Commodity Calendar Spread Options : Replication, Hedging and Implied Correlations

Author

Ziqi Yuan
Litai Ren

Supervisor

Prof. Roza Galeeva

New York University
Tandon School of Engineering
Department of Finance and Risk Engineering

Contents

Chapter 1

Introduction

1.1 Introduction

The main objective of our project is study of Calendar Spread Options. Our data source are CME group website for calendar spread option market quotes, and Barchart for near-the-money vanilla option implied volatilities. We would like to acknowledge the work and help of Litai Ren, NYU Tandon School of Engineering, who did extensive preparations for this project early from January 2020, collecting weekly market quotes and investigating into the implied correlation calibration.

1.1.1 Spread Options

Calendar spread option, our main investigation objective, is one particular type of spread options. Spread options have widespread applications in commodity and beyond. A spread option is a kind of option that derives value from the spread of two or more assets. We list a variety of spread instruments [2] and provide a brief explanation.

- **Spread Options in the Energy Markets**
  - **Crack Spreads** A crack spread is the simultaneous purchase or sale of crude against the sale or purchase of refined petroleum products.
  - **Spark Spreads** The primary cross-commodity transaction in the electricity markets. A proxy for the cost of converting a specific fuel into electricity at a specific facility.
  - **Locational Spreads** Used for hedging transportation risk exposure from futures contracts on the same commodity with physical deliveries at two different locations.

  - **Temporal Spreads** Differences in the prices of the same commodity at two different dates in the future.

- **Spread Options in the Currency and Fixed Income Markets** Spread options are quite common in the foreign exchange markets where spread involve interest rates in different countries.
- **Spread Options in the Agricultural Futures Markets** The diverse selection of options on Grains, Oilseeds, Livestock and Dairy products range from outright options and spread options, to cost-effective short-term alternatives.

1.1.2 Calendar Spread Options

Calendar Spread Options (CSOs) are options on the spread between two different futures expirations. The Energy futures term structure represents the time value of Energy market variables such as storage costs, seasonality, and supply/demand conditions. Calendar Spread Options provide a leveraged means of hedging against, or speculating on, a change in the shape of the futures term structure. Readers can refer to https://www.cmegroup.com/trading/agricultural/grain-and-oilseed/grain-calendar-spreads-options.html for more information.

CME Group has a diverse product offering that includes Crude Oil, Natural Gas, and Refined Products CSOs. We will be using their CSO settlements for WTI and NG futures.

1.2 Goals of Project

In our project, we concentrate on the following objectives:

- Develop calibration procedures for implied calendar correlations from calendar spread options for Natural Gas and Wti, given by the CME group.
- Calibrate the implied correlations for oil contracts to the model of decay and growth and analyze recent trends.
- Develop methodology of calendar correlation calculation based on replicating and hedging performance with **Black model** and **Bachelier model**, run it on historical data of Wti and Natural Gas contracts, and compare the results.

1.3 Literature Review

1.3.1 Spread Option Pricing

There are many remarkable papers on this topic. Our primary reference for this topic is [2] since it consists of all the analytics we need for the project, which we explicitly use in later sections. The paper investigated the theoretical and computational problems associated with the pricing and hedging of spread options. The authors researched several models of pricing spread options, Bachelier model and Kirk's approximation included. We applied both models and figured out Bachelier model works better in this case.

CME group uses Bachelier model for Calendar Spread Options. According to the CME group website announcement, the clearinghouse also switched the pricing and valuation model of options on futures to Bachelier on April 22, 2020. For more information, please go to https://www.cmegroup.com/notices/clearing/2020/04/Chadv20-171.html.

1.3.2 Correlation Analytics

R. Galeeva, Research Professor at NYU Tandon School of Engineering and Th. Haversang, Graduate of NYU Tandon School of Engineering, [3] propose a novel two-parameter framework to parameterize calendar correlation. We adopted the analytics and formulae for implied correlation modeling. The model captures the key fundamental properties of calendar correlations: the decay of correlations with time between contracts, and growth for contracts farther away. The growth of correlation is captured through the dynamics of instantaneous correlations, and it represents the famous Samuelson effect for commodity futures. In terms of Samuelson effect, our colleagues, Yuxiao Cheng and Qi Yan, researched Samuelson effect in detail this summer. They modeled historical volatility of oil future contracts with several models and compared the results.

1.3.3 Bachelier Model

The research on Bachelier model dates back to 1900. Bachelier [1] is a pioneering paper on option pricing. He examined the properties of the price of the stock, finding that it follows the normal distribution. He derived Bachelier pricing formula under this assumption. In [5] , one of the latest papers on Bachelier model, the author derives an option pricing formula based on the Bachelier model and compares it with the prior researches. When the interest rate is non-zero, the formula becomes much more complicated. We decided to use the formula of zero interest rate situation for calendar spread options, especially when the interest rate is close to 0 for 2020 extraordinary spring.

1.3.4 Options Replication

In reality, we pay transaction costs when we trade futures, which makes the replication more complex. Transactions costs invalidate the Black-Scholes arbitrage argument for option pricing. [4] figures out that discrete revision using Black-Scholes deltas generates errors that do not approach zero with more frequent revision when transaction costs are non-zero. In short, we don't want to hedge too often since we have to pay transaction costs.

1.4 Structure of Report

The structure of our report is as below.

- **Chapter 1**: Introduction and literature reviews for the core concepts of our project.
- **Chapter 2&3**: Theory and Methodology of two main topics:
  - Theory and Methodology of Implied Correlations
  - Theory and Methodology of Calendar Spread Option Replication
- **Chapter 4**: Simulation Chapter for the verification of our methodology on the simulated data.
- **Chapter 5&6**: Detailed results of two main topics
- **Chapter 7**: Summary and Conclusions
- **Reference and Appendix**

Chapter 2

Theory and Methodology: Implied Correlation

In this chapter, we demonstrate theory and methodology for two main topics: implied correlation calibration and implied correlation modeling.

2.1 Implied Correlation Calibration

We calibrate calendar spread option implied correlations according to Bachelier model discussed in [2]. In this section, we will discuss the theory of Bachelier model and the procedures of implied correlation calibration.

2.1.1 Bachelier Model Set Up

Readers can refer to [2] for more detailed set up of Bachelier model. As mentioned in the reference paper, "future prices normally distributed" is not directly adopted in industry since the dynamics for F_1 and F_2 are totally unrealistic since their marginal distributions are Gaussian and can therefore be negative with positive probability. Instead, practitioners usually assume that the dynamics for F_1 and F_2 are given by geometric Brownian motions and that the dynamics of the spread can be approximated by an arithmetic Brownian motion. The formulae are as below,

$$dS_1(t) = \mu S_1(t)dt + \sigma_1 S_1(t)dW_1(t) \tag{2.1}$$

$$dS_2(t) = \mu S_2(t)dt + \sigma_2 S_2(t)dW_2(t) \tag{2.2}$$

If the value of the spread at maturity is assumed to have the Gaussian distribution, the price p of the call spread option with maturity T and strike K is given by:

$$p = (m(T) - Ke^{-rT})\Phi(\frac{m(T) - Ke^{-rT}}{s(T)}) + s(T)\phi(\frac{m(T) - Ke^{-rT}}{s(T)}) \tag{2.3}$$

where we used the notation:

$$m(T) = (x_2 - x_1)e^{(\mu - r)T}$$

$$s^2(T) = e^{2(\mu - r)T}(x_1^2(e^{\sigma_1^2 T} - 1) - 2x_1 x_2(e^{\rho\sigma_1\sigma_2 T} - 1) + x_2^2(e^{\sigma_2^2 T} - 1))$$

We use the notation $\Phi(x)$ and $\phi(x)$ for the density and the cumulative distribution function of the standard normal $\mathcal{N}(0, 1)$ distribution throughout the report. Here, we would like to mention that the formulae are based on spot prices; however, our study is about future prices. We assume interest rate to be 0 throughout our project so that spot prices and future prices are the same numerically.

2.1.2 Implied Correlation Calibration Procedures

1. **Pricing Formula** The Bachelier pricing formula for calendar spread option is Formula 2.3. We calibrate ρ according to this formula.

2. **Inputs and Sources** Given a spread call option, below are the parameters needed in order to price it with Formula 2.3.

   - x_1: Price of future 1 with earlier expiration (CME group website)
   - x_2: Price of future 2 with later expiration (CME group website)
   - σ_1: Implied Volatility (IV) of future 1 (IV of near ATM vanilla option on future 1 from Barchart)
   - σ_2: Implied Volatility (IV) of future 2 (IV of near ATM vanilla option on future 2 from Barchart)
   - T: Maturity of the calendar spread option
   - K: Strike price of the calendar spread option
   - r: We assume $r = \mu = 0$
   - ρ: Implied correlation: the only unknown parameter

3. ρ **Calibration** Given a spread call option settlement price $C_{observed}$ for strike K and expiration T, it is easy to collect the future prices F_1 and F_2 as well as their corresponding implied volatilities σ_1 and σ_2 (ATM vanilla option volatility) from websites. Since the only unknown parameter in the pricing formula is ρ, we can calibrate ρ by minimizing the difference between $C_{observed}$ and $p(\rho)$. The objective function is as below,

$$\rho = arg\,min(C_{observed} - p(\rho))^2 \tag{2.4}$$

4. **Open Interest Weighted Aggregate Implied Correlation** There are many contracts with different strike prices for one month. For each month, we aggregate the implied correlation with the weight of open interest of each contract. Let ρ_{agg} be the aggregated implied correlation and ρ_i be the implied correlation calibrated in step 3 for each contract. V_i represents the open interest for each contract. The formula of calculating ρ_{agg} is as below,
$$\rho_{agg} = \sum_i \frac{\rho_i V_i}{\sum_i V_i} \tag{2.5}$$

2.2 Implied Correlation Modeling

The modeling of implied correlations is crucial for proper evaluation and hedging of energy derivatives. The two-parameter framework comes from paper [3]. The model shows perfect fit for WTI realized correlation. Inspired by the paper, our main goal is to apply the framework into modeling implied correlation, and then further stabilize the parameters if it works.

2.2.1 Two-Parameter Framework Set Up

For more detailed derivations, readers can consult [3].

- **Notation**
  - t_1: Current time
  - t_2: Expiration date of Calendar Spread Option
  - T_i: Expiration of earlier future contract
  - T_j: Expiration of later future contract ($T_i < T_j$)
  - $C(t_1, t_2, T_i, T_j)$: Implied Correlation between t_1, t_2
  - $C^0_{i,j}$: Instantaneous correlation at $t = T_i$
  - $\rho(t, T_i, T_j)$: Instantaneous correlation between the two contracts at time $t < T_i$
  - B: Samuelson parameter
  - α: Monthly rate of growth of correlations
- **Formulae** We provide the formulae we used for implied correlation modeling. For the detailed derivations, readers can refer to paper [3].
  - The important formula based on which we calibrate the parameters is shown below,
$$C(t_1, t_2, T_i, T_j) = K^i_s C^0_{i,j} + 1 - K^i_s \tag{2.6}$$

where K_s^i is

$$K_s^i = \frac{2Be^{-\alpha(T_i - t_2)}}{\alpha + 2B} \frac{1 - e^{-(\alpha+2B)(t_2 - t_1)}}{1 - e^{-2B(t_2 - t_1)}} \tag{2.7}$$

– We label the future contracts in the order of expiration date. Here, we examine $C(t_1, t_2, T_1, T_j)$ using Equation 2.6,

$$C(t_1, t_2, T_1, T_j) = K_s^1 C_{1,j}^0 + (1 - K_s^1) \tag{2.8}$$

Since the instantaneous correlations $C_{i,j}^0$ depend only on the time between contracts, the implied correlation can then be written as,

$$C(t_1, t_2, T_i, T_j) = K_s^i C_{1,j-i+1}^0 + 1 - K_s^i \tag{2.9}$$

With this form, we express $C_{i,j}^0 = C_{|i-j|}^0$ and get the following expression,

$$C(t_1, t_2, T_i, T_j) = \frac{K_s^i}{K_s^1} C(t_1, t_2, T_1, T_{j-i+1}) + 1 - \frac{K_s^i}{K_s^1} \tag{2.10}$$

2.2.2 Implied Correlation Parameter Calibration Procedures

In the paper [3], the authors assume analytical models for the historical correlation; meanwhile, we tried to determine three parameters directly using Equation 2.10.

1. **Determine Samuelson Parameter B** The beauty of the model is that we separate the Samuelson parameter from the growth of implied correlation since they have the opposite effect for the model. More specifically, K_s^i is an increasing function with respect to B and is a decreasing function with respect to α.

   The result of Samuelson parameter B comes from the other part of our capstone project: Samuelson effect investigation. It is done by our colleagues: Qi Yan and Yuxiao Cheng. A little tricky here is that they calibrate parameter B based on historical data; thus B is a realized parameter, which is backward-looking and; meanwhile, we need B to be forward-looking here. Despite the historical and implied difference, we verified it with the contour plot method, showing it is within a reasonable range. The details of analytics of the contour plot method are appended in Appendix A.1

2. **Determine Decay Parameter $Corr_0$**

   - **Method 1**: Choose "Decay" as First Implied Correlation Method 1 is that we use the "first" implied correlation (prompt contract) for each date to represent decay. We fix first implied correlation, in other words, take $C(t_1, t_2, T_1, T_2)$ as what it is.

The formula we derived in section 2.2 is,

$$C(t_1, t_2, T_i, T_j) = \frac{K_s^i}{K_s^1} C(t_1, t_2, T_1, T_{j-i+1}) + 1 - \frac{K_s^i}{K_s^1}$$

We seek to calibrate monthly growth rate α of implied correlation of Wti 1 month correlation so that the month apart $j - i$ is always 1. Naturally, the original formula becomes,

$$C^{est}(t_1, t_2, T_i, T_j) = \frac{K_s^i}{K_s^1} C(t_1, t_2, T_1, T_2) + 1 - \frac{K_s^i}{K_s^1}$$

where K_s^i is

$$K_s^i = \frac{2Be^{-\alpha(T_i - t_2)}}{\alpha + 2B} \frac{1 - e^{-(\alpha+2B)(t_2 - t_1)}}{1 - e^{-2B(t_2 - t_1)}}$$

and $C(t_1, t_2, T_1, T_2)$ is known. The superscript "est" stands for "estimated". To avoid confusion, t_2 changes according to different option contracts.

- **Method 2**: Parameterize "Decay" For this part, we parameterize first implied correlation, in other words, treat $C(t_1, t_2, T_1, T_2)$ as a parameter. Logic of calibration is the similar to that we mentioned above. The only difference is that we treat $C(t_1, t_2, T_1, T_2)$ as a parameter; therefore, the optimization becomes two-dimensional so that the original equation becomes,

$$C^{est}(t_1, t_2, T_i, T_j) = \frac{K_s^i}{K_s^1} Corr_0 + 1 - \frac{K_s^i}{K_s^1}$$

  where C_0 and α are parameters to be calibrated.

- **Method 3**: Normal calibration methods do not give us stable and reasonable parameters. The difference between implied correlation and realized correlation is that when we calculate realized correlation, we usually stop one month before expiration; however, implied correlation directly goes to the expiration, which brings additional noise into the data. The implied correlation of the first nearby contract is quite low so that when we try to calibrate Corr0 and α using normal methods, Corr0 tends to be lower than usual and α tends to be higher in order to fit the first "outlier".

  - **Search for** $Corr$ Firstly, we determine the what we called $Corr_0$ before adjustment, denoted as $Corr$. $Corr$ is defined as the "first" correlation that is greater than 0.99. We search for that from the very beginning and if we find the $Corr$ that is greater than 0.99, we stop.

- **Extrapolate** $Corr_0$ After that, we extrapolate $Corr_0$ through the reference table. $Corr_0$ is defined as the $Corr$ minus 1 standard deviation. The reference table is appended in Appendix.

3. **Determine Growth Parameter** α Once Samuelson parameter B and Decay parameter α are determined, we can apply the following objective function to determine Growth parameter α. Denote open interest as V.

$$\hat{\alpha} =_{\alpha} \sum W \cdot (C^{est}(t_1, t_2, T_i, T_j) - C^{real}(t_1, t_2, T_i, T_j))^2 \tag{2.11}$$

where,

$$W_{call} = \frac{V_{call}}{V_{call} + V_{put}}$$

$$W_{put} = \frac{V_{put}}{V_{call} + V_{put}}$$

Chapter 3

Theory and Methodology: Spread Option Replication

When some particular types of spread options are not available on the market but we need the options for the purpose of risk management or speculation, it is natural that we seek to replicate the options through the underlyings. The problem is that what kind of parameters we ought to use into the replication. Of course, implied parameters, such as implied correlations would be the most ideal to plug into pricing formula. However, it is a tough task to back up implied parameters, especially when no relevant derivatives are available on the market. Therefore, we attempt to use historical information to derive the optimal parameters. In the later sections, we will be showing the results of replication using the historical realized values as ”benchmarks”.

Structures

- Replication Theory
- Generic Spread Option Replication Set Up
- Replication Models and Corresponding Parameters

3.1 Multi-asset Replication Theory

We aim to replicate options through trading underlying assets. In this subsection, we verify the multi-asset option replication through trading underlying assets separately according to Δ. The derivation was done by our colleague, Litai Ren, who did extensive preparation for this project.

3.1.1 Pricing Formulae

Assume we have a derivative $f(t, S(t))$, where $S(t) = \begin{pmatrix} S_1(t) \\ \vdots \\ S_n(t) \end{pmatrix}$. And $\frac{dS(t)}{S(t)} = \mu dt + \sigma R dW(t)$, where $\mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_n \end{pmatrix}$, $RR_T = \Omega$, $\sigma = \begin{pmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n \end{pmatrix}$, $dW(t) = \begin{pmatrix} dW_1(t) \\ \vdots \\ dW_n(t) \end{pmatrix}$, $dW^T dW = Idt$. (If consider the case that n=2, $\Omega = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$, $R = \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix}$.) The differential of the derivative $f(t, S(t))$is

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} dS + \frac{1}{2} dS^T H dS \tag{3.1}$$

$$where \quad H = \begin{pmatrix} \frac{\partial^2 f}{\partial S_1^2} & \cdots & \frac{\partial^2 f}{\partial S_1 \partial S_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial S_n \partial S_1} & \cdots & \frac{\partial^2 f}{\partial S_n^2} \end{pmatrix} \tag{3.2}$$

Therefore, we construct risk-neutral portfolio

$$\Pi = f - \frac{\partial f^T}{\partial S} S \tag{3.3}$$

We can write

$$d\Pi = df - \frac{\partial f^T}{S} dS = \frac{\partial f}{\partial t} dt + \frac{1}{2} dS^T H dS \tag{3.4}$$

$$dSdS^T = SRdWdW^T R^T S^T = S \sum S^T dt \tag{3.5}$$

Obviously, there is no cross term of $dSdS^T$; therefore, all the terms are deterministic since there is no dW term.

Through this method, we eliminate diffusion term of Π, and then we should have the equation by risk-neutral principle

$$d\Pi = r\Pi dt \tag{3.6}$$

From another perspective, $df = d\Pi + \frac{\partial f^T}{\partial S} dS$, so we can replicate f by holding cash Π (a typical risk-neutral asset) and $\frac{\partial f^T}{\partial S}$ shares of S at every moment.

If we choose $\Pi(0) + \frac{\partial f^T}{\partial S}(0)S(0) = f(0)$, then we should end with $\Pi(T) + \frac{\partial f^T}{\partial S}(T)S(T) = f(T)$; vice versa.

3.1.2 Replication with Underlying Assets

Consider calendar spread option with $K = 0$, $c = S_1\mathcal{N}(d_1) - S_2\mathcal{N}(d_2)$, where $d_1 = \frac{ln(\frac{S_1}{S_2}) + \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}$, $d_2 = \frac{ln(\frac{S_1}{S_2}) - \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}$. If we view this as a vanilla call with parameters of $\begin{cases} S = S_1 \\ K = S_2 \\ r = 0 \end{cases}$, we can easily deduce that

$$\frac{\partial c}{\partial S_1} = \mathcal{N}(d_1)$$

However, there's no existing Greeks w.r.t K, so we have to derive $\frac{\partial c}{\partial S_2}$ from the very beginning.

$$\frac{\partial c}{\partial S_2} = S_1 n(d_1)\frac{\partial d_1}{\partial S_2} - (\mathcal{N}(d_2) + S_2 n(d_2)\frac{\partial d_2}{\partial S_2})$$

$$\frac{\partial d_1}{\partial S_1} = \frac{\partial d_1}{\partial S_1} = \frac{1}{\sigma\sqrt{\tau}S_1}$$

$$S_1 n(d_1) - S_2 n(d_2) = 0$$

Finally, we get

$$\frac{\partial c}{\partial S_2} = -\mathcal{N}(d_2)$$

3.2 Spread Option Replication Set Up: Model Independent

3.2.1 Generic Replication Procedures

We provide a generic replication set up. The set up is model-independent so that we can implement different models as long as pricing formula and first-order derivatives with respect to underlyings are known. The parameter that needed to be calibrated is usually embedded in the pricing and delta formulae.

Replication Assumptions

- Interest rate and dividend rate are zero.
- It is possible to purchase or sell any number of future shares.
- No transaction costs.

Replication Notation

- Implicit pricing formula $C(t, F_t^1, F_t^2)$
- Implicit Δ positions for futures at time t Δ_t^i, $i = 1, 2$
- Maturity T
- Future prices at time t F_t^i, $i = 1, 2$
- Cash position at time t $cash_t$
- Replication Portfolio value at time t P_t

Replication Steps

1. At time 0 t_0
   (a) Use the formula $C(t, F_t^1, F_t^2)$ to calculate the calendar spread option price. (Parameter list: $F_1 : F_0^1$, $F_2 : F_0^2$, $expiration : T$)
   (b) Assume the option price to be the initial wealth X_0.
   (c) Calculate future positions Δ_0^1 and Δ_0^2 at t_0. (Parameter List: $F_1 : F_0^1$, $F_2 : F_0^2$, $expiration : T$)
   (d) Buy futures with price F_0^1 and F_0^2 according to the position calculated in (c) and calculate cash position $cash_0 = X_0 - \Delta_0^1 F_0^1 - \Delta_0^2 F_0^2$.
   (e) Calculate today's portfolio value $P_0 = cash_0 + \Delta_0^1 F_0^1 + \Delta_0^2 F_0^2$.
2. At time 1 t_1
   (a) Calculate future positions Δ_1^1 and Δ_1^2 at t_1 using delta formulae. (Parameter List: $F_1 : F_1^1$, $F_2 : F_1^2$, $\sigma_S : \sigma_{opt}$, $expiration : T-(t_1-t_0)$)
   (b) Calculate $(\Delta_1^1 - \Delta_0^1)$ and $(\Delta_1^2 - \Delta_0^2)$. If the number is negative, then we sell $(\Delta_1^i - \Delta_0^i)$ positions at price F_1^i, $i = 1, 2$. If the number is positive, then we buy $(\Delta_1^i - \Delta_0^i)$ positions at price F_1^i, $i = 1, 2$.
   (c) Calculate cash position. $cash_1 = cash_0 * exp(r * (t_1 - t_0)) - (\Delta_1^1 - \Delta_0^1)F_1^1 - (\Delta_1^2 - \Delta_0^2)F_1^2$.
   (d) Calculate portfolio value $P_1 = cash_1 + F_1^1\Delta_1^1 + F_1^2\Delta_1^2$
3. At time 2 t_2

(a) Calculate future positions Δ_2^1 and Δ_2^2 at t_2 using delta equation. (Parameter List: $F_1 : F_2^1$, $F_2 : F_2^2$, $expiration : T - (t_2 - t_0)$)

(b) Calculate $(\Delta_2^1 - \Delta_1^1)$ and $(\Delta_2^2 - \Delta_1^2)$. If the number is negative, then we sell $(\Delta_2^i - \Delta_1^i)$ positions at price F_2^i, $i = 1, 2$. If the number is positive, then we buy $(\Delta_2^i - \Delta_1^i)$ positions at price F_2^i, $i = 1, 2$.

(c) Calculate cash position. $cash_2 = cash_1 * exp(r * (t_2 - t_1)) - (\Delta_2^1 - \Delta_1^1)F_2^1 - (\Delta_2^2 - \Delta_1^2)F_2^2$.

(d) Calculate portfolio value $P_2 = cash_2 + F_2^1\Delta_2^1 + F_2^2\Delta_2^2$

4. t_3, t_4 ...... the same logic with t_1 and t_2. Each time we update the delta function parameters list with new F_t^i, $i = 1, 2$ and new expiration.

5. At expiration

   Repeat steps (a)-(d) (Actually there is no need to do that because the portfolio value remains the same after changing position. The only thing that we should do at the expiration is to calculate the portfolio value. Considering the generality of the algorithm, we repeat steps (a)-(d) at the expiration which makes no change if not). The final portfolio value we calculate should be equal to the value of the option that we are trying to replicate at expiration under specific future price paths. Let's denote the final portfolio value as P_T.

3.2.2 Parameter Optimization: Breaking Good and Hedging Good

When it comes to optimization, the key concept is cost function (or objective function). We defined two different cost functions, which lead to two different kinds of "optimal" parameters: breaking good parameters and hedging good parameters. We denote calendar spread option value as $C(t, F_t^1, F_t^2)$ and the replication portfolio value to be P_t.

1. **Breaking Good Parameters**

$$breaking\,good\,parameter = argmin\;(R_T - P_T)^2 \tag{3.7}$$

2. **Hedging Good Parameters**

$$hedging\,good\,parameter = argmin\;\sum_t (R_t - P_t)^2 \tag{3.8}$$

3.3 Replication Models and Application

3.3.1 Black Model (Kirk's Approximation)

- **Pricing Formula** The Appendix A.3.1 contains all the mathematical details of derivations. Let $c(F_1, F_2, \tau)$ represents the price of a call option with the boundary condition $c(F_1, F_2, 0) = (F_1 - F_2 - K)^+$. Similarly, we have $p(F_1, F_2, 0) = (K - F_1 + F_2)^+$.

  Assume S_1, S_2 move as GBM in risk-neutral space, i.e. $\frac{dS_i}{S_i} = rdt + \sigma_i dW_i$ and $\rho = corr(dW_1, dW_2)$. Since $F = Se^{r\tau}$, the diffusion of future prices becomes $\frac{dF_i}{F_i} = \sigma_i dW_i$.

  By Kirk's approximation, when $\frac{F_2}{\sigma_{eff}} |\frac{\partial \sigma_{eff}}{\partial F_2}| \ll 1$ and $\frac{1}{r\sigma_{eff}} |\frac{\partial \sigma_{eff}}{\partial \tau}| \ll 1$, viewing σ_{eff} as a constant, we have,

$$c(F_1, F_2, \tau) = e^{-r\tau}[F_1 \Phi(d_1) - (F_2 + K)\Phi(d_2)] \tag{3.9}$$

  where, $d_1 = \frac{ln(\frac{F_1}{F_2+K}) + \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}$, $d_2 = \frac{ln(\frac{F_1}{F_2+K}) - \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}$, $\sigma = \sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_{eff} + \sigma_{eff}^2}$, $\sigma_{eff} = \sigma_2 \frac{F_2}{F_2 + K}$.

- **Greek: Delta**

$$\begin{cases} \Delta_{F_1} = e^{-r\tau}\Phi(d_1) \\ \Delta_{F_2} = -e^{-r\tau}\Phi(d_2) + \tilde{F_1} e^{-r\tau}\sqrt{\tau}\phi(d_1)\frac{\tilde{\sigma_2} - \rho\sigma_1}{\sqrt{\sigma_1^2 + \tilde{\sigma_2}^2 - 2\rho\sigma_1\tilde{\sigma_2}}}\frac{\sigma_2 K}{F_2 + K} \end{cases} \tag{3.10}$$

  where $\tilde{\sigma_2} = \frac{\sigma_1 F_2}{F_2 + K}$, $\tilde{F_1} = \frac{F_1}{F_2 + K}$

- **Replication Parameters** In the previous subsection, we set up the generic model-independent calendar spread option replication procedures. Here, we apply Black model into the framework.

  1. $K \neq 0$ **Case** According to formula A.16, we plug historical volatilities of future 1 and future 2 into σ_1 and σ_2 so that the parameter needed to be optimized is ρ.

     – **Breaking Good** ρ

$$\rho^{BG} = argmin\ (R_T - P_T)^2$$

– **Hedging Good** ρ

$$\rho^{HG} = argmin \sum_t (R_t - P_t)^2$$

2. $K = 0$ **Case** $K = 0$ is a special case. σ in formula A.5 is fixed since $\sigma_{eff} = \sigma_2$. Denote the special fixed σ as σ_S. Besides, Delta formulae become much simpler,

$$\begin{cases} \Delta_{F_1} = e^{-r\tau}\Phi(d_1) \\ \Delta_{F_2} = -e^{-r\tau}\Phi(d_2) \end{cases} \tag{3.11}$$

where, $d_1 = \dfrac{ln(\dfrac{F_1}{F_2+K}) + \dfrac{\sigma_S^2}{2}\tau}{\sigma\sqrt{\tau}}$, $d_2 = \dfrac{ln(\dfrac{F_1}{F_2+K}) - \dfrac{\sigma_S^2}{2}\tau}{\sigma\sqrt{\tau}}$.

Under this circumstance, we can optimize σ_S directly and then back up correlation. We did the simulation on this case and detailed information will be displayed in later sections,

– **Breaking Good** $_S$

$$\sigma_S = argmin\ (R_T - P_T)^2$$

– **Hedging Good** σ_S

$$\sigma_S = argmin \sum_t (R_t - P_t)^2$$

3.3.2 Bachelier Model

- **Pricing Formula** Generic Bachelier model pricing formula is complicated. For simplicity, we discuss the case of $r = 0$ here. In this case, future prices equate spot prices $F = S$. Therefore, Bachelier pricing formula becomes,

$$\begin{aligned} C_0^B &= E[(F_T^B - K)^+] \\ &= \int_{K-F_0}^{\infty} (F_0 + x - K)\frac{1}{\sigma_B\sqrt{2\pi T}} exp(-\frac{x^2}{2(\sigma_B)^2 T})dx \\ &= (F_0 - K)\Phi(\frac{F_0 - K}{\sigma_B\sqrt{T}}) + \sigma_B\sqrt{T}\phi(\frac{F_0 - K}{\sigma_B\sqrt{T}}) \end{aligned} \tag{3.12}$$

- **Greek: Delta**

$$\begin{aligned} \Delta = \frac{\partial C}{\partial F} &= \Phi(\frac{F_0 - K}{\sigma_B\sqrt{T}}) + \frac{F - K}{\sigma_B\sqrt{T}}\phi(\frac{F_0 - K}{\sigma_B\sqrt{T}}) + \frac{\sigma_B\sqrt{T}}{\sigma_B\sqrt{T}}\phi(\frac{F_0 - K}{\sigma_B\sqrt{T}})(-\frac{F_0 - K}{\sigma_B\sqrt{T}}) \\ &= \Phi(\frac{F_0 - K}{\sigma_B\sqrt{T}}) \end{aligned} \tag{3.13}$$

- **Replication Parameters** We model price spread with Bachelier model and the "underlying" is actually spread. In order to "trade" spread, the Δ_1 and Δ_2 have the exactly same absolute but opposite sign. The parameter needed to be optimized is σ_B. It is embedded in pricing and delta formulae.

$$\begin{cases} \Delta_{F_1} = \Phi(\dfrac{F_0 - K}{\sigma_B \sqrt{T}}) \\ \Delta_{F_2} = -\Phi(\dfrac{F_0 - K}{\sigma_B \sqrt{T}}) \end{cases} \tag{3.14}$$

  - **Breaking Good σ_B (Daily)**

$$\sigma_B^{BG} = argmin \ (R_T - P_T)^2$$

  - **Hedging Good σ_B (Daily)**

$$\sigma_B^{HG} = argmin \ \sum_t (R_t - P_t)^2$$

Chapter 4

Simulation

It is wise to test and verify the theory on the simulated data before applying it to real-world data. We did the same thing for spread option replication breaking good method and also the comparison of breaking good and hedging good methods under the same simulated future price series. This chapter demonstrates the simulation steps and visualizes the results.

4.1 Simulation Results for Breaking Good Method

Expectation We did simulation with correlation of 0.1, 0.5, 0.9, 0.95 and 0.99. For each case, if the methodology is practical, the optimized parameters would distribute among the "real" values.

4.1.1 Set up for Case $K = 0$

We did the simulation on the case of $K = 0$. As mentioned before, it is a very special case that σ_S does not change throughout the whole replication period so that we can optimize σ_S at first and then back up the correlation using historical volatility σ_1 and σ_2. The payoff of the spread option is,

$$max(F_1(T) - F_2(T) - K, 0) \tag{4.1}$$

Since we assume $K = 0$, the spread option value becomes an exchange option, whose value can be calculated exactly.

$$C = F_1\mathcal{N}(d_1) - F_2\mathcal{N}(d_2), d_1 = \frac{ln(\frac{F_1}{F_2}) + \frac{\sigma_S^2 T}{2}}{\sigma_S\sqrt{T}}, d_2 = d_1 - \sigma_S\sqrt{T} \tag{4.2}$$

It employs spread volatility σ_S, given by

$$\sigma_S = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{4.3}$$

4.1.2 Path Generation

1. **Assumptions** Black model assumes that the underlying assets follow log normal distribution. Given initial prices F_0^i, $i = 1, 2$, volatility σ_1, σ_2, correlation ρ, set of independent standard normal random variables x_k, y_k, $k = 1, ..N$. We generate paths using following formulae,

$$F_k^1 = F_{k-1}^1 exp(-\frac{1}{2}\sigma_1^2(t_k - t_{k-1}) + \sigma_1\sqrt{t_k - t_{k-1}}x_k), k = 1, 2, ...N \tag{4.4}$$

$$F_k^2 = F_{k-1}^2 exp(-\frac{1}{2}\sigma_2^2(t_k - t_{k-1}) + \sigma_2\sqrt{t_k - t_{k-1}}[\rho x_k + \sqrt{1-\rho^2}y_k]), k = 1, 2, ...N \tag{4.5}$$

2. **Parameters** The table below shows the parameters we used in simulation. As for the correlation ρ, we simulated five cases: 0.1, 0.5, 0.9, 0.95 and 0.99.

