CSM 165: Discrete Mathematics for Computer Science

Chapter 1: Propositional and first order predicate logic

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Content

Propositional Equivalence

Inference

First Order Predicate Logic

Definition 1 (Tautology)

A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology

Definition 2 (Contradiction)

A compound proposition that is always false is called a contradiction

Definition 3 (Contingency)

A compound proposition that is neither a tautology nor a contradiction is called a contingency.

Example 1

Table 1 : A tautology and a Contradiction

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

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Definition 4 (Logical Equivalence)

*Compound propositions that have the same truth values in all possible cases are called **logically equivalent**.*

The compound propositions p and q are also called **logically equivalent** if $p \leftrightarrow q$ is a **tautology**. The notation $p \equiv q$ denotes that p and q are logically equivalent.

De Morgan's Laws

1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

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1. $\neg(p \land q) \equiv \neg p \lor \neg q$
2. $\neg(p \lor q) \equiv \neg p \land \neg q$

Logical Equivalence

Example 2

1. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent

Table 2 : Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

2. Show that $p \to q$ and $\neg p \vee q$ and equivalent.
3. Show that $p \wedge (q \vee r)$ and $(p \vee q) \wedge (p \vee r)$.

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3. Show that $p \wedge (q \vee r)$ and $(p \vee q) \wedge (p \vee r)$.

Logical Equivalence

Solution to example 2 question 3

Table 3 : Truth Table for $p \wedge (q \vee r)$ and $(p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Precedence of Logical Operators

Table 4 : Precedence of Logical Operators

Operators	Names	Precedence
$\neg$	Negation	1
$\wedge$	Conjunction	2
$\vee$	Disjunction	3
$\rightarrow$	Implication	4
$\leftrightarrow$	Biconditional	5

Table 5 : Logical Equivalences

Equivalence	***Name***
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
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$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
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Table 6 : Logical Equivalences Involving Conditional Statements.

$p \rightarrow q \equiv \neg p \vee q$
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$p \wedge q \equiv \neg(p \rightarrow \neg q)$
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Inference

Definition 5

Premise: *It is the proposition on the basis of which we would be able to draw a conclusion.*
It can be thought of as an evidence or assumption.

Conclusion: *It is the a proposition that is reached from a given set of premises.*

Argument: *Sequence of statements that ends with a conclusion.*

Valid Argument: *An argument is valid if and only if it is impossible for all the premises to be true and the conclusion to be false. OR*

Let A and B be two statement formulas. We say that "B ***logically follows from*** *A" or "B is a valid conclusion of the premise A" iff* $A \to B$ *is a tautology.*

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Validity Using Truth Table

Example 3

Determine whether the following conclusion C follows logically from the premises H_1 and H_2.

1. $H_1 \ P \rightarrow Q \quad H_2{:}P \quad C{:}Q$
2. $H_1 \ P \rightarrow Q \quad H_2{:}\neg P \quad C{:}Q$
3. $H_1{:}\ P \rightarrow Q \quad H_2{:}\neg(p \wedge Q) \quad \mathrm{C}{:}\neg P$
4. $H_1{:}\neg P \quad H_2{:}P \leftrightarrow Q \quad \mathrm{C}{:}\neg(P \wedge Q)$

P	Q	$P \rightarrow Q$	$\neg P$	$\neg Q$	$\neg(P \wedge Q)$	P$\leftrightarrow Q$
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4. $H_1{:}\neg P \quad H_2{:}P \leftrightarrow Q \quad \mathrm{C}{:}\neg(P \wedge Q)$

P	Q	$P \rightarrow Q$	$\neg P$	$\neg Q$	$\neg(P \wedge Q)$	P$\leftrightarrow Q$
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T	F	F	F	T	T	F
F	T	T	T	F	T	F
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Rules of Inference

Example 4

Consider:
"If you have a current password, then you can log onto the network".

"You have a current password".

Therefore, "You can log onto the network."

Let P = you have a current password
q = you can log onto the network

Argument form:

$$\begin{array}{l} p \to q \\ \underline{p \quad\quad} \\ \therefore q \end{array} \qquad\qquad ((p \to q) \land p) \to q$$

This form of argument is valid because whenever all its premises are true, the conclusion must also be true

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Rules of Inference

Rule	Tautology	Name
p $\underline{p \rightarrow q}$ $\therefore q$	$(p \wedge (p \rightarrow)) \rightarrow q$	Modus ponens
$\neg q$ $\underline{p \rightarrow q}$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $\underline{q \rightarrow r}$ $\therefore p \rightarrow r$	$(p \rightarrow q) \wedge (\rightarrow r) \rightarrow (p \rightarrow r)$	Hypothetical syllogism

Rules of Inference

$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p p $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Rules of Inference

Example 6

State which rule of inference is the basis of each of the following argument:

(i) " It is below freezing now. Therefore, it is either below freezing or raining now."

(ii) If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow.

Using Rules of Inference to Build Arguments

Example 7

1. Show that the premises “It is not sunny this afternoon and it is colder than yesterday,” “We will go swimming only if it is sunny,” “If we do not go swimming, then we will take a canoe trip,” and “If we take a canoe trip, then we will be home by sunset” lead to the conclusion “We will be home by sunset.”

