

Visual Learning

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Volume 4

András Benedek / Kristóf Nyíri (eds.)

The Power of the Image

Emotion, Expression, Explanation

Trace, Writing, Diagram: Reflections on Spatiality, Intuition, Graphical Practices and Thinking

Sybille Krämer

1. Diagrammatics and Cognition

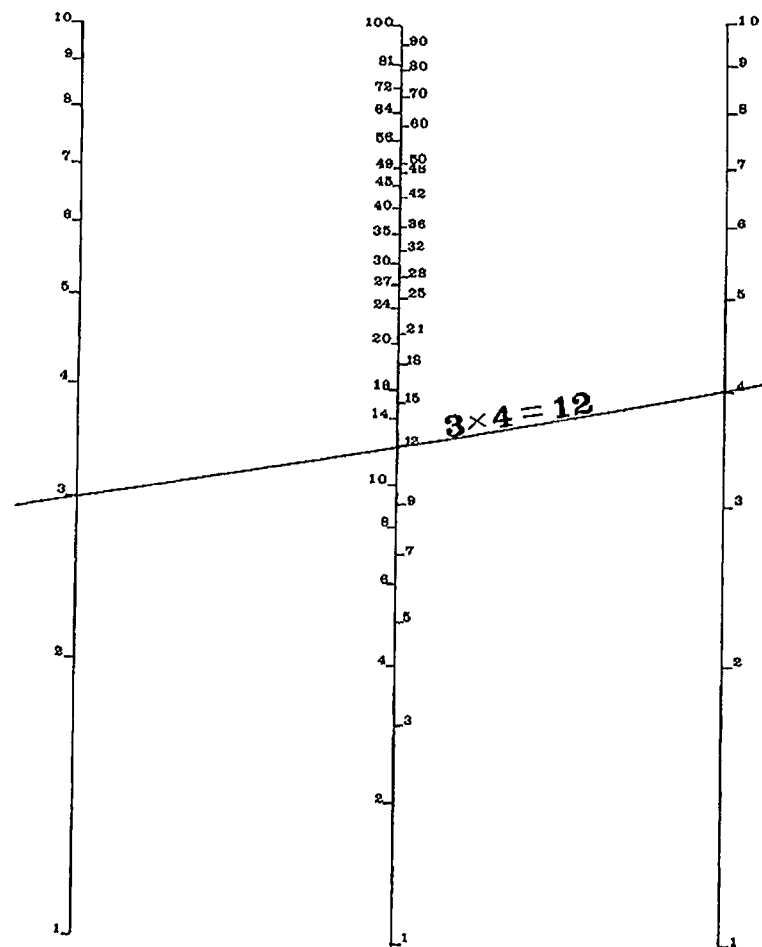
If there is a crucial dichotomy in the human sciences, it is that between word and image, between representation and presentation, between the discursive and iconic forms of the symbolic. Our speaking of a “linguistic” turn or an “iconic” or “pictorial” turn confirms and strengthens this conceptual dichotomy. However there exists a sizable class of representational tools which challenge this binary ordering of the symbolic: consider writing, tables, graphs, diagrams or maps. They arise as a conjunction of language and image; let us call this class the “diagrammatic”. Diagrammatics’ smallest common denominator arises from the interaction between point, line and plane. “Saying” and “showing” work together in the diagrammatic to create an “operative iconicity”.¹ These graphical representations open up a field of both aesthetic and epistemic experience. They are not only a medium for the representation of the objects of knowledge, but also at the same time an instrument through which those very objects can be *generated* and *explored*. Knowledge is not only represented, transmitted and disseminated through the diagrammatic; it is produced and expanded by it.

This paper will describe the role that the diagrammatic plays for thinking and understanding, and how it realizes its part. Our assumption is that the cognitive function associated with the diagrammatic exists in order to allow imperceptible theoretical objects to become visible and tactile. Intuition – what Kant calls “*Anschauung*” – and thought become so closely tied in the diagrammatic, that it allows us to undertake the perceptual investigation of otherwise imperceptible objects. The hypothesis which underlies this paper is that the epistemic function of operative iconicity is grounded in the particular connection that exists between visuality, spatiality and graphism. Look at a simple example, the Nomogram (Figure 1).

1 Sybille Krämer, “Operative Bildlichkeit: Von der ‘Grammatologie’ zu einer ‘Diagrammatologie’? Reflexionen über erkennendes ‘Sehen’”, in Martina Heßler and Dieter Mersch (eds.), *Logik des Bildlichen: Zur Kritik der ikonischen Vernunft*, Bielefeld: transcript, pp. 94–122.

Nomograms are operative visualizations of mathematical functions. This nomogram transforms the procedure of multiplication to be carried out by schematic operation on a plane.² The outside lines represent the terms to be multiplied (the factors); the inside lines show the result of the equation (the product). To realize the operation of multiplication, a thread or ruler is laid between the two factors; the product is to be found where it crosses the middle line.

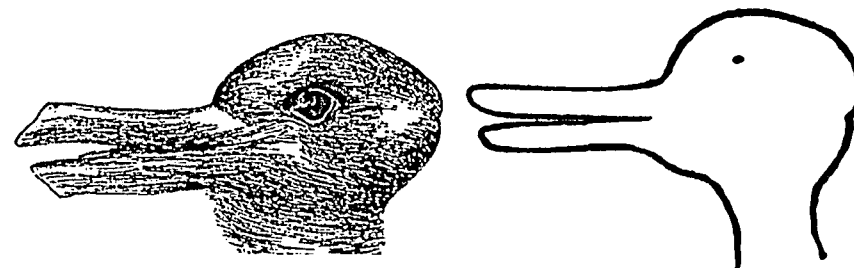
Figure 1: Nomogram of the multiplication table



2 Thomas Hankins – Robert Silverman, *Instruments and the Imagination*, Princeton, NJ: Princeton University Press, 1999, p. 53.

