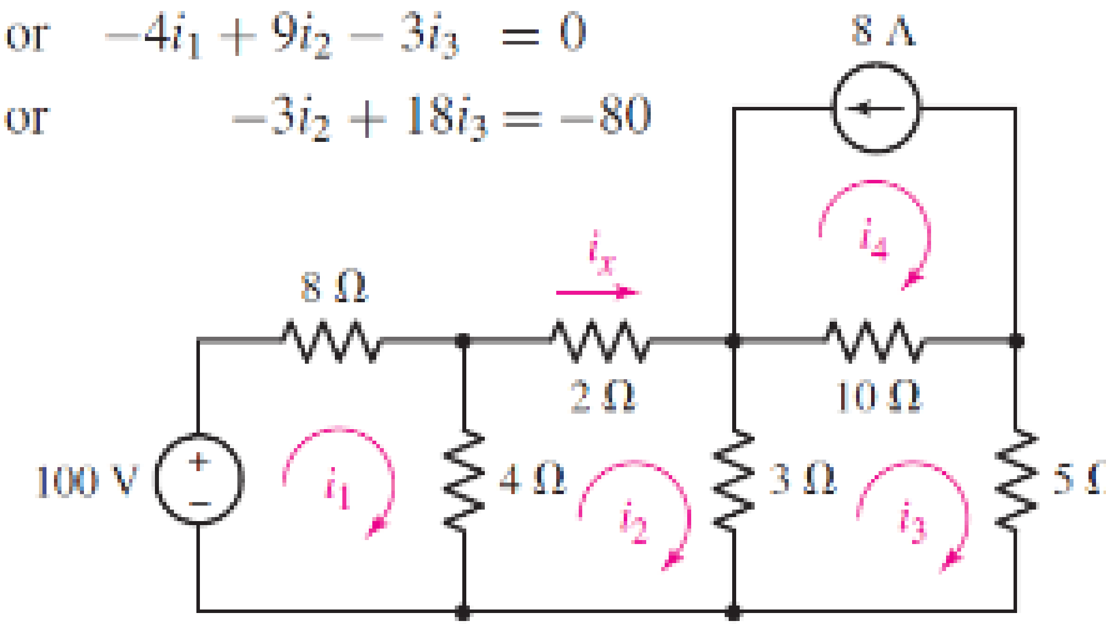
- Determine the current I_x



Soln

$$-100 + 8i_1 + 4(i_1 - i_2) = 0 \quad \text{or} \quad 12i_1 - 4i_2 = 100$$
$$4(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0 \quad \text{or} \quad -4i_1 + 9i_2 - 3i_3 = 0$$
$$3(i_3 - i_2) + 10(i_3 + 8) + 5i_3 = 0 \quad \text{or} \quad -3i_2 + 18i_3 = -80$$

- $i_x = i_2 = 2.79\ A$

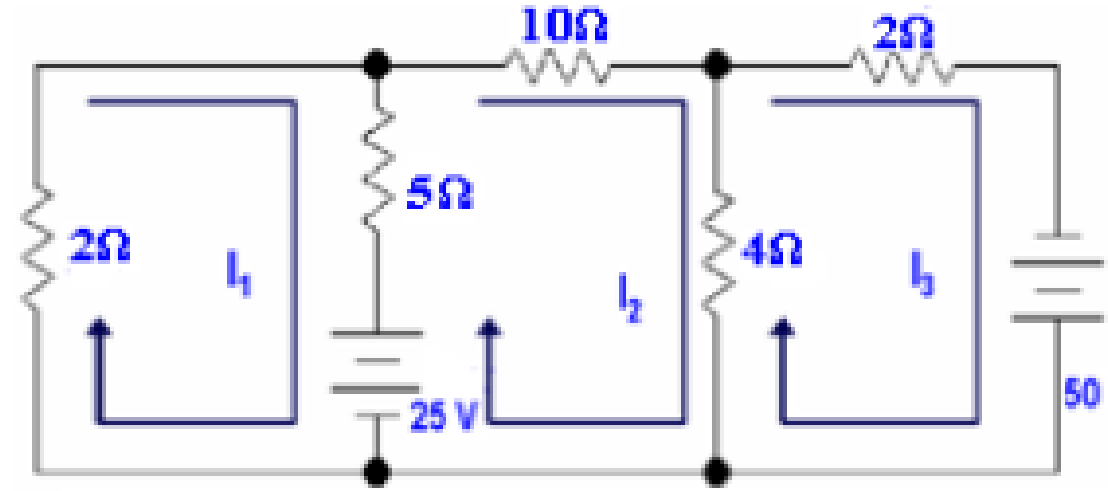


Circuit Analysis

- Find the mesh currents

I [redacted]

[redacted]



KVL equation for loop 1

$$2I_1 + 5(I_1 - I_2) = -25$$
$$2I_1 + 5I_1 - 5I_2 = -25$$
$$7I_1 - 5I_2 = -25$$

KVL equation for loop 2

$$10I_2 + 4(I_2 - I_3) + 5(I_2 - I_1) = 25$$
$$10I_2 + 4I_2 - 4I_3 + 5I_2 - 5I_1 = 25$$
$$-5I_1 + 19I_2 - 4I_3 = 25$$

KVL equation for loop 3

$$2I_3 + 4(I_3 - I_2) = 50$$
$$2I_3 + 4I_3 - 4I_2 = 50$$
$$-4I_2 + 6I_3 = 50$$

Direct Circuit Analysis
Series and Parallel Circuits
Circuit Analysis

[redacted]

