# CSM 165: Discrete Mathematics for Computer Science

Chapter 1: Propositional and first order predicate logic

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### **Content**

Propositional Equivalence

Inference

A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology

### Definition 2 (Contradiction)

A compound proposition that is always false is called a contradiction

### Definition 3 (Contingency)

A compound proposition that is neither a tautology nor a contradiction is called a contingency.

Table 1: A tautology and a Contradiction

p	$\neg p$	$p \lor \neg p$	$p \land \neg p$
Т	F	Т	F
F	Т	Т	F

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### Definition 4 (Logical Equivalence)

Compound propositions that have the same truth values in all possible cases are called **logically equivalent**.

The compound propositions p and q are also called **logically** equivalent if  $p \leftrightarrow q$  is a **tautology**. The notation  $p \equiv q$  denotes that p and q are logically equivalent.

### De Morgan's Laws

- 1.  $\neg (p \land q) \equiv \neg p \lor \neg q$
- 2.  $\neg (p \lor q) \equiv \neg p \land \neg c$

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# **Logical Equivalence**

### Example 2

1. Show that  $\neg (p \lor q)$  and  $\neg p \land \neg q$  are logically equivalent

Table 2: Truth Tables for  $\neg (p \lor q)$  and  $\neg p \land \neg q$ 

p	q	$p \vee q$	$\neg (p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	Т	Т	F	T	F	F
F	F	F	T	T	T	T

- 2. Show that  $p \rightarrow q$  and  $\neg p \lor q$  and equivalent.
- 3. Show that  $p \land (q \lor r)$  and  $(p \lor q) \land (p \lor r)$ .

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T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	Т	Т	F	T	F	F
F	F	F	T	T	T	T

- 2. Show that  $p \rightarrow q$  and  $\neg p \lor q$  and equivalent.
- 3. Show that  $p \land (q \lor r)$  and  $(p \lor q) \land (p \lor r)$ .

# **Logical Equivalence**

### Solution to example 2 question 3

Table 3: Truth Table for  $p \land (q \lor r)$  and  $(p \lor q) \land (p \lor r)$ 

p	q	r	q∧r	$p \lor (q \land r)$	$p \lor q$	$p \lor r$	$(p \lor q) \land (p \lor r)$
T	Т	Т	T	T	T	T	T
T	Т	F	F	T	T	T	T
T	F	Т	F	T	T	T	T
T	F	F	F	T	T	T	T
F	Т	Т	T	T	T	T	T
F	Т	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

# **Precedence of Logical Operators**

Table 4: Precedence of Logical Operators

Operators	Names	Precedence
_	Negation	1
٨	Conjunction	2
V	Disjunction	3
$\rightarrow$	Implication	4
$\leftrightarrow$	Biconditional	5