A paper-tool is born, an instrument of arithmetical problem solving. In this simple example we see an important aspect of operative iconicity: It is the phenomenon of problem-solving through spatialized ordering. Numbers are theoretical entities; they have no locale in the world. But with the help of three lines these abstract entities are given a quasi-physical place. Through the ascending number line this numerical paper-space acquires a clear directionality and orientation. The higher the number is placed on the line, the bigger its numerical value. The topological quality of “higher or lower” corresponds to a quantitative, arithmetic quality. An arithmetic law – embodied by the vertical lines – as well as its application in specific instances – embodied by the horizontal lines – are made visible as part of a technique for problem-solving. In this case as in many others, we do not merely think *WITH* paper; we think *ON* paper.

Figures 2 and 3: Duck-Rabbit Head from Jastrow; Duck-Rabbit Head from Wittgenstein



Now let us look at a second example (Figures 2 and 3): Wittgenstein's duck-rabbit.³ This flip-flop-picture with its two aspects, first discussed by the psychologist Joseph Jastrow,⁴ appears in a passage in the *Philosophical Investigations* in which Wittgenstein explains just how complex our use of the word “see” is: aspect-seeing is as much an act of thinking as it is of seeing per se. Looking at the drawing we see two different possibilities even though the picture itself remains identical. While we can always see *either* the head of the rabbit or the duck, we can never actually perceive the alternation between one “gestalt” and

3 Ludwig Wittgenstein, *Philosophical Investigations*, Oxford: Basil Blackwell, 1953, Part II, sect. xi.

4 Joseph Jastrow, “The Mind's Eye”, *Popular Science Monthly*, vol. 54, January 1899, pp. 299–312.

the other. For Wittgenstein, the invisibility of aspect-change is symptomatic of the thinking involved in aspect-seeing.

Let us have a closer look at this kind of aspect-change. The alteration from duck-head to rabbit-head means re-organizing not only gestalt but also – and this is a fundamental prerequisite for gestalt-change – to take into consideration the *alignment* of structures on the surface. As a duck-schema, the drawing is oriented (if we think of the beak as the front) towards the left: the point representing the duck's eye “looks” toward the leftward margin of the paper. When, however, we imagine the drawing to represent a rabbit, then it is re-oriented rightwards: the same point, now the rabbit-eye, gazes in the direction of the right margin. What remains invisible – the sudden change which Wittgenstein associates with thinking itself – is rooted in the complete re-orientation of *directionality* that the viewer carries out on the surface.

2. Inscribed Surfaces

We live in a three-dimensional world, and yet we are surrounded by surfaces covered with two-dimensional images and two-dimensional scripts. We constantly represent the fullness of the real world and the phantasms of fictional worlds on flat surfaces as drawings, figures, schemata and descriptions. So self-evident is the existence of inscribed surfaces to us that we barely realize what a special form of spatiality they embody.⁵ The technique of “flattening out” – of making the three-dimensional two-dimensional – is a major principle of modern technology: think of the flat screen, the computer monitor, the cell-phone interface, etc.; yet the cultural technique of flattening out has a long and rich tradition as an aesthetic and epistemic principle too. It is not accidental that the silhouette and the shadow is a phenomenon as important for visual art as for science and philosophy.⁶

⁵ On flatness see David Summers, *Real Spaces: World Art History and the Rise of Western Modernism*, London: Phaidon Press, 2003.

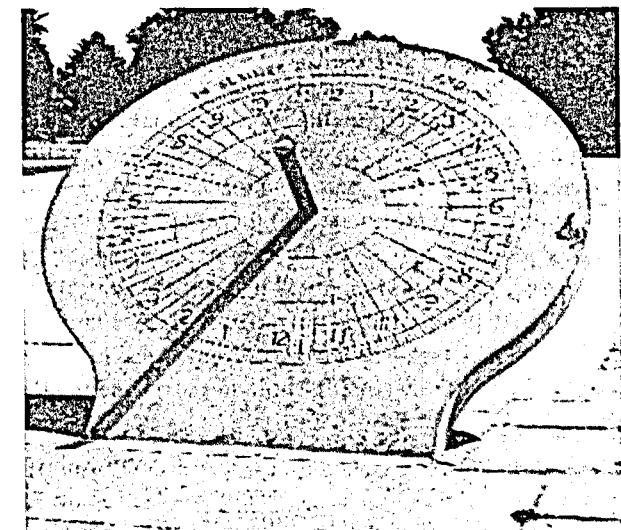
⁶ Steffen Bogen, “Schattenriss und Sonnenuhr: Überlegungen zu einer kunsthistorischen Diagrammatik”, *Zeitschrift für Kunstgeschichte*, vol. 68, no. 2 (2005), pp. 153–176.

Figure 4: *The Legend of Butades*. Etching in Joachim von Sandrart's “*Teutsche Academie*”, 1675



Let us recall the legend, recounted in Pliny, of the daughter of the potter Butades (Figure 4), who traces the shadow of her departing beloved on the wall. This event symbolizes the birth of the creation of the pictorial by “going to flatness”.

