Random Variables

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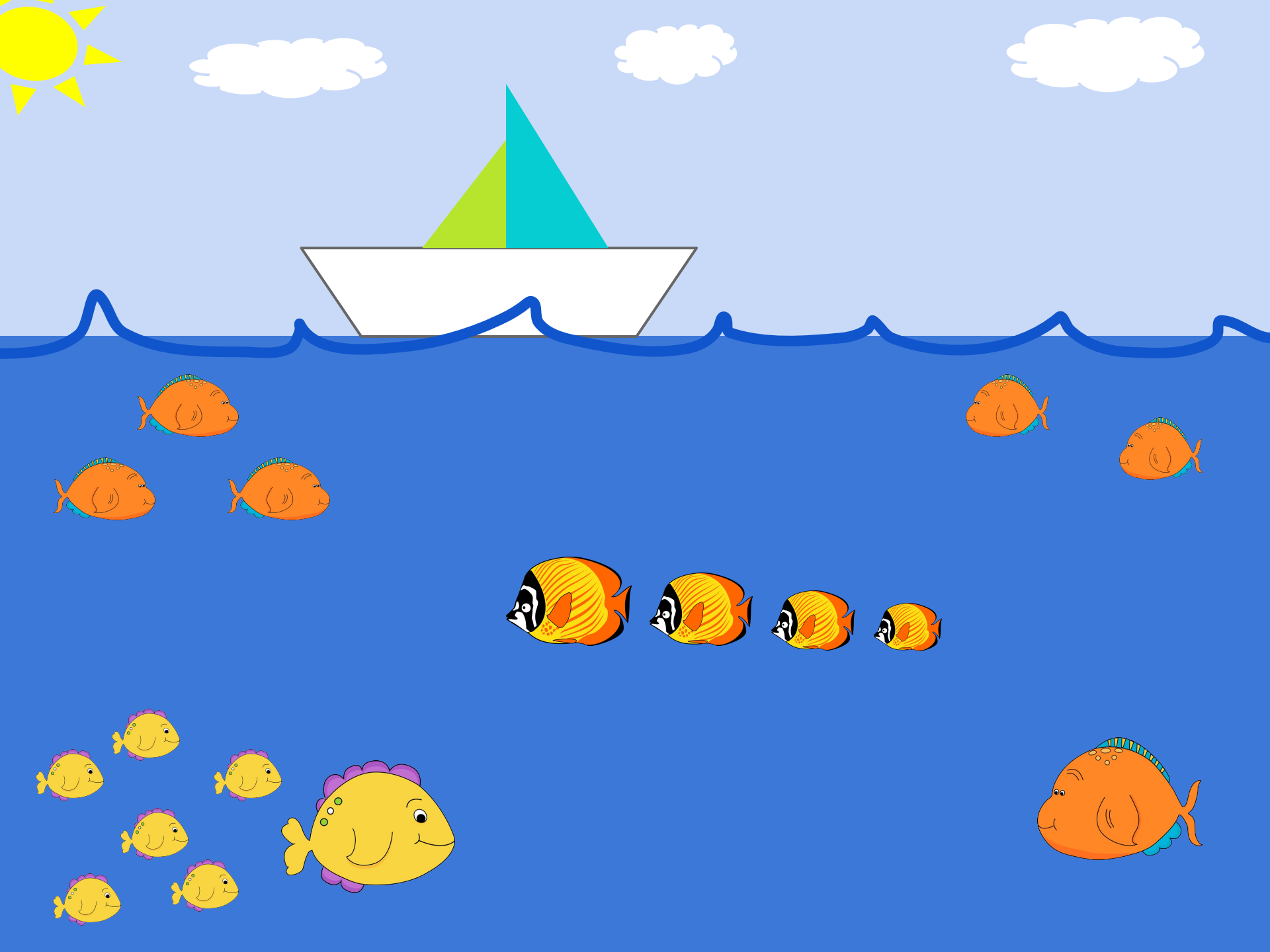
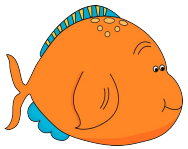
Reminder

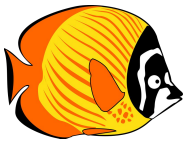
We've seen the concept of data variable: a characteristic measured on individuals

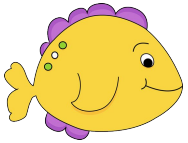


Data: individuals and variables

Individuals







Variables

weight
length
color
type
etc

What is a Random Variable?