### Table 5: Logical Equivalences

$\begin{array}{ll} p \wedge \mathbf{T} \equiv p \\ p \vee \mathbf{F} \equiv p \\ \end{array} \qquad \qquad$	Equivalence	Name
$\begin{array}{lll} p\vee\mathbf{T}\equiv\mathbf{T} \\ p\wedge\mathbf{F}\equiv\mathbf{F} \\ \end{array} \qquad \begin{array}{ll} \text{Domination laws} \\ p&\neq p\equiv p \\ p\wedge p\equiv p \\ \hline & \neg(\neg p)\equiv p \\ \end{array} \qquad \begin{array}{ll} \text{Idempotent laws} \\ p&\neq q\equiv q\vee p \\ p\wedge q\equiv q\wedge p \\ p\wedge q\equiv q\wedge p \\ (p\vee q)\vee r\equiv p\vee (q\vee r) \\ (p\wedge q)\wedge r\equiv p\wedge (q\wedge r) \\ p\neq (q\wedge r)\equiv (p\vee q)\wedge (p\vee r) \\ p\wedge (q\vee r)\equiv (p\wedge q)\vee (p\wedge r) \\ \hline & p\wedge (q\vee r)\equiv (p\wedge q)\vee (p\wedge r) \\ \hline & \neg(p\wedge q)\equiv \neg p\wedge \neg q \\ \hline & p\vee (p\wedge q)\equiv \neg p\wedge \neg q \\ \hline & p\vee (p\wedge q)\equiv p \\ \hline & p\wedge (p\vee q)\equiv p \\ \hline & p\wedge (p\wedge q)\equiv p \\ \hline & p\wedge (p\wedge q)\equiv p \\ \hline & p\wedge (p\wedge q)\equiv p \\ \hline & p\wedge p\wedge p\equiv \mathbf{T} \\ \end{array} \qquad \begin{array}{ll} \text{Negation laws} \\ \end{array}$		Identity laws
$\begin{array}{lll} p \wedge \mathbf{F} \equiv \mathbf{F} \\ \\ p \vee p \equiv p \\ p \wedge p \equiv p \\ \\ \neg (\neg p) \equiv p \\ \\ \hline \\ \neg (\neg p) \equiv p \\ \\ \hline \\ p \vee q \equiv q \vee p \\ p \wedge q \equiv q \wedge p \\ \\ p \wedge q \equiv q \wedge p \\ \\ (p \vee q) \vee r \equiv p \vee (q \vee r) \\ (p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \\ \\ (p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \\ \\ p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) \\ p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \\ \hline \\ \neg (p \wedge q) \equiv \neg p \vee \neg q \\ \neg (p \vee q) \equiv \neg p \wedge \neg q \\ \\ \hline \\ p \vee (p \wedge q) \equiv p \\ \hline \\ p \wedge (p \vee q) \equiv p \\ \\ \hline \\ p \wedge (p \vee q) \equiv p \\ \\ \hline \\ p \wedge (p \vee q) \equiv p \\ \\ \hline \\ \end{array}  \begin{array}{l} \text{Idempotent laws} \\ \\ \text{Commutative laws} \\ \\ \text{Distributive laws} \\ \\ \text{De Morgan's laws} \\ \\ \hline \\ p \vee (p \wedge q) \equiv p \\ \\ \\ p \wedge (p \vee q) \equiv p \\ \\ \\ \hline \\ p \vee p \rightarrow p \equiv \mathbf{T} \\ \\ \end{array}  \text{Negation laws}$	$p \lor \mathbf{F} \equiv p$	
$\begin{array}{c} p\vee p\equiv p\\ p\wedge p\equiv p\\ p\wedge p\equiv p\\ \hline \neg(\neg p)\equiv p\\ \hline \\ \neg(\neg p)\equiv p\\ \hline \\ (p\vee q)\equiv q\vee p\\ (p\wedge q)\vee r\equiv p\vee (q\vee r)\\ (p\wedge q)\wedge r\equiv p\wedge (q\wedge r)\\ \hline \\ (p\vee q)\wedge r\equiv p\wedge (q\wedge r)\\ \hline \\ (p\wedge q)\wedge r\equiv p\wedge (q\wedge r)\\ \hline \\ p\vee (q\wedge r)\equiv (p\vee q)\wedge (p\vee r)\\ \hline \\ p\wedge (q\vee r)\equiv (p\vee q)\wedge (p\vee r)\\ \hline \\ p\wedge (q\vee r)\equiv (p\wedge q)\vee (p\wedge r)\\ \hline \\ \neg(p\wedge q)\equiv \neg p\wedge \neg q\\ \hline \\ \neg(p\vee q)\equiv \neg p\wedge \neg q\\ \hline \\ p\vee (p\wedge q)\equiv p\\ \hline \\ p\wedge (p\vee q)\equiv p\\ \hline \\ p\wedge (p\vee q)\equiv p\\ \hline \\ p\wedge (p\vee q)\equiv p\\ \hline \\ \hline \end{array}$		Domination laws
$\begin{array}{lll} p \wedge p \equiv p & & & \\ \hline \neg (\neg p) \equiv p & & & \\ \hline p \vee q \equiv q \vee p & & & \\ \hline p \wedge q \equiv q \wedge p & & & \\ \hline (p \vee q) \vee r \equiv p \vee (q \vee r) & & \\ \hline (p \wedge q) \wedge r \equiv p \wedge (q \wedge r) & & \\ \hline p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) & & \\ \hline p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) & & \\ \hline p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) & & \\ \hline \neg (p \wedge q) \equiv \neg p \wedge \neg q & & \\ \hline \neg (p \vee q) \equiv \neg p \wedge \neg q & \\ \hline p \vee (p \wedge q) \equiv p & & \\ \hline p \wedge (p \vee q) \equiv p & & \\ \hline p \wedge (p \vee q) \equiv p & & \\ \hline \end{array}$	$p \wedge \mathbf{F} \equiv \mathbf{F}$	
