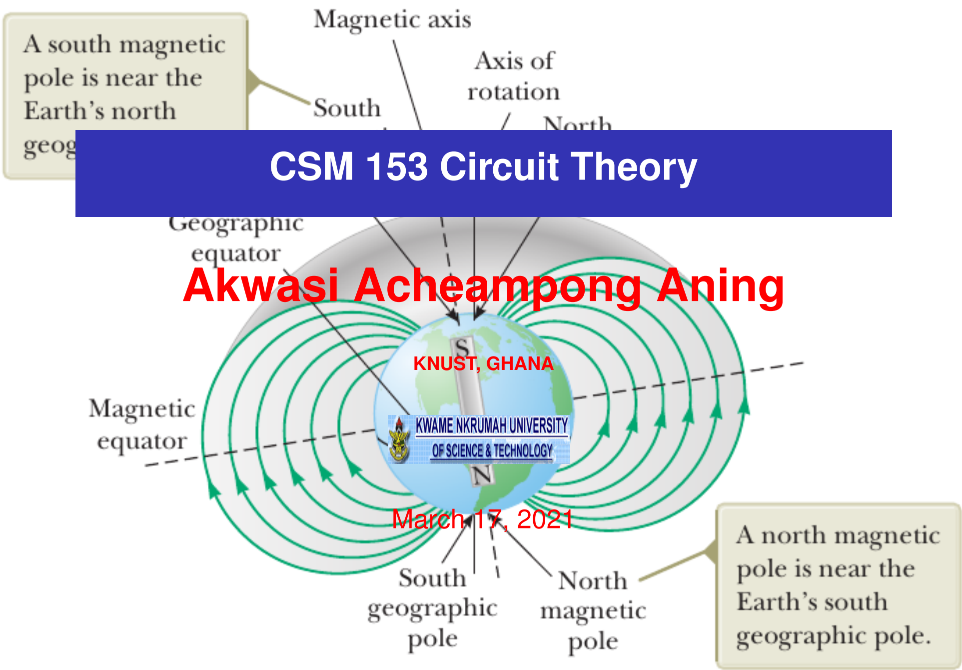
CSM 153 Circuit Theory



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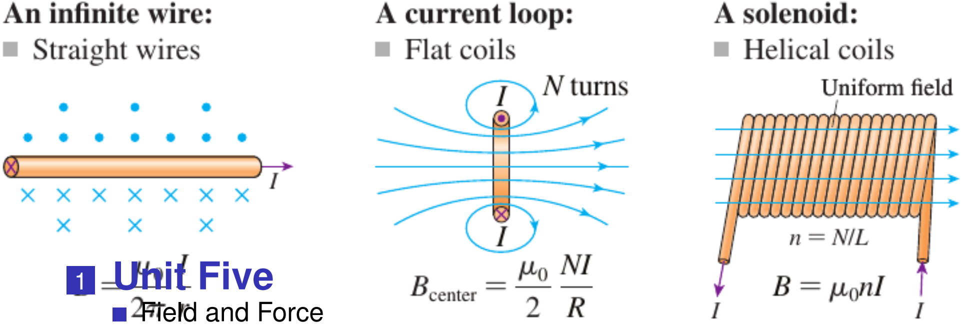


1 Unit Five

- Field and Force
- Ampere's and Biot-Savart Laws
- Inductance

UNIT FIVE
Magnetism

- Field and Force
- Ampere’s and Biot-Savart Laws
- Inductance and Induction

Field and Force

Field and Force

- **Magnetism** - It is a phenomenon associated with magnetic fields, which arise from the motion of electric charges. This motion can take many forms:
  - It can be an electric current in a conductor or charged particles moving through space, or
  - It can be the motion of an electron in an atomic orbital
- Magnetism is also associated with elementary particles, such as the electron, that have a property called spin.
- **Magnetic field** - It is a vector field in the neighbourhood of a magnet, electric current, or changing electric field, in which magnetic forces are observable. Magnetic fields such as that of Earth cause magnetic compass needles and other permanent magnets to line up in the direction of the field.
- **Magnetic force** - It is an attraction or repulsion that arises between electrically charged particles because of their motion. It is the basic force responsible for such effects as the action of electric motors and the attraction of magnets for iron.
- Electric forces exist among stationary electric charges
- A magnet has two poles, north and south. The north pole is that end which points toward geographic north when the magnet is freely suspended. Like poles of two magnets repel each other, whereas unlike poles attract