Table 4.1: Parameters for Future Price Paths Simulation

F_0^1	100
F_0^2	102
σ_1	0.55
σ_2	0.60

4.1.3 Replication and Optimization

We demonstrated the very detailed steps of spread option replication as well as the methods to optimize the breaking good parameters in Chapter 2.

1. **Optimization** We did the simulation with the "breaking good" method. A brief recap: R_T is option value at maturity T and P_T is the replication portfolio value at maturity T. The cost function (objective function) is shown as follow,

$$cost\ function = (R_T - P_T)^2$$

and the breaking good σ_S (in this case) is,

$$\sigma_S^{BG} = argmin\ (R_T - P_T)^2$$

2. **Evaluation** It is time consuming to run the optimization many times especially when the frequency of adjusting positions is quite high. Therefore, we repeated the process of simulation and optimization 10000 times so that it reduces much noise, in the meanwhile, it does not take a remarkable amount of time to run the 1H case. We take **expectations** and **standard deviation** of the optimized parameters. For each correlation case, we compare the expectation with "real" parameters.

4.1.4 Results

Below is the parameter list we applied into the Calendar Spread Option pricing and hedging. To clarify, 1H, 12H, 1D, 3D, 5D and 10D represent the frequency (position adjustment) of 1 hour, 12 hours, 1 day, 3 days, 5 days and 10 days, respectively. We present the results for 5 cases: $\rho = 0.1$, $\rho = 0.5$, $\rho = 0.9$, $\rho = 0.95$ and $\rho = 0.99$ below. **The tables** show σ_S^{BG} expectation and the corresponding standard deviation. $|\sigma_S^{BG} - \sigma_S^{real}|$ is the absolute number of difference between σ_S^{BG} and σ_S^{real}. **The graphs** display the distribution of parameter σ_S^{BG} and **Red Dashed Lines** label the true values.

Table 4.2: Calendar Spread Option Parameters

F_0^1	100
F_0^2	102
K	0
σ_1	0.55
σ_2	0.60
r	0.0
τ	1.0

1. **Result of $\rho = 0.1$ (Low Correlation)** Real σ_S is 0.77233.

Table 4.3: Results with $\rho = 0.1$

Replication Frequency	σ_S^{BG} **mean**	σ_S^{BG} **std**	$\lvert\sigma_S^{BG} - \sigma_S^{real}\rvert$
1H (1 hour)	0.77224	0.01287	0.00009
12H (12 hours)	0.77157	0.03206	0.00076
1D (1 day)	0.77196	0.03930	0.00037
3D (3 days)	0.77155	0.06466	0.00078
5D (5 days)	0.77404	0.07976	0.00171
10D (10 days)	0.77452	0.10922	0.00219

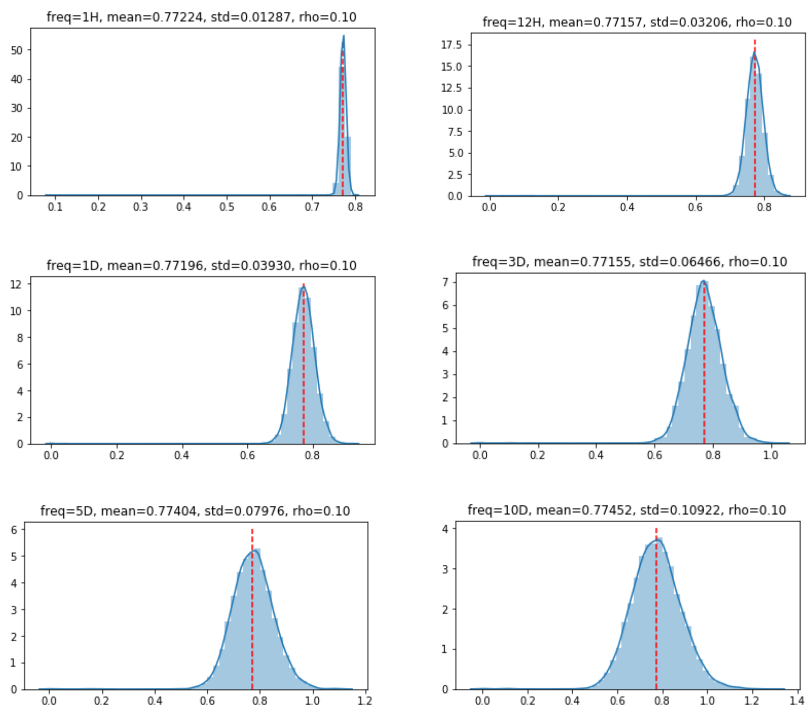

Figure 4.1: Graphs of σ Error for the case $\rho = 0.1$

2. **Result of** $\rho = 0.5$ **(Median Correlation)** Real σ_S is 0.57663.

Table 4.4: Results with $\rho = 0.5$

Replication Frequency	σ_S^{BG} **mean**	σ_S^{BG} **std**	$\lvert\sigma_S^{BG} - \sigma_S^{real}\rvert$
1H (1 hour)	0.57666	0.00522	0.00003
12H (12 hours)	0.57678	0.01846	0.00015
1D (1 day)	0.57658	0.02620	0.00005
3D (3 days)	0.57807	0.04479	0.00144
5D (5 days)	0.57726	0.05753	0.00063
10D (10 days)	0.57864	0.08014	0.00201

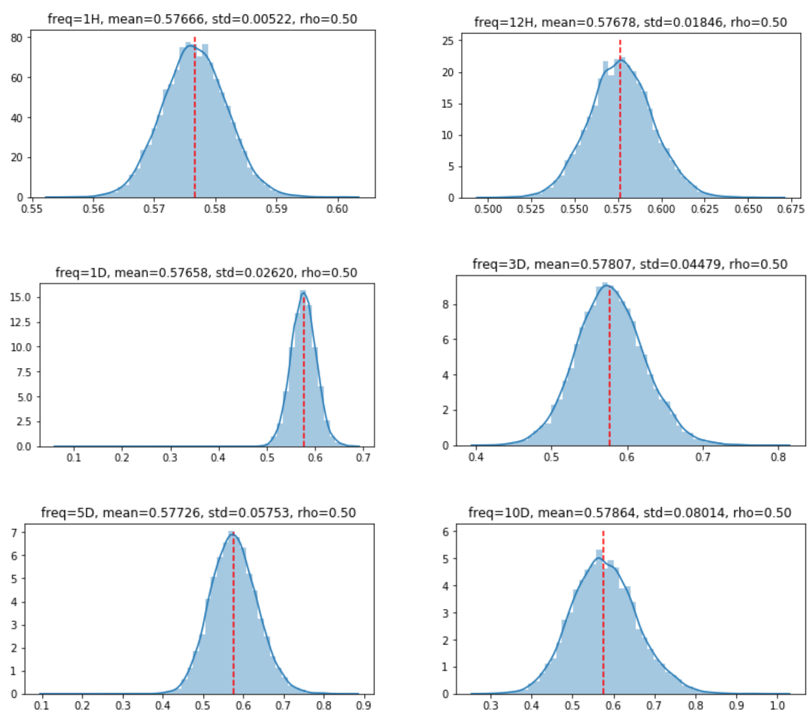

Figure 4.2: Graphs of σ Error for the case $\rho = 0.5$

3. **Result of** $\rho = 0.9$ **(High Correlation)** Real σ_S is 0.26173.

Table 4.5: Results with $\rho = 0.9$

Replication Frequency	σ_S^{BG} **mean**	σ_S^{BG} **std**	$\lvert\sigma_S^{BG} - \sigma_S^{real}\rvert$
1H (1 hour)	0.26173	0.00241	$5.79e^{-6}$
12H (12 hours)	0.26173	0.00819	$7.97e^{-6}$
1D (1 day)	0.26196	0.01156	0.00024
3D (3 days)	0.26169	0.01996	$3.09e^{-5}$
5D (5 days)	0.26218	0.02548	0.00045
10D (10 days)	0.26260	0.03648	0.00088

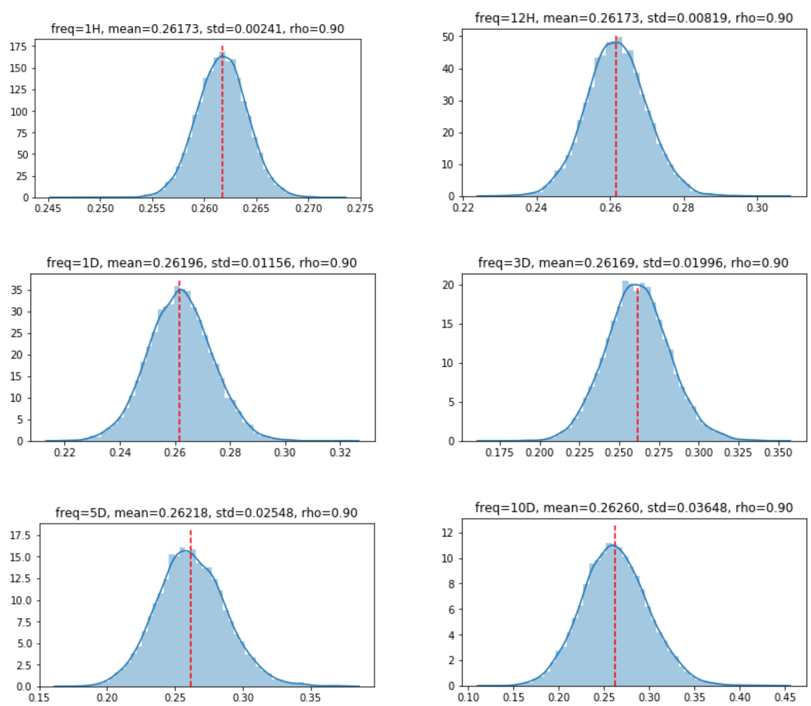

Figure 4.3: Graphs of σ Error for the case $\rho = 0.9$

4. **Result of** $\rho = 0.95$ **(High Correlation)** Real σ_S is 0.18841.

Table 4.6: Results with $\rho = 0.95$

Replication Frequency	σ_S^{BG} **mean**	σ_S^{BG} **std**	$\lvert\sigma_S^{BG} - \sigma_S^{real}\rvert$
1H (1 hour)	0.18844	0.00173	$2.66e^{-5}$
12H (12 hours)	0.18852	0.00593	0.00011
1D (1 day)	0.18853	0.00843	0.00011
3D (3 days)	0.18841	0.01462	$9.12e^{-6}$
5D (5 days)	0.18855	0.01855	0.00013
10D (10 days)	0.18949	0.02678	0.00107

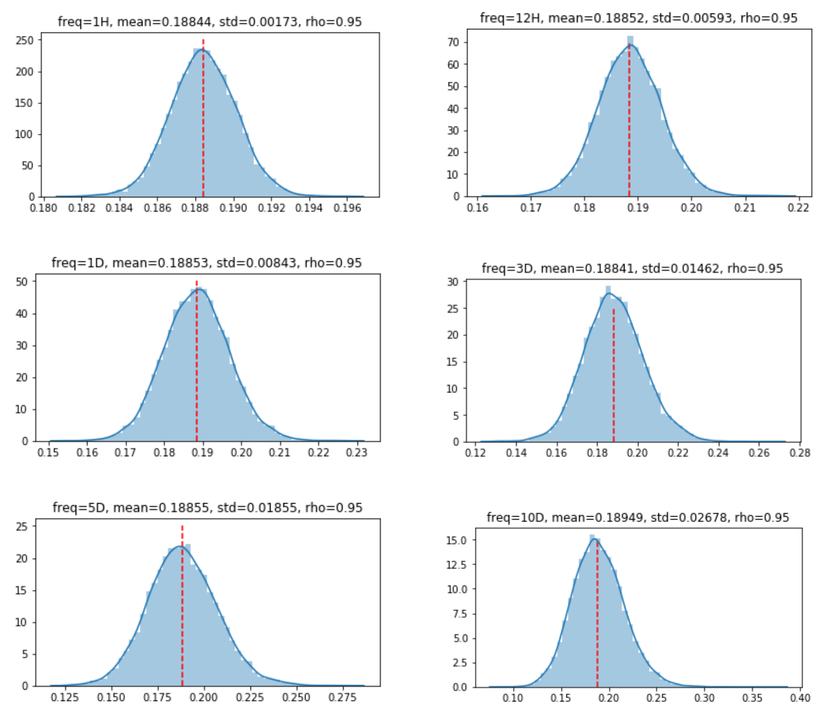

Figure 4.4: Graphs of σ Error for the case $\rho = 0.95$

5. **Result of** $\rho = 0.99$ **(High Correlation)** Real σ_S is 0.09539.

Table 4.7: Results with $\rho = 0.99$

Replication Frequency	σ_S^{BG} **mean**	σ_S^{BG} **std**	$\lvert\sigma_S^{BG} - \sigma_S^{real}\rvert$
1H (1 hour)	0.09539	0.00087	$3.05e^{-7}$
12H (12 hours)	0.09540	0.00299	$1.86e^{-6}$
1D (1 day)	0.09541	0.00420	$1.14e^{-5}$
3D (3 days)	0.09552	0.00734	0.00012
5D (5 days)	0.09556	0.00936	0.00017
10D (10 days)	0.09563	0.01332	0.00024

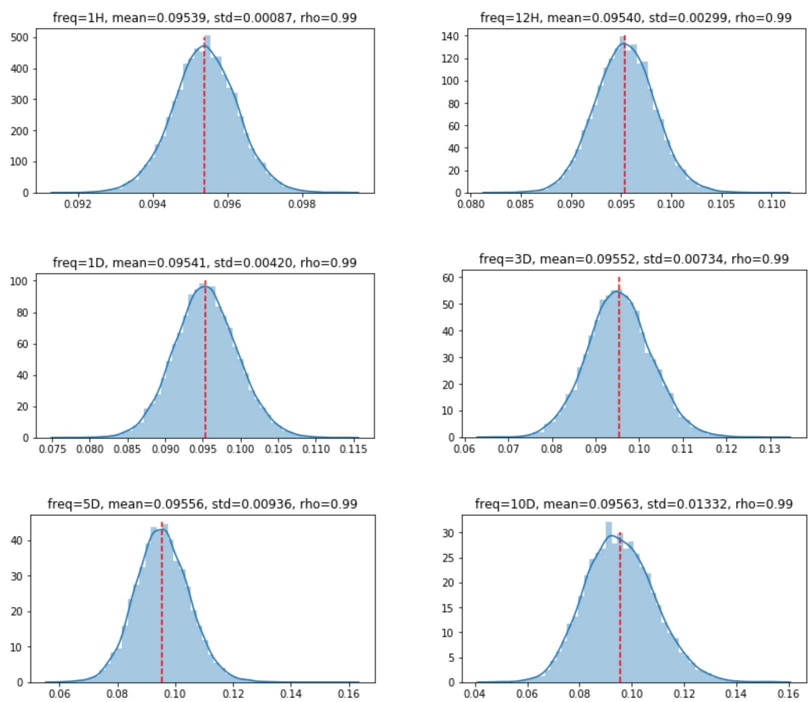

Figure 4.5: Graphs of σ Error for the case $\rho = 0.99$

4.1.5 Analysis for Simulation Results

According to the results of all cases, whether the correlation is low, medium, or high, we can observe that the simulated "optimal" parameters are distributed around the real value (red dashed line). The expectation converges to the true value with decreasing standard deviation as the frequency of position change is higher. Therefore, we can conclude that breaking good parameters converge to the "real" values as long as no transaction costs are involved.

4.2 Breaking Good and Hedging Good Simulation

Recap of Breaking Good vs Hedging Good The definitions of breaking good and hedging good are illustrated in section **??** and **??**. The reason that we define hedging good other than breaking good is that we observe some interesting facts. When there is a huge spike or plunge in the spread value, the breaking good parameter tends to replicate the options "purposely" in the wrong direction and then make up for the replication error with the "expected" jump. In this case, replication error would be increase steadily before the jump comes and then dramatically plunge after the jump. However, when our goal is to hedge the option or replicate the spread options in a more "honest" way, it is natural to think of the cost function that minimizes the replication errors along the way. We name the method "Hedging Good" since the underlying portfolio value is designed to be close to option value all the time instead of only caring about the single replication error at expiration.

Expectation In the simulation world, future prices follow the log-normal distribution. Even we simulate daily price data, a dramatic jump is rare or never happens. Therefore, breaking good method and hedging good method are supposed to have similar performance.

Analysis for the Results The parameters of simulation and results are shown below. The distributions of Breaking Good parameters and Hedging Good parameters are similar under the "ideal" assumption when future prices follow the log normal distribution and there is no obvious jump.

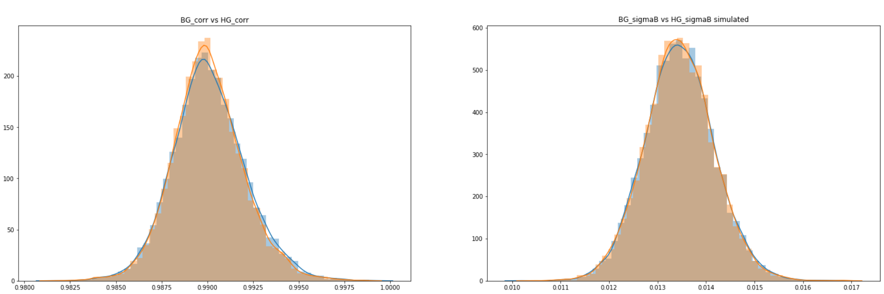

Figure 4.6: NG MAR19&APR19 BG vs HG Replication

Chapter 5

Results: Implied Correlation

Theory and methodology of implied correlation calibration and implied correlation modeling are illustrated in Chapter 2. This chapter focuses on the display of results for real-world data. Before presenting the results, we will provide some brief and straightforward explanation for the terminology; for more details, readers can refer to Chapter 2 accordingly.

5.1 Implied Correlation Calibration Results

As we mentioned before, the basic model for calibration is Bachelier model [2]. The only unknown parameter in Formula **??** is ρ so that we can back up implied correlation by minimizing the difference between observed data and model data. The brief process is shown as below. For more detailed information, readers can go to section 2.1.2.

- Collect future contracts data from CME group and Barchart, including F_1, F_2, σ_1, σ_2, etc.
- Calibrate ρ according to Bachelier formula through minimizing the difference between model value and observed value.
- Drop the unreasonable implied correlations (greated than 1.0). For each month, we aggregate the implied correlations with the weight of **open interests**.

We calibrated implied correlations weekly from Jan 10, 2020 until present except for the unfortunately missing data in March. In this section, we only provide one example reason. The rest of results will be submitted along with the report.

Table 5.1: Calibrated Implied Correlations (Partially) for Each Call Option on 0626

	S1	S2	K	r	tau	sigma_1	sigma_2	Expire_date	Month	Month_diff	Price	Prior_interest	ImCorr	ImVol	ImCorr2	ImCorr3	ImCorr4	Agg_ImCorr
0	38.49	38.65	-3	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	2.84	0.01		1.26E-12	1.000022		1.00251	0.996273
1	38.49	38.65	-1.5	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	1.35	0.01	0.99232	0.075938	0.99232	0.994412	0.994412	0.996273
2	38.49	38.65	-1	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.86	1500	0.9959	0.060065	0.9959	0.997376	0.997376	0.996273
3	38.49	38.65	-0.5	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.38	550	0.999599	0.039228	0.999599		1.000291	0.996273
4	38.49	38.65	-0.3	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.18	100		0.02388	1.001356		1.001683	0.996273
5	38.49	38.65	-0.25	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.14	4000		0.022313	1.001549		1.001787	0.996273
6	38.49	38.65	-0.2	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.11	1000		0.022505	1.001624		1.001774	0.996273
7	38.49	38.65	-0.1	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.06	1000		0.022006	1.001838		1.001804	0.996273
8	38.49	38.65	0	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.03	4000		0.022242	1.002009		1.001787	0.996273
9	38.49	38.65	0.1	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.02	1500		0.025551	1.001966		1.001553	0.996273
10	38.49	38.65	0.25	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.02	5650		0.034862	1.001421		1.000723	0.996273
11	38.49	38.65	0.5	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.01	2400		0.04181	1.001141	0.999923	0.999923	0.996273
12	38.49	38.65	0.75	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.01	2550	0.999949	0.053828	0.999949	0.998195	0.998195	0.996273
13	38.49	38.65	1	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.01	3000	0.998435	0.065318	0.998435	0.996116	0.996116	0.996273
14	38.49	38.65	1.5	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.01	2250	0.994511	0.087179	0.994511	0.990975	0.990975	0.996273
15	38.49	38.65	2	0	0.065753	0.6005	0.5576	2020-07-20 00:00:00	AUG20	1	0.01	250	0.98947	0.107943	0.98947	0.984603	0.984603	0.996273
16	38.65	38.8	-1	0	0.147945	0.5576	0.5366	2020-08-19 00:00:00	SEP20	1	0.88	1500	0.996842	0.044565	0.996842	0.997535	0.997535	0.998663
17	38.65	38.8	-0.75	0	0.147945	0.5576	0.5366	2020-08-19 00:00:00	SEP20	1	0.64	100	0.99784	0.037769	0.99784	0.998392	0.998392	0.998663
18	38.65	38.8	-0.5	0	0.147945	0.5576	0.5366	2020-08-19 00:00:00	SEP20	1	0.4	500	0.99894	0.028955	0.99894	0.999305	0.999305	0.998663
19	38.65	38.8	-0.3	0	0.147945	0.5576	0.5366	2020-08-19 00:00:00	SEP20	1	0.23	0.01	0.999506	0.024096	0.999506	0.999707	0.999707	0.998663
20	38.65	38.8	-0.25	0	0.147945	0.5576	0.5366	2020-08-19 00:00:00	SEP20	1	0.19	3000	0.999658	0.022642	0.999658	0.999814	0.999814	0.998663
21	38.65	38.8	0	0	0.147945	0.5576	0.5366	2020-08-19 00:00:00	SEP20	1	0.06	4400		0.020267	1.000051	0.999972	0.999972	0.998663
22	38.65	38.8	0.1	0	0.147945	0.5576	0.5366	2020-08-19 00:00:00	SEP20	1	0.04	1000		0.021512	1.000064	0.999886	0.999886	0.998663
23	38.65	38.8	0.25	0	0.147945	0.5576	0.5366	2020-08-19 00:00:00	SEP20	1	0.03	4650	0.999882	0.025832	0.999882	0.999555	0.999555	0.998663

Table 5.2: Prior Open Interest Weighted Implied Corr 0626 (Call Options)

	Expire_date	tau	Month_diff	Agg_ImCorr
0	2020-07-20 00:00:00	0.065753425	1	0.996273425
1	2020-08-19 00:00:00	0.147945205	1	0.998662613
2	2020-09-21 00:00:00	0.238356164	1	0.998841748
3	2020-10-19 00:00:00	0.315068493	1	0.998548167
4	2020-11-19 00:00:00	0.4	1	0.998679106
5	2020-12-18 00:00:00	0.479452055	1	0.99878058
6	2021-01-19 00:00:00	0.567123288	1	0.998930548
7	2021-02-19 00:00:00	0.652054795	1	0.99909769
8	2021-03-19 00:00:00	0.728767123	1	0.998875419
9	2021-04-19 00:00:00	0.81369863	1	
10	2021-05-19 00:00:00	0.895890411	1	0.999216375
11	2021-06-21 00:00:00	0.98630137	1	0.999264784
12	2021-07-19 00:00:00	1.063013699	1	0.998967665
13	2021-08-19 00:00:00	1.147945205	1	
14	2021-09-20 00:00:00	1.235616438	1	0.998579639
15	2021-10-19 00:00:00	1.315068493	1	0.999719962
16	2021-11-18 00:00:00	1.397260274	1	0.998742966
17	2021-12-17 00:00:00	1.476712329	1	
18	2022-01-19 00:00:00	1.567123288	1	0.998129487

5.2 Implied Correlation Two-Parameter Modeling Results

Detailed theory and methodology have already been presented in Chapter 2, section 2. In this section, we first give a brief explanation of the steps of processing the real-world data and then show the results of each parameter with the same order of the calibration steps in Theory and Methodology part.

5.2.1 Data Processing

Create A Calendar Spread Option Table

Firstly, we create a table with 6 columns: t_1, t_2, T_i, T_j, corr, weight. In the data processing, we use "Daily" convention since expiration of calendar spread option can be one day or three days before of earlier future contract (sometimes there is a weekend). Due to the discrepancy, we calibrate the parameters on daily basis in order to achieve more stable parameters. The explanation of each column is provided below:

- t_1: "today"
- t_2: Expiration date of Calendar Spread Option
- T_i: Expiration of earlier future contract
- T_j: Expiration of later future contract ($T_i < T_j$)
- corr: Implied Correlation we calculated weekly (aggregate correlation)
- weight: Prior interest weighted
    1. call option weight = call option prior interest / all prior interest
    2. put option weight = put option prior interest / all prior interest

To be more specific, we use JLY20 contract, the "first" contract on May 22, 2020 as an example to explain what we did in the first step. We assume "today" to be May 22, 2020. Obviously, t_1 is "today", May 22, 2020. The expiration of JLY20 1 month calendar spread option is Jun 19, 2020, which is represented as t_2. The option has two underlyings: WTI JLY20 future contract with expiration date of Jun 22, 2020, and AUG20 future contract with expiration date of July 21, 2020. Therefore, T_i is Jun 22, 2020 and T_j is July 21, 2020. Total prior interests of call and put options for JLY20 contracts are 30200.09 and 48555 respectively, thus the weights for call and put options are 0.3835 and 0.6165, respectively. As for corr, aggregated implied correlation is 0.995953 for call options and is 0.988668 for put options.

First, we drop the rows where there is missing data. If implied correlation for a call option is missing, put option implied correlation becomes the final aggregated correlation; vice versa.

5.2.2 Contour Plot Method

The reasonable number of Samuelson parameter B 0.001 daily was from my colleagues' work and then we verified it on the contour plot. It works for almost all the cases. They calibrated the parameter B on the historical volatility, which is backward looking. When applying pricing formulae, we should use implied parameters, which are forward looking. Regardless of the fact, B = 0.001 works well in calibration.

Plotting all the results, we observe that $B = 0.001$ daily (0.36 annually) basically works all the time.

5.2.3 Determination of $Corr_0$

We described three methods in the previous section : fixing $Corr_0$, parameterizing $Corr_0$ and a novel method to determine $Corr_0$. Given the fact that the "first" implied correlation is usually low, it introduces additional challenge to determining $Corr_0$. Under this circumstance, we tried to apply a new method to determine $Corr_0$ in order to give a more reasonable level of decay parameter, thus leading to a realistic growth parameter.

Method Description Firstly, we find the first implied correlation that is greater than 0.99, denoted as $Corr$. Then, we extrapolate $Corr_0$ according to the reference table below. $Corr_0$ is $Corr$ minus "1 standard deviation". We only list partial reference table. For the complete table, readers can go to Appendix A.2.1.

Table 5.3: Flexible C_0 Calibration Results with Different B

$Corr$	$Corr_0$
0.9925	0.990301
0.99275	0.990624
0.993	0.990947
0.99325	0.99127
0.9935	0.991593
......	......
0.99925	0.999029
0.9995	0.999353
0.99975	0.999676

Results According to the numbers in the reference table, we are able to extrapolate Corr0 from Corr around 0.99. We present a table with all the results in the following section and avoid redundant lengthy list of the results of $Corr_0$ here.

5.2.4 Calibration of Growth Parameter α

Once we have determined the decay parameter $Corr_0$ and the Samuelson parameter B, the only parameter to be determined is the growth parameter α.

We calibrate α by minimizing the difference between the model and real values.

$$\hat{\alpha} =_{\alpha} \sum W \cdot (C^{est}(t_1, t_2, T_i, T_j) - C^{real}(t_1, t_2, T_i, T_j))^2 \tag{5.1}$$

Explanation According to the results, we observe that 0.02 is the typical α parameter. Fixing B to be 0.001 and α to be 0.02, we check the root mean squared error (RMSE) and find it reasonable. $\alpha = 0.02(daily)$ is not the "best" parameter to capture the growth of implied correlation. Given some level of error tolerance, it fits the implied correlations reasonably well.

All results will be presented in the following section.

5.2.5 General Results

In this paper, we only present the results by the end of July and we will keep tracking the further results in the future.

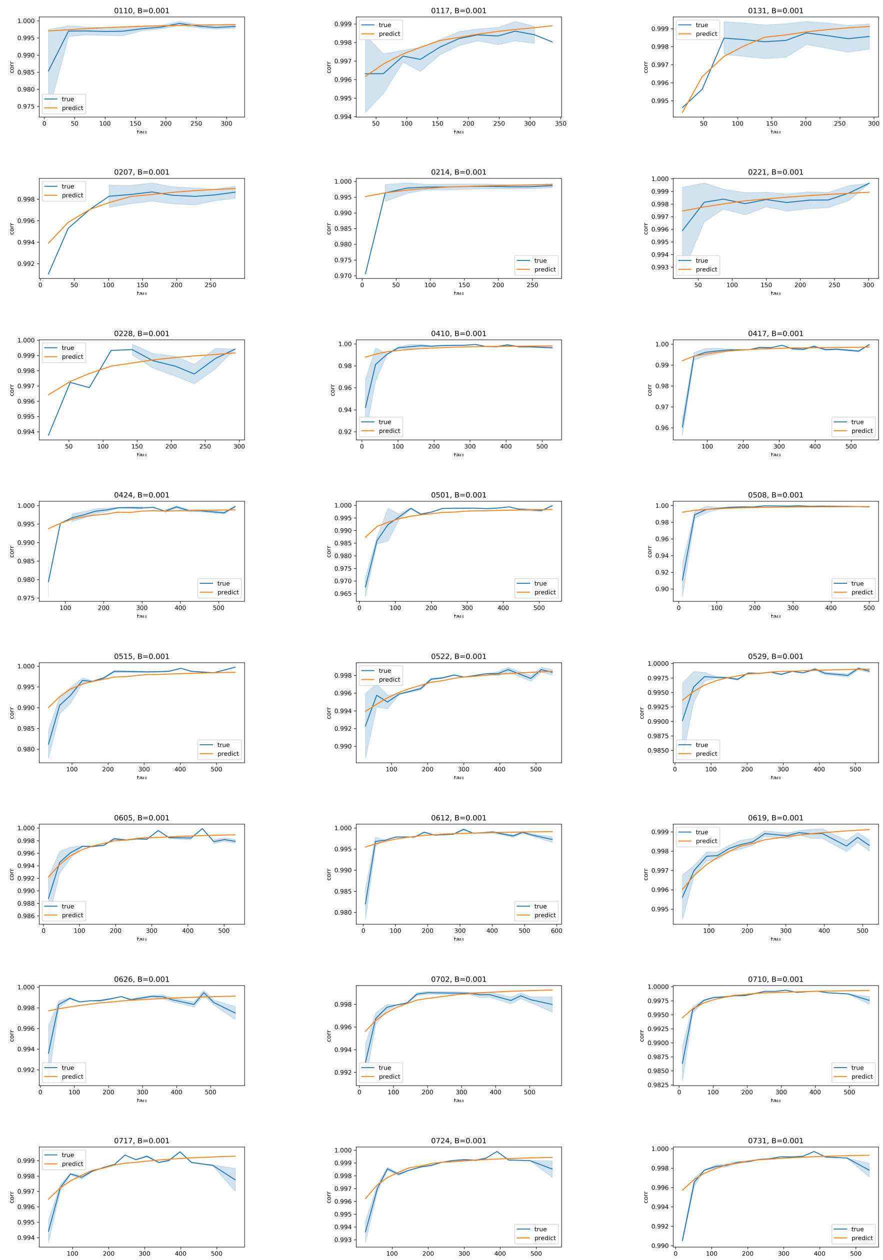

Figure 5.1: Ensure Corr0 before standard deviation adjustment greater than 0.99

Table 5.4: Flexible C_0 Calibration Results with Different B

Date	α	$Corr_0$	RMSE
0110	0.010330106	0.997052932	0.003909534
0117	0.015611892	0.996170173	0.000807524
0131	0.038730392	0.994359325	0.000824078
0207	0.031309747	0.993909462	0.001150371
0214	0.021401798	0.9951813	0.007860504
0221	0.009378668	0.997447396	0.001447103
0228	0.02052573	0.996420448	0.001124754
0410	0.019161257	0.987879823	0.010945633
0417	0.021618307	0.992117443	0.008057516
0424	0.066779498	0.993764587	0.003987034
0501	0.026542028	0.987346604	0.005424604
0508	0.020665706	0.99202746	0.020888087
0515	0.03533659	0.990017383	0.002994072
0522	0.012330545	0.993938379	0.001174073
0529	0.024992654	0.993671428	0.002115081
0605	0.025988913	0.992202774	0.001459222
0612	0.0146184	0.995492141	0.00353338
0619	0.015505997	0.996019701	0.000527733
0626	0.006281192	0.997709885	0.001494552
0702	0.018734722	0.995623829	0.001097348
0710	0.025825881	0.9944792	0.002362358
0717	0.017399668	0.996494527	0.000783552
0724	0.025157972	0.996220858	0.000825044
0731	0.021673786	0.995724835	0.001369803

Table 5.5: Fix $B = 0.001$ and $\alpha = 0.02$

Date	α	$Corr_0$	RMSE
0110	0.02	0.997052932	0.00393522
0117	0.02	0.996170173	0.000828847
0131	0.02	0.994359325	0.00107185
0207	0.02	0.993909462	0.00128048
0214	0.02	0.9951813	0.00786083
0221	0.02	0.997447396	0.00150839
0228	0.02	0.996420448	0.00112492
0410	0.02	0.987879823	0.0109463
0417	0.02	0.992117443	0.00805818
0424	0.02	0.993764587	0.00406742
0501	0.02	0.987346604	0.00547337
0508	0.02	0.99202746	0.0208882
0515	0.02	0.990017383	0.00311352
0522	0.02	0.993938379	0.00130203
0529	0.02	0.993671428	0.00212687
0605	0.02	0.992202774	0.00150011
0612	0.02	0.995492141	0.00353274
0619	0.02	0.996019701	0.00055753
0626	0.02	0.997709885	0.00157263
0702	0.02	0.995623829	0.00109849
0710	0.02	0.9944792	0.00237626
0717	0.02	0.996494527	0.00078815
0724	0.02	0.996220858	0.00083799
0731	0.02	0.995724835	0.00137126

Chapter 6

Results: Spread Option Replication

We aim to compare replication results according to two different models: Black model and Bachelier model. Since our topic is mainly about calendar spread option replication, we treat "Breaking Good" parameters, which is designed to minimize replication errors as our key parts and "Hedging Good" parameters as a supplementary when presenting detailed results.