	$p \lor p \equiv p$	Idempotent laws
$\begin{aligned} p \lor q &\equiv q \lor p \\ p \land q &\equiv q \land p \end{aligned} & \text{Commutative laws} \\ (p \lor q) \lor r &\equiv p \lor (q \lor r) \\ (p \land q) \land r &\equiv p \land (q \land r) \end{aligned} & \text{Associative laws} \\ p \lor (q \land r) &\equiv (p \lor q) \land (p \lor r) \\ p \land (q \lor r) &\equiv (p \land q) \lor (p \land r) \end{aligned} & \text{Distributive laws} \\ \neg (p \land q) &\equiv \neg p \lor \neg q \\ \neg (p \land q) &\equiv \neg p \lor \neg q \end{aligned} & \text{De Morgan's laws} \\ \neg (p \lor q) &\equiv \neg p \land \neg q \end{aligned} & p \lor (p \land q) &\equiv p \\ p \land (p \lor q) &\equiv p \end{aligned} & \text{Absorption laws} \\ p \lor \neg p \Rightarrow \mathbf{T} & \text{Negation laws} \end{aligned}$	$p \wedge p \equiv p$	
$\begin{array}{ll} p \wedge q = q \wedge p \\ \\ (p \vee q) \vee r \equiv p \vee (q \vee r) \\ (p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \\ \\ p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) \\ p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \\ \\ \neg (p \wedge q) \equiv \neg p \vee \neg q \\ \neg (p \vee q) \equiv \neg p \wedge \neg q \\ \\ p \vee (p \wedge q) \equiv p \\ p \wedge (p \vee q) \equiv p \\ \\ p \wedge p \vee p \equiv \mathbf{T} \end{array} \qquad \begin{array}{ll} \text{Associative laws} \\ \text{Distributive laws} \\ \text{De Morgan's laws} \\ \text{De Morgan's laws} \\ \text{De Morgan's laws} \\ \text{Perposition laws} \\ Perposit$	$\neg(\neg p) \equiv p$	Double negation law
$ (p \lor q) \lor r \equiv p \lor (q \lor r) $ (p \land q) \land r \sim p \land (q \land r)	$p \vee q \equiv q \vee p$	Commutative laws
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $\neg (p \wedge q) \equiv \neg p \vee \neg q$ $\neg (p \vee q) \equiv \neg p \wedge \neg q$ $p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$ $p \wedge (p \vee q) \equiv p$ $p \wedge (p \vee q) \equiv T$ Negation laws	$p \wedge q \equiv q \wedge p$	
$\begin{array}{ll} p\vee (q\wedge r)\equiv (p\vee q)\wedge (p\vee r) \\ p\wedge (q\vee r)\equiv (p\wedge q)\vee (p\wedge r) \\ \hline \neg (p\wedge q)\equiv \neg p\vee \neg q \\ \neg (p\vee q)\equiv \neg p\wedge \neg q \\ \hline p\vee (p\wedge q)\equiv p \\ p\wedge (p\vee q)\equiv p \\ \hline p\wedge (p\vee q)\equiv p \\ \hline \end{array} \qquad \begin{array}{ll} \text{De Morgan's laws} \\ \hline \text{Absorption laws} \\ \hline p \vee \neg p \equiv \mathbf{T} \\ \hline \end{array}$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative laws
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $\neg (p \wedge q) \equiv \neg p \vee \neg q$ $\neg (p \vee q) \equiv \neg p \wedge \neg q$ $p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$ $p \wedge (p \vee q) \equiv p$ $p \vee \neg p \equiv \mathbf{T}$ De Morgan's laws  Absorption laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	
	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	Distributive laws
$\neg (p \lor q) \equiv \neg p \land \neg q$ $p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$ $p \lor \neg p \equiv \mathbf{T}$ Absorption laws $p \lor \neg p \equiv \mathbf{T}$ Negation laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	
$\begin{array}{ll} p\vee (p\wedge q)\equiv p & \text{Absorption laws} \\ p\wedge (p\vee q)\equiv p & \\ \\ p\vee \neg p\equiv \mathbf{T} & \text{Negation laws} \end{array}$	$\neg (p \land q) \equiv \neg p \lor \neg q$	De Morgan's laws
$p \wedge (p \vee q) \equiv p$ $p \vee \neg p \equiv \mathbf{T}$ Negation laws	$\neg(p\vee q)\equiv \neg p\wedge \neg q$	
$p \lor \neg p \equiv \mathbf{T}$ Negation laws	$p \lor (p \land q) \equiv p$	Absorption laws
	$p \wedge (p \vee q) \equiv p$	
$p \land \neg p \equiv \mathbf{F}$	$p \lor \neg p \equiv \mathbf{T}$	Negation laws
	$p \land \neg p \equiv \mathbf{F}$	

