CSM 165: Discrete Mathematics for Computer Science

Chapter 1: Propositional and first order predicate logic

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Content

Introduction

What is discrete mathematics?

- Discrete mathematics is the part of mathematics devoted to study discrete objects.
  - Discrete Means not continuous or unconnected :

Discrete Verses Continuous (examples)

1. Natural Numbers are discrete
2. Digital clock are discrete

1. Real numbers are continuous
2. Analog clock are continuous

NB: here we mean the analog clock with the second, minute and hour hand moving/sweeping continuously.

Introduction Cont'd

Discrete Mathematics helps in solving Problems such as:

- Is there a link between two computers in a network?
- Sorting a list of integers.
- Finding the shortest path from your home to your friend's house.
- How many different combinations of passwords are possible with just 9 alphanumeric characters?
- How can I identify spam e-mail messages?

Introduction Cont'd

Why Discrete Mathematics

- It develops your mathematical thinking
- Improves problem solving ability.
- Many problems can be solved using discrete mathematics.
- Foundation for many computer science courses:
  - data structures
  - algorithms
  - database theory
  - automata theory
  - computer security
  - operating systems.

Course Outline

1. Propositional and first order predicate logic
2. Set Theory.
3. Relations and Functions
4. First Principle of induction.
5. Number Systems and arithmetic (complement number system)

Propositional and first order predicate logic

Definition 1 (Proposition)

*A proposition is a **declarative** sentence that is either **true** or **false**, but not both*

Example 1

1. COVID-19 is a communicable disease
2. Wearing of nose mask is the only preventive measure for COVID-19.
3. $2 + 3 = 5$
4. $1 + 1 = 11$

Propositional Logic

Example 2

1. Kindly send me the code snippet for the assignment.
2. What is your name?
3. Remember to observe all the COVID-19 protocols.
4. x+5=10
5. $\sqrt{16} + y = z$

NB: None of the above examples is a proposition since none is a declaration nor True or False.

Propositional logic

Definition 2 (Logic)

Logic is the science of reasoning.

It helps in understanding and reasoning about different mathematical statements.

The area of logic that deals with propositions is called the **propositional logic**.

Definition 3 (Propositional Variables)

Propositional Variables are variables used to represent propositions.

Example 3

p = My PC runs Linux

q = Hannah's smart phone has 256GB of memory.

Logical Connectives (operators)

Definition 4 (Negation $\neg$)

Let p be a proposition. The negation of p, denoted by $\neg p$ (also denoted by ~p), is the statement “It is not the case that p.

Table 1 : Truth table for $\neg p$

p	$\neg p$
T	F
F	T

Example 4

Find the negation of the following propositions.

1. Hannah’s PC runs linux.
2. Data science is the sexiest job of 21st century.
3. Africa is the richest continent in the world.

Logical Connectives (operators)

Definition 5 (Conjunction $\wedge$)

Let p and q be propositions. The conjunction of p and q, denoted by $p \wedge q$, is the proposition "p and q".

The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Table 2 : Truth Table for $P \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example 5

Let p = "Hannah's PC has more than 16 GB free hard disk space"

q = "The processor in Hannah's PC runs faster than 1 GHz."

Definition 6 (Disjunction $\vee$)

Let p and q be propositions. The disjunction of p and q, denoted by $p \vee q$, is the proposition "p or q".

The disjunction p ∨ q is false when both p and q are false and is true otherwise.

Table 3 : Truth Table for $P \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example 6

Let: p = Students who have taken algebra can enroll in this course.

$p \vee q$ = Students who have taken

q = students who have taken

Logical Connectives (operator)

Definition 7 (Exclusive OR (XOR))

Let p and q be two propositions. The exclusive OR of p and q (denoted by $p \oplus q$) is the proposition that is true when exactly one of p and q is true and is false otherwise.

Table 4 : Truth table for $p \oplus q$

p	q	p ⊕q
T	T	F
T	F	T
F	T	T
F	F	F

Example 7

1. Coffee or Tea comes with dinner
2. Students who have taken calculus or computer science, but not both, can enroll in this class.

Conditional Statements

Definition 8

For proposition p and q,the conditional sentence $p \to q$ is the proposition "If p, then q". Proposition p is called the ***antecedent*** *and q is the* ***consequence****.*

Table 5 : Truth table for $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example 8

(a) If you try hard for your exams, then you will pass.

Exercise A:

1. Let p be the statement “Hannah learns discrete mathematics” and q the statement “Hannnah will find a good job”. Express the statement $p \to q$ as a statement in English.
2. What is the value of the variable x after the statement:
   “if $5+7=12$ then $x = x+1$ ”,
   if $x = 0$ before this statement

Converse and Contrapositive

Let p and q be propositions.