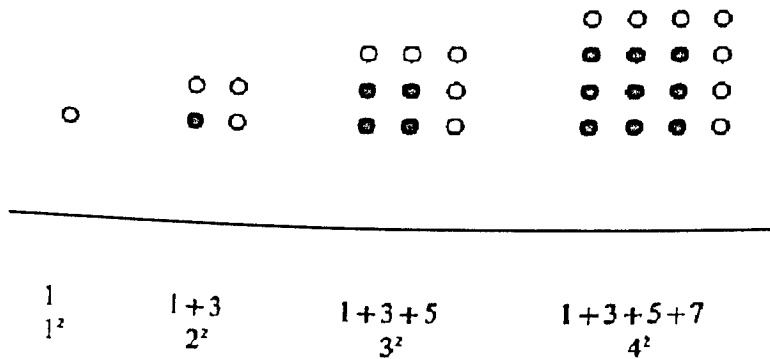
Figure 5: *Gnomon* (Source: <http://mysite.du.edu/~jcalvert/astro/crandial.jpg>)



Let us at the same time bring to mind the “Gnomon”, the shadow-hand of the sundial, which, interacting with an inscribed and image-laden surface, was a cognitive instrument for the measuring of time in many ancient cultures, playing both a practical and theoretical role. This image shows how on a sundial, the shadow of the rod interacts with the inscribed surface in order to mark the advance of time (Figure 5).

The Pythagoreans employed the Gnomon as a number-angle, which they used to construct figurate numbers and so discover and test the qualities and laws of arithmetic (cf. Figure 6).⁷

Figure 6: Pythagorean Number-Figuration (Source: Krämer, *Berechenbare Vernunft*, 1991, pp. 17 ff.)



The epistemic use of inscribed and illustrated surfaces is a cultural achievement of the highest order. Let us ask a speculative question: was the invention of the surface as important for the mobility and creativity of the mind as the invention of the wheel was for the mobility and creativity of the body?

The following reflections seek better to clarify the relationship of spatiality to graphism within an inscribed, “operative iconicity”. We will lay out the conceptual foundations of this special form of spatiality, and will bring to your attention examples of the relation between spatiality and graphism from the history of philosophy.

⁷ Oscar Becker, *Das mathematische Denken der Griechen*, Göttingen: Vandenhoeck & Ruprecht, 1966; Sybille Krämer, *Symbolische Maschinen*, Darmstadt: Wissenschaftliche Buchgesellschaft, 1988, pp. 28–31.

3. Aspects of Spatiality

There are at least four aspects of spatiality which are especially helpful to understanding its cognitive functions.

3.1. Spatiality as Planarity

In the reality that surrounds us, our bodies create a three-part matrix of orientation: we distinguish between above and below, right and left, and in front of and behind. With this powerful body-based structure we can endow our surroundings with an elementary order. The advantage of the inscribed plane is that this elementary orientation-matrix can be projected into two dimensions – with the crucial exception that there is no longer any “behind” or “beneath”. Depth is eliminated in the silhouette and in graphism.

It is essential to bear in mind that there are *no* planes in our reality; rather, we handle the surfaces of voluminous objects *as if* they had no depth. Through artificial two-dimensionality a small terrain is produced which is mobile, manipulable by hand, and fully available to inspection by the eye. In the synoptic experience of planarity the “overview” is generated; a “cartographic perspective” or bird’s eye view becomes possible for the external observer. This is a position which is inaccessible to us as participants in the three-dimensional life-world. With the inscribed surface we discover a distinct space which seems entirely subject to our visual perception and capacity for examination, tactile manipulation and controlling.

3.2. Spatiality as Directionality/Orientation

A consequence of the projection of the asymmetric body-matrix of above/below and right/left onto a plane is that the plane acquires a directional alignment or orientation. This orientation is important for making graphic objects, but also for reading, examining and interpreting inscriptions. Look for a moment at scripts: all scripts follow a two-dimensional matrix, in that they move right to left (or vice versa), or from top to bottom (or vice versa); a piece of paper with writing on it becomes unreadable when turned 90 degrees. Likewise a topographic map requires an indication of the north-south and east-west axes; and Wittgenstein’s duck-rabbit springs forth in the flipping of directionality.

It was Immanuel Kant who recognized that spatiality necessarily included a directional alignment or orientation:⁸ this is why there are objects which can

⁸ Immanuel Kant, “Von dem ersten Grunde des Unterschiedes der Gegenden im Raume” (1768), in Kant, *Vorkritische Schriften bis 1768*, ed. by Wilhelm Weischedel, Frankfurt/M.: Suhrkamp, 1977, vol. 2, pp. 991–1000.

resemble one another in all attributes, like the contours of the right and left hands, and yet are still not congruent, and therefore still not identical: the left glove does not fit the right hand. Such "incongruent objects"⁹ are only describable when their rotational direction – towards the right or left – is added to the description. Kant is convinced that such a right-left orientation and the rotational direction cannot be *verbally* described, but rather can only be *shown* through intuition ("Anschauung").

3.3. Spatialization of Time

Space and time normally function as equally-weighted forms of our perception of the world. Our claim here, however, is that whenever we must orient ourselves either practically or theoretically in an uncertain situation, space takes pride of place over time. Graphical inscriptions and drawings are temporal processes – and yet their value comes from their ability to transform the succession of temporal experience ("one thing after another") into *the simultaneity of spatial experience* ("one thing next to the other"). Even though the examining and reading of inscriptions takes place temporally, the fundamental effect of the diagrammatic is the spatialization of an act executed in time – which can then once again be made fluid and temporal. One consequence is the deceleration and slackening of time. Our nerve cells work at lightning speed – and so does our faculty of speech. But the techniques of writing and drawing are burdensome and take time. And this is precisely their cognitive advantage: when we make, examine and correct traces of thought in the sluggish materiality of the inscribed surface, then we gain a critical distance from the erratic flux of the spontaneous, mercurial inner life of the mind. At the same time in the two-dimensional plane we have created a domain in which the irreversibility of time, to which we must succumb as a mortal species has been undone: all that is ordered can be otherwise re-ordered.

