VECTORS

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Outline

1. Introduction To Vectors
   - Introduction
   - Vectors in $\mathbb{R}^n$

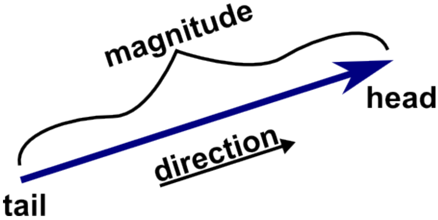
2. Vector Products
   - Dot Product

Outline of Presentation

Introduction To Vectors

Definition (Vector)

A vector is an object that has both a magnitude and a direction.



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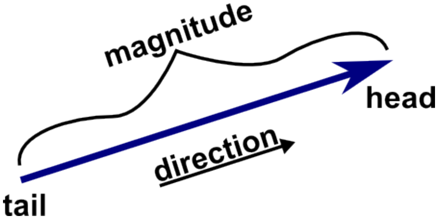

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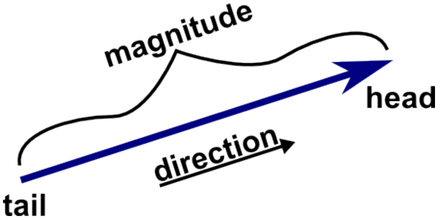
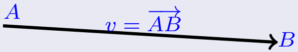


1. An example of a vector quantity is velocity. This is speed, in a particular direction. An example of velocity might be 60 mph due north.
2. A quantity with magnitude alone, but no direction, is not a vector. It is called a scalar instead. One example of a scalar is distance.

Geometric representation

Definition (Geometric representation)

A vector $\mathbf{v}$ is represented by a directed line segment denoted by $\overrightarrow{AB}$.



1. A is the **initial point/origin/tail** and B is the **terminal point/endpoint/tip**.
2. The length of the segment is the **magnitude** of $\mathbf{v}$ and is denoted by $|\mathbf{v}|$.
3. A and B are any points in space.

Vectors are represented with arrows on top ($\vec{v}$ or $\overrightarrow{OP}$) or as boldface $\mathbf{v}$.

Definition

Two vectors are equal if they have the same magnitude and direction.

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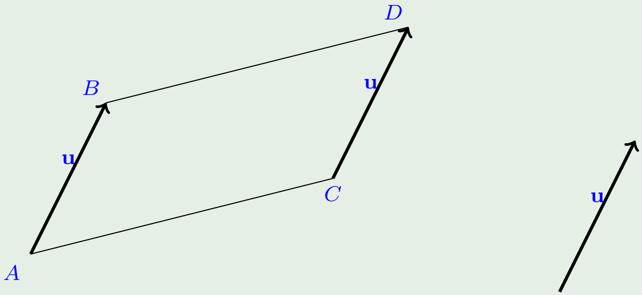
Example



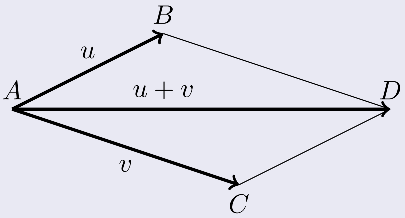
$$\overrightarrow{AB} = \mathbf{u} = \overrightarrow{DC}$$

Addition of Vectors

Theorem (Parallelogram law)

Vector $\mathbf{u}+\mathbf{v}$ *is the diagonal of the parallelogram formed by* $\mathbf{u}$ *and* $\mathbf{v}$.