6.1 Data Selection and Processing

6.1.1 Historical Data Selection

Let F_1 and F_2 denote two futures with expiry T_1 and T_2 ($T_1 < T_2$). Since the volatility of a contract could increase dramatically during the last few days of its trading, which is a well-known phenomenon called Samuelson effect, we choose historical prices stopping **30 days** before the expiry of the earlier contract.

6.1.2 Future Contract Pairs

We use Crude Oil (WTI) and Natural Gas future contracts from JAN19 to MAY20. There are 102 combinations with month difference from 1 to 6. For example, we constructed 6 pairs with JAN19 contract: JAN19&FEB19, JAN19&MAR19, JAN19&APR19, JAN19&MAY19, JAN19&JUN19 and JAN19&JLY19.

6.2 Benchmark: Historical Parameters

It is a naive way that we plug historical (realized) parameters into pricing and hedging models when implied parameters are not available. Therefore, we use historical parameters and their corresponding replication errors as benchmarks to see whether breaking good (hedging good) parameters outperform realized parameters. Below presents a definition of historical parameters of Black and Bachelier model. Since we have a large number of pairs, we will enumerate and compare realized parameters with breaking good parameters in the later sections instead of listing all of them here.

6.2.1 Historical Correlation for Black Model

Historical correlation can be calculated through two log return series. Log returns can be calculated using the formula

$$R_n = \frac{ln(\frac{F_n}{F_{n-1}})}{\sqrt{t_n - t_{n-1}}} \tag{6.1}$$

Once we define the daily log returns, we can calculate historical correlation between two log return series. Annual volatility can be calculated through

$$\sigma = std(R_n) * \sqrt{365} \tag{6.2}$$

With the formula, we can estimate the historical volatility of two future price paths, denoted as σ_1 and σ_2. σ_1 and σ_2 are the volatility parameters we plug into Black model replication.

6.2.2 Historical Spread Volatility for Bachelier Model

As for Bachelier model, we assume the spread of two prices follows normal distribution. Even though we trade two separate futures, the model is basically uni-variant. Spread at t, denoted as $Spread_t$, is defined as $F_{1t} - F_{2t}$.

Time adjusted daily spread change is

$$D_n = \frac{Spread_n - Spread_{n-1}}{\sqrt{t_n - t_{n-1}}} \tag{6.3}$$

Here is a little difference in "volatility" treatment than in Black model. The time convention we used for σ_B is **daily** so that it is not necessary to annualize the spread volatility. Accordingly, the maturity T in Bachelier formula should also be measured with days.

6.3 Bachelier vs Black: Breaking Good

We did the replication both on Crude Oil and Natural Gas contracts. We visualize several typical results (ATM cases) and then we use normalized moneyness table to see which model works for more cases.

As we mentioned, we did replication on a variety of future contract pairs (Crude Oil and Natural Gas contracts) using both Bachelier and Black model. The structure of this section is that we provide several typical results first and then compare the results through **Normalized Moneyness**.

6.3.1 Typical Results

As we mentioned, we did replication on a variety of future contract pairs using both Bachelier and Black model.

1. **Natural Gas Typical Results**

    - **Inter-Storage Season JAN19&APR19** Unlike Crude Oil, natural gas is a seasonal commodity. In the periods of low consumption, April through October, excess supply can be injected into storage. In winter months, from November through March, natural gas would be withdrawn from storage to meet the high demand. Therefore, we provide three typical examples for April is the start of storage season when weather becomes mild and the demand for natural gas decreases. The production, i.e, the supply of natural gas is inelastic so in storage season, people put the extra natural gas into storage. Meanwhile, January is the one of the coldest months among the whole year; people need natural gas for house heating. JAN19&APR19 is a typical example for Inter-Storage season case. We chose the 252 days of historical data stopping 30 trading days before JAN19 contract expires (NOV, 16, 2017 - NOV, 15, 2018). On NOV, 16, 2017, "issuing date", F_1 is 3.274 and F_2 is 2.808; the strike price is set to be 0.466 so that the option is ATM. The graphs below present the replication process in the period as well as the replication error. It is clear that portfolio value is closer to option value all the time when replicating according to Bachelier model. The optimized parameters along with replication errors at expiration are listed in table below. Breaking good correlation is almost on the boundary 1.0 and gives much larger replication error at expiration.

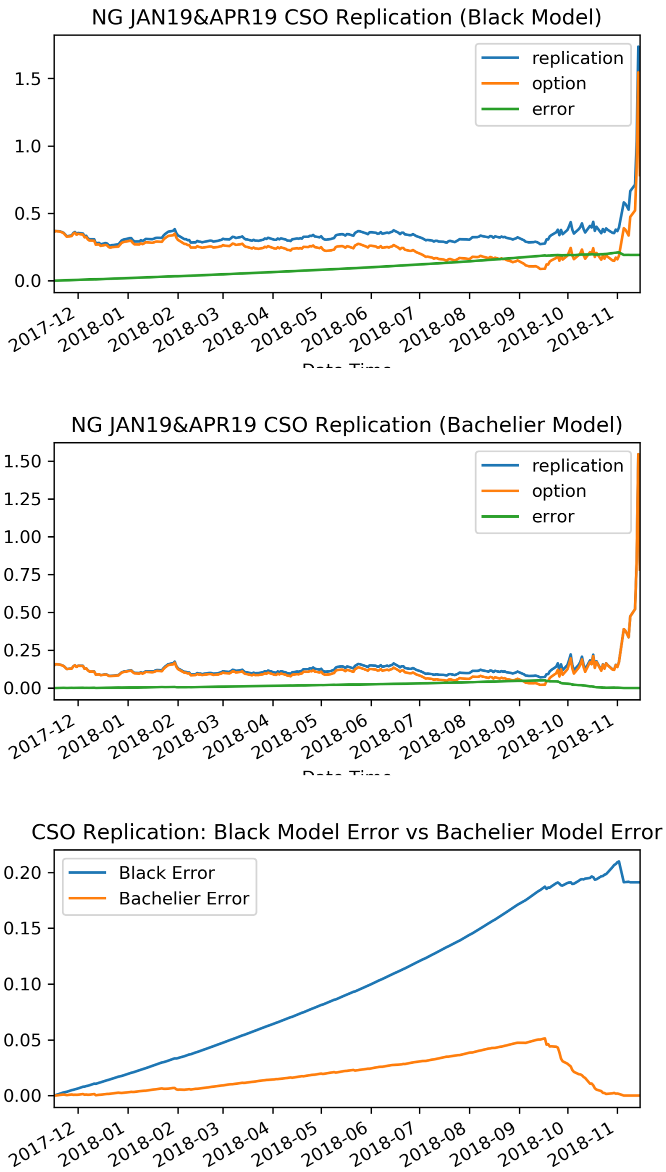

Figure 6.1: Inter-Storage Season

Table 6.1: Natural Gas JAN19&APR19 Pair Results

F_1	3.274
F_2	2.808
K	0.466
Breaking Good Correlation	0.999999985
Replication Error at Expiration (BG Correlation)	0.191065
Breaking Good σ_B	0.019522056
Replication Error at Expiration (BG σ_B)	1.219891E-11
Historical Correlation	0.707311
Replication Error at Expiration (Historical Correlation)	0.237194
Historical Spread Vol	0.072453
Replication Error at Expiration (His spread vol)	0.379249

- **Intra-Storage Season JAN20&MAR20**

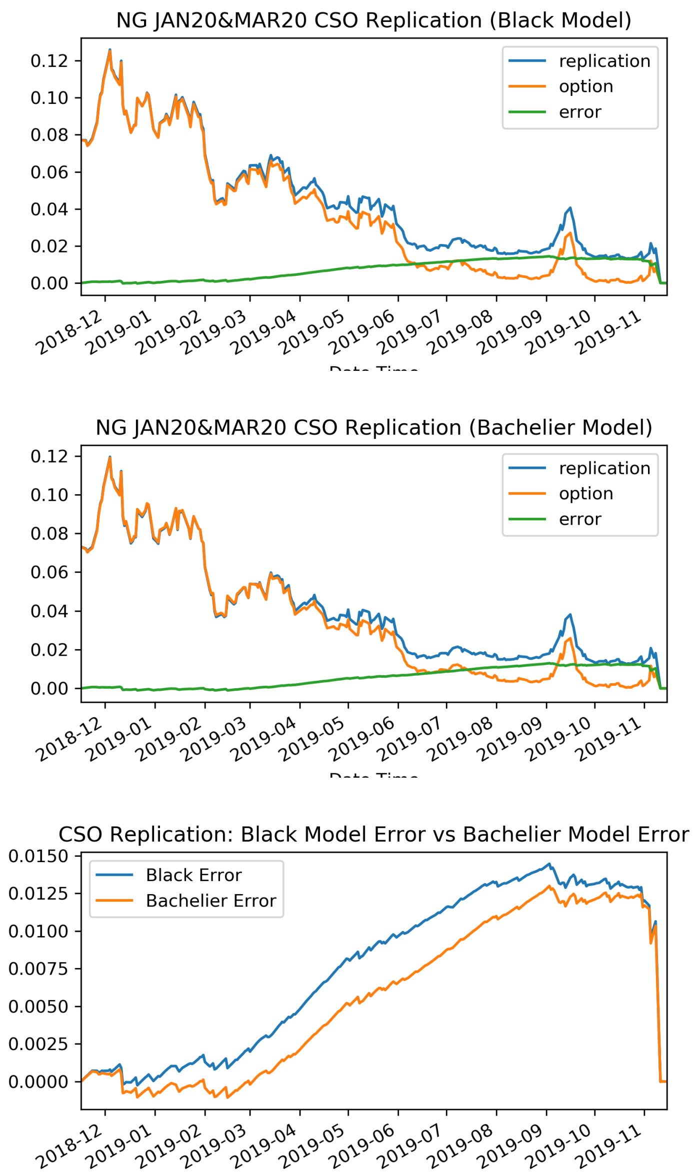

Figure 6.2: Intra Storage Season: JAN20&MAR20

Table 6.2: Natural Gas JAN20&MAR20 Pair Results

F_1	3.015
F_2	2.798
K	0.217
Breaking Good Correlation	0.969741
Replication Error at Expiration (BG Correlation)	4.06904E-09
Breaking Good σ_B	0.009609
Replication Error at Expiration (BG σ_B)	-5.82952E-12
Historical Correlation	0.975568
Replication Error at Expiration (Historical Correlation)	-0.005047
Historical Spread Vol	0.008942
Replication Error at Expiration (His spread vol)	-0.0062

- **Inter-Storage Season AUG19&SEP19**

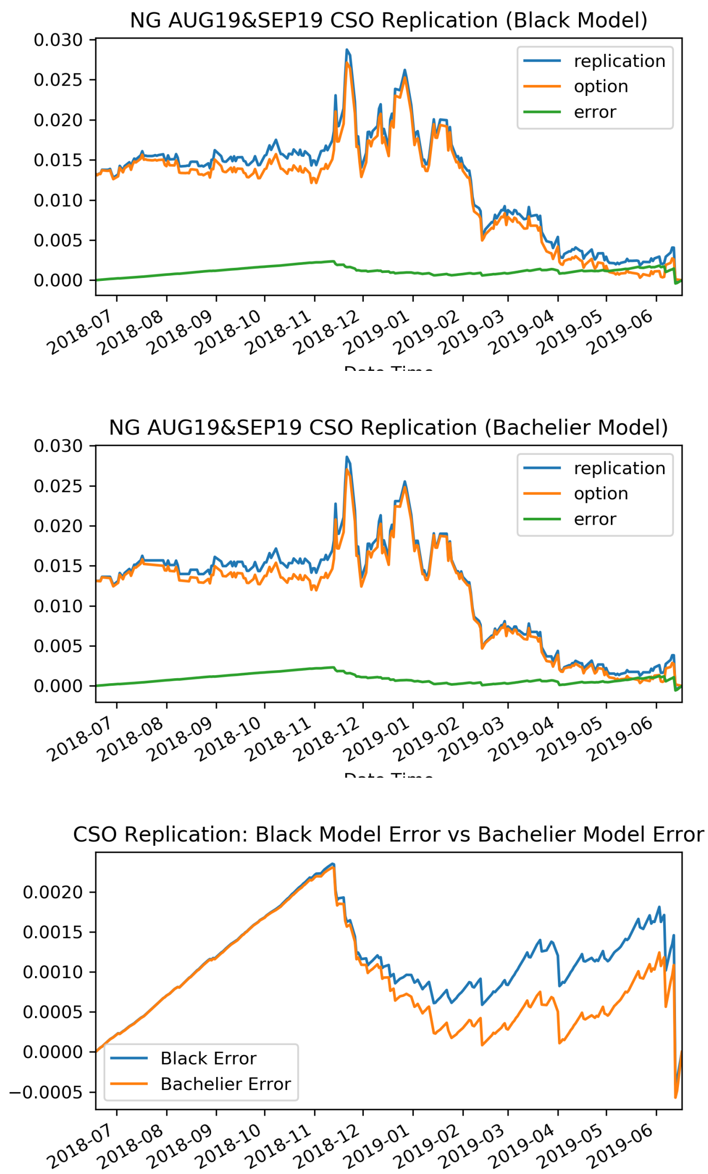

Figure 6.3: Intra Storage Season: AUG19&SEP19

Table 6.3: Natural Gas AUG19&SEP19 Pair Results

F_1	2.701
F_2	2.687
K	0.014
Breaking Good Correlation	0.998275
Replication Error at Expiration (BG Correlation)	-2.75945E-09
Breaking Good σ_B	0.001725
Replication Error at Expiration (BG σ_B)	-2.99464E-11
Historical Correlation	0.998170
Replication Error at Expiration (Historical Correlation)	0.0005713
Historical Spread Vol	0.001809
Replication Error at Expiration (His spread vol)	0.001057

2. **Crude Oil Typical Results** Crude oil commodity is not seasonal. The most calendar spread options traded on the market is one month difference so that we provide a typical example of crude oil one-month calendar spread option (MAR19&APR19) replication result. The option is set to be **At the Money**. According to graphs and the result table, breaking good correlation is 0.999897 and breaking good daily σ_B is 0.024622. The replication error using Bachelier model is dramatically smaller than that of Black model. From the graphs, we can observe that Bachelier model absolute replication error is smaller along the way. We also list the results of "benchmark": historical correlation and historical spread volatility in order to compare. It is obvious that breaking good parameters significantly outperform historical parameters.

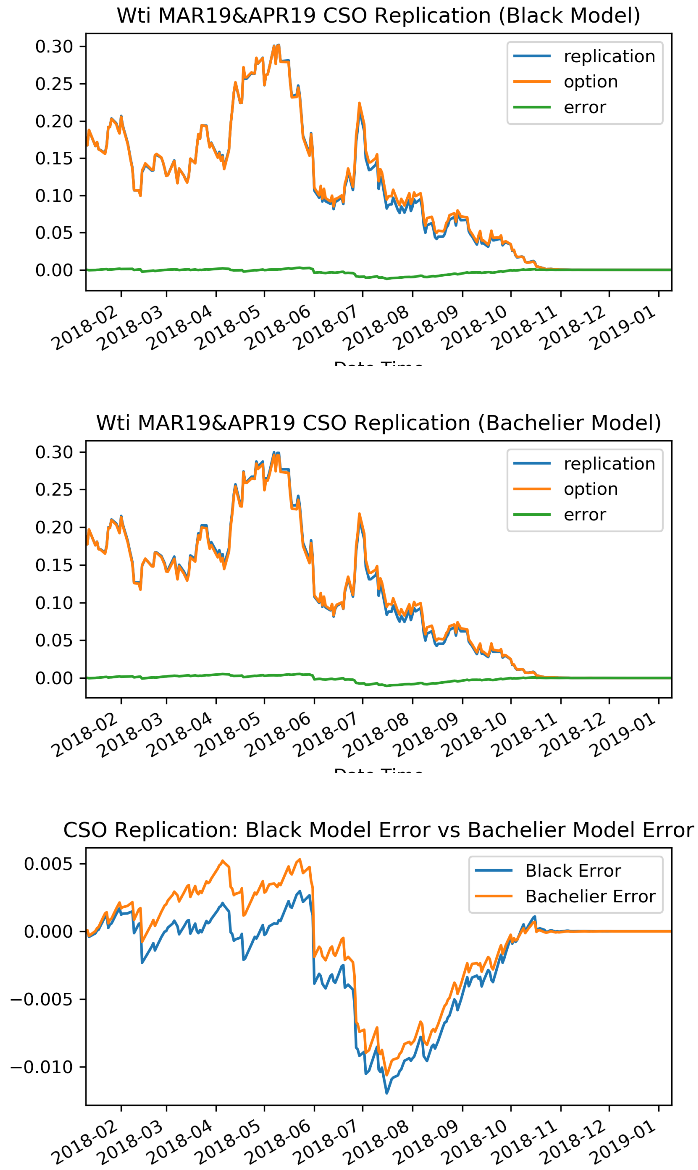

Figure 6.4: Wti: MAR19&APR19

Table 6.4: Crude Oil MAR19&APR19 Pair Results

F_1	59.14
F_2	58.74
K	0.4
Breaking Good Correlation	0.999897
Replication Error at Expiration (BG Correlation)	1.37531E-06
Breaking Good σ_B	0.024622
Replication Error at Expiration (BG σ_B)	6.22928E-11
Historical Correlation	0.999765
Replication Error at Expiration (Historical Correlation)	0.039212
Historical Spread Vol	0.026905
Replication Error at Expiration (His spread vol)	0.0161565

The results above show that Bachelier model mostly replicates better than Black model, leading to smaller replication errors. The third subfigure of each graph shows the replication errors of two models during the period. Absolute replication errors are usually smaller with Bachelier along the way. Besides, breaking good parameters, correlation or σ_B outperform significantly their corresponding historical benchmarks.

6.3.2 Normalized Moneyness

Even though Bachelier model beats from all aspects from the typical results, we cannot jump to a conclusion that it is better to replicate with Bachelier model. We applied the normalized moneyness to compare the results of a large range of strike price cases. In this subsection, we provide the definition of the normalized moneyness and the way we applied it into checking our results.

Options with different strikes and maturities are usually incomparable. In order to "standardized" and compare the options, we calculate normalized moneyness and find out ranges of strikes under which the methods work.

1. **Definition**
   - **Black Model Case**

$$z = \frac{ln(\frac{F_1}{F_2 + K})}{\sigma_{ATM}\sqrt{\tau}} \tag{6.4}$$

where F_1, F_2 are the underlying future prices at time0. σ_{ATM} =

$\sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_{eff} + \sigma_{eff}^2}$, $\sigma_{eff} = \sigma_2 \dfrac{F_2}{F_2 + K_{ATM}}$, where ρ is Breaking good correlation optimized from ATM case. τ is time to expiration. Since everything is measured in years, τ should be measured in years.

$$K = \frac{F_1}{exp(z \cdot \sigma_{ATM}\sqrt{\tau})} - F_2 \tag{6.5}$$

- **Bachelier Model Case**

$$z = \frac{F_1 - F_2 - K}{\sigma_B^{ATM}\sqrt{\tau}} \tag{6.6}$$

where F_1, F_2 are the underlying future prices. τ is time to expiration. Since σ_B used in Bachelier model is measured daily, τ should be measured by days too. σ_B^{ATM} is the Breaking good Bachelier spread volatility found in the analysis for ATM option (measured daily)

$$K = F_1 - F_2 - z \cdot \sigma_B^{ATM} \cdot \sqrt{\tau} \tag{6.7}$$

2. **Process**

   - Take z from -1.0 to 1.0 with step size 0.1.
   - We converted each z to K using Formula 6.5 for Black model and Formula 6.7 for Bachelier model.
   - Conduct the optimization and back up breaking good parameters for each K.
   - Compare the corresponding replication errors for different models.

3. **Comparison Results** Here we provide partial results of replication errors with Bachelier model and Black model. The complete tables can be found in Appendix B.1.

   According to the two tables listing the results under different normalized moneyness, it is obvious that Bachelier Model works for a larger range of strikes and gives smaller replication errors. Even though Black model replication enables different absolute numbers for Delta, which seems to enable the model more freedom, actually, one disadvantage of the model is that correlation has a natural boundary [-1.0, 1.0] and the natural but strict bound would affect the accuracy of replication.

   However, when we reduce the number of dimension from 2 to 1, we do not need to take the relation between two variables, i.e., "correlation" into consideration. For the spread volatility, simple positivity constraint enables more freedom of replication, leading to smaller replication errors.

Table 6.5: Partial Bachelier Breaking Good σ_B Under Different Normalized Moneyness (Wti)

	Month	Month_diff	z=-1.0	z=-0.9	z=-0.8	z=-0.7	z=-0.6	z=-0.5	z=-0.4	z=-0.3	z=-0.2	z=-0.1	z=0.0	z=0.1	z=0.2	z=0.3	z=0.4	z=0.5	z=0.6	z=0.7	z=0.8	z=0.9	z=1.0
0	JAN19	1	0.02835502	0.02844024	0.02846614	0.02843945	0.0283675	0.02825771	0.02811681	0.0279499	0.02776003	0.02754921	0.02732142	0.02708746	0.02686897	0.02669839	0.02661154	0.02662221	0.0266213	0.02624259	0.0266946	0.02884216	0.0257298
1	FEB19	1	0.02651329	0.02645433	0.02636662	0.02625256	0.02611492	0.02595699	0.02578271	0.02559689	0.02540538	0.02521516	0.02503429	0.02487175	0.02473671	0.02463682	0.0245751	0.02454762	0.02453592	0.02448125	0.02418094	0.02348675	0.02360614
2	MAR19	1	0.02529126	0.0253094	0.02530438	0.02527718	0.02522912	0.02516192	0.02507766	0.02497876	0.02486793	0.02474811	0.02462236	0.02449375	0.02436531	0.02424033	0.02412327	0.02402198	0.02395139	0.02393778	0.02401841	0.02422613	0.02455615
3	APR19	1	0.02472951	0.02459889	0.02445647	0.02430343	0.02414116	0.02397125	0.02379548	0.02361584	0.02343456	0.02325416	0.02307769	0.02290901	0.02275345	0.02261875	0.02251612	0.02246108	0.02247231	0.02256683	0.02275158	0.02302053	0.02338564
4	MAY19	1	0.02554899	0.02544588	0.02533875	0.02522898	0.02511819	0.02500828	0.02490151	0.02480057	0.02470872	0.02462985	0.02456851	0.02452987	0.02451972	0.02454459	0.02461235	0.02473321	0.02491995	0.02518456	0.02552824	0.02592821	0.02633349
5	JUN19	1	0.02314285	0.02306098	0.02297605	0.02288931	0.02280215	0.02271619	0.02263332	0.02255576	0.02248621	0.02242796	0.022385	0.02236221	0.02236543	0.02240146	0.02247793	0.02260291	0.02278522	0.02305829	0.02501768	0.03138439	0.03280998
6	JLY19	1	0.02373698	0.02362044	0.02350763	0.02339955	0.02329729	0.02320212	0.02311554	0.0230393	0.02297564	0.02292756	0.02289972	0.0229007	0.02294476	0.02308656	0.02341771	0.02418352	0.02576907	0.02804695	0.03011115	0.03154999	0.03239839
7	AUG19	1	0.02035456	0.0203298	0.02034737	0.02045075	0.0207485	0.0215064	0.02317575	0.02546766	0.02731409	0.02851592	0.02923117	0.02961074	0.02977435	0.0297998	0.02968548	0.0293627	0.02884541	0.02810535	0.02739432	0.02827113	0.02806999
8	SEP19	1	0.01992486	0.02007213	0.02042522	0.02113723	0.02230373	0.02366336	0.02483488	0.02571151	0.02632784	0.02672915	0.02694382	0.02700646	0.02701649	0.02713156	0.02733124	0.02718281	0.02603214	0.02645602	0.02673966	0.02651361	0.02611427
9	OCT19	1	0.0209713	0.02288313	0.02424487	0.02517558	0.02583176	0.02630941	0.02666877	0.02695084	0.02718605	0.02739787	0.02761973	0.03332987	0.03450024	0.03344796	0.03283758	0.03107602	0.02955451	0.02811701	0.02705923	0.02644537	0.02603202
10	NOV19	1	0.02636477	0.02676961	0.02745462	0.02865796	0.03031078	0.0319697	0.03320958	0.03378256	0.03352877	0.03228078	0.03068018	0.03111346	0.03067475	0.03018704	0.02969717	0.02916978	0.02861549	0.02807765	0.02760792	0.02723674	0.0269641
11	DEC19	1	0.0245591	0.02816367	0.03949694	0.04849541	0.05428698	0.0577548	0.05945816	0.05970785	0.05864763	0.05630907	0.05261549	0.04722612	0.03873558	0.03215345	0.0291869	0.0280701	0.02751566	0.02714176	0.0269113	0.02679443	0.02675373
12	JAN20	1	0.0566775	0.05615342	0.05530999	0.05414145	0.05263907	0.0507885	0.04853381	0.04545818	0.04267709	0.04031429	0.03802752	0.03450862	0.02951769	0.02690497	0.02603999	0.02565592	0.02545479	0.0253553	0.02532574	0.0253469	0.02540405
13	FEB20	1	0.05385773	0.05305128	0.05205653	0.05086554	0.04944001	0.04764013	0.0452859	0.0435766	0.04324117	0.04151585	0.03933116	0.03675618	0.03340349	0.02969609	0.02713097	0.02584876	0.02523599	0.02494673	0.02483004	0.02481512	0.02486388
14	MAR20	1	0.04891981	0.04799529	0.04685847	0.04545691	0.04390191	0.04281638	0.04253231	0.04145355	0.04085664	0.04020993	0.03895941	0.03706993	0.03436071	0.03069128	0.02682766	0.0243183	0.0229851	0.02222206	0.02174409	0.02143377	0.0051442
15	APR20	1	0.04349981	0.04301048	0.04250203	0.04197736	0.04143039	0.04084775	0.0402115	0.03950561	0.03873868	0.03799786	0.03749434	0.03732956	0.03688895	0.03497228	0.03287448	0.02999292	0.02777016	0.02490271	0.02297261	0.02180722	0.02104725
16	MAY20	1	0.03968191	0.03946031	0.03922588	0.03898406	0.03874007	0.03849969	0.03826972	0.03805718	0.03786674	0.03769571	0.03752461	0.03730297	0.03699432	0.03687304	0.03785047	0.04111144	0.04590008	0.04839486	0.04586346	0.04169467	0.02652961
17	JAN19	2	0.05294205	0.05307458	0.05309985	0.05302678	0.05286564	0.05262731	0.05232244	0.05196023	0.05154767	0.05109021	0.05059572	0.05008259	0.04959043	0.04918772	0.04896707	0.04901406	0.04929447	0.04933312	0.04800889	0.0500561	0.05274301
18	FEB19	2	0.05000954	0.04989918	0.0497322	0.04951301	0.04924714	0.04894144	0.04860448	0.04824677	0.04788096	0.04752155	0.04718376	0.04688139	0.04662327	0.04640981	0.04623719	0.04612432	0.04613877	0.04631152	0.04647373	0.04596141	0.04750851

Table 6.6: Partial Bachelier Replication Errors Under Different Normalized Moneyness (Wti)

	Month	Month_diff	z=-1.0	z=-0.9	z=-0.8	z=-0.7	z=-0.6	z=-0.5	z=-0.4	z=-0.3	z=-0.2	z=-0.1	z=0.0	z=0.1	z=0.2	z=0.3	z=0.4	z=0.5	z=0.6	z=0.7	z=0.8	z=0.9	z=1.0
0	JAN19	1	3.8675E-10	4.3944E-10	2.5674E-10	-1.386E-09	2.3982E-11	6.0184E-10	2.0701E-10	8.2603E-10	1.7107E-09	9.2752E-10	1.5322E-10	-6.337E-11	1.2659E-09	1.0983E-09	-6.557E-10	2.6062E-11	-5.573E-10	2.491E-10	5.4794E-11	6.1655E-12	1.0222E-10
1	FEB19	1	1.4931E-10	4.9412E-11	-2.765E-11	-3.313E-11	-9.907E-10	-1.145E-11	2.7822E-11	9.7182E-11	2.6953E-11	1.315E-09	3.3336E-11	6.8248E-10	7.0385E-11	-2.763E-11	-9.073E-12	1.5336E-10	-3.244E-10	-9.127E-11	6.8344E-11	2.1226E-10	-5.947E-10
2	MAR19	1	-1.429E-10	3.1804E-10	7.5112E-12	-4.058E-12	1.8126E-10	1.273E-10	1.057E-11	2.2172E-11	-9.341E-10	2.5849E-11	6.2293E-11	4.6983E-11	4.3432E-10	-3.092E-10	-3.715E-12	3.1366E-12	2.2831E-11	6.1487E-11	1.0394E-11	-6.013E-10	-2.454E-11
3	APR19	1	7.5886E-11	4.4957E-10	-5.572E-10	-2.59E-10	1.465E-11	-1.902E-10	-5.503E-10	-2.725E-11	-1.21E-10	6.4612E-10	-6.802E-12	4.0932E-11	-3.589E-10	-4.211E-12	2.5662E-11	-9.658E-11	1.0421E-10	1.4981E-10	2.2111E-11	6.9382E-11	-8.773E-10
4	MAY19	1	-6.128E-11	2.2613E-11	1.6976E-10	5.564E-10	5.1043E-11	-1.782E-11	8.0196E-10	1.1272E-11	6.0524E-12	3.1362E-11	1.5886E-10	1.4711E-10	-1.041E-11	-2.516E-10	-2.512E-12	-2.537E-10	3.4534E-10	6.7992E-11	2.0906E-11	1.2631E-11	-2.67E-10
5	JUN19	1	1.5779E-10	8.1618E-11	9.9406E-11	-3.353E-12	2.1319E-11	8.5793E-12	3.4684E-11	1.9455E-10	-1.404E-11	-6.648E-10	3.0398E-10	1.9677E-11	2.7603E-11	5.3436E-11	1.332E-11	6.6029E-10	1.4121E-10	4.233E-10	-1.499E-10	-1.143E-10	-1.336E-10
6	JLY19	1	9.5509E-12	2.7419E-11	-1.982E-11	2.1681E-10	8.0461E-11	4.4991E-12	-2.267E-11	6.3485E-12	2.7915E-11	1.1867E-10	8.254E-11	-8.025E-11	3.7013E-10	4.946E-11	1.1631E-10	8.5947E-13	8.1784E-12	-1.646E-11	-4.467E-11	2.9038E-11	4.4646E-10
7	AUG19	1	-2.905E-12	-1.149E-11	1.3595E-11	-1.385E-11	1.0343E-10	8.3581E-12	1.04E-11	2.2025E-11	-1.751E-11	8.7273E-10	-2.296E-10	-6.108E-11	6.3229E-11	-6.441E-10	2.2757E-09	-6.395E-12	2.1474E-09	-3.583E-10	1.6045E-12	-4.73E-12	-9.106E-11
8	SEP19	1	-3.951E-11	2.7782E-11	4.3887E-12	-7.982E-11	1.9868E-11	-2.613E-11	1.6E-10	4.2292E-11	1.9849E-11	5.6072E-11	-1.166E-10	7.1921E-10	1.0594E-09	-1.463E-10	-2.154E-11	3.5394E-10	-1.64E-09	3.1314E-11	-1.345E-11	-9.614E-10	-3.393E-11
9	OCT19	1	-1.497E-11	6.6278E-12	4.4852E-11	-7.848E-11	-1.151E-10	1.8023E-10	-2.283E-10	4.081E-11	-6.719E-10	-1.939E-11	-2.386E-12	-2.691E-10	-6.317E-11	7.0197E-11	1.0648E-09	1.5871E-11	-1.611E-09	-1.395E-09	-2.716E-10	-1.505E-10	3.2456E-12
10	NOV19	1	-1.302E-11	2.8864E-12	-1.122E-10	4.0008E-12	-6.741E-10	-7.45E-12	-8.241E-10	-8.363E-11	3.956E-13	9.4321E-12	-9.742E-11	-6.982E-12	-6.91E-12	-2.29E-10	3.8983E-11	2.3294E-10	4.7028E-10	2.597E-11	4.4852E-10	-4.091E-10	-1.669E-10
11	DEC19	1	4.4586E-12	8.7207E-12	1.1675E-11	9.8938E-10	-6.141E-10	-1.847E-10	1.6996E-10	9.7931E-11	9.8382E-10	-2.087E-09	-1.567E-09	-5.639E-10	-1.219E-10	5.6093E-10	1.0479E-09	-2.238E-10	-6.607E-11	5.0405E-12	-3.158E-10	3.3383E-10	-5.048E-12
12	JAN20	1	-1.894E-11	-3.222E-11	5.3874E-10	2.083E-11	-1.002E-10	5.8858E-10	-7.652E-10	3.2968E-09	-1.937E-11	8.2065E-11	-2.384E-10	2.8415E-11	-7.522E-10	4.2385E-10	-3.772E-10	5.2037E-10	2.235E-10	8.8638E-11	-1.078E-10	9.4091E-12	-1.408E-10
13	FEB20	1	1.9146E-11	-2.246E-09	2.5981E-10	-2.204E-10	-9.259E-13	8.7231E-11	1.1086E-09	1.4729E-10	9.5882E-10	7.377E-10	1.1508E-09	2.5904E-10	1.433E-11	1.2023E-10	8.3296E-11	8.3283E-11	-1.195E-11	2.2671E-10	-4.903E-11	1.0373E-10	-1.754E-11
14	MAR20	1	-6.223E-10	-1.153E-10	1.4367E-09	2.4678E-10	3.8432E-09	1.5646E-10	2.1555E-11	-5.192E-10	3.9864E-10	1.9406E-11	8.5129E-12	-1.025E-11	1.1764E-11	1.5735E-11	1.8072E-11	4.1406E-11	5.4607E-11	1.7839E-11	-2.199E-12	1.5205E-12	-3.331E-16
15	APR20	1	2.1325E-09	3.4171E-10	-2.923E-10	-1.358E-10	1.9023E-09	2.2601E-10	2.7496E-10	1.2509E-10	7.9743E-10	1.3415E-09	-4.835E-11	-1.819E-09	-4.405E-10	-8.015E-10	1.0571E-11	8.8124E-11	2.9128E-11	3.2074E-13	1.0747E-11	2.6311E-12	3.2263E-13
16	MAY20	1	2.7561E-11	6.199E-12	-1.085E-09	8.343E-11	4.474E-10	3.7101E-10	-3.897E-11	-7.039E-10	2.9838E-10	1.8502E-09	1.966E-09	-7.638E-11	3.9217E-10	-1.573E-09	-9.122E-10	3.1553E-10	1.5799E-09	8.1775E-11	2.0304E-09	9.0783E-10	7.1954E-13
17	JAN19	2	1.4078E-11	1.8947E-11	1.0813E-11	2.6447E-09	5.4314E-11	1.385E-10	1.6819E-09	4.0773E-10	2.3074E-10	1.9223E-10	-1.848E-09	-1.314E-09	1.2717E-09	-2.521E-09	9.6823E-10	5.9842E-10	-1.903E-09	-5.682E-11	1.0379E-10	1.6494E-10	1.2112E-10
18	FEB19	2	9.6393E-12	1.6953E-09	5.3977E-11	7.6872E-10	-2.653E-09	9.636E-11	2.6871E-10	1.4366E-10	2.6656E-11	1.5612E-09	3.6694E-11	6.2467E-12	-2.516E-10	9.3095E-11	-2.286E-09	-1.17E-10	-1.48E-09	-2.069E-11	-2.036E-10	-5.191E-11	2.1368E-09

