ELECTRICITY AND MAGNETISM

2012/2013

LECTURER: **DR. SAMUEL N.A. DODOO**
OFFICE: **DEPT OF PHYSICS**

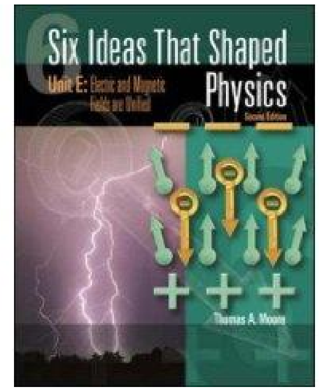
Email: **sandodoo@yahoo.com**

REQUIRED TEXT BOOKS

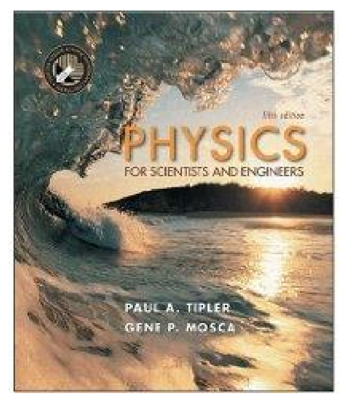
- Physics for Scientists and Engineers with Modern Physics - Paperback (12 Sep 2008) by Douglas C. Giancoli **ISBN-13:** 978-0136074809

REQUIRED TEXT BOOKS

- Six Ideas That Shaped Physics: Unit E - Electromagnetic Fields by Thomas A Moore (Paperback - 1 Jan 2003) by Thomas A Moore (Author)**ISBN-13:** 978-0072397116

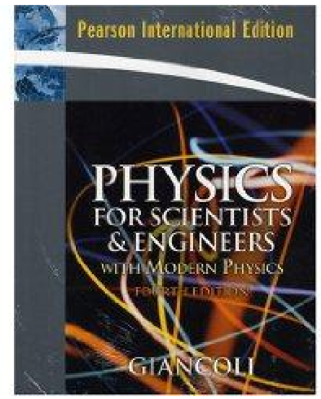


- Physics for Scientists and Engineers: Extended Version by Paul A. Tipler and Gene P. Mosca (Hardcover - 11 Sep 2003) **ISBN-13:** 978-0716743897



REQUIRED TEXT BOOKS

- Physics for Scientists and Engineers with Modern Physics - Paperback (12 Sep 2008) by Douglas C. Giancoli **ISBN-13:978-0136074809**



ASSESSMENT SCHEDULE

10 ASSIGNMENTS (best 8 to be assessed)	10%
MID SEMESTER EXAMINATION	20%
END OF SEMESTER EXAMINATION	70%

COURSE OUTLINE

- ELECTROSTATICS
- ELECTRIC FIELD & GAUSS' LAW
- ELECTRIC POTENTIAL
- CAPACITORS & DIELECTRICS
- STEADY CURRENTS & DIRECT CURRENT CIRCUITS
- MAGNETISM & MAGNETIC FIELDS
- ELECTROMAGNETIC INDUCTION
- MAGNETIC PROPERTIES OF MATTER
- A C THEORY

THE ELECTRIC CHARGE & ELECTRON TRANSFER

- Materials differ in their ability to **lose** and **capture electrons**
- Rubber molecules, for example, have greater potential to capture and hold electrons than that of fur or cloth
- Hence, a piece of rubber gains a **net negative charge** when rubbed with fur while the fur gains a **net positive charge**.
- **Charging** is the process whereby excess electrons are added or taken from a body.
- There is in principle **conservation of charge**

METHODS OF CHARGING A BODY

There are **3 methods** of charging a body These are by:

- Friction
- Contact
- Induction

SI UNIT OF CHARGE

Coulomb [C]

1 C = charge of 6.242×10^{18} protons = $(6.242 \times 10^{18})e$

1 e = 1.602×10^{-19} C

COULOMB´S LAW

The electrostatic force of attraction or repulsion exerted by a point charge q_1 on another point charge q_2 is given by

$$F = \frac{k\,|\,q_1 q_2\,|}{r^2}$$

where $k = 8.99 \times 10^9\ Nm^2C^{-2}$ or

$$k = \frac{1}{4\pi\varepsilon_0}$$

where ε_0 = permittivity of free space = $8.8542 \times 10^{-12}\ C^2N^{-1}m^{-2}$ is the permittivity of free space.

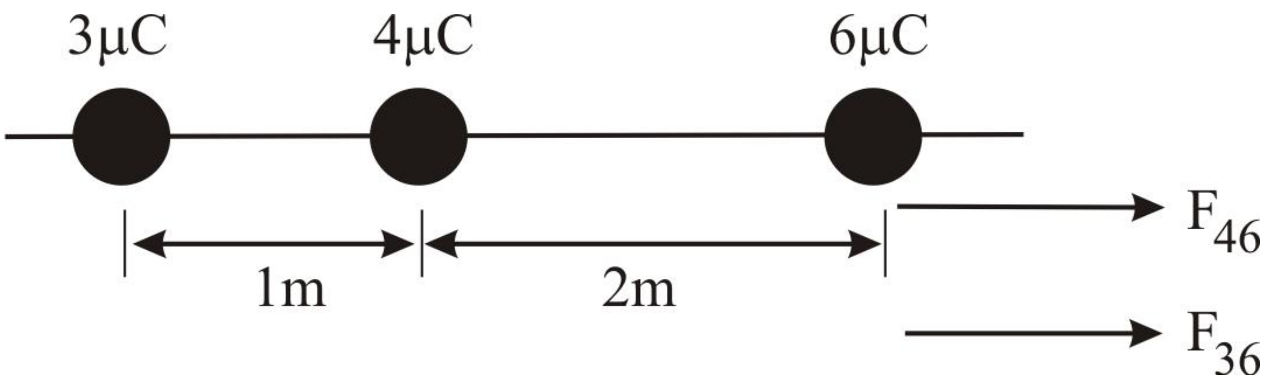
PERMITTIVITY ε & RELATIVE PERMITTIVITY ε_r

- Permittivity ε is defined as the reluctance of a medium to transmit an electrostatic force
- $\varepsilon = 1/4\pi k$
- Relative permittivity ε_r is defined as
- $\varepsilon_r = \varepsilon/\varepsilon_0$

COULOMB´S LAW

Sample Problem 1

Charges are distributed along a straight line as shown. What is the total force exerted on the 6 μC charge by the charges 3 μC and 4 μC ?



Solution

$F = F_{36} + F_{46}$

SOLVING PROBLEMS INVOLVING COULOMB´S LAW AND VECTORS

Principle of Superposition

The net force exerted on a point charge q_1 by a number of point charges q_2, q_3, q_4 ..... q_n is the vector sum of the forces exerted separately by the individual point charges q_2 .... q_n.

Methods

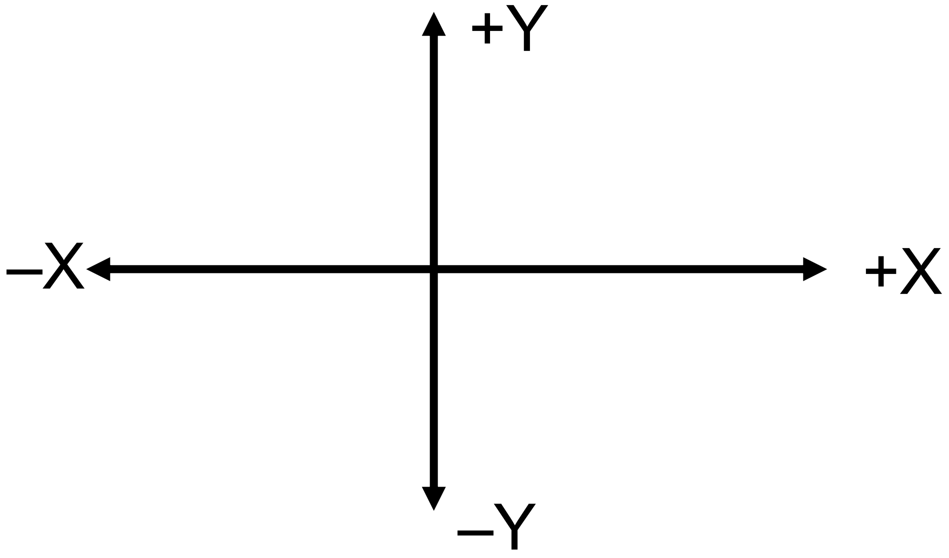
- Parallelogram Law of Vectors
- Resolution of Vectors

Rectangular Resolution Method

- Choose two rectangular perpendicular axes e.g. x and y axes.
- Assign a sign convention along the axes e.g. vectors in the +x (i.e. to the right) or +y (i.e. upward) are **positive** while vectors in the –x (i.e. to the left) or –y (i.e. downward) are **negative.**
- Resolve all vectors along the x and y axes taking note of their signs.
- Find the sum $\sum X$ of all vectors in the x direction
- Find the sum $\sum Y$ of all vectors in the y direction.
- Find the resultant vector $R = \sqrt{[(\sum X)^2 + (\sum Y)^2]}$
- The direction of R is given by $\tan \theta = \sum Y/\sum X$

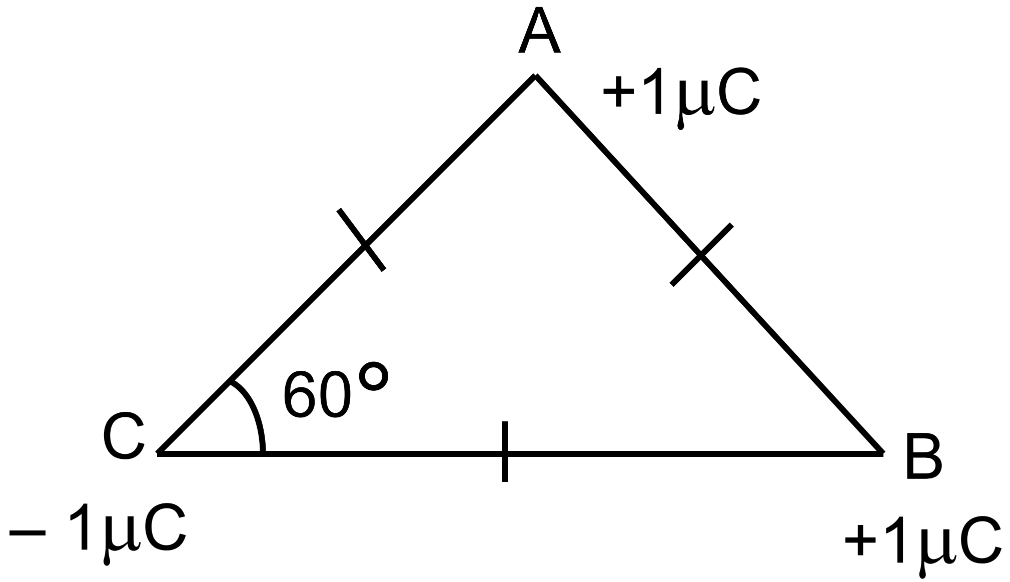
Rectangular Resolution Method

- Or $\theta = \tan^{-1}[\Sigma Y / \Sigma X]$
- θ is the direction of the resultant vector R relative to the x-axis

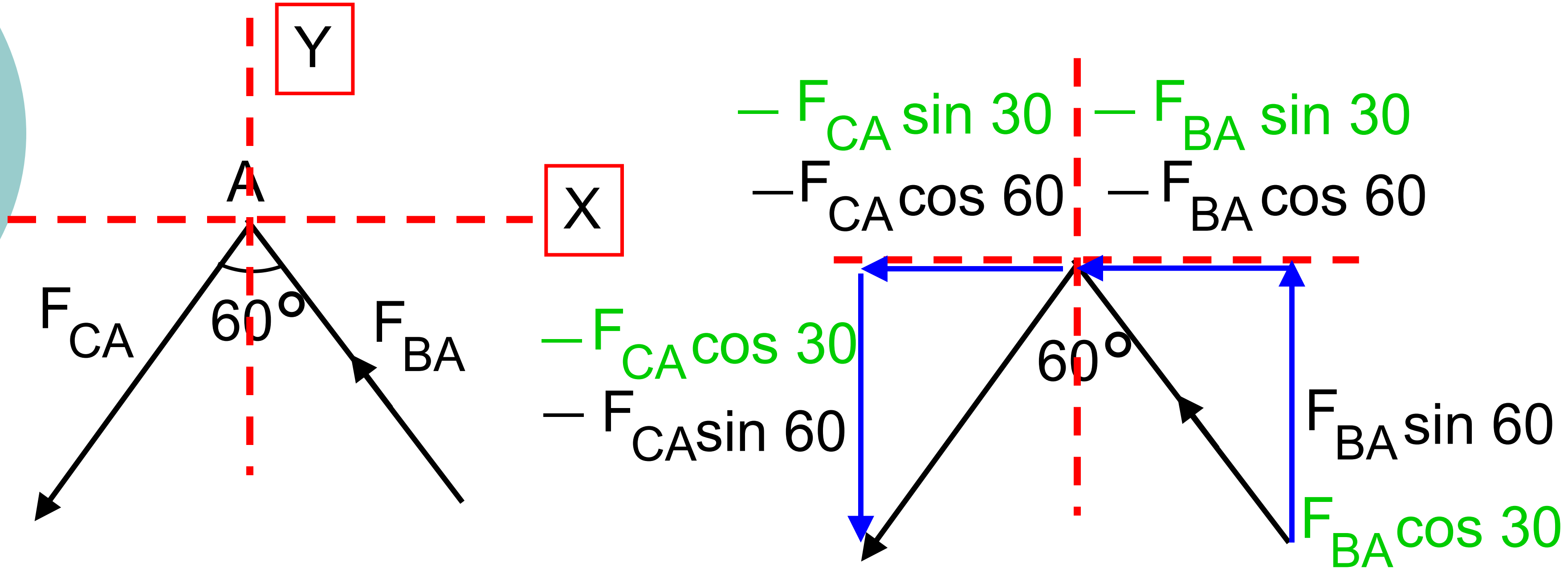


Sample Problem 2

What is the force on particle A due to particles B and C?



Solution



Y
A
X
F CA
60°
F BA
− F CA sin 30
− F BA sin 30
−F CA cos 60
− F BA cos 60
−F CA cos 30
− F CA sin 60
60°
F BA sin 60
F BA cos 30

Solution

$$\sum F_X = -F_{BA}\cos 60 - F_{CA}\cos 60 = -(F_{BA} + F_{CA})\cos 60$$

$$\sum F_Y = F_{BA}\sin 60 - F_{CA}\sin 60 = (F_{BA} - F_{CA})\sin 60$$

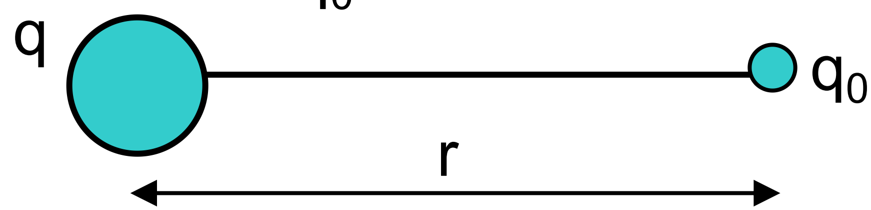
$$F_R = \sqrt{[(F_{BA} + F_{CA})\cos 60]^2 + [(F_{BA} - F_{CA})\sin 60]^2}$$

Let the direction of the resultant force be ϕ, then,

$$\phi = \tan^{-1}\frac{\sum F_Y}{\sum F_X}$$

Electric Field

The electric field ***E,*** at a point in an electric field is defined *as the electric force experienced per unit positive charge q_0* placed at that point where q_0 is assumed infinitesimally small.



Mathematically,

$$E = F/q_0$$

But from Coulomb's law, $F = kqq_0/r^2$

$$\Rightarrow E = kq/r^2$$

Electric Field

NOTE

- The **SI units** of the electric field are *newtons per coulomb* (NC^{-1}) or *volts per meter* (Vm^{-1}).

- For any distribution of point charges, the electric field can be calculated by superposition of the individual fields of the point charges; in much the same way as is done for electric forces. That is,

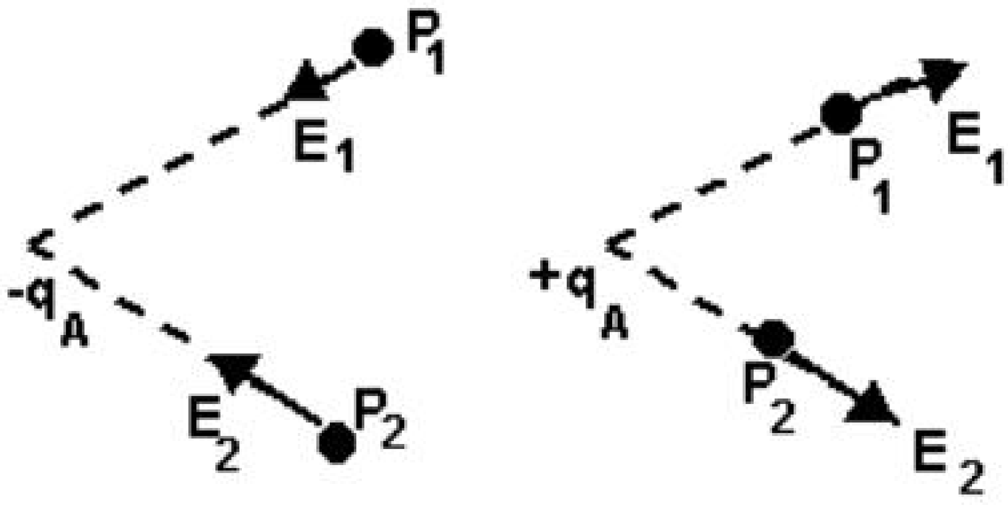
$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_3 + \ldots\ldots\ldots\ldots$$

- This is the principle of superposition

Direction of Electric Field

In the figure below, an infinitesimally small positive test charges are places at the points P_1 and P_2.

Note the directions of the E fields indicated by arrows.

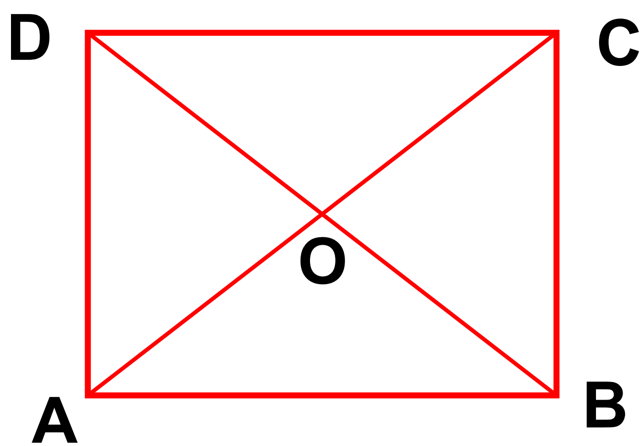


Calculation of E-field

- Determine the directions of the fields due to all the charges under electrostatic consideration.
- Once these directions are marked use the magnitudes of all charges in the computation of the electric field due to the charges under consideration.
- The resultant electric field at the given point may then be calculated by invoking the Rectangular resolution method.