Addition Laws



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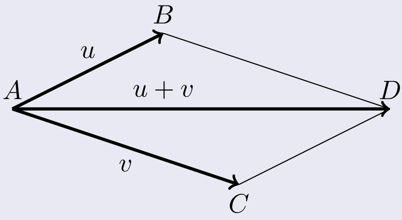

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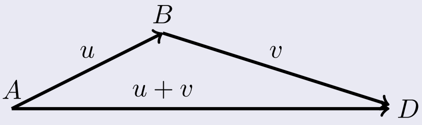

(b) Triangle law of vector addition: The tip of $\mathbf{u}$ coincides with the tail of $\mathbf{v}$. (Also called head to tail rule)

Addition

If the two vectors do not have a common point, we can always coincide them by shifting one of the vectors.
By observing that

$$\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD} \tag{1}$$

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Zero Vector

1. $\overrightarrow{AB} + \overrightarrow{BA} = \overrightarrow{AA}$. This is the **zero vector**. It has zero magnitude and is denoted by $\mathbf{0}$ or $\overrightarrow{0}$. Its origin is equal to its endpoint.
2. For any vector $\mathbf{w}, \mathbf{w} + \mathbf{0} = \mathbf{w}$ [if we let $\mathbf{w} = \overrightarrow{MN}$ and $\mathbf{0} = \overrightarrow{NN}$ then $\mathbf{w} + \mathbf{0} = \overrightarrow{MN} + \overrightarrow{NN} = \overrightarrow{MN} = \mathbf{w}$.]

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Negative vector

$\overrightarrow{BA}$ and $\overrightarrow{AB}$ have the same magnitude but opposite directions and satisfy $\overrightarrow{AB} + \overrightarrow{BA} = \overrightarrow{AA} = \vec{0}$. $\overrightarrow{BA}$ is the **negative** of $\overrightarrow{AB}$ i.e. $\overrightarrow{BA} = -\overrightarrow{AB}$.

1. In general, we have $k\overrightarrow{AB}$. For instance $\underbrace{\overrightarrow{AB}+\overrightarrow{AB}+\cdots+\overrightarrow{AB}}_{k}=k\overrightarrow{AB}$ if $k \in \mathbb{Z}$. The coefficient k is a **scalar** and the product between a scalar and a vector is called **scalar multiplication**.

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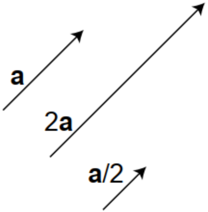
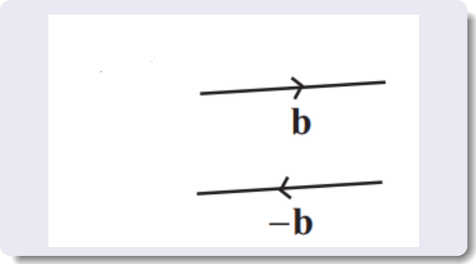
2. $\mathbf{w}$ and $k\mathbf{w}$ are said to be **parallel**; they have the same direction if $k > 0$ and opposite direction if $k < 0$.

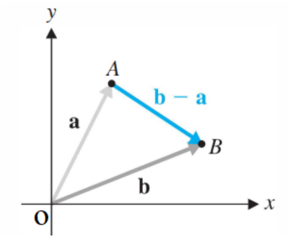




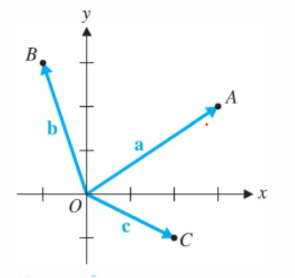
Position Vectors

Definition (Position Vectors)

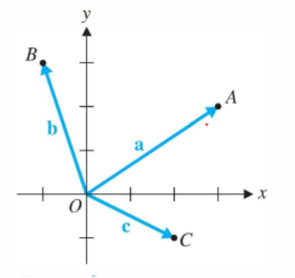
While vectors can exist anywhere in space, a point is always defined relative to the origin, O. Vectors defined from the origin are called Position Vectors



$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} \quad (2)$$
$$= [-\vec{a}] + \vec{b} \quad (3)$$
$$= \vec{b} - \vec{a} \quad (4)$$
$$= \overrightarrow{OB} - \overrightarrow{OA} \quad (5)$$