Table 6.7: Partial Black Replication Errors Under Different Normalized Moneyness (Wti)

	Month	Month_diff	z=-1.0	z=-0.9	z=-0.8	z=-0.7	z=-0.6	z=-0.5	z=-0.4	z=-0.3	z=-0.2	z=-0.1	z=0.0	z=0.1	z=0.2	z=0.3	z=0.4	z=0.5	z=0.6	z=0.7	z=0.8	z=0.9	z=1.0
0	JAN19	1	-1.389E-07	-8.289E-08	-7.424E-07	2.0156E-07	7.2221E-07	-6.404E-07	-6.725E-08	2.5128E-07	1.8087E-07	1.2928E-06	-7.558E-07	1.5534E-08	4.2291E-08	-3.28E-07	1.2415E-06	-6.879E-07	5.8207E-07	9.8247E-08	1.2236E-07	2.5814E-09	1.3322E-08
1	FEB19	1	6.0408E-07	4.1273E-07	1.6502E-06	3.0162E-08	5.771E-08	9.0193E-08	1.2182E-07	1.4737E-07	1.5813E-07	1.4963E-07	1.2222E-07	8.2496E-08	-3.419E-07	7.6016E-07	-4.952E-07	5.3501E-07	4.098E-08	2.9167E-08	-6.058E-07	-1.183E-06	-6.462E-07
2	MAR19	1	2.5976E-07	3.4644E-07	2.4651E-07	2.6594E-08	-7.635E-07	3.1446E-07	-3.023E-08	-1.218E-07	-1.921E-07	-1.886E-07	1.3753E-06	8.818E-07	-5.736E-07	7.2426E-08	-2.877E-07	2.5404E-06	7.0081E-07	-8.427E-07	5.4791E-07	-3.282E-08	1.4017E-06
3	APR19	1	0.00390318	0.00234461	0.00057831	9.2345E-07	-5.703E-08	-5.118E-07	-2.612E-07	-1.992E-08	1.4631E-06	-2.35E-07	-1.82E-07	-1.102E-07	1.9981E-06	-1.059E-06	-8.773E-07	-2.256E-06	-2.735E-07	1.1756E-06	-2.413E-09	1.471E-07	5.9033E-09
4	MAY19	1	0.00150903	-9.37E-07	-1.695E-07	-1.572E-07	-3.219E-07	4.2885E-07	-7.208E-07	-9.111E-07	-1.004E-06	4.8173E-07	1.7579E-06	-1.063E-06	-3.263E-07	1.6142E-06	1.2622E-06	5.9413E-07	-9.315E-07	-8.053E-08	9.6531E-07	-9.274E-08	-7.357E-07
5	JUN19	1	0.01421922	0.01390827	0.01348199	0.01292409	0.0122174	0.01134426	0.01028715	0.00902941	0.00755626	0.00585595	0.00392092	0.00174883	4.1277E-07	-3.245E-07	2.8458E-07	1.2683E-06	-1.053E-06	4.9998E-07	9.8483E-07	-6.377E-07	-5.018E-08
6	JLY19	1	0.01190818	0.01188834	0.01179449	0.01161089	0.0113193	0.01089877	0.0103254	0.00957201	0.00860759	0.00739592	0.00589202	0.0040327	0.00171229	-5.381E-08	2.1889E-08	5.1431E-07	-3.391E-07	3.0768E-08	2.1043E-07	1.766E-06	6.1759E-07
7	AUG19	1	0.01225324	0.01178185	0.01098504	0.0097075	0.00771378	0.00463525	4.1307E-07	3.6958E-08	2.5905E-08	-1.911E-07	-4.166E-10	-9.55E-08	-2.352E-07	7.4115E-08	2.3731E-07	3.2971E-07	-3.423E-07	-5.103E-07	1.5134E-07	3.6999E-08	-1.825E-06
8	SEP19	1	0.01059345	0.00992358	0.00890764	0.00741649	0.00527786	0.00225996	5.6703E-07	4.7091E-08	7.9207E-07	2.383E-07	-5.773E-08	-3.758E-07	2.232E-06	-1.89E-06	1.5204E-07	-3.143E-06	4.4306E-08	-3.169E-07	2.933E-06	4.7955E-07	-6.537E-07
9	OCT19	1	0.01232039	0.01101679	0.00920671	0.00673519	0.0034012	1.6285E-07	-4.706E-08	1.3007E-06	-8.899E-08	-2.207E-06	-2.41E-07	1.1562E-07	-3.117E-07	2.5968E-07	1.0127E-06	1.2473E-06	-1.223E-07	-2.203E-06	-1.355E-06	6.8395E-08	-3.56E-08
10	NOV19	1	0.01291639	0.0118449	0.00998992	0.00664404	0.00041515	4.2201E-08	8.2911E-08	-1.023E-07	-2.06E-07	1.3714E-06	-2.248E-06	1.6389E-06	-9.903E-08	-5.022E-07	1.6317E-07	1.3528E-07	-1.36E-06	9.2455E-07	7.2331E-07	-3.805E-07	2.4252E-07
11	DEC19	1	0.02114564	0.02078679	0.01805408	0.0094403	2.1405E-07	1.2373E-07	-3.867E-09	1.8336E-07	-5.374E-08	-1.739E-06	9.8122E-07	1.2002E-07	-1.056E-06	-2.048E-06	2.1021E-06	1.069E-06	7.1107E-08	-2.658E-07	2.7079E-07	3.8088E-08	-1.148E-08
12	JAN20	1	8.576E-07	-6.549E-08	9.5613E-07	3.1797E-07	8.344E-07	3.8162E-07	4.2275E-07	-1.552E-07	-1.767E-06	1.404E-06	8.8779E-07	2.8539E-08	8.823E-08	-8.653E-07	3.1649E-07	-6.145E-09	7.9892E-08	-3.038E-07	2.0746E-07	3.1929E-08	4.8165E-08
13	FEB20	1	1.97E-07	3.0836E-07	-4.442E-10	4.752E-08	-1.712E-06	-4.394E-07	2.9329E-07	2.621E-07	1.8301E-06	1.4356E-06	-1.286E-06	-7.691E-08	2.1057E-07	1.9934E-07	3.594E-07	-1.684E-07	3.4766E-07	6.9929E-08	1.5683E-07	-9.817E-08	8.2572E-09
14	MAR20	1	-7.012E-07	6.6465E-07	-1.712E-06	-2.151E-07	1.171E-07	3.0823E-07	2.9845E-07	1.2091E-06	1.3887E-06	-5.278E-08	6.3474E-07	-2.615E-07	-1.244E-07	4.7338E-08	1.9536E-07	-1.769E-07	7.2967E-08	4.0439E-08	2.8635E-08	1.7047E-08	9.4673E-09
15	APR20	1	4.969E-07	-2.932E-08	8.3065E-08	2.0011E-07	3.5592E-07	4.4147E-07	2.8204E-07	1.273E-07	-1.191E-06	-6.779E-07	-1.878E-08	2.7875E-07	8.544E-09	1.6349E-07	7.7767E-08	0.00060739	0.00063224	0.00034125	0.00012157	2.8997E-05	3.8136E-06
16	MAY20	1	0.04438135	0.04163928	0.0381369	0.03378333	0.02851507	0.02231729	0.01524822	0.00746204	2.0609E-07	-3.211E-07	-4.391E-07	-1.003E-06	-1.901E-06	-4.609E-07	1.3425E-06	6.9924E-07	-9.314E-07	-2.243E-07	2.0432E-07	1.772E-07	0.00106613
17	JAN19	2	7.0234E-09	3.2226E-07	-2.245E-08	-7.996E-08	-2.153E-07	-5.08E-07	4.3729E-07	3.5811E-08	8.377E-08	1.7154E-07	2.8748E-07	3.5908E-07	2.9326E-07	1.2955E-07	7.3044E-07	-4.961E-08	-2.844E-12	1.0808E-07	2.6099E-09	5.1708E-09	-1.726E-07
18	FEB19	2	-7.527E-08	-3.583E-07	7.6312E-07	7.8205E-08	1.3983E-07	2.1621E-07	2.9034E-07	3.3323E-07	3.1671E-07	2.3833E-07	1.3382E-07	1.4365E-07	-6.667E-07	-1.136E-07	8.2087E-07	8.0232E-09	4.2078E-09	3.931E-07	-2.655E-07	3.8757E-08	6.4915E-08

Table 6.8: Partial Black Breaking Good Correlation Under Different Normalized Moneyness (Wti)

	Month	Month_diff	z=-1.0	z=-0.9	z=-0.8	z=-0.7	z=-0.6	z=-0.5	z=-0.4	z=-0.3	z=-0.2	z=-0.1	z=0.0	z=0.1	z=0.2	z=0.3	z=0.4	z=0.5	z=0.6	z=0.7	z=0.8	z=0.9	z=1.0
0	JAN19	1	0.99966245	0.9996554	0.99965025	0.99964678	0.99964482	0.99964419	0.99964472	0.99964624	0.99964861	0.99965165	0.99965514	0.99965871	0.99966189	0.99966397	0.99966395	0.99966068	0.99965328	0.99964376	0.99964095	0.9996398	0.99952464
1	FEB19	1	0.9998284	0.99982339	0.99981929	0.99981603	0.9998135	0.9998116	0.99981023	0.99980927	0.9998086	0.99980807	0.99980753	0.99980676	0.99980551	0.99980331	0.99979946	0.99979283	0.99978222	0.99976695	0.99974796	0.99972907	0.99971797
2	MAR19	1	0.99999464	0.99997911	0.99996522	0.99995282	0.99994175	0.99993187	0.9999231	0.99991531	0.99990841	0.99990231	0.9998969	0.99989211	0.99988782	0.99988391	0.99988022	0.99987652	0.99987243	0.99986728	0.99985991	0.99984853	0.9998311
3	APR19	1	0.99999998	0.99999998	0.99999998	0.99999377	0.99998553	0.99997803	0.9999712	0.99996499	0.99995935	0.99995421	0.99994952	0.99994519	0.99994111	0.99993718	0.99993317	0.99992879	0.99992353	0.99991665	0.99990711	0.99989385	0.99987653
4	MAY19	1	0.99999998	0.99999808	0.99998692	0.99997659	0.99996703	0.99995814	0.99994985	0.99994208	0.99993471	0.9999276	0.99992062	0.99991355	0.99990611	0.99989796	0.99988872	0.99987786	0.99986481	0.99984876	0.99982877	0.99980419	0.99977586
5	JUN19	1	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999722	0.99998666	0.99997567	0.99996379	0.99995041	0.99993418	0.99986345	0.99965755	0.99960895
6	JLY19	1	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999188	0.99996413	0.99991092	0.99981813	0.99972843	0.9996662	0.99962844	0.99961109
7	AUG19	1	0.99999998	0.99999998	0.99999997	0.99999998	0.99999998	0.99999997	0.99999887	0.99991871	0.99985032	0.99980164	0.99976811	0.99974507	0.99972845	0.99971538	0.99970621	0.99970426	0.99970745	0.99972137	0.99970534	0.99970098	0.99971092
8	SEP19	1	0.99999998	0.99999997	0.99999997	0.99999998	0.99999998	0.99999998	0.99997837	0.99993453	0.99989963	0.99987201	0.99985052	0.99983361	0.99981752	0.9997986	0.99978255	0.99978644	0.99978961	0.99976987	0.99977039	0.99977715	0.99978434
9	OCT19	1	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999234	0.99995913	0.99992953	0.99990272	0.99987833	0.99985588	0.99967946	0.9996572	0.99966304	0.99969672	0.9997404	0.99977719	0.9998021	0.99981207	0.99981752	0.99982173
10	NOV19	1	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99991873	0.99983735	0.99977769	0.99975134	0.99976557	0.99980015	0.99977497	0.99977816	0.99978575	0.99979633	0.99980773	0.9998168	0.99982226	0.99982536	0.99982765	0.99982981
11	DEC19	1	0.99999998	0.99999998	0.99999998	0.99999998	0.99972991	0.9993235	0.99908526	0.99897309	0.99896526	0.99904906	0.99921556	0.99946384	0.99980756	0.99986969	0.99987865	0.99987189	0.9998589	0.99984141	0.99982051	0.99979848	0.99977813
12	JAN20	1	0.99906816	0.99907171	0.99909278	0.99913034	0.99918331	0.99925055	0.99933163	0.99943494	0.9995377	0.99960487	0.99967374	0.99978511	0.99989242	0.99991044	0.99990842	0.99990342	0.9998973	0.99988963	0.99987949	0.99986683	0.99985255
13	FEB20	1	0.99917578	0.99920798	0.99924739	0.99929517	0.99935478	0.99942946	0.99952429	0.99951044	0.9995404	0.99960139	0.99967327	0.9997583	0.99985806	0.99992657	0.99994806	0.9999504	0.99994705	0.99994214	0.99993625	0.99992845	0.99991783
14	MAR20	1	0.99925633	0.99930314	0.99935805	0.99941391	0.99945288	0.99946424	0.99947642	0.99954395	0.99952917	0.99956444	0.99962753	0.99972564	0.99986593	0.99996726	0.99998883	0.9999855	0.99997394	0.999959	0.99994258	0.99992564	0.99990866
15	APR20	1	0.99945022	0.99945645	0.99946511	0.99947601	0.99948909	0.99950448	0.99952248	0.99954347	0.99956712	0.99959035	0.99960456	0.99959941	0.99959259	0.99964366	0.99976764	0.99999997	0.99999997	0.99999998	0.99999998	0.99999998	0.99999998
16	MAY20	1	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999998	0.99999784	0.99997525	0.99995279	0.9999297	0.9999061	0.99987835	0.9998205	0.99971655	0.99948706	0.9993414	0.99945677	0.99964825	0.99999998
17	JAN19	2	0.9989306	0.99890647	0.99888922	0.99887819	0.99887276	0.99887233	0.99887629	0.99888406	0.99889501	0.99890845	0.9989235	0.99893896	0.99895308	0.99896329	0.99896596	0.9989562	0.99892838	0.99888095	0.99883007	0.99882631	0.99887275
18	FEB19	2	0.99942014	0.99940131	0.99938589	0.99937354	0.99936389	0.99935656	0.99935115	0.99934724	0.99934436	0.99934201	0.99933964	0.99933662	0.99933222	0.99932537	0.99931439	0.99929642	0.99926676	0.99921814	0.99914338	0.99905045	0.99896589

6.4 Bachelier vs Black: Hedging Good

For the purpose of replication, we only pay attention to the replication error at expiration. Breaking good parameters work well at the expiration. However, we observe some facts that we dislike when there is a jump, replication error is purposely big before the jump. To cope with the problem, we came up with "hedging good" parameters.

We use an example to demonstrate our points in breaking good and hedging good methods. The first graph shows the spread of MAR20 and APR20 pair. It is obvious that there is an obvious spike in the spread around mid September 2019. If the spike is not expected, replication error would become large immediately due to "under replication". However, when we minimize the replication error, we already know the price series which means we have already expected a huge spike and taken that spike into consideration, it is natural that in the very beginning we replicate the spread option in a wrong direction and then take advantage of that expected spike to make up for the error, thus the replication error at expiration is small.

However, when we apply this method into hedging options, it sounds like "looking ahead" because in reality, we cannot know exactly whether and when there is going to be a spike or a plunge. Therefore, only paying attention to the replication error at expiration and allowing replicating in a wrong direction is not an ideal method. For the purpose of replicating the options "well" all the time, we come up with a brand new cost function: sum of squared error for each day. The second graphs presents the comparison of replication error for Breaking Good Method and replication error for Hedging Good Method. Accordingly, the replication error remains small, around 0.0 until the dramatic spike happens. It seems to be a more "honest" method.

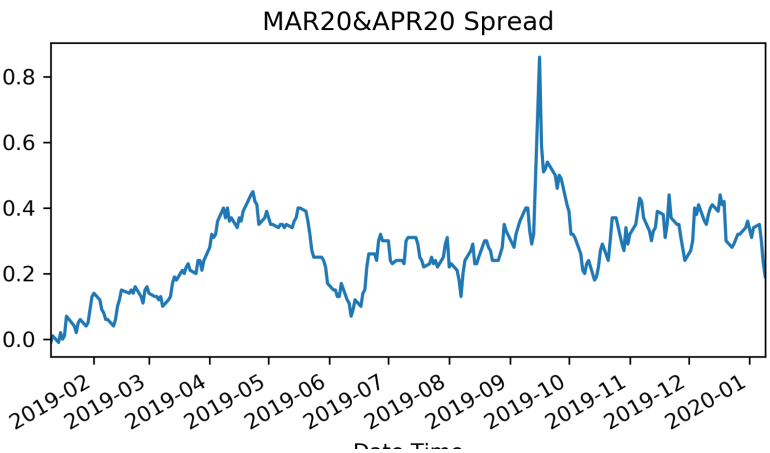

Figure 6.5: NG MAR20&APR20 Price Spread

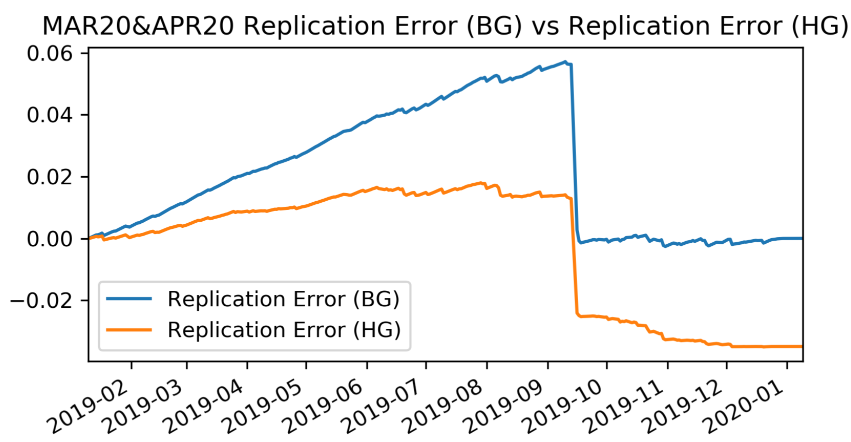

Figure 6.6: NG MAR20&APR20 Replication Errors (BG vs HG)

6.5 Cases Not Applicable

Breaking good and hedging good methods are widely applicable. As long as the option is not always deep in the money or out the money, optimal parameters are possible to be found through appropriate cost functions. If the spread option is deep in the money or out the money, breaking good or hedging good methods are not applicable. When the spread of futures is always deep in the money, we are able to perfectly replicate the spread option by entering 1 long position of future 1 and 1 short position of future 2 and holding the positions until the expiration. Both breaking good and hedging good replication errors are 0.0.

Chapter 7

Summary and Conclusion

7.1 Summary

There are two main topics of our project: pricing of calendar spread options and replication of calendar spread options. We calibrated implied correlations according to the Bachelier formula in Carmona's paper [2]. Then calibrate the results for oil correlations to the model of decay and growth. As for the replication part, we compared Bachelier model with Black model. From the results, Bachelier model outperforms Black model in replicating calendar spread options. Moreover, we developed two different optimization methods leading to various "optimal" parameters: breaking good and hedging good parameters. Breaking good parameters are designed to replicate the spread options, and hedging good parameters are found to be more suitable for option hedge.

7.1.1 Implied Correlation Calibration and Modeling

Firstly, we calibrated implied correlations using WTI weekly settlement price data from the beginning of 2020 until recently. Using these implied correlations as inputs, we applied the novel two-parameter framework [3] into modeling implied correlation. In the paper, they show excellent fits for WTI and Brent historical correlation with Angular Decay model. As for implied correlation, we calibrated the growth and decay parameters directly instead of applying "simple model" or "angular model" mentioned in the paper. We managed to stabilize the Samuelson parameter after having a thorough investigation of each parameter. We fix Samuelson parameter B at 0.001 (daily), which is calibrated by our colleagues Qi Yan and Yuxiao Cheng. Since the implied correlation for the earliest contract is usually outrageously low, which brings additional "noise" to the growth parameter calibration. To solve the problem, we came up with a new method to determine $Corr_0$. Finally, we calibrated the growth parameter α and observed that 0.02 (daily) works for almost all cases. In summary, we managed to stabilize both Samuelson parameter and growth parameter, narrowing three dimensions to only one.

Most calendar spread options on the market have one-month difference. If we would like to capture the whole picture of growth and decay of implied correlations, data is far from enough.

7.1.2 Option Replicating and Hedging

As for the spread option replication, we illustrate the multi-asset pricing model and verify the method that replicates the spread option by trading the underlying assets according to Δ position. Moreover, we present detailed replication steps for spread option replication. The framework is model-independent. Later on, we replicate spread options according to two different models: Black model and Bachelier model. The theory of each model is demonstrated, followed by two distinct cost functions for optimization, which lead to two types of optimal parameters: breaking good parameters and hedging good parameters.

- **Bachelier Model vs Black Model** Even though Black model has more dimensions than Bachelier model, which seemingly enables more freedom, it is actually restricted with natural correlation bound [-1.0, 1.0]. In the meanwhile, spread volatility has a larger range to adjust replication errors despite the fact the model is uni-variant. According to the normalized moneyness result table, we can conclude that Bachelier model applies to more cases than Black model and gives smaller replication errors in most cases.

- **Breaking Good vs Hedging Good** For Wti cases, breaking good parameters and hedging good parameters are close except for a few cases when there is a dramatic spike or plunge. Hedging good method is designed to minimize replication error along the way, which is defined as replication error 2; therefore, hedging good optimal parameters always lead to smaller replication error 2. The good thing is that replication error 2 of breaking good method is comparable to hedging good, meaning optimal parameters derived from these two methods are close and comparable. We observe more difference in breaking good and hedging good parameters for natural gas cases, especially for inter season months.

In conclusion, Bachelier model with breaking good daily σ_B is the better way to replicate calendar spread options and Bachelier model with hedging good daily σ_B is a better method to hedging options in order to avoid dramatic PnL swings.

7.1.3 Further Research

1. We expect to apply new Samuelson models into correlation framework in the future. The model used to capture Samuelson effect is a simple exponentially decay model, which implies an unrealistic fact that implied volatility would converge to 0. In reality, implied volatility would go to some specific level (above 0) instead of 0. My colleagues have investigated into several more complicated models, such as adding a σ_{inf} term into formula so that volatility converges to $\sigma_0\sigma_{inf}$ instead of 0.

2. Bachelier model has different formulae according to interest rate. When interest rate is not equal to 0.0, Bachelier formula becomes more complicated. We expect to cover the situation when interest rate is not necessarily equal to 0.0 in our future research.

Bibliography

[1] L. Bachelier. "Theory of Speculation". In: 1964.

[2] René Carmona and Valdo Durrleman. "Pricing and Hedging Spread Options". In: *SIAM Review* 45.4 (2003), pp. 627–685. DOI: 10.1137/S0036144503424798. eprint: https://doi.org/10.1137/S0036144503424798. URL: https://doi.org/10.1137/S0036144503424798.

[3] Roza Galeeva and Thomas Haversang. "Parameterized Calendar Correlations: Decoding Oil and Beyond". In: *The Journal of Derivatives* (2020). ISSN: 1074-1240. DOI: 10.3905/jod.2019.1.093. eprint: https://jod.pm-research.com/content/early/2020/02/14/jod.2019.1.093.1.full.pdf. URL: https://jod.pm-research.com/content/early/2020/02/14/jod.2019.1.093.1.

[4] Hayne E. Leland. "Option Pricing and Replication with Transactions Costs". In: *The Journal of Finance* 40.5 (1985), pp. 1283–1301. ISSN: 00221082, 15406261. URL: http://www.jstor.org/stable/2328113.

[5] Satoshi Terakado. "On the Option Pricing Formula Based on the Bachelier Model". In: (2019). URL: http://dx.doi.org/10.2139/ssrn.3428994.

Appendix A

Appendix for Methodology

A.1 Coutour Plot Method

Contour plot method has widespread application especially when it comes to the model of which two parameters have opposite effects. As we all know, it is rather costly to change parameters in reality. Under this circumstance, contour plot for empirical data can help discover the reasonable ranges and even stabilize and fix one parameter, thus reducing the number of dimensions.

Definitions of Empirical K_s and K_s in Model Fix τ_i at one time, and we can know K_i^{emp} correspondingly. Draw the contour of function K_s^i with the level of K_i^{emp}, which shows the different combination of α and B.

K_s is defined as,

$$K_i^{emp} = \frac{1 - Corr_i}{1 - Corr_0} \tag{A.1}$$

where, $Corr_i$ is implied correlation for each contract and $Corr_0$ is a parameter.

The original equation for K_s^i is

$$K_s^i = \frac{2Be^{-\alpha(T_i - t_2)}}{\alpha + 2B} \frac{1 - e^{-(\alpha+2B)(t_2 - t_1)}}{1 - e^{-2B(t_2 - t_1)}} \tag{A.2}$$

For each contract, $T_i - t_2$ might be different (3 days or 1 day); however, the difference is small enough to be ignored without loss of generality.

Therefore, the Ks for model becomes,

$$K_s^i = \frac{\dfrac{1 - e^{-(\alpha+2B)\tau_i}}{1 - e^{-2B\tau_i}}}{\dfrac{1 - e^{-(\alpha+2B)\tau_0}}{1 - e^{-2B\tau_0}}} \tag{A.3}$$

Let the model "cater to" the empirical data, $K_s^i = K_s^{emp}$; and the equation becomes,

$$\frac{1 - Corr_i}{1 - Corr_0} = \frac{\dfrac{1 - e^{-(\alpha+2B)\tau_i}}{1 - e^{-2B\tau_i}}}{\dfrac{1 - e^{-(\alpha+2B)\tau_0}}{1 - e^{-2B\tau_0}}} \tag{A.4}$$

Left hand is empirical data for each date. There is a remarkable number of combinations of α and B that make the right hand equate the left hand data. Each period would present a contour line with α and B as two axes. If there is some crossing, it means the number works for many dates (many cases). Therefore, we can find a reasonable range and stabilize the parameter.

A.2 Determination of $Corr_0$

A.2.1 Reference Table for $Corr_0$ Extrapolation

Table A.1: Reference Table for $Corr_0$ Extrapolation

$Corr$	$Corr_0$
0.9925	0.990301
0.99275	0.990624
0.993	0.990947
0.99325	0.99127
0.9935	0.991593
0.99375	0.991916
0.994	0.992239
0.99425	0.992562
0.9945	0.992885
0.99475	0.993209
0.995	0.993532
0.99525	0.993855
0.9955	0.994178
0.99575	0.994501
0.996	0.994825
0.99625	0.995148
0.9965	0.995471
0.99675	0.995795
0.997	0.996118
0.99725	0.996441
0.9975	0.996765
0.99775	0.997088
0.998	0.997412
0.99825	0.997735
0.9985	0.998059
0.99875	0.998382
0.999	0.998706
0.99925	0.999029
0.9995	0.999353
0.99975	0.999676

A.3 Replication Models: Black and Bachelier

A.3.1 Black Model Pricing Formula (Kirk's Approximation)

Let $c(F_1, F_2, \tau)$ represents the price of a call option with the boundary condition $c(F_1, F_2, 0) = (F_1 - F_2 - K)^+$. Similarly, we have $p(F_1, F_2, 0) = (K - F_1 + F_2)^+$.

Assume S_1, S_2 move as GBM in risk-neutral space, i.e. $\frac{dS_i}{S_i} = rdt + \sigma_i dW_i$ and $\rho = corr(dW_1, dW_2)$. Since $F = Se^{r\tau}$, the diffusion of future prices becomes $\frac{dF_i}{F_i} = \sigma_i dW_i$.

