

3. [5 pts] Let $n \in \mathbb{Z}$. Prove that n^2 leaves a remainder of 0 or 1 when divided by 3. (Hint : Proof by cases. There are three possibilities for n. $n = 3k$, $n = 3k + 1$, $n = 3k + 2$)

- ① Let $n \in \mathbb{Z}$ [Assumption]
- ② $\exists k \in \mathbb{Z}$, for $\forall n \in \mathbb{Z}$, $n = 3k$ or $n = 3k + 1$ or $n = 3k + 2$ [from ①]
- ③ If $n = 3k$, $n^2 = 9k^2$, $n^2 \bmod 3 = 0$ [from ②]
- ④ If $n = 3k + 1$, $n^2 = 9k^2 + 6k + 1$, $n^2 \bmod 3 = 1$ [from ②]
- ⑤ If $n = 3k + 2$, $n^2 = 9k^2 + 12k + 4$, $n^2 \bmod 4 = 1$ [from ②]
- ⑥ $n^2 \bmod 3 = 0$ or $n^2 \bmod 3 = 1$ [from ③④⑤]