Field and Force

- Electric and magnetic forces exist among moving electric charges. The magnetic force between two moving charges may be described as the effect exerted upon either charge by a magnetic field created by the other
- The magnetic field around elementary particles is a basic characteristic of the particles, just as their mass and electric charge are intrinsic properties
- The magnetic field of all the electrons in certain materials add together to give a resultant magnetic field around the material
- However, in other materials the magnetic fields of all the electrons cancel out, giving no resultant magnetic field surrounding the material.
- Magnetic field $\overrightarrow{B}$ is a vector quantity that can be defined at a point in terms of the:
  - velocity $\overrightarrow{v}$ of a moving charged particle, and
  - force $\overrightarrow{F_B}$ that acts on the particle at that point
- When an electrically charged particle of charge q moves with a velocity, $\overrightarrow{v}$ through a point in a magnetic field, a magnetic force, $\overrightarrow{F_B}$ is exerted on the moving particle provided the velocity is inclined to an axis through the point

Field and Force

- The magnetic force is proportional to the charge q of the particle
- The magnetic force on a negative charge is directed opposite to the force on a positive charge moving in the same direction
- The magnetic force is proportional to the magnitude of the magnetic field vector $\overrightarrow{B}$
- The magnetic force is proportional to the speed v of the particle
- If the velocity vector makes an angle θ with the magnetic field, the magnitude of the magnetic force is proportional to sin θ

$$\overrightarrow{F_B} = q\overrightarrow{v}x\overrightarrow{B}sin\theta \tag{1}$$

- When a charged particle moves parallel to the magnetic field vector, the magnetic force on the charge is zero
- When a charged particle moves in a direction not parallel to the magnetic field vector, the magnetic force acts in a direction perpendicular to both $\overrightarrow{v}$ and $\overrightarrow{B}$; that is, the magnetic force is perpendicular to the plane formed by $\overrightarrow{v}$ and $\overrightarrow{B}$.

$$\overrightarrow{F_B} = q\overrightarrow{v}x\overrightarrow{B} \tag{2}$$

- This means the force, $\overrightarrow{F_B}$ on the charged particle is equal to the charge q times the cross product of its velocity, $\overrightarrow{v}$ and the magnetic field, $\overrightarrow{B}$

Field and Force

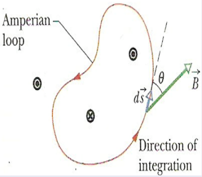
Why is magnetism important?

- Motors and generators are based on magnetic forces
- Many forms of data storage, from hard disks to the stripe on your credit card, are magnetic
- Magnetic resonance imaging (MRI) is essential to modern medicine
- Magnetic levitation trains are being built around the world.
- The earth's magnetic field keeps the solar wind from sterilizing the surface
- Exploration of gold and other minerals are based on their magnetic properties
- Magnets are used to make a tight seal on the doors to refrigerators and freezers
- There would be no life and no modern technology without magnetism

Ampere's and Biot-Savart Laws

Ampere's and Biot-Savart Laws

Ampere's Law



- **Ampere's Law** states that the line integral of the tangential component of the magnetic field, $\vec{B}$ around a closed loop (path) is equal to the current I enclosed by the loop

$$\oint \vec{B}.\vec{ds} = \mu_0 I \quad (3)$$

- Consider a concentric circular Amperian loop of radius r that lies outside a wire carrying a current I
- Let the magnetic field, $\vec{B}$ have the same magnitude B at every point on the loop

Ampere's Law

- Assume that $\vec{B}$ and $\vec{ds}$ are tangential to the loop at every point along the loop
- Then at every point, the angle θ between them is zero, so that cos θ = cos 0 = 1

$$\oint \vec{B}.\vec{ds} = \oint Bcos\theta ds = B(2\pi r) = \mu_0 I$$

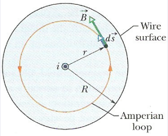
$$\therefore B = \frac{\mu_0 I}{2\pi r} \quad (4)$$

Please note:

- $\oint \vec{ds}$ is the summation of all the line segment lengths ds around the circular loop
- $\oint \vec{ds}$ gives the circumference $2\pi r$ of the circular loop

Ampere's and Biot-Savart Laws

Ampere's Law



- Ampere's law can be used to find magnetic field that a current produces inside a long straight wire of circular cross section
- The current is uniformly distributed over the cross section of the wire and emerges from the page
- The Amperian loop is a concentric circle that lies inside the wire
- Consider circular Amperian loop of radius r drawn inside a wire or radius R carrying a current I
- Let the current be uniformly distributed over the cross section of the wire so that the magnetic field $\vec{B}$ has cylindrical symmetry

Ampere's Law

- Assuming that magnetic field $\vec{B}$ and $\vec{ds}$ are tangential to the loop, we have $\oint \vec{B}.d\vec{s} = B \oint ds = B(2\pi R) = \mu_0 I$
- But since the current is uniformly distributed the current I encircled by the loop is proportional to the area encircled by the loop.