By Kirk's approximation, when $\frac{F_2}{\sigma_{eff}}|\frac{\partial \sigma_{eff}}{\partial F_2}| \ll 1$ and $\frac{1}{r\sigma_{eff}}|\frac{\partial \sigma_{eff}}{\partial \tau}| \ll 1$, viewing σ_{eff} as a constant, we have,

$$c(F_1, F_2, \tau) = e^{-r\tau}[F_1\Phi(d_1) - (F_2 + K)\Phi(d_2)] \tag{A.5}$$

where, $d_1 = \frac{ln(\frac{F_1}{F_2+K}) + \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}$, $d_2 = \frac{ln(\frac{F_1}{F_2+K}) - \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}$, $\sigma = \sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_{eff} + \sigma_{eff}^2}$, $\sigma_{eff} = \sigma_2\frac{F_2}{F_2+K}$. Trivially,

$$p(F_1, F_2, \tau) = e^{-r\tau}[(F_2 + K)\Phi(-d_2) - F_1\Phi(-d_1)] \tag{A.6}$$

Change of Numeraire Moreover, if we view the CSO as a vanilla European option of F_1 with numeraire $F_2 + K$ and strike $= 1$, we can also derive the above pricing formula. In this perspective, set $\tilde{F_1} = \frac{F_1}{F_2+K}$, we have

$$d\tilde{F_1} = \frac{1}{F_2+K}dF_1 - \frac{F_1}{(F_2+K)^2}dF_2 - \frac{1}{(F_2+K)^2}dF_1dF_2 + \frac{F_1}{(F_2+K)^3}(dF_2)^2 \tag{A.7}$$

$$\begin{aligned}\frac{d\tilde{F_1}}{\tilde{F_1}} &= \frac{dF_1}{F_1} - \frac{dF_2}{F_2+K} - \frac{dF_1}{F_1}\frac{dF_2}{F_2+K} + (\frac{dF_2}{F_2+K})^2 \\ &= \sigma_1 dW_1 - \frac{F_2}{F_2+K}\sigma_2 dW_2 - \frac{F_2}{F_2+K}\sigma_1\sigma_2 dW_1 dW_2 + (\frac{F_2}{F_2+K})^2\sigma_2^2 dt\end{aligned} \tag{A.8}$$

Note that $\frac{F_2}{F_2+K}$ is always a coefficient of σ_2, then we can set $\tilde{\sigma_2} = \frac{F_2}{F_2+K}\sigma_2$ and derive following approximation:

$$\frac{d\tilde{F_1}}{\tilde{F_1}} = \sigma_1 dW_1 - \tilde{\sigma_2} dW_2 - \sigma_1 \tilde{\sigma_2} dW_1 dW_2 + \tilde{\sigma_2}^2 dt \tag{A.9}$$

Since we know $corr(dW_1, dW_2) = \rho$, we can decompose dW_1, dW_2 into $dW_1 = d\tilde{W_1}$, $dW_2 = \rho d\tilde{W_1} + \sqrt{1-\rho^2}d\tilde{W_2}$, where $\tilde{W_1}$, $\tilde{W_2}$ are independent BM

$$\frac{d\tilde{F_1}}{\tilde{F_1}} = (\sigma_1 - \rho\tilde{\sigma_2})d\tilde{W_1} - \tilde{\sigma_2}\sqrt{1-\rho^2}d\tilde{W_2} + \tilde{\sigma_2}(\tilde{\sigma_2} - \rho\sigma_1)dt \tag{A.10}$$

Since $ln\tilde{F_1}(t)$ is a normal distribution, $\Phi(\tilde{\sigma_2}(\tilde{\sigma_2} - \rho\sigma_1)t, \sigma_1^2 - 2\rho\sigma_1\tilde{\sigma_2} + \tilde{\sigma_2}^2)$, we can substitute corresponding parameter into vanilla Black pricing formula.

$$\begin{aligned} c(F_1, F_2, \tau) &= (F_2 + K) \cdot c(\tilde{F_1}, \tau) \\ &= (F_2 + K) \cdot e^{-r\tau}(\tilde{F_1}\Phi(d_1) - \Phi(d_2)) \\ &= e^{-r\tau}[F_1\Phi(d_1) - (F_2 + K)\Phi(d_2)] \end{aligned} \tag{A.11}$$

Greeks

$$f(F_1, F_2, \tau) = (F_2 + K) \cdot \tilde{f}(\tilde{F_1}, \tau) \tag{A.12}$$

where $\tilde{F_1} = \frac{F_1}{F_2+K}$, $\tilde{f}$ is Black model for $\begin{cases} F = \tilde{F_1}, \\ K = 1, \\ r = 0 \end{cases}$

$$\frac{\partial f}{\partial F_1} = \frac{\partial f}{\partial \tilde{F_1}}\frac{\partial \tilde{F_1}}{\partial F_1} = \Delta_{\tilde{f}} = e^{-r\tau}\Phi(d_1) \tag{A.13}$$

$$\frac{\partial f}{\partial F_2} = \begin{cases} \frac{\partial f}{\partial \tilde{F_1}}\frac{\partial \tilde{F_1}}{\partial F_2} = \tilde{f}(\tilde{F_1}, t) + (F_2 + K)\frac{\partial \tilde{f}}{\partial \tilde{F_1}}\frac{\partial \tilde{F_1}}{\partial F_2} = \tilde{f}(\tilde{F_1}, \tau) - \tilde{F_1}\Delta_{\tilde{f}} = -e^{-r\tau}\Phi(d_2) \\ \frac{\partial f}{\partial \sigma}\frac{\partial \sigma}{\partial F_2} = (F_2 + K)\tilde{F_1}e^{-r\tau}\phi(d_1)\sqrt{\tau}\frac{\tilde{\sigma_2} - \rho\sigma_1}{\sqrt{\sigma_1^2 + \tilde{\sigma_2}^2 - 2\rho\sigma_1\tilde{\sigma_2}}}\frac{\sigma_2 K}{(F_2+K)^2} \end{cases} \tag{A.14}$$

$$\Rightarrow \frac{\partial f}{\partial F_2} = -e^{-r\tau}\Phi(d_2) + \tilde{F_1}e^{-r\tau}\sqrt{\tau}\phi(d_1)\frac{\tilde{\sigma_2} - \rho\sigma_1}{\sqrt{\sigma_1^2 + \tilde{\sigma_2}^2 - 2\rho\sigma_1\tilde{\sigma_2}}}\frac{\sigma_2 K}{F_2+K} \tag{A.15}$$

where $\tilde{\sigma_2} = \frac{\sigma_1 F_2}{F_2+K}$, $\tilde{F_1} = \frac{F_1}{F_2+K}$

Delta summary

$$\begin{cases} \Delta_{F_1} = e^{-r\tau}\Phi(d_1) \\ \Delta_{F_2} = -e^{-r\tau}\Phi(d_2) + \tilde{F_1}e^{-r\tau}\sqrt{\tau}\phi(d_1)\dfrac{\tilde{\sigma_2} - \rho\sigma_1}{\sqrt{\sigma_1^2 + \tilde{\sigma_2}^2 - 2\rho\sigma_1\tilde{\sigma_2}}}\dfrac{\sigma_2 K}{F_2 + K} \end{cases} \tag{A.16}$$

where $\tilde{\sigma_2} = \dfrac{\sigma_1 F_2}{F_2 + K}$, $\tilde{F_1} = \dfrac{F_1}{F_2 + K}$

Appendix B

Appendix for Results

B.1 Normalized Moneyness Results

Table B.1: WTI Bachelier Model Breaking Good σ_B

	Month	Month_diff	z=-1.0	z=-0.9	z=-0.8	z=-0.7	z=-0.6	z=-0.5	z=-0.4	z=-0.3	z=-0.2	z=-0.1	z=0.0	z=0.1	z=0.2	z=0.3	z=0.4	z=0.5	z=0.6	z=0.7	z=0.8	z=0.9	z=1.0
0	JAN19	1	0.028355	0.02844	0.028466	0.028439	0.028367	0.028258	0.028117	0.02795	0.02776	0.027549	0.027321	0.027087	0.026869	0.026698	0.026612	0.026622	0.026621	0.026243	0.026695	0.028842	0.02573
1	FEB19	1	0.026513	0.026454	0.026367	0.026253	0.026115	0.025957	0.025783	0.025597	0.025405	0.025215	0.025034	0.024872	0.024737	0.024637	0.024575	0.024548	0.024536	0.024481	0.024181	0.023487	0.023606
2	MAR19	1	0.025291	0.025309	0.025304	0.025277	0.025229	0.025162	0.025078	0.024979	0.024868	0.024748	0.024622	0.024494	0.024365	0.02424	0.024123	0.024022	0.023951	0.023938	0.024018	0.024226	0.024556
3	APR19	1	0.02473	0.024599	0.024456	0.024303	0.024141	0.023971	0.023795	0.023616	0.023435	0.023254	0.023078	0.022909	0.022753	0.022619	0.022516	0.022461	0.022472	0.022567	0.022752	0.023021	0.023386
4	MAY19	1	0.025549	0.025446	0.025339	0.025229	0.025118	0.025008	0.024902	0.024801	0.024709	0.02463	0.024569	0.02453	0.02452	0.024545	0.024612	0.024733	0.02492	0.025185	0.025528	0.025928	0.026333
5	JUN19	1	0.023143	0.023061	0.022976	0.022889	0.022802	0.022716	0.022633	0.022556	0.022486	0.022428	0.022385	0.022362	0.022365	0.022401	0.022478	0.022603	0.022785	0.023058	0.025018	0.031384	0.03281
6	JLY19	1	0.023737	0.02362	0.023508	0.0234	0.023297	0.023202	0.023116	0.023039	0.022976	0.022928	0.0229	0.022901	0.022949	0.023087	0.023418	0.024184	0.025769	0.028047	0.030111	0.03155	0.032398
7	AUG19	1	0.020355	0.02033	0.020347	0.020451	0.020749	0.021506	0.023176	0.025468	0.027314	0.028516	0.029231	0.029611	0.029774	0.0298	0.029685	0.029363	0.028845	0.028105	0.027394	0.028271	0.02807
8	SEP19	1	0.019925	0.020072	0.020425	0.021137	0.022304	0.023663	0.024835	0.025712	0.026328	0.026729	0.026944	0.027006	0.027016	0.027132	0.027331	0.027183	0.026032	0.026456	0.02674	0.026514	0.026114
9	OCT19	1	0.020971	0.022883	0.024245	0.025176	0.025832	0.026309	0.026669	0.026951	0.027186	0.027398	0.02762	0.03333	0.0345	0.033448	0.032838	0.031076	0.029555	0.028117	0.027059	0.026445	0.026032
10	NOV19	1	0.026365	0.02677	0.027455	0.028658	0.030311	0.03197	0.03321	0.033783	0.033529	0.032281	0.03068	0.031113	0.030675	0.030187	0.029697	0.02917	0.028615	0.028078	0.027608	0.027237	0.026964
11	DEC19	1	0.024559	0.028164	0.039497	0.048495	0.054287	0.057755	0.059458	0.059708	0.058648	0.056309	0.052615	0.047226	0.038736	0.032153	0.029187	0.02807	0.027516	0.027142	0.026911	0.026794	0.026754
12	JAN20	1	0.056677	0.056153	0.05531	0.054141	0.052639	0.050789	0.048534	0.045458	0.042677	0.040314	0.038028	0.034509	0.029518	0.026905	0.02604	0.025656	0.025455	0.025355	0.025326	0.025347	0.025404
13	FEB20	1	0.053858	0.053051	0.052057	0.050866	0.04944	0.04764	0.045286	0.043577	0.043241	0.041516	0.039331	0.036756	0.033403	0.029696	0.027131	0.025849	0.025236	0.024947	0.02483	0.024815	0.024864
14	MAR20	1	0.04892	0.047995	0.046858	0.045457	0.043902	0.042816	0.042532	0.041454	0.040857	0.04021	0.038959	0.03707	0.034361	0.030691	0.026828	0.024318	0.022985	0.022222	0.021744	0.021434	0.005144
15	APR20	1	0.0435	0.04301	0.042502	0.041977	0.04143	0.040848	0.040212	0.039506	0.038739	0.037998	0.037494	0.03733	0.036889	0.034972	0.032874	0.029993	0.02777	0.024903	0.022973	0.021807	0.021047
16	MAY20	1	0.039682	0.03946	0.039226	0.038984	0.03874	0.0385	0.03827	0.038057	0.037867	0.037696	0.037525	0.037303	0.036994	0.036873	0.03785	0.041111	0.0459	0.048395	0.045863	0.041695	0.02653
17	JAN19	2	0.052942	0.053075	0.0531	0.053027	0.052866	0.052627	0.052322	0.05196	0.051548	0.05109	0.050596	0.050083	0.04959	0.049188	0.048967	0.049014	0.049294	0.049333	0.048009	0.050056	0.052743
18	FEB19	2	0.05001	0.049899	0.049732	0.049513	0.049247	0.048941	0.048604	0.048247	0.047881	0.047522	0.047184	0.046881	0.046623	0.04641	0.046237	0.046124	0.046139	0.046312	0.046474	0.045961	0.047509
19	MAR19	2	0.048065	0.048011	0.047915	0.047777	0.047602	0.047391	0.047149	0.046881	0.046593	0.046292	0.045983	0.045676	0.045376	0.04509	0.044828	0.044599	0.044426	0.044352	0.044451	0.044824	0.045533
20	APR19	2	0.048165	0.047951	0.047717	0.047465	0.047199	0.046921	0.046635	0.046346	0.046059	0.045779	0.045513	0.045271	0.045065	0.044912	0.044835	0.044861	0.045015	0.045311	0.04574	0.046273	0.046899
21	MAY19	2	0.046527	0.046339	0.046147	0.045954	0.045765	0.045584	0.045418	0.045274	0.045162	0.045093	0.04508	0.045138	0.045283	0.045529	0.045889	0.046376	0.047002	0.047774	0.048683	0.049681	0.050679
22	JUN19	2	0.044966	0.044777	0.044587	0.044399	0.044215	0.044039	0.043877	0.043732	0.043612	0.043525	0.04348	0.043488	0.043564	0.043723	0.043984	0.044366	0.044898	0.046372	0.055077	0.061038	0.067908
23	JLY19	2	0.043369	0.043156	0.04295	0.042751	0.042563	0.042388	0.042229	0.04209	0.041975	0.041892	0.04185	0.041873	0.042007	0.042379	0.043319	0.045622	0.05006	0.054998	0.05867	0.061019	0.062308
24	AUG19	2	0.039712	0.039658	0.039762	0.040218	0.04148	0.044309	0.04833	0.051706	0.05394	0.055272	0.055974	0.056271	0.056311	0.056078	0.055383	0.054286	0.052682	0.052069	0.053202	0.052692	0.05175
25	SEP19	2	0.040354	0.040853	0.041929	0.043807	0.046239	0.048526	0.050308	0.051581	0.052424	0.052902	0.053076	0.0531	0.053309	0.053895	0.054258	0.053287	0.051013	0.052811	0.05264	0.051816	0.050849
26	OCT19	2	0.043497	0.045962	0.047843	0.049203	0.050187	0.050914	0.051474	0.051936	0.052348	0.052745	0.056557	0.066317	0.064899	0.062661	0.059867	0.05604	0.054093	0.052648	0.051635	0.050898	0.050315
27	NOV19	2	0.047827	0.048983	0.050308	0.052309	0.055194	0.058363	0.060917	0.062246	0.061891	0.059316	0.059235	0.059347	0.058677	0.058036	0.057284	0.056372	0.055335	0.054264	0.053283	0.052481	0.051881
28	DEC19	2	0.046471	0.002341	0.085923	0.10145	0.111131	0.116637	0.118921	0.118491	0.115579	0.110224	0.102213	0.090502	0.070748	0.061029	0.054955	0.053124	0.052183	0.051595	0.051315	0.051264	0.051352
29	JAN20	2	0.112054	0.110775	0.108851	0.106256	0.10292	0.098573	0.091699	0.088026	0.083798	0.07964	0.073465	0.063718	0.055586	0.052164	0.050714	0.05001	0.049654	0.049504	0.049493	0.049581	0.049735
30	FEB20	2	0.10328	0.101642	0.099648	0.09729	0.094441	0.090666	0.085175	0.085962	0.083604	0.079816	0.075403	0.069836	0.062581	0.055399	0.051049	0.048982	0.048021	0.047602	0.047478	0.047531	0.047695
31	MAR20	2	0.091204	0.089269	0.086851	0.084252	0.082751	0.083075	0.082346	0.078881	0.078464	0.076712	0.074015	0.07014	0.064722	0.057476	0.050005	0.045295	0.042809	0.041367	0.040442	0.039825	0.039421
32	APR20	2	0.081065	0.08032	0.079555	0.078757	0.077901	0.076953	0.075882	0.074687	0.073493	0.072678	0.07276	0.073423	0.072796	0.068536	0.059497	0.054573	0.049609	0.044419	0.041213	0.039273	0.005748
33	MAY20	2	0.075683	0.075393	0.075063	0.074704	0.074331	0.07396	0.073615	0.073326	0.073128	0.073055	0.073103	0.073157	0.073067	0.073353	0.07573	0.081917	0.092167	0.099904	0.100135	0.092331	0.078604
34	JAN19	3	0.075925	0.076093	0.076116	0.076003	0.075765	0.075413	0.07496	0.074417	0.073796	0.073109	0.072376	0.071635	0.070955	0.070447	0.070236	0.070401	0.070854	0.071066	0.069652	0.066888	0.076498
35	FEB19	3	0.072026	0.071773	0.07144	0.071035	0.070566	0.070042	0.069479	0.06889	0.068298	0.067723	0.06719	0.06672	0.066326	0.066005	0.065737	0.065511	0.065392	0.065525	0.065948	0.066386	0.065199
36	MAR19	3	0.070154	0.070079	0.069941	0.069746	0.069496	0.069199	0.06886	0.068487	0.068089	0.067677	0.067259	0.066846	0.06645	0.066082	0.065759	0.065503	0.065356	0.065391	0.065711	0.066429	0.067577
37	APR19	3	0.06924	0.068938	0.068611	0.068262	0.067897	0.067523	0.067145	0.06677	0.066407	0.066068	0.065764	0.065513	0.065339	0.065274	0.065355	0.065624	0.06611	0.066816	0.067708	0.068726	0.06984
38	MAY19	3	0.06715	0.066858	0.066567	0.066285	0.066018	0.065775	0.065568	0.065411	0.06532	0.065316	0.065421	0.065661	0.066061	0.066646	0.067438	0.068458	0.069719	0.071203	0.072833	0.074473	0.075956
39	JUN19	3	0.064074	0.063813	0.063552	0.063294	0.063045	0.062811	0.062598	0.062414	0.06227	0.062179	0.062157	0.062224	0.062405	0.062732	0.063243	0.063981	0.065144	0.071279	0.080658	0.094668	0.098448
40	JLY19	3	0.064484	0.064146	0.063818	0.063503	0.063206	0.062929	0.062677	0.062459	0.062283	0.062166	0.062143	0.062294	0.062829	0.064289	0.067862	0.074247	0.080824	0.085565	0.088505	0.089955	0.090036
41	AUG19	3	0.05955	0.05959	0.060023	0.061329	0.064408	0.069528	0.074591	0.078197	0.080434	0.081653	0.082166	0.082208	0.081863	0.080963	0.079447	0.077875	0.073837	0.076704	0.077004	0.076079	0.074885
42	SEP19	3	0.061407	0.062803	0.065123	0.068106	0.07102	0.073385	0.07512	0.076293	0.07698	0.077257	0.077297	0.07755	0.078504	0.079638	0.079229	0.07593	0.077375	0.077875	0.077031	0.075824	0.074642
43	OCT19	3	0.061308	0.065465	0.068459	0.070576	0.072115	0.073285	0.074231	0.075057	0.075834	0.076595	0.079629	0.094619	0.095312	0.091026	0.088342	0.082236	0.079253	0.077462	0.076155	0.075196	0.074436
44	NOV19	3	0.068197	0.069736	0.071211	0.073178	0.076234	0.080309	0.084314	0.086893	0.08682	0.083139	0.086237	0.085283	0.084186	0.083459	0.082595	0.081456	0.080083	0.078631	0.077305	0.07626	0.075536
45	DEC19	3	0.068871	0.00366	0.134078	0.155099	0.168041	0.175126	0.177635	0.176252	0.171299	0.162815	0.150371	0.132135	0.101363	0.088189	0.079866	0.077455	0.076195	0.075479	0.075236	0.075326	0.075605
46	JAN20	3	0.162841	0.160496	0.157243	0.152965	0.147207	0.138085	0.132155	0.127434	0.121814	0.114454	0.103044	0.089323	0.080157	0.075921	0.073984	0.073036	0.072574	0.072404	0.072432	0.072599	0.072861
47	FEB20	3	0.146367	0.143995	0.141157	0.137809	0.133504	0.127533	0.12568	0.124201	0.119732	0.114286	0.107877	0.099438	0.088835	0.079211	0.073712	0.071153	0.07002	0.069594	0.069552	0.069737	0.070062
48	MAR20	3	0.126456	0.123368	0.120283	0.119032	0.120322	0.119559	0.113119	0.113358	0.111488	0.108604	0.104469	0.098762	0.091019	0.08099	0.070839	0.064275	0.060696	0.058576	0.057198	0.056264	0.055638
49	APR20	3	0.11608	0.115305	0.114466	0.113517	0.112405	0.111093	0.109585	0.108015	0.106783	0.10655	0.107545	0.108379	0.106681	0.101258	0.091203	0.081471	0.074185	0.066837	0.061767	0.058548	0.01241
50	MAY20	3	0.108622	0.108321	0.107948	0.107527	0.107086	0.106663	0.106308	0.106093	0.106107	0.106436	0.107041	0.107622	0.107946	0.109202	0.114156	0.123678	0.139125	0.153344	0.158357	0.149296	0.142665
51	JAN19	4	0.097385	0.097471	0.09738	0.097119	0.096697	0.096126	0.09542	0.094596	0.093672	0.092673	0.091631	0.090602	0.08968	0.089002	0.088729	0.088964	0.089596	0.089998	0.088621	0.08345	0.092837
52	FEB19	4	0.093079	0.092745	0.09231	0.091783	0.091176	0.090505	0.089787	0.089045	0.088305	0.087594	0.086941	0.08637	0.085894	0.085509	0.085201	0.084995	0.08505	0.085623	0.086785	0.088081	0.087331
53	MAR19	4	0.090848	0.090699	0.090476	0.090185	0.089833	0.089428	0.088979	0.088497	0.087996	0.087486	0.086984	0.086502	0.086058	0.08567	0.085366	0.085186	0.0852	0.08551	0.086245	0.08751	0.089291
54	APR19	4	0.089762	0.089341	0.088895	0.088429	0.087953	0.087475	0.087004	0.086552	0.086133	0.085764	0.085466	0.085268	0.085208	0.085331	0.085687	0.086317	0.087237	0.088422	0.089819	0.091384	0.093168
55	MAY19	4	0.085663	0.085296	0.084937	0.084594	0.084278	0.084003	0.083783	0.083641	0.083599	0.083689	0.083946	0.08441	0.085126	0.086145	0.087519	0.089292	0.091454	0.093895	0.0964	0.098715	0.10062
56	JUN19	4	0.084939	0.084565	0.084194	0.08383	0.083482	0.083156	0.082862	0.082613	0.082422	0.08231	0.0823	0.082424	0.082725	0.083252	0.084067	0.08546	0.092574	0.107039	0.122303	0.125489	0.12222
57	JLY19	4	0.084961	0.084537	0.084125	0.083731	0.083359	0.083016	0.082709	0.08245	0.082263	0.08219	0.082339	0.082982	0.084814	0.089329	0.097356	0.105604	0.11154	0.115181	0.116889	0.116767	0.114894
58	AUG19	4	0.079823	0.080311	0.081747	0.085001	0.090478	0.096389	0.100919	0.103848	0.105489	0.106188	0.106243	0.10583	0.104876	0.10313	0.101137	0.098759	0.097522	0.100163	0.100013	0.098994	0.097834
59	SEP19	4	0.078634	0.080617	0.083866	0.08793	0.09177	0.094814	0.097018	0.0985	0.099377	0.099772	0.1	0.100768	0.102558	0.10417	0.103191	0.097996	0.101744	0.102217	0.101205	0.099733	0.098278
60	OCT19	4	0.080221	0.0849	0.088243	0.090645	0.092437	0.093853	0.095066	0.096203	0.097345	0.098507	0.100804	0.119356	0.12034	0.117339	0.114398	0.105857	0.101778	0.099855	0.098573	0.097687	0.096985
61	NOV19	4	0.08619	0.088286	0.090039	0.091939	0.094746	0.099194	0.104782	0.109325	0.110436	0.107477	0.111874	0.110409	0.108979	0.1081	0.107102	0.105767	0.104114	0.102324	0.100671	0.099378	0.098514
62	DEC19	4	0.094248	0.14153	0.18168	0.206796	0.222049	0.230118	0.232507	0.230014	0.222999	0.211446	0.194658	0.170263	0.133272	0.113659	0.103384	0.100535	0.099104	0.098372	0.098246	0.098535	0.099055
63	JAN20	4	0.207453	0.204252	0.199831	0.19377	0.184436	0.174091	0.170213	0.163997	0.156518	0.145649	0.13043	0.11447	0.103932	0.098896	0.096641	0.095626	0.095219	0.095163	0.095335	0.095664	0.096098
64	FEB20	4	0.182403	0.17924	0.175569	0.170983	0.16475	0.160538	0.160663	0.156052	0.149873	0.143051	0.13466	0.123513	0.110153	0.09885	0.092543	0.089529	0.088142	0.087587	0.087498	0.087691	0.088059
65	MAR20	4	0.158949	0.155391	0.153507	0.155386	0.156306	0.150788	0.147127	0.145678	0.143173	0.139493	0.134375	0.127514	0.118483	0.107013	0.094624	0.085308	0.079796	0.076442	0.074208	0.072648	0.071559
66	APR20	4	0.148042	0.147308	0.14641	0.145282	0.143879	0.142205	0.140374	0.138757	0.138075	0.138948	0.140568	0.140316	0.13637	0.130469	0.11013	0.105886	0.095846	0.085996	0.078659	0.073705	0.017121
67	MAY20	4	0.137384	0.13708	0.136662	0.136173	0.135665	0.135209	0.134898	0.134865	0.135277	0.136243	0.137595	0.138809	0.139766	0.142892	0.152009	0.165085	0.184008	0.204779	0.216321	0.213629	0.201536
68	JAN19	5	0.117777	0.117813	0.117638	0.117258	0.116684	0.11593	0.115012	0.113953	0.112781	0.111529	0.110249	0.109021	0.107975	0.107281	0.107104	0.107485	0.108154	0.1082	0.10572	0.098614	0.117372
69	FEB19	5	0.112881	0.112422	0.111852	0.111186	0.11044	0.109634	0.108793	0.107944	0.107118	0.106346	0.105658	0.10508	0.104627	0.104304	0.104129	0.104196	0.104761	0.106099	0.108124	0.110062	0.109187
70	MAR19	5	0.110904	0.110631	0.110278	0.109854	0.109368	0.108832	0.108258	0.107663	0.10706	0.106468	0.105905	0.105392	0.104953	0.104622	0.104444	0.104489	0.104857	0.105668	0.107036	0.108992	0.111417
71	APR19	5	0.10809	0.107574	0.107033	0.106475	0.105912	0.105353	0.104812	0.104304	0.103847	0.103465	0.103188	0.103056	0.103118	0.103433	0.10406	0.105044	0.106389	0.108067	0.110045	0.11236	0.115235
72	MAY19	5	0.105579	0.105111	0.104658	0.10423	0.103841	0.10351	0.103257	0.103109	0.1031	0.10327	0.103672	0.104366	0.105428	0.106946	0.108994	0.111576	0.11455	0.117626	0.120464	0.122784	0.1244
73	JUN19	5	0.104925	0.104487	0.104053	0.10363	0.103227	0.102854	0.102525	0.102257	0.102069	0.101991	0.102061	0.102328	0.102858	0.103732	0.105344	0.113827	0.132657	0.149296	0.152049	0.147605	0.141
74	JLY19	5	0.104461	0.103989	0.103534	0.103101	0.102697	0.10233	0.102014	0.101772	0.101654	0.101774	0.102427	0.104366	0.109232	0.118224	0.127952	0.135121	0.139542	0.141586	0.141308	0.138806	0.135424
75	AUG19	5	0.097668	0.098607	0.101013	0.105886	0.112808	0.119246	0.123844	0.126684	0.128177	0.12872	0.128633	0.12806	0.126818	0.124703	0.12283	0.11967	0.119955	0.122531	0.122402	0.121438	0.120345
76	SEP19	5	0.095891	0.098739	0.102949	0.10767	0.111833	0.11503	0.117305	0.118802	0.119655	0.120047	0.120483	0.121901	0.124464	0.126067	0.123987	0.120129	0.12424	0.124367	0.123117	0.12151	0.120013
77	OCT19	5	0.097968	0.102968	0.106583	0.109246	0.111311	0.113036	0.114613	0.116177	0.117795	0.119485	0.121287	0.14334	0.146542	0.14161	0.138331	0.128485	0.123693	0.121735	0.120456	0.119551	0.118846
78	NOV19	5	0.10578	0.107914	0.109704	0.11154	0.11407	0.118191	0.124069	0.129463	0.130793	0.130821	0.134554	0.132975	0.131678	0.130817	0.129791	0.128363	0.126534	0.124524	0.122667	0.121243	0.120339
79	DEC19	5	0.008007	0.174758	0.221572	0.250794	0.268488	0.277779	0.280402	0.277274	0.268787	0.254847	0.234523	0.205296	0.164642	0.138071	0.12578	0.122696	0.121376	0.120851	0.120982	0.121563	0.122391
80	JAN20	5	0.244951	0.24052	0.234303	0.224824	0.210005	0.207353	0.201287	0.193932	0.183914	0.169714	0.151883	0.134548	0.123265	0.117752	0.115177	0.113961	0.113441	0.113336	0.113506	0.113866	0.114354
81	FEB20	5	0.216551	0.212866	0.208447	0.20266	0.194994	0.195441	0.193291	0.187185	0.179989	0.172194	0.16258	0.150096	0.135418	0.122478	0.114654	0.110735	0.108893	0.108142	0.108005	0.108237	0.108701
82	MAR20	5	0.187793	0.185387	0.187074	0.189902	0.185835	0.177989	0.177363	0.175221	0.172063	0.167554	0.161451	0.153441	0.143078	0.130014	0.115449	0.103569	0.096121	0.09151	0.088468	0.086416	0.085067
83	APR20	5	0.17587	0.175176	0.174143	0.172713	0.170892	0.168803	0.166797	0.165603	0.166059	0.168011	0.169244	0.166904	0.162035	0.157214	0.13568	0.12863	0.118109	0.107363	0.098392	0.091645	0.086356
84	MAY20	5	0.162728	0.162511	0.162142	0.161689	0.161233	0.160875	0.160753	0.161058	0.162015	0.163721	0.165865	0.167765	0.169453	0.174473	0.187495	0.20386	0.225443	0.251961	0.269994	0.272862	0.25451
85	JAN19	6	0.137294	0.1372	0.136871	0.136316	0.135552	0.134596	0.133476	0.132224	0.130877	0.129481	0.128103	0.126839	0.125837	0.125273	0.125285	0.125835	0.126477	0.126065	0.122378	0.114104	0.144694
86	FEB19	6	0.131865	0.131238	0.130498	0.129665	0.128762	0.127816	0.126858	0.125922	0.125043	0.124258	0.123598	0.123094	0.122777	0.122693	0.122932	0.123672	0.12515	0.127422	0.130115	0.132078	0.128793
87	MAR19	6	0.128685	0.128308	0.127848	0.127313	0.126716	0.126072	0.125397	0.124709	0.124026	0.12337	0.122765	0.122236	0.12182	0.121564	0.121536	0.121829	0.122568	0.123883	0.12586	0.128464	0.131493
88	APR19	6	0.127454	0.126822	0.126167	0.125499	0.124831	0.124177	0.123552	0.122975	0.12247	0.122066	0.121803	0.121733	0.121918	0.12243	0.123333	0.124662	0.126418	0.128589	0.131232	0.134588	0.139251
89	MAY19	6	0.124536	0.124011	0.123505	0.123032	0.122609	0.122259	0.122009	0.121895	0.121962	0.122268	0.122892	0.123935	0.125524	0.127788	0.130779	0.134352	0.138135	0.141663	0.144551	0.146559	0.14754
90	JUN19	6	0.12346	0.122992	0.12253	0.122085	0.121669	0.121293	0.120977	0.120742	0.12062	0.120653	0.120898	0.121432	0.122357	0.124196	0.134654	0.156394	0.174473	0.176669	0.171008	0.163587	0.158433
91	JLY19	6	0.122773	0.122207	0.121666	0.121158	0.120691	0.12028	0.119945	0.119726	0.119714	0.120131	0.121554	0.125375	0.133668	0.14501	0.154543	0.160827	0.164218	0.164895	0.162761	0.159068	0.153766
92	AUG19	6	0.114946	0.116734	0.12067	0.127311	0.134872	0.140977	0.145048	0.147402	0.148496	0.148741	0.148436	0.147643	0.146044	0.143673	0.142238	0.137526	0.140976	0.143473	0.143422	0.142572	0.141606
93	SEP19	6	0.113605	0.117161	0.12191	0.126815	0.130972	0.13412	0.136343	0.13779	0.138616	0.139092	0.13996	0.142321	0.145734	0.147171	0.144104	0.142102	0.145961	0.145753	0.144238	0.142492	0.141005
94	OCT19	6	0.116528	0.121492	0.125152	0.127918	0.130132	0.132061	0.133909	0.135809	0.137828	0.140029	0.142502	0.164425	0.16938	0.165099	0.160722	0.148602	0.143523	0.141899	0.140911	0.140173	0.139577
95	NOV19	6	0.123724	0.12586	0.127715	0.129622	0.132147	0.13618	0.142194	0.148057	0.14959	0.152361	0.155653	0.153877	0.1526	0.151787	0.150826	0.149432	0.147601	0.145584	0.143756	0.142407	0.141621
96	DEC19	6	0.138039	0.007746	0.255273	0.288445	0.308468	0.318974	0.321948	0.318426	0.308804	0.292839	0.269338	0.235965	0.193218	0.159895	0.145286	0.141584	0.140008	0.139381	0.139532	0.140214	0.141189
97	JAN20	6	0.281439	0.276402	0.268956	0.256835	0.243467	0.239301	0.232209	0.223697	0.212287	0.19684	0.177962	0.159077	0.146025	0.139392	0.136269	0.134818	0.134227	0.134141	0.134386	0.134858	0.135484
98	FEB20	6	0.247275	0.243142	0.237961	0.23139	0.222296	0.226912	0.222822	0.215614	0.207736	0.199146	0.188403	0.174592	0.15847	0.14393	0.134729	0.129991	0.12776	0.126882	0.126775	0.127133	0.12778
99	MAR20	6	0.213232	0.21464	0.219245	0.217151	0.203667	0.204798	0.202913	0.200331	0.196576	0.191385	0.184555	0.175812	0.164797	0.151292	0.136271	0.123078	0.113909	0.107828	0.103618	0.100666	0.098663
100	APR20	6	0.200494	0.199866	0.198726	0.197051	0.194936	0.192676	0.190911	0.190619	0.192414	0.195226	0.195609	0.1915	0.187315	0.179249	0.158966	0.14971	0.13863	0.126851	0.018457	0.018631	0.100807
101	MAY20	6	0.185556	0.185424	0.185105	0.184694	0.184306	0.184081	0.184214	0.184987	0.186712	0.189433	0.192576	0.195201	0.197447	0.203973	0.220798	0.240619	0.263996	0.294982	0.319407	0.327858	0.315234