Worked Example

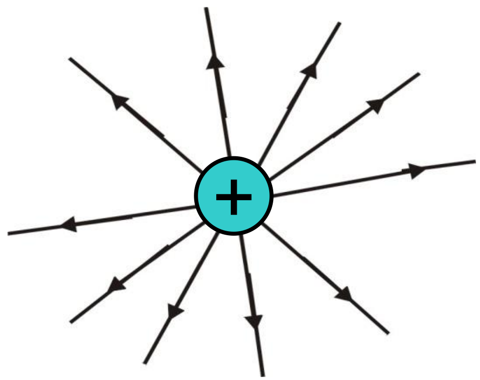
ABCD is a rectangle whose diagonals AC and BD intersect at O. Point charges of 40 μC and $-40\ \mu C$ are placed at A and B respectively. If AB = 8 cm and BC = 6 cm. Calculate The magnitude and direction of the electric Intensity at O.

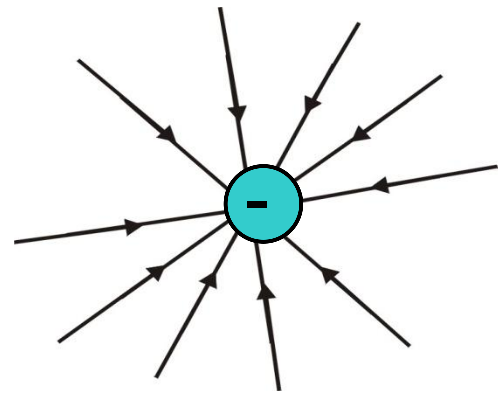


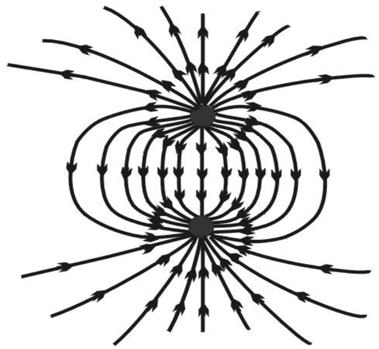
ELECTRIC FIELD LINES

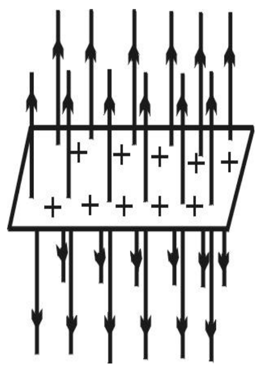
- Electric field lines originate on positive charges and terminate on negative charges (or go to infinity). They do not begin or end on a charge-free point in finite space.
- At each point in space, the direction of the electric field vector $\boldsymbol{E}$ is represented by a tangent to the line of force at that point.
- They never cross.
- The density of the lines of force is a measure of the magnitude of the electric field.

ELECTRIC FIELD LINES









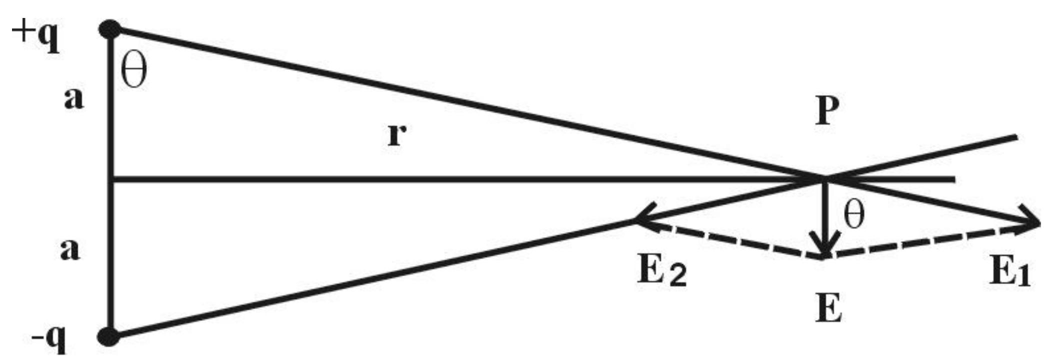
Electric Field of an Electric Dipole

Electric dipole

Configuration of equal and opposite charges (+*q* and -*q*) separated by a small distance *d*

Electric dipole moment p

The product of the positive charge *q* and the distance *d* between the charges *q* and -*q* i.e. *p* = *qd*



Field of an Electric Dipole Cont.

Determination of the E-field at the point P distance *r* along the perpendicular bisector of the line joining the charges

Basic Assumption: ***r* » *a***

Resultant field **E** : **E** = **E**1 + **E**2

but $E1 = E2 = [1/4\pi\varepsilon_o]q/[a^2 + r^2]$

Vector sum of **E1** and **E**2 points vertically downwards

→ $E = 2E1\cos\theta$ but $\cos\theta = a/(a^2 + r^2)^{1/2}$

→ $E = (2/4\pi\varepsilon_o)[q/(a^2 + r^2)]a/(a^2 + r^2)^{1/2}$

$= [1/4\pi\varepsilon_o]2aq/(a^2 + r^2)^{3/2}$

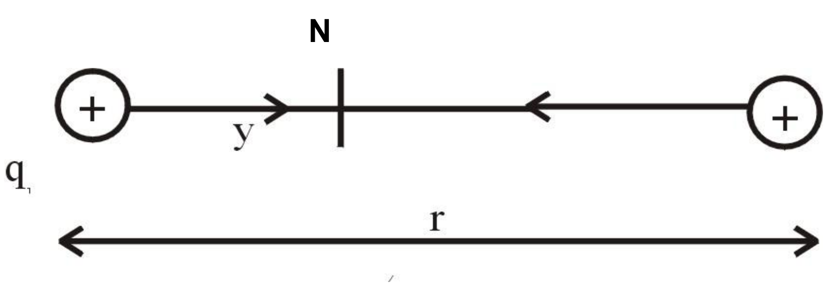
Electric Field of an Electric Dipole

For *r* » *a*

$$E = (1/4\pi\varepsilon_o)2aq/r^3 = (1/4\pi\varepsilon_o)dq/r^3$$

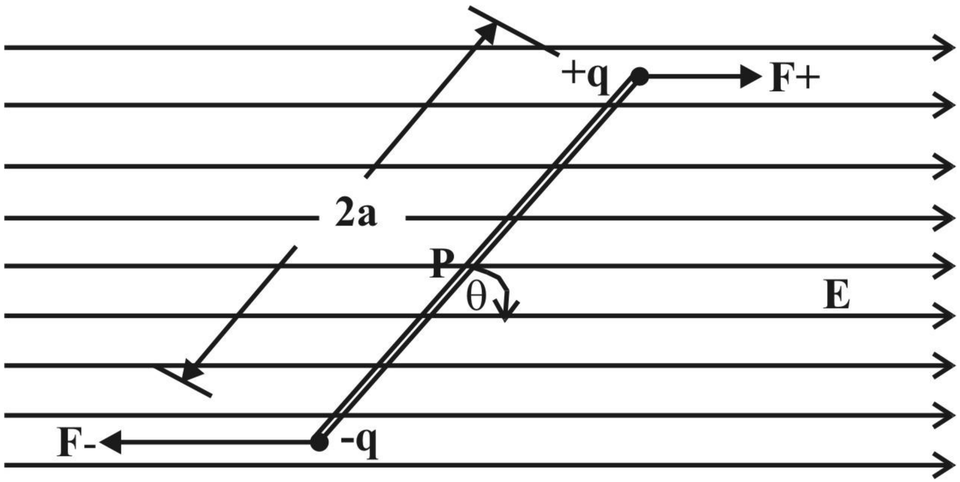
where 2*aq* = *dq* is the electric dipole moment

The Neutral Point (N)



$$y = r\left[\left(\frac{q_2}{q_1}\right)^{\frac{1}{2}} + 1\right]^{-1}$$

ELECTRIC DIPOLE IN AN ELECTRIC FIELD



Magnitude of Dipole Moment p

$$\mathbf{p} = 2aq$$

Net force on Dipole F = F+ + F–

$$= q\mathbf{E} - q\mathbf{E} = 0$$

ELECTRIC DIPOLE IN AN ELECTRIC FIELD

Net Torque τ

$$\tau = 2F(a \sin\theta) = 2aF \sin\theta$$

$$= 2aqE \sin\theta$$

$$= pE \sin\theta = \mathbf{p \times E}$$

Work done by Torque

$$W = \int dW = \int \tau d\theta = Up$$

$$Up = \int pE \sin\theta \, d\theta = pE\int \sin\theta \, d\theta$$

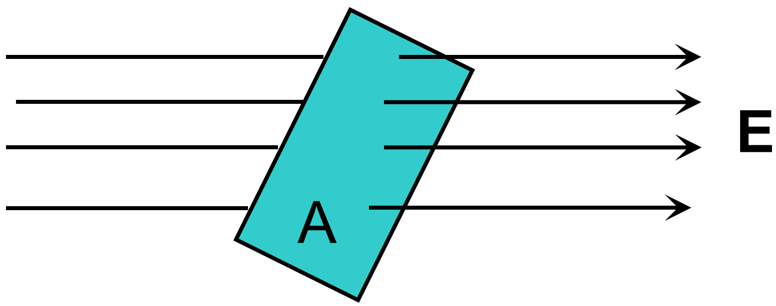
$$= \mathbf{p}E \left| -\cos\theta \right|^{\theta}$$

$$= -pE \cos\theta = \mathbf{-p.E}$$

ELECTRIC FLUX

Electric Flux Φ

Product of the area A and the magnitude of the normal component of the electric field i.e. $\Phi = E_n A$



If the electric field E mamkes an angle θ with the area A, then $E_n = E\cos\theta$ and hence,

$$\Phi = EA\cos\theta$$

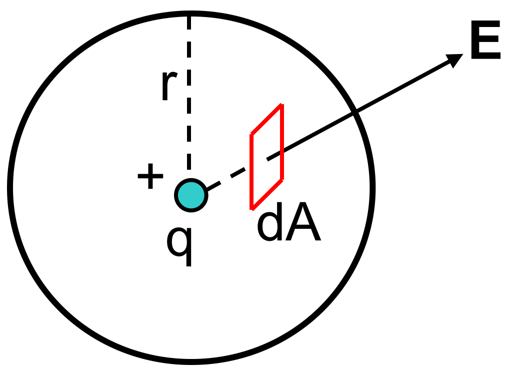
The electric flux Φ through an area is, therefore, equal to the number of lines of force intercepted by the area

Gauss´s Law

The total flux summed over any closed surface (considered to be made up of small areas dS) is equal to the ratio of the total charge Q_{encl} enclosed by the surface to the permittivity of free space ε_0

$$\oint E_n \, dS = \frac{Q_{encl}}{\varepsilon_o}$$

Gauss’s law and Coulomb’s Law



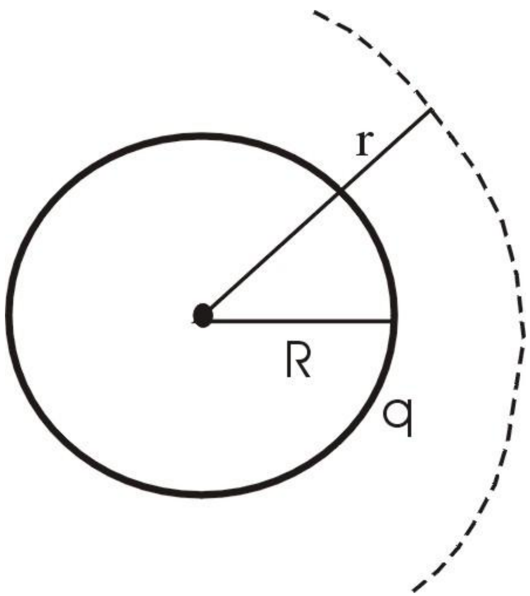
Application of Gauss´s Law

Worked Example

A sphere of radius R has a total charge q which is uniformly distributed over its volume

(a) What is the electric field at points inside the sphere?

(b) What is the electric field at points outside the sphere?



Application of Gauss´s Law

(a)

Charge density within the sphere = charge/volume

$= q/(4/3\pi R^3)$

Charge within area of radius r = $q/(4/3\pi R^3) \cdot 4/3\,\pi r^3$

$= qr^3/R^3$

Then Gauss's law gives

$$E.4\pi r^2 = q/\varepsilon_o = (qr^3/R^3)/\varepsilon_o$$

$\rightarrow E = (1/4\pi\varepsilon_o)qr/R^3$, for $r \leq R$

(b)

$E = (1/4\pi\varepsilon_o)q/r^2$, for $r \geq R$.

Gauss´s Law & Coulomb´s Law

From Gauss`s law

$$\oint \mathbf{E}.d\mathbf{A} = q/\varepsilon_o$$

$$\mathbf{E}.4\pi r^2 = q/\varepsilon_o$$

$$\mathbf{E} = (1/4\pi\varepsilon_o)q/r^2$$

From the definition of **E**, the force on the test charge q_o is

$$\mathbf{F} = (1/4\pi\varepsilon_o)qq_o/r^2$$

which is Coulomb´s law

THE ELECTRIC POTENTIAL (V)

Potential at a Point

The potential at a point in an electric field is the work done in moving a unit positive charge from infinity to that point.

$$V = \text{Work/Charge} = W/q$$

$$V = kq/r$$

Potential is a scala quantity

Potential and Electric Field (E)

$E = - V/d$ $\quad$ d = distance

Capacitors

Definition of capacitance (C)

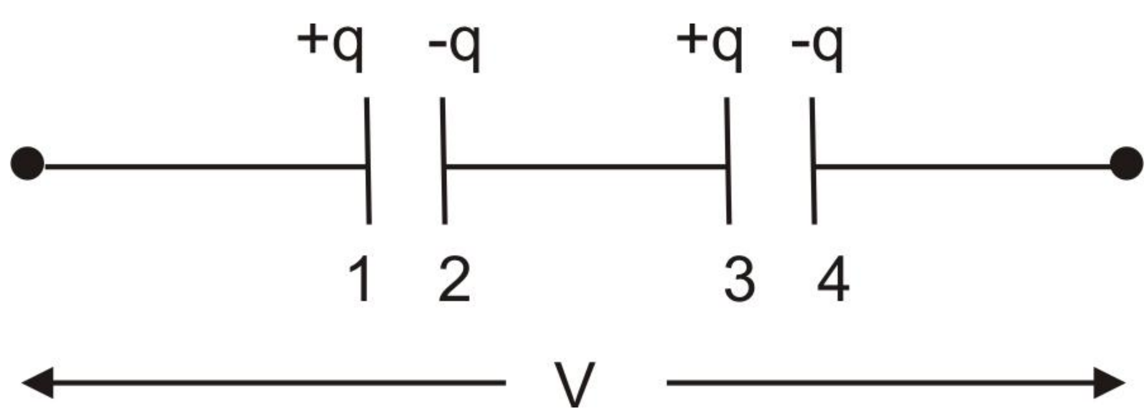
$$C = \text{Charge/voltage} = q/V$$

Capacitance of a parallel plate capacitor (in vacuum)