It is natural to represent position vectors using coordinates. For example, in $A = (3, 2)$ and we write the vector $\vec{a} = \overrightarrow{OA} = [3, 2]$ using square brackets. Similarly, $\vec{b} = [-1, 3]$ and $\vec{c} = [2, -1]$



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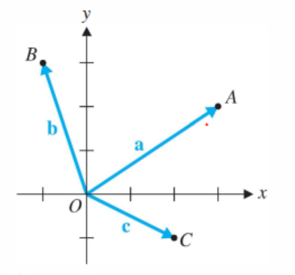

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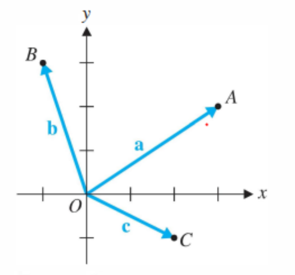
2. A vector is sometimes said to be an ordered pair of real numbers. That is

$$[3, 2] \neq [2, 3] \tag{6}$$



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3. Vectors can be represented either as row vector $[a,b,c]$ of a column vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Some 2D Properties

Given $\mathbf{u} = [u_1, u_2]$ and $\mathbf{v} = [v_1, v_2]$ then

1. Addition

$$\mathbf{u} + \mathbf{v} = [u_1 + v_1, u_2 + v_2] \tag{7}$$

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4. Magnitude / Length / Norm

$$|\mathbf{u}| = ||\mathbf{u}|| = \sqrt{u_1^2 + u_2^2} \tag{10}$$

Standard Basis Vectors

Let $\vec{i} = [1, \ 0]$ and $\vec{j} = [0, \ 1]$, the i, j are called standard basis vectors in $\mathbb{R}^2$.

Each vector have length 1 and point in the directions of the positive x, and $y-$axes respectively.

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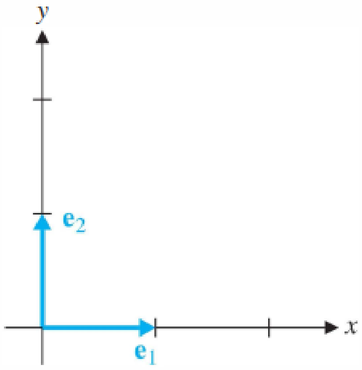
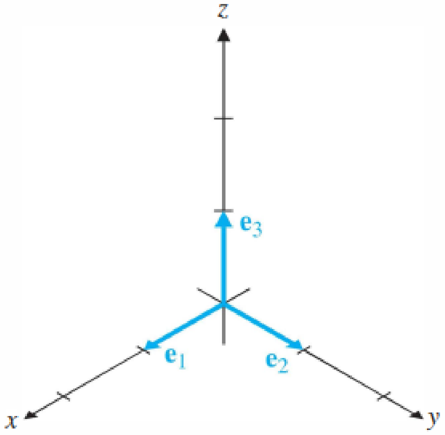
Similarly, in the three-dimensional plane, the vectors $\vec{i} = [1, 0, 0], \vec{j} = [0, 1, 0]$ and $\vec{k} = [0, 0, 1]$ are also called the standard basis vectors.

Again they have length 1 and point in the directions of the positive x, y, and $z-$axes.

Standard Basis Vectors

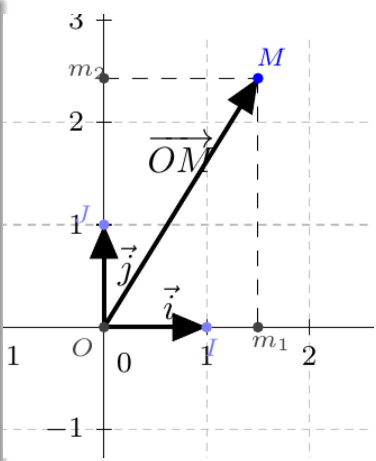




Unit Vector

Definition

1. A vector of magnitude 1 is called a **unit vector**. In the Cartesian coordinate system, $\mathbf{i}$ and $\mathbf{j}$ are reserved for the unit vector along the positive x-axis and y-axis respectively.
2. The Cartesian coordinate system is therefore defined by three reference points (O, I, J) such that the origin O has coordinates $(0,0)$, $\mathbf{i} = \overrightarrow{OI}$ and $\mathbf{j} = \overrightarrow{OJ}$, and any vector $\mathbf{w} = \overrightarrow{OM}$ can be expressed as $\mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j}$. w_1 and w_2 are the coordinates of $\mathbf{w}$.



