# CSM 166: Discrete Mathematics for Computer Science

Fundamentals of Counting

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# Content

Course Outline

Fundamentals of Counting

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1. Fundamentals of Counting
2. Multinomial Coefficients
3. Complex Numbers
4. Theory of Difference Equations/Recurrence Relations.
5. Boolean Algebra and Boolean Functions

# Fundamentals of Counting

## Definition 1 (additive principle)

*The additive principle states that if event A can occur in $m$ ways, and event B can occur in $n$ **disjoint** ways, then the event "A or B" can occur in $m + n$ ways.*

Since A and B are disjoint :

$$|A \cup B| = |A| + |B|$$

# Fundamentals of Counting

## Example 1

i How many ways can a student be selected from a class of 18 boys and 20 girls

ii How many two letter “words” start with either A or B?

# Fundamentals of Counting

Definition 2 (Multiplicative Principle.)

*The multiplicative principle states that if event A can occur in m ways, and each possibility for A allows for exactly n ways for event B, then the event "A and B" can occur in $m \cdot n$ ways.*

# Example 2

i. How many ways can 1 boy and 1 girl be selected from a class of 18 boys and 20 girls?

ii. There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?

iii. A new company with just two employees, Sanchez and Patel, rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?

# Fundamentals of Counting

## Definition 3

*Any arrangement of a set of n object in a given order is called a permutation of the objects (taken all at a time).*

## Example 3

How many permutations are there of the letters a, b, c, d, e, f ?

**Solution:**

There are $6 \cdot 5 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 1 = 720$ permutations of the 6 letters.

# Permutation of $n$ elements

There are $n!\cdot = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$ permutations of n (distinct) elements.

# k-permutation of n elements

$P(n, k)$ is the number of k-permutations of n elements, the number of ways to arrange k objects chosen from n distinct objects.

$$P(n,k) = \frac{n!}{(n-k)!} = n(n-1)(n-2)\ldots(n-(k-1))$$

# Example 4

i How many 4 letter “words” can you make from the letters a through f, with no repeated letters?

ii How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest

**Exercise A:**

Suppose that there are eight runners in a race. The winner receives a gold medal, the second-place finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?

# Fundamentals of Counting

## Definition 4

*Suppose we have a collection of n objects. A combination of these n objects taken r at a time is any selection of r of the objects without taking order in account.*

*An r - combination of n objects, denoted* $C(n,r)$ *is an* ***unordered*** *selection of r of the n objects.*

$$C(n,r) = \frac{P(n,r)!}{r!} = \frac{n!}{r!(n-r)!}$$

# Example 5

1. Find the number of combinations of four objects, a,b c, d taken three at a time

2. How many different committees of three students can be formed from a group of five (5) students?

3. In how many different ways can a hand of 5 cards be selected from a deck of 52 cards?

# Fundamentals of Counting

**Exercise B:**

Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

# Fundamentals of Counting

**Exercise C:**

Prove the following identities:

a) $C(n,0) = C(n,n) = 1$ and
$C(n,1) = C(n,n-1) = n$

b) Symmetry property: $C(n,r) = C(n,n-r)$, $r \leq n$.

c) Pascal's identity:
$C(n+1,k) = C(n,k-1) + C(n,k)$, $n \leq k$

End of Lecture

Questions...???

Thanks

# Reference Books


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2. H. Levy, F. Lessman Finite Difference Equations.Dover books on mathematices
3. Gary Chartrand. Ping Zhang. Discrete Mathematics $1^{th}$
4. Oscar Leven. Discrete Mathematics: An open introduction. 2nd Edition. 2013