  Solve for B:

$$\therefore B = \frac{\mu_0 I}{2\pi R} \tag{5}$$

  (for $R \geq r$)

- Now consider the interior of the wire, where $r < R$. Here the current I' passing through the plane of Amperian loop is less than the total current I

Ampere's and Biot-Savart Laws

Ampere's Law

- Set the ratio of the current I' enclosed by the Amperian loop to the entire current I equal to the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire:

$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2} \quad \therefore I' = \frac{r^2}{R^2} I$$

Apply Ampère's law to the loop:

$$\oint \vec{B}.d\vec{s} = B \oint ds = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I\right)$$

$$\therefore B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r \qquad (6)$$

(for $r<R$)

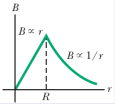
- Inside the wire, the magnitude B of the magnetic field is proportional to r
- The magnitude B is zero at the centre and maximum at the surface, where $r = R$

Ampere's Law

- Expressions for the magnetic field outside and inside the wire yield the same result at the surface of the wire at $r = R$

Magnetic field variation inside and outside a cylindrical conductor

- The magnetic field B is zero at the centre of the conductor, and then increases linearly with r up to $r = R$
- However, with $r>R$, B decreases inversely with r as $1/r$.



- The field is proportional to r inside the wire and varies as $1/r$ outside the wire

Ampere's and Biot-Savart Laws

Ampere's Law

Question A long straight wire of radius 3.0 cm is carrying current 1.5 A out of the page. Calculate the magnitude of the magnetic field at

- i 2 cm radius from the centre
- ii Mid-point of the wire
- iii Surface of the wire (B_{max})
- iv 3.5 cm outside the wire

Solution

- i Using $B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r = \left[\frac{4\pi x 10^{-7} x 1.5}{2\pi x (3x10^{-2})^2}\right](2x10^{-2})$

  $= 6.7\ x\ 10^{-6}\ T = 6.7\ \mu\ T$
- ii At mid-point

  $$B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r = \left[\frac{4\pi x 10^{-7} x 1.5}{2\pi x (3x10^{-2})^2}\right](1.5x10^{-2})$$

  $= 5.0\ x\ 10^{-6}\ T = 5.0\ \mu\ T$

Ampere's Law

- i At $r = R$

  $$B = \left(\frac{\mu_0 I}{2\pi R^2}\right) R = \left[\frac{4\pi x 10^{-7} x 1.5}{2\pi x 3 x 10^{-2}}\right]$$

  $= 1.0\ x\ 10^{-6}\ T = 10.0\ \mu\ T$
- ii Outside the surface

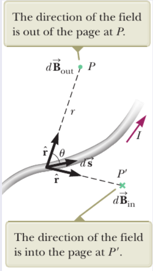
  $$B = \left(\frac{\mu_0 I}{2\pi r}\right) = \left[\frac{4\pi x 10^{-7} x 1.5}{2\pi x 3.5 x 10^{-2}}\right]$$

  $= 8.57\ x\ 10^{-6}\ T = 8.57\ \mu\ T$

Ampere's and Biot-Savart Laws

Biot-Savart Law

The expression for the **Biot–Savart law** is based on the following experimental observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{s}$ of a wire carrying a steady current I as shown in the figure below:



- The vector $d\vec{B}$ is perpendicular both to $d\vec{s}$ (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from $d\vec{s}$ toward P.

Biot-Savart Law

- The magnitude of $d\vec{B}$ is inversely proportional to r^2 , where r is the distance from $d\vec{s}$ to P.
- The magnitude of $d\vec{B}$ is proportional to the current I and to the magnitude ds of the length element $d\vec{s}$
- The magnitude of $d\vec{B}$ is proportional to $sin\theta$, where θ is the angle between the vectors $d\vec{s}$ and $\hat{r}$

  These observations are summarized in the **Biot–Savart law** as

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s}\, x \hat{r}}{r^2} \tag{7}$$

  where μ_0 is a constant called permeability of free space $\mu_0 = 4\pi x 10^{-7}\ Tm/A$

- The Biot–Savart law describes the magnetic field produced by a current element.