Table B.2: WTI Bachelier Replication Error

	Month	Month_diff	z=-1.0	z=-0.9	z=-0.8	z=-0.7	z=-0.6	z=-0.5	z=-0.4	z=-0.3	z=-0.2	z=-0.1	z=0.0	z=0.1	z=0.2	z=0.3	z=0.4	z=0.5	z=0.6	z=0.7	z=0.8	z=0.9	z=1.0
0	JAN19	1	3.87E-10	4.39E-10	2.57E-10	-1.4E-09	2.4E-11	6.02E-10	2.07E-10	8.26E-10	1.71E-09	9.28E-10	1.53E-10	-6.3E-11	1.27E-09	1.1E-09	-6.6E-10	2.61E-11	-5.6E-10	2.49E-10	5.48E-11	6.17E-12	1.02E-10
1	FEB19	1	1.49E-10	4.94E-11	-2.8E-11	-3.3E-11	-9.9E-10	-1.1E-11	2.78E-11	9.72E-11	2.7E-11	1.32E-09	3.33E-11	6.82E-10	7.04E-11	-2.8E-11	-9.1E-12	1.53E-10	-3.2E-10	-9.1E-11	6.83E-11	2.12E-10	-5.9E-10
2	MAR19	1	-1.4E-10	3.18E-10	7.51E-12	-4.1E-12	1.81E-10	1.27E-10	1.06E-11	2.22E-11	-9.3E-10	2.58E-11	6.23E-11	4.7E-11	4.34E-10	-3.1E-10	-3.7E-12	3.14E-12	2.28E-11	6.15E-11	1.04E-11	-6E-10	-2.5E-11
3	APR19	1	7.59E-11	4.5E-10	-5.6E-10	-2.6E-10	1.46E-11	-1.9E-10	-5.5E-10	-2.7E-11	-1.2E-10	6.46E-10	-6.8E-12	4.09E-11	-3.6E-10	-4.2E-12	2.57E-11	-9.7E-11	1.04E-10	1.5E-10	2.21E-11	6.94E-11	-8.8E-10
4	MAY19	1	-6.1E-11	2.26E-11	1.7E-10	5.56E-10	5.1E-11	-1.8E-11	8.02E-10	1.13E-11	6.05E-12	3.14E-11	1.59E-10	1.47E-10	-1E-11	-2.5E-10	-2.5E-12	-2.5E-10	3.45E-10	6.8E-11	2.09E-11	1.26E-11	-2.7E-10
5	JUN19	1	1.58E-10	8.16E-11	9.94E-11	-3.4E-12	2.13E-11	8.58E-12	3.47E-11	1.95E-10	-1.4E-11	-6.6E-10	3.04E-10	1.97E-11	2.76E-11	5.34E-11	1.33E-11	6.6E-10	1.41E-10	4.23E-10	-1.5E-10	-1.1E-10	-1.3E-10
6	JLY19	1	9.55E-12	2.74E-11	-2E-11	2.17E-10	8.05E-11	4.5E-12	-2.3E-11	6.35E-12	2.79E-11	1.19E-10	8.25E-11	-8E-11	3.7E-10	4.95E-11	1.16E-10	8.59E-13	8.18E-12	-1.6E-11	-4.5E-11	2.9E-11	4.46E-10
7	AUG19	1	-2.9E-12	-1.1E-11	1.36E-11	-1.4E-11	1.03E-10	8.36E-12	1.04E-11	2.2E-11	-1.8E-11	8.73E-10	-2.3E-10	-6.1E-11	6.32E-11	-6.4E-10	2.28E-09	-6.4E-12	2.15E-09	-3.6E-10	1.6E-12	-4.7E-12	-9.1E-11
8	SEP19	1	-4E-11	2.78E-11	4.39E-12	-8E-11	1.99E-11	-2.6E-11	1.6E-10	4.23E-11	1.98E-11	5.61E-11	-1.2E-10	7.19E-10	1.06E-09	-1.5E-10	-2.2E-11	3.54E-10	-1.6E-09	3.13E-11	-1.3E-11	-9.6E-10	-3.4E-11
9	OCT19	1	-1.5E-11	6.63E-12	4.49E-11	-7.8E-11	-1.2E-10	1.8E-10	-2.3E-10	4.08E-11	-6.7E-10	-1.9E-11	-2.4E-12	-2.7E-10	-6.3E-11	7.02E-11	1.06E-09	1.59E-11	-1.6E-09	-1.4E-09	-2.7E-10	-1.5E-10	3.25E-12
10	NOV19	1	-1.3E-11	2.89E-12	-1.1E-10	4E-12	-6.7E-10	-7.4E-12	-8.2E-10	-8.4E-11	3.96E-13	9.43E-12	-9.7E-11	-7E-12	-6.9E-12	-2.3E-10	3.9E-11	2.33E-10	4.7E-10	2.6E-11	4.49E-10	-4.1E-10	-1.7E-10
11	DEC19	1	4.46E-12	8.72E-12	1.17E-11	9.89E-10	-6.1E-10	-1.8E-10	1.7E-10	9.79E-11	9.84E-10	-2.1E-09	-1.6E-09	-5.6E-10	-1.2E-10	5.61E-10	1.05E-09	-2.2E-10	-6.6E-11	5.04E-12	-3.2E-10	3.34E-10	-5E-12
12	JAN20	1	-1.9E-11	-3.2E-11	5.39E-10	2.08E-11	-1E-10	5.89E-10	-7.7E-10	3.3E-09	-1.9E-11	8.21E-11	-2.4E-10	2.84E-11	-7.5E-10	4.24E-10	-3.8E-10	5.2E-10	2.23E-10	8.86E-11	-1.1E-10	9.41E-12	-1.4E-10
13	FEB20	1	1.91E-11	-2.2E-09	2.6E-10	-2.2E-10	-9.3E-13	8.72E-11	1.11E-09	1.47E-10	9.59E-10	7.38E-10	1.15E-09	2.59E-10	1.43E-11	1.2E-10	8.33E-11	8.33E-11	-1.2E-11	2.27E-10	-4.9E-11	1.04E-10	-1.8E-11
14	MAR20	1	-6.2E-10	-1.2E-10	1.44E-09	2.47E-10	3.84E-09	1.56E-10	2.16E-11	-5.2E-10	3.99E-10	1.94E-11	8.51E-12	-1E-11	1.18E-11	1.57E-11	1.81E-11	4.14E-11	5.46E-11	1.78E-11	-2.2E-12	1.52E-12	-3.3E-16
15	APR20	1	2.13E-09	3.42E-10	-2.9E-10	-1.4E-10	1.9E-09	2.26E-10	2.75E-10	1.25E-10	7.97E-10	1.34E-09	-4.8E-11	-1.8E-09	-4.4E-10	-8E-10	1.06E-11	8.81E-11	2.91E-11	3.21E-13	1.07E-11	2.63E-12	3.23E-13
16	MAY20	1	2.76E-11	6.2E-12	-1.1E-09	8.34E-11	4.47E-10	3.71E-10	-3.9E-11	-7E-10	2.98E-10	1.85E-09	1.97E-09	-7.6E-11	3.92E-10	-1.6E-09	-9.1E-10	3.16E-10	1.58E-09	8.18E-11	2.03E-09	9.08E-10	7.2E-13
17	JAN19	2	1.41E-11	1.89E-11	1.08E-11	2.64E-09	5.43E-11	1.39E-10	1.68E-09	4.08E-10	2.31E-10	1.92E-10	-1.8E-09	-1.3E-09	1.27E-09	-2.5E-09	9.68E-10	5.98E-10	-1.9E-09	-5.7E-11	1.04E-10	1.65E-10	1.21E-10
18	FEB19	2	9.64E-12	1.7E-09	5.4E-11	7.69E-10	-2.7E-09	9.64E-11	2.69E-10	1.44E-10	2.67E-11	1.56E-09	3.67E-11	6.25E-12	-2.5E-10	9.31E-11	-2.3E-09	-1.2E-10	-1.5E-09	-2.1E-11	-2E-10	-5.2E-11	2.14E-09
19	MAR19	2	3.3E-11	1.5E-10	6.7E-10	-9.5E-10	8.11E-12	-1.9E-10	4.63E-12	4.41E-10	1.34E-09	7.67E-10	-8E-11	1.72E-10	1.71E-11	-3.8E-10	-4.9E-12	-2.8E-10	3.56E-10	2.92E-11	1.8E-10	6.7E-11	1.73E-09
20	APR19	2	5.11E-10	-1.1E-10	3.77E-10	-3.3E-11	-2.4E-10	-5.3E-10	1.69E-10	-1.8E-09	3.11E-11	6.68E-11	5.64E-11	1.65E-11	-7E-10	-5.7E-12	6.32E-11	9.93E-12	4.76E-11	7.53E-11	-9.6E-10	5.54E-11	-1.8E-09
21	MAY19	2	1.21E-11	1.33E-10	-1.4E-09	5.32E-10	-5.5E-10	7.5E-12	1.49E-11	1.04E-10	6.54E-10	1.38E-09	9.8E-10	-6.3E-11	1.38E-09	1.04E-09	-1.7E-10	1.19E-09	3.08E-11	2.83E-10	4.63E-10	3.07E-10	3.38E-11
22	JUN19	2	1.91E-11	-5.2E-10	-2.6E-11	2.45E-11	2.41E-10	3.46E-12	2.79E-11	6.26E-12	-2.7E-11	5.37E-10	1.11E-09	1.02E-10	1.14E-10	8.01E-11	6.51E-10	-1.1E-09	-8.7E-13	4.41E-11	7.99E-11	-2.7E-09	2.79E-09
23	JLY19	2	1.32E-10	1.01E-11	-2.5E-11	5.13E-11	-7.1E-11	2.45E-12	-5.6E-11	1.01E-11	9.04E-11	4.29E-10	4.46E-10	-8.8E-12	5.79E-10	4.42E-10	6.01E-10	1.03E-10	-1.1E-09	-6.5E-11	-4.7E-11	1.51E-09	-1.8E-09
24	AUG19	2	-1.7E-11	7.17E-12	-5.6E-11	1.85E-12	-1.5E-12	2.92E-11	5.11E-10	1.26E-09	7.53E-12	-1.9E-09	-1.6E-10	-4.6E-10	-7.2E-10	-5.3E-10	-6.6E-11	-6.9E-11	-3.1E-11	2.26E-09	-1.2E-10	2.05E-09	-1.4E-09
25	SEP19	2	2.19E-12	1.63E-11	1.07E-10	4.32E-11	4.3E-11	-6.7E-10	-3.9E-10	1.46E-09	3.98E-10	1.99E-09	-2.3E-10	-1.2E-10	-1.2E-09	-5.8E-11	1.62E-10	2.46E-10	-2.2E-09	8.32E-10	-1.7E-09	-8.9E-10	-2.4E-11
26	OCT19	2	-5.4E-11	1.86E-10	-5.6E-11	1.52E-10	9.86E-10	-2E-10	-3.1E-10	4.33E-10	1.62E-09	7.35E-10	1.63E-11	-2.7E-10	2.05E-09	1.95E-10	5.46E-11	-2.3E-10	3.86E-10	-1.5E-11	-1.6E-11	1.12E-09	5.3E-10
27	NOV19	2	2.23E-10	-2.1E-10	-2.4E-10	8.44E-11	7.86E-11	-1.5E-09	-2.4E-09	-5.4E-10	5.95E-10	1.79E-09	6.94E-11	2.23E-10	-2.8E-09	6.83E-11	3.01E-09	-1.5E-11	3.81E-13	1.07E-11	2.02E-10	9.03E-10	-1E-10
28	DEC19	2	-2.6E-13	0	2.15E-11	-1.2E-10	-1.8E-09	-4.5E-09	1.77E-09	4.27E-09	-1.4E-09	-4.2E-09	-6.8E-11	1.77E-11	-2.6E-09	1.58E-09	2.53E-10	-1.6E-09	6.02E-11	2.05E-12	-7.3E-10	-7.9E-12	1.15E-10
29	JAN20	2	2.49E-10	-3.1E-11	-2E-09	-2.9E-09	-3.9E-09	-5.2E-11	-1.6E-10	7.48E-10	8.74E-12	3.79E-09	1.2E-11	1.82E-09	7.64E-10	4.91E-12	-7.5E-11	2.41E-11	2.92E-11	5.69E-10	-1.5E-10	-2.3E-10	3.38E-11
30	FEB20	2	-8.6E-12	3.21E-09	-2.7E-09	-1E-09	4.84E-11	-1.3E-10	6.53E-09	-2.2E-10	-2.5E-09	-1.6E-10	-1.4E-09	-1.7E-10	-4.9E-11	2.24E-10	4.08E-11	4.72E-11	3.27E-10	-2.6E-10	-2.6E-11	8.33E-11	5.36E-12
31	MAR20	2	-1.4E-09	-1.2E-10	-2.8E-09	-2.8E-09	8.46E-11	2.84E-09	3.34E-10	1.61E-11	-9.9E-11	8.32E-11	3.78E-11	6.67E-10	-3.2E-10	-2.6E-11	1.43E-11	-1.8E-11	2.28E-11	1.28E-10	-8E-11	-1.7E-11	1.69E-11
32	APR20	2	2.89E-09	1.77E-10	-9.9E-10	-3.2E-10	4.63E-10	2.66E-10	1.82E-10	-3.5E-09	-5.4E-09	2.68E-09	4.25E-10	2.24E-11	-1.1E-10	-1.6E-11	-1.3E-09	1.73E-10	1.31E-11	-4E-11	1.04E-12	-3.2E-11	0
33	MAY20	2	1.68E-09	1.76E-10	-5E-10	1.52E-10	-4.5E-09	2.1E-10	-2.7E-11	-2.3E-10	-4.3E-11	-1.9E-09	-5.4E-10	-3.3E-10	2.87E-10	2.99E-10	-7E-10	-8.4E-11	1.42E-09	-1.4E-10	-1.5E-10	4.01E-10	1.04E-11
34	JAN19	3	2.91E-10	4.31E-10	2.66E-10	-1.3E-11	5.28E-10	1.52E-10	6.81E-10	2.38E-10	-2.1E-10	-9.5E-10	-1.6E-10	-3.7E-09	1.47E-09	-2.9E-12	1.31E-10	1.78E-09	2.57E-11	-1.9E-09	-2.7E-09	-1.6E-11	6.78E-11
35	FEB19	3	9.55E-11	5.92E-10	6.06E-12	2.19E-09	2.02E-11	2.19E-11	-3.5E-09	4.22E-10	2.1E-10	1.73E-09	-1.1E-10	3.77E-10	6.71E-11	2.79E-10	-4.8E-10	-1E-10	5.11E-10	1.27E-11	-1.1E-09	-2.4E-10	3.06E-11
36	MAR19	3	3.28E-10	1.5E-09	1.63E-11	7E-11	1.15E-10	3.43E-10	6.15E-10	-5.6E-10	1.87E-09	1.03E-10	1.41E-10	7.37E-11	1.6E-11	1.18E-09	1.75E-11	-1.1E-10	5.04E-11	2.85E-09	4E-11	2.88E-11	3.08E-10
37	APR19	3	8.67E-12	7.38E-11	4.28E-10	1.27E-09	-5.8E-11	1.79E-12	2.58E-10	1.04E-09	1.23E-09	6.75E-10	2.48E-10	8.18E-11	-6.5E-10	3.08E-10	5.6E-10	-2.4E-09	5.51E-11	7.97E-11	-2.9E-09	1.32E-11	4.55E-10
38	MAY19	3	1.32E-09	1.91E-11	8.17E-11	-5.8E-10	2.86E-10	2.42E-09	2.18E-09	5.36E-10	-5.2E-11	-2.1E-11	2.32E-11	8.35E-12	3.08E-11	8.13E-12	1.94E-10	7.95E-10	1.17E-09	5.67E-10	5.14E-11	9.99E-11	1.65E-10
39	JUN19	3	4.58E-12	1E-11	2.18E-10	-1.2E-11	1.85E-10	1.21E-09	5.62E-12	-1.7E-12	4.63E-11	2.95E-11	2.11E-10	5.08E-10	6E-10	2.93E-10	2.25E-11	-1.6E-09	7.05E-10	1.65E-10	2.01E-09	2.7E-10	7.34E-10
40	JLY19	3	1.48E-10	1.14E-11	-4.8E-10	5.26E-11	2.48E-12	-1.9E-10	1.33E-11	1.13E-10	7.01E-10	6.08E-10	9.42E-12	-1.4E-09	6.05E-11	-1.6E-11	-2.6E-10	5.83E-11	1.44E-09	1.34E-11	-5.1E-11	-5.7E-09	-3.9E-12
41	AUG19	3	1.46E-10	3.36E-11	1.19E-11	3.72E-11	7.16E-10	-1.2E-10	8.03E-11	2.2E-11	-1.3E-10	-2.5E-09	1.06E-09	-5.2E-09	-1.3E-09	3.86E-09	-8.4E-11	6.13E-09	9.37E-10	-6.6E-10	4.02E-09	-2E-10	1.02E-11
42	SEP19	3	6.02E-12	3.2E-10	3.32E-10	1.81E-10	3.39E-11	-3E-11	3.68E-11	1.7E-10	2.47E-09	3E-09	-1E-09	-1.6E-11	-5.1E-11	-4.3E-10	-6.8E-09	1.26E-10	-1.6E-10	6.1E-11	5.35E-11	-6.4E-10	-1.4E-10
43	OCT19	3	1.54E-10	-3.9E-10	-6.9E-12	-1.7E-10	-5.8E-10	1.51E-11	-3.2E-10	1.93E-11	5.92E-10	1.58E-09	2.83E-09	-3.1E-09	5.23E-09	1.24E-09	1.94E-09	-8.9E-10	-1.1E-10	-8.7E-11	-2E-10	-6.2E-10	4.67E-10
44	NOV19	3	-3.1E-10	4.29E-11	-5.5E-10	4.6E-11	-7E-10	-2.9E-10	1.57E-09	2.23E-10	-5.3E-10	-1.9E-10	1.45E-10	-7.3E-10	-5.1E-10	4.42E-10	-2.6E-09	-2.6E-11	9.16E-11	-1.3E-09	9.66E-11	1.02E-10	4.12E-11
45	DEC19	3	1.89E-12	-8E-283	-1.3E-10	3.15E-09	3.68E-10	2.78E-10	4.07E-09	6.51E-11	-2.4E-10	-2.6E-09	-2E-10	-7.1E-09	-2.3E-10	-2.1E-09	-4.7E-10	6.84E-11	7.55E-11	5.67E-10	9.97E-12	-4.7E-10	2.01E-10
46	JAN20	3	1.78E-10	3.97E-11	-4.3E-09	-1.7E-10	-5.3E-10	1.43E-10	3.12E-10	5.66E-09	5.31E-09	-6.3E-10	-1.4E-10	-6.3E-10	1.93E-09	4.41E-10	-1.2E-10	-7.3E-10	1.37E-10	-1E-11	-4.4E-10	-4.4E-10	-2E-10
47	FEB20	3	1E-08	-3.7E-11	-1.4E-11	-5.6E-09	8.99E-09	3.34E-09	3.57E-10	-1.6E-10	-7.8E-10	-2.9E-10	-1.2E-09	1.74E-11	6.73E-10	1.05E-09	-5.8E-10	8.59E-10	-3.9E-10	-1.2E-10	4.79E-11	1.69E-10	-4E-12
48	MAR20	3	-6.1E-11	-9.7E-10	8.61E-11	5.67E-10	2.32E-10	2.72E-09	-5.9E-09	-3.5E-10	-5.4E-11	2.81E-09	5.14E-10	-2.2E-09	-2.1E-11	-2.7E-11	1.16E-10	9.9E-12	4.12E-11	1.19E-10	8.65E-12	-1.4E-12	1.17E-12
49	APR20	3	-7.8E-10	-7.6E-10	-6.9E-12	2.35E-11	5E-10	5.66E-10	6.35E-10	1.85E-10	2.24E-10	-1.1E-09	2.86E-09	-3.8E-09	2.04E-11	-3.5E-11	6.63E-11	2.87E-10	3.63E-12	1.19E-11	-2.1E-12	2.9E-12	0
50	MAY20	3	2.6E-10	3.11E-11	-5.4E-11	1.74E-10	9.03E-11	-2.3E-10	1.18E-11	6.85E-11	4.98E-09	-1.1E-09	-4E-10	6.15E-11	4.39E-11	-5.6E-11	-5.4E-11	-4.9E-09	-1.7E-10	-1.6E-10	7.72E-09	2.98E-09	1.85E-09
51	JAN19	4	3.22E-09	-3.3E-09	1.54E-09	2.82E-09	4.78E-10	-1.9E-10	-2.1E-10	-6.7E-10	-3E-11	-4.5E-10	5.67E-10	-3E-11	2.28E-10	1.54E-09	-6.4E-12	-1.6E-10	4.5E-11	9.43E-11	7.7E-10	3.77E-10	2.24E-09
52	FEB19	4	2.67E-09	-1.1E-09	2.93E-09	1.31E-10	-3.5E-10	1.77E-09	1.01E-09	1.44E-10	5.4E-10	1.97E-09	1.38E-10	3.01E-09	-4.2E-09	-2.5E-09	-1.2E-09	3.67E-09	-1.2E-10	-4.7E-10	3.02E-11	1.23E-10	7.8E-10
53	MAR19	4	-1.1E-09	4.96E-11	2.77E-10	1.04E-09	1.92E-09	-3.3E-11	4.02E-11	1.57E-09	-4.3E-09	4.33E-09	1.33E-09	-1.6E-11	9.8E-12	-3.7E-09	3.39E-10	-4.1E-11	1.37E-11	6.15E-10	3.11E-09	3.13E-09	2.9E-10
54	APR19	4	1.01E-10	7.8E-10	-1.7E-09	1.55E-11	6.92E-11	1.36E-09	9.38E-10	-3.6E-09	2.5E-10	4.09E-09	3.55E-09	5.94E-10	-4.1E-11	3.23E-09	1.44E-11	1.8E-09	1.69E-11	1.99E-11	1.06E-09	-3.3E-09	-2.1E-09
55	MAY19	4	3.74E-11	2.72E-10	9.05E-10	-3.7E-11	2.6E-11	-1.3E-11	2.27E-10	5.39E-10	4.04E-10	1.11E-10	4.88E-12	-6E-10	2.81E-09	-1.4E-09	1.22E-10	3.87E-10	3.02E-10	4.32E-11	6.05E-10	9.6E-11	3.2E-09
56	JUN19	4	-5.7E-10	-6.4E-10	5.02E-11	9.37E-10	4.22E-11	8.92E-10	1.15E-11	5.35E-10	-6.4E-10	-4.6E-10	9.29E-10	1.56E-09	1.21E-09	2.87E-10	-1.6E-09	2.38E-09	2.37E-10	-3.7E-11	-6.1E-09	-6.2E-10	-1.3E-09
57	JLY19	4	-4.3E-10	1.15E-11	-4.1E-10	-7.3E-10	5.61E-11	8.74E-10	4.76E-11	1.39E-09	7.2E-12	8.25E-11	6.06E-10	-4.3E-11	-1.2E-10	-1.1E-10	7.07E-10	-1.8E-11	-4.8E-09	3.97E-09	-6.9E-10	-2E-10	-2.5E-10
58	AUG19	4	-1.4E-10	2.97E-10	-3.2E-12	5.89E-11	1.35E-09	2.13E-11	2.78E-09	-3.5E-09	1.87E-09	-1.4E-10	-1.1E-10	7.33E-10	1.03E-09	-1.7E-10	-5.9E-09	3.8E-10	4.14E-10	-4.9E-10	4.35E-09	-8.6E-10	2.88E-10
59	SEP19	4	-2.6E-11	-1.6E-10	1.43E-11	2.1E-10	4.53E-12	-1.3E-09	3.44E-11	6.39E-10	-2E-09	-2.7E-11	-5E-11	-1.9E-11	-2.7E-10	8.03E-09	4.51E-09	2.65E-09	-6.1E-09	4.95E-10	1.13E-11	7.71E-10	8.73E-10
60	OCT19	4	-2.2E-10	-6.2E-10	-3.5E-10	-1E-09	-1.7E-09	2.32E-11	1.94E-10	1.75E-09	5.61E-10	8.27E-11	6.14E-11	-6.6E-12	-1.8E-09	5.16E-09	2.56E-09	-5.8E-10	8.57E-11	-1.5E-09	-8.3E-10	-4.8E-10	-1.4E-09
61	NOV19	4	-3.1E-12	4.1E-11	-8E-10	1.29E-10	-8.4E-11	-3E-10	-3.4E-09	2.88E-09	-7.5E-09	-2.3E-10	9.15E-09	4.36E-09	-3.1E-09	2.26E-11	6.92E-10	-6.4E-09	1.03E-09	1.43E-09	1.73E-11	-1.7E-09	1.86E-09
62	DEC19	4	-2.4E-11	-5E-10	6.89E-10	1.76E-09	1.94E-09	-2E-09	-2E-09	3.61E-09	-4.3E-10	-3E-09	1.2E-09	8.93E-11	3.79E-09	6.83E-10	1.02E-10	3.66E-09	7.75E-10	9.36E-12	3.58E-11	-6.8E-10	6.46E-11
63	JAN20	4	1.12E-09	-7.2E-09	-3.3E-10	-3.8E-09	1.71E-10	3.6E-09	5.59E-10	1.19E-10	2.74E-10	-7.9E-11	4.53E-09	4.84E-11	-2.2E-09	-4.5E-11	4.18E-10	-2E-09	4.56E-10	4.13E-11	9.49E-10	7.98E-11	8.52E-12
64	FEB20	4	1.9E-09	4.49E-11	1.56E-10	-1.2E-08	2.89E-09	3.79E-09	2.7E-09	-5.1E-09	4.4E-09	-2.9E-11	1.47E-09	-1.4E-09	9.58E-11	1.52E-09	1.66E-10	-1E-10	2.5E-11	-1.5E-10	-4.7E-10	-2.6E-10	-7.6E-12
65	MAR20	4	7.14E-11	1.91E-09	1.58E-10	9.44E-09	8.61E-09	7.59E-10	-8.9E-11	1.31E-10	2.12E-10	5.29E-11	1.88E-10	2.7E-09	5.26E-11	1.24E-09	-6.7E-10	6.61E-11	2.07E-10	1.31E-10	-2.5E-11	-1.4E-11	2.33E-12
66	APR20	4	-1.5E-10	2.36E-09	1.02E-10	-9.2E-10	1.84E-09	1.18E-08	3.47E-09	-5.7E-09	-6.3E-09	6.41E-10	-9.8E-10	2.11E-11	-4E-09	2.31E-09	-1.5E-09	-4.8E-10	3.53E-10	-1.8E-10	-2.4E-11	3.35E-13	0
67	MAY20	4	3.25E-11	8.09E-11	-2.6E-09	3.91E-11	1.34E-09	-1.3E-10	-3.4E-10	-2.7E-09	-1.6E-10	-6.1E-09	-3.7E-11	5.41E-10	3.25E-10	-4.2E-10	4.12E-09	3.05E-09	1.76E-09	1.19E-10	5.07E-09	1.43E-09	-4.5E-09
68	JAN19	5	1.53E-10	1.85E-10	-9.4E-10	-1.8E-11	-2.5E-09	6.99E-09	-9.8E-11	-6.7E-10	-2.2E-09	-2.6E-09	-3.9E-10	-1.1E-10	-2.5E-11	-6.2E-11	2.42E-09	1.68E-09	-3.1E-09	5.22E-10	2.16E-09	2.46E-09	-8.1E-10
69	FEB19	5	1.11E-10	-7.6E-11	-2.1E-11	-4.7E-09	3.9E-10	1.12E-09	1.17E-09	5.08E-10	7.64E-11	2.25E-09	3.56E-09	1.95E-12	2.36E-11	7.25E-11	-3.5E-11	-6.7E-10	-9.5E-10	-5.6E-10	-2.2E-11	-1.5E-10	1.94E-09
70	MAR19	5	1.11E-10	6.75E-10	3.37E-09	2.5E-11	1.62E-11	-3.5E-12	2.74E-11	3.53E-11	2.38E-10	1.27E-10	2.78E-09	3.5E-11	-3.2E-09	9.95E-10	-2.1E-09	3.52E-09	1.88E-09	1.05E-10	3.82E-10	2.42E-10	2.29E-09
71	APR19	5	2.42E-09	4.4E-11	2.58E-10	5.38E-10	5.66E-12	2.5E-11	1.74E-09	-1.4E-09	3.93E-11	3.87E-11	-1.2E-09	2.36E-09	1.29E-10	-4.4E-09	-2.7E-09	7.46E-10	4.36E-09	-3.8E-09	1.09E-09	2.25E-11	-2.1E-09
72	MAY19	5	-1.9E-10	-3E-10	-3.9E-12	-3.8E-11	-3.7E-09	3.72E-11	1E-10	1.13E-10	7.09E-11	2.2E-11	2.86E-09	-1.3E-11	1.1E-11	3.47E-09	3.61E-11	4.55E-11	4.98E-09	1.37E-10	6.08E-11	4.97E-10	6.52E-09
73	JUN19	5	2.21E-10	1.1E-10	1.17E-09	3.11E-11	2.29E-09	3E-10	2.37E-09	-5.9E-11	-1.5E-09	1E-10	2.72E-10	3.67E-10	-4.5E-10	-9.9E-12	-9.7E-11	3.99E-09	-2E-09	-6.7E-10	9.64E-09	-1.8E-10	-4.9E-11
74	JLY19	5	-4.1E-10	-3.6E-10	2.77E-10	2.57E-11	2.06E-11	3.27E-10	1.09E-11	1.65E-10	1.68E-09	1.12E-09	1.19E-10	-1.4E-11	-2.2E-11	-2.9E-10	9.56E-11	-8.8E-10	-1.3E-09	-1.2E-10	-2.4E-09	1.12E-08	3.97E-11
75	AUG19	5	1.32E-10	-1.5E-11	8.87E-11	2.5E-11	-4.4E-11	5.05E-11	2.47E-10	-2.1E-10	-6.8E-10	6.23E-09	-2.2E-11	1.69E-10	5.49E-09	1.02E-08	-1E-08	-2.1E-11	1.2E-09	5.07E-09	2.69E-09	8.08E-10	5.31E-10
76	SEP19	5	-4E-12	-8.4E-11	8.69E-11	4.41E-11	1.79E-10	7.3E-11	2.72E-09	2.61E-10	2.54E-11	1.04E-09	3.44E-10	4.97E-09	-1.2E-10	-2.3E-10	6.87E-11	1.08E-09	-2E-10	3E-10	-3.2E-09	4.39E-11	-4.1E-09
77	OCT19	5	-4.9E-12	1.62E-11	1.23E-10	-1.9E-10	6.69E-11	2.31E-09	3.44E-10	4.38E-10	1.09E-10	2.51E-09	3.06E-11	-1.2E-11	-2.3E-09	7.63E-09	2.21E-09	-1.4E-09	1.53E-11	-1.1E-10	8.13E-10	-2.9E-10	4.31E-10
78	NOV19	5	1.79E-10	-1.2E-11	8.93E-11	-9.7E-10	-4.3E-11	1.07E-10	1.81E-09	-3.5E-10	-3.3E-10	8.46E-11	5.9E-10	-5E-11	5.43E-09	-1E-09	9.19E-09	1.39E-09	-3.4E-10	7.4E-10	4.23E-09	1.61E-09	2.95E-10
79	DEC19	5	-7E-180	-2.7E-10	1.94E-10	-7.2E-10	-3.5E-10	2.88E-09	2.8E-09	1.6E-09	-1.5E-08	-1.4E-10	7.43E-10	2.82E-09	1.53E-10	4.49E-09	6.45E-11	1.41E-09	-1.2E-09	1.14E-10	-1.1E-09	5.36E-10	8.23E-11
80	JAN20	5	4.58E-10	5.44E-10	1.03E-09	-2.8E-10	1.49E-08	-1.8E-08	-2E-09	3.77E-11	-8.2E-09	6.35E-10	8.48E-11	2.14E-09	3.63E-11	1.36E-09	-6.1E-11	-5.1E-10	6.57E-10	-7.5E-10	-6.9E-11	-5.5E-10	1.34E-10
81	FEB20	5	1.56E-10	-4.4E-09	-3E-09	9.96E-10	1.98E-10	1.26E-08	-8.7E-09	1.31E-08	8.76E-09	8.63E-11	1.14E-09	-2.6E-10	-2.3E-09	1.15E-10	1.13E-10	7.55E-10	3.1E-10	6.22E-11	6.05E-12	-2.5E-11	-3.4E-11
82	MAR20	5	-1.3E-08	-8.7E-09	1.48E-08	5.48E-10	-1.3E-08	1.9E-09	-9.8E-10	3.3E-11	4.44E-09	-4.4E-09	4.97E-11	-3.2E-09	9.99E-11	1.79E-09	1.76E-10	-1.1E-11	-3.4E-12	-6.4E-12	-2.2E-11	1.1E-11	2.37E-11
83	APR20	5	1.31E-10	3.46E-10	-2.3E-11	2.17E-09	1.69E-10	4.46E-10	1.41E-10	-2.3E-09	-2.5E-10	1.98E-11	1.3E-09	8.17E-09	-6.4E-09	2.19E-10	3.48E-11	-6.9E-10	1.81E-11	4.59E-12	-1.3E-10	4.1E-11	2.48E-12
84	MAY20	5	-1E-09	1.55E-09	4.08E-10	2.93E-09	5.04E-09	-1.1E-10	-1.4E-10	8.85E-09	-9E-09	-1.8E-10	2.55E-09	-3.2E-09	1.09E-08	7.61E-10	1.33E-10	1.98E-10	2.79E-09	-1.8E-10	-1.2E-09	-7.7E-09	-2.3E-09
85	JAN19	6	-5.3E-09	4.1E-09	1.71E-09	-5.5E-12	1.66E-09	-9.6E-11	-6.4E-10	-9.1E-10	1.38E-10	-1.5E-10	-4.6E-10	-4.2E-11	-2.7E-11	-1.1E-10	-1.2E-09	2.12E-10	-5.5E-10	-8.8E-10	8.46E-10	-2.4E-09	1.01E-10
86	FEB19	6	1.4E-09	-2.9E-09	4.17E-10	-1.9E-09	4.83E-11	-4.3E-10	-3.2E-09	1.45E-09	-5.3E-09	6.91E-10	-2.7E-09	2.91E-11	1.21E-11	3.88E-09	-5.4E-10	-4.8E-09	1.91E-11	2.56E-09	-5.8E-09	-5.7E-09	-3.9E-11
87	MAR19	6	2.62E-09	4.05E-11	2.52E-10	7.86E-10	3.62E-10	7.53E-12	-3.3E-10	-5.3E-09	2.36E-10	4.6E-10	3.89E-10	1.58E-10	-1.3E-09	9.68E-10	1.98E-09	-5E-10	9.1E-10	-6.5E-10	9.26E-11	5.69E-09	1.72E-10
88	APR19	6	1.93E-10	1.52E-09	-1.2E-09	1.71E-11	5.48E-11	4.22E-11	3.69E-10	6.63E-10	6.02E-10	6.46E-10	6.77E-10	2.96E-10	2.24E-11	3E-11	3.86E-09	2.31E-10	1.31E-09	1.86E-09	2.68E-10	2.23E-09	-1.5E-09
89	MAY19	6	-9.7E-10	-1.5E-09	1.15E-10	-1E-09	-1.4E-10	4.66E-10	-3.7E-11	-1E-10	4.83E-10	8.95E-10	3.47E-10	3.9E-09	6.44E-11	2.79E-09	-4.5E-09	2.35E-09	7.07E-11	-2E-10	2.94E-11	1.19E-09	1.89E-09
90	JUN19	6	3.36E-10	6.61E-11	4.9E-10	-6E-10	-5E-11	1.06E-09	1.71E-11	-1.2E-09	1.82E-10	9.91E-10	1.77E-09	1.26E-09	2.48E-10	2.94E-09	3.15E-11	5.57E-09	-2.9E-10	3.15E-10	1.15E-09	1.61E-09	3.02E-10
91	JLY19	6	1.08E-11	4E-10	4.1E-11	1.18E-09	9.11E-11	6.35E-10	1.24E-11	-4.7E-10	-1.7E-09	3.91E-10	2.33E-11	1.05E-11	-2.7E-11	-4.1E-10	4.39E-10	-5.2E-11	3.27E-09	2.83E-10	-3.1E-09	-7.7E-11	-9E-11
92	AUG19	6	3.08E-10	1.27E-11	-7E-11	6.97E-12	5.93E-11	-1.7E-09	2.77E-09	1.48E-09	2.73E-09	4.43E-10	-1.5E-11	-3.5E-10	-9.1E-10	2.84E-10	1.27E-10	-2.7E-10	1.63E-09	1.69E-10	8.49E-10	9.57E-10	1.34E-10
93	SEP19	6	1.01E-11	-3.8E-11	2.34E-11	1.57E-10	-1.1E-10	2.92E-10	-1.1E-09	8.81E-11	3.6E-11	9.65E-10	8.29E-11	1.69E-09	-1.2E-11	-1.6E-10	-3.4E-09	2.71E-09	6.86E-10	4.87E-11	-2.5E-10	1.61E-09	-2.6E-09
94	OCT19	6	9.85E-12	-4.5E-11	-6.6E-10	9.12E-10	2.12E-10	-1.1E-09	1.61E-10	7.56E-11	7.74E-10	1.22E-11	3.1E-10	-4.4E-11	1.73E-09	5.28E-10	-1.4E-08	-9.7E-09	1.94E-10	9.85E-11	5.58E-11	2.86E-09	7.63E-10
95	NOV19	6	-7.8E-11	2.51E-11	3.2E-10	2.87E-11	2.17E-09	6.7E-11	9.42E-11	-8.6E-10	1.33E-10	1.19E-10	8.95E-09	-1.1E-09	-6.3E-10	-5.2E-09	-2E-09	1.24E-10	-2.6E-09	6.06E-10	1.68E-09	3.57E-10	3.46E-11
96	DEC19	6	-1.3E-11	-2E-202	-3.4E-10	-1.8E-10	-3.6E-09	1.94E-08	-8.6E-09	-8.2E-10	-1.1E-08	-1.9E-09	9.83E-09	-3.6E-09	-2.5E-09	1.28E-09	-2.9E-11	-2.2E-10	9.95E-10	-3.4E-09	-1.1E-09	-6.2E-10	2.6E-11
97	JAN20	6	4.55E-11	1.11E-09	9.27E-09	-1.5E-10	2.02E-08	-5.1E-09	6.91E-09	-1.4E-08	2.15E-09	1.4E-10	6.54E-11	1.03E-10	-1.1E-09	-2.2E-10	-1.4E-11	-3.9E-10	1.39E-09	-2.6E-11	1.51E-10	-1.7E-11	-1.2E-10
98	FEB20	6	-3E-10	5.25E-10	-1.6E-09	-1.9E-09	8.22E-10	-8.1E-09	-1.2E-10	-2.2E-12	2.21E-09	2.79E-09	1.39E-10	1.28E-09	6.52E-10	7.15E-10	-1.5E-09	5.66E-10	-3.6E-11	-4.1E-11	1.15E-11	3.67E-10	1.48E-10
99	MAR20	6	-3.3E-09	-2.9E-09	3.19E-10	3.16E-10	5.28E-10	-4.3E-09	5.21E-09	2.65E-09	-1.5E-09	1.16E-10	5.84E-09	1.23E-10	-3.8E-10	-6.4E-10	-2E-11	1.08E-11	-2.5E-11	-1.7E-11	1.02E-10	3.07E-11	-1.3E-11
100	APR20	6	6.7E-11	1.93E-11	-4E-11	2E-10	7.26E-09	-1.4E-08	5.42E-09	1.3E-10	3.49E-11	-1.7E-09	4.98E-09	5.65E-11	8.38E-09	-4.7E-10	-3.1E-09	-6.8E-11	-1.3E-11	8.41E-12	0	0	7.92E-12
101	MAY20	6	-6.4E-09	1.87E-09	-8.2E-10	3.85E-10	9.17E-10	6.34E-09	5.71E-09	-6.4E-10	-4.1E-10	3.39E-11	5.5E-09	-1.4E-08	-9.2E-09	1.46E-09	2.39E-09	-1E-09	8.15E-10	-3.1E-10	6.51E-10	1.4E-09	-2.6E-09

