

# Lab 7a: Probability Rules II

Stat 131A, Spring 2019

## Learning Objectives:

- Probability Rules.
- Understand mutually exclusive (i.e. disjoint) events.
- Addition rule:  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

## General Instructions

- Write your solutions in an `Rmd` (R markdown) file.
  - Name this file as `lab07a-first-last.Rmd`, where `first` and `last` are your first and last names (e.g. `lab07a-gaston-sanchez.Rmd`).
  - Knit your `Rmd` file as an html document (default option).
  - Submit your `Rmd` and `html` files to bCourses, in the corresponding lab assignment.
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## Problem 1

You roll two fair dice, a green one and a red one.

- What is the probability of getting a sum of 6?
- What is the probability of getting a sum of 4?
- What is the probability of getting a sum of 6 *or* 4?
- Are the outcomes in (c) mutually exclusive?

## Problem 2

A (fair) coin is tossed 100 times, landing heads 53 times. However, the last seven tosses are all heads. True or false: the chance that the next toss will be heads is somewhat less than 50%. Explain.

## Problem 3

Two cards will be dealt off the top of a well-shuffled deck. You have a choice:

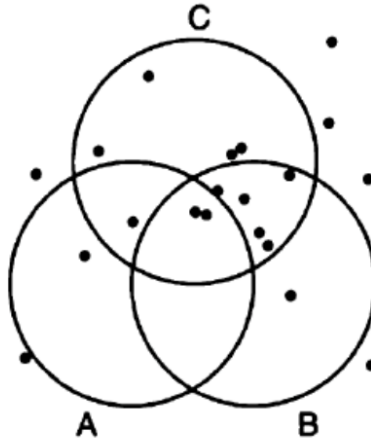
- To win \$1 if the first is a king.

- ii. To win \$1 if the first is a king and the second is a queen.

Which option is better? Or are they equivalent? Explain.

#### Problem 4

There are 20 dots in the diagram below, and 3 circles. The circles are labeled A, B, and C. One of the dots will be chosen at random.



- What is the probability that the dot falls inside circle A?
- What is the probability that the dot falls inside circle B?
- What is the probability that the dot falls inside circle C?
- What is the probability that the dot falls (simultaneously) inside both circles A and B?
- What is the probability that the dot falls (simultaneously) inside both circles A and C?
- What is the probability that the dot falls inside at least one of the circles?

#### Problem 5

True or False, and explain.

- If a die is rolled three times, the chance of getting at least one ace is  $1/6 + 1/6 + 1/6 = 1/2$ .
- If a coin is tossed twice, the chance of getting at least one head is 100%.

#### Problem 6

Assume A and B are events such that  $0 < P(A) < 1$  and  $0 < P(B) < 1$ . Answer the following questions true or false.

- a.  $P(A \text{ and Not } A) = 0$
- b.  $P(A \text{ or Not } A) = 1$
- c.  $P(A | \text{Not } A) = 0$
- d.  $P(A \text{ or } B) = P(A) + P(B)$
- e.  $P(A | B) \geq P(A \text{ and } B)$
- f.  $P(A \text{ or } B) \geq P(A)$  if A and B are independent events.
- g.  $P(A \text{ and } B) \leq P(A)$ .
- h.  $P(A|B) > P(A)$  if A and B are independent events.
- i.  $P(\text{Not } A \text{ and Not } B) \leq 1 - P(A)$ .
- j.  $P(\text{Not } A \text{ or Not } B) \leq 2 - P(A) - P(B)$

### Problem 7

The unconditional probability of event A is  $1/2$ . The unconditional probability of event B is  $1/3$ . Say whether each of the following is true or false, and explain briefly.

- a. The chance that A and B both happen must be  $1/2 \times 1/3 = 1/6$ .
- b. If A and B are independent, the chance that they both happen must be  $1/2 \times 1/3 = 1/6$ .
- c. If A and B are mutually exclusive, the chances that they both happen must be  $1/2 \times 1/3 = 1/6$ .
- d. The chance that at least one of A or B happens must be  $1/2 + 1/3 = 5/6$ .
- e. If A and B are independent, the chance that at least one of them happens must be  $1/2 + 1/3 = 5/6$ .
- f. If A and B are mutually exclusive, the chance that at least one of them happens must be  $1/2 + 1/3 = 5/6$ .

### Problem 8

The committee on Student Life did a survey of 417 students regarding satisfaction with student government and class standing. The results follow:

	Freshman	Sophomore	Junior	Senior	Row Totals
Not Satisfied	17	19	23	12	71
Neutral	61	35	32	38	166
Satisfied	23	49	43	65	180
Column Totals	101	103	98	115	417

Find the probability that a student selected at random is:

- a. Satisfied.
- b. Junior.
- c. Satisfied, given that the student is a senior.
- d. Neutral and freshman.
- e. Senior, given satisfied.
- f. Neutral or satisfied.
- g. At least a sophomore.

### Problem 9

The following table is based on information about Customer Loyalty. The rows represent regions of the United States. The columns represent length of customer loyalty at a primary supermarket.

	< 1 yr	1-2 yrs	3-4 yrs	5-9 yrs	10-14 yrs	15+ yrs	Row Totals
East	32	54	59	112	77	118	452
Midwest	31	68	68	120	63	173	523
South	53	92	93	158	106	158	660
West	41	56	67	78	45	86	373
Column Totals	157	270	287	468	291	535	2008

What is the probability that a customer chosen at random:

- a. has been loyal 10 to 14 years?
- b. has been loyal 10 to 14 years, given that he or she is from the east?
- c. has been loyal at least 10 years?
- d. has been loyal at least 10 years, given that he or she is from the west?
- e. is from either the west or south?
- f. is from the west and south?
- g. is from the west, given that he or she has been loyal less than 1 year?
- h. is from the south, given that he or she has been loyal less than 1 year?
- i. has been loyal 1 or more years, given that he or she is from the east?
- j. has been loyal 1 or more years, given that he or she is from the west?
- k. Are the events “from the east” and “loyal 15 or more years” independent? Explain.

### Problem 10

Is enrollment in the Health Science program independent of whether a student is female? Or is there a relationship between these two events?

	Arts-Sci	Bus-Econ	InfoTech	Health	Graphics	Culinary	Row Totals
Female	4,660	435	494	421	105	83	6,198
Male	4,334	490	564	223	97	94	5,802
Column Totals	8,994	925	1,058	644	202	177	12,000