One sample t-test

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Tests of Significance

(FPP chapter 26)

Can the result be explained by chance or is another explanation necessary?

2 competing ideas

2 competing ideas

Just chance

Null hypothesis

Other explanation

Alternative hypothesis

To make a test of significance you have to:

- Set up the null hypothesis (in terms of a box model for the data)
- Set up the alternative hypothesis
- Pick a test statistic, to measure the difference between the data and what is expected on the null hypothesis
- Compute the observed significance level P
- Make a conclusion

To make a test of significance you have to:

How small the observed significance level has to be before rejecting the null hypothesis?

If P is less than 5%, the result is called statistically significant

If P is less than 1%, the result is called highly significant

One sample z-test

One sample z-test

Test statistic:

$$Z = \frac{\text{Observed} - \text{EV}}{\text{SE}}$$

How many SEs away an observed value is from its expected value (computed under the null hypothesis)

Another Example

From past year the national values:

Avg of MSAT = 519, SD of MSAT = 110

expected

In this year, a SRS of 100 students who took MSAT in CA:

Sample Avg = 504, SD = 100

observed

Did all CA students have a lower score avg MSAT score or can results be explained by chance?

MSAT scores example

Null:

a) Results can be exaplined by chance
b) Avg of all CA students = **519**

Alternative:

a) Not just chance
b) Avg of all CA students is **lower than 519**

MSAT scores example

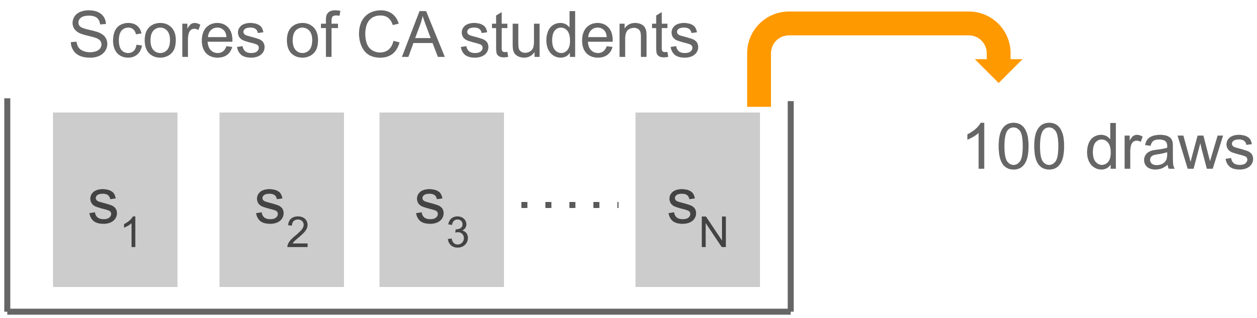
$$z = \frac{\text{Obs} - \text{EV}}{\text{SE}} = \frac{504 - 519}{\text{SE}}$$

SE avg = (SE sum) / # draws

SE sum = $\sqrt{100}$ (SD box)

SD box = ?

What is the box?



SD of box is unknown

But we can use SD of sample (Bootstrap method)

What SD should you use?

If you know SD of box or it is implied by the null hypothesis, then use it.

If not, use sample SD fo sample to estimate SD of box (bootstrap method)