Table 6: Logical Equivalence Involving Conditional Statements.

$$p \rightarrow q \equiv \neg p \lor q$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$p \lor q \equiv \neg p \rightarrow q$$

$$p \land q \equiv \neg (p \rightarrow \neg q)$$

$$\neg (p \rightarrow q) \equiv p \land \neg q$$

$$(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$$

$$(p \rightarrow r) \land (q \rightarrow r) \equiv (p \lor q) \rightarrow r$$

$$(p \rightarrow q) \lor (p \rightarrow r) \equiv p \rightarrow (q \lor r)$$

$$(p \rightarrow r) \lor (q \rightarrow r) \equiv (p \land q) \rightarrow r$$

Table 7: Equivalences Involving

$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$$

$$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Table 5: Logical Equivalences

Equivalence	Name
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \lor \mathbf{T} \equiv \mathbf{T}$ $p \land \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \lor p \equiv p$ $p \land p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative laws
$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	Associative laws
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive laws
$\neg (p \land q) \equiv \neg p \lor \neg q$ $\neg (p \lor q) \equiv \neg p \land \neg q$	De Morgan's laws
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption laws
$p \lor \neg p \equiv \mathbf{T}$ $p \land \neg p \equiv \mathbf{F}$	Negation laws

Table 6: Logical Equivalences Involving Conditional Statements.

```
p \rightarrow q \equiv \neg p \lor q
p \rightarrow q \equiv \neg q \rightarrow \neg p
p \lor q \equiv \neg p \rightarrow q
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\neg (p \rightarrow q) \equiv p \land \neg q
(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)
(p \rightarrow r) \land (q \rightarrow r) \equiv (p \lor q) \rightarrow r
(p \rightarrow q) \lor (p \rightarrow r) \equiv p \rightarrow (q \lor r)
(p \rightarrow r) \lor (q \rightarrow r) \equiv p \rightarrow (q \lor r)
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$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$

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Table 5: Logical Equivalences

Equivalence	Name
$p \wedge \mathbf{T} \equiv p$	Identity laws
$p \vee \mathbf{F} \equiv p$	
$p \vee \mathbf{T} \equiv \mathbf{T}$	Domination laws
$p \wedge \mathbf{F} \equiv \mathbf{F}$	
$p \lor p \equiv p$	Idempotent laws
$p \wedge p \equiv p$	
$\neg(\neg p) \equiv p$	Double negation law
$p \lor q \equiv q \lor p$	Commutative laws
$p \wedge q \equiv q \wedge p$	
$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative laws
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	Distributive laws
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	
$\neg (p \land q) \equiv \neg p \lor \neg q$	De Morgan's laws
$\neg(p\vee q)\equiv \neg p\wedge \neg q$	
$p \lor (p \land q) \equiv p$	Absorption laws
$p \wedge (p \vee q) \equiv p$	
$p \lor \neg p \equiv \mathbf{T}$	Negation laws
$p \land \neg p \equiv \mathbf{F}$	

Table 6: Logical Equivalences Involving Conditional Statements.

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$$\neg (p \rightarrow q) \equiv p \land \neg q$$

$$(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$$

$$(p \rightarrow r) \land (q \rightarrow r) \equiv (p \lor q) \rightarrow r$$

$$(p \rightarrow q) \lor (p \rightarrow r) \equiv p \rightarrow (q \lor r)$$

$$(p \rightarrow r) \lor (q \rightarrow r) \equiv (p \land q) \rightarrow r$$

Table 7: Equivalences Involving Biconditional Statements.

$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$$

$$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

#### **Definition 5**

**Premise:** It is the proposition on the basis of which we would be able to draw a conclusion.

It can be thought of as an evidence or assumption.

**Conclusion:** It is the a proposition that is reached from a given set of premises.

Argument: Sequence of statements that ends with a conclusion.