3.4. Structural-space and Movement-space

Often two representational forms of space are set in opposition to one another: there is on one side stable *structural space*, characterized by the coexistence of places and the relations between them, and on the other there is *movement space*, which comes into being through the motions of actors, and which is

⁹ For some interesting essays on incongruency and Kant see James van Cleve and Robert Frederick (eds.), *The Philosophy of Right and Left: Incongruent Counterparts and the Nature of Space*, Dordrecht: Kluwer Academic Publishers, 1990.

temporally bound to them.¹⁰ The synoptic, topographical map – the bird's eye view of a landscape – is the model for structural space; the successive, participant-oriented description of a series of movements ("turn right, then left...") is the prototypical representational modality for movement space. Customarily, structural space and movement space are thought of as dichotomous principles. And yet when we look at the practice of orienteering with a map, we notice that with the help of the map a user (thanks to his indexical self-projection onto the map with his finger) can transform an unknown terrain in his *own* realm of movement and action. This metamorphosis of a structural spatiality into a movement-based spatiality is the decisive reason for and goal of modern overview-maps. Let us call this the "cartographic impulse". By this we mean the *interaction* – rather than opposition – of the two spatial principles. Structure and movement "work" together, not against one another! The decisive point here is that the cartographic impulse can be applied to mental orientation in the domain of knowledge – that is, to thinking and understanding. But in order to understand this mode of enabling intellectual actions through the spatiality of planar inscriptions, we must first discuss some of the principle characteristics of graphism.

4. The Intellectual Potential of Graphism

4.1. Medial Bilingualism

Our point of departure is the observation that from an anthropological point of view human beings exhibit a fundamental bilingualism in media: we "have" not only an acoustic language, but also a visual one, which includes mimicry, gesture and the ability to produce and interpret graphical marks. The paleontologist André Leroi-Gourhan reminds us that in the animal kingdom there seem to be parallels to human speech in the signals some species use to communicate, but there is nothing that approximates the engraving of images and drawing of configurations of lines.¹¹ We mean to say here that when we look at language not only from a communicative perspective but also from the point of view of its cognitive import, graphical language acquires a deep operative meaning and an epistemic priority.

¹⁰ Michel de Certeau, *L'Invention du Quotidien*, vol. 1: *Arts de Faire* (1980), here quoted after the German edition: *Die Kunst des Handelns*, Berlin: Merve, 1988, p. 217.

¹¹ André Leroi-Gourhan, *Hand und Wort: Die Evolution von Technik, Sprache und Kunst*, Frankfurt/M.: Suhrkamp, 1980.

4.2. Intellectuality of the Schematic

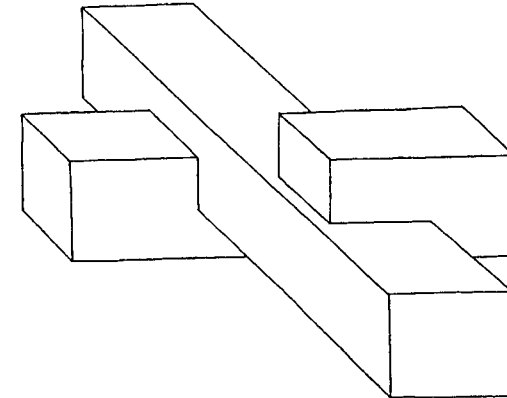
How does the line take on the powers of cognition, and where does the intellectual potential of graphism take root? The empirical line has not only length but also width and often depth; and of course aesthetically speaking the physicality of the line is highly consequential. But when we treat the physical dash schematically as a line, we turn an empirical fact into a non-empirical circumstance: in the schematic the one-dimensionality of the line as a theoretical construct first comes into being. This metamorphosis of the dash into the line seems unassuming and trivial. And yet it contains within itself the seed of the potential for *symbolic transfiguration*, by which something still only thinkable comes into being in the perceptible. Every border-line testifies to this transfiguring power, in that a borderline enables a distinction between territories without belonging to them. The line is the condition for the possibility of differentiation. Have we stumbled upon the origin of our capacity for transcendentalism?

4.3. Heteronomy and Autonomy

Two cognitive achievements intersect in the inscription: the capacity for trans-natural figuration and the potential for unprecedented conceptualization. The line is at first the trace of a gesture; the line contains the potential of mapping, the capacity to be a representation of something that is itself not a line. The transference of the three-dimensional into the two-dimensional through projection already testifies to the creative power of these acts of transference. We can also graphically represent the non-existent or even the utterly impossible. Here we come across the imaginative design character of the line: on the plane, the real can be stood on its head and the logically impossible can be made visible to the eye – think of Escher's drawings or Reutersvärd's impossible figures (Figure 7).¹²

¹² Oscar Reutersvärd, *Unmögliche Figuren: Vom Abenteuer der Perspektiven*, Augsburg: Augustus-Verlag, 1990.

Figure 7: Reutersvärd, impossible figure (Source: *Unmögliche Figuren: Vom Abenteuer der Perspektiven*, Augsburg: Augustus Verlag, repr. of Augsburg: Weltbild Verlag, 1988, p. 22)



In the line, heteronomy and autonomy – external determination and self-determination – cross.