Inductance

Faraday and Lens' Laws

- **Faraday's law** of induction states that the emf induced in a loop is directly proportional to the time rate of change of magnetic flux through the loop, or

$$\varepsilon = -\frac{d\Phi_B}{dt} \tag{8}$$

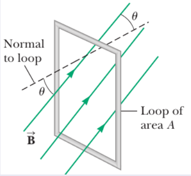
where $\Phi_B = \int \overrightarrow{B}.d\overrightarrow{A}$ is the magnetic flux through the loop

- If a coil consists of N loops with the same area, and Φ_B is the magnetic flux through one loop, an emf is induced in every loop. If the loops are in series, their emfs add; therefore, the total induced emf in the coil is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} \tag{9}$$

- If the area A lies in a uniform magnetic $\overrightarrow{B}$ as shown in figure on the right, the magnetic flux through the loop is equal to $BA\cos\theta$

Faraday and Lens' Laws



$$\therefore \varepsilon = -\frac{d}{dt}(BA\cos\theta) \tag{10}$$

where θ is the angle between the magnetic field and the normal to loop

- **Lenz's law** states that the induced current and induced emf in a conductor are in such a direction as to set up a magnetic field that opposes the change that produced them.

Faraday and Lens' Laws

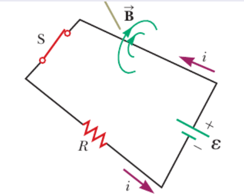
From Eqn. 10, we see that an emf can be induced in the circuit in several ways:

- The magnitude of $\vec{B}$ can change with time
- The area enclosed by the loop can change with time
- The angle θ between $\vec{B}$ and the normal to the loop can change with time
- Any combination of the above can occur
- Faraday's law also tells us that a changing magnetic field produces an electric field; and that a straight wire of length l moving with speed v perpendicular to a magnetic field of strength B has an emf induced between its ends equal to

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{\Delta\Phi_B}{\Delta t} = \frac{B\Delta A}{\Delta t} = \frac{Blv\Delta t}{\Delta t}$$

$$\therefore \varepsilon = Blv \tag{11}$$

Self and Mutual Inductance



- Consider a circuit consisting of a switch, a resistor, and a source of emf as shown above. When the switch is closed the current does not immediately jump from zero to its maximum value.
- After the switch is closed, the current produces a magnetic flux through the area enclosed by the loop. As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop

Self and Mutual Inductance

- The increasing flux creates an induced emf in the circuit. The direction of the induced emf is such that it would cause an induced current in the loop (if the loop did not already carry a current), which would establish a magnetic field opposing the change in the original magnetic field.
- The direction of the induced emf is opposite the direction of the emf of the battery, which results in a gradual rather than instantaneous increase in the current to its final equilibrium value
- This effect is called **self-induction** because the changing flux through the circuit and the resultant induced emf arise from the circuit itself. The emf ε_L set up in this case is called a **self-induced emf**
- The magnetic flux is proportional to the magnetic field, which in turn is proportional to the current in the circuit.

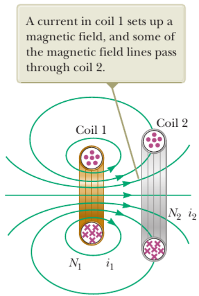
Self and Mutual Inductance

- Self-induced emf in a circuit is always proportional to the time rate of change of the current in the circuit. For any loop of wire, we can write this proportionality as

$$\varepsilon_L = -L\frac{di}{dt} \tag{12}$$

  where L is a proportionality constant called the inductance of the loop and it depends on the geometry of the loop and other physical characteristics
- The SI unit of inductance is the **henry** (H)
- The inductance is a measure of the opposition to a change in current
- The magnitude of L depends on the size and shape of the coil and on the presence of an iron core

Self and Mutual Inductance



- If two coils of wire are near one another, as in fig. above, a changing current in one will induce an emf in the other
- We apply Faraday's law to coil 2: the emf ε_2 induced in coil 2 is proportional to the rate of change of magnetic flux passing through it
- A changing flux in coil 2 is produced by a changing current I_1 in coil 1