$$C = \varepsilon_o A/d$$

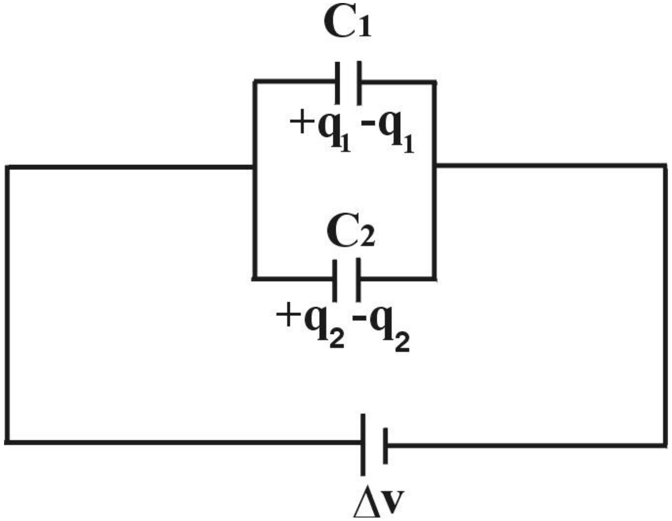
Capacitors in Series

RECIPROCAL ADDITION OF CAPACITANCES



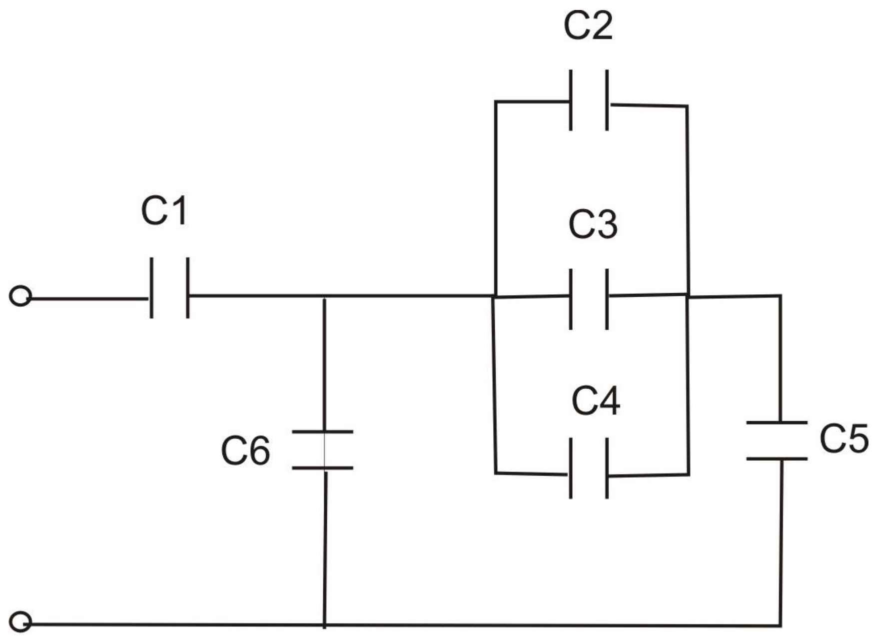
Parallel Combinations

Capacitors in Parallel



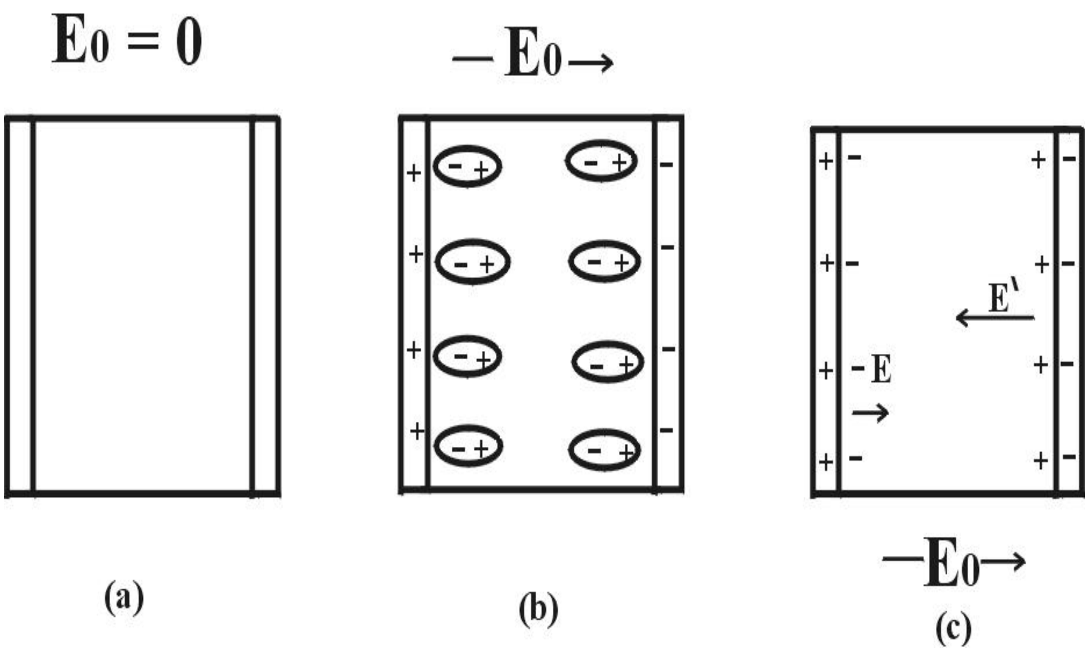
ALGEBRAIC ADDITION OF CAPACITANCES

Series-Parallel Combinations

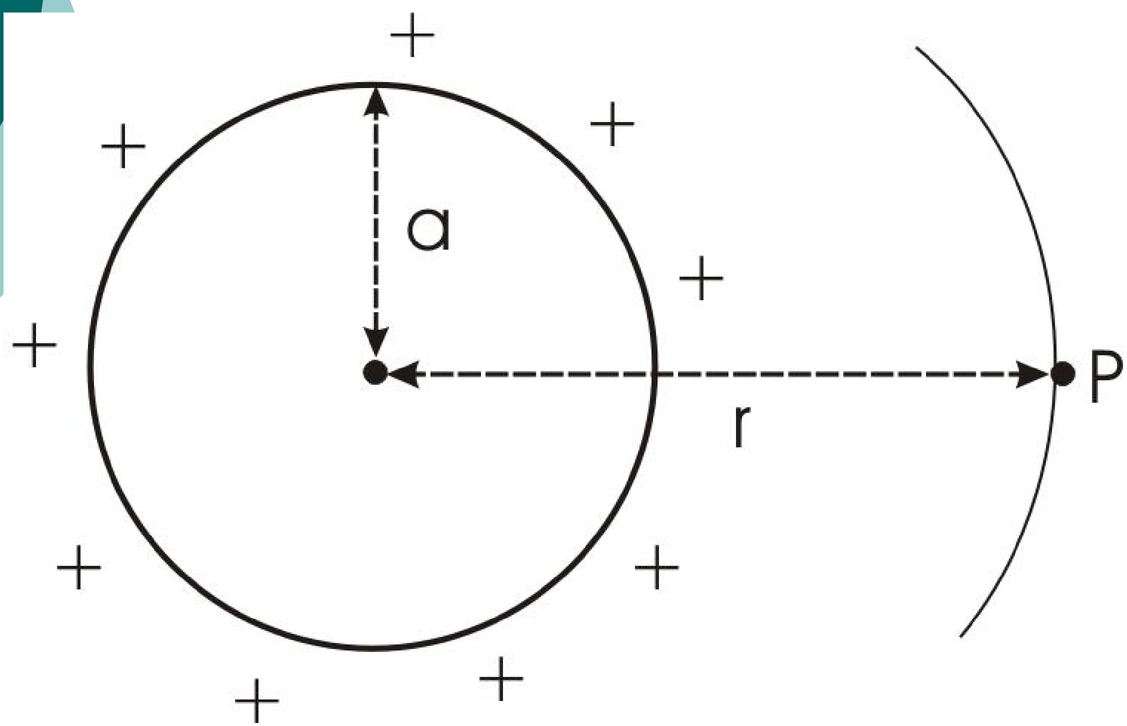


Dielectric View Of Capacitors

The function of the dielectric in capacitors is to increase the capacitance by reducing the potential between the plates



CAPACITANCE OF AN ISOLATED SPHERE



At the point P

$\sigma = Q/A = Q/4\pi r^2$

and $E = \sigma/\varepsilon$

$\rightarrow E = Q/4\pi\varepsilon r^2$

But $E = -dV/dr$

Or $dv = -Edr$

$$\Rightarrow \quad V = -\int_{\infty}^{r} E dr = -kQ \int_{\infty}^{r} r^{-2} dr = -kQ\left[-r^{-1}\right]_{\infty}^{r}$$

CAPACITANCE OF AN ISOLATED SPHERE

At the surface of the sphere r = a

$$\therefore \qquad V = kQ\left[\frac{1}{a}\right]_{\infty}^{a} = \frac{kQ}{a} = \frac{Q}{4\pi\,\varepsilon a}$$

C = $4\pi\varepsilon a$

ENERGY STORAGE IN CAPACITORS

When the plates carry charges $\pm q$, the potential difference between the plates is q/C.
Work that must be done to increase the charge on the plates by $\pm dq$ is

$$dU = Vdq = \frac{q}{C}dq$$

$$U = \int \frac{q}{C}dq = \int \frac{1}{C}qdq$$

$$\frac{1}{2}\frac{Q^2}{C}$$

ENERGY STORAGE IN CAPACITORS

Using Q = CV gives

$$U = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2$$

Using C = Q/V gives

$$U = \frac{1}{2}\frac{Q^2V}{Q} = \frac{1}{2}QV$$

For a parallel-plate capacitor *C* = $A\varepsilon_o/d$ and V = Ed. Hence

$$U = \frac{1}{2}\frac{A\varepsilon_0}{d}\cdot(Ed)^2 = \frac{1}{2}Ad\varepsilon_0 E^2$$

APPLICATION OF CAPACITORS

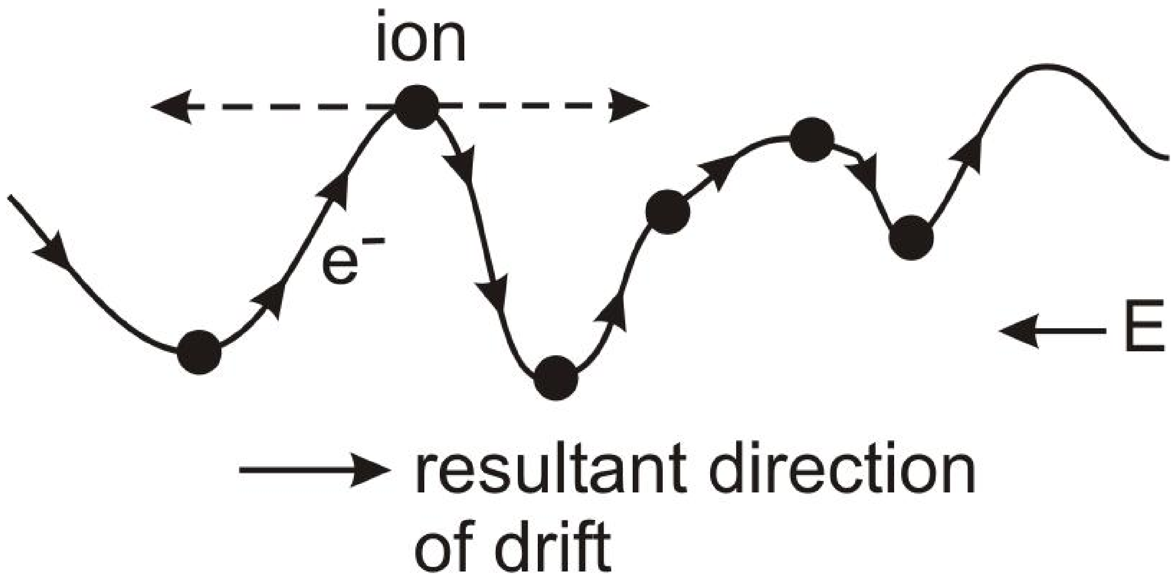
- Used as a `tuner´ in radio sets to tune the radio to various frequencies.
- Very large currents can be produced by a capacitor. By quickly discharging the capacitor such large currents can be used very high temperatures needed in thermonuclear reactions.

APPLICATION OF CAPACITORS

- Used to operate flash bulbs for cameras when they are allowed to discharge through the bulbs.
- Used to eliminate sparks between contact points in electrical circuits.
- Used to smoothen or make the pd constant when the AC is rectified to produce a DC

CURRENT ELECTRICITY

Mechanism of conduction through Metals



Current & Charge

Current (I) is defined as the rate of flow of charge (Q)

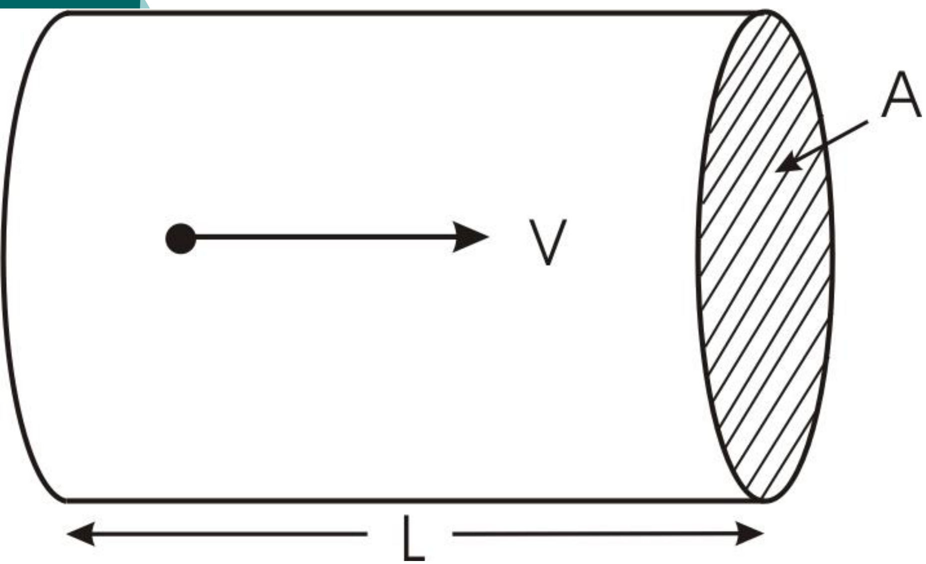
$$I = Q/t$$

$$Q = It$$

where

t = time of flow of the quantity of charge Q

Drift Velocity



A = Cross-sectional area

V = drift velocity

L = Length of conductor

n = number of free electrons per unit volume

Volume of conductor	=	AL
Number of free electrons	=	nAL
Total charge Q	=	nALe
Drift velocity V	=	L/t

Drift Velocity

Current flowing I = Q/t

$$I = \frac{Q}{t} = \frac{nALe}{t}$$

$$\therefore \qquad I = neVA$$

$$V = \frac{I}{neA}$$

Ohm´s Law

The potential difference (pd) across the ends of a conductor is directly proportional to the current flowing through the condutor provided temperature and other physical conditions of the conductor remain constant.

Mathematically

$$V = IR$$

Where V = pd, I = current and R = resistance.

Battery & Emf

A **battery** is a source of electrical energy. If no internal energy losses occur in the battery, then the potential difference between its terminals is called the electromotive force (emf).

Battery & Emf

NOTE

1. When delivering current (on discharge):
Terminal Voltage = emf – voltage drop in internal resistance

$$V = \xi - Ir$$

2. When receiving current (on charge)
Terminal Voltage = emf + voltage drop in internal resistance

$$V = \xi + Ir$$

3. When no current exists
Terminal Voltage = Emf of battery

Resistivity

The resistance R of a piece of wire of length L and cross sectional area A is given by

$$R = \rho \frac{L}{A}$$

where ρ = resistivity [Ωm]. Resistivity is characteristic of the material from which the wire was made.

Resistivity: Worked Example

- A wire of resistivity 1.7×10^{-8} Ωm commonly used for electrical installation in homes has a radius of 1.3×10^{-3} m.
- (a) What is the resistance of a piece of this wire 25 m long?
- (b) What is the potential drop along this length of wire if it carries a current of 10 A?

Temperature variation of Resistance

If we consider a wire of resistance R_0 at an initial T_0 then the new resistance at a temperature T is given by

$$R = R_0 + \alpha R_0(T - T_0)$$

where α = temperature coefficient of resistance [K^{-1}].

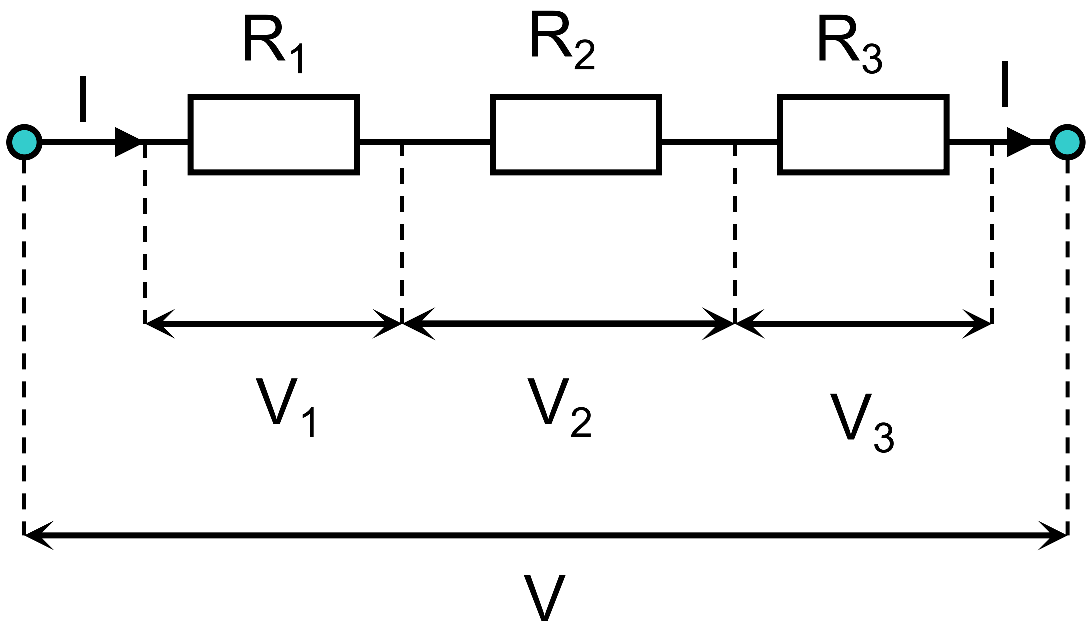
NOTE

1. α Varies with temperature, hence, the relationship is only valid over a small temperature range.

2. Similarly, resistivity varies with temperature according to

$$\rho = \rho_0 + \alpha \rho_0(T - T_0)$$

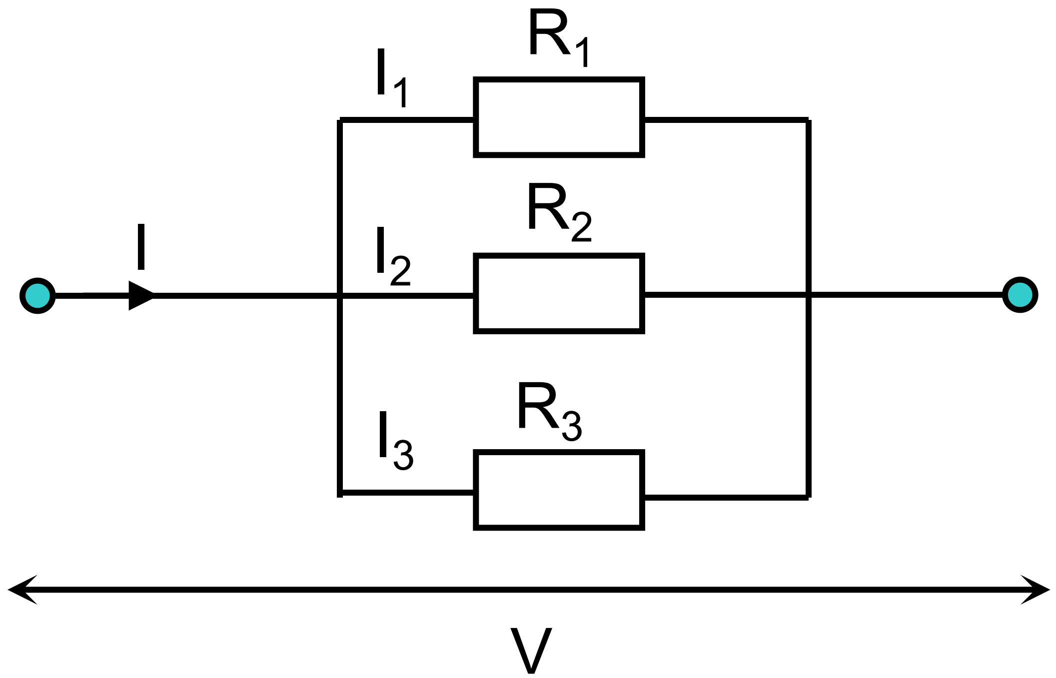
Combination of Resistances: Series



$V = V_1 + V_2 + V_3 \rightarrow IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$

$\rightarrow R = R_1 + R_2 + R_3$

Combination of Resistances: Parallel



$I = I_1+I_2+I_3 \rightarrow V/R_1+V/R_2+V/R_3 = V(1/R_1 + 1/R_2 + 1/R_3)$
$\rightarrow 1/R = 1/R1 + 1/R2 + 1/R3$

Electrical Power

$$\mathrm{Power} = \frac{\mathrm{Work(qV)}}{\mathrm{time(t)}} = \mathrm{IV} = \mathrm{I} \cdot \mathrm{IR} = \mathrm{I}^2\mathrm{R} = \frac{\mathrm{V}^2}{\mathrm{R}}$$

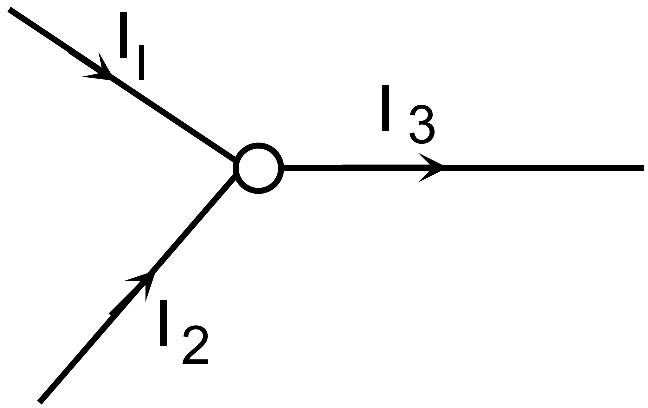
The thermal energy generated in a resistor per second = the energy loss in the resistor.