4.4. Syntax of the Linear Configuration

The line contains an elementary repertoire of forms. We can access this repertoire if we ask ourselves what forms we can produce on a surface with a rod – forerunner of the straight line – and a thread – forerunner of the curved line. Rod and thread comprise the material and physical fundamentals for the technique of flattening out. The rod becomes the “stylus” for scratching and the brush for filling in spaces; it becomes the gnomon for the shadow-hand of the sundial, and is used as a universal instrument of measure and understanding – and indeed, even as a single universal measure in the form of the prototype meter (le mètre des archives) in Paris. The thread can be laid along curves, tied together into a net, or woven tightly with other threads to create a surface, as with canvas.¹³

5. Cartographic Impulse

At this point we return to the idea of a *cartographic impulse*. The “cartographic impulse” means that a structural space is used in order to create a movement-space,

¹³ Tim Ingold in his *Lines: A Brief History*, London: Routledge, 2007, interprets not the rod, but trace and thread as forerunners of the line.

that is, to open up a complex terrain to the possibility of movement and action. We are interested in the transference of this process into the domain of the cognitive and into the many fields of knowledge. Just as a topographical map provides orientation in unexplored spaces of the life-world, so do diagrams enable orientation and movement in the spaces of thought. The idea that understanding and thinking is a *path* (as Plato says); that it is a *direction* (as Wittgenstein says); that we must *orient* ourselves in thought (Kant); and that searching for knowledge is like following Ariadne's thread through the labyrinth of understanding (Descartes and Leibniz), all bear witness to an implicit cartographic dimension in many philosophers. The cartographic impulse unspools like a red thread through the history of philosophy. Its epistemic role is implicit in many epistemologies and explicit also in many of the visualization practices of philosophers (and scholars in general).

Opening up this connection between the graphic and philosophy is the goal of a *diagrammatology*. With the term diagrammatology we imply a transformation of Derrida's *Grammatologie*.¹⁴ While Derrida extended the term of writing to any use of signs, diagrammatology amends the subset of "script" by "table", "list", "graph", "diagram", and "map", and defines these under the expression "*diagrammatics as operative iconicity*". This operative iconicity should be clearly differentiated not only from spoken language but from technical pictures as x-ray or scanning microscopy etc. as well.

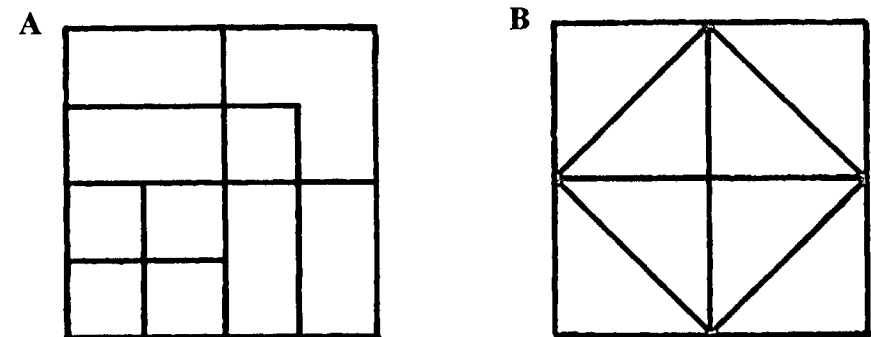
Frederik Stjernfelt introduced the term "diagrammatology" as an epistemic concept;¹⁵ he also rehabilitated Charles Sanders Peirce as a pioneer of "diagrammatical reasoning".¹⁶ And yet, moving beyond Stjernfelt, it can be demonstrated that "diagrammatical spatialization" appears in the work of many philosophers – Plato, Aristotle, von Kues, Ramus, Lullus, Descartes, Leibniz, Lambert, Kant, Frege and Wittgenstein among them. We will only show how this spatialization is manifested in two canonical philosophers: Plato and Kant.

6. Plato

Plato is most often associated with the discrediting of the pictorial; and yet a closer look at his writings fails utterly to confirm this presumed hostility to the

image. In fact, Plato is convinced of the unavoidability of epistemic encounters with visio-spatial forms. This can clearly be seen in two important scenes from his dialogues.

Figure 8: Meno's Scenario



6.1. Meno

In the *Meno* (82b-85c), Socrates wishes to demonstrate that a person may acquire mathematical knowledge without it being either communicated through language or explicitly taught. He asks a mathematically illiterate young slave to double the area of a square with a side-length of 2 (cf. Figure 8). His immediate reaction is to double the lengths of the sides. He quickly realizes, however, that the new square is four times too big. His next idea is to try a square with a side-length of 3: it, however, is still more than twice as big as the original. The translation of his spontaneous thoughts into a drawing shows him directly his own failure and ignorance. The young slave finds himself on unfamiliar ground, he loses his self-confidence and falls into an aporia; he tells Socrates: "I don't know." With the help of a new drawing and further questions from Socrates, the young slave finally succeeds in solving the problem: it is possible to double a square – by using its diagonal as the side-length for the larger square.

Plato intends to show here that knowledge cannot be transmitted; acquiring knowledge is possible in a play between diagram and dialogue without this knowledge being verbally communicated within that dialogue. The kind of knowledge acquired in the *Meno* is procedural knowledge – a "being able to", a "knowing how", rather than a "knowing that". "Knowing how" can in fact never be fully verbalized: for learning to be able to do something is produced through

¹⁴ Jacques Derrida, *De la grammatologie* (1967).

¹⁵ Frederik Stjernfelt, *Diagrammatology: An Investigation on the Borderlines of Phenomenology, Ontology, and Semiotics*, Dordrecht: Springer, 2007.

