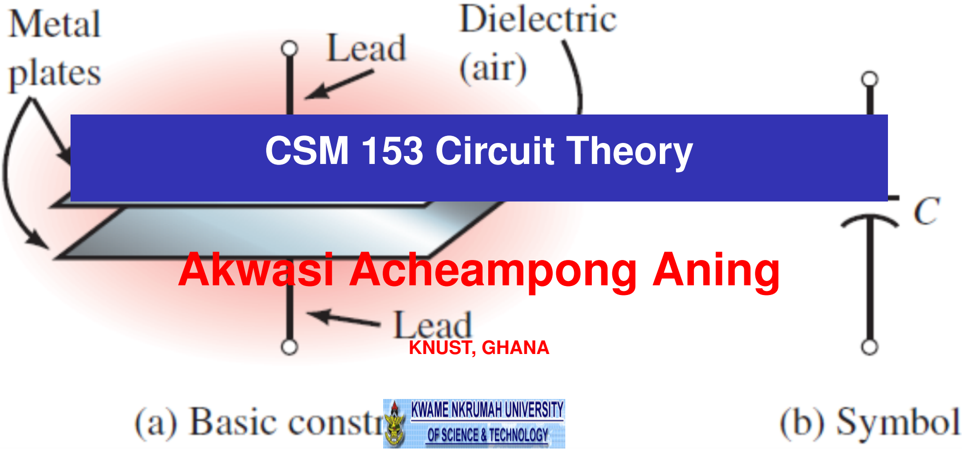

CSM 153 Circuit Theory

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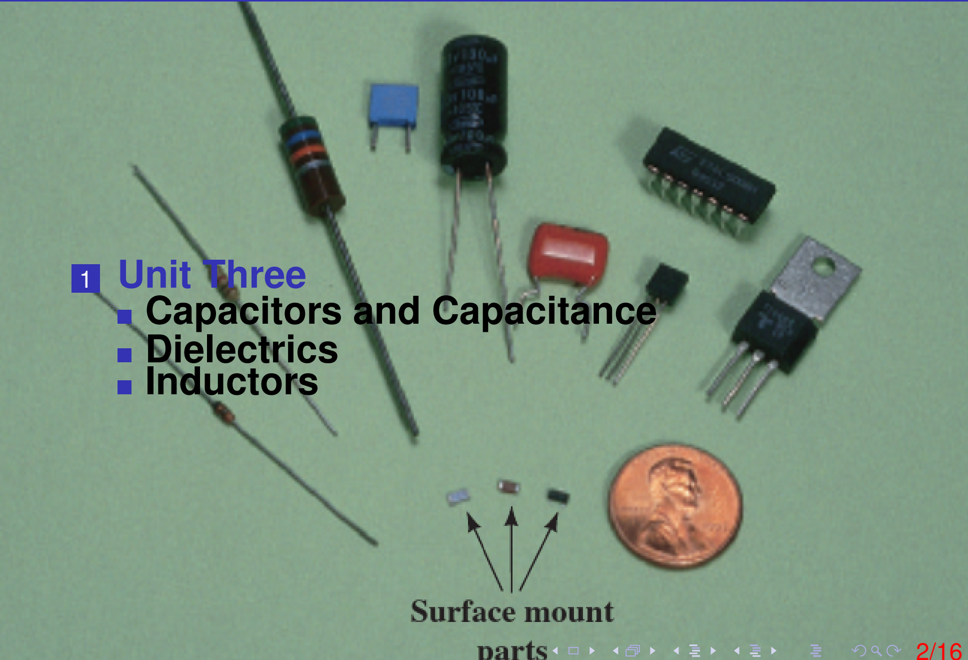
1. Unit Three
   - Capacitors and Capacitance
   - Dielectrics
   - Inductors



UNIT THREE
Capacitors and Inductors

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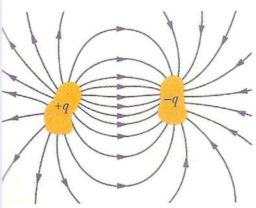
Capacitors and Inductors

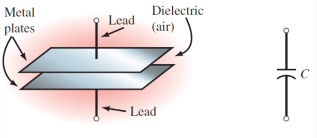
- Capacitors and Capacitance
- Dielectrics
- Inductors

Capacitors

A capacitor is an electronic device for storing electrical energy as potential energy in an electric field



- An arrangement of two isolated conductors of any shape form a capacitor
- Conventionally, an arrangement consisting of two parallel conducting plates of area, A separated by a distance, d form a parallel-plate capacitor



(a) Basic construction (b) Symbol

- When the capacitor is charged the plates acquire equal but opposite charges of +q and –q. However, we refer to the absolute charge q of a capacitor

Capacitors

- When a capacitor is charge, a potential difference, V is set up between the plates
- The charge Q and the potential difference V for a capacitor are proportional to each other i.e. Q $\propto$ V

$$Q = CV \tag{1}$$

where C is a proportional constant, called capacitance of the capacitor $\therefore$ $C = \dfrac{Q}{V}$

- For a conductor of any geometrical shape the capacitance, C is defined as the ratio of charge on the conductor to the potential it is raised i.e. C = (Charge on conductor)/(Potential it is raised)
- For a parallel-plate capacitor, capacitance C is defined as the ratio of charge on each (either) plate to the potential difference between the plates
- Capacitance is a measure of the charge a capacitor can store. Thus, the higher the capacitance, the greater or more charge it can store
- SI Unit of capacitance: coulomb per volt CV^{-1} = 1 Farad (1F)
- Practical unit are: microfarad (1mF = 10^{-6} F) and (1pF = 10^{-12} F)

Capacitors

Gauss' law says that the electric flux through a closed surface is proportional to the amount of charge Q enclosed within the surface.