If you are confused, think about what the box is

MSAT scores example

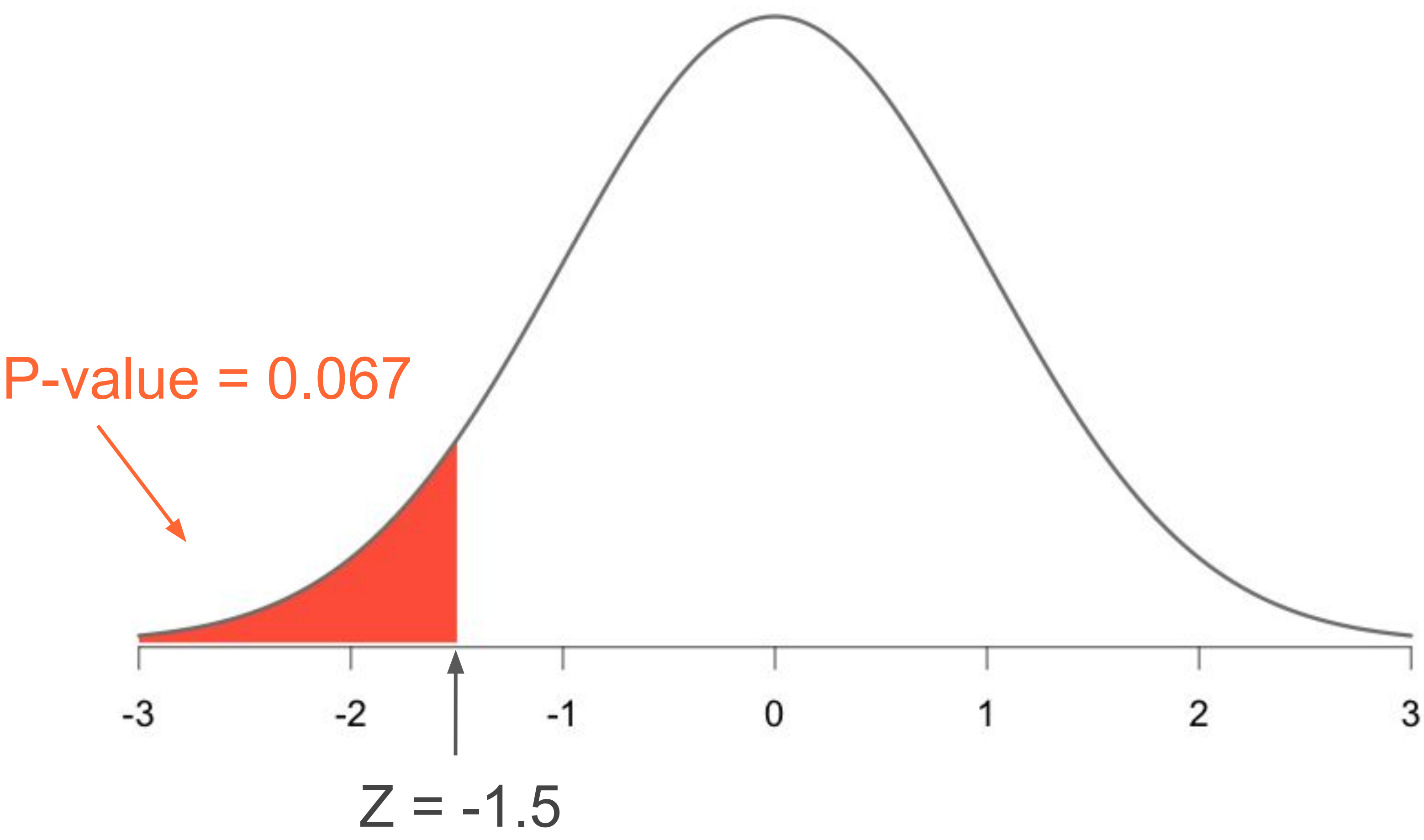
SD box = 100 (SD of sample)

SE sum = $\sqrt{100}$ (100) = 1000

SE avg = 1000 / 100 = 10

$$z = \frac{504 - 519}{10} = -1.5$$

MSAT scores example



MSAT scores example

Conclusion:

- At a 5% significance level, the null hypothesis can’t be rejected.
- The difference (504 -vs- 519) could be due to chance.
- CA students have same avg MSAT as nation

One sample t-test

About t-test

Less than 25 draws (small samples)

SD of box unknown (bootstrap)

Data is not too different from the normal curve

Requires using Student's ***t*** distribution, i.e. the t-table (Not the Normal table)

In R, use function: `pt(x, df)`

Differences between z-test and t-test

The t-statistic uses sample std deviation: SD^+

$$SD^+ = \sqrt{\frac{\text{\# draws}}{\boxed{\text{\# draws - 1}}}} \times \text{SD sample}$$

`sd()` *in R*

(dividing by n)