¹⁶ Frederik Stjernfelt, "Diagrams as Centerpiece of a Peircean Epistemology", *Transactions of the Charles S. Peirce Society*, vol. 36, issue 3 (Summer 2000), pp. 357–384.

doing and rarely through verbal description – think of swimming or riding a bicycle. The schematism of an activity is acquired. And the young slave acquires this intellectual “being able to” to solve a mathematical problem through an explorative interaction with an inscribed surface. Plato himself compares this problem-solving knowledge with the kind of knowledge about movement in space needed to find the “road to Larissa” (97a/b) – a highly practical, highly spatial analogy which has vexed many an interpreter.

Mathematics is often thought of as the incarnation of a non-empirical, propositional knowledge. And it is of course true that the young slave will have acquired the skills of mathematical problem-solving only when he realizes that it is not only in the case of the two-foot long square that using the diagonal as a side allows the square to be doubled but that rather the same rule applies to every square. And yet how can such mathematical insights be gained through experimentation in concrete visualizations without mathematics losing its purely non-empirical status? A second scenario in Plato answers this question. This is the simile of the line from the seventh book of the *Republic* (509d–511e).

6.2. The Simile of the Line

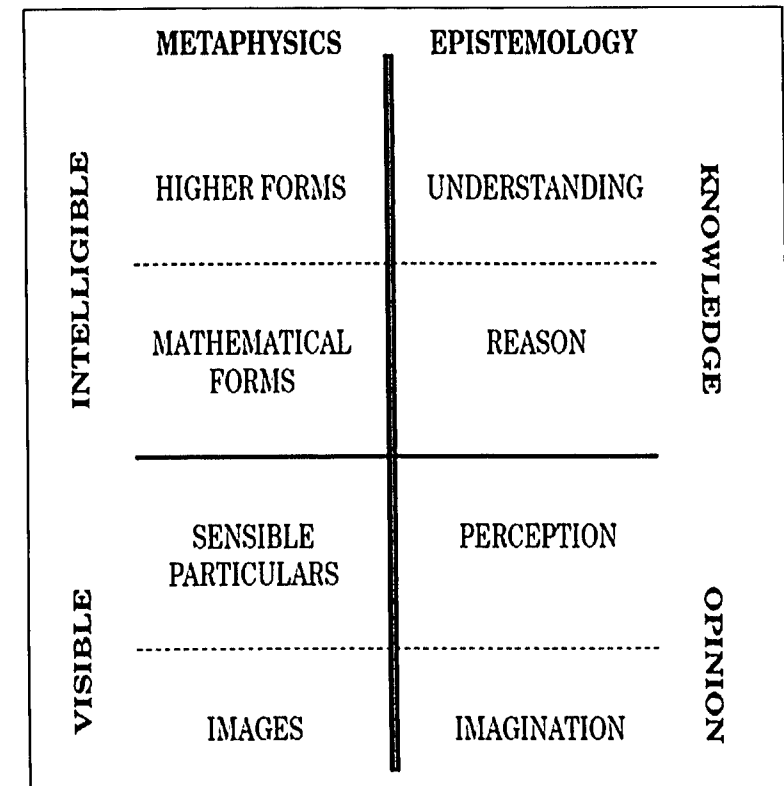
Plato lays out the central features of his ontology – which is above all based on the distinction between the perceptible and the intelligible, between the copy and its original – in a linear figure, the simile of the line (Figure 9). Socrates’ instructions for building the line are as follows: a line is drawn and divided into two unequal segments; the smaller segment represents the perceptible, the larger the intelligible; the intelligible gives us access to the original object, while the perceptible gives us access to the image. Then, each of the segments is once again divided according to the same asymmetrical proportion and to the relation between the original and the copy. The line is now divided into four segments, each of which has a double meaning – both ontological and epistemological. Each segment corresponds to a domain of being, as well as to the form of knowledge that coincides with this domain.

The realm of the visible contains two domains: first, the domain of images and shadows, which make up the epistemic state of *eikasia* – that is, conjecture. The second domain encompasses the originals of these images: things, plants, animals; the corresponding cognitive activity is *pistis*, faith or belief. Together these segments make up the larger realm of *doxa*, that is, of opinion. The third segment of the line contains mathematical objects and general concepts; its related form of knowledge is *dianoia*, or understanding. The fourth segment

concerns ideas which are only accessible through *noesis*, that is, pure intellection. The third and fourth segments together make up – in contrast to *doxa* – the realm of *episteme*.

Unfortunately, the length of this paper does not allow for a full explication of the simile of the line, so we will limit ourselves to two observations of particular importance to the diagrammatic perspective on philosophy.

Figure 9: Simile of the divided line



(1) *The potential for understanding in visualization*: visual and spatial attributes are given the task of representing non-spatial, non-perceptible concepts in analogical proportionality. Length and position represent ontological and epistemic circumstances. The longer a portion of the line is, and the farther it lies from the beginning of the line, the more intelligible and full of reality

the objects associated with it are – at least in relation to Platonic philosophy, which is precisely what the segments make visible. The crucial fact here is that through the examination of visual configurations, insights are to be gained which are not explicit in the text. An example of this is the role played by proportion in the conception of the relationship between the perceptible and the intelligible. Although this does not appear in Socrates' instructions, we can see that a middle proportion exists nested within the simile (A is to B as C is to D; which can be reformulated as: A plus B is proportional to C plus D; therefore: B equals C), and therefore the two middle segments must in fact be of equal length. The visual equivalence of the two line segments brings to light one of Plato's far-reaching insights into the nature of mathematical understanding. Plato characterizes the *dianoia*, the cognitive state which corresponds to the third segment of the simile, in such a way that when mathematicians "make use of the visible forms and reason about them, they are thinking not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameter." Mathematicians – and by extension scientists of all kinds – must render the non-sensible objects of their study in the form of images and drawings perceivable to the eye even though their kind of knowing is directed toward non-sensible, theoretical objects. In the particular we see the general; in the phenomenal we see the conceptual: this is – metaphorically speaking – the task of the "mind's eye", insofar as seeing and interpreting are here inseparable. A "seeing thinking" and a "thoughtful seeing" take place. And it is exactly this interplay between the sensuous and the intelligible, this double-life of the mathematical diagram, which embodies the bodiless, and is in turn made manifest in the third segment of the simile of the line, whose *length* is equal to that of sensuous things, but whose *location* is situated within the realm of the intelligible.