- To calculate capacitance for different geometrical shapes the following procedures must be followed
- Assume a charge q on the plates or conductor.
- Calculate the electric field E between the plates or due to the conductor in terms of the charge q using Gauss' law. i.e.

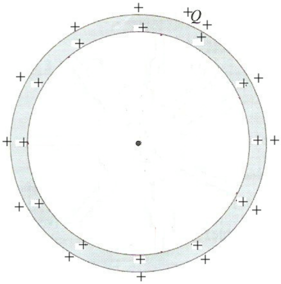
$$\varepsilon_0 \oint \vec{E}.d\vec{A} = q \tag{2}$$

  where E is the electric field
- Proceed to calculate the potential difference V between the plates or conductor using

$$V = -\int E dx \tag{3}$$

- Then calculate the capacitance C using $C = \dfrac{q}{V}$

Capacitors



- Consider a single isolated spherical conductor of radius R and charge Q on its surface
- Using Gauss' law the electric field is given by $\varepsilon_0 \oint \vec{E}.d\vec{A} = q$ $\Rightarrow \varepsilon_0 E(4\pi R^2)$

- Thus E is expressed as $E = \dfrac{Q}{4\pi\varepsilon_0 R^2}$
- The potential V of the conductor is given by $dV = -EdR \Rightarrow V = -\int_0^R EdR$ we've $V = -\int_0^R \dfrac{Q}{4\pi\varepsilon_0 R^2} dR$ = - $\dfrac{Q}{4\pi\varepsilon_0}\int_0^R \left(\dfrac{1}{R^2}\right) dR$

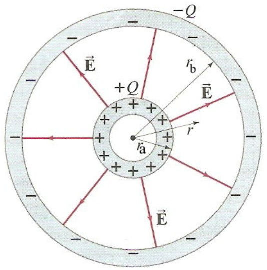
$$\Rightarrow V = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{R}\right]_0^R = \frac{Q}{4\pi\varepsilon_0 R}$$

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 R \qquad (4)$$

- C is independent of the charge on the spherical conductor but depends only on the radius R

method

Capacitors



- Consider a cross-section of a long cylindrical capacitor consisting of two concentric spherical shells of radii r_a and r_b
- Let a Gaussian surface be a sphere of radius r concentric with the two shells

- Using Gauss' law the electric field E is expressed as

$$\varepsilon_0 \oint \overrightarrow{E}.d\overrightarrow{A} = Q$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

- The potential V is given by

$$V = -\int_{r_b}^{r_a} \frac{Q}{4\pi\varepsilon_0 r^2} dr = - \frac{Q}{4\pi\varepsilon_0} \int_{r_b}^{r_a} \left(\frac{1}{r^2}\right) dr$$

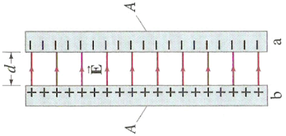
$$\Rightarrow V = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_b}^{r_a} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_b - r_a}{r_a r_b}\right)$$

- If we set $r_a = a$ and $r_b = b$ then the capacitance C is

$$\therefore C = \frac{Q}{V} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) \quad (5)$$

Capacitors



- Consider parallel-plates of a capacitor each of area A and charge magnitude Q on plates
- Assuming the plates are so large and close together, we can neglect edge effects of the electric field
- The electric field E between the plates is given by

$$E = \frac{\sigma}{\varepsilon_0} \quad (6)$$

and

$$\sigma = \frac{Q}{A} \quad (7)$$

and σ is the surface charge density

- this implies

$$E = \frac{Q}{A\varepsilon_0} \quad (8)$$

- The potential difference between plates is given by

$$V = -\int_0^d E dr = Ed \quad (9)$$

$$E = \frac{V}{d} \quad (10)$$

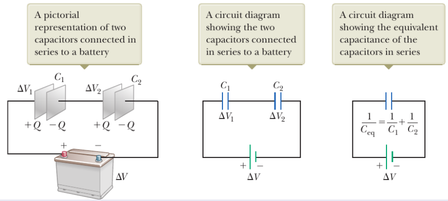
Thus $\frac{Q}{A\varepsilon_0} = \frac{V}{d} \Rightarrow \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$

$$C = \frac{\varepsilon_0 A}{d} \quad (11)$$

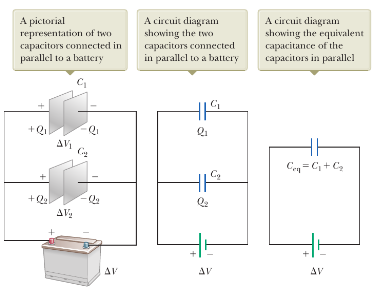
- C increases as we increase the area A or decrease separation d of the plates