Degrees of freedom

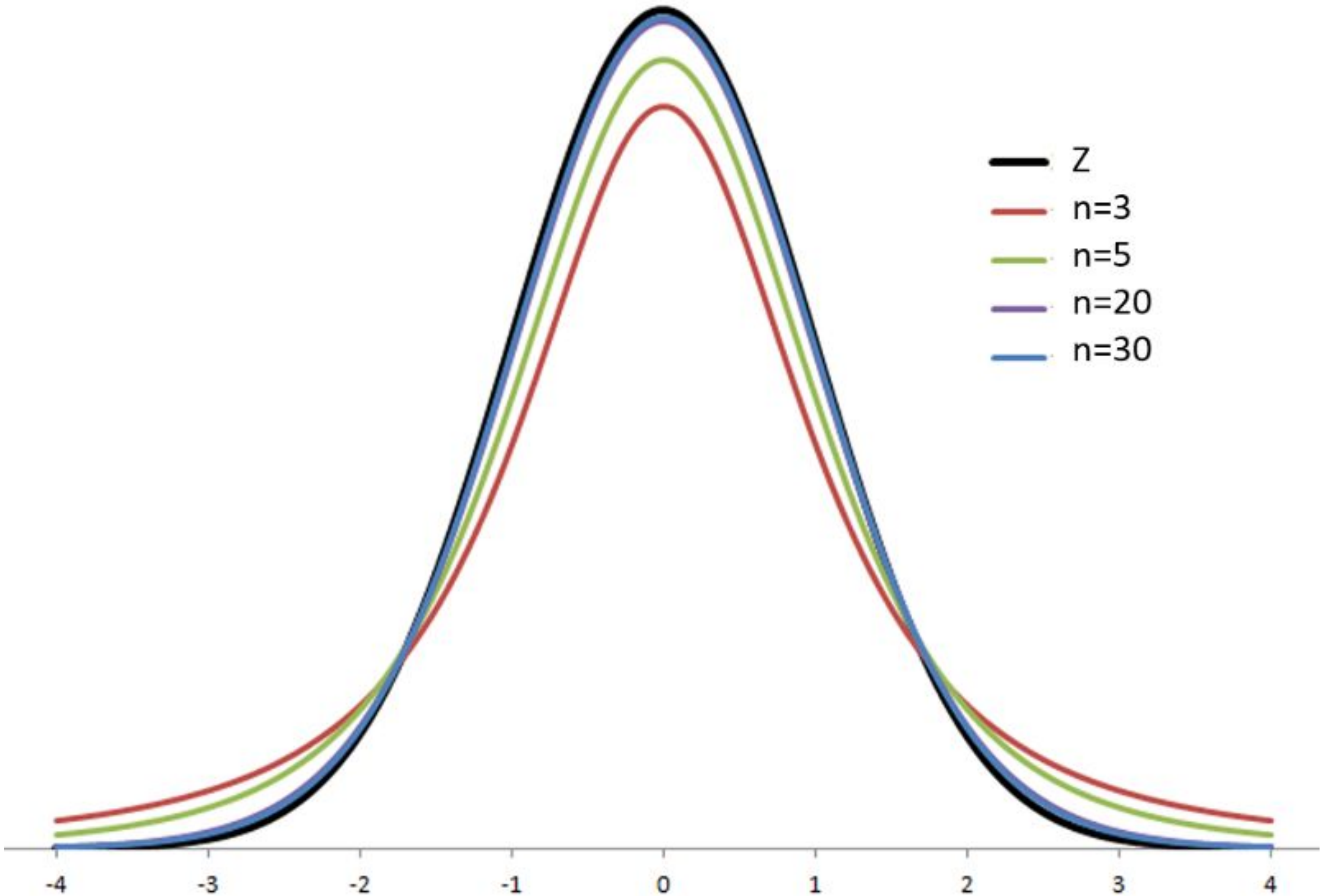
Why we need SD^+? Because when samples are very small you tend to underestimate the SD

t-statistic

$$t = \frac{\text{Observed - Expected}}{\text{SE (from SD}^{+}\text{)}}$$

t-statistic follows a Student's t-distribution with degrees of freedom = # draws -1

Normal and Student's *t* distributions



Example

Water purification

A new experimental process is employed to purify drinking water. This procedure must not change the acidity of the treated water (i.e. maintain neutral pH of 7.0)

A random sample of 24 pH-values results in an average pH = 6.806, and a SD = 0.3586

Does the new process is acceptable?