Solution

Let $p =$ It is sunny this afternoon, $q =$ it is colder than yesterday, $r =$ We will go swimming $s =$ we will take a canoe trip, t we will be home by sunset.

Premises: $\neg p \wedge q$, $r \to p$, $\neg r \to s$ and $s \to s$

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   Premises: $\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$ and $s \rightarrow s$

Step	**Reason**
(1) $\neg p \wedge q$	Premise
(2) $\neg p$	Simplification using (1)
(3) $r \rightarrow p$	Premise
(4) $\neg r$	Modus tollens using (2) and (3)
(5) $\neg r \rightarrow s$	Premise
(6) s	Modus ponens using (4) and (5)
(7) $s \rightarrow t$	Premise
(8) t	Modus ponens using (6) and (7)

Example 8

Demonstrate that R is a valid inference from the premises $P \rightarrow Q$, $Q \rightarrow P$.

Solution

(1) $P \rightarrow Q$	Premise
(2) P	Premise
(3) Q	Modus ponens using (1) and (2)
(4) $Q \rightarrow R$	Premise
(5) R	modus ponens using (3), (4)

Inference

Exercise A:

1. Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

2. Show that $R \vee S$ follows logically from the premises $(C \vee D) \to \neg H$, $\neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to (R \vee S)$.

Predicates and Quantifiers

Definition 6

A predicate or propositional function is a statement containing variable(s) which are neither true nor false until the values of the variables are specified .

A predicate is represented by a letter followed by the variables enclosed between parenthesis: $P(x)$, $Q(x, y)$, *etc*

A propositional function has two parts:

1. A **Subject**
2. A **predicate**

x is the subject

is greater than 10 is the predicate

Example 9

x is greater than 10

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(a) Let $P(x)$ denote the statement "x > 3." What are the truth values of $P(4)$ and $P(2)$?

(b) Let $A(x)$ denote the statement "Computer x is under attack by an intruder." Suppose that of the computers on campus, only CS2 and MATH1 are currently under attack by intruders. What are truth values of A(CS1), A(CS2), and A(MATH1)?

(c) Let $Q(x, y)$ denote the statement "x = y + 3." What are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$?

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Predicates

Example 10

(a) Let $P(x)$ denote the statement "x > 3." What are the truth values of $P(4)$ and $P(2)$?

(b) Let $A(x)$ denote the statement "Computer x is under attack by an intruder." Suppose that of the computers on campus, only CS2 and MATH1 are currently under attack by intruders. What are truth values of A(CS1), A(CS2), and A(MATH1)?

(c) Let $Q(x,y)$ denote the statement "x = y + 3." What are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$?

Quantifiers

Definition 7 (Universal Quantifier $\forall$)

The universal quantification of $P(x)$ is the statement

"P(x) for all values of x in the domain".

The notation $\forall x\, P(x)$ denotes the universal quantification of $P(x)$. $\forall x\, P(x)$ is read as "for all x $P(x)$" or "for every x $P(x)$".

An element for which $P(x)$ is false is called a **counterexample** of $\forall x\, P(x)$.

"all of", "for each", " given any", "for arbitrary", "for each", and "for any".

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Example 11

1. Let $P(x)$ be the statement "$x > x - 1$". What is the truth value of the quantification $\forall x\, P(x)$, where the domain consists of all real numbers?

2. Let $Q(x)$ be the statement "$x < 5$". What is the truth value of the quantification $\forall x\, P(x)$, where $x \in \mathscr{R}$?

3. What is the truth value of $\forall x\, P(x)$, where $P(x)$ is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?

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Quantifiers

Definition 8 (Existential Quantifier ($\exists$))

The existential quantification of $P(x)$ is the proposition

"There exists an element x in the domain such that $P(x)$."

We use the notation $\exists(x)P(x)$ for the existential quantification of $P(x)$.

$\exists(x)P(x)$ is read as "There is an x such that $P(x)$"

Alternatives: "for some", "for at least one" or "there is".

NB: The statement $\exists x\, P(x)$ is false *iff* there is no element x in the domain for which $P(x)$ is true.

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Quantifiers

Example 12

(i) Let $P(x)$ denote the statement “x > 3”. What is the truth value of the quantification $\exists x\, P(x)$, where the domain consists of all real numbers?

(ii) Let $Q(x)$ denote the statement “x = x + 1.” What is the truth value of the quantification $\exists x\, P(x)$, where the domain consists of all real numbers?

(iii) What are the truth values for the statements $\forall x < 0 (x^2 > 0)$, $\forall y \neq 0 (y^3 \neq 0)$, and $\exists z > 0 (z^2 = 2)$ mean, where the domain in each case consists of the real numbers?

Quantifiers

Precedence of Quantifiers

The quantifiers $\forall$ and $\exists$ have higher precedence than all logical operators from propositional calculus

For instance, $\forall x\, P(x) \vee Q(x)$ is the disjunction of $\forall x\, P(x)$ and $Q(x)$

Table 8 : De Morgan's Laws for Quantifiers.

Negation	***Equivalent Statement***	***When Is Negation True?***	***When False?***
$\neg\exists x\, P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x\, P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

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Assignment

To be posted on the class Telegram Channel: CSM 165 A

End of Lecture

Questions...???

Thanks

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