**Valid Argument:** An argument is valid if and only if it is impossible for all the premises to be true and the conclusion to be false. OR

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# **Validity Using Truth Table**

### Example 3

Determine whether the following conclusion C follows logically from the premises  $H_1$  and  $H_2$ .

- 1.  $H_1 P \rightarrow Q \quad H_2:P \quad C:Q$
- 2.  $H_1 P \rightarrow Q \quad H_2: \neg P \quad C:Q$
- 3.  $H_1: P \rightarrow Q$   $H_2: \neg (p \land Q)$   $C: \neg P$
- 4.  $H_1: \neg P \quad H_2: P \leftrightarrow Q \quad C: \neg (P \land Q)$

P	Q	$P \rightarrow Q$	$\neg P$	$\neg Q$	$\neg (P \land Q)$	$P \leftrightarrow Q$
Т	Т	Т	F	F	F	Т
Т	F	F	F	Т	Т	F
F	Т	Т	Т	F	Т	F
F	F	Т	Т	Т	Т	Т

# **Validity Using Truth Table**

### Example 3

Determine whether the following conclusion C follows logically from the premises  $H_1$  and  $H_2$ .

- 1.  $H_1 P \rightarrow Q \quad H_2:P \quad C:Q$
- 2.  $H_1 P \rightarrow Q \quad H_2: \neg P \quad C:Q$
- 3.  $H_1: P \rightarrow Q$   $H_2: \neg (p \land Q)$   $C: \neg P$
- 4.  $H_1: \neg P \quad H_2: P \leftrightarrow Q \quad C: \neg (P \land Q)$

P	Q	$P \rightarrow Q$	$\neg P$	$\neg Q$	$\neg (P \land Q)$	$P \leftrightarrow Q$
T	Т	T	F	F	F	T
T	F	F	F	T	T	F
F	T	T	T	F	T	F
F	F	T	T	T	T	T

Example 4

Consider:

"If you have a current password, then you can log onto the network".

"You have a current password".

Therefore, "You can log onto the network."

Let P = you have a current password q = you can log onto the network

Argument form

$$p \to q$$

$$p \qquad ((p \to q) \land p) \to q$$

$$\therefore q$$

This form of argument is valid because whenever all its premises are true, the conclusion must also be true

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CSM 165: Discrete Mathematics Chapter 1: Propositional and first order predicate logic, 05/02/2021

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Argument form:

$$\begin{array}{ccc}
p \to q \\
\hline
p & ((p \to q) \land p) \to q
\end{array}$$

This form of argument is valid because whenever all its premises are true, the conclusion must also be true

Example 5

Now Consider:

"If you have a current password, then you can log onto the network".

"you can log onto the network".

Therefore, "You have a current password"

Let *P* = you have a current password *q* = you can log onto the network

Argument form

$$\begin{array}{ccc}
p \to q \\
q \\
\vdots, p
\end{array}$$

$$((p \to q) \land q) \to p$$

This form of argument is invalid since we can make all premises true and conclusion false.

Example 5

Now Consider:

"If you have a current password, then you can log onto the network".

"you can log onto the network".

Therefore, "You have a current password"

Let P = you have a current password q = you can log onto the network

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q \\
\vdots p
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Rule	Tautology	Name
$ \begin{array}{c} p \\ \underline{p \to q} \\ \therefore q \end{array} $	$(p \land (p \rightarrow)) \rightarrow q$	Modus ponens
$   \begin{array}{c}     \neg q \\     \underline{p \rightarrow q} \\     \vdots \neg p   \end{array} $	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$(p \to q) \land (\to r) \to (p \to r)$	Hypothetical syllogism

$ \begin{array}{c} p \lor q \\ \neg p \\ \therefore q \end{array} $	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
$\frac{p}{\therefore p \vee q}$	$p \rightarrow (p \lor q)$	Addition
$\frac{p \wedge q}{\therefore p}$	$(p \land q) \rightarrow p$	Simplification
<i>p</i>	$((p) \land (q)) \to (p \land q)$	Conjunction
$ \begin{array}{c} p \lor q \\ \neg p \lor r \\ \therefore q \lor r \end{array} $	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

#### End of Lecture

Questions...???

Thanks

**End of Lecture** 

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Thanks