- The proposition $q \rightarrow p$ is called the **converse** of $p \rightarrow q$.
- The **contrapositive** of $p \rightarrow q$ is the proposition $(\neg q) \rightarrow (\neg p)$

Example 9

Consider: "If n is an integer, then 14 is even"

1. **Converse**: If 14 is even, then n is an integer
2. **Contraposition**: If 14 is not even, then n is not an integer.

NB: The converse is false, but the sentence and its contrapositive are true

Theorem 1

For propositions p and q,

(a) $p \rightarrow q$ *is equivalent to its contrapositive* $(\neg q \rightarrow \neg p)$.

(b) $p \rightarrow q$ *is not equivalent to its converse* $q \rightarrow p$

Proof.

The proofs are carried out by examination of the truth tables

p	q	$p \rightarrow q$	$\neg p$	$\neg q$	$(\neg q) \rightarrow (\neg p)$	$q \rightarrow p$
T	T	T	F	F	T	T
F	T	T	T	F	T	F
T	F	F	F	T	F	T
F	F	T	T	T	T	T

$\square$

Definition 9 (Biconditional)

For propositions p and q, the biconditional sentence $p \leftrightarrow q$ is the proposition "p if and only if q".
$p \leftrightarrow q$ is true exactly when p and q have the same truth values.

Table 6 : Truth table for $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example 10

1. Let
   p = "You can take the flight" and
   q = "You buy a ticket."
   Then $p \leftrightarrow q$ is:
   "You can take the flight if and only if you buy a ticket"

NB: We also write **p iff q** to abbreviate p if and only if q.

Definition 10 (Tautology)

A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology

Definition 11 (Contradiction)

A compound proposition that is always false is called a contradiction

Definition 12 (Contingency)

A compound proposition that is neither a tautology nor a contradiction is called a contingency.

Example 11

Table 7 : A tautology and a Contradiction

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Definition 13 (Logical Equivalence)

*Compound propositions that have the same truth values in all possible cases are called **logically equivalent**.*

The compound propositions p and q are also called **logically equivalent** if $p \leftrightarrow q$ is a **tautology**. The notation $p \equiv q$ denotes that p and q are logically equivalent.

De Morgan's Laws

1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Logical Equivalence

Example 12

1. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent

Table 8 : Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

2. Show that $p \rightarrow q$ and $\neg p \vee q$ and equivalent.
3. Show that $p \wedge (q \vee r)$ and $(p \vee q) \wedge (p \vee r)$.

Logical Equivalence

Solution to example 12 question 3

Table 9 : Truth Table for $p \wedge (q \vee r)$ and $(p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Precedence of Logical Operators

Table 10 : Precedence of Logical Operators

Operators	Names	Precedence
$\neg$	Negation	1
$\wedge$	Conjunction	2
$\vee$	Disjunction	3
$\rightarrow$	Implication	4
$\leftrightarrow$	Biconditional	5

Table 11 : Logical Equivalences

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Table 12 : Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Table 13 : Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Inference

Definition 14

Premise: *It is the proposition on the basis of which we would be able to draw a conclusion.*
It can be thought of as an evidence or assumption.

Conclusion: *It is the a proposition that is reached from a given set of premises.*

Argument: *Sequence of statements that ends with a conclusion.*

Valid Argument: *An argument is valid if and only if it is impossible for all the premises to be true and the conclusion to be false. OR*

Let A and B be two statement formulas. We say that "B ***logically follows from*** *A" or "B is a valid conclusion of the premise A" iff* $A \rightarrow B$ *is a tautology.*

Validity Using Truth Table

Example 13

Determine whether the following conclusion C follows logically from the premises H_1 and H_2.

1. $H_1\ P \to Q \quad H_2{:}P \quad C{:}Q$
2. $H_1\ P \to Q \quad H_2{:}\neg P \quad C{:}Q$
3. $H_1{:}\ P \to Q \quad H_2{:}\neg(p \wedge Q) \quad$ C:$\neg P$
4. $H_1{:}\neg P \quad H_2{:}P \leftrightarrow Q \quad$ C:$\neg(P \wedge Q)$

P	Q	$P \to Q$	$\neg P$	$\neg Q$	$\neg(P \wedge Q)$	P$\leftrightarrow Q$
T	T	T	F	F	F	T
T	F	F	F	T	T	F
F	T	T	T	F	T	F
F	F	T	T	T	T	T

Rules of Inference

Example 14

Consider:

"If you have a current password, then you can log onto the network".

"You have a current password".

Therefore, "You can log onto the network."

Let P = you have a current password

q = you can log onto the network

Argument form:

$$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad} \\ \therefore q \end{array} \qquad\qquad ((p \rightarrow q) \wedge p) \rightarrow q$$

This form of argument is valid because whenever all its premises are true, the conclusion must also be true

Rules of Inference

Example 15

Now Consider:

"If you have a current password, then you can log onto the network".