Example

Given the points $E = (2, 7)$ and $F = (3, -1)$, then the vector $\overrightarrow{EF}$ is given as

$$\begin{aligned}
\overrightarrow{EF} &= \overrightarrow{OF} - \overrightarrow{OE} && (11)\\
&= [(3, -1) - (2, 7)] && (12)\\
&= [1, -8] && (13)
\end{aligned}$$

Example

Calculate the magnitude of vector $\mathbf{a} = \lambda_1\mathbf{a}_1 - \lambda_2\mathbf{a}_2 - \lambda_3\mathbf{a}_3$ in case $\mathbf{a}_1 = [2, -3], \mathbf{a}_2 = [-3, 0], a_3 = [-1, -1]$ and $\lambda_1 = 2, \lambda_2 = -1, \lambda_3 = 3$.

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$$\mathbf{a} = \lambda_1\mathbf{a}_1 - \lambda_2\mathbf{a}_2 - \lambda_3\mathbf{a}_3 \tag{14}$$
$$= 2[2, -3] - [-1][-3, 0] - 3[-1, -1] \tag{15}$$
$$= [4, -6] + [-3, 0] + [3, 3] \tag{16}$$
$$= [4, -3] \tag{17}$$

Thus

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$$\begin{aligned}
\mathbf{a} &= \lambda_1\mathbf{a}_1 - \lambda_2\mathbf{a}_2 - \lambda_3\mathbf{a}_3 && (14)\\
&= 2[2, -3] - [-1][-3, 0] - 3[-1, -1] && (15)\\
&= [4, -6] + [-3, 0] + [3, 3] && (16)\\
&= [4, -3] && (17)
\end{aligned}$$

Thus

$$|\mathbf{a}| = \sqrt{4^3 + [-3]^2} = \sqrt{25} = 5 \qquad (18)$$

Example

A car is pulled by four men. The components of the four forces are $\mathbf{F}_1 = [20, 25], \mathbf{F}_2 = [15, 5], \mathbf{F}_3 = [25, -5]$ and $\mathbf{F}_4 = [30, -15]$. Find the resultant force $\mathbf{R}$.

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The resultant force

$$\begin{aligned}
\mathbf{F} &= F_1 + F_2 + F_3 + F_4 && (19)\\
&= [20, 25] + [15, 5] + [25, -5] + [30, -15] && (20)\\
&= [90, 10] && (21)
\end{aligned}$$

Definition

1. If $v_1, v_2, \ldots, v_n$ are n real numbers:
2. $v = [v_1, v_2]$ is a two dimension vector. It belongs to $\mathbb{R}^2$.
3. $v = [v_1, v_2, v_3]$ is a three dimension vector. It belongs to $\mathbb{R}^3$.
4. $v = [v_1, v_2, \ldots, v_n]$ is an n dimension vector. It belongs to $\mathbb{R}^n$. We may use $\mathbf{e}_1, \mathbf{e}_2, \ldots \mathbf{e}_n$ to denote the unit vectors of the coordinate system.

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2. $\mathbf{v} = [3, -1, 5, 0, 4] = 3\mathbf{e}_1 - \mathbf{e}_2 + 5\mathbf{e}_3 + 4\mathbf{e}_5$ is a five dimension vector; $\mathbf{v} \in \mathbb{R}^5$.

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3. The coordinate system is defined by $\mathbf{e}_1 = [1, 0, 0, 0, 0]$, $\mathbf{e}_2 = [0, 1, 0, 0, 0]$, $\mathbf{e}_3 = [0, 0, 1, 0, 0]$, $\mathbf{e}_4 = [0, 0, 0, 1, 0]$, and $\mathbf{e}_5 = [0, 0, 0, 0, 1]$.

