$$P\Big(\sup_{t\in[0,T]} |\hat{u}_t - u_t| > \varepsilon_N^* \Big) \circleddash$$

$$\underbrace{\hat{u}_t - \hat{u}_t^B}_{\text{↳ truncated}} + \hat{u}_t^B - u_t = \underbrace{\hat{u}_t - \hat{u}_t^B - E(\hat{u}_t - \hat{u}_t^B)}_{①} + E(\hat{u}_t - \hat{u}_t^B) + \hat{u}_t^B - u_t .$$

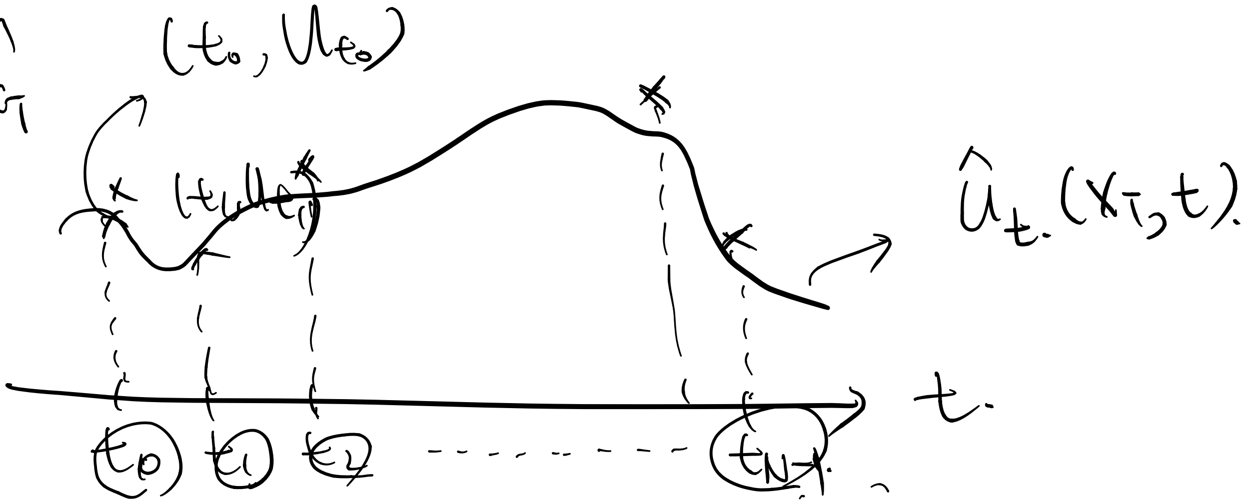
$$= ① + \underbrace{E(\hat{u}_t) - u_t}_{②} + \underbrace{\hat{u}_t^B - E(\hat{u}_t^B)}_{③} .$$

$$\circleddash\, P\Big(\sup_{t\in[0,T]} | ① + ② + ② | > \varepsilon_N^* \Big)$$

$$\le \underline{P\Big(\sup_{t\in[0,T]} | ① + ③ | > \frac{\varepsilon_N^*}{2} \Big)} + \underline{P\Big(\sup_{t\in[0,T]} | ② | > \frac{\varepsilon_N^*}{2} \Big)}$$

$$E(\hat{u}_t) - u_t = \underline{C^*(X_T)\, h_N^2}$$

for each fixed X_T



$n \in \{0, 1, \cdots, N-1\}$.

$$\hat{u}_t^B(X_i, t) = \frac{1}{Nh_n^2}\sum_{n=0}^{N-1} K^*\left(\frac{t_n - t}{h_n}\right) U_i^n \mathbb{1}\left(|U_i^n| < B_N\right).$$

$$= \frac{1}{h_n^2}\int_{t\in\mathbb{R}}\int_{|y|<B_n} K^*\left(\frac{t_n - t}{h_n}\right) y \, dF_n(t, y)$$

$F_n(t,y) \rightarrow$ Empirical measure of t & y..
(Distribution)

$$\hat{u}_t^B(X_i, t) - E\left(\hat{u}_t^B(X_i, t)\right).$$

$$= \frac{1}{\sqrt{n}\, h_n^2}\int_{t\in\mathbb{R}}\int_{|y|<B_n} K^*\left(\frac{t_n - t}{h_n}\right) y \, d\,\underbrace{\sqrt{n}\left(F_n(t,y) - F(t,y)\right)}_{Z_n(t,y).}$$

$$= \frac{1}{\sqrt{n}\, h_n^2}\int_{t\in\mathbb{R}}\int_{|y|<B_n} K^*\left(\frac{t_n - t}{h_n}\right) y \, dZ_n(t,y).$$

$$= \frac{1}{\sqrt{n}\, h_n^2}\int_{t\in\mathbb{R}} K^*\left(\frac{t_n - t}{h_n}\right) \boxed{\int_{|y|\le B_n} y \, dZ_n(t,y)}$$

By using Rosenblatt transformation.

$$T(T, y) = \left(F_T(t),\ F_{Y|T}(y|t)\right),$$

$$\int_{|y|<B_n} y \, dZ_n(t,y)$$

solution path of
2-dimensional

$\int_{|y| \le B_n}$

$$= \underbrace{\int_{|y| \le B_n} y \, d\{ Z_n(t,y) - B_0(T(t,y)) \}}_{ⓐ}.$$

$$+ \underbrace{\int_{|y| \le B_n} y \, d \, B_0(T(t,y))}_{ⓑ}.$$

Brown Br Bridge

$$\hat{U}_t^B(x_i,t) - E(\hat{U}_t^B(x_i,t))$$

$$= \underbrace{\frac{1}{\sqrt{n}\, h_n^2} \int_{t \in R} k^*\left(\frac{t_n - t}{h_n}\right) ⓐ}_{} + \underbrace{\frac{1}{\sqrt{n}\, h_n^2} \int_{t \in R} k^*\left(\frac{t_n - t}{h_n}\right) ⓑ}_{}$$

$$\frac{1}{\sqrt{n}} \underbrace{\frac{1}{h_n^2} \int_{t \in R} k^*\left(\frac{t_n - t}{h_n}\right) ⓑ}_{\ell_N(x_i,t) \text{ in } (28)} \checkmark$$

$\rightarrow$ Page 28. Bottom part

$$\frac{1}{\sqrt{n}\, h_n^2} \int_{t \in R} k^*\left(\frac{t_n - t}{h_n}\right) dt \int_{|y| \le B_n} y \, d\{ Z_n(t,y) - B_0(T(t,y)) \}.$$

$\Rightarrow$ Integration By Part. $\int f'g = fg - \int fg'$.

$$= -\frac{1}{\sqrt{n}\, h_n} \int \int_{|y| < B_n} y \, d_y [Z_n(z,y) - B^0(T(z,y))] \, dz \; k^*\left(\frac{z-t}{h_n}\right)$$