We will write equation A ,B and C in matrix form

$$\begin{bmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -25 \\ 25 \\ 50 \end{bmatrix}$$

The determinant of the coefficient matrix is

$$|A| = \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}$$

$$|A| = 536$$

By Cramer's rule

$$I_1 = \begin{vmatrix} -25 & -5 & 0 \\ 25 & 19 & -4 \\ 50 & -4 & 6 \end{vmatrix} \div \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}$$

$$I_1 = (-700) \div 536$$

$$\mathbf{I_1 = -1.31\ A}$$

Series and Parallel Circuits

Circuit Analysis

[redacted]

$$I_2 = \begin{vmatrix} 7 & -25 & 0 \\ -5 & 25 & -4 \\ 0 & 50 & 6 \end{vmatrix} \div \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}$$

$I_2 = 1700/536$

$\mathbf{I_2 = 3.17A}$

$$I_3 = \begin{vmatrix} 7 & -5 & -25 \\ -5 & 19 & 25 \\ 0 & -4 & 50 \end{vmatrix} \div \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}$$

$I_3 = 5600/536$

$\mathbf{I_3 = 10.45\ A}$

Cramer's Rule

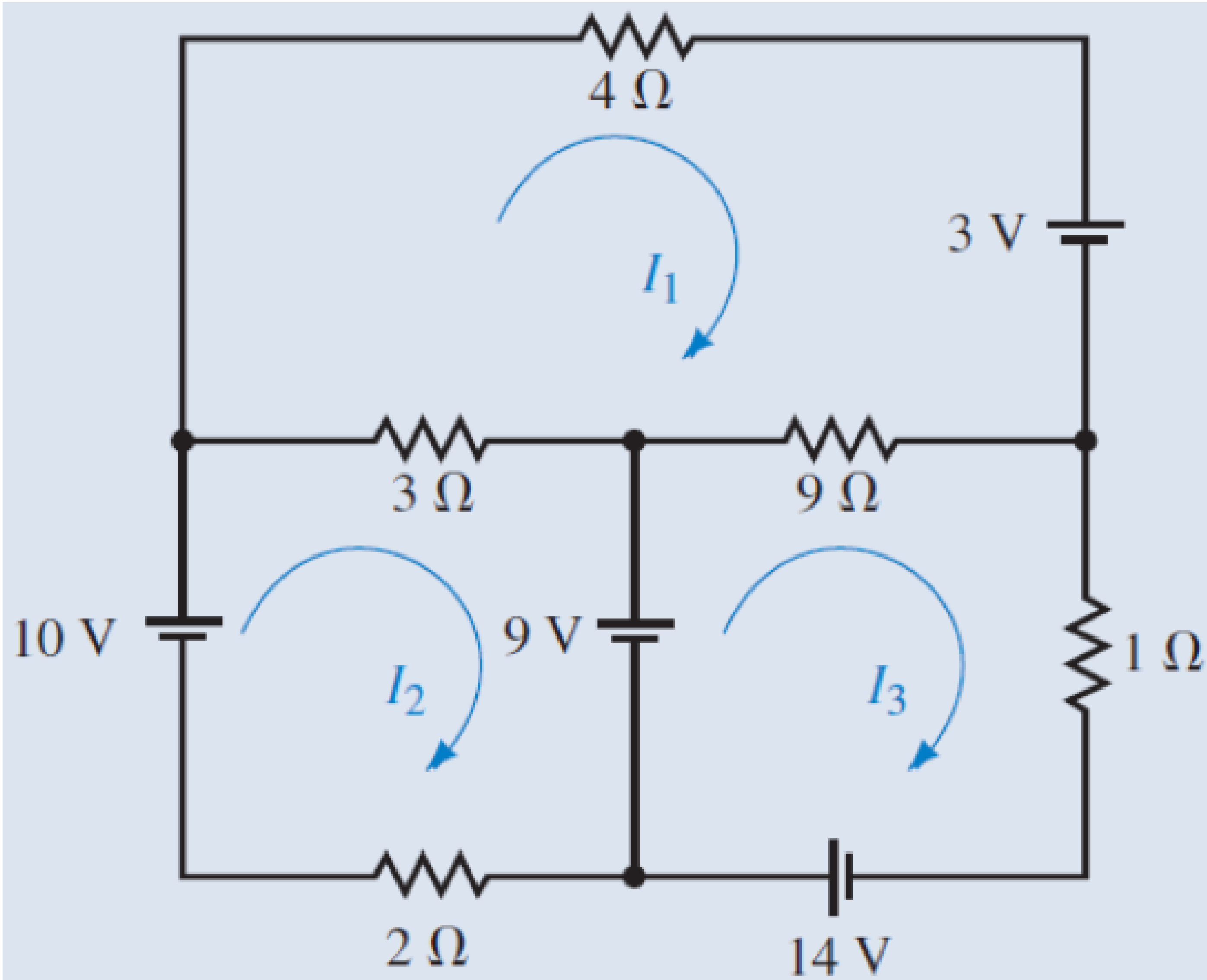
The solution of the system of equations

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

is given by $x = \dfrac{D_x}{D}$, $y = \dfrac{D_y}{D}$, and $z = \dfrac{D_z}{D}$, where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\quad D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},\quad D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix},\quad D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}, \text{ and } D \neq 0.$$

Use the mesh analysis to find the loop currents



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