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6.046 Problem 1-1

Collaborators: *none*

(a) The farthest distance between two points within a $\frac{1}{2} \times \frac{1}{2}$ square is $\frac{\sqrt{2}}{2} < 1$. Then FCC must reject the set of requests if two requests are in or on the boundary of the same square.

(b) The problem is that given two lists of points (x, y): L_x and L_y, sorted by x coordinate and y coordinate respectively, and contain the same set of points, we wish to check whether there are two points within Euclidean distance 1.

To solve this problem, we first find $L_x[\frac{n}{2}]$, and devide L_x into L_{xl} and L_{xr}, based on whether a point is before or after $L_x[\frac{n}{2}]$ in L_x. Then we get the corresponding L_{yl} and L_{yr}, by going through L_y. Then check the left half and the right half seperately. If both sides return true, we need to check whether there is a point from the left and a point from the right that the Euclidean distance between them is less than 1. We can do this by finding all points whose x coordinate is within $\frac{1}{2}$ of $L_x[\frac{n}{2}]$'s x coordinate and putting them in a list L' with increasing y coordinate (we can do so by going through L_y). Calculate the Euclidean distances between neighboring points in L': $L'[0]$ and $L'[1]$, ... If there is a pair whose distance is less than 1, return true; else, return false.

Let's prove that only checking neighboring points works. Otherwise, we can find $L'[i]$ and $L'[j]$, $j - i > 1$, and the distance between them is less than 1; while the distance between $L'[i]$ and $L'[i+1]$, the distance between $L'[i+1]$ and $L'[j]$ are both greater than 1 (if the distance between $L'[i+1]$ and $L'[j]$ is less than 1, replace $L'[i]$ with $L'[i+1]$). Then in the triangle formed by $L'[i]$, $L'[i+1]$ and $L'[j]$, the angle of $L'[i] - L'[i+1] - L'[j]$ must be less than $\frac{\pi}{3}$. As we have checked points of the same side before, $L'[i]$ and $L'[j]$ must be from diferrent side of $L_x[\frac{n}{2}]$. Then $L'[i+1]$ is at the same side with either $L'[i]$ or $L'[j]$, which indicates that the distance between x coordinates is less than $\frac{1}{2}$, then the distance between y coordinates is greater than $\frac{\sqrt{2}}{2}$. Then the angle made by either $L'[i]L'[i+1]$ or $L'[i+1]L'[j]$ with x axis is greater than $\frac{\pi}{3}$, which contradicts with what we got before.

As all we do every time is going through L_y twice, we have

$$T(n) = 2T(\frac{n}{2}) + O(n)$$

Then the overall running time is $O(n \lg n)$.