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First printing, March 2022



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Part One Title

1	Pre-requisite knowledge:Set Theory 11
1.1	Explanation of symbols
1.2	mapping
2	In-text Element Examples
2.1	Referencing Publications
2.2	Link Examples
2.3	Lists
2.4	International Support
2.5	Ligatures



1.1 Explanation of symbols

在抽象代数的学习中,会使用到集合中的知识,当然也会使用到集合中的各种符号, 在这一小节中,对这些符号进行一遍复习。

1.1.1 不同数的符号表示

- ℝ 全体实数 (有理数和无理数) 的集合
- № 全体自然数集合
- № 全体非负整数排除 0 的集合
- ② 全体有理数 (整数和分数) 的集合
- ℤ 全体整数的集合
- ℂ 全体复数的集合

1.1.2 集合中的常见概念符号

- 集合 A,B,C
- 元素 a,b,c
- 空集 ∅
- 元素与集合之间的从属关系:∈,∉
- 集合与集合之间的从属关系:⊂,⊆,⊄
- 交集,补集,并集: $A \cup B A \cap B,A^{\complement}$

1.1.3 集合中的常见概念

要证明两个集合相等,只需要证明这两个集合相互包含即可,这是一个非常常见的证明集合相等的手段。

Theorem 1.1 定理一: 两个集合 A,B 相等的充要条件: $A = B \iff A \subset B \iff B \subset A$

在抽象代数中,我们将使用集合的笛卡尔积来定义映射,这是一个非常重要的概念。

Theorem 1.2 我们称: $A_1 \times A_2 \times ... \times A_n = \{(a_1, a_2, ..., a_n) | a_i \in A_i\}$ 为 n 个集合 $A_1, A_2, ... A_n$ 的笛卡尔积。

() 内表示的是有序数组,而 a_1 则表示的是 A_1 里的元素。

■ Example 1.1

$$A = \{a, b, c\}, B = \{1, 2\}, A \times B = \{(a, 1), (b, 1), (c, 1), (a, 2), (b, 2), (c, 2)\}$$

根据上面这个例题,我们可以推断出一个结论。

Theorem 1.3 一般地,如果 |A| = m, |B| = n 那么 $|A \times B| = mn$

R 一般,我们使用 |A| 来表示 A 集合中元素的个数。比如上面的 $A \times B$ 表示的就是 AB 笛卡尔积所构成的集合的元素的个数。

1.2 mapping

Theorem 1.4 映射的定义

设 \emptyset 是从笛卡尔积 $A_1 \times A_2 \times ... \times A_n$ 到集合 D 的一个法则,如果 $A_1 \times A_2 \times ... \times A_n$ 中的每一个元素 $(a_1,a_2,...a_n)$ 都有 D 中唯一的元素 d 与之对应,那么我们称 \emptyset 是从 $A_1 \times A_2 \times ... \times A_n$ 到 D 的一个映射。

- Example 1.2 设 $A_1 = \{ \text{东, m} \}, A_2 = \{ \text{南} \}, D = \{ \text{高, K} \}, \text{则 } \emptyset_1(\text{m, h}) = \text{高不是 } A_1 \times A_2$ 到 D 的映射,因为只定义了一种情况,总共有两种情况,没有进行一一对应。如果改为 $\emptyset_2(\text{m, h}) = \text{高,} \emptyset_2(\text{r, h}) = \text{K, 符合定义,所以是 } A_1 \times A_2$ 到 D 的映射。

$$\emptyset(a) = a, a \neq 1$$

$$\emptyset(1) = b, b^2 = 1$$

不是 A_1 到 D 的映射。

虽然这个映射对每一个定义域内的变量都进行了映射,但是在自变量为 1 的时候 b 可以等于 +1 也可以等于 –1, 不符合一一对应的条件,所以不是映射。 ■

■ Example 1.4 设 $A_1 = D = \mathbb{Z}_+$, 则

$$\emptyset(a) = a - 1$$

不是 A_1 到 D 的映射。

由于 A 和 D 都是属于正整数集合,所以当 a=1 的时候映射结果不在 D 集合内,所以不是。

1.2 mapping 13

Theorem 1.5 映射相等

设 \emptyset_1,\emptyset_2 都是从笛卡尔积 $A_1\times A_2\times ...\times A_n$ 到集合 D 的映射,如果对于 $A_1\times A_2\times ...\times A_n$ 中的每一个元素 $(a_1,a_2,...,a_n)$ 都有

