CSM 165: Discrete Mathematics for Computer Science

Lecture 1: Propositional and first order predicate logic

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Content

Introduction

Course Outline

Propositional and first order predicate logic

Introduction

What is discrete mathematics?

- Discrete mathematics is the part of mathematics devoted to study discrete objects.
  - Discrete Means not continuous or unconnected :

Discrete Verses Continuous (examples)

1. Natural Numbers are discrete
2. Digital clock

1. Real numbers are continuous
2. Analog clock:

NB: here we mean the analog clock with the second, minute and hour hand moving/sweeping continuously.

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Discrete Mathematics helps in solving Problems such as:

- Is there a link between two computers in a network?
- Sorting a list of integers.
- Finding the shortest path from your home to your friend's house.
- How many different combinations of passwords are possible with just 9 alphanumeric characters?
- How can I identify spam e-mail messages?

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Why Discrete Mathematics

- It develops your mathematical thinking
- Improves problem solving ability.
- Many problems can be solved using discrete mathematics.
- Foundation for many computer science courses:
  - data structures
  - algorithms
  - database theory
  - automata theory
  - computer security
  - operating systems.

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Course Outline

1. Propositional and first order predicate logic
2. Set Theory.
3. Relations and Functions
4. First Principle of induction.
5. Number Systems and arithmetic (complement number system)

Propositional and first order predicate logic

Definition 1 (Proposition)

*A proposition is a **declarative** sentence that is either **true** or **false**, but not both*

Example 1

1. COVID-19 is a communicable disease
2. Wearing of nose mask is the only preventive measure for COVID-19.
3. $2 + 3 = 5$
4. $1 + 1 = 11$

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Propositional Logic

Example 2

1. Kindly send me the code snippet for the assignment.
2. What is your name?
3. Remember to observe all the COVID-19 protocols.
4. x+5=10
5. $\sqrt{16} + y = z$

NB: None of the above examples is a proposition since none is a declaration nor True or False.

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Propositional logic

Definition 2 (Logic)

Logic is the science of reasoning.

It helps in understanding and reasoning about different mathematical statements.

The area of logic that deals with propositions is called the **propositional logic**.

Definition 3 (Propositional Variables)

Propositional Variables are variables used to represent propositions.

Example 3

p = My PC runs Linux
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Logical Connectives (operators)

Definition 4 (Negation ¬)

Let p be a proposition. The negation of p, denoted by ¬p (also denoted by ~p), is the statement "It is not the case that p.

Table 1 : Truth table for $\neg p$

p	$\neg p$
T	F
F	T

Example 4

Find the negation of the following propositions.

1. Hannah's PC runs linux.
2. Data science is the sexiest job of 21st century.
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Let p and q be propositions. The conjunction of p and q, denoted by $p \wedge q$, is the proposition "p and q".

The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Table 2 : Truth Table for $P \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
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Example 5

Let p = "Hannah's PC has more than 16 GB free hard disk space"

q = "The processor in Hannah's PC runs faster than 1 GHz."

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Example 6

Let: p = Students who have taken algebra can enroll in this course.

q = students who have taken calculus can enroll in this course.

$p \vee q$ = Students who have taken algebra **or** calculus can enroll in this course.

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Logical Connectives (operator)

Definition 7 (Exclusive OR (XOR))

Let p and q be two propositions. The exclusive OR of p and q (denoted by $p \oplus q$) is the proposition that is true when exactly one of p and q is true and is false otherwise.

Table 4 : Truth table for $p \oplus q$

p	q	p ⊕q
T	T	F
T	F	T
F	T	T
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Example 7

1. Coffee or Tea comes with dinner
2. Students who have taken calculus or computer science, but not both, can enroll in this class.

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Conditional Statements

Definition 8

For proposition p and q,the conditional sentence p⇒q is the proposition "If p, then q". Proposition p is called the ***antecedent*** *and q is the* ***consequence****.*

Table 5 : Truth table for $p \Rightarrow q$

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Example 8

(a) If you try hard for your exams, then you will pass.

(b) If you score 90% in CSM 165, then you will get free accommodation.

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Exercise A:

1. Let p be the statement "Hannah learns discrete mathematics" and q the statement "Hannnah will find a good job". Express the statement $p \Rightarrow q$ as a statement in English.

2. What is the value of the variable x after the statement:
"if $5 + 7 = 12$ then $x := x + 1$ ",
if $x = 0$ before this statement

Assignment

To be posted on the class Telegram Channel:
CSM 165 B

End of Lecture

Questions...???

Thanks

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