$\mathbf{u} = [u_1, u_2, \ldots, u_n]$, and $\mathbf{v} = [v_1, v_2, \ldots, v_n]$ then

1. Addition

$$\mathbf{u} + \mathbf{v} = [u_1 + v_1, u_2 + v_2, \ldots, u_n + v_n] \tag{22}$$

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$$|\mathbf{u}| = \sqrt{u_1^2 + u_2^2 + \ldots + u_n^2} \tag{25}$$

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$$|\mathbf{u}| = \sqrt{u_1^2 + u_2^2 + \ldots + u_n^2} \tag{25}$$

5. Unit vector in the direction of $\mathbf{u}$

$$\mathbf{e_u} = \frac{\mathbf{u}}{|\mathbf{u}|} = \left(\frac{u_1}{|\mathbf{u}|}, \frac{u_2}{|\mathbf{u}|}, \ldots, \frac{u_n}{|\mathbf{u}|}\right) \tag{26}$$

Example

If $\mathbf{a} = [4, 0, 3]$ and $\mathbf{b} = [-2, 1, 5]$, find $|\mathbf{a} - \mathbf{b}|$ and the unit vector $\mathbf{e}$ in the direction of $\mathbf{a}$.

Example

If $\mathbf{a} = [4,0,3]$ and $\mathbf{b} = [-2,1,5]$, find $|\mathbf{a}-\mathbf{b}|$ and the unit vector $\mathbf{e}$ in the direction of $\mathbf{a}$.

$$\vec{a}-\vec{b} = [4,0,3]-[-2,1,5] \tag{27}$$
$$= [6,-1,-2] \tag{28}$$

and

$$|\vec{a}-\vec{b}| = \sqrt{6^2+[-1]^2+[-2]^2} = \sqrt{41} \tag{29}$$

Example

If $\mathbf{a} = [4, 0, 3]$ and $\mathbf{b} = [-2, 1, 5]$, find $|\mathbf{a} - \mathbf{b}|$ and the unit vector $\mathbf{e}$ in the direction of $\mathbf{a}$.

$$\vec{a} - \vec{b} = [4, 0, 3] - [-2, 1, 5] \tag{27}$$

$$= [6, -1, -2] \tag{28}$$

and

$$|\vec{a} - \vec{b}| = \sqrt{6^2 + [-1]^2 + [-2]^2} = \sqrt{41} \tag{29}$$

The unit in the direction of $\mathbf{a}$ is

$$\mathbf{e} = \frac{1}{\sqrt{4^2 + 0^2 + 3^2}}[4, 0, 3] \tag{30}$$

$$= \frac{1}{5}[4, 0, 3] \tag{31}$$

$$= \left[\frac{4}{5}, 0, \frac{3}{5}\right] \tag{32}$$

Properties of vectors

For any vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and any scalars $k, k' \in \mathbb{R}$,

1. Closure $\mathbf{u} + \mathbf{v} \in \mathbb{R}^n$

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8. Scalar Multiplication $[k + k']\mathbf{u} = k\mathbf{u} + k'\mathbf{u}$

Definition (Linear Combination)

A vector $\mathbf{v}$ is a linear combination of vectors $\mathbf{v_1}, \mathbf{v_2}, \cdots, v_n$ if there are scalars $k_1, k_2, \cdots, k_n$ such that

$$\mathbf{v} = k_1\mathbf{v_1} + k_2\mathbf{v_2} + \cdots + k_n\mathbf{v_n} \tag{33}$$

The scalars $k_1, k_2, \cdots, k_n$ are called the **coefficients** of the linear combination.