$$\emptyset_1(a_1, a_2, ..., a_n) = \emptyset_2(a_1, a_2, ..., a_n),$$

则称这两个映射 θ_1, θ_2 是相等的。



特别注意:两个映射相等,实际上的要求是:

- 它们的定义域相等
- 它们的作用效果是相通的
- Example 1.5 设 A=D 都表示正整数的集合, $\emptyset_1: A \to D$ 定义为: $\emptyset_1(a) = 1, \emptyset_2: A \to$ 定义为: $\emptyset_2(a) = a^0$,则 $\emptyset_1 = \emptyset_2$

由于前提条件是正整数的集合,而不是自然数,元素自然不可能是 0,所以定义域一样,而作用效果也一样,映射的结果都是 1,所以映射是相等的,但是如果将本例改为自然数,则是错误的。

Unnumbered Section

Unnumbered Subsection

Unnumbered Subsubsection



2.1 Referencing Publications

This statement requires citation [Smith:2022jd]; this one is more specific [Smith:2021qr].

2.2 Link Examples

This is a URL link: LaTeX Templates. This is an email link: example@example.com. This is a monospaced URL link: https://www.LaTeXTemplates.com.

2.3 Lists

Lists are useful to present information in a concise and/or ordered way.

2.3.1 Numbered List

- 1. First numbered item
 - a. First indented numbered item
 - b. Second indented numbered item
 - i. First second-level indented numbered item
- 2. Second numbered item
- 3. Third numbered item

2.3.2 Bullet Point List

- First bullet point item
 - First indented bullet point item
 - Second indented bullet point item
 - First second-level indented bullet point item
- Second bullet point item

• Third bullet point item

2.3.3 Descriptions and Definitions

Name Description
Word Definition
Comment Elaboration

2.4 International Support

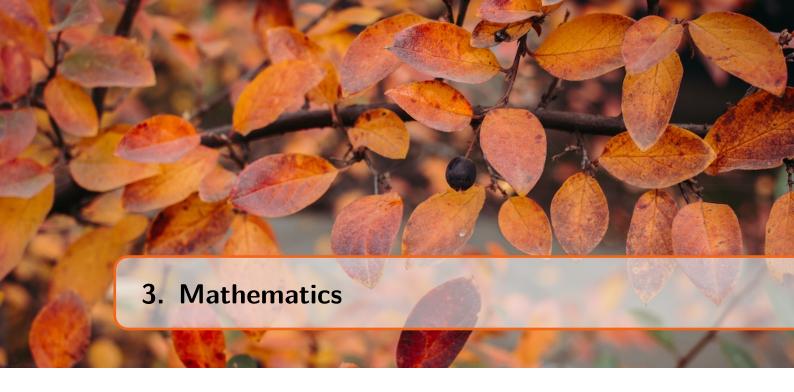
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2.5 Ligatures

fi fj fl ffl ffi Ty

Part Two Title

3	Mathematics
3.1	Theorems
3.2	Definitions
3.3	Notations
3.4	Remarks
3.5	Corollaries
3.6	Propositions
3.7	Examples
3.8	Exercises
3.9	Problems
3.10	Vocabulary
4	
4	Presenting Information and Results with a
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3.1 Theorems

3.1.1 Several equations

This is a theorem consisting of several equations.

Theorem 3.1 — Name of the theorem. In $E = \mathbb{R}^n$ all norms are equivalent. It has the properties:

$$|||\mathbf{x}|| - ||\mathbf{y}||| \le ||\mathbf{x} - \mathbf{y}|| \tag{3.1}$$

$$||\sum_{i=1}^{n} \mathbf{x}_i|| \le \sum_{i=1}^{n} ||\mathbf{x}_i||$$
 where n is a finite integer (3.2)

3.1.2 Single Line

This is a theorem consisting of just one line.

