Question 2:

```r
#Loading Data
q2data <- data.matrix(read.csv(file = "Q2_Data.csv"))
q2frame <- read.csv(file="Q2_Data.csv")
y <- matrix(q2data[,1],7,1)
y
```

```
##      [,1]
## [1,] 37.9
## [2,] 42.2
## [3,] 47.3
## [4,] 43.1
## [5,] 54.8
## [6,] 47.1
## [7,] 40.3
```

```r
x <- matrix(c(rep(1,7),q2data[,-1]),7,4)
x
```

```
##      [,1] [,2]   [,3] [,4]
## [1,]    1 32.0   84.9   19
## [2,]    1 19.5  306.6    9
## [3,]    1 13.3  562.0    5
## [4,]    1 13.3  562.0    5
## [5,]    1  5.0  390.6    5
## [6,]    1  7.1 2175.0    3
## [7,]    1 34.5  623.5    7
```

```r
df <- 7-4
```

a: Fit a linear model to the data and estimate the parameters and variance.

```r
#Finding Beta using BLUE
b <- solve(t(x)%*%x,t(x)%*%y)
b
```

```
##              [,1]
## [1,] 54.776606226
## [2,] -0.389598784
## [3,] -0.001973937
## [4,] -0.242767764
```

```r
#Finding variance
#sum-Square
e <- (y-x%*%b)
SSres <- sum(e^2)
s2 <- SSres/(df)
s <- sqrt(s2)

#Beta Variance
C2x <- solve(t(x)%*%x)*s2
diag(C2x)
```

```
## [1] 1.964791e+01 3.378471e-02 7.330554e-06 1.870117e-01
```

```r
#Beta standard Error
sqrt(diag(C2x))
```

```
## [1] 4.4325965 0.1838062 0.0027075 0.4324485
```

Thus the model is given by

$$\begin{array}{lllll} y = & 54.776606226 & -0.389598784X_1 & -0.001973937X_2 & -0.242767764X_3 \\ & (4.4325965) & (0.1838062) & (0.0027075) & (0.4324485) \end{array}$$

b. Find a 90% confidence interval for the expected price per square metre of a 10 year old apartment that is 100 meters away from the train station and has 6 convenience stores nearby.

```
#Part B Computing CI

alpha <- 0.1
x.star <- c(1,10,100,6)
y.star <- x.star%*%b
ta <- qt(1-alpha/2, df)

#Computing 90CI for x1=10, x2= 100 ,x3 =6
CI = c(y.star - s*sqrt(t(x.star)%*%solve(t(x)%*%x)%*%x.star),
       y.star + s*sqrt(t(x.star)%*%solve(t(x)%*%x)%*%x.star))
CI
```

```
## [1] 46.59336 51.85988
```

The 90% confidence invterval of 10 years old appeartment that is 100 meters away from train station and has 6 convenience stores nearby is $(46.59336, 51.85988)$.

c. Find the standard error of $\beta_1 - \beta_3$

```
#General Linear Hypothesis for B1-B3
#Setting C and delta star
C <- c(0,1,0,-1)
cdelta.star <- matrix(0)

#Computing the variance and standard error for B1-B3
Cb.var <- t(C)%*%solve(t(x)%*%x)%*%C*s2
Cb.var
```

```
##          [,1]
## [1,] 0.316463
```

```
Cb.ste <- sqrt(Cb.var)
Cb.ste
```

```
##           [,1]
## [1,] 0.5625504
```

The standard error of $\beta_1 - \beta_3$ is 0.5625504.

d. Test the hypothesis that the price per square metre falls by $1000 for every year that the apartment ages, at the 5% significance level.

Testing $H_0 = \beta_1 = -1$ vs $H_1 = \beta_1 \neq -1$

```
#General Linear Hypothesis
#General Linear Hypothesis
C <- matrix(c(0,1,0,0),1,4)
dst <- matrix(-1)
```

```r
#Conducting an F-test for y=-1 given B1=1
num <- (t(C%*%b-dst)*solve((C%*%solve(t(x)%*%x)%*%t(C)))%*%(C%*%b-dst))
Fstat <- num/(SSres/df)
pf(Fstat,1,3, lower.tail = FALSE)
```

```
##            [,1]
## [1,] 0.04502395
```

Since the P-value is 0.04502395, which is less than 0.05, which should reject the null that the price will fall by \$1000 for each year the apartment age at 5% statistical significant.2

e. Test for model relevance using a corrected sum of squares.

