Project 3: Catalan and Motzkin Numbers

Due: Tuesday May 29, 11:59 pm

1   Assignment Overview

This project focuses on writing functions that do mathematical computations. You will compute Catalan and Motzkin numbers.

2   Assignment Deliverables

You must turn in completed versions of the following files:

- proj03/catalan.cpp

Be sure to use the specified file name and to submit your files for grading via **Mimir** before the project deadline.

3   Background

In mathematics, the binomial coefficient $\binom{n}{k}$ represents the number of ways to select a committee of k people from a total population of n people. There are many ways to compute $\binom{n}{k}$ but one of the most computationally efficient is with the product

$$\binom{n}{k} = \prod_{i=1}^{k} \frac{n+1-i}{i}.$$

The Catalan numbers C_n counts the number of valid ways to nest n pairs of parenthesis. They can be computed with binomial coefficients with

$$C_n = \frac{1}{n+1}\binom{2n}{n}.$$

Finally, the Motzkin numbers M_n count the number of ways to draw non-intersecting chords on n points equally spaced around a circle. These can be computed from Catalan numbers by the sum

$$M_n = \sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n}{2k} C_k.$$

4   Assignment Specifications

In this and in future projects we will provide *exactly* our function specifications: the function name, its return type, its arguments, and each arguments type. The functions will be tested individually in Mimir using these exact function specifications. If you do not follow the function specifications, these independent tests of your functions will fail. Do not change the function declarations!

What you test on Mimir is a file that contains only the functions. You do not turn in a main program. We can test the functions individually on Mimir. However, you should write your own main program to test your functions separate from Mimir. It is more flexible and you can debug more easily.

function `Binom`
    params: n: long, k: long
    returns: long

function `Catalan`
    params: n : long
    returns: long

function `Motzkin`
    params: n : long
    returns: long

5   Assignment Notes

- You will receive no points if your solution does not compile on Mimir.

- Points will be deducted if your solution has any compiler warnings.

- Points will be deducted if your solution has the line "using namespace std;" or any variant that broadly includes a namespace instead of individual functions

- The `<cmath>` library contains a number of useful methods.

- You *do not* need to check for bad input values for this project. In future projects we will explicitly indicate the errors we are looking for.

- The smallest inputs are $n = 0$ and $k = 0$.

- $\binom{5}{2} = 10$ and $\binom{6}{3} = 20$.

- The additive identity is 0 and the multiplicative identity is 1.

- The first ten Catalan numbers are 1, 1, 2, 5, 14, 42, 132, 429, 1430, and 4862.

- The first ten Motzkin numbers are 1, 1, 2, 4, 9, 21, 51, 127, 323, and 835.