(2) *The inherent spatiality of thought*: the simile of the line can be thought of as the inauguration of the distinctions between sensuousness and intellection, and between the phenomenal and the intelligible; it established a division which would become for 2000 years the lifeblood of western philosophy. And yet for Plato, this conceptual divide is not at all an absolute, disjunctive "either-or": for it is made relative through the simile's method of construction and its visual mode of explanation. The differing positions of course show how epistemologically and ontologically distant these domains and cognitive abilities are from one another; and yet at the same time they show that the phenomenal and intelligible segments are nevertheless part of one and the same continuously drawn line, and that there is a family resemblance between them. The graphism of the line homogenizes the heterogeneity of perceptible and intelligible and thus makes

visible not only their differences, but also their relationship. When an interpreter suggests that the perceptible and intelligible should be drawn as two parallel lines, he misunderstands the crucial implication of the simile of the line – which is that the distinction between the visible and the intelligible comes into being along a single graduated line.

Thus it becomes possible for the succession of various line segments to show us the direction of movement towards understanding, which in Plato is understood as an ascent from the lowest copies to the realm of ideas. It is of little importance whether the line is drawn horizontally – as is the case in the oldest manuscript copy – or vertically – which would be closer to Plato's philosophy. Either way it has a direction, either from left to right or from bottom to top. And in this directionality of the line, the idea of an orientation in the movement of thought is set in place. Understanding begins in images; and yet it does not remain with them – for then they would be illusions – but rather identifies them as copies of an original image; so the thinking person must pass over these and strive to reach the next highest level. In Plato's philosophy the "Good" is the highest of all ideas; its strategic meaning, however, inheres in the fact that it is not a de facto state of understanding at which one arrives, but rather a compass and mover for the understanding: the Good is a category of orientation in practical and theoretical matters; no more and no less.

The Simile of the Line visualizes the structure of the world along with the direction of movement for understanding. The allegory of the cave, which follows directly after the simile of the line in Plato, offers a representation of this rising through levels of knowledge in a three-dimensional configuration – which the simile of the line represents through an ordering on a flat surface. Let us, with Plato, hold fast to the claim that *thinking is directed, oriented movement*. In thinking and understanding we find an inherent spatiality, which anticipates Immanuel Kant's question: "What does it mean to orient oneself in thinking?"

7. Kant

In the text "What Does It Mean to Orient Oneself in Thinking"¹⁷, Kant transforms the notion of orientation into an epistemological concept. Orientation in space is for Kant something which relies on the one hand on what he calls *Anschauung* – the intuition of visual space – and on the other on our own embodiment.

17 Immanuel Kant, "Was heißt: sich im Denken orientieren" (1786), in Kant, *Schriften zur Metaphysik und Logik*, vol. 1, ed. by Wilhelm Weischedel, pp. 263–283, Frankfurt/M.: Suhrkamp, 1977.

Kant takes orientation from the domain of movement in space and transfers it to the cognitive faculties, and on this way he reveals "*Anschauung*" to be a primary and irreplaceable source for thought. *Anschauung* gains – beneath conceptual thought – the status of a freestanding source of knowledge. Kant's train of thought can be reconstructed in two steps:

(i) In the text *Concerning the Ultimate Ground of the Differentiation of Directions in Space*, written before the Critiques in 1768, Kant develops his standard-setting distinction between "Lage" ("position") and "Gegend" ("direction").¹⁸ "Position" ("Lage") is the form of spatiality determined by relations between things; this is an insight that goes back to Leibniz. Kant's addition to the Leibnizian idea of spatiality is the term "Gegend", which means a direction or orientation which reveals itself – and this is what it depends upon – *only* in the relation to the body's method of orientation, that is, to the distinction between above and below, right and left, front and back. Kant explains the difference between "Lage" and "Gegend" with the example of an inscribed plane: when we have writing on a page and we turn the page, the "Lage" or position of the signs remains the same, while the "Gegend", the direction, changes: the text becomes illegible. The plane of the inscription is always a plane with a *directional inclination*, with which reader and observer enter into a bodily relation.

(ii) It is essential that this direction or inclination be something that cannot be verbally described; it can only be shown. Kant explains this with the example of incongruent objects. If we trace the contours of our hands, we can turn and tilt the drawings we make: yet we cannot make the right and left hand cover to one another simply by a change of position – we cannot make them truly congruent. This is also the case for actual, three-dimensional hands: thus the left hand does not fit into the right glove. Crucial for Kant is that left- and right-handedness cannot be made explicit through pure linguistic description. Thus, for an adequate explanation and understanding of chirality, we absolutely require visual perception. A thought experiment makes this clear: if we had to explain linguistically the difference between right and left to an extra-terrestrial intelligence, our effort would fail. A common perceptual space (*Anschauungsraum*) is indispensable for explaining direction. As a consequence of this Kant establishes *Anschauung* as a genuine source of knowledge. The *perceptible* furnishes – along with the conceptual – its own source of knowing and cognition. No epistemic task – including in mathematics – is possible without *perception*.