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Example

The vectors $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ -4 \\ 0 \end{bmatrix}$ Since

$$3\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} - \begin{bmatrix} 5 \\ -4 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \tag{34}$$

Exercise

1. Let $M = (m_1, m_2)$ and $N = (n_1, n_2)$ be two points in the Cartesian coordinate system (O, I, J). Express $\mathbf{u} = \overrightarrow{MN}, \mathbf{v} = \overrightarrow{MO} - 2\overrightarrow{ON}, \mathbf{w} = \overrightarrow{MN} + 2\overrightarrow{NM} + \overrightarrow{ON} - \overrightarrow{OI} + 3\overrightarrow{IJ}$ in terms of $\overrightarrow{OI}$ and $\overrightarrow{OJ}$.
2. Find a unit vector that has the same direction as
   a. $\mathbf{u} = 8\mathbf{i} - \mathbf{j} + \mathbf{k}$
   b. $\mathbf{v} = [-2, 1, 2]$.
3. Show that $\mathbf{u} = 2\mathbf{e}_1 - 4\mathbf{e}_2 + \mathbf{e}_3 - \sqrt{2}\mathbf{e}_4$ and $\mathbf{v} = -4\mathbf{e}_1 + 8\mathbf{e}_2 - 2\mathbf{e}_3 + \sqrt{8}\mathbf{e}_4$ are parallel vectors.

Outline of Presentation

Dot Product

For $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$,

1. If
$$\mathbf{u} = [u_1, u_2, \ldots, u_n], \quad \text{and} \quad \mathbf{v} = [v_1, v_2, \ldots, v_n]$$
then
$$\langle \mathbf{u},\ \mathbf{v} \rangle = \mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + \ldots + u_nv_n \tag{35}$$
is the **dot product** of the two vectors.
2. The dot product is also called **inner or scalar product**.

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$[1, 2, 3] \cdot [-1, 0, 1] = 1(-1) + 2(0) + 3(1) = 2.$

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Example

$[1, 2, 3] \cdot [-1, 0, 1] = 1(-1) + 2(0) + 3(1) = 2.$

Example

$[\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}] \cdot [2\mathbf{j} - \mathbf{k}] = 0 + 4 + 3 = 7.$

Example

In $\mathbb{R}^3, \mathbf{i} = [1, 0, 0], \mathbf{j} = [0, 1, 0]$ and $\mathbf{k} = [0, 0, 1]$ so that

$\cdot$	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$
$\mathbf{i}$	1	0	0
$\mathbf{j}$	0	1	0
$\mathbf{k}$	0	0	1

Properties

For any vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and any scalar $k \in \mathbb{R}$:

1. $\mathbf{u} \cdot \mathbf{v} \in \mathbb{R}$

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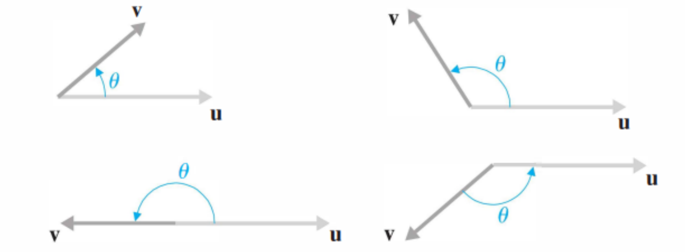
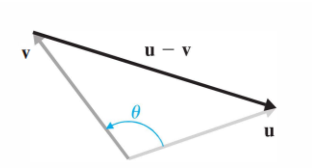
5. $[k\mathbf{u}]\cdot\mathbf{v} = k[\mathbf{u} \cdot \mathbf{v}]$
6. $\mathbf{u}\cdot[\mathbf{v} + \mathbf{w}] = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$.

Angles

The dot product can also be used to calculate the angle between a pair of vectors. In R^2 or R^3, the angle between the nonzero vectors u and v will refer to the angle θ determined by these vectors that satisfies $0 \leq \theta \leq 180$.