Unit of Power = Watt = J/s

Kirchoff´s Laws

Kirchoff´s Node or Junction Rule

The algebraic sum of the currents flowing into a junction equals the sum of the currents flowing out of the junction.



$$I_1 + I_2 = I_3 \quad \text{OR} \quad I_1 + I_2 - I_3 = 0$$

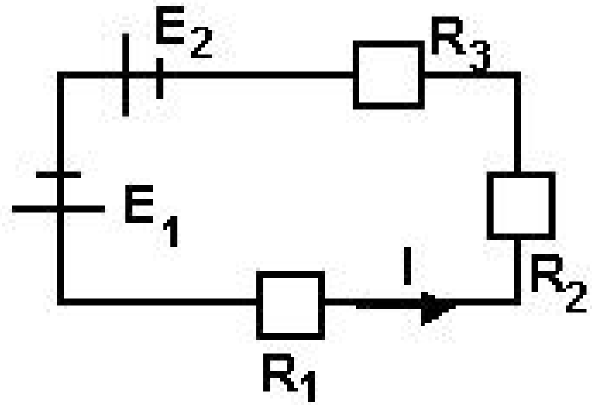
Kirchoff´s Laws

Kirchoff´s Loop or Circuit Rule

For a closed circuit of loop, Sum of potential drops (IRs) = Sum of emf i.e . $\sum(IR) = \sum V$

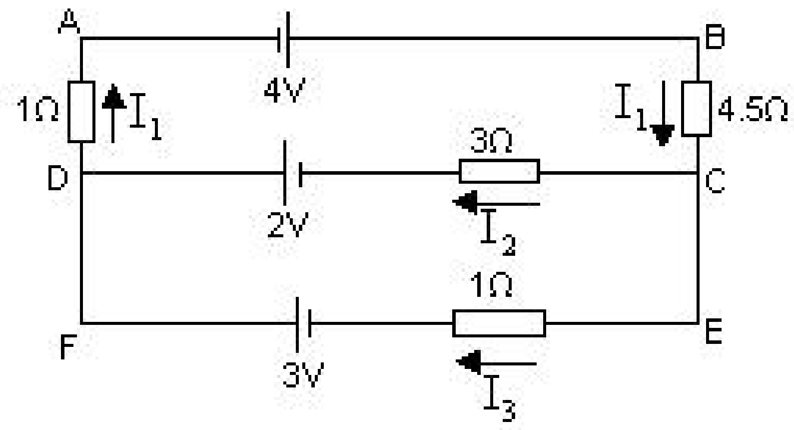
Illustration

$$E_2 - E_1 = IR_1 + IR_2 + IR_3$$



Kirchoff´s Laws:Example

Determine the current strengths in the circuit



For current loop ABCDA

$$0.5i_1 \quad + \quad 3i_2 \quad + \quad 1i_1 \quad = \quad 4 + 2$$

For loop FDCEF

$$1i_3 - 3i_2 \quad = \quad 3 - 2$$

At the junction C

$$i_1 \quad = \quad i_2 \; + i_3$$

Magnetic Field

A magnetic field **B** is exists in a region where a moving charge experiences a force by virtue of its motion.

The magnitude of the magnetic force F acting on a moving charge in a magnetic field depends on a number of factors:

1. The charge q [C].
2. The magnitude of the velocity v [m/s]
3. Strength B of the magnetic field
4. Sin θ where θ is the angle between the field lines and the velocity.

Magnetic Induction **B**

The magnetic field at a point is represented by **B** and it is variously called etic **Magnetic Induction**, **Magnetic Flux Density** or simply **Magnetic Field**

Mangnetic Force F [T]

$$F= qvB\sin \theta$$

1 Tesla = 1T = 1 Weber per square metre [1 Wb/m^2]

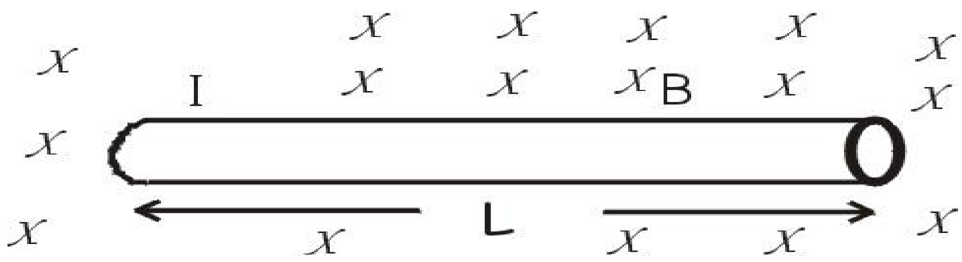
Sample Caculation

A 5.0-MeV proton moves perpendicularly to a uniform magnetic field of strength 1.5 T.

Determine the

- (i) Speed of the proton, and
- (ii) force that acts on the proton. (Mass of proton = 1.7×10^{-27} kg).

Force on a Current in a Magneitc Field



$$F = BIL\sin\theta$$

where θ is the angle between the direction of the current I and the field B.

Sample Calculation

A 0.3 m long wire carrying a current of 25 A makes an angle of 60o to a magnetic field of flux density 8.0 x 10^{-4} T. What is the magnitude of the force on this wire?

Torque τ on a Flat Coil

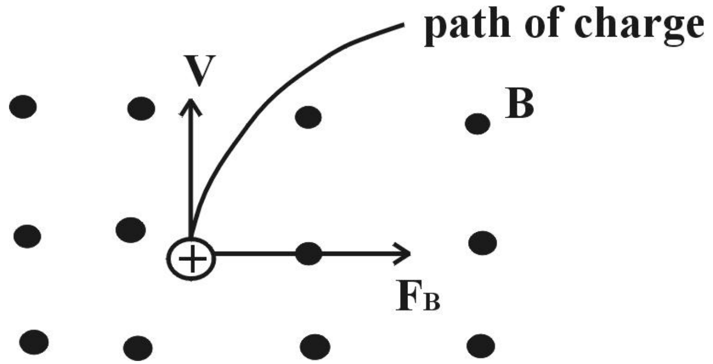
$$\tau = BANI\sin\theta$$

where

A = crossectional area of coil

θ = angle between the field lines and perpendicular to the plane

Motion of Charges in Fields



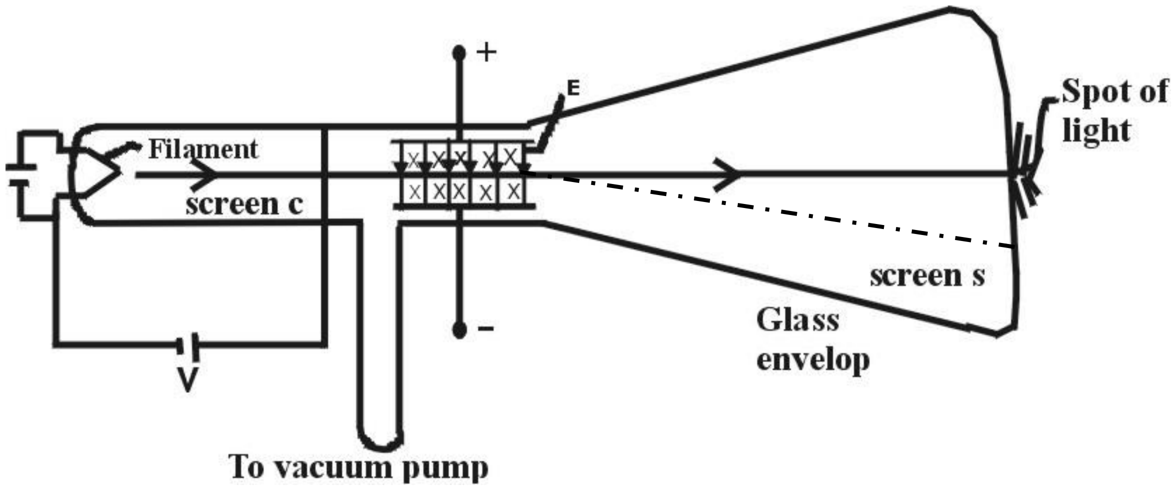
Electric Force

$$F_E = qE$$

Magnetic Force (causes charge to describe a circular motion)

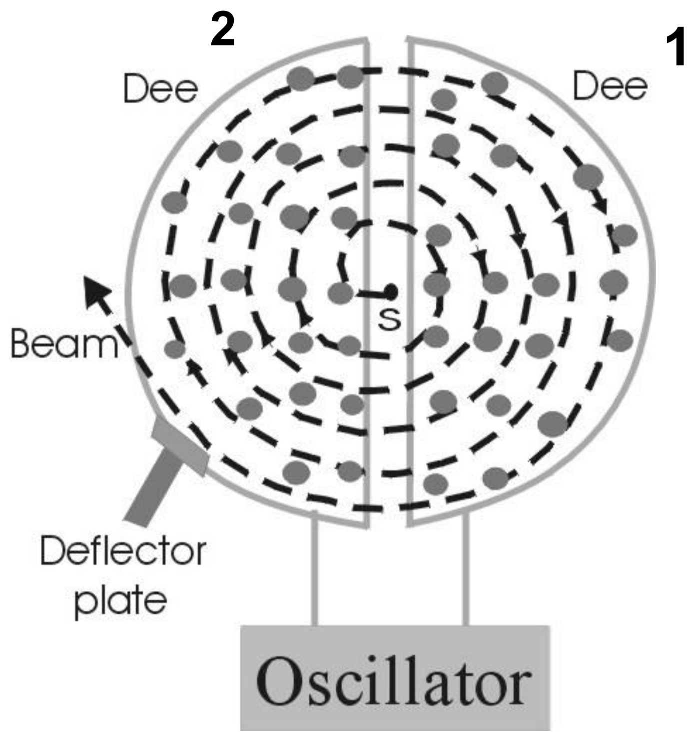
$$F_M = qVB \quad \text{i.e} \quad MV^2/r = qVB$$

Mass Spectrometer: Measurement of mass to charge ratio



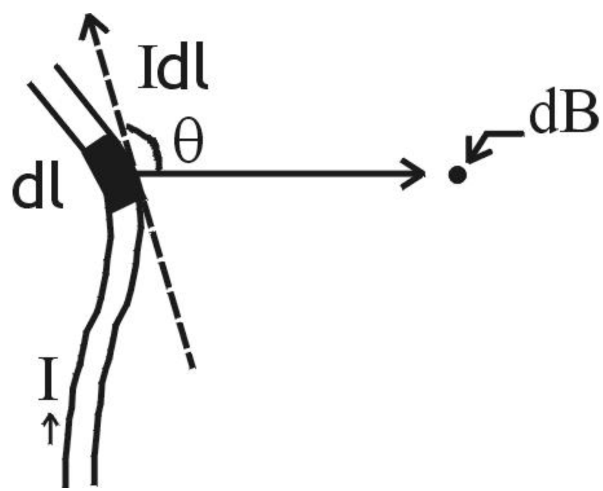
m/q = rB/V

The Cyclotron



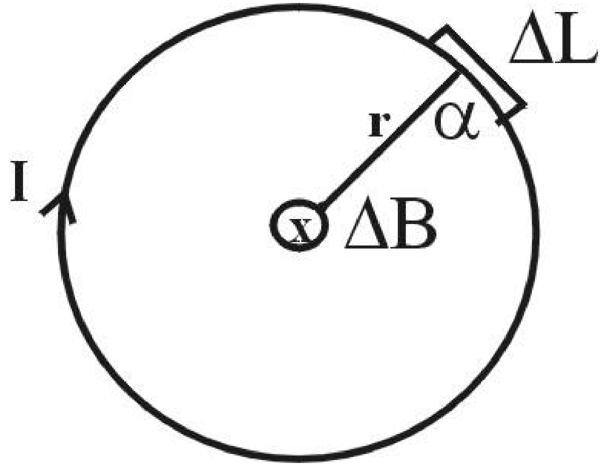
MAGNETIC FIELD OF A CURRENT-CARRYING CONDUCTOR:

THE BIOT-SAVART LAW



$$\delta B \;\; \alpha \;\; \frac{IdL\sin\theta}{r^2} \quad \text{or} \quad \delta B = \frac{KIdL\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{IdL\sin\theta}{r^2}$$

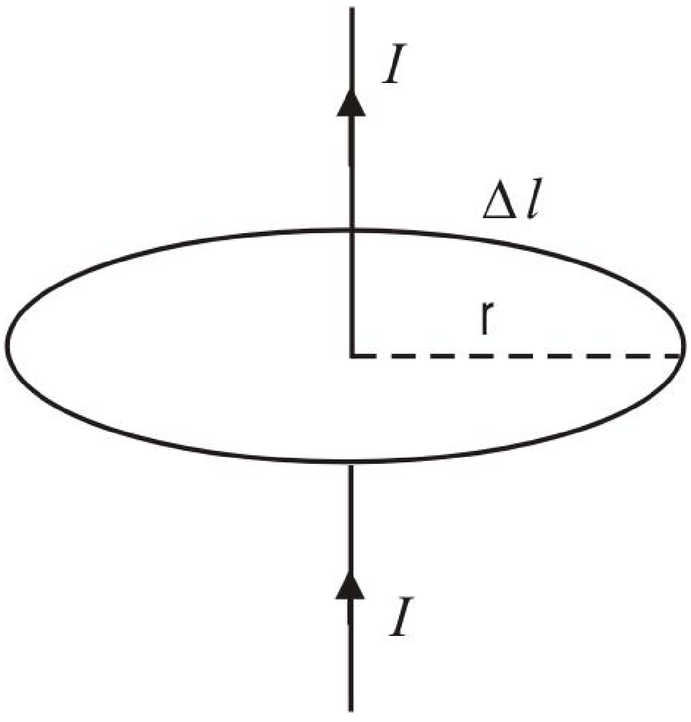
A ciricular Coil



AMPERE´S LAW

$$\Sigma B_{\|}\Delta l = \mu_0 I_{encl}$$

FIELD OF A LONG STRAIGHT WIRE



Application of Ampere´s Law

Ampere´s law is very useful in finding the magnetic field situations of high geometrical symmetry.

- When the loop encloses the conductor or a current the line integral equals $\mu_o i$ i.e.

$$\int B.dL = \mu_o i$$

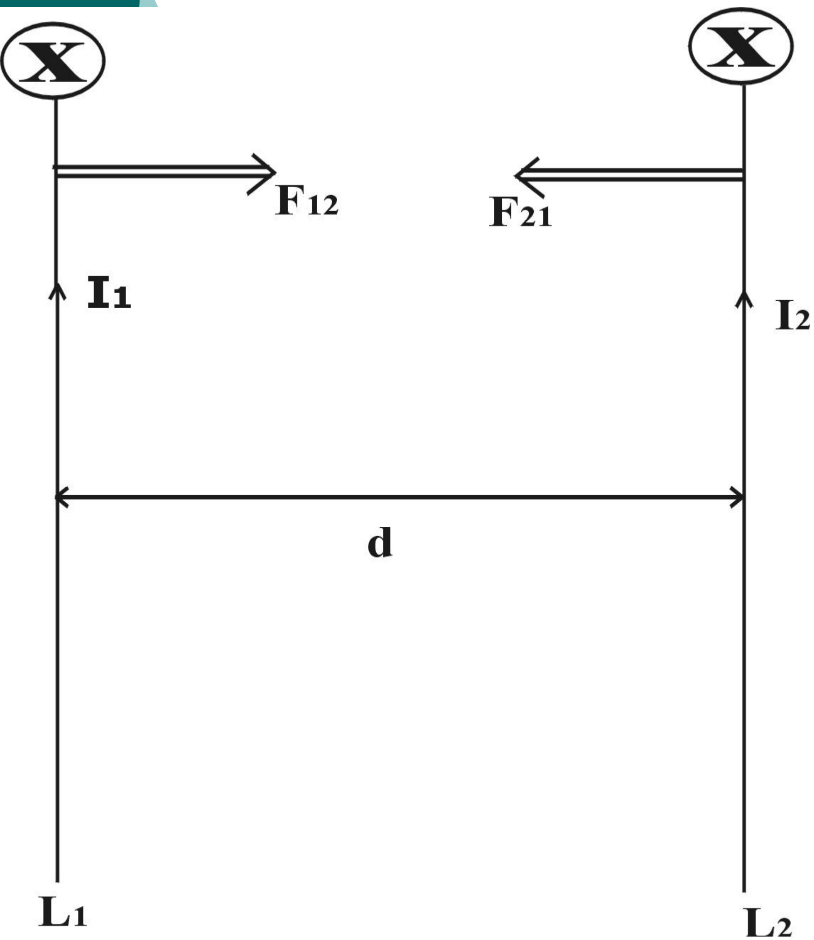
- When the loop does not enclose any current or when the loop does not enclose the conductor the line integral equals zero

$$\int B.dL = 0$$

- In the diagram below the current enclosed by the Amperian loop (i.e. for r < R) is given by $\left(\pi r^2 / \pi R^2\right) i$ Hence Ampere´s law becomes

$$B = \left(\frac{\mu_o i}{2R^2} \right) r$$

Magnetic Force between Two Current-Carrying Conductors



Consider two long straight parallel conductors *C* and *D*, a distance *d* apart. One carries a current I_1 and the other carries a current I_2

Such wires exert a force on each other.