Random Variable

A variable is said to be a **random variable** when it comes from a random experiment

Simply put

Think of "**random variable**" as a formalizing term that allows us to wrap a variable around probability ideas

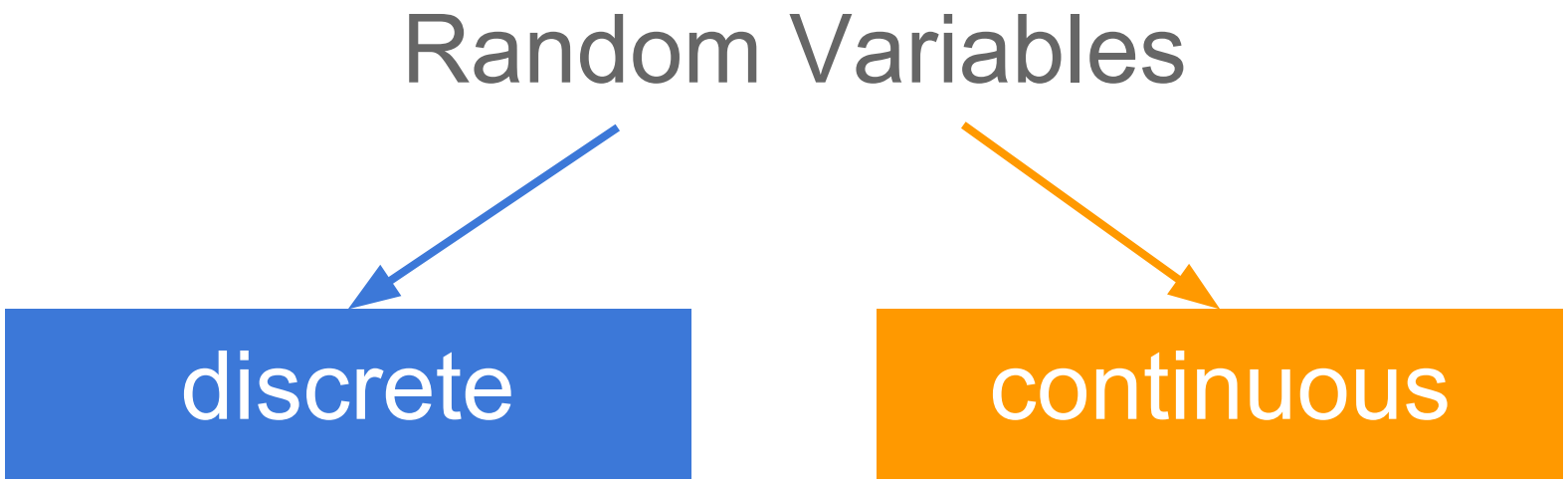
Notation

$$X, Y, Z \ldots$$

random variables

Types of Random Variables

Types of random variables



We'll consider only quantitative variables

Discrete random variable

Can take on only a finite number of values or a countable number of values

Discrete RVs will be

the **result of a count**

Discrete random variable example

tossing 2 coins

Counting results

X: number of heads when tossing two coins

Continuous random variable

Can take on any of the countless number of values in a line interval

Continuous RVs will be the **result of measurement** on a continuous scale

Continuous random variable example

Measurement

X: the height of an individual

Distributions of a Random Variable

A random variable has a probability distribution

Random Variables & Probabilities

Since the value of a random variable is determined by the outcome of the experiment, we may assign probabilities to its possible values

Toss a die

Experiment: toss a die and observe the value that appears on the top face

Random Variable: number on the top face

Toss a die (cont'd)

Possible realizations x = 1, 2, 3, 4, 5, 6

value	*probability*	*value*	*probability*
X = 1		X = 4	
X = 2		X = 5	
X = 3		X = 6	

Toss a die (cont'd)

Possible realizations $x = 1, 2, 3, 4, 5, 6$

value	*probability*	*value*	*probability*
$X = 1$	$P(X = 1) = 1/6$	$X = 4$	$P(X = 4) = 1/6$
$X = 2$	$P(X = 2) = 1/6$	$X = 5$	$P(X = 5) = 1/6$
$X = 3$	$P(X = 3) = 1/6$	$X = 6$	$P(X = 6) = 1/6$

Discrete Random Variables

Example: 3 children

Suppose we are about to learn the sexes of the 3 children of a certain family

oldest *middle* *youngest*

X: number of female children in the family

Example: 3 children

Suppose we are about to learn the sexes of the 3 children of a certain family

Sample Space (i.e. all possible outcomes):

Example: 3 children

Suppose we are about to learn the sexes of the 3 children of a certain family

Sample Space (i.e. all possible outcomes):

(b, b, b)	(g, g, g)
(b, b, g)	(g, g, b)
(b, g, b)	(g, b, g)
(b, g, g)	(g, b, b)

