Chapter 9. Generalized linear models and the exponential family

9.1.
For univariate Gaussian,

$$p(\boldsymbol{\theta}|\mu,\lambda) \propto \lambda^{-\frac{n}{2}} e^{-\frac{n}{2}\mu^2\lambda+\mu\lambda\sum_i x_i-\frac{1}{2}\lambda\sum_i x_i^2}$$

To represent it as a form of $\mathcal{N}(\mu|\gamma,\lambda(2\alpha-1))\mathrm{Ga}(\lambda|\alpha,\beta)$, we deduce that

$$\gamma = \bar{\mathbf{x}}, \alpha = -\frac{n}{2}+1, \beta = -\frac{n\lambda}{2}s_x^2.$$

9.2.

$$\mathcal{N}(\mathbf{x}|\boldsymbol{\xi},\boldsymbol{\Lambda}) = (2\pi)^{-\frac{D}{2}}|\boldsymbol{\Lambda}|^{\frac{1}{2}}e^{-\frac{1}{2}(\mathbf{x}^T\boldsymbol{\Lambda}\mathbf{x}+\boldsymbol{\xi}^T\boldsymbol{\Lambda}^{-1}\boldsymbol{\xi}-2\mathbf{x}^T\boldsymbol{\xi})}.$$

Letting

$$\mathbf{y} = [-\mathbf{x}\mathbf{x}^T, \mathbf{x}], \boldsymbol{\theta} = [\frac{1}{2}\boldsymbol{\Lambda}^T, \boldsymbol{\xi}], \frac{1}{Z(\boldsymbol{\theta})} = (\pi)^{-\frac{D}{2}}|\boldsymbol{\theta}_1|^{\frac{1}{2}}e^{-\boldsymbol{\theta}_2^T(\boldsymbol{\theta})^{-1}\boldsymbol{\theta}_2}$$

gives an expression of MVN in exponential family form.