18 Kant, "Von dem ersten Grunde des Unterschiedes der Gegenden im Raume" (cf. note 8 above), pp. 993–1000.

(iii) With this comes a problem: *Perception* is always perception of something singular, situated spatiotemporally. And yet how can imperceptible concepts and the perceptible world of experience relate to one another? How can abstract generalizations and concrete empirical objects come into contact? For Kant tells us that there is no doubt as to the fundamental heterogeneity of the two: he calls the conceptual and the perceptible completely and utterly "unlike" (B 176).¹⁹ Kant's solution is as follows: in order to bring together the heterogeneous domains of the perceptible and the intelligible, a third domain must be introduced which contains qualities of both. And this third domain, which mediates between concept and percept (that is, between *Begriff* and *Anschauung*) and between perceptibility and intelligibility, is for Kant schematism: schemata have the quality of "being at once intellectual and perceptible" (B 178).

Schematism (*Schematismus der reinen Verstandesbegriffe* (B 176–187, *Schematism of the Pure Concepts of Understanding*) – is a method of imagining which allows us to create certain images. To give a concept its proper image (B 179) means to develop the schema for that concept. Yet how can this apply to concepts that are independent of experience, like for example the mathematical concept of a triangle or that of a circle, to which an image in our reality can never correspond?

(iv) Here Kant clearly distinguishes between image and schema. A schema is not simply a visual structure, but rather an action, which he characterizes as "figural synthesis". The paradigmatic example for this form of schematization through figural synthesis is for Kant the line: the line conceived of not as a stable demarcation, but rather as the temporal action of its own production (the line is its drawing). I cannot imagine any line – so Kant reasons – without drawing it in my mind "to make a record of our *Anschauung*" (B 156). The schema, which guarantees that abstract concepts take on a meaning within perceptible experience, can itself not be explained through concepts. Thus is our graphical, constructive faculty (our capacity for figuration) the originating location for the work of the imagination. Kant calls schematism a "hidden art in the depths of the human soul" (B 181).

Kant also uses the term "monogram" (B 181) for figural synthesis through imagination. Our hypothesis is: the term "monogram", which constructs the hinge between thinking and *Anschauung* in Kant, is a conceptual forerunner to the term "diagram" we use today.

19 Immanuel Kant, *Kritik der reinen Vernunft* (*Critique of Pure Reason*), 1st ed. ("A") 1781, 2nd ed. ("B") 1787.

8. Conclusion

Let us conclude: there are two widespread misperceptions about the relation between philosophy and image: (1) Minerva's owl – so Hegel thought – symbolizes philosophy because it flies at twilight. This story illustrates a common topos: the more the bodily eye is closed, the better the “mind's eye” sees. (2) Philosophy, it is thought by many, is a discipline that forgets and discredits images. Thinking and knowing – so goes the assumption – are the hoard of a purely linguistic zone, far from all perception. The reflections we have presented in this paper – so we hope – have shown that these assumptions are empty; they ignore some of the fundamental aspects of philosophical thinking. In philosophers' intellectual practices and theories of knowledge, graphical and spatial figuration – whether explicit or implicit – play an important epistemic role. By interacting with inscribed surfaces we engage in the epistemology of the line.

Diagramming: Connecting Cognitive Systems to Improve Reasoning

Valeria Giardino

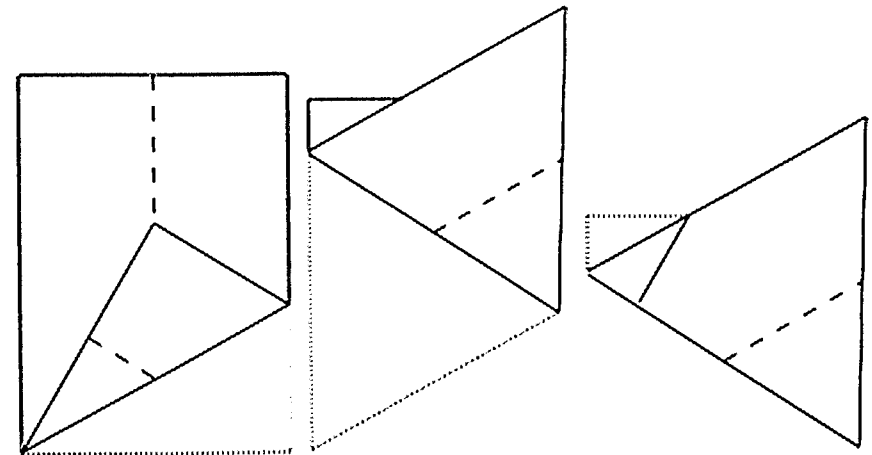
Introduction: A Manipulation Experiment

I shall start from an example. Consider the following exercise. Take a common A4 sheet, fold it in half along the longer side, and then open it up again. Now there is a fold in the middle of the sheet. Orient the rectangular sheet in such a way that it has its longer side towards you. Hold firm the bottom left corner and grab the bottom right corner, folding it so that it touches the fold in the middle and the figure obtained is a trapeze (Figure 1a). Then, grab the bottom left corner and fold it on the other side of the virtual line that the previous fold has created (Figure 1b). At this point, you should recognize a triangle: two of its sides are already there. To obtain the last one, refold what is left outside the area of the triangle (Figure 1c).

Figure 1a

Figure 1b

Figure 1c



The experiment has come to its end: there was a rectangle, and now we have a triangle, which is equilateral by construction. Hence, a rectangle (but indeed