Definition

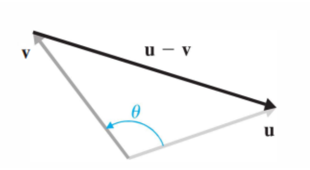
For nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ in R^n

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{u}||\ ||\mathbf{v}||} \tag{36}$$

This implies that

$$\mathbf{u}\cdot\mathbf{v} = ||\mathbf{u}||\ ||\mathbf{v}|| \cos\theta \tag{37}$$

$0 \le \theta \le \pi$ is the internal angle.



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Orthogonal vectors

Two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ are said to be orthogonal or perpendicular if

$$\mathbf{u}\cdot\mathbf{v} = 0 \tag{38}$$

Example

Let $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$ be three vectors of $\mathbb{R}^3$. Show that $\mathbf{u}$ and $\mathbf{w}$ are orthogonal but $\mathbf{u}$ and $\mathbf{v}$ are not. Find the angle θ between the direction of $\mathbf{u}$ and $\mathbf{v}$.

Example

Let $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$ be three vectors of $\mathbb{R}^3$. Show that $\mathbf{u}$ and $\mathbf{w}$ are orthogonal but $\mathbf{u}$ and $\mathbf{v}$ are not. Find the angle θ between the direction of $\mathbf{u}$ and $\mathbf{v}$.

$$\mathbf{u} \cdot \mathbf{v} = [1, -2, 3] \cdot [1, 0, -1] \tag{39}$$

$$= 1 + 0 - 3 = -2 \quad \text{Not orthogonal} \tag{40}$$

Example

Let $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$ be three vectors of $\mathbb{R}^3$. Show that $\mathbf{u}$ and $\mathbf{w}$ are orthogonal but $\mathbf{u}$ and $\mathbf{v}$ are not. Find the angle θ between the direction of $\mathbf{u}$ and $\mathbf{v}$.

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$$= 1 + 0 - 3 = -2 \quad \text{Not orthogonal} \tag{40}$$

$$\mathbf{u} \cdot \mathbf{w} = [1, -2, 3] \cdot [2, 7, 4] \tag{41}$$
$$= 2 - 14 + 12 = 0 \quad \text{orthogonal} \tag{42}$$

Example

Let $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$ be three vectors of $\mathbb{R}^3$. Show that $\mathbf{u}$ and $\mathbf{w}$ are orthogonal but $\mathbf{u}$ and $\mathbf{v}$ are not. Find the angle θ between the direction of $\mathbf{u}$ and $\mathbf{v}$.

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$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||\ ||\mathbf{v}||} = \frac{-2}{\sqrt{14} \times \sqrt{2}} = \frac{-1}{\sqrt{7}} \tag{43}$$
$$\theta = 112.2078 \tag{44}$$

Example

Find k so that $\mathbf{u} = [1, k, -3, 1]$ and $\mathbf{v} = [1, k, k, 1]$ are orthogonal.

Example

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For orthogonal

$$\mathbf{u} \cdot \mathbf{v} = 0 \tag{45}$$

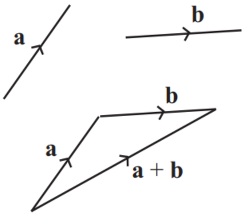
$$[1, k, -3, 1] \cdot [1, k, k, 1] = 0 \tag{46}$$

$$1 + k^2 - 3k + 1 = 0 \tag{47}$$

$$k^2 - 3k + 2 = 0 \tag{48}$$

$$[k - 2][k - 1] = 0 \tag{49}$$

Hence $k = 2$ or $k = 1$



Theorem

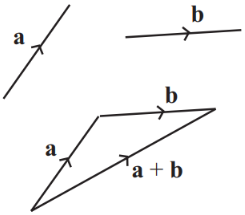
For any two vectors $\mathbf{a}$ *and* $\mathbf{b}$

1. *Schwartz's inequality:*

$$|\mathbf{a}\cdot\mathbf{b}| \leq |\mathbf{a}|\,|\mathbf{b}| \tag{50}$$

2. *Triangle inequality:*

$$|\mathbf{a}+\mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}| \tag{51}$$