Testing for $H_0 = \beta_1 = \beta_2 = \beta_3 = 0$ vs $H_1 = \beta_1$ or β_2 or β_3 is non-zero using corrected sum of squares.

```r
#Computing model 2
x2 <- x[,-1]
b2 <- solve(t(x2)%*%x2,t(x2)%*%y)

#Breaking Rg1g2 and Rg2 for correct sum squared
SSres2 <- sum((y-x2%*%b2)^2)
Rg2 <- t(y)%*%x2%*%b2
SSreg <- t(y)%*%y
Rg1g2 <- SSreg - Rg2
Rg1g2
```

```
##          [,1]
## [1,] 2158.632
```

```r
#F Test
r <- 1
Fstat <- (Rg1g2/r)/(SSres/(df))
Fstat
```

```
##          [,1]
## [1,] 155.7122
```

```r
pf(Fstat,r,df, lower.tail=FALSE)
```

```
##             [,1]
## [1,] 0.001109267
```

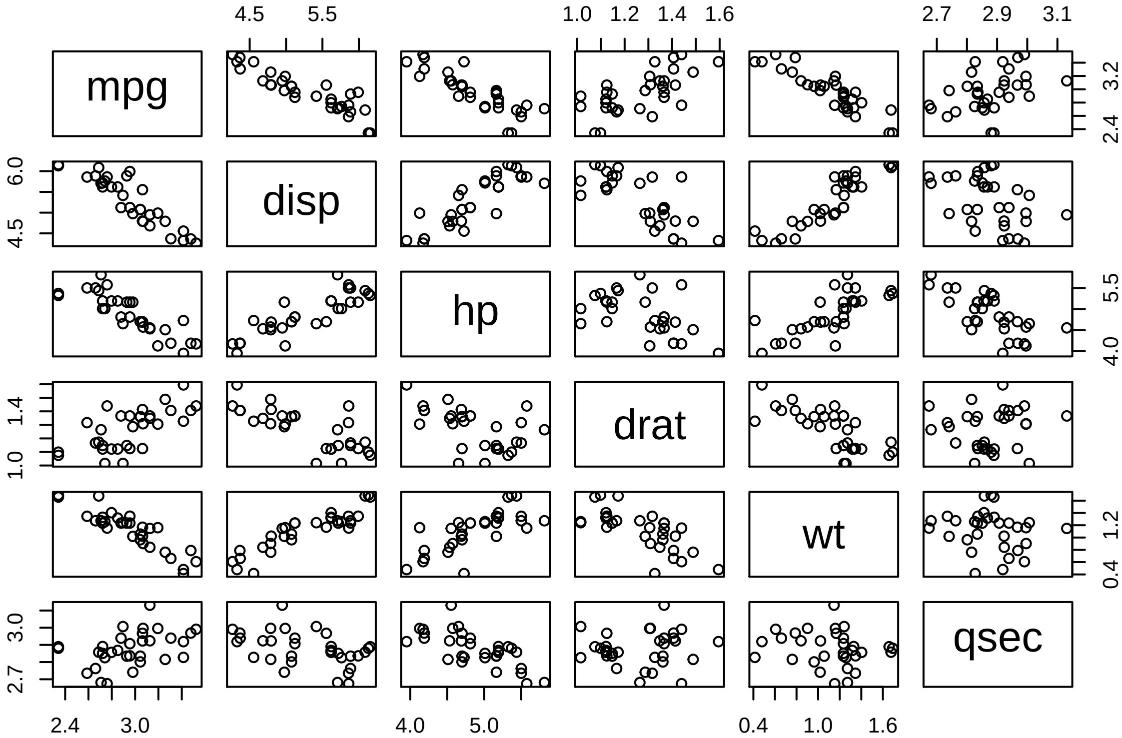
Since the p-value for the test is $0.001109267 < 0.05$, we can say the model is statically significant using the corrected sum of squared method. Hence, we should reject the null

Question 4:

```r
#Loading and Scaling Data
data(mtcars)
mtcars.new = log(mtcars[, c(1,3:7)])
```

a. Plot the data and Comment

```r
#Plotting Pair Graph
pairs(mtcars.new)
```



Form the pairs plots we can see there is a negative linear relationship between **mpg** and **disp**; **mpg** and **hp**; **mpg** and **wt** with all of thems having a small additive error

Between **mpg** and **drat** there is a positive linear relationship; however there seem to be a big error additive error. Similarly, the positive linear relationship also exist between **mpg** and **qsec** but with a multiplicative error instead of an additive one like the other explanatory variables.

For **disp** it is positively correlated with **hp** and **wt** but is negatively correlated with wt. There is a linear relationship between weight and gross horse power.