"you can log onto the network".

Therefore, "You have a current password"

Let P = you have a current password

q = you can log onto the network

Argument form:

$$\begin{array}{l} p \rightarrow q \\ \underline{q \qquad} \\ \therefore p \end{array} \qquad\qquad ((p \rightarrow q) \land q) \rightarrow p$$

This form of argument is invalid since we can make all premises true and conclusion false.

Rules of Inference

Rule	Tautology	Name
p $\underline{p \rightarrow q}$ $\therefore q$	$(p \wedge (p \rightarrow)) \rightarrow q$	Modus ponens
$\neg q$ $\underline{p \rightarrow q}$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $\underline{q \rightarrow r}$ $\therefore p \rightarrow r$	$(p \rightarrow q) \wedge (\rightarrow r) \rightarrow (p \rightarrow r)$	Hypothetical syllogism

Rules of Inference

$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p p $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Rules of Inference

Example 16

State which rule of inference is the basis of each of the following argument:

(i) " It is below freezing now. Therefore, it is either below freezing or raining now."

(ii) If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow.

Using Rules of Inference to Build Arguments

Example 17

1. Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

   Solution
   Let p = It is sunny this afternoon, q = it is colder than yesterday, r = We will go swimming s = we will take a canoe trip, t = we will be home by sunset.
   Premises: $\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$ and $s \rightarrow s$

Step	**Reason**
(1) $\neg p \wedge q$	Premise
(2) $\neg p$	Simplification using (1)
(3) $r \to p$	Premise
(4) $\neg r$	Modus tollens using (2) and (3)
(5) $\neg r \to s$	Premise
(6) s	Modus ponens using (4) and (5)
(7) $s \to t$	Premise
(8) t	Modus ponens using (6) and (7)

Example 18

Demonstrate that R is a valid inference from the premises $P \to Q$, $Q \to P$.

Solution

(1) $P \to Q$	Premise
(2) P	Premise
(3) Q	Modus ponens using (1) and (2)
(4) $Q \to R$	Premise
(5) R	modus ponens using (3), (4)

Inference

Exercise B:

1. Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

2. Show that $R \vee S$ follows logically from the premises $(C \vee D) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ and $(A \wedge \neg B) \rightarrow (R \vee S)$.

Predicates and Quantifiers

Definition 15

A predicate or propositional function is a statement containing variable(s) which are neither true nor false until the values of the variables are specified .

A predicate is represented by a letter followed by the variables enclosed between parenthesis: $P(x)$, $Q(x,y)$, *etc*

A propositional function has two parts:

1. A **Subject**
2. A **predicate**

Example 19

x is greater than 10

x is the subject

is greater than 10 is the predicate

Predicates

Example 20

(a) Let $P(x)$ denote the statement "x > 3." What are the truth values of $P(4)$ and $P(2)$?

(b) Let $A(x)$ denote the statement "Computer x is under attack by an intruder." Suppose that of the computers on campus, only CS2 and MATH1 are currently under attack by intruders. What are truth values of A(CS1), A(CS2), and A(MATH1)?

(c) Let $Q(x, y)$ denote the statement "x = y + 3." What are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$?

Quantifiers

Definition 16 (Universal Quantifier $\forall$)

The universal quantification of $P(x)$ is the statement

"P(x) for all values of x in the domain".

The notation $\forall x\, P(x)$ denotes the universal quantification of $P(x)$.

$\forall x\, P(x)$ is read as "for all $x\, P(x)$" or "for every $x\, P(x)$".

An element for which $P(x)$ is false is called a **counterexample** of $\forall x\, P(x)$.

"all of", "for each", " given any", "for arbitrary", "for each", and "for any".

Example 21

1. Let $P(x)$ be the statement "$x > x - 1$". What is the truth value of the quantification $\forall x\, P(x)$, where the domain consists of all real numbers?

2. Let $Q(x)$ be the statement "$x < 5$". What is the truth value of the quantification $\forall x\, P(x)$, where $x \in \mathscr{R}$?

3. What is the truth value of $\forall x\, P(x)$, where $P(x)$ is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?

Quantifiers

Definition 17 (Existential Quantifier ($\exists$))

The existential quantification of $P(x)$ is the proposition

"There exists an element x in the domain such that $P(x)$."

We use the notation $\exists(x)P(x)$ *for the existential quantification of* $P(x)$.

$\exists(x)P(x)$ is read as "There is an x such that $P(x)$"

Alternatives: "for some", "for at least one" or "there is".

NB: The statement $\exists x\, P(x)$ is false *iff* there is no element x in the domain for which $P(x)$ is true.

Quantifiers

Example 22

(i) Let $P(x)$ denote the statement "x > 3". What is the truth value of the quantification $\exists x\, P(x)$, where the domain consists of all real numbers?