Table B.3: WTI Black Model Breaking Good Rho

	Month	Month_diff	z=-1.0	z=-0.9	z=-0.8	z=-0.7	z=-0.6	z=-0.5	z=-0.4	z=-0.3	z=-0.2	z=-0.1	z=0.0	z=0.1	z=0.2	z=0.3	z=0.4	z=0.5	z=0.6	z=0.7	z=0.8	z=0.9	z=1.0
0	JAN19	1	0.999662	0.999655	0.99965	0.999647	0.999645	0.999644	0.999645	0.999646	0.999649	0.999652	0.999655	0.999659	0.999662	0.999664	0.999664	0.999661	0.999653	0.999644	0.999641	0.99964	0.999525
1	FEB19	1	0.999828	0.999823	0.999819	0.999816	0.999814	0.999812	0.99981	0.999809	0.999809	0.999808	0.999808	0.999807	0.999806	0.999803	0.999799	0.999793	0.999782	0.999767	0.999748	0.999729	0.999718
2	MAR19	1	0.999995	0.999979	0.999965	0.999953	0.999942	0.999932	0.999923	0.999915	0.999908	0.999902	0.999897	0.999892	0.999888	0.999884	0.99988	0.999877	0.999872	0.999867	0.99986	0.999849	0.999831
3	APR19	1	1	1	1	0.999994	0.999986	0.999978	0.999971	0.999965	0.999959	0.999954	0.99995	0.999945	0.999941	0.999937	0.999933	0.999929	0.999924	0.999917	0.999907	0.999894	0.999877
4	MAY19	1	1	0.999998	0.999987	0.999977	0.999967	0.999958	0.99995	0.999942	0.999935	0.999928	0.999921	0.999914	0.999906	0.999898	0.999889	0.999878	0.999865	0.999849	0.999829	0.999804	0.999776
5	JUN19	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999997	0.999987	0.999976	0.999964	0.99995	0.999934	0.999863	0.999658	0.999609
6	JLY19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999992	0.999964	0.999911	0.999818	0.999728	0.999666	0.999628	0.999611
7	AUG19	1	1	1	1	1	1	1	0.999999	0.999919	0.99985	0.999802	0.999768	0.999745	0.999728	0.999715	0.999706	0.999704	0.999707	0.999721	0.999705	0.999701	0.999711
8	SEP19	1	1	1	1	1	1	1	0.999978	0.999935	0.9999	0.999872	0.999851	0.999834	0.999818	0.999799	0.999783	0.999786	0.99979	0.99977	0.99977	0.999777	0.999784
9	OCT19	1	1	1	1	1	1	0.999992	0.999959	0.99993	0.999903	0.999878	0.999856	0.999679	0.999657	0.999663	0.999697	0.99974	0.999777	0.999802	0.999812	0.999818	0.999822
10	NOV19	1	1	1	1	1	1	0.999919	0.999837	0.999778	0.999751	0.999766	0.9998	0.999775	0.999778	0.999786	0.999796	0.999808	0.999817	0.999822	0.999825	0.999828	0.99983
11	DEC19	1	1	1	1	1	0.99973	0.999324	0.999085	0.998973	0.998965	0.999049	0.999216	0.999464	0.999808	0.99987	0.999879	0.999872	0.999859	0.999841	0.999821	0.999798	0.999778
12	JAN20	1	0.999068	0.999072	0.999093	0.99913	0.999183	0.999251	0.999332	0.999435	0.999538	0.999605	0.999674	0.999785	0.999892	0.99991	0.999908	0.999903	0.999897	0.99989	0.999879	0.999867	0.999853
13	FEB20	1	0.999176	0.999208	0.999247	0.999295	0.999355	0.999429	0.999524	0.99951	0.99954	0.999601	0.999673	0.999758	0.999858	0.999927	0.999948	0.99995	0.999947	0.999942	0.999936	0.999928	0.999918
14	MAR20	1	0.999256	0.999303	0.999358	0.999414	0.999453	0.999464	0.999476	0.999544	0.999529	0.999564	0.999628	0.999726	0.999866	0.999967	0.999989	0.999986	0.999974	0.999959	0.999943	0.999926	0.999909
15	APR20	1	0.99945	0.999456	0.999465	0.999476	0.999489	0.999504	0.999522	0.999543	0.999567	0.99959	0.999605	0.999599	0.999593	0.999644	0.999768	1	1	1	1	1	1
16	MAY20	1	1	1	1	1	1	1	1	1	0.999998	0.999975	0.999953	0.99993	0.999906	0.999878	0.99982	0.999717	0.999487	0.999341	0.999457	0.999648	1
17	JAN19	2	0.998931	0.998906	0.998889	0.998878	0.998873	0.998872	0.998876	0.998884	0.998895	0.998908	0.998924	0.998939	0.998953	0.998963	0.998966	0.998956	0.998928	0.998881	0.99883	0.998826	0.998873
18	FEB19	2	0.99942	0.999401	0.999386	0.999374	0.999364	0.999357	0.999351	0.999347	0.999344	0.999342	0.99934	0.999337	0.999332	0.999325	0.999314	0.999296	0.999267	0.999218	0.999143	0.99905	0.998966
19	MAR19	2	1	0.999997	0.99995	0.999907	0.999869	0.999836	0.999805	0.999779	0.999755	0.999734	0.999715	0.999698	0.999683	0.99967	0.999657	0.999644	0.99963	0.999613	0.99959	0.999556	0.999502
20	APR19	2	1	1	1	1	0.999975	0.999944	0.999915	0.999888	0.999864	0.999841	0.99982	0.999801	0.999781	0.999762	0.999741	0.999718	0.99969	0.999654	0.999607	0.999549	0.999479
21	MAY19	2	1	1	1	1	1	0.999968	0.999937	0.999907	0.999878	0.99985	0.999821	0.999792	0.99976	0.999724	0.999683	0.999635	0.999577	0.999509	0.999427	0.999329	0.999218
22	JUN19	2	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999977	0.999929	0.999875	0.999811	0.999661	0.999045	0.998566	0.998175
23	JLY19	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999921	0.999549	0.999149	0.998878	0.998715	0.99864	0.998649
24	AUG19	2	1	1	1	1	1	1	0.99979	0.999543	0.999362	0.999235	0.999146	0.999081	0.99903	0.998993	0.998984	0.998997	0.99905	0.998982	0.998963	0.998994	0.999033
25	SEP19	2	1	1	1	1	1	1	0.999892	0.999745	0.999627	0.999534	0.99946	0.999393	0.999312	0.99922	0.999173	0.999222	0.999169	0.999134	0.999157	0.999191	0.99922
26	OCT19	2	1	1	1	1	1	1	0.999987	0.999867	0.999757	0.999656	0.999399	0.998777	0.998906	0.998925	0.999151	0.999262	0.999317	0.999346	0.999363	0.999377	0.99939
27	NOV19	2	1	1	1	1	1	0.999886	0.99958	0.999343	0.999231	0.999283	0.999253	0.99919	0.999184	0.999196	0.999227	0.999269	0.999306	0.99933	0.999342	0.999348	0.999353
28	DEC19	2	1	1	1	1	0.998435	0.996986	0.996166	0.99583	0.995897	0.996315	0.997047	0.998123	0.999313	0.99964	0.99965	0.999615	0.999562	0.999497	0.999414	0.99932	0.999228
29	JAN20	2	0.996295	0.996344	0.99646	0.996641	0.996886	0.997205	0.99768	0.997981	0.998245	0.998509	0.998915	0.999476	0.999704	0.99973	0.999715	0.999692	0.999666	0.999636	0.999598	0.999549	0.999492
30	FEB20	2	0.996981	0.997107	0.997262	0.997453	0.997703	0.998031	0.998085	0.998121	0.998316	0.99857	0.998864	0.999236	0.999633	0.999838	0.999886	0.99988	0.999858	0.999834	0.999808	0.999778	0.999737
31	MAR20	2	0.997509	0.997691	0.997863	0.997947	0.997911	0.997898	0.9981	0.998156	0.998235	0.998403	0.998669	0.999068	0.999628	0.999981	1	1	0.999981	0.999924	0.999862	0.999798	0.999732
32	APR20	2	0.998173	0.998175	0.998186	0.998206	0.998235	0.998273	0.998323	0.998383	0.998444	0.998477	0.99844	0.998326	0.998285	0.998544	0.999241	1	1	1	1	1	1
33	MAY20	2	1	1	1	1	1	1	1	1	1	1	1	0.999891	0.999758	0.999615	0.999348	0.998941	0.998051	0.997141	0.997156	0.997758	1
34	JAN19	3	0.997886	0.997836	0.9978	0.997777	0.997766	0.997766	0.997776	0.997793	0.997817	0.997846	0.997878	0.997909	0.997935	0.99795	0.997946	0.997915	0.997849	0.997748	0.997637	0.997593	0.997735
35	FEB19	3	0.99883	0.998797	0.998771	0.998751	0.998737	0.998727	0.99872	0.998716	0.998714	0.998713	0.998711	0.998707	0.998701	0.998689	0.99867	0.998638	0.998587	0.998505	0.998375	0.998193	0.997989
36	MAR19	3	1	1	0.99996	0.999868	0.999786	0.999712	0.999646	0.999587	0.999534	0.999487	0.999445	0.999407	0.999372	0.99934	0.999308	0.999276	0.999239	0.999193	0.999129	0.999035	0.998898
37	APR19	3	1	1	1	1	1	1	0.999935	0.999875	0.999819	0.999766	0.999716	0.999668	0.99962	0.99957	0.999516	0.999453	0.999376	0.99928	0.999159	0.999014	0.998849
38	MAY19	3	1	1	1	1	1	1	0.999966	0.999898	0.999832	0.999766	0.999698	0.999625	0.999545	0.999454	0.999348	0.999223	0.999074	0.998895	0.998679	0.99843	0.998163
39	JUN19	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999946	0.999812	0.999602	0.998696	0.997379	0.996414	0.996253
40	JLY19	3	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999934	0.999236	0.998416	0.997828	0.997464	0.997285	0.997283	0.997406
41	AUG19	3	1	1	1	1	1	0.999833	0.999281	0.998851	0.998542	0.998323	0.998168	0.998053	0.997966	0.997925	0.997938	0.997946	0.998056	0.99786	0.997864	0.997926	0.998001
42	SEP19	3	1	1	1	1	1	1	0.999807	0.999537	0.999318	0.999141	0.998986	0.998808	0.998575	0.998379	0.998384	0.998441	0.998235	0.99823	0.998282	0.998339	0.998383
43	OCT19	3	1	1	1	1	1	1	1	0.999825	0.999578	0.999348	0.998977	0.997583	0.997643	0.99775	0.998146	0.99846	0.99855	0.998595	0.998621	0.998639	0.998655
44	NOV19	3	1	1	1	1	1	1	0.999543	0.998991	0.998695	0.998779	0.998465	0.99842	0.998401	0.998398	0.998441	0.998513	0.998581	0.998618	0.998628	0.998627	0.998625
45	DEC19	3	1	1	1	1	0.995903	0.992826	0.991131	0.990512	0.990787	0.991832	0.99357	0.996106	0.998675	0.999397	0.99938	0.99928	0.999152	0.999004	0.998818	0.9986	0.998382
46	JAN20	3	0.99203	0.992229	0.992562	0.993031	0.993673	0.994662	0.995374	0.995786	0.996332	0.997061	0.998176	0.999156	0.999459	0.99949	0.99945	0.999391	0.999328	0.99926	0.999181	0.999085	0.998971
47	FEB20	3	0.993944	0.994207	0.994537	0.994974	0.99555	0.995914	0.995792	0.996092	0.996559	0.997091	0.997742	0.998579	0.99937	0.999715	0.999779	0.999749	0.999694	0.999634	0.999574	0.999511	0.999433
48	MAR20	3	0.995494	0.995753	0.99578	0.995577	0.995527	0.995942	0.996121	0.996213	0.996461	0.996861	0.997451	0.9983	0.99942	1	1	1	1	0.999977	0.999851	0.999718	0.999582
49	APR20	3	0.996404	0.996361	0.996345	0.996355	0.996392	0.996454	0.996539	0.996628	0.996678	0.996615	0.996396	0.996161	0.996243	0.996861	0.997831	1	1	1	1	1	1
50	MAY20	3	1	1	1	1	1	1	1	1	1	1	1	1	0.999713	0.999293	0.998414	0.997339	0.994986	0.992674	0.992653	0.993942	0.997387
51	JAN19	4	0.996658	0.996592	0.996549	0.996529	0.996527	0.996544	0.996575	0.996618	0.996671	0.996731	0.996792	0.99685	0.996898	0.996925	0.996919	0.996866	0.996752	0.996576	0.996378	0.996266	0.996427
52	FEB19	4	0.99806	0.998006	0.997964	0.997932	0.997908	0.997892	0.997881	0.997874	0.99787	0.997867	0.997863	0.997856	0.997843	0.997822	0.997786	0.997728	0.997631	0.997469	0.997207	0.996844	0.996438
53	MAR19	4	1	1	1	0.999931	0.999791	0.999664	0.99955	0.999447	0.999354	0.99927	0.999193	0.999123	0.999057	0.998994	0.998931	0.998865	0.998787	0.998688	0.998552	0.998358	0.998089
54	APR19	4	1	1	1	1	1	1	1	0.999913	0.999812	0.999716	0.999623	0.999531	0.999437	0.999337	0.999224	0.99909	0.998928	0.998729	0.99849	0.998216	0.997914
55	MAY19	4	1	1	1	1	1	1	1	0.999999	0.99988	0.99976	0.999635	0.9995	0.99935	0.999176	0.998972	0.998725	0.998422	0.998047	0.997598	0.997106	0.996623
56	JUN19	4	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999999	0.999781	0.999474	0.998324	0.996001	0.994352	0.994032	0.994399
57	JLY19	4	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999073	0.997697	0.996631	0.995943	0.995574	0.995507	0.995661	0.995938
58	AUG19	4	1	1	1	1	1	0.999295	0.998547	0.997987	0.997581	0.99729	0.997074	0.996907	0.996794	0.996772	0.996766	0.996845	0.996653	0.996466	0.996484	0.996568	0.99667
59	SEP19	4	1	1	1	1	1	1	0.99991	0.999445	0.999064	0.998748	0.998453	0.998088	0.997631	0.997295	0.997359	0.997313	0.996993	0.996971	0.997046	0.997132	0.997196
60	OCT19	4	1	1	1	1	1	1	1	0.999919	0.999487	0.999082	0.998601	0.996361	0.99639	0.996321	0.996989	0.997594	0.997698	0.997728	0.99773	0.997729	0.997732
61	NOV19	4	1	1	1	1	1	1	0.999942	0.998892	0.998235	0.998222	0.997566	0.997514	0.997478	0.997452	0.997498	0.997595	0.997689	0.997734	0.997733	0.997711	0.997688
62	DEC19	4	1	1	1	1	0.992086	0.987029	0.984301	0.983401	0.984024	0.985959	0.989097	0.9936	0.997854	0.999119	0.999046	0.998839	0.99859	0.998316	0.997985	0.997597	0.997201
63	JAN20	4	0.986972	0.987351	0.987959	0.988842	0.990206	0.991911	0.992237	0.993017	0.993993	0.995446	0.997353	0.998735	0.999161	0.999189	0.999104	0.998992	0.998875	0.998757	0.998631	0.998484	0.99831
64	FEB20	4	0.990702	0.991168	0.991794	0.992621	0.993244	0.992885	0.993263	0.993944	0.994712	0.995587	0.996716	0.998111	0.999248	0.999683	0.999751	0.999695	0.999604	0.999507	0.999413	0.999311	0.999187
65	MAR20	4	0.993035	0.993125	0.992703	0.992346	0.992694	0.993581	0.993449	0.993673	0.994092	0.994748	0.995691	0.997002	0.998725	1	1	1	1	1	0.999921	0.999699	0.999468
66	APR20	4	0.994255	0.994135	0.99407	0.994059	0.994096	0.994172	0.994265	0.99432	0.99423	0.993903	0.993436	0.993283	0.99388	0.994849	0.996513	1	1	1	1	1	1
67	MAY20	4	1	1	1	1	1	1	1	1	1	1	1	1	1	0.998965	0.996768	0.994721	0.990194	0.985648	0.985338	0.988197	0.991343
68	JAN19	5	0.995257	0.995174	0.995126	0.99511	0.995122	0.995159	0.995216	0.995291	0.995378	0.995471	0.995564	0.995647	0.995708	0.995732	0.9957	0.995592	0.995398	0.99513	0.994864	0.994763	0.995084
69	FEB19	5	0.99722	0.997143	0.997082	0.997033	0.996996	0.996968	0.996947	0.996932	0.996918	0.996905	0.996888	0.996866	0.996834	0.996786	0.996714	0.996601	0.996416	0.996112	0.99564	0.995013	0.994339
70	MAR19	5	1	1	1	1	0.999875	0.999687	0.999516	0.99936	0.999218	0.999087	0.998966	0.998853	0.998745	0.998637	0.998525	0.9984	0.998249	0.998052	0.997782	0.997417	0.996949
71	APR19	5	1	1	1	1	1	1	1	0.999962	0.99981	0.999661	0.999512	0.999357	0.999189	0.998999	0.998775	0.998503	0.998174	0.997789	0.997352	0.996865
72	MAY19	5	1	1	1	1	1	1	1	0.999984	0.999799	0.999608	0.999407	0.999185	0.998933	0.998635	0.998272	0.997818	0.997249	0.996563	0.995812	0.99508	0.994435
73	JUN19	5	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999823	0.999396	0.997896	0.994495	0.991908	0.991345	0.991811	0.99248
74	JLY19	5	1	1	1	1	1	1	1	1	1	1	1	1	0.999429	0.997445	0.995749	0.994611	0.993967	0.9938	0.994006	0.994369	0.993614
75	AUG19	5	1	1	1	1	1	0.998892	0.997876	0.99712	0.996565	0.996154	0.995834	0.995577	0.995405	0.995359	0.995256	0.995455	0.994991	0.994731	0.994727	0.994815	0.994933
76	SEP19	5	1	1	1	1	1	1	1	0.999491	0.998941	0.998469	0.997993	0.997371	0.996649	0.996222	0.996402	0.996022	0.995629	0.995601	0.995683	0.995769	0.995825
77	OCT19	5	1	1	1	1	1	1	1	1	0.999532	0.998883	0.998259	0.994955	0.994742	0.99488	0.995756	0.996628	0.996722	0.996721	0.996692	0.996659	0.996632
78	NOV19	5	1	1	1	1	1	1	1	0.999882	0.999717	0.999788	0.996673	0.996574	0.996472	0.996399	0.996425	0.996531	0.996638	0.996678	0.996645	0.996581	0.996516
79	DEC19	5	1	1	1	1	0.988316	0.980873	0.976858	0.975542	0.976473	0.979356	0.98406	0.990722	0.996919	0.998863	0.998691	0.998331	0.997927	0.997505	0.997017	0.996444	0.995841
80	JAN20	5	0.981683	0.98239	0.983482	0.985197	0.98812	0.988301	0.989176	0.990328	0.991956	0.994266	0.996838	0.998475	0.998962	0.998978	0.998855	0.998697	0.998534	0.998371	0.998199	0.998001	0.997767
81	FEB20	5	0.986996	0.987698	0.988646	0.989866	0.989687	0.989552	0.990265	0.991268	0.992341	0.993578	0.995174	0.997098	0.998719	0.999425	0.999568	0.999503	0.999372	0.99923	0.99909	0.998946	0.998779
82	MAR20	5	0.990092	0.989517	0.988701	0.988873	0.990721	0.990167	0.990299	0.990672	0.991322	0.992305	0.993682	0.995564	0.998027	1	1	1	1	1	1	0.999676	0.999311
83	APR20	5	0.992128	0.991917	0.991805	0.991781	0.991827	0.991911	0.991961	0.991844	0.991419	0.990718	0.99017	0.990474	0.991523	0.992422	0.999331	1	1	1	1	1	1
84	MAY20	5	1	1	1	1	1	1	1	1	1	1	1	1	1	0.998534	0.994566	0.991379	0.984095	0.976447	0.975409	0.981211	0.982842
85	JAN19	6	0.993706	0.993616	0.993571	0.993566	0.993595	0.993655	0.993739	0.99384	0.993952	0.994067	0.994173	0.994259	0.994307	0.994295	0.994202	0.994004	0.993697	0.993322	0.992996	0.99294	0.993485
86	FEB19	6	0.996303	0.996204	0.996125	0.996061	0.99601	0.99597	0.995936	0.995907	0.995878	0.995845	0.995806	0.995755	0.995682	0.995585	0.995437	0.995203	0.99483	0.994251	0.993444	0.992489	0.991538
87	MAR19	6	1	1	1	1	1	0.999905	0.999667	0.999449	0.999248	0.999062	0.998887	0.99872	0.998558	0.998393	0.998218	0.998018	0.997771	0.997448	0.997015	0.996448	0.995748
88	APR19	6	1	1	1	1	1	1	1	1	0.99999	0.999774	0.99956	0.999342	0.999111	0.998857	0.998563	0.998213	0.99779	0.997286	0.996703	0.996042	0.995272
89	MAY19	6	1	1	1	1	1	1	1	0.999964	0.999695	0.999415	0.999113	0.998773	0.998375	0.997888	0.99727	0.996479	0.995504	0.994422	0.993362	0.992427	0.991667
90	JUN19	6	1	1	1	1	1	1	1	1	1	1	1	1	1	0.999607	0.997441	0.992768	0.989321	0.988669	0.989351	0.990196	0.990614
91	JLY19	6	1	1	1	1	1	1	1	1	1	1	1	1	0.998268	0.995661	0.993708	0.992461	0.991867	0.991915	0.992223	0.992038	0.991047
92	AUG19	6	1	1	1	1	0.999987	0.998436	0.997229	0.996325	0.995647	0.995123	0.994693	0.994336	0.994101	0.994003	0.99373	0.994228	0.99316	0.992815	0.992774	0.992851	0.992973
93	SEP19	6	1	1	1	1	1	1	1	0.999783	0.999037	0.998368	0.997632	0.996645	0.995596	0.995094	0.995396	0.99459	0.994127	0.994101	0.994184	0.994249	0.994264
94	OCT19	6	1	1	1	1	1	1	1	1	0.999521	0.998577	0.997647	0.993655	0.993121	0.993158	0.994496	0.995642	0.995657	0.99557	0.995473	0.995387	0.995311
95	NOV19	6	1	1	1	1	1	1	1	0.999052	0.997776	0.997151	0.995826	0.995691	0.99552	0.995375	0.995352	0.995431	0.995514	0.995513	0.995425	0.995299	0.995174
96	DEC19	6	1	1	1	1	0.984669	0.974592	0.969141	0.967319	0.96852	0.972376	0.978778	0.988717	0.995897	0.998669	0.998438	0.997943	0.997389	0.996816	0.996161	0.995386	0.994554
97	JAN20	6	0.97569	0.97658	0.978076	0.98059	0.984035	0.984344	0.985565	0.987138	0.989329	0.992296	0.995552	0.997774	0.998495	0.998536	0.998369	0.998146	0.997913	0.997681	0.997439	0.997167	0.996846
98	FEB20	6	0.983055	0.98403	0.985258	0.986942	0.985662	0.985872	0.986943	0.988251	0.989627	0.991262	0.99339	0.995934	0.998114	0.99912	0.999341	0.999252	0.999058	0.998837	0.998615	0.99839	0.998145
99	MAR20	6	0.98656	0.985159	0.984916	0.986865	0.9871	0.987045	0.987302	0.987842	0.988743	0.990056	0.991844	0.994216	0.997206	0.999839	1	1	1	1	1	0.999755	0.999257
100	APR20	6	0.990206	0.989871	0.989688	0.98963	0.989652	0.989676	0.989558	0.989099	0.988194	0.987065	0.986575	0.987512	0.988498	0.989549	0.998401	1	1	1	1	1	1
101	MAY20	6	1	1	1	1	1	1	1	1	1	1	1	1	1	0.998057	0.991818	0.987303	0.977396	0.965559	0.962772	0.970591	0.971654