Self and Mutual Inductance

- The mutual inductance M_{12} of coil 2 with respect to coil 1:

$$M_{12} = \frac{N_2 \Phi_{12}}{i_1} \tag{13}$$

- Mutual inductance depends on the geometry of both circuits and on their orientation with respect to each other. As the circuit separation distance increases, the mutual inductance decreases because the flux linking the circuits decreases
- From Faraday's law and Eqn. 13, if the current i_1 varies with time, then the emf induced by coil 1 in coil 2 is

$$\varepsilon_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} i_1}{N_2}\right) = -M_{12} \frac{di_1}{dt} \tag{14}$$

Self and Mutual Inductance

- If the current i_2 varies with time, then the emf induced by coil 2 in coil 1, will be

$$\varepsilon_1 = -M_{21}\frac{di_2}{dt} \quad (15)$$

- In mutual induction, the emf induced in one coil is always proportional to the rate at which the current in the other coil is changing. Although the proportionality constants M_{12} and M_{21} have been treated separately, it can be shown that they are equal $M_{12} = M_{21} = M$
- A transformer is an example of mutual inductance in which the coupling is maximized so that nearly all flux lines pass through both coils.
- The transformer consists of two coils: the primary and the secondary arranged close together. The emf supplied to the primary produces a current which induces an emf in the secondary.

Self and Mutual Inductance

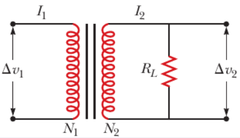
- This induced emf will be larger or smaller than the original emf, depending on the number of turns in the coils
- Mutual inductance has other uses as well, including inductive charging of cell phones, electric cars, and other devices with rechargeable batteries
- The energy stored in a magnetic field can be expressed as:

$$U_B = \frac{1}{2}LI^2 \quad (16)$$

  This energy is the magnetic counterpart to the energy stored in the electric field of a charged capacitor.
- The energy density at a point where the magnetic field is B is:

$$u_B = \frac{B^2}{2\mu_0} \quad (17)$$

The Transformer



- A transformer is a device used to increase or decrease the voltage in a circuit without an appreciable loss of power
- The figure above shows a simple transformer consisting of two wire coils around a common iron core
- The coil carrying the input power is called the primary, and the other coil is called the secondary. Either coil of a transformer can be used for the primary or secondary.
- The transformer operates on the principle that an alternating current in one circuit induces an alternating emf in a nearby circuit due to the mutual inductance of the two circuits

The Transformer

- If the potential difference across the secondary coil is larger than the potential difference across the primary coil, the current in the secondary coil is smaller than the current in the primary coil, and vice versa
- Actual power distribution transformers often have efficiencies of 98% or more
- If ϕ_{turn} is the magnetic flux per turn of the primary coil of N_1 turns, then the potential drop V_1 across the primary coil is:

$$V_1 = N_1 \frac{d\phi_{turn}}{dt} \tag{18}$$

- If there is no flux leakage out of the iron core, the flux through each turn is the same for both coils.

$$V_2 = N_2 \frac{d\phi_{turn}}{dt} \tag{19}$$

$$V_2 = \frac{N_2}{N_1} V_1 \tag{20}$$

The Transformer

- If N_2 is greater than N_1, the potential difference across the secondary coil is greater than the potential drop across the primary coil, and the transformer is called a step-up transformer
- If N_2 is less than N_1, the potential difference across the secondary coil is less than the potential drop across the primary coil, and the transformer is called a step-down transformer
- In an ideal transformer where there are no losses, the power supplied by the source is equal to the power in the secondary circuit.Thus:

$$I_1 \Delta V_1 = I_2 \Delta V_2 \tag{21}$$

- The value of the load resistance R_L determines the value of the secondary current

The Transformer

- $$I_2 = \frac{\Delta V_2}{R_L} \tag{22}$$
- The current in the primary is:

$$I_1 = \frac{\Delta V_1}{R_{eq}} \tag{23}$$

where

$$R_{eq} = \left(\frac{N_1}{N_2}\right)^2 R_L \tag{24}$$

R_{eq} is the equivalent resistance of the load resistance when viewed from the primary side

- The ratio of secondary to primary current is in the inverse ratio of turns:

$$I_2 = \frac{N_1}{N_2} I_1 \tag{25}$$

THANK YOU FOR YOUR ATTENTION