b. Preform using forward Selection

```r
#Preforming forward selection of mtcars model
basemodel <- lm(mpg~1, data=mtcars.new)
add1(basemodel, scope = ~.+disp+hp+drat+wt+qsec, test="F")
```

```
## Single term additions
##
## Model:
## mpg ~ 1
##        Df Sum of Sq     RSS      AIC  F value    Pr(>F)
## <none>              2.74874  -76.547
## disp    1   2.25596 0.49277 -129.550 137.3427 1.006e-12 ***
```

```
## hp      1   1.96733 0.78140 -114.797  75.5310 1.080e-09 ***
## drat    1   1.23131 1.51742  -93.559  24.3435 2.807e-05 ***
## wt      1   2.21452 0.53422 -126.966 124.3596 3.406e-12 ***
## qsec    1   0.47755 2.27119  -80.654   6.3079   0.01763 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
q4model2 <- lm(mpg ~ disp, data=mtcars.new)
add1(q4model2, scope = ~.+hp+drat+wt+qsec, test="F")
```

```
## Single term additions
##
## Model:
## mpg ~ disp
##        Df Sum of Sq     RSS     AIC F value  Pr(>F)
## <none>              0.49277 -129.55
## hp      1  0.045531 0.44724 -130.65  2.9523 0.09641 .
## drat    1  0.001383 0.49139 -127.64  0.0816 0.77711
## wt      1  0.098796 0.39398 -134.71  7.2722 0.01154 *
## qsec    1  0.000308 0.49247 -127.57  0.0181 0.89382
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
q4model3 <- lm(mpg~disp+wt, data=mtcars.new)
add1(q4model3, scope = ~.+hp+drat+qsec, test="F")
```

```
## Single term additions
##
## Model:
## mpg ~ disp + wt
##        Df Sum of Sq     RSS     AIC F value  Pr(>F)
## <none>              0.39398 -134.71
## hp      1  0.078605 0.31537 -139.83  6.9789 0.01334 *
## drat    1  0.007358 0.38662 -133.31  0.5329 0.47146
## qsec    1  0.057788 0.33619 -137.79  4.8130 0.03671 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
q4model4 <- lm(mpg~disp+hp+wt, data=mtcars.new)
add1(q4model4, scope = ~.+drat+qsec, test="F")
```

```
## Single term additions
##
## Model:
## mpg ~ disp + hp + wt
##        Df Sum of Sq     RSS     AIC F value Pr(>F)
## <none>              0.31537 -139.83
## drat    1 0.0000095 0.31536 -137.83  0.0008 0.9774
## qsec    1 0.0033067 0.31206 -138.17  0.2861 0.5971
```

```r
summary(q4model4)
```

```
##
## Call:
## lm(formula = mpg ~ disp + hp + wt, data = mtcars.new)
##
## Residuals:
```

```
##       Min        1Q    Median        3Q       Max
## -0.196932 -0.086109  0.005329  0.073336  0.220450
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.94620    0.26867  18.410  < 2e-16 ***
## disp        -0.07792    0.10152  -0.768  0.44919
## hp          -0.21299    0.08063  -2.642  0.01334 *
## wt          -0.47880    0.13993  -3.422  0.00193 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1061 on 28 degrees of freedom
## Multiple R-squared:  0.8853, Adjusted R-squared:  0.873
## F-statistic: 72.01 on 3 and 28 DF,  p-value: 2.805e-13
```

Model 4 is the optimal model using forward selection as **drat** and **qsec** no longer have any significant after adding **disp**, **hp** and **wt**.

c. Starting from the full model, perform model selection using stepwise selection with AIC}

```
AICbasemodel <- lm(mpg ~ disp+hp+drat+wt+qsec ,data=mtcars)
q4modelAIC <- step(AICbasemodel, scope = ~., steps=4)
```

```
## Start:  AIC=65.47
## mpg ~ disp + hp + drat + wt + qsec
##
##        Df Sum of Sq    RSS    AIC
## - disp  1     3.974 174.10 64.205
## <none>              170.13 65.466
## - hp    1    11.886 182.01 65.627
## - qsec  1    12.708 182.84 65.772
## - drat  1    15.506 185.63 66.258
## - wt    1    81.394 251.52 75.978
##
## Step:  AIC=64.21
## mpg ~ hp + drat + wt + qsec
##
##        Df Sum of Sq    RSS    AIC
## - hp    1     9.418 183.52 63.891
## - qsec  1     9.578 183.68 63.919
## <none>              174.10 64.205
## - drat  1    11.956 186.06 64.331
## + disp  1     3.974 170.13 65.466
## - wt    1   113.882 287.99 78.310
##
## Step:  AIC=63.89
## mpg ~ drat + wt + qsec
##
##        Df Sum of Sq    RSS    AIC
## <none>              183.52 63.891
## - drat  1    11.942 195.46 63.908
## + hp    1     9.418 174.10 64.205
## + disp  1     1.506 182.02 65.627
```

```
## - qsec  1    85.720 269.24 74.156
## - wt    1   275.686 459.21 91.241
```

The best model for AIC was achieved after 4 steps; doing nothing allow us to have the lowest possible value for $AIC = -141.16$

d. Write down the final fitted model from stepwise selection.

```r
summary(q4modelAIC)
```

```
##
## Call:
## lm(formula = mpg ~ drat + wt + qsec, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.1152 -1.8273 -0.2696  1.0502  5.5010
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  11.3945     8.0689   1.412  0.16892
## drat          1.6561     1.2269   1.350  0.18789
## wt           -4.3978     0.6781  -6.485 5.01e-07 ***
## qsec          0.9462     0.2616   3.616  0.00116 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.56 on 28 degrees of freedom
## Multiple R-squared:  0.837,  Adjusted R-squared:  0.8196
## F-statistic: 47.93 on 3 and 28 DF,  p-value: 3.723e-11
```

The model final fitted model from stepwise using AIC as a goodness of fit is given by:

$$\begin{array}{llll} mpg = & 4.83469 & -0.25532hp & -0.56228wt \\ & (0.22440) & (0.05840) & (0.08742) \end{array}$$