Hence, the force per unit length on the conductor *D* due to the field around *C* is,

$$F = BI_2 l = \frac{\mu_o I_1 I_2}{2\pi d}$$

for l = 1 m. Now if $I_1 = I_2 = I$, the force per unit length on conductor *D* is

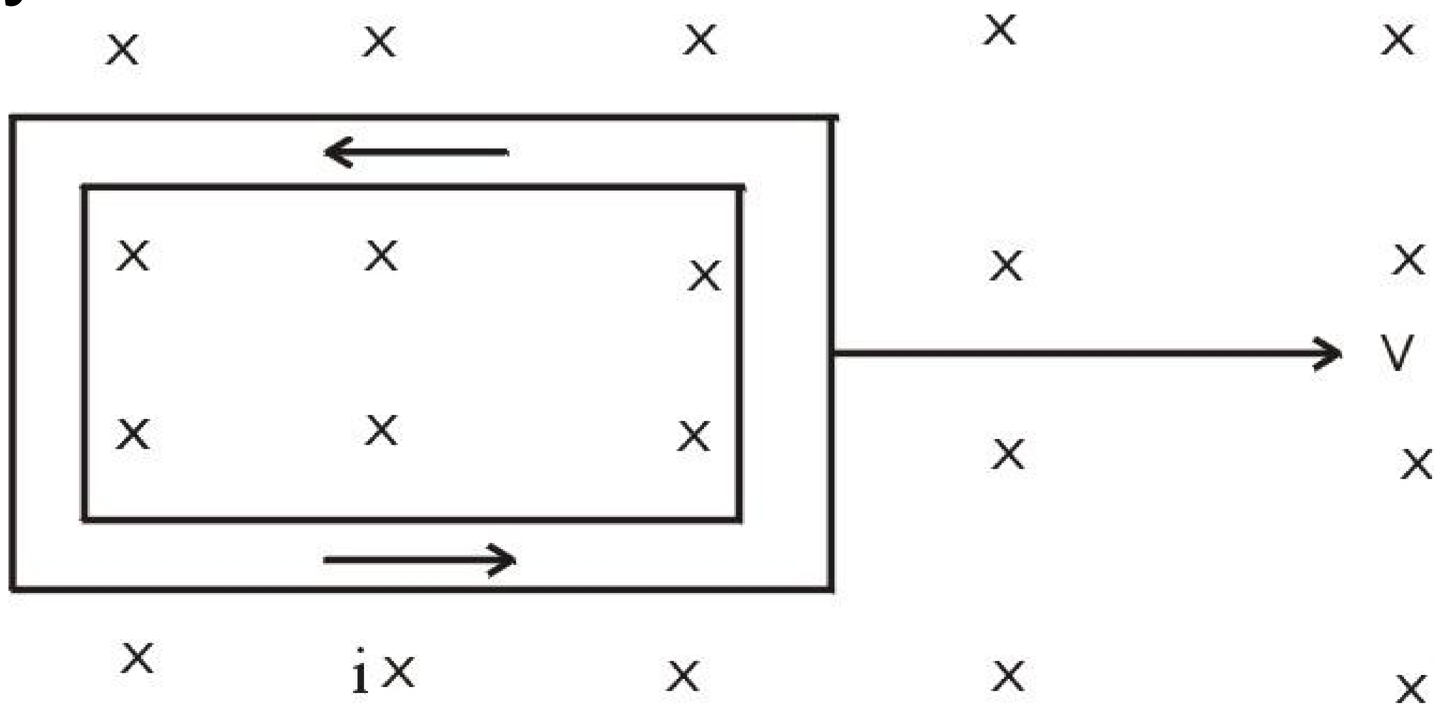
$$F = BI_2 l = \frac{\mu_o I^2}{2\pi d}$$

MOTIONAL ELECTROMOTIVE FORCE

Current could be generated magnetically but such an effect is observed *only when the magnetic flux through the circuit changes with time.* This effect is referred to as ***electromagnetic induction***, and the currents and emfs that are generated this way are called ***induced currents*** and ***induced emfs***.

A rod in a magnetic field

Consider a conductor of length *l* moving perpendicular to a magnetic field *B* with velocity *v*.



- The charges in the moving conductor move relative to the field ***B*** and therefore experience a force

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}.$$

- The emf associated with the rod is the work done by the driving force on a unit positive charge that passes from the negative end of the rod to the positive end $\varepsilon = lvB$
- This is called a *motional emf* because it is generated by the motion of the rod through the magnetic field.

Maxwell's equations

$$\oint_{GS} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_o}$$

$$\oint_{GS} \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{s} = \frac{1}{c^2}\left(\varepsilon_o I_{net} + \frac{d\Phi_E}{dt} \right)$$

Faraday's law of induction

The equation $\varepsilon = lvB$ is an expression of Faraday's law of induction which states that:

The induced emf along any moving path in a constant or changing magnetic field equals the time rate at which magnetic flux sweeps across the path.

Faraday’s law

The induced emf around a closed path in a magnetic field is equal to the time rate of change of the magnetic flux intercepted by the area of the path.

That is

$$\varepsilon = -d\,\Phi_B/dt.$$

The minus sign is an indication of the direction of the induced emf .

Expression of Faraday's Law

The magnitude of the flux is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA = BLx$$

The time rate of change of this quantity is

$$\frac{d\Phi_B}{dt} = BL\frac{dx}{dt} = BLv$$

$$\mathcal{E} = \frac{d\Phi_B}{dt}$$

This is Faraday's law, with a minor but important correction of sign.

Line integral of the induced emf

The electric force is precisely equal to the magnetic force observed by someone at rest relative to the rails:

$$q\vec{E} = q\vec{v} \times \vec{B} \quad or$$

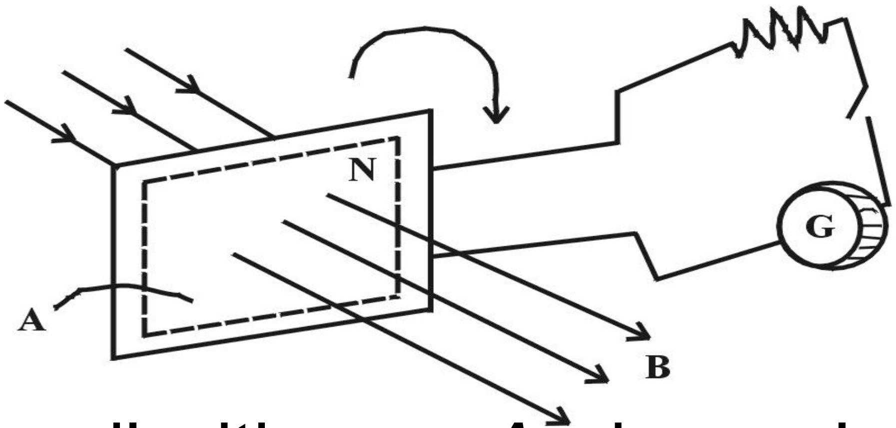
$$E = vB$$

$$\mathcal{E} = BvL = EL = \int_P^Q \vec{E} \cdot d\vec{s}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}$$

Application of Faraday’s Law

- **Flip-Coil Measurement of Magnetic Field**

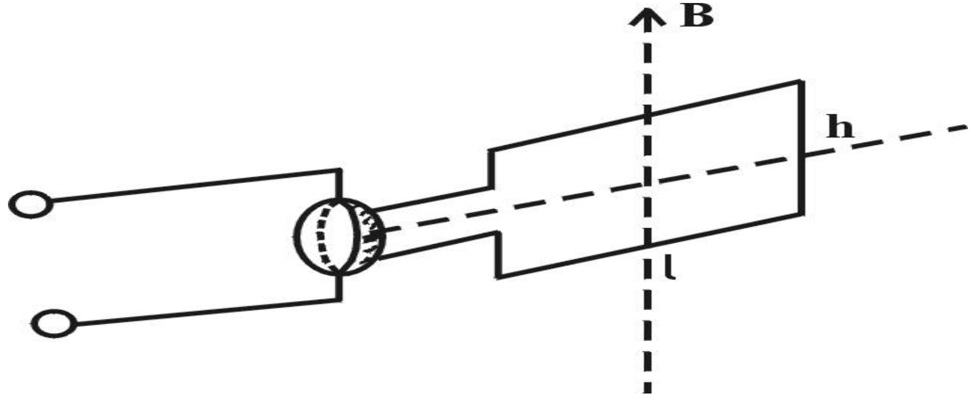


Consider a coil with area *A* whose plane is perpendicular to a uniform magnetic field *B.* The flux through the coil is

$$\Phi_B = NAB.$$

Where N is the number of turns on the coil

• A Rectangular Loop Generator



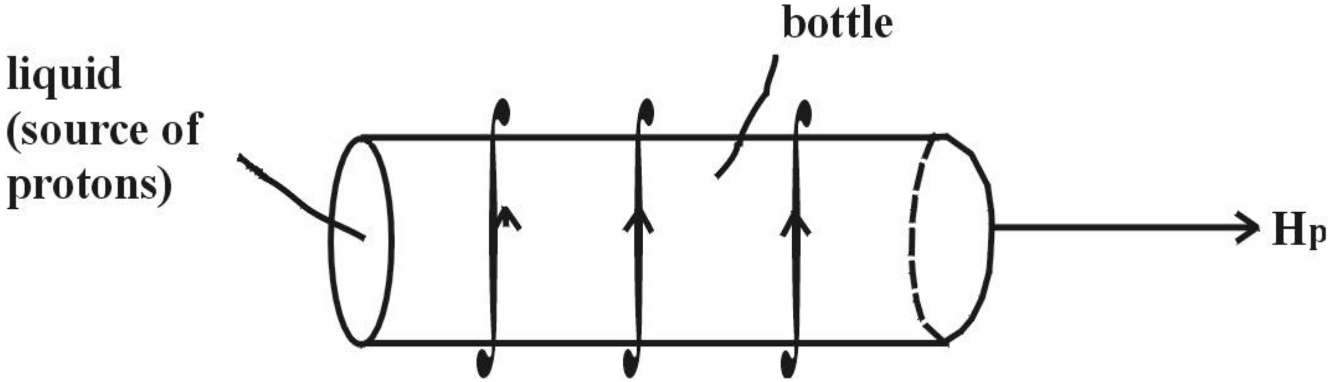
A rectangular loop of wire of length *l* and height *h* is rotated about an axis perpendicular to an external field ***B.*** The flux through the loop is given as

$$\Phi_B = Blh\cos\omega t$$

$$\frac{d\Phi_B}{dt} = -\omega Blh\sin\omega t = -\varepsilon$$

The current is $I = \varepsilon / R = (\omega Blh/R)\ \sin\omega t.$

- **The Spin-Echo Magnetometer**



The spin-echo magnetometer is a magnetic field measuring device. One of its applications is in measuring magnetic anomalies in the ocean floor. A measurement of the frequency of the induced emf thus gives oceanographers and geophysicists a measurement of the strength of the magnetic field on the ocean floor.

MAGNETIC PROPERTIES OF MATTER

2.0 MACROSCOPIC MAGNETIC PROPERTIES OF MATTER

This is due to the following atomic properties:

- **An atom is made up of a number of charged particles in constant motion. Electrons orbit round the nucleus continually whilst within the nucleus protons orbit round each other.**

- **These orbital motions may be considered flowing electric currents which generate corresponding magnetic fields.**

- The electrons, protons and neutrons all spin about their axes which creates flowing electric currents generating magnetic fields.

- These magnetic fields can be described in terms of their corresponding magnetic dipole moments.

- **These small magnetic dipole moments can produce a strong magnetic field especially in the presence of an external magnetic field.**

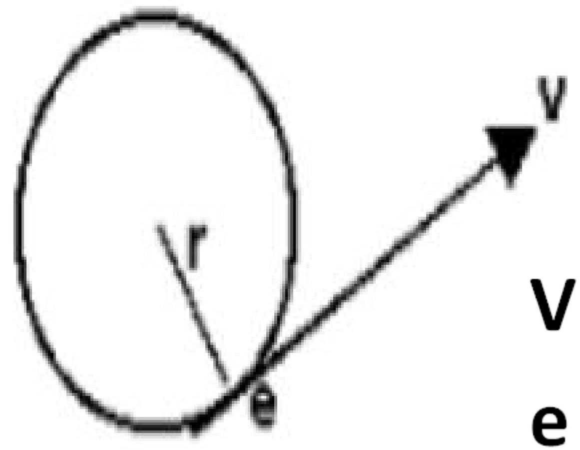
- The strength of the magnetic field produced, however, depends on how readily the atomic and subatomic dipoles respond to the external magnetic field.

Depending on their magnetic response, materials may be put into the following categories:

- **Diamagnetic Materials**
- **Paramagnetic Materials**
- **Ferromagnetic Materials**

2.1 ATOMIC AND NUCLEAR MAGNETIC MOMENTS

An electron moving in an orbit around a nucleus produces an average current *I* along its orbit. For an electron in a circular path,



An electron in a circular orbit

V = speed   r = radius of circular path

e = electron with charge *e*

The time for one complete cycle $T = 2\pi r/v$

$I = \text{charge/time} = e/(2\pi r/v) = ev/2\pi r$ (2.1)

orbital magnetic dipole moment $\mu_m = I \times$ (area of orbit)

$$\boldsymbol{\mu_m = ev/2\pi r \times \pi r^2 = evr/2}$$ (2.2)

$$\boldsymbol{m_e \mu_m = m_e evr/2}$$ but $\mathbf{L = m_e vr}$

$$\boldsymbol{\mu_m = (e/2m_e).L}$$

m_e = mass of the orbiting electron

L = orbital angular momentum of the orbiting electron.

- The orbital magnetic dipole moment for an orbiting electric charge is thus proportional to the orbital angular momentum

The magnitude of L is always some integer multiple of the constant $\hbar$. Thus, the possible values of the orbital angular momentum L are

$$L = 0, \hbar, 2\hbar, 3\hbar, 4\hbar, \ldots\ldots\ldots \qquad (2.4)$$

Because angular momentum exists only in discrete packets, it is said to be quantized.

- **The net magnetic moment of the atom is obtained by combining the orbital and spin moments of all the electrons, taking into account the directions of these moments.**

NB

The nucleus of the atom also has a magnetic moment. This is due to

(i) the orbital motion of the protons inside the nucleus, and

- **(ii) the rotational motion of individual protons and neutrons.**

The magnetic moment of a proton or neutron is small compared with that of an electron, and in reckoning the total magnetic moment of an atom, the nucleus can usually be neglected.

2.2 CLASSIFICATION OF MAGNETIC MATERIALS

From above, $\mu = I\,\boldsymbol{A}$.

Thus, for a solenoid with ***N*** turns, the net magnetic dipole moment is $\mu = \boldsymbol{NIA}$

- If the solenoid is placed in a magnetic field, then it experiences a torque $\tau = \mu \; \boldsymbol{x}\, \boldsymbol{B}$.
- In a vacuum, the magnetic field inside a long solenoid is $\boldsymbol{B_E = \mu_o nI}$ or $\boldsymbol{B_E = \mu_o NIA/Al}$

where $\boldsymbol{n = N/l}$. Thus using $\mu = \boldsymbol{NIA}$, we have $\boldsymbol{B_E = \mu_o \mu /V}$

- where $\boldsymbol{V = A\,l}$ is the volume contained within the windings of the solenoid.

- **The magnetic moment changes with changing current. When the inside of the solenoid is filled with a material and a magnetic dipole moment is then induced, the magnetic dipole moment of the solenoid changes. That is, when the interior of the solenoid is filled with some material, the induced magnetic moments produce an additional contribution to the magnetic moment. This additional contribution may be denoted as** μi. Thus, $B = \mu o (\mu + \mu i)/V$. ....*

- Experimentally the induced magnetic moment depends on the current in the solenoid. This dependence can be written as

$$\mu_i = \chi_m \mu$$

where χm is the magnetic susceptibility. Depending on the type of material inside the solenoid, χ m may be constant (at a particular temperature) or may depend on the current *I*. In terms of χ m, Eq. * becomes

$B = \mu o (1 + \chi m) \mu /V$

where μ is determined from the characteristics of the solenoid and the current in the solenoid. The magnitude of *B* is determined from measurements of magnetic flux. Thus the magnetic susceptibility is determined from the equation

$$\chi_m = (BV/\mu_o \mu) - 1.$$

- Now, $\boldsymbol{1 + \chi_m}$ is the relative permeability and is denoted by κ_m. That is

$$\kappa_m = 1 + \chi_m.$$

Equation $\boldsymbol{B = \mu_o (1 + \chi_m) \mu / V}$ becomes

$$\boldsymbol{B = \mu_o \kappa_m \mu / V}.$$

2.3 DIAMAGNETISM

Diamagnetic materials

- interact weakly with an imposed magnetic field.
- weaken the existing magnetic field.
- have negative values of χ_m.
- The magnetic susceptibility is independent of temperature and solenoid current.

A change in magnetic field lines threading a current loop causes a current to be induced in the loop. The magnetic flux produced by the induced current always acts to oppose the change.

- **Diamagnetism is a property of all materials, but it is a very weak property and is observed in materials made of atoms that have permanent magnetic dipole moments.**

- **When a diamagnetic material is placed in a magnetic field *B*, the force experienced by the electron is $-ev \times B$ in addition to the usual electric force within the atom**
- **Assume that the nucleus produces an electric field *E*. Then the net force on the electron is $-eE - ev \times B$.**
- **To keep the electron in a circular orbit of radius *r*,**

$$eE + evB = m_e v^2/r \quad \ldots\ldots\ldots \quad (1)$$

- Using $\omega = v/r$, Eq. (1)can be written as

$$eE + e\omega rB = m_e\omega^2 r \quad .....(2)$$

In the absence of the magnetic field

$$eE = m_e\omega_o^2 r. \quad ....(3)$$

- Subtracting Eq.(3) from Eq.(2), we have

$$e\omega B = m_e(\omega^2 - \omega_o^2). \quad ....(4)$$

The increment of frequency is

$$\Delta\omega = \omega - \omega_o. \qquad .....(5)$$

For small magnetic field, $\Delta\omega$ is small compared with ω_o.

Hence

$$\omega^2 - \omega_o^2 = (\omega_o + \Delta\omega)^2 - \omega_o^2$$

$$= 2\omega_o\Delta\omega + (\Delta\omega)^2 \cong 2\omega_o\Delta\omega \qquad .....(6)$$

Eq.(4) is now

$$e\omega B \cong 2m_e\omega_o\,\Delta\omega. \qquad ......(7)$$

- ω and ω_o are nearly equal.

Thus Eq.(7) becomes

$$\Delta\omega = eB/2m_e. \qquad ...(8)$$

This frequency is called the *Lamor frequency*; it tells how much faster the electron moves around its orbit as a result of the presence of the magnetic field.

There is a change in the orbital magnetic moment corresponding to the change $\Delta\omega$ in the orbital frequency.

From Eq. 2.2

$$\mu = evr/2 = er2\omega_o/2.$$

Hence

$$\Delta \quad \mu \quad = (er^2/2)\, \Delta\omega.$$

Thus the fractional change in the magnetic moment is

$$\Delta \; \mu \; / \; \mu \quad = \Delta\omega \,/\, \omega_o$$

……………….(9)

Typically, the frequency of an electron in an atom is $\omega_o \cong 10^{16}$ s^{-1}. If the magnetic field is B = 1.5 T, then

$\Delta\omega = \boldsymbol{e\,B\,/2\,m_e}$ = 1.6 x 10^{-19} x 1.5/(2 x 9.1 x 10^{-31})

= 1.32 x 10^{11} s^{-1}.

As a result,

$\Delta\mu / \mu = \Delta\omega / \omega_o$ = 1.32 x $10^{11}/10^{16}$

= 1.32 x $10^{-5} \cong 10^{-5}$.