Table B.4: WTI Black Replication Error

	Month	Month_diff	z=-1.0	z=-0.9	z=-0.8	z=-0.7	z=-0.6	z=-0.5	z=-0.4	z=-0.3	z=-0.2	z=-0.1	z=0.0	z=0.1	z=0.2	z=0.3	z=0.4	z=0.5	z=0.6	z=0.7	z=0.8	z=0.9	z=1.0
0	JAN19	1	-1.4E-07	-8.3E-08	-7.4E-07	2.02E-07	7.22E-07	-6.4E-07	-6.7E-08	2.51E-07	1.81E-07	1.29E-06	-7.6E-07	1.55E-08	4.23E-08	-3.3E-07	1.24E-06	-6.9E-07	5.82E-07	9.82E-08	1.22E-07	2.58E-09	1.33E-08
1	FEB19	1	6.04E-07	4.13E-07	1.65E-06	3.02E-08	5.77E-08	9.02E-08	1.22E-07	1.47E-07	1.58E-07	1.5E-07	1.22E-07	8.25E-08	-3.4E-07	7.6E-07	-5E-07	5.35E-07	4.1E-08	2.92E-08	-6.1E-07	-1.2E-06	-6.5E-07
2	MAR19	1	2.6E-07	3.46E-07	2.47E-07	2.66E-08	-7.6E-07	3.14E-07	-3E-08	-1.2E-07	-1.9E-07	-1.9E-07	1.38E-06	8.82E-07	-5.7E-07	7.24E-08	-2.9E-07	2.54E-06	7.01E-07	-8.4E-07	5.48E-07	-3.3E-08	1.4E-06
3	APR19	1	0.003903	0.002345	0.000578	9.23E-07	-5.7E-08	-5.1E-07	-2.6E-07	-2E-08	1.46E-06	-2.3E-07	-1.8E-07	-1.1E-07	2E-06	-1.1E-06	-8.8E-07	-2.3E-06	-2.7E-07	1.18E-06	-2.4E-09	1.47E-07	5.9E-09
4	MAY19	1	0.001509	-9.4E-07	-1.7E-07	-1.6E-07	-3.2E-07	4.29E-07	-7.2E-07	-9.1E-07	-1E-06	4.82E-07	1.76E-06	-1.1E-06	-3.3E-07	1.61E-06	1.26E-06	5.94E-07	-9.3E-07	-8.1E-08	9.65E-07	-9.3E-08	-7.4E-07
5	JUN19	1	0.014219	0.013908	0.013482	0.012924	0.012217	0.011344	0.010287	0.009029	0.007556	0.005856	0.003921	0.001749	4.13E-07	-3.2E-07	2.85E-07	1.27E-06	-1.1E-06	5E-07	9.85E-07	-6.4E-07	-5E-08
6	JLY19	1	0.011908	0.011888	0.011794	0.011611	0.011319	0.010899	0.010325	0.009572	0.008608	0.007396	0.005892	0.004033	0.001712	-5.4E-08	2.19E-08	5.14E-07	-3.4E-07	3.08E-08	2.1E-07	1.77E-06	6.18E-07
7	AUG19	1	0.012253	0.011782	0.010985	0.009708	0.007714	0.004635	4.13E-07	3.7E-08	2.59E-08	-1.9E-07	-4.2E-10	-9.5E-08	-2.4E-07	7.41E-08	2.37E-07	3.3E-07	-3.4E-07	-5.1E-07	1.51E-07	3.7E-08	-1.8E-06
8	SEP19	1	0.010593	0.009924	0.008908	0.007416	0.005278	0.00226	5.67E-07	4.71E-08	7.92E-07	2.38E-07	-5.8E-08	-3.8E-07	2.23E-06	-1.9E-06	1.52E-07	-3.1E-06	4.43E-08	-3.2E-07	2.93E-06	4.8E-07	-6.5E-07
9	OCT19	1	0.01232	0.011017	0.009207	0.006735	0.003401	1.63E-07	-4.7E-08	1.3E-06	-8.9E-08	-2.2E-06	-2.4E-07	1.16E-07	-3.1E-07	2.6E-07	1.01E-06	1.25E-06	-1.2E-07	-2.2E-06	-1.4E-06	6.84E-08	-3.6E-08
10	NOV19	1	0.012916	0.011845	0.00999	0.006644	0.000415	4.22E-08	8.29E-08	-1E-07	-2.1E-07	1.37E-06	-2.2E-06	1.64E-06	-9.9E-08	-5E-07	1.63E-07	1.35E-07	-1.4E-06	9.25E-07	7.23E-07	-3.8E-07	2.43E-07
11	DEC19	1	0.021146	0.020787	0.018054	0.00944	2.14E-07	1.24E-07	-3.9E-09	1.83E-07	-5.4E-08	-1.7E-06	9.81E-07	1.2E-07	-1.1E-06	-2E-06	2.1E-06	1.07E-06	7.11E-08	-2.7E-07	2.71E-07	3.81E-08	-1.1E-08
12	JAN20	1	8.58E-07	-6.5E-08	9.56E-07	3.18E-07	8.34E-07	3.82E-07	4.23E-07	-1.6E-07	-1.8E-06	1.4E-06	8.88E-07	2.85E-08	8.82E-08	-8.7E-07	3.16E-07	-6.1E-09	7.99E-08	-3E-07	2.07E-07	3.19E-08	4.82E-08
13	FEB20	1	1.97E-07	3.08E-07	-4.4E-10	4.75E-08	-1.7E-06	-4.4E-07	2.93E-07	2.62E-07	1.83E-06	1.44E-06	-1.3E-06	-7.7E-08	2.11E-07	1.99E-07	3.59E-07	-1.7E-07	3.48E-07	6.99E-08	1.57E-07	-9.8E-08	8.26E-09
14	MAR20	1	-7E-07	6.65E-07	-1.7E-06	-2.2E-07	1.17E-07	3.08E-07	2.98E-07	1.21E-06	1.39E-06	-5.3E-08	6.35E-07	-2.6E-07	-1.2E-07	4.73E-08	1.95E-07	-1.8E-07	7.3E-08	4.04E-08	2.86E-08	1.7E-08	9.47E-09
15	APR20	1	4.97E-07	-2.9E-08	8.31E-08	2E-07	3.56E-07	4.41E-07	2.82E-07	1.27E-07	-1.2E-06	-6.8E-07	-1.9E-08	2.79E-07	8.54E-09	1.63E-07	7.78E-08	0.000607	0.000632	0.000341	0.000122	2.9E-05	3.81E-06
16	MAY20	1	0.044381	0.041639	0.038137	0.033783	0.028515	0.022317	0.015248	0.007462	2.06E-07	-3.2E-07	-4.4E-07	-1E-06	-1.9E-06	-4.6E-07	1.34E-06	6.99E-07	-9.3E-07	-2.2E-07	2.04E-07	1.77E-07	0.001066
17	JAN19	2	7.02E-09	3.22E-07	-2.2E-08	-8E-08	-2.2E-07	-5.1E-07	4.37E-07	3.58E-08	8.38E-08	1.72E-07	2.87E-07	3.59E-07	2.93E-07	1.3E-07	7.3E-07	-5E-08	-2.8E-12	1.08E-07	2.61E-09	5.17E-09	-1.7E-07
18	FEB19	2	-7.5E-08	-3.6E-07	7.63E-07	7.82E-08	1.4E-07	2.16E-07	2.9E-07	3.33E-07	3.17E-07	2.38E-07	1.34E-07	1.44E-07	-6.7E-07	-1.1E-07	8.21E-07	8.02E-09	4.21E-09	3.93E-07	-2.7E-07	3.88E-08	6.49E-08
19	MAR19	2	0.003529	3.51E-08	-3.3E-09	-2.9E-07	3.97E-07	6.03E-07	1.38E-07	-3.5E-08	-3.2E-08	6.21E-07	8.6E-07	3.03E-08	-4.6E-08	-2.2E-07	9.81E-07	2.22E-07	-6.6E-07	-4.4E-08	-1.8E-07	-1.9E-07	4.2E-07
20	APR19	2	0.011001	0.008128	0.004833	0.001102	-6.2E-08	-3.6E-07	1.03E-06	-3.2E-07	2.48E-07	-4.8E-07	5.53E-07	-1.3E-06	-5.7E-07	-1.8E-07	-3.4E-08	-4.2E-07	-8.3E-07	-2.8E-08	-2.6E-07	2.26E-07	-1.5E-08
21	MAY19	2	0.013088	0.010438	0.007406	0.003981	0.000162	-1.5E-07	-5.2E-07	7.4E-08	-9.8E-07	6.76E-07	4.7E-07	-4.3E-07	-7.6E-08	-3.3E-07	1.75E-07	-2.7E-07	-2.2E-08	6.95E-07	-1.3E-07	-8.1E-07	2.13E-07
22	JUN19	2	0.030109	0.029711	0.029091	0.028216	0.027048	0.025548	0.023678	0.021397	0.018668	0.015455	0.011726	0.007457	0.002627	-4E-08	-3.6E-07	2.51E-08	-1.2E-07	2.99E-08	1.35E-07	-1E-07	-5.8E-07
23	JLY19	2	0.027695	0.02811	0.028416	0.028581	0.028572	0.028347	0.027855	0.027038	0.025821	0.024107	0.02175	0.018501	0.013866	0.006803	-1.9E-07	-5.6E-08	-1.6E-07	6.89E-08	-3.4E-07	1.49E-07	4.62E-07
24	AUG19	2	0.022448	0.021222	0.01916	0.015848	0.010653	0.002637	-3.1E-07	-5.1E-07	-6.7E-09	-5.3E-07	1.28E-07	1.13E-06	-2.3E-08	-7.3E-08	-4.2E-07	-2.9E-07	1.57E-06	-1.5E-06	8.57E-08	1.07E-06	-1.4E-07
25	SEP19	2	0.021364	0.019881	0.017632	0.014348	0.00967	0.003142	1.92E-07	2.92E-08	-4.6E-08	5.3E-07	9.8E-08	-1.3E-07	-1.7E-07	1.92E-07	7E-07	7.29E-07	8.26E-07	2.5E-07	1.64E-07	-3.3E-08	5.44E-07
26	OCT19	2	0.029367	0.027594	0.024966	0.021207	0.015949	0.008702	3.23E-07	2E-08	-4.1E-07	-7.8E-07	-7E-07	1.09E-07	1.32E-07	-1.6E-07	6.33E-07	1.37E-07	-1E-06	-8.4E-08	-7.7E-08	-5.7E-07	-6.7E-07
27	NOV19	2	0.027179	0.025727	0.023101	0.018321	0.009418	6.48E-08	2.97E-07	-1.5E-09	9.34E-07	-6.2E-08	1.94E-08	1.79E-07	4.11E-07	6.69E-07	3.92E-07	1.04E-06	-4E-07	-3.8E-08	9.77E-08	3.05E-09	1.61E-07
28	DEC19	2	0.043079	0.041944	0.034445	0.011955	5.97E-08	-5.3E-08	5.27E-07	-1.5E-08	5.54E-07	-7E-07	4.95E-07	-5.6E-07	-1E-07	2.74E-07	-4.8E-08	2.15E-07	7.41E-08	-2.9E-07	1.8E-08	1.11E-07	4.2E-08
29	JAN20	2	-8.7E-08	3.21E-07	-6.6E-07	-3.8E-07	2.59E-07	-3E-07	1.71E-07	4.89E-07	1.22E-07	6.62E-07	7.59E-07	3.72E-07	-7.5E-07	1.8E-08	5.38E-07	5.42E-08	2.63E-07	6.44E-08	-8.1E-08	9.87E-09	2.03E-08
30	FEB20	2	4.52E-07	1.18E-07	-2.8E-07	-2.8E-07	3.76E-07	-3.2E-07	-1.6E-07	-7.2E-07	-7.2E-07	-2.3E-09	1.65E-07	7.18E-09	-1.6E-08	1.94E-07	4.49E-08	-3.2E-07	2.21E-07	1.53E-08	3.29E-08	3.86E-08	4.62E-08
31	MAR20	2	-2.6E-08	1.47E-07	8.08E-07	9.18E-07	-7.8E-07	-1E-06	-4.3E-07	-3.4E-07	-7.3E-08	3.91E-07	-2.4E-07	1.16E-07	1.38E-07	3.36E-08	0.001265	0.000579	1.76E-08	2.01E-08	1.17E-08	2.52E-08	5.86E-09
32	APR20	2	-5.1E-08	-2.9E-07	2.63E-07	7.75E-08	3.25E-07	-7.7E-07	-3E-08	-5.1E-08	-2.6E-08	3.92E-07	-7.1E-08	4.84E-07	-1.5E-07	-2.2E-09	-2.5E-07	0.003379	0.002288	0.001165	0.000425	0.000109	1.7E-05
33	MAY20	2	0.119055	0.116303	0.112077	0.106114	0.098164	0.088018	0.075545	0.060729	0.043682	0.024616	0.003756	6.86E-08	1.06E-07	8.77E-07	5.97E-07	-4.7E-08	5.16E-07	-4.9E-07	-5.3E-08	5.19E-07	0.004977
34	JAN19	3	4.06E-08	-8E-08	1.25E-07	-1.2E-07	-1.6E-08	-8.1E-08	1.38E-07	2.14E-07	3E-07	3.66E-07	3.74E-07	3.13E-07	2.09E-07	1.02E-07	3.16E-08	2.51E-07	4.97E-09	2.76E-07	8.88E-10	-2.6E-07	1.79E-07
35	FEB19	3	-8.8E-09	-5.1E-07	1.85E-08	5.11E-08	9.95E-08	1.57E-07	2.11E-07	2.49E-07	2.58E-07	2.33E-07	1.77E-07	1.11E-07	5.18E-08	6.68E-07	-2.7E-07	1.6E-08	1.55E-07	-9.1E-08	-8.8E-08	4.16E-07	2.5E-08
36	MAR19	3	0.008621	0.003483	9.55E-08	-1.3E-08	2.86E-07	2.79E-07	-5E-07	1.79E-07	7.65E-07	2.59E-07	2.04E-07	1.21E-07	4.85E-07	-6.6E-07	-1.9E-07	8.01E-07	-2.7E-07	-8.1E-07	6.68E-08	5.64E-09	2.24E-07
37	APR19	3	0.023906	0.020322	0.01616	0.011395	0.006018	3.81E-05	-7.1E-07	1.18E-07	7.45E-07	-9E-08	-4.6E-07	-1.8E-07	-4.8E-08	6.79E-07	6.68E-07	-2.8E-07	-8.6E-09	1.97E-07	-5E-07	1.81E-07	-3.6E-07
38	MAY19	3	0.025602	0.0222	0.018262	0.013766	0.008701	0.003073	-2.6E-07	-4.8E-07	6.95E-07	-6.9E-08	-1.2E-07	-1.9E-08	-4.4E-07	-4.3E-07	-3.8E-08	-4E-07	-3.3E-07	-3.9E-07	-2.3E-07	-9.7E-08	-3.7E-07
39	JUN19	3	0.048954	0.048825	0.048385	0.04758	0.046352	0.044637	0.042368	0.039474	0.035882	0.03152	0.026316	0.020201	0.013101	0.004937	-1.4E-07	5.19E-07	2.17E-07	3.65E-07	-3.4E-07	-1.7E-08	-7.3E-08
40	JLY19	3	0.038377	0.038945	0.03934	0.039518	0.039424	0.038992	0.038144	0.036778	0.034759	0.031884	0.027802	0.021833	0.012585	9.7E-08	8.49E-08	1.17E-07	1.35E-08	-2.7E-07	1.05E-07	-7.5E-07	-8.1E-08
41	AUG19	3	0.030886	0.028623	0.024866	0.018871	0.009538	8.54E-09	-1.9E-07	-3.6E-07	4.09E-07	-5E-07	-9.2E-08	-4.2E-09	4.36E-07	-1.1E-06	-8.4E-07	-9.6E-07	3.51E-07	2.6E-07	-1.8E-07	-2.3E-08	5.8E-08
42	SEP19	3	0.033015	0.030615	0.027022	0.021848	0.014607	0.004717	-2.5E-07	2.19E-07	8.64E-08	5.25E-07	7.27E-09	-1.8E-08	-5.1E-07	-5.9E-07	-1.1E-07	-3.6E-07	3.93E-07	1.44E-07	8.79E-08	7.79E-07	-3.5E-08
43	OCT19	3	0.046852	0.044605	0.041165	0.036147	0.029038	0.01915	0.005544	-2.1E-07	-7E-08	3.5E-07	-7.2E-08	-4.7E-08	-5.1E-07	-9.4E-07	1.01E-07	-2.3E-07	-6.4E-08	7.03E-07	-2.8E-07	-1.3E-07	-5.4E-07
44	NOV19	3	0.043466	0.042188	0.039565	0.034581	0.025172	0.007319	4.91E-09	4.04E-08	-5E-08	1.34E-08	-1.1E-06	9.56E-07	-2.9E-08	-1.1E-07	-1.8E-07	-1E-07	-1.5E-08	1.84E-07	1.85E-07	7E-09	1.79E-07
45	DEC19	3	0.064666	0.062608	0.04937	0.010605	9.67E-08	2.11E-09	-1.5E-09	-4.2E-07	4.94E-08	-9.1E-09	2.54E-07	-1.7E-08	6.13E-07	-5E-07	-1.2E-07	7.55E-09	5.6E-09	9.85E-08	-8.8E-08	2.56E-08	3.03E-08
46	JAN20	3	4.72E-09	8.31E-08	-7.6E-09	-1.3E-07	-6.1E-07	-1.7E-07	-6.7E-07	-6.6E-08	-2.2E-07	-1.4E-08	2.47E-08	9.19E-08	-6.1E-08	-4.1E-07	2.08E-08	-3.2E-07	2.05E-08	1.55E-07	-4.9E-08	7.25E-09	1.03E-08
47	FEB20	3	-2.6E-07	5.45E-07	-2.3E-07	2.98E-08	-6.1E-08	-7.7E-07	-7E-08	-2.6E-08	-3.8E-09	-1.7E-08	-2.8E-08	9.49E-08	1.01E-07	1.16E-07	-3.1E-08	-5E-08	2.7E-08	3.59E-08	9.68E-09	3.05E-08	3.85E-09
48	MAR20	3	2.2E-07	-3.4E-08	-1.5E-07	-1.1E-07	-5.1E-08	-2.1E-07	-7.5E-08	-1.3E-08	2.38E-07	4.75E-08	1.85E-07	-1.8E-07	1.43E-08	0.001676	0.004166	0.002826	0.000988	3.92E-08	-1.6E-08	-1.4E-09	7.31E-09
49	APR20	3	8.47E-08	-4.2E-09	-7.2E-08	2.12E-07	4.19E-07	-5.7E-07	-4.7E-07	6.35E-07	3.08E-07	9.65E-08	1.45E-07	-3.7E-08	5.27E-08	-1.7E-07	9.11E-09	0.004029	0.003605	0.001992	0.000782	0.000218	3.84E-05
50	MAY20	3	0.214262	0.214439	0.212396	0.20755	0.199275	0.186933	0.169933	0.147783	0.120153	0.086982	0.048772	0.007449	1.92E-07	1.73E-07	-8.5E-09	-4.7E-08	-3.5E-07	1.78E-08	-1E-09	-2.5E-07	8.41E-09
51	JAN19	4	8.01E-08	-2.1E-07	2.41E-07	-4.4E-09	-3.3E-08	1.18E-07	1.88E-07	2.52E-07	2.96E-07	2.61E-07	1.23E-07	4.41E-08	4.01E-07	-6.3E-08	1.71E-07	1.21E-07	1.24E-07	-4.9E-08	-5.2E-08	5.93E-08	7.71E-09
52	FEB19	4	-7.9E-09	-1.3E-08	3.32E-08	7.47E-08	1.34E-07	2.03E-07	2.66E-07	3.08E-07	3.15E-07	2.8E-07	2.1E-07	1.26E-07	-2.6E-08	3.9E-07	-1.1E-07	2.64E-08	4.62E-08	-1.7E-07	-4.2E-08	-3.5E-07	2.6E-08
53	MAR19	4	0.017206	0.011259	0.004279	-6.9E-09	-2.2E-07	8.88E-08	6.97E-08	-5.1E-07	-4.7E-07	-1E-08	5.81E-07	3.49E-07	1.5E-07	2.54E-07	-3.5E-07	-4.9E-08	-9.9E-09	1.47E-07	3.8E-07	-1.9E-07	4.1E-07
54	APR19	4	0.03763	0.033514	0.02867	0.023059	0.016659	0.009472	0.001531	-1.4E-07	-3.8E-07	1.2E-07	-1.5E-07	-2.5E-07	-4.3E-08	-9.6E-09	2.16E-07	-6E-08	3.92E-07	2.98E-08	-1.6E-08	-4.9E-09	4.43E-07
55	MAY19	4	0.042216	0.038361	0.033817	0.028546	0.022521	0.015732	0.008185	-3.6E-07	3.95E-08	1.18E-07	-2.7E-07	5.51E-09	1.14E-07	3.52E-07	-3.5E-08	2.65E-07	-2.9E-08	-9.1E-08	-1.6E-07	-3.2E-07	6.98E-08
56	JUN19	4	0.060733	0.060483	0.05981	0.058643	0.056903	0.054505	0.051358	0.047368	0.042437	0.036463	0.029343	0.02097	0.011222	2.15E-07	1.04E-07	-3.6E-07	2.31E-09	8.47E-09	-8.4E-09	-3.5E-08	1.29E-07
57	JLY19	4	0.048124	0.04883	0.049296	0.049459	0.049241	0.048547	0.047259	0.045213	0.042165	0.037699	0.031032	0.02059	0.003283	6.44E-09	2.11E-08	1.93E-08	-1.3E-08	2.97E-08	2.75E-07	-1.7E-08	-1.9E-07
58	AUG19	4	0.039524	0.035553	0.029136	0.019143	0.004015	1.64E-08	7.74E-08	-3E-08	3.92E-07	-5.7E-08	-3.1E-07	-7.1E-08	1.96E-07	3.71E-07	4.63E-07	6.13E-07	4.45E-07	-4.2E-07	-9.8E-08	-1.2E-08	-1.6E-07
59	SEP19	4	0.046742	0.043986	0.039743	0.033545	0.024814	0.012864	7.61E-09	2.14E-07	3.01E-07	1.69E-08	5.91E-08	-1.6E-08	6.82E-07	6.54E-07	-4.4E-08	-3.2E-08	3.34E-08	1.02E-08	1.45E-07	-4.4E-07	-5.5E-08
60	OCT19	4	0.066098	0.063798	0.060045	0.054359	0.046095	0.034375	0.017989	3.71E-08	-1.9E-07	4.96E-07	-6.6E-08	-2.4E-07	3.96E-07	-6E-07	5.64E-08	-2.2E-07	-4.2E-08	-6.3E-07	2.96E-07	6.45E-08	4.09E-09
61	NOV19	4	0.063228	0.062651	0.060649	0.056255	0.047535	0.030526	1.84E-08	-4E-07	1.41E-07	-4.6E-08	-2.3E-07	-7.4E-08	-3.3E-08	-8.3E-08	-1.1E-07	-5E-08	-7.8E-09	1.89E-07	1.07E-08	2.01E-08	-1.3E-09
62	DEC19	4	0.082455	0.079386	0.060111	0.004342	1.94E-08	-1.7E-07	-1.1E-09	3.02E-07	-4E-07	8.32E-08	9.57E-08	-6.2E-09	-3.8E-07	4.18E-07	5E-07	3.94E-09	4.29E-08	3.04E-08	3.02E-09	1.69E-09	-2E-09
63	JAN20	4	1.2E-07	6.29E-08	-6.4E-09	-1.6E-07	5.06E-07	2.38E-07	2.43E-07	-2.8E-08	-3.6E-08	2E-07	3E-08	7.35E-08	-5.5E-08	-3.7E-07	4.45E-08	-2.2E-07	6.5E-09	3.72E-08	-2.3E-08	-1.2E-08	-3E-08
64	FEB20	4	-3.3E-07	1.52E-08	-4.6E-07	-2.1E-08	-5.3E-07	-4.2E-08	-2.7E-08	-1.5E-08	-6.9E-09	7.98E-08	-4.7E-09	1.28E-07	-6.7E-08	4.7E-08	3.93E-08	-8.5E-08	-6.7E-08	3.67E-08	-5.6E-09	4.8E-08	-3.2E-08
65	MAR20	4	-3.7E-07	-2.1E-08	-1.8E-08	-2.6E-07	-1.7E-08	-2E-07	-4.3E-08	-1.3E-08	-3.4E-07	1.36E-08	-9.9E-08	6.38E-08	5.93E-08	0.000507	0.005817	0.004901	0.002508	0.00066	2.43E-09	1.95E-09	4.18E-10
66	APR20	4	2.07E-07	-3.1E-08	-3.5E-08	-3.6E-07	3.7E-07	-3.2E-07	-1.6E-07	-2.7E-07	3.31E-07	-1.7E-08	3.81E-08	-2E-07	-2.4E-08	-2.1E-07	8.16E-09	0.005837	0.005658	0.003252	0.001351	0.000408	8.2E-05
67	MAY20	4	0.320765	0.326492	0.32939	0.328473	0.322606	0.310549	0.291019	0.262801	0.224959	0.177294	0.121257	0.061253	0.002319	1.8E-07	-6.2E-09	-4.7E-09	8.71E-09	1.52E-07	1.62E-07	2.89E-08	1.19E-07
68	JAN19	5	1.63E-07	2.98E-09	-1.1E-08	-3.9E-08	-1.2E-07	2.92E-07	9.05E-09	3.19E-08	9.18E-08	2.04E-07	3.23E-07	3.26E-07	1.78E-07	4.06E-08	-1.2E-07	4.34E-08	1.36E-07	1.35E-09	-1.6E-07	1.91E-08	8.1E-08
69	FEB19	5	-2E-08	-7.5E-08	-1.6E-07	-2.8E-07	-4E-07	3.92E-07	3.77E-07	-3.9E-07	-2.1E-07	-6.4E-08	-2.1E-07	1.62E-08	1.53E-08	8.32E-09	9.48E-08	1.25E-07	2.91E-07	-3.1E-07	-1E-07	-1.2E-07	4.19E-10
70	MAR19	5	0.026967	0.020442	0.012745	0.003805	-1.6E-07	2.91E-08	4.69E-08	2.53E-07	5.62E-08	-5E-08	-5.5E-08	-1.8E-08	-1.3E-08	-3.1E-07	1.63E-07	-3.9E-07	-2.1E-08	5.95E-08	5.57E-11	3.15E-09	1.73E-08
71	APR19	5	0.056389	0.052224	0.047203	0.041268	0.034378	0.026515	0.017688	0.007947	-1.8E-08	7.86E-08	-4.4E-07	4.5E-07	9.37E-08	-2.2E-08	-1.7E-08	-4.8E-07	8.9E-09	-2.8E-07	-1.4E-07	-4.4E-08	-1E-08
72	MAY19	5	0.051327	0.046651	0.041105	0.034636	0.027205	0.018792	0.009395	-2.4E-07	5.5E-08	1.57E-07	-2.7E-07	-3.5E-08	-7.5E-09	1.02E-07	-2.5E-08	-8.9E-08	6.91E-08	9.19E-08	-2E-07	-4.2E-09	5.66E-08
73	JUN19	5	0.071821	0.071401	0.070433	0.068827	0.066483	0.06329	0.059134	0.053893	0.047438	0.039634	0.030336	0.019383	0.006579	4.82E-08	-2.8E-07	-2.3E-07	4.43E-08	1.29E-07	1.06E-07	8.02E-09	2.32E-08
74	JLY19	5	0.060697	0.061791	0.062583	0.062986	0.062894	0.06217	0.060627	0.057984	0.053775	0.047135	0.036379	0.01826	9.22E-10	9.71E-08	2.21E-08	8.6E-08	3.79E-07	-3.3E-08	1.99E-07	5.45E-07	-1.1E-07
75	AUG19	5	0.04949	0.04421	0.035697	0.022545	0.002902	-1.8E-07	-1.5E-07	2.32E-08	6.89E-08	8.76E-08	1.76E-07	2.74E-07	3.59E-07	6.36E-07	2.74E-07	-4.7E-07	-8.7E-08	2.38E-07	1.88E-07	-1.9E-07	-2.4E-07
76	SEP19	5	0.060173	0.057177	0.052464	0.045517	0.035707	0.022301	0.00449	9.97E-08	6.2E-09	1.8E-07	2.61E-07	-1E-08	-3.8E-08	-1.8E-07	4.19E-07	-2.6E-07	-1.1E-08	-3.7E-07	4.65E-07	1.33E-07	3.46E-07
77	OCT19	5	0.088306	0.086439	0.082899	0.077097	0.068234	0.055211	0.036513	0.010083	-4.6E-08	7.6E-08	1.27E-07	3.49E-08	5.12E-07	-1.3E-08	-5.3E-08	3.18E-07	9.5E-08	-2.4E-08	-1.3E-08	6.64E-08	2.37E-08
78	NOV19	5	0.079931	0.079655	0.077723	0.073122	0.063918	0.046178	0.012124	1.81E-07	1.58E-07	1.04E-07	1.16E-07	2.03E-07	3.38E-07	4.06E-07	3.79E-07	2.13E-07	6.77E-08	-1.2E-08	-4E-07	1.88E-08	2.53E-07
79	DEC19	5	0.098977	0.095812	0.072658	0.004582	2.15E-08	5.41E-09	5.91E-08	8.78E-09	1.05E-07	-1.6E-07	3.73E-07	-8.3E-08	7.65E-09	3.26E-07	7.31E-08	1.01E-09	1.15E-07	-5E-09	1.75E-09	1.44E-08	-2.1E-08
80	JAN20	5	-1E-08	-1.2E-07	1.99E-08	3.52E-08	-2.6E-08	-6.6E-08	-3.9E-07	1.21E-07	-2.7E-08	4.18E-08	1.09E-08	5.9E-08	9.5E-08	1.39E-07	-2.8E-07	-2.1E-07	1.37E-07	1.14E-08	-8.4E-09	-3.4E-08	-4.8E-08
81	FEB20	5	9.95E-08	1.68E-07	-5.6E-08	2.11E-07	-3.3E-07	1.52E-07	3.21E-07	1.4E-07	-3.1E-08	-1.9E-07	-1.2E-07	2.09E-08	3.41E-09	3.43E-08	-4.7E-08	2.28E-08	-1.1E-08	7E-08	6.42E-09	-1.3E-09	-2.6E-08
82	MAR20	5	-5.9E-08	-1.9E-07	-1.6E-07	-2.1E-07	-5.2E-09	1.44E-08	1.83E-08	-1.8E-07	-1E-08	-2.5E-08	1.82E-09	9.75E-09	8.35E-08	0.000931	0.008411	0.007585	0.004382	0.001618	7.62E-05	2.42E-09	7.82E-10
83	APR20	5	2.17E-07	-6.8E-09	6.39E-09	4.62E-08	1.28E-07	-3.5E-07	5.46E-09	1.11E-07	-3.6E-07	-9E-08	-2.4E-07	1.3E-08	-1.9E-07	9.88E-09	-4.3E-08	0.005311	0.006343	0.004022	0.001833	0.000614	0.000141
84	MAY20	5	0.417697	0.429868	0.43875	0.44293	0.440733	0.430263	0.409506	0.376513	0.329801	0.269158	0.197005	0.119603	0.040389	3.25E-07	-7.5E-09	-5.2E-09	2.19E-08	2.29E-07	-2.3E-07	2.59E-08	1.03E-09
85	JAN19	6	3.06E-08	1.02E-09	2.02E-08	1.1E-07	-7.5E-08	-6.2E-09	-2.6E-08	-8.4E-08	-2E-07	1.61E-07	1.94E-07	1.64E-07	-5.5E-08	-3.9E-09	4.77E-08	2.3E-09	3.03E-08	-4E-08	1.37E-08	7.47E-09	-4.1E-09
86	FEB19	6	6.01E-10	1.24E-07	-3.1E-07	-3E-09	-7.7E-09	-1.5E-08	-2.3E-08	-2.6E-08	-2.1E-08	-1.1E-08	-5.4E-09	2.37E-07	3.73E-08	-1.3E-07	9.03E-08	1.41E-09	1.49E-08	-7.4E-08	1.51E-07	3.65E-09	6.78E-10
87	MAR19	6	0.041398	0.03485	0.027034	0.017869	0.007302	-2.6E-08	2.18E-08	2.62E-07	1.1E-07	-2.6E-08	-1.2E-08	-2.5E-07	6.38E-08	-3.1E-07	-4.9E-08	-1.3E-08	-2.9E-07	-9.3E-08	2.06E-08	4.39E-08	6.86E-10
88	APR19	6	0.067433	0.062788	0.057128	0.050379	0.042487	0.033423	0.023188	0.011827	7.62E-08	1.38E-07	5.27E-08	2.13E-07	3.46E-08	8.36E-08	2.22E-08	3.84E-07	-5.3E-08	-4.6E-09	-3.6E-07	3.62E-07	-2.7E-08
89	MAY19	6	0.06078	0.055283	0.048723	0.041031	0.032156	0.022064	0.010747	-5.5E-09	1.31E-07	2.65E-07	-1.2E-07	1.72E-07	-2.7E-08	-5.5E-08	1.3E-07	-2.7E-07	6.18E-08	8.53E-09	-1.4E-07	-3E-07	8.91E-08
90	JUN19	6	0.088468	0.088221	0.0873	0.085585	0.082947	0.079242	0.074314	0.067995	0.060098	0.05042	0.038723	0.024721	0.008006	3.14E-08	-8E-08	2.62E-07	2.15E-08	4.46E-08	2.6E-07	-2.1E-08	-3.9E-07
91	JLY19	6	0.070983	0.072369	0.073382	0.073911	0.073816	0.072916	0.070947	0.067499	0.061857	0.052663	0.037309	0.01103	1.74E-08	9.75E-09	2.06E-07	-4.2E-09	7.16E-08	-1.2E-07	1.9E-08	3.99E-09	-4E-07
92	AUG19	6	0.058759	0.051819	0.040808	0.024118	2.37E-08	5.36E-09	4.97E-09	5.95E-08	2.06E-07	-1.6E-08	7.81E-08	2.69E-07	3.72E-07	3.5E-07	1.58E-07	5.38E-07	-6.1E-09	2.47E-07	1.49E-07	6.61E-08	-1.8E-07
93	SEP19	6	0.07409	0.071224	0.066525	0.059454	0.049361	0.035495	0.01703	3.6E-10	-2E-07	5.34E-08	2.65E-08	-1.2E-07	2.48E-07	5.2E-07	1.24E-07	6E-07	4.02E-07	3.97E-07	-2.3E-07	-4.2E-08	-2.7E-09
94	OCT19	6	0.107159	0.105702	0.102319	0.096298	0.086667	0.072079	0.05068	0.019962	-2.5E-08	4.08E-07	9.84E-08	-2.5E-08	-3.6E-08	3.75E-07	5.56E-07	2.12E-07	5.4E-08	-4E-07	-5.8E-08	4.19E-09	-3E-07
95	NOV19	6	0.097818	0.098058	0.0964	0.091738	0.082065	0.063401	0.02788	-3.7E-08	-1.6E-07	-1.5E-08	1.89E-07	2.22E-07	1.9E-07	1.21E-07	7.29E-08	3.71E-08	1.95E-07	-1.3E-07	-5E-09	5.42E-09	-2.1E-08
96	DEC19	6	0.11256	0.109974	0.084676	0.007923	8.99E-09	1.02E-07	7.27E-09	1.16E-07	-4.8E-08	-1.7E-08	8.18E-09	1.4E-08	1.59E-07	-1.6E-07	1.24E-07	-1.1E-08	-8.4E-08	2.26E-08	-6.6E-08	1.16E-09	-2.5E-08
97	JAN20	6	4.51E-09	1.91E-08	-2.1E-07	-1.4E-08	3.7E-07	8.93E-09	3.98E-08	6.95E-09	9.45E-08	-9.7E-08	-1.7E-07	5.9E-08	2.84E-07	1.05E-08	-7E-10	-6.6E-09	-5.1E-09	-2.8E-08	-7.5E-09	-4.8E-09	1.51E-08
98	FEB20	6	-2.7E-08	7.68E-09	2.19E-08	9.36E-09	4.81E-08	2.3E-08	2.99E-07	4.84E-08	3.26E-08	3.97E-08	1.08E-07	1.84E-09	-3E-09	6.57E-08	4.04E-09	-7.7E-08	1.14E-07	-3.9E-08	-3.4E-08	2.81E-09	-2.6E-08
99	MAR20	6	8.86E-08	1.6E-08	1.88E-08	-1.2E-07	1.31E-08	8.95E-09	1.98E-08	2.9E-08	1.46E-08	2.26E-08	3.06E-08	-5.1E-08	5.08E-08	5.83E-08	0.008456	0.008749	0.005678	0.002555	0.000541	3.71E-09	-1.5E-09
100	APR20	6	5.15E-08	3.99E-08	-1.2E-09	-1.6E-08	-9.3E-08	-2.4E-07	-2.2E-07	-4.2E-08	2.58E-07	-2.4E-07	1.81E-08	-4.5E-08	-5.1E-08	5.04E-08	5.21E-09	0.005194	0.007383	0.004993	0.002392	0.000847	0.00021
101	MAY20	6	0.506957	0.52592	0.54118	0.550874	0.55275	0.5442	0.522373	0.484449	0.428286	0.353768	0.264864	0.170713	0.073914	2.19E-08	1.07E-09	-1.5E-08	1.56E-07	-1.1E-07	7.24E-08	3.85E-09	2.89E-09