Theorem 3.2 A set $\mathcal{D}(G)$ in dense in $L^2(G)$, $|\cdot|_0$.

3.2 Definitions

A definition can be mathematical or it could define a concept.

Definition 3.1 — **Definition name.** Given a vector space E, a norm on E is an application, denoted $||\cdot||$, E in $\mathbb{R}^+ = [0, +\infty[$ such that:

$$||\mathbf{x}|| = 0 \Rightarrow \mathbf{x} = \mathbf{0} \tag{3.3}$$

$$||\lambda \mathbf{x}|| = |\lambda| \cdot ||\mathbf{x}|| \tag{3.4}$$

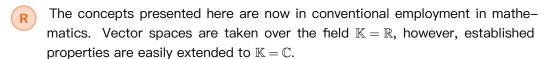
$$||x+y|| \le ||x|| + ||y|| \tag{3.5}$$

3.3 Notations

- Notation 3.1 Given an open subset G of \mathbb{R}^n , the set of functions φ are:
 - 1. Bounded support G;
 - 2. Infinitely differentiable;
- a vector space is denoted by $\mathcal{D}(G)$.

3.4 Remarks

This is an example of a remark.



3.5 Corollaries

Corollary 3.1 — Corollary name. The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field $\mathbb{K} = \mathbb{R}$, however, established properties are easily extended to $\mathbb{K} = \mathbb{C}$.

3.6 Propositions

3.6.1 Several equations

Proposition 3.1 — **Proposition name.** It has the properties:

$$|||\mathbf{x}|| - ||\mathbf{y}||| \le ||\mathbf{x} - \mathbf{y}|| \tag{3.6}$$

$$||\sum_{i=1}^{n} \mathbf{x}_i|| \le \sum_{i=1}^{n} ||\mathbf{x}_i||$$
 where n is a finite integer (3.7)

3.6.2 Single Line

Proposition 3.2 Let $f,g \in L^2(G)$; if $\forall \varphi \in \mathcal{D}(G)$, $(f,\varphi)_0 = (g,\varphi)_0$ then f = g.

3.7 Examples

3.7.1 Equation Example

■ Example 3.1 Let $G = \{x \in \mathbb{R}^2 : |x| < 3\}$ and denoted by: $x^0 = (1,1)$; consider the function:

$$f(x) = \begin{cases} e^{|x|} & \text{si } |x - x^0| \le 1/2\\ 0 & \text{si } |x - x^0| > 1/2 \end{cases}$$
 (3.8)

The function f has bounded support, we can take $A=\{x\in\mathbb{R}^2:|x-x^0|\leq 1/2+\varepsilon\}$ for all $\varepsilon\in]0;5/2-\sqrt{2}[$.

3.8 Exercises 21

3.7.2 Text Example

■ Example 3.2 — Example name. Aliquam arcu turpis, ultrices sed luctus ac, vehicula id metus. Morbi eu feugiat velit, et tempus augue. Proin ac mattis tortor. Donec tincidunt, ante rhoncus luctus semper, arcu lorem lobortis justo, nec convallis ante quam quis lectus. Aenean tincidunt sodales massa, et hendrerit tellus mattis ac. Sed non pretium nibh. Donec cursus maximus luctus. Vivamus lobortis eros et massa porta porttitor.

3.8 Exercises

Exercise 3.1 This is a good place to ask a question to test learning progress or further cement ideas into students' minds.

3.9 Problems

Problem 3.1 What is the average airspeed velocity of an unladen swallow?

3.10 Vocabulary

Define a word to improve a students' vocabulary.

■ Vocabulary 3.1 — Word. Definition of word.

4.1 Table

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Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

表 4.1: Table caption.

Referencing Table 4.1 in-text using its label.

4.2 Figure

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Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

表 4.2: Floating table.



图 4.1: Figure caption.

Referencing Figure 4.1 in-text using its label.



Bibliography

Articles

Books



A.1 Appendix Section Title

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B.1 Appendix Section Title

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