That is, the magnetic moment changes by about 1 part in 10^5. This gives an indication of the small size of the diamagnetic effect.

2.4 PARAMAGNETISM

Paramagnetic materials

interact weakly with the imposed magnetic field

strengthen the existing magnetic field

- **have positive values of χ_m.**
- **χ_m depends on temperature and is essentially independent of solenoid current**

- **A paramagnetic material is composed of a uniform distribution of atomic magnetic dipoles sufficiently separated so that the magnetic field of any given dipole does not influence any of its neighbours.**

- **In the absence of magnetic field, the dipoles are randomly oriented as a result of thermal motions.**

The net magnetic moment of a paramagnetic material is, therefore, zero.

- **However, when an external magnetic field is applied, the dipoles align themselves with the field and produce a net magnetic moment in the material.**

Magnetic alignment can be achieved in two ways: (i) by lowering the temperature of the specimen or (ii) by increasing the applied magnetic field.

<u>How does such an increase of magnetic field come about?</u>

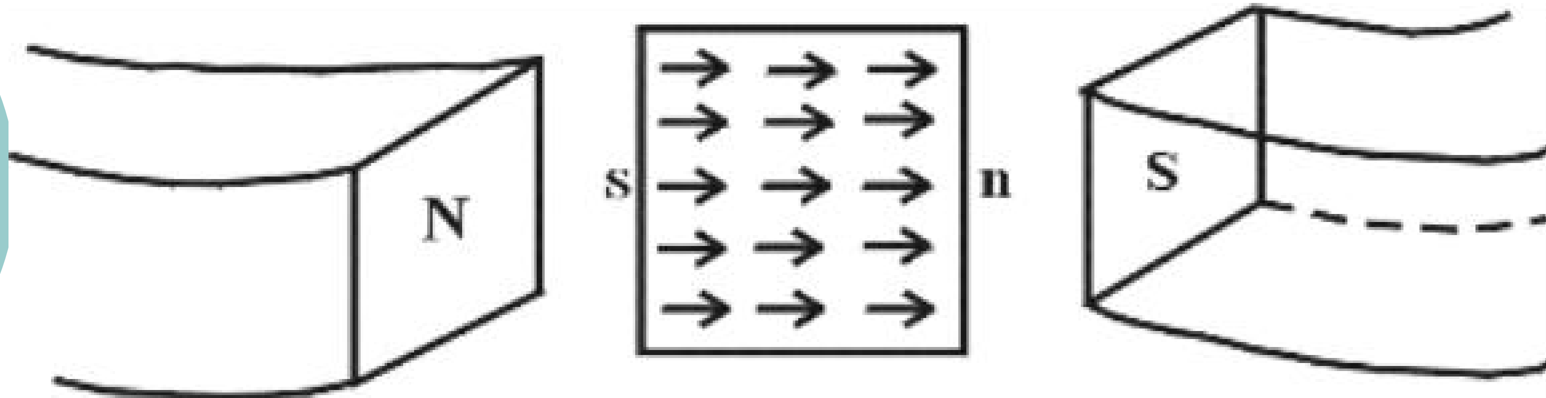
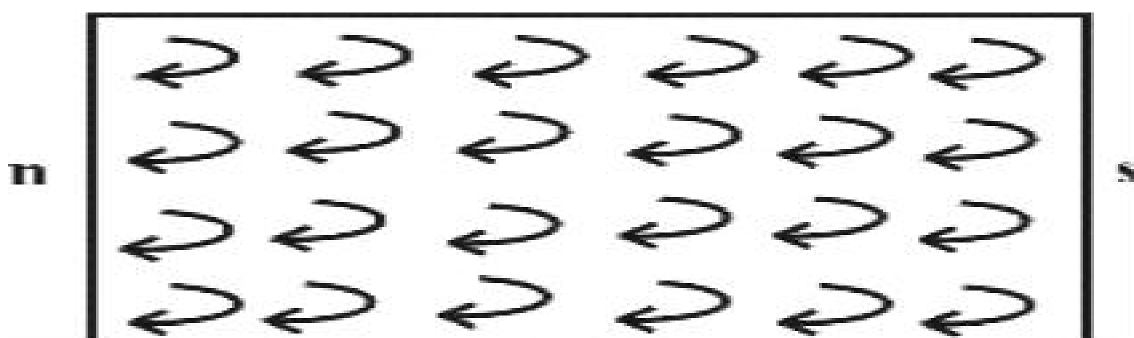


Fig. 2.1: A piece of paramagnetic material in an electromagnet.



oops.

Figure 2.1 shows the alignment of the magnetic

dipole in such a material

Figure 2.2 shows the alignment of current loops.

- **Consider a piece of paramagnetic material placed between the poles of an electromagnet.**
- **The magnetic dipoles are due to small current loops within the atoms.**

- Now look at any point inside the material where two of these current loops (almost) touch. The currents at this point are opposite and cancel. Thus, everywhere inside the material, the current is effectively zero.

- However, at the surface of the material, the current does not cancel. The net result of the alignment current loops is therefore a current running along the surface of the magnetized material.

- **The material consequently behaves like a solenoid; it produces an extra magnetic field in its interior.**

This extra magnetic field has the *same* direction as the original, external magnetic field. Hence, the total magnetic field in a paramagnetic material is larger than the original magnetic field produced by the currents of the electromagnet.

NB:

The alignment of atomic dipole moment in a paramagnetic specimen enhances the magnetic dipole moment,

- **and the magnetic field increases.**
- **It follows that χ_m is positive.**

2.5 FERROMAGNETISM

Ferromagnetic materials

- **interact strongly with an imposed magnetic field**
- **strengthen the existing magnetic field**
- **have magnetic susceptibilities that depend sensitively on the solenoid current.**

- Ferromagnetism is exhibited by five elements - **iron** (Fe), **nickel** (Ni), **cobalt** (Co), **dysprosium** (Dy), and **gadolinium** (Gd) - **and some alloys, which usually contain one or more of these five elements.**

The intense magnetization in ferromagnetic materials is due to a strong alignment of the spin magnetic moments of electrons.

- **In these materials, there exists a special force that couples the spins of the electrons in adjacent atoms in the crystal.**

- **This force (known as *exchange coupling*) couples magnetic moments of adjacent atoms together in rigid parallelism.**

- **There are regions in every ferromagnetic specimen that have near perfect alignment of magnetic dipole moments even when there is no applied magnetic field.**

- **These regions are called *magnetic domains*. The direction of alignment of the dipoles varies from one domain to the next** (Fig. 2.3).

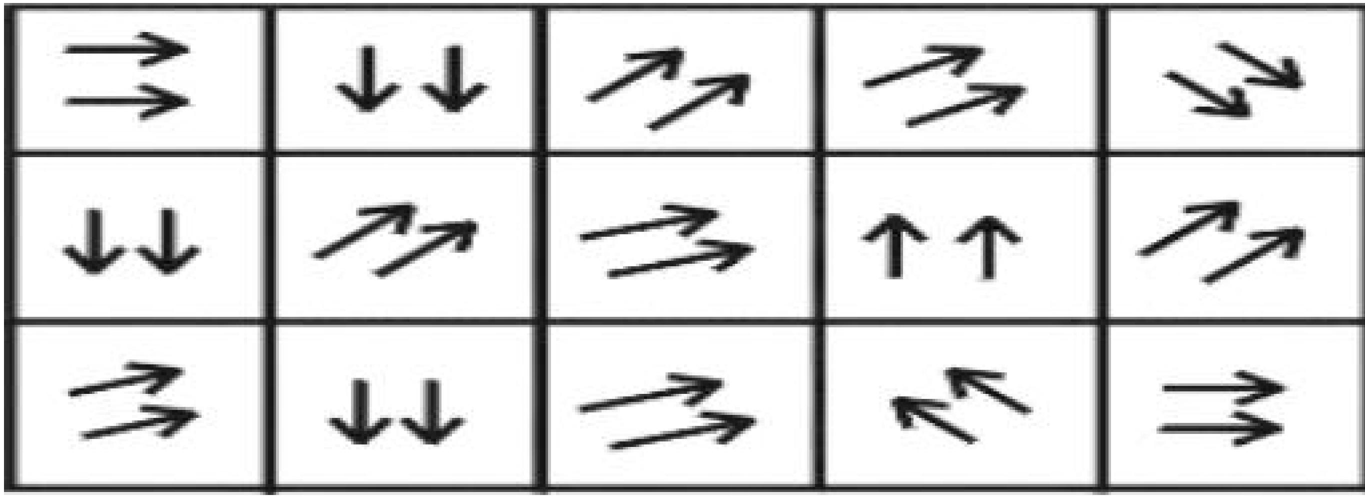


Fig. 2.3: Magnetic domains in a ferromagnetic material.

- **However, if the material is immersed in an external magnetic field, all dipoles tend to align along this field. The domains then change in two ways:**

a) **Those domains with magnetic dipole moments parallel to the magnetic field grow at the expense of the neighboring domains (Fig. 2.3). This effect is responsible for producing a net magnetic dipole moment in a weak applied magnetic field.**

b) **The magnetic dipole moments of the domains rotate toward alignment with the applied magnetic field. This is the mechanism of magnetic dipole alignment when the applied magnetic field is strong**

NB:

If the direction of the current in the solenoid is reversed, the magnetic field (*B*) within the specimen is reduced steadily from the remanent value B_r

- **At a critical value of B_E, called the *coercive force (B_c)*, the magnetic field is zero.**
- **The larger the coercive force, the more difficult it is to demagnetize a ferromagnetic specimen.**
- ***Ferromagnetic materials having a large coercive force are said to be magnetically "hard", those having a small coercive force are said to be magnetically "soft".***

Self-induction and Self-inductance

Electromagnetic forces generated by a circuit's own current are called *self-induced emfs*, and their generation is referred to as ***self-induction***.

According to Faraday's law, $\varepsilon = - d\Phi_B/dt.$

For a coil the flux linkage $N\Phi_B$ is proportional to the current I, or

$$N\Phi_B = LI.$$

L is the proportionality constant, and is called the *self-inductance* or simply *inductance* of the device.

$$\varepsilon = \frac{d\Phi_B}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

units of L = (units of ε) /(units of dI/dt)

= volts / (ampere/second)

= ohm second = Ω.s

= henry (H).

Self-inductance, like capacitance, depends on geometric factors.

Self-Inductance of a Solenoid

The axial magnetic induction ***B*** within the turns of a solenoid is

$$B = \mu_o n I, \qquad \text{........ } 1$$

$$\Phi_m = BA = \mu_o n I A \qquad \text{........ } 2$$

$$\frac{d\Phi_m}{dt} = \mu_o n A \frac{dI}{dt} \qquad \text{....... } 3$$

The self-induced emf is now obtained from Faraday's law as

$$\varepsilon = -N\frac{d\Phi_m}{dt} = -\mu_o NnA\frac{dI}{dt} \qquad \text{..........} \quad 4$$

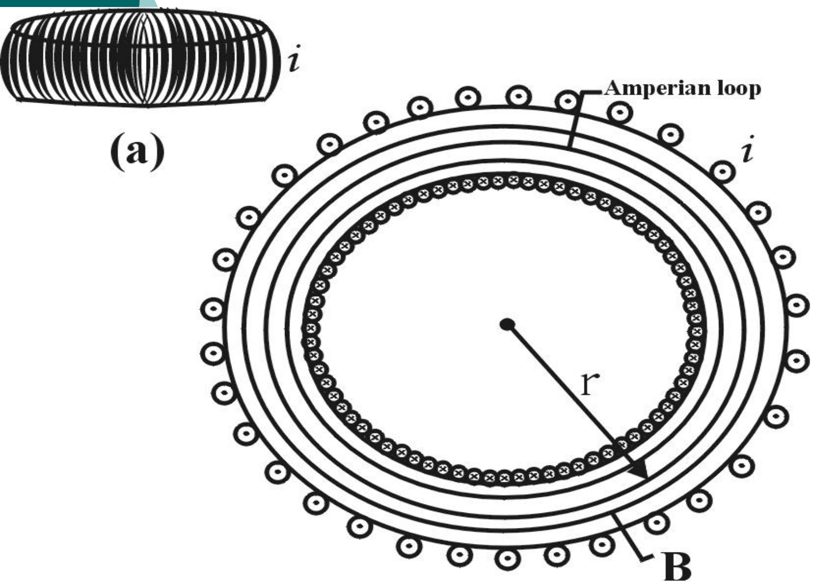
The self-induced emf can then be written as

$$\varepsilon = -\mu_o n^2 IA\left(\frac{dI}{dt}\right) \qquad \text{......} \quad 5$$

the self-inductance is found to be

$$L = \mu_o n^2 IA \qquad \text{......} \quad 6$$

Self-inductance of a Toroid of a Rectangular Cross Section



The lines of the magnetic field ***B*** for the Toroid are concentric circles. Applying Ampere's law,

$$\int \boldsymbol{B}.d\boldsymbol{l} = \mu_o I$$

to a circular path of radius *r* yields

$$B(2\pi r) = \mu_o I_o N,$$

Hence $$B = \frac{\mu_o I_o N}{2\pi r}$$

the flux $\Phi_{\mathbf{B}}$ for the cross section of the toroid is

$$\Phi_{\mathrm{B}} = \int B.dS = \int_a^b B(hdr)$$

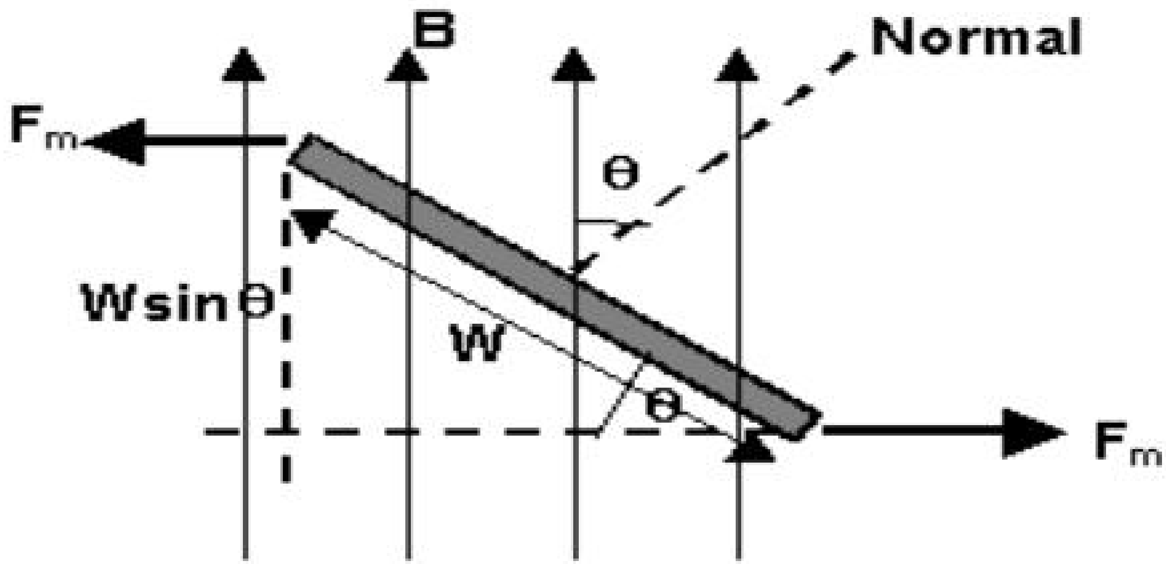
$$= \int_a^b \left(\frac{\mu_o I_o N}{2\pi r}\right) hdr$$

$$\Phi_B = \left(\frac{\mu_o I_o N h}{2\pi}\right) \int_a^b \frac{dr}{r} = \left(\frac{\mu_o I_o N h}{2\pi}\right) \ln\left(\frac{b}{a}\right)$$

Thus the self inductance is

$$\mathrm{L} = \frac{\mathrm{N}\Phi_{\mathrm{B}}}{\mathrm{I_o}} = \left(\frac{\mu_o N^2 h}{2\pi}\right) \ln\left(\frac{b}{a}\right)$$

A.C. THEORY



N is the number of turns, magnetic field B, the magnetic flux $\phi = BAN\cos\theta$, A = surface area of the coil and θ is angle between the field B and the normal to the surface.

$$\text{Induced emf } E = -\frac{d\phi}{dt} = -\frac{d\left(BAN\cos\theta\right)}{dt} = -\frac{d\left(BAN\cos\omega t\right)}{dt}$$

$$\therefore\ E = -\omega BAN\left(-\sin\omega t\right) = \omega BAN\left(\sin\omega t\right) \quad \cdots\cdots\cdots\cdots\ 1$$

Let E_{max} be maximum emf. Peak volta ge occurs at $\theta = \omega t = 90^{o}$

So

$$E_{max} = \omega BAN \quad \cdots\cdots\cdots\ 2$$

Equation 1 and 2 gives

$$E = E_{max}\sin\omega t \quad \cdots\cdots\cdots\ 3$$

likewise

$$i = i_{max}\sin\omega t \quad \cdots\cdots\cdots\ 4$$

Writing $\omega = 2\pi f$ in eqn 3 gives

$$E = E_{max}\sin 2\pi f$$

$$= E_{max}\sin\left(\frac{2\pi}{T}\right) \quad \text{where } f = \frac{1}{T} \quad T = \text{period}$$

relationship between the I_{rms}, V_{rms}, i_{max} and v_{max}

When a direct (i_d) and alternating (i) currents pass through a resistor, i produces a root mean square (I_{rms}) value equal to I_d

So

$$\text{Average of } i^2 = i_d{}^2 = I_{rms}^2$$

likewise $$\text{Average of } v^2 = v_d^2 = V_{rms}^2$$

For an alternating current,

$$i = i_{\max} \sin \omega t$$

$$\Rightarrow \quad I_{\text{rms}}^2 R = \left(\text{Average of } i^2\right)R = \left(\text{Average of } i_{\max}^2 \sin^2 \omega t\right)R$$

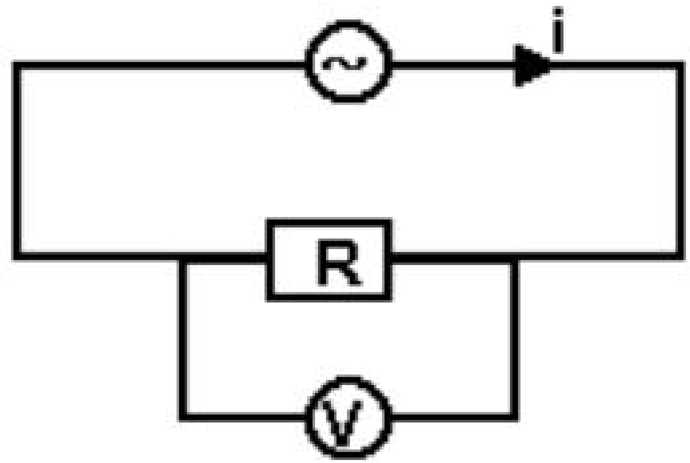
$$\Rightarrow \quad I_{\text{rms}}^2 = \left(\text{Average of } i^2\right) = \left(\text{Average of } i_{\max}^2 \sin^2 \omega t\right)$$