Capacitors

Capacitors in Series



Capacitors in Parallel



- For three capacitors in series the equivalent capacitance C_{eq} is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad (12)$$

- For three capacitors in parallel the equivalent capacitance C_{eq} is given by

$$C_{eq} = C_1 + C_2 + C_3 \quad (13)$$

- Potential energy U stored in a capacitor is given by any of the following

$$U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2} \quad (14)$$

Dielectrics

Effect of Dielectric in a Capacitor

- A dielectric is an insulating material such as mica, paper, mineral oil or plastic, which can be used to fill the space between the plates of a capacitor
- When a dielectric slab is inserted between the plates of a capacitor, the charge Q stored increases by a factor k, called dielectric constant of the insulating material
- In effect, the potential difference V between the plates rather decreases by a factor k
- In general, in a region or space completely filled by a dielectric material of dielectric constant k, all electrostatic equations containing ε_0 are to be replaced by $k\varepsilon_0$
- Thus, a point charge inside a dielectric produces an electric field E given by

$$E = \frac{Q}{4\pi k\varepsilon_0 r^2} \tag{15}$$

- This shows that for a fixed distribution of charges the effect of dielectric is to weaken the electric field that would have been present between the plates

- Dielectric constant (relative permittivity) ε_r of a material is the ratio of the capacitance with dielectric to capacitance without dielectric between the plates
- Potential energy U stored in a capacitor is given by any of the following

$$\varepsilon_r = \frac{C}{C_0} \tag{16}$$

where C is capacitance with plates filled with dielectric material and C_0 is capacitance of the same capacitance with plates in free space (vacuum) or air.

Dielectrics

Effect of Dielectric in a Capacitor

- For parallel plate capacitor $C_0 = \frac{\varepsilon_0 A}{d}$ and $C = \frac{\varepsilon A}{d}$

$$\varepsilon_r = \frac{C}{C_0} = \frac{\varepsilon}{\varepsilon_0} \tag{17}$$

$\therefore \varepsilon = \varepsilon_r \varepsilon_0 = k\varepsilon_0$

- Hence, the dielectric constant or relative permittivity is the ratio of the permittivity of a material to permittivity of free space and has no dimensions
- Dielectric strength: The strength of a dielectric is the potential gradient (electric field) at which its insulation breaks down and a spark passes through the material
- Every dielectric material has a characteristic dielectric strength, which is the maximum value of electric field that it can withstand without breakdown

Uses of Capacitors

- Capacitors are widely used in electronic circuits in devices. They are used to store charge and released later when needed
- Capacitors are used to block power surges of charge and energy to protect devices
- Used in filter circuits in rectifiers to obtain d.c. outputs
- Can be made in the form of very tiny capacitors to serve as memory for binary code in the RAM of computers

Inductors

Effect of Dielectric in a Capacitor

- **Inductance** is the name given to the property of a circuit where there is an emf induced into the circuit by the change of flux linkages produced by a current change
- When the emf is induced in the same circuit as that in which the current is changing, the property is called self inductance, L
- When the emf is induced in a circuit by a change of flux due to current changing in an adjacent circuit, the property is called mutual inductance, M. The unit of inductance is the henry, H
- Inductor is used when the property of inductance is required in a circuit. The basic form of an inductor is simply a coil of wire

Factors which affect the inductance of an inductor

- the number of turns of wire – the more the turns the higher the inductance
- The cross-sectional area of the coil of wire – the greater the cross-sectional area the higher the inductance
- The presence of a magnetic core – when the coil is wound on an iron core the same current sets up a more concentrated magnetic field and the inductance is increased
- The way the turns are arranged – a short, thick coil of wire has a higher inductance than a long, thin one

We will look at inductance and induction in detail under electromagnetism

Multiple Choice Question

1. A conductor is distinguished from an insulator with the same number of atoms by the number of:
   1. nearly free atoms
   2. electrons
   3. nearly free electrons ANS
   4. protons
   5. molecules
2. Two small charged objects attract each other with a force F when separated by a distance d. If the charge on each object is reduced to one-fourth of its original value and the distance between them is reduced to d/2 the force becomes:
   1. F/16
   2. F/8
   3. F/4 ANS
   4. F/2
   5. F

1. If the potential difference across a resistor is doubled:
   1. only the current is doubled ANS
   2. only the current is halved
   3. only the resistance is doubled
   4. only the resistance is halved
   5. both the current and resistance are doubled
2. A certain wire has resistance R. Another wire, of the same material, has half the length and half the diameter of the first wire. The resistance of the second wire is:
   1. R/4
   2. R/2
   3. R
   4. 2R ANS
   5. 4R

Thank You