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Definition

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$$d(\mathbf{a}, \mathbf{b}) = |\mathbf{a} - \mathbf{b}| \tag{52}$$

Theorem (Pythagoras)

For all vectors $\mathbf{u}$ *and* $\mathbf{v}$ *in* R^n,

$$|\mathbf{u}+\mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 \tag{53}$$

if and only if $\mathbf{u}$ *and* $\mathbf{v}$ *are orthogonal.*

Note

$$|\mathbf{u}+\mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 + 2\mathbf{u}\cdot\mathbf{v} \tag{54}$$

For orthogonal $\mathbf{u}\cdot\mathbf{v} = 0$

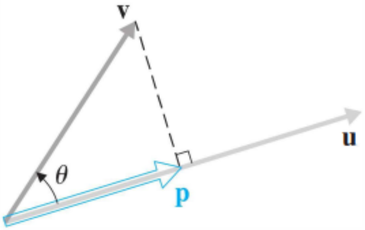
Projection of v onto u

Consider two nonzero vectors $\mathbf{u}$and $\mathbf{v}$. Let $\mathbf{p}$ be the vector obtained by dropping a perpendicular from the head of $\mathbf{v}$onto $\mathbf{u}$ and let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$

Scaler Projection

The **scalar projection** of a vector $\mathbf{v}$ onto a nonzero vector $\mathbf{u}$ is defined and denoted by the scalar

$$comp_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|} \qquad (55)$$



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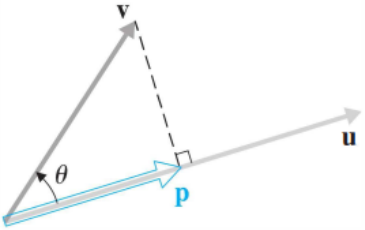

Vector Projection

The **[vector] projection** of $\mathbf{v}$ onto $\mathbf{u}$ is

$$proj_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|}\frac{\mathbf{u}}{|\mathbf{u}|} = comp_{\mathbf{u}}\mathbf{v}\frac{\mathbf{u}}{|\mathbf{u}|} \tag{56}$$

Example

Find the projection of $\mathbf{v}$ onto $\mathbf{u}$ if $\mathbf{u} = [2, 1]$ and $\mathbf{v} = [-1, 3]$

Example

Find the projection of $\mathbf{v}$ onto $\mathbf{u}$ if $\mathbf{u} = [2, 1]$ and $\mathbf{v} = [-1, 3]$

$$\mathbf{u} \cdot \mathbf{v} = -2 + 3 = 1 \tag{57}$$

$$||\mathbf{u}|| \cdot ||\mathbf{u}|| = \sqrt{5}(\sqrt{5}) = 5 \tag{58}$$

Example

Find the projection of $\mathbf{v}$ onto $\mathbf{u}$ if $\mathbf{u} = [2, 1]$ and $\mathbf{v} = [-1, 3]$

$$\mathbf{u} \cdot \mathbf{v} = -2 + 3 = 1 \tag{57}$$

$$||\mathbf{u}|| \cdot ||\mathbf{u}|| = \sqrt{5}(\sqrt{5}) = 5 \tag{58}$$

$$proj_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|} \frac{\mathbf{u}}{|\mathbf{u}|} = \frac{1}{5}\mathbf{u} \tag{59}$$

$$= \frac{1}{5}[2, 1] \tag{60}$$

$$= \left[\frac{2}{5}, \frac{1}{5}\right] \tag{61}$$

Exercise

Find the vector projection of $\mathbf{v}$ *onto* $\mathbf{u}$

1. $\mathbf{u} = [-1,\ 1], \quad \mathbf{v} = [-2,\ 4]$
2. $\mathbf{u} = [3/5,\ -4/5], \quad \mathbf{v} = [1,\ 2]$
3. $\mathbf{u} = [1/2,\ -1/4, / -1/2], \quad \mathbf{v} = [2,\ 2, / -2]$

END OF LECTURE

THANK YOU