Significance level = 0.05

Sample of 24 pH values

5.95	7.39	6.88	6.54	6.50	6.73
6.69	6.95	7.58	6.62	6.96	6.90
6.93	6.32	7.22	6.36	6.54	6.67
7.25	6.94	7.21	6.83	6.80	6.59

Avg = 6.806

SD = 0.3586

(dividing by n)

EV = 7.0

Water purification

Null:

a) Only random variation causes the 24 observed values to differ from pH = 7.0
b) Avg of all pH values = **7.0**

Alternative:

a) Not just random variation
b) Avg of all pH values is **lower than 7.0**

$$t = \frac{\text{Obs} - \text{EV}}{\text{SE}} = \frac{6.806 - 7.0}{\text{SE}}$$

SD box = ?

$$\underset{\text{Sample SD}}{SD^{+}} = \sqrt{\frac{\#\text{ draws}}{\boxed{\#\text{ draws} - 1}}} \times \underset{\text{(dividing by n)}}{\text{SD sample}}$$

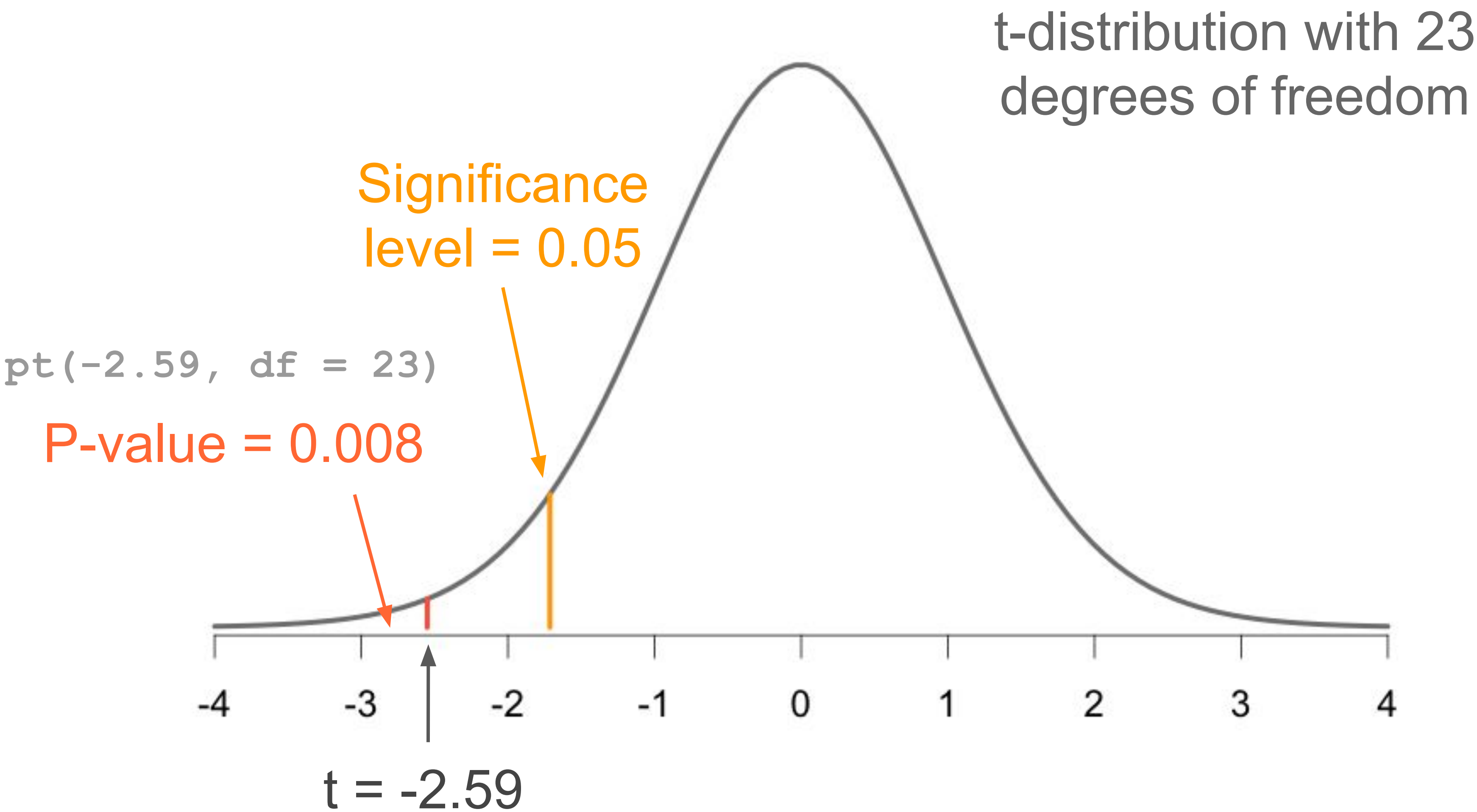