(ii) Let $Q(x)$ denote the statement "x = x + 1." What is the truth value of the quantification $\exists x\, P(x)$, where the domain consists of all real numbers?

(iii) What are the truth values for the statements $\forall x < 0(x^2 > 0)$, $\forall y \neq 0(y^3 \neq 0)$, and $\exists z > 0(z^2 = 2)$ mean, where the domain in each case consists of the real numbers?

Quantifiers

Precedence of Quantifiers

The quantifiers $\forall$ and $\exists$ have higher precedence than all logical operators from propositional calculus

For instance, $\forall x\, P(x) \vee Q(x)$ is the disjunction of $\forall x\, P(x)$ and $Q(x)$

Table 14 : De Morgan's Laws for Quantifiers.

Negation	***Equivalent Statement***	***When Is Negation True?***	***When False?***
$\neg\exists x\, P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x\, P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

Introduction to Proofs

Definition 18

*A **Theorem** is a statement that can be shown to be true.*

*A **proof** is a valid argument that establishes the truth of a theorem.*
The statements used in a proof can include axioms (or postulates).

Direct Proofs

A direct proof of a conditional statement $p \to q$ is constructed when the first step is the assumption that p is true.

Definition 19

The integer n is even if there exists an integer k such that $n = 2k$, and n is odd if there exists an integer k such that $n = 2k + 1$.

Direct Proof

Example 23

Give a direct proof of the theorem "If n is odd integer, then n^2 is odd".

Proof.

Assume that n is odd.

$n = 2k + 1$ where $k \in \mathbb{Z}$ (from definition of an odd integer)

$\rightarrow n^2 = (2k+1)^2$ (Squaring both sides of the equation)

$\rightarrow n^2 = (2k+1)(2k+1)$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

Hence if n is odd integer, then n^2 is odd $\square$

Direct Proof

Example 24

Proof that, If n is even integer, then n^2 is even.

Proof.

Assume n is an even integer.

$\rightarrow n = 2k$ where $k \in \mathbb{Z}$ (by definition)

$\rightarrow n^2 = (2k)^2$

$\rightarrow n^2 = 4k^2$

$\rightarrow n^2 = 2(2k^2)$

Which means that n^2 is even.

Hence If n is even integer, then n^2 is even. $\square$

Proof by Contraposition

This makes use of the fact that the conditional statement $p \rightarrow q$ is equivalent to its contrapositive, $\neg q \rightarrow \neg p$

Here we take $\neg q$ as a premise, and using axioms, definitions, and previously proven theorems, together with rules of inference, we show that $\neg p$ must follow.

Proof by Contraposition

Example 25

Prove that if n is an integer and $3n+2$ **is odd**, then n is **odd**.

Proof.

Assume that the conclusion of the conditional statement
if $3n+2$ is odd, then n is false;
Assume n is even
By definition of an integer,
Let $n = 2k$ for some integer k.

$$\begin{aligned} 3n+2 &= 3(2k)+2 (\text{substitute } 2k) \\ &= 6k+2 \\ &= 2(3k+1) \end{aligned}$$

This shows that $3n+2$ is even (since it is a multiple of 2)

Because the negation of the conclusion of the conditional statement implies that the hypothesis is false, the original conditional statement is true. Hence the proof □

Proof by Contraposition

Exercise C:
Prove that if n is an integer and n^2 is odd, then n is odd

Proof by Contradiction

In a proof by contradiction, the contrary is assumed to be true at the start of the proof. After logical reasoning at each step, the assumption is shown not to be true.

Definition 20

*The real number r is **rational** if there exist integers p and q with $q \neq 0$ such that $r = \frac{a}{b}$.*
*A real number that is not rational is called **irrational***

Proof by Contradiction

Example 26

Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction

Proof.

Let assume $\sqrt{2}$ is rational.
Then $\sqrt{2} = a/b$ where $a, b \in \mathbb{Z}$, $b \neq 0$ and a, b are co-prime
$\rightarrow 2 = \frac{a^2}{b^2}$ (squaring both sides)
Hence $2b^2 = a^2$
It follows that a^2 is even. (by definition of an even integer)
Now since a^2 is even it means a **is also even**.
This implies $a = 2k$ for some $k \in Z$ by definition of an even integer.
$\rightarrow 2b^2 = (2k)^2$
$\rightarrow 2b^2 = 4k^2$
$\rightarrow b^2 = 2k^2$
This means b^2 is even from which follows again that b **itself is even** by definition. And this a contradiction.

Proof by Contradiction

Exercise D:

Using a proof by contradiction, Prove that

1. $\sqrt{5}$ is irrational.
2. if $3n+2$ is odd, then n is odd.

End of Lecture

Questions...???

Thanks

Reference Books

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