## 内容简介

编程实现用 Richardson 外推计算 $f'(x)$ 的值，$h=1$。函数 $f(x)$ 分别取

1. $\ln x \qquad x=3 \qquad M=3$
2. $\tan x \qquad x=\arcsin 0.8 \qquad M=4$
3. $\sin(x^2+\frac{1}{3}x) \qquad x=0 \qquad M=5$

输出相应的三角阵列

$$
\begin{array}{ccccc}
D(0,0) & & & & \\
D(1,0) & D(1,1) & & & \\
D(2,0) & D(2,1) & D(2,2) & & \\
\vdots & \vdots & \vdots & \ddots & \\
D(M,0) & D(M,1) & D(M,2) & \cdots & D(M,M)
\end{array}
$$

## 工作环境

程序所用语言：**python**
软件：**JupyterLab**
使用的包：**numpy, matplotlib, bisect**

## 输出结果

```
log(x), x = 3, h = 1, M = 3
D(*, 0) = [0.346573590280, 0.336472236621, 0.334108169326, 0.333526435756]
D(*, 1) = [0.333105118735, 0.333320146895, 0.333332524566]
D(*, 2) = [0.333334482105, 0.333333349744]
D(*, 3) = [0.333333331770]
error = 0.000000001563060181286602

tan(x), x = arcsin 0.8, h = 1, M = 4
D(*, 0) = [-1.306186251360, 6.465336386487, 3.209099924788, 2.872980093931, 2.800901808516]
D(*, 1) = [9.055843932436, 2.123687770888, 2.760940150312, 2.776875713378]
D(*, 2) = [1.661544026785, 2.803423642273, 2.777938084249]
D(*, 3) = [2.821548715535, 2.777533551582]
D(*, 4) = [2.777360943096]
error = 0.000416834681740141377304

sin(x^2 + x / 3), x = 0, h = 1, M = 5
D(*, 0) = [0.176784049147, 0.321477647361, 0.332297588048, 0.333196213584, 0.333306678258, 0.333327146260]
D(*, 1) = [0.369708846765, 0.335904234944, 0.333495755430, 0.333343499816, 0.333333968927]
D(*, 2) = [0.333650594156, 0.333335190129, 0.333333349442, 0.333333333534]
```

```
D(*, 3) = [0.333330183716, 0.333333320224, 0.333333333282]
D(*, 4) = [0.333333332524, 0.333333333333]
D(*, 5) = [0.333333333334]
error = 0.000000000000408784117667
```

## 分析

各组试验的真实导数值与偏差分别为

1. $$(\ln x)'\Big|_{x=3} = \frac{1}{3} \qquad error = 1.56306 \times 10^{-9}$$

2. $$(\tan x)'\Big|_{x=\arcsin 0.8} = \frac{1}{1-\sin^2 x}\Big|_{\sin x=0.8} = \frac{25}{9} \qquad error = 4.16835 \times 10^{-4}$$

3. $$\left(\sin(x^2+\frac{x}{3})\right)'\Big|_{x=0} = \left(2x+\frac{1}{3}\right)\cos(x^2+\frac{x}{3})\Big|_{x=0} = \frac{1}{3} \qquad error = 4.08784 \times 10^{-13}$$

其中 $error = |D(M, M) - f'(x)|$。试验 2 的误差明显过高。

容易观察出 $x+h = \arcsin 0.8 + 1 > \frac{\pi}{2}$，与其他结点相隔一个第二类间断点，取到了负值，导致计算出的第一个导数值为负，其不合理性导致了误差的增大。另外，$x+\frac{h}{2} = \arcsin 0.8 + 0.5 = 1.4273 \approx 1.5708 = \frac{\pi}{2}$，非常靠近该间断点，其函数值相比其他结点而言过大，对最后的计算也产生了负面影响。

计算出 $\left|D(3, 3) - \frac{25}{9}\right| = 2.44226 \times 10^{-4}$，$\left|D(2, 2) - \frac{25}{9}\right| = 1.60306 \times 10^{-4}$，更证明了这一点。

取 $h = 0.25$，重新应用 Richardson 外推法得到如下结果：

```
tan(x), x = arcsin 0.8, h = 0.25, M = 4
D(*, 0) = [3.209099924788, 2.872980093931, 2.800901808516, 2.783518000094, 2.779210306821]
D(*, 1) = [2.760940150312, 2.776875713378, 2.777723397286, 2.777774409064]
D(*, 2) = [2.777938084249, 2.777779909547, 2.777777809849]
D(*, 3) = [2.777777398837, 2.777777776520]
D(*, 4) = [2.777777778001]
error = 0.000000000223547402811164
```

可见在一定条件下，选取恰当结点会使计算精度大幅提高。

## 参考资料

[1] David R. Kincaid & E. Ward Cheney. *Numerical Analysis: Mathematics of Scientific of Computing Third Edition*, Brooks/Cole, 2002.