Example: 3 children

X: number of female children in the family

Possible values of X:

0

1

2

3

Example: 3 children

X: number of female children in the family

Possible values of X:

- 0 ➡ (b, b, b)
- 1 ➡ (b, b, g), (b, g, b), (g, b, b)
- 2 ➡ (b, g, g), (g, b, g), (g, g, b)
- 3 ➡ (g, g, g)

Example: 3 children

X: number of female children in the family

Probability Distribution of X:

$P(X = 0) =$

$P(X = 1) =$

$P(X = 2) =$

$P(X = 3) =$

Example: 3 children

X: number of female children in the family

Probability Distribution of X:

$P(X = 0) = 1/8$

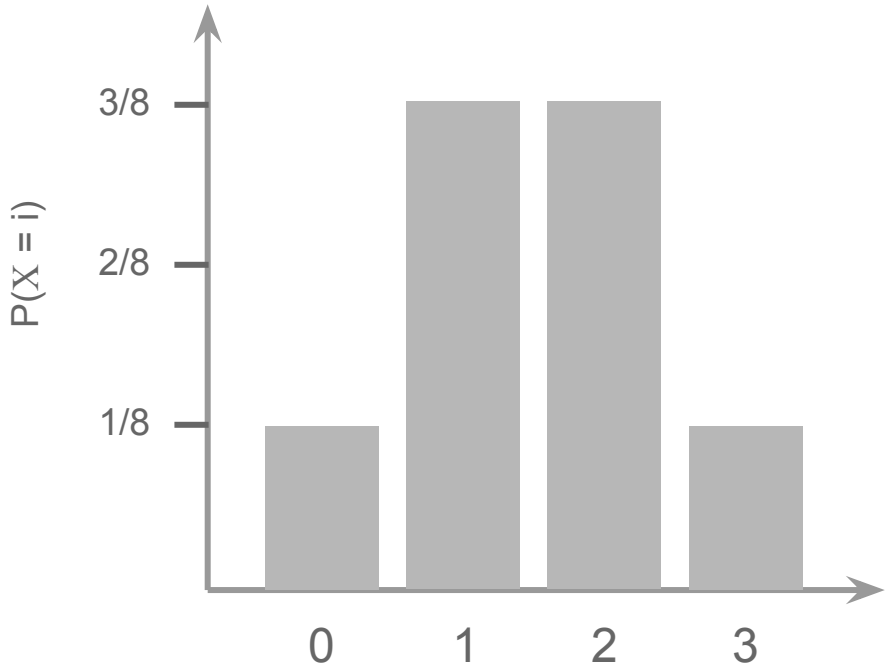
$P(X = 1) = 3/8$

$P(X = 2) = 3/8$

$P(X = 3) = 1/8$

Example: 3 children

X: number of female children in the family



A Box Model

True / False questions

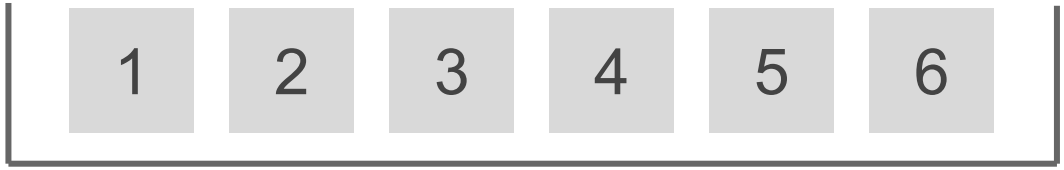
Box Models provide an analogy for many chance processes which help to analyze chance variability

A chance problem is like drawing (with replacement) from a box with numbered tickets and looking at the sum of the draws

Example

Roll a die 5 times, and add up the points

What to put in a box that will mimic the behavior of rolling a die?



Draw one ticket 5 times

Example

Toss a coin 5 times, and count # heads

```
T H T T H
0 1 0 0 1
```

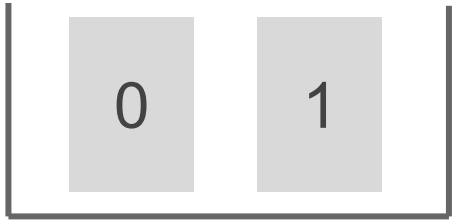
sum(heads) = 2

```
H T H H T
1 0 1 1 0
```

sum(heads) = 3

```
T T T H T
0 0 0 1 0
```

sum(heads) = 1



Draw one ticket 5 times

Making a Box Model

What numbers go in the box?

What is the quantity of interest?

What could happen to that quantity on each draw?

How many tickets of each number?

What are the chances for each draw?

How many draws?