By integrating w.r.t to time, I_{rms}^2 and V_{rms}^2 are reduced to

$$I_{rms} = \frac{i_{\max}}{\sqrt{2}}$$

likewise $$V_{rms} = \frac{v_{\max}}{\sqrt{2}}$$

a.c circuit with pure resistance only



For the a.c circuit,

$$V = v_{\max} \sin \omega t \qquad \text{but} \quad V = iR$$

$$\therefore \quad v_{\max} \sin \omega t = iR$$

$$\Rightarrow \qquad i = \frac{v_{\max} \sin \omega t}{R} \qquad \text{but} \quad \frac{v_{\max}}{R} = i_{\max}$$

$$\therefore \qquad i = i_{\max} \sin \omega t$$

Thus, for a circuit with a pure resistance only,

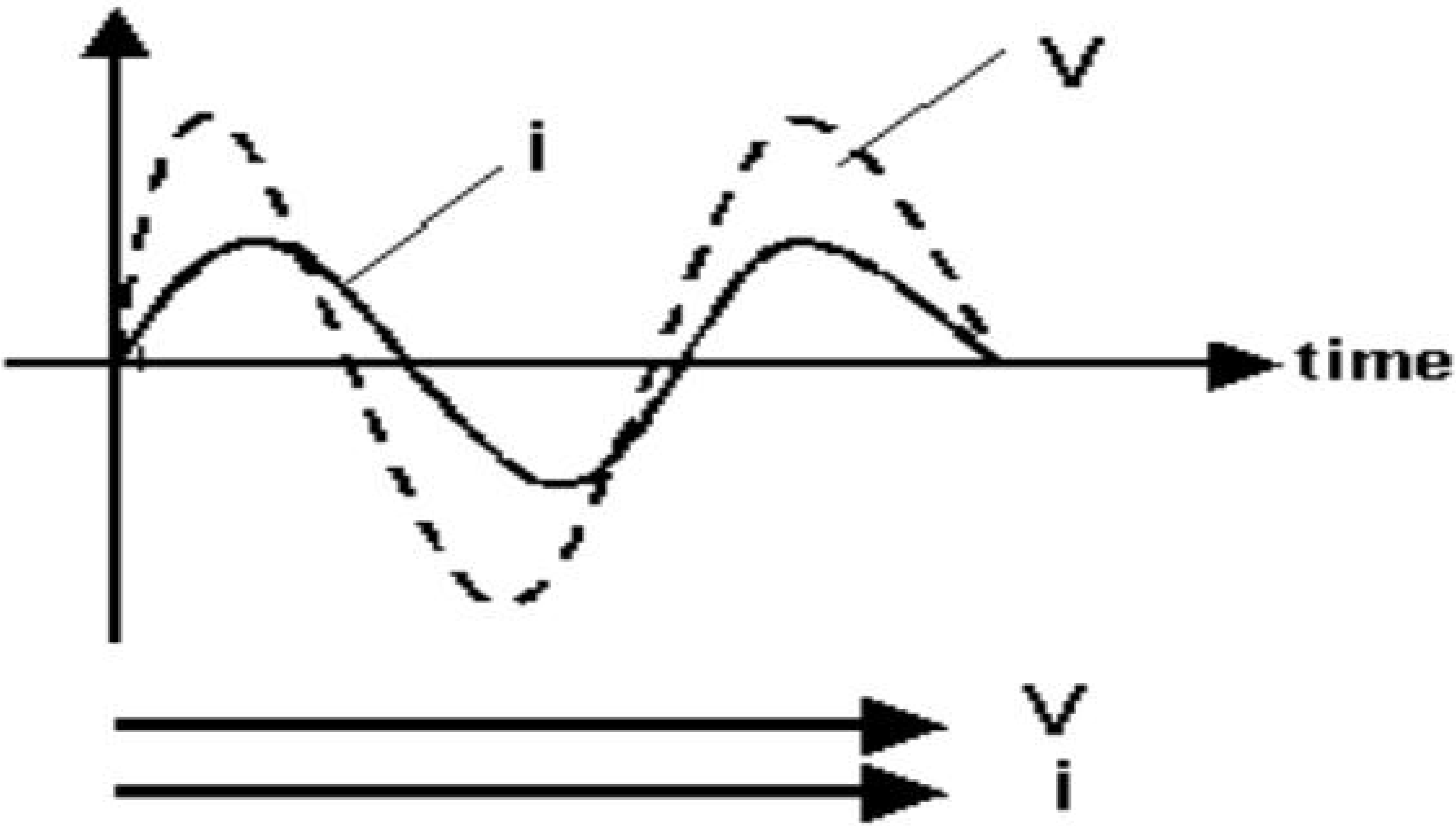
$$v = v_{\max} \sin \omega t$$

and

$$i = i_{\max} \sin \omega t$$

This means that the current is exactly in phase with the voltage across the resistance.

graph of i and v for a capacitor



Using $i_{\max} = \dfrac{v_{\max}}{R}$ and dividing through by $\sqrt{2}$

gives

$$\frac{i_{\max}}{\sqrt{2}} = \frac{v_{\max}}{R\sqrt{2}}$$

$$\Rightarrow \qquad I_{\text{rms}} = \frac{V_{rms}}{R}$$

power dissipated in the resistor of a pure resistive circuit

Instantaneous power, $P = iV$ where $V = v_{\max}\sin\omega t$ and $i = i_{\max}\sin\omega t$.

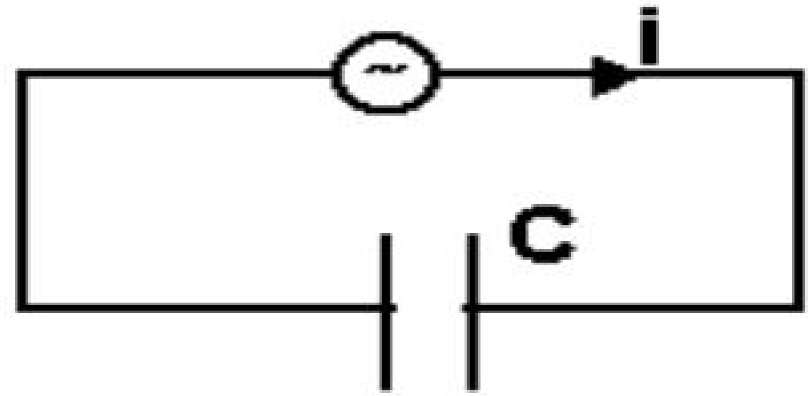
$$\Rightarrow \qquad P = i_{\max} v_{\max} \sin^2 \omega t$$

Average power dissipated over one cycle is given as

$$P_{ave} = i_{\max} v_{\max} \left[\int^{2\pi/\omega} \frac{\left(\sin^2 \omega t\right)}{2\pi/\omega} \right] dt = \frac{1}{2} i_{\max} v_{\max}$$

$or \qquad P_{ave} = i_r^2 R \qquad\qquad i_r = \text{current through resistor}$

a.c circuit containing a capacitance only



Let the voltage across the capacitor be v, then :

$$v = v_{\max} \sin \omega t$$

if q is the charge on the capacitor, then, $q = Cv$

$$\Rightarrow \qquad q = Cv_{\max} \sin \omega t \qquad \cdots\cdots\cdots \; 1$$

Now writing $i = \dfrac{dq}{dt}$ gives

$$i = \frac{d\left(Cv_{\max} \sin \omega t\right)}{dt} = C\omega v_{\max} \cos \omega t \quad \cdots \cdots 2$$

comparing eqn. 2 with the general equation

$$i = i_{\max} \cos \omega t$$

gives

$$i_{\max} = C\omega v_{\max}$$

Let X_c = capacitive reactance, then :

$$X_c = \frac{v_{\max}}{i_{\max}} = \frac{1}{C\omega} = \frac{1}{2\pi fC} \qquad (\text{with } \omega = 2\pi \text{f})$$

Capacitive reactance refers to the opposition of an a.c. circuit to a capacitor.

power dissipated in a capacitance

We recall that for the capacitor

$$v = v_{\max} \sin \omega t \qquad \text{and} \qquad i = i_{\max} \cos \omega t$$

Instantaneous power dissipated $\quad P = iv$

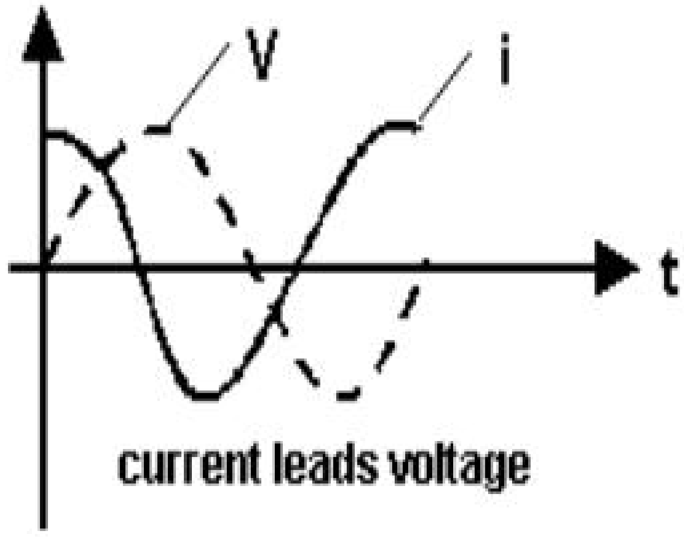
$$\therefore P = i_{\max} \cos \omega t . v_{\max} \sin \omega t = i_{\max} v_{\max} \sin \omega t \cos \omega t$$

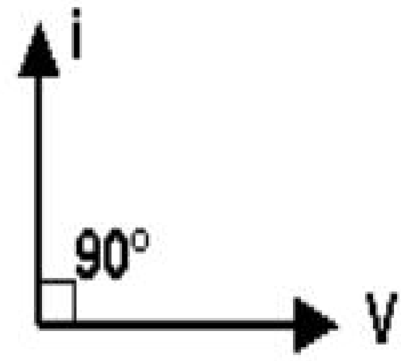
Using the relation $\quad \sin 2\theta = 2 \sin \theta \cos \theta$

$$\Rightarrow \qquad \sin \theta \cos \theta = \tfrac{1}{2} \sin 2\theta$$

$$P = \tfrac{1}{2} i_{\max} v_{\max} \sin 2\omega t$$

phase relation between current and voltage





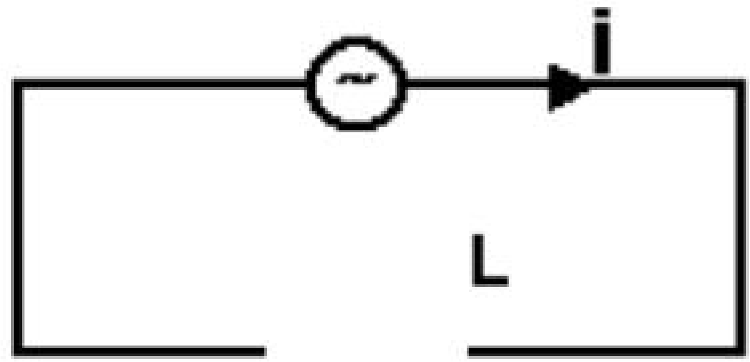
$$v = v_{\max} \sin \omega t$$

$$i = i_{\max} \cos \omega t$$

$$\Rightarrow i = i_{\max} \sin\left(\omega t + \tfrac{1}{2}\pi\right)$$

- Comparing the phases of the voltage and current in the figure above, we see that the **current leads** (i.e. it is ahead of) the voltage across the capacitance by a phase difference of ½ π or said otherwise the **voltage lags behind** the current by a phase difference of ½ π.

a.c. circuit containing inductance only



$V = applied\ \ voltage\,, E = induced\ \ emf\,.$

The back emf will oppose the variation of current through it and it is equal and opposite to the applied voltage V. i.e $\quad V = -E$

Let $$i = i_{\max} \sin \omega t \qquad \cdots\cdots 1$$

$$E = -L\frac{di}{dt} = -L\frac{d\left(i_{\max} \sin \omega t\right)}{dt} = -\omega L i_{\max} \cos \omega t$$

$But \quad V = -E$

$$\Rightarrow \qquad V = \omega L i_{\max} \cos \omega t \qquad \cdots\cdots 2$$

comparing eqn. 2 with the standard equation :

$$v = v_{\max} \cos \omega t \qquad \text{gives}$$

$$v_{\max} = \omega L i_{\max}$$

$$X_L = \frac{v_{\max}}{i_{\max}} = \omega L = 2\pi f L$$

X_L is inductive reactance; the opposition to an a.c due to an inductance .

power dissipated in an inductance

Comparing the power dissipated in a capacitor we see that the average power dissipated in a pure inductor is 0.

This is explained as follows: At one time when the current increases the induced emf produced opposes the increase. In another instance when the current decreases energy s released to restore the decrease. Hence no energy is dissipated.

We recall that for the inductor,

$$v = v_{\max} \cos \omega t$$

and

$$i = i_{\max} \sin \omega t$$

Instantane ous power dissipated $\quad \mathrm{P = iv}$

$$\therefore \quad P = i_{\max} \sin \omega t . v_{\max} \cos \omega t$$

$$P = i_{\max} v_{\max} \sin \omega t \cos \omega t$$

$$P = \frac{1}{2} i_{\max} v_{\max} \sin 2\omega t$$

phase relation between current and voltage

For an inductor,

$$i = i_{\max} \sin \omega t$$

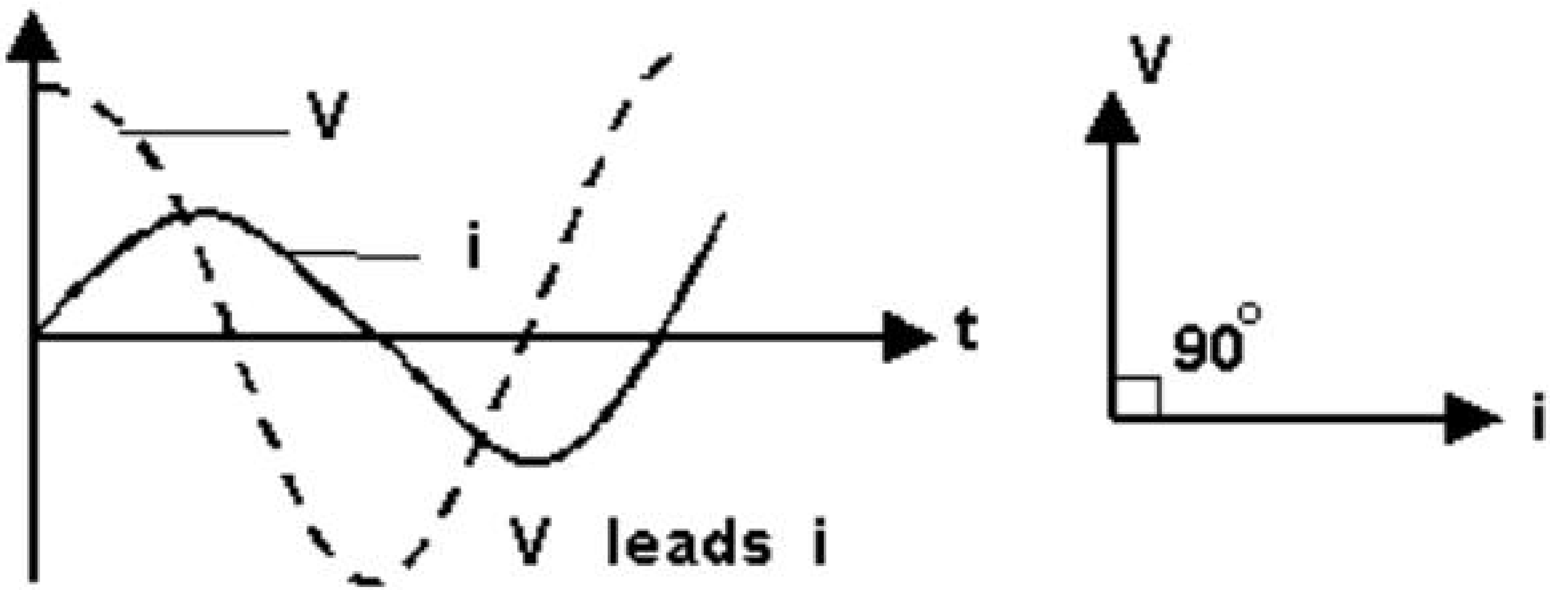
and

$$v = v_{\max} \cos \omega t$$

$$\Rightarrow \qquad v = v_{\max} \sin\left(\omega t + \tfrac{1}{2}\pi\right)$$

Hence, the voltage always leads the current by a phase difference of $\tfrac{1}{2}\pi$

graph of i and v for an inductor



Reactance: This refers to the opposition to electrical current due to the storage of magnetic or electrical energy in the circuit.

inductive reactance and capacitive reactance

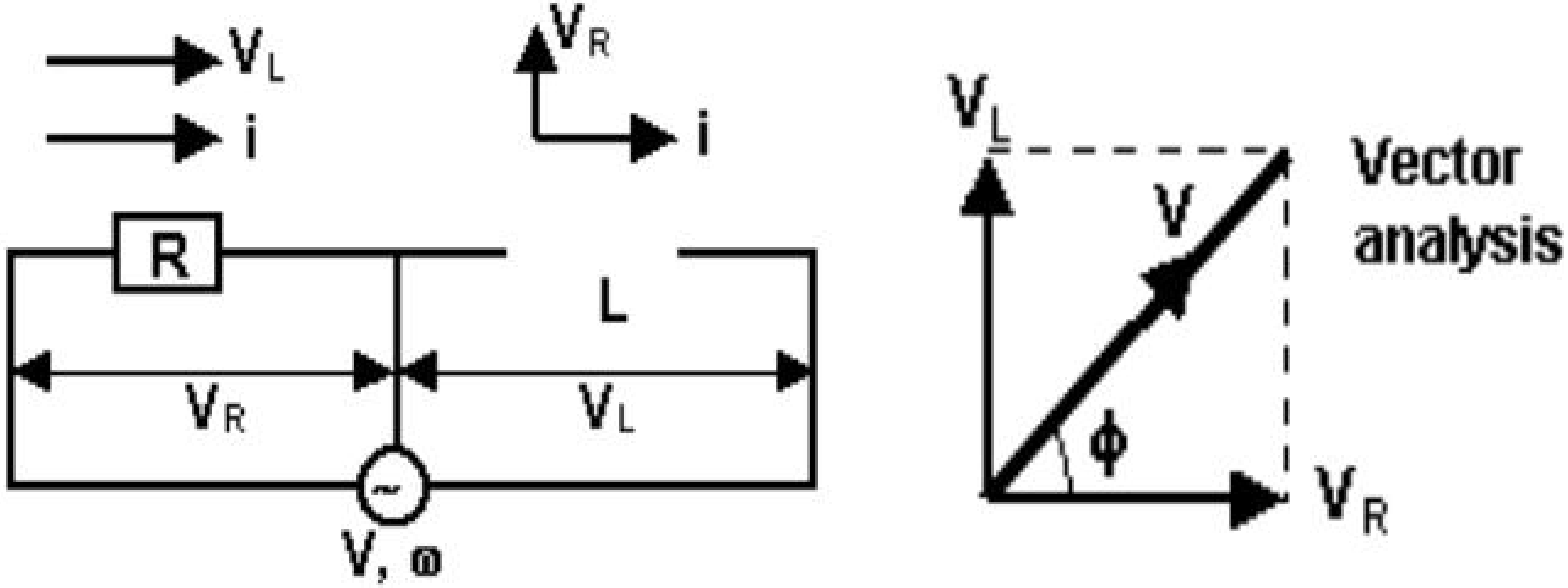
$x_L \propto \omega \qquad x_c \propto 1/\omega$

- For the same current the voltage in a pure inductive circuit leads the current by 90^o whilst the voltage in a pure capacitive circuit lags behind the current by 90^o.
- For a circuit containing a capacitance and an inductor, x_L is considered positive and x_C negative.

- An aid for remembering the relationship between current (I) and voltage (V) in both capacitors (C) and inductors (L) is CIVIL; that is for a capacitor (C), current (I) leads voltage (V) and for an inductor (L) ,voltage (V) leads current(I).

ac series circuits

i. R – L Series Circuit



The total reactance z is found by vector analysis.

From the vector diagram above,

$$V^2 = V_R^2 + V_L^2$$

$$\Rightarrow \qquad V = \left[V_R^2 + V_L^2\right]^{1/2}$$

Writing $\quad V_R = iR,$ and $\quad V_R = i\omega L \quad$ gives

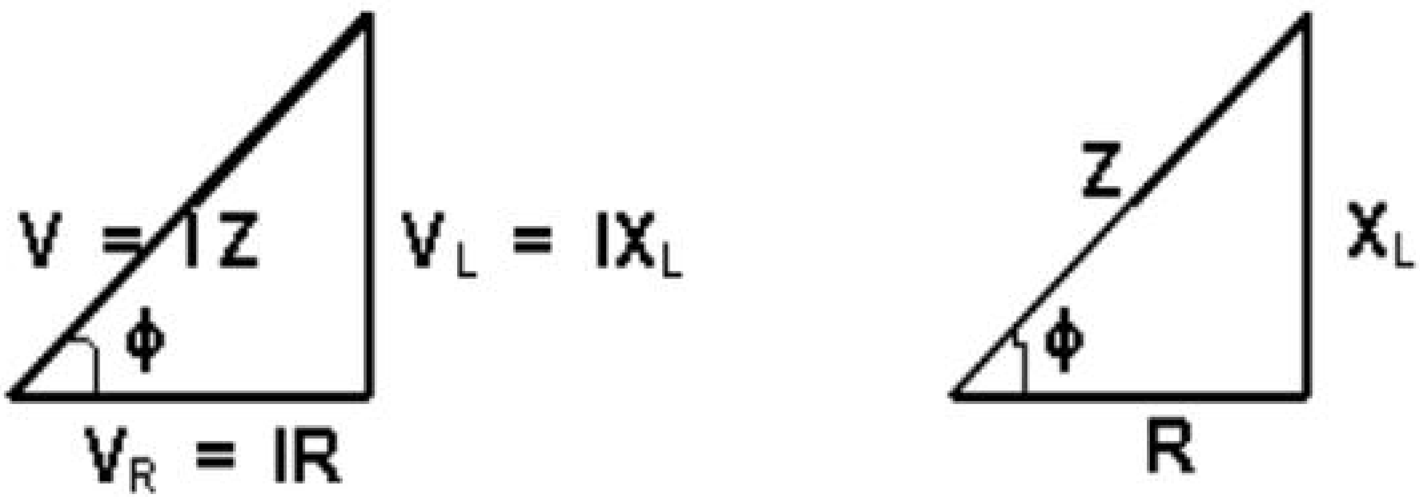
$$V = \left[(iR)^2 + (i\omega L)^2\right]^{1/2}$$

$$= i\left[(R)^2 + (\omega L)^2\right]^{1/2}$$

$$i = \frac{V}{\left[R^2 + (\omega L)^2\right]^{1/2}} = \frac{V}{\left[R^2 + (X_L)^2\right]^{1/2}}$$

The quantity $\left[R^2 + (X_L)^2\right]^{1/2}$ has a unit of resistance and is called impedance (z) : defined as the effective (resultant) opposition to the flow of an a.c. in a combination of resistance and reactance.

voltage triangle & impedance triangle



The vector V leads the vector I by an angle of ϕ. The cosine of this angle (i.e. $\cos\phi$) is called the **power factor** (pf).

$\Rightarrow$ Pf = $\cos\phi$ = R/Z

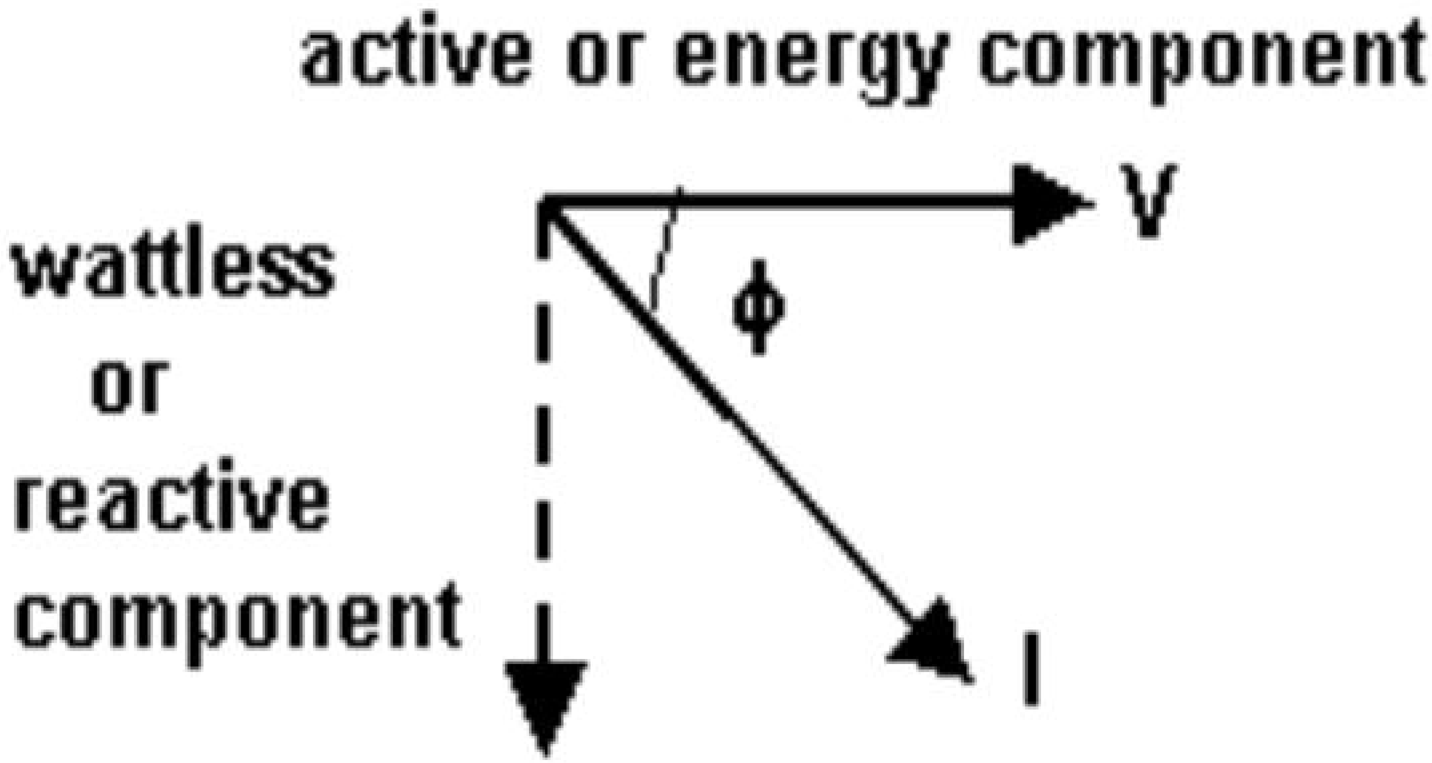
ii. Power dissipated in an R – L Series Circuit

We recall that the average power dissipated in L is zero. Hence for a R–L series circuit all the power is dissipated in R.

$$\text{Power } P = VI\cos\phi$$

= voltage × component of current in phase with V

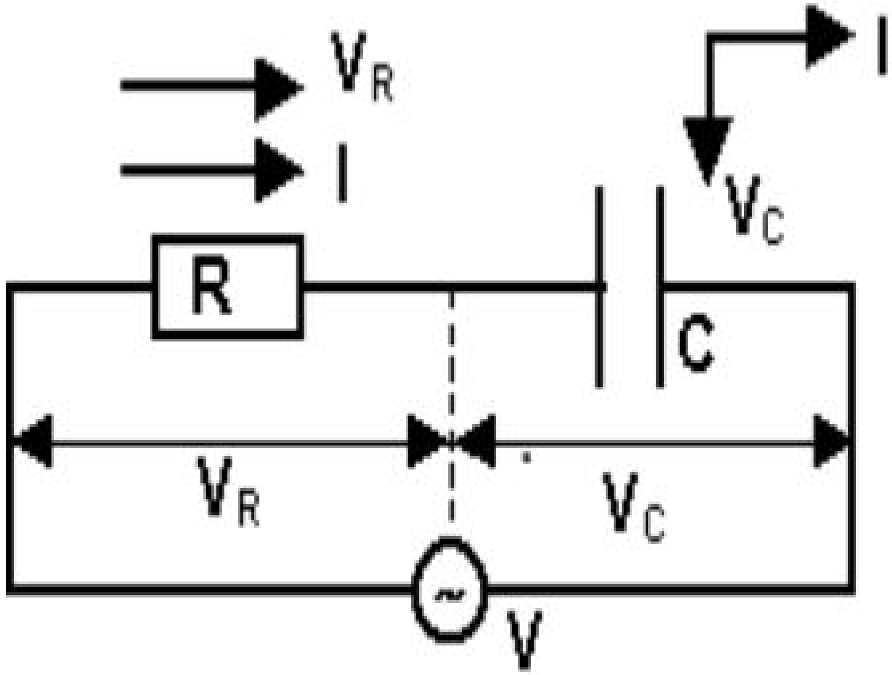
iii. Active and Reactive Components of a Circuit

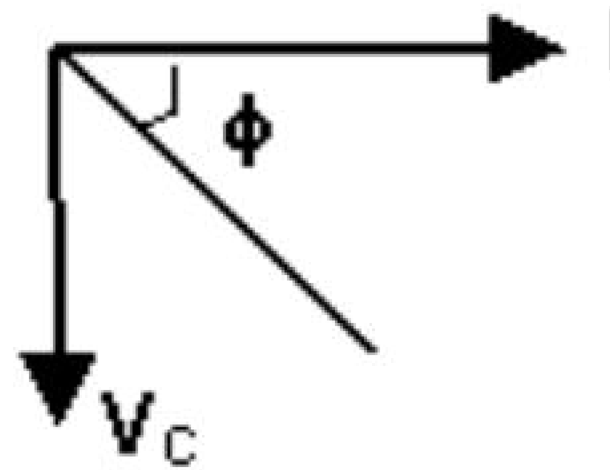


The figure above shows the vector diagram of a circuit in which the voltage V leads the current I by an angle ϕ. The current may be resolved into two parts:

1. $I\cos\phi$ = voltage-in-phase component.
2. $I\sin\phi$ = voltage-out-of-phase component

R – C SERIES CIRCUIT





$$V = \sqrt{\left(V_R^2 + V_C^2\right)}$$

where $V_R = IR$, $V_C = \dfrac{1}{\omega C}$ or $X_C = \dfrac{1}{\omega C}$

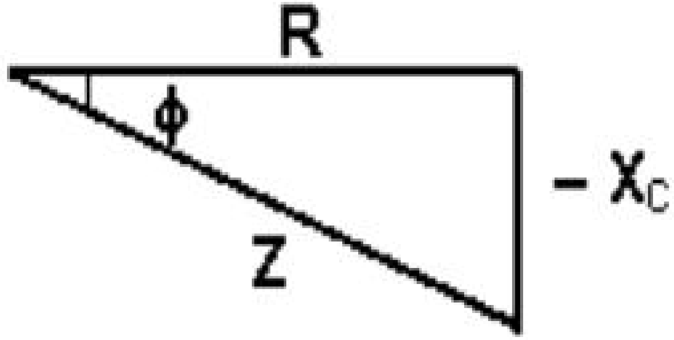
$$\tan\phi = \frac{X_C}{R} = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$X_C =$ capacitive reactance.

note :

1. If the voltage leads the current then, the angle is in the first quadrant and is considered positive.

2. If the current leads the voltage, then, the angle ϕ is in the 4th quadrant and is considered **negative**.

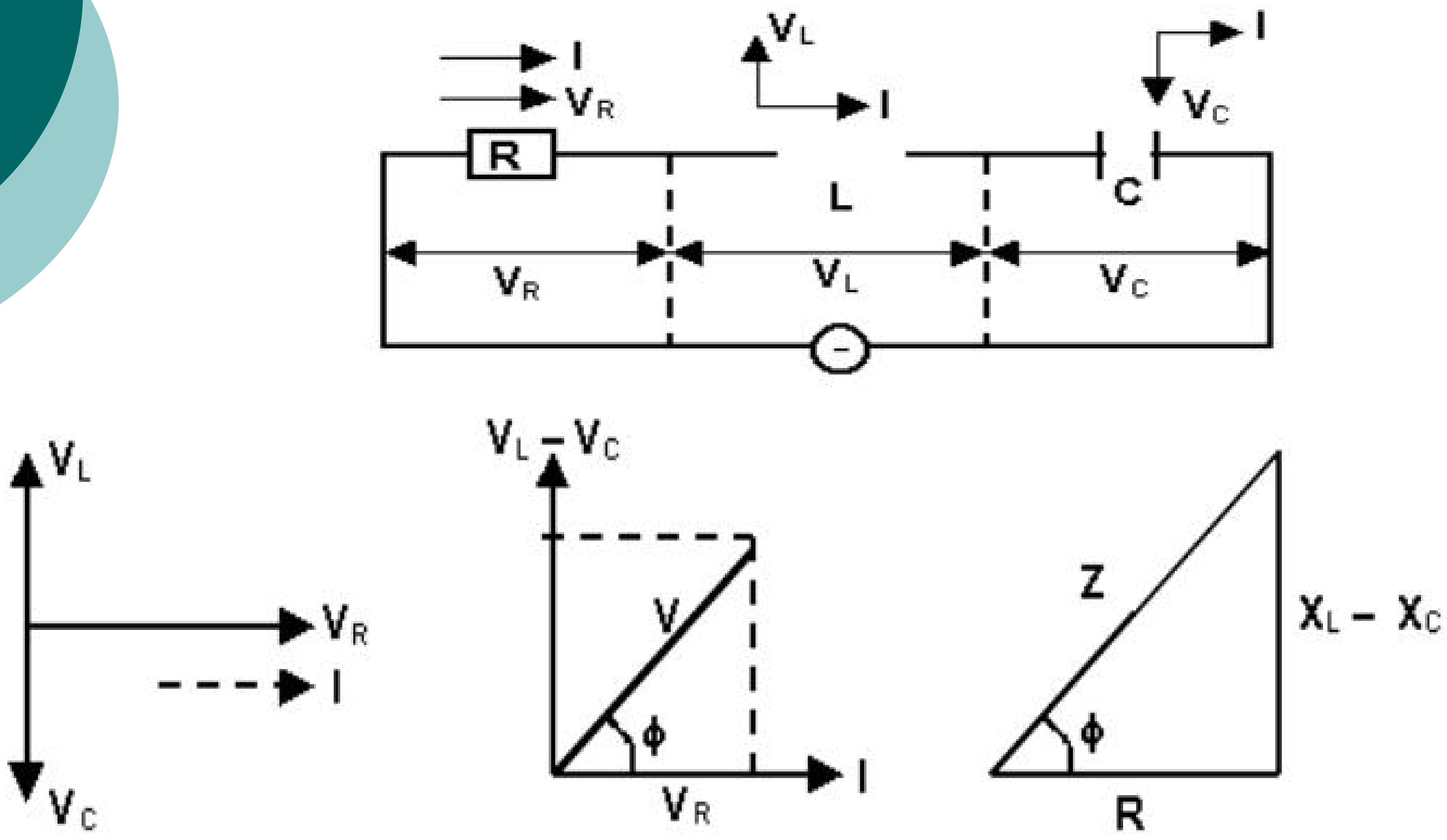


$$Z^2 = R^2 + X_c^2$$

$\Rightarrow$

$$Z = \sqrt{\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]}$$

R – L – C SERIES CIRCUIT



V_L leads I by a phase of 90°. V_C lags behind I by a phase of 90°. The resultant $V_L - V_C$ leads I by 90° assuming that $V_L > V_C$.

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$\Rightarrow (IZ)^2 = (IR)^2 + (IX_L - IX_C)^2$$

$$Z = \sqrt{\left[R^2 + (X_L - X_C)^2\right]}$$

$$= \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

Where Z = impedance

$$I = \frac{V}{Z} = \frac{V}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$$

I lags behind V by an angle ϕ where

$$\cos\phi = \frac{R}{Z} = \frac{V}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$$

R – L – C SERIES RESONANCE

We recall that for an R - L - C series circuit,

$$Z = \sqrt{\left[R^2 + \left(X_L - X_C\right)^2\right]}$$

$$= \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

For various frequencies,

$$\omega L \neq \frac{1}{\omega C}$$

but at a specific angular frequency ω_o (or frequency f_o) called the resonance frequency

$$\omega \mathrm{L} = \frac{1}{\omega C}$$ which gives Z = R.

at resonance,

$$\omega_\mathrm{o} L = \frac{1}{\omega_\mathrm{o} C}$$

$$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \omega_o^2 LC = 1 \qquad \Rightarrow \omega_o = \sqrt{\frac{1}{LC}}$$

$$\Rightarrow 2\pi f_o = \sqrt{\frac{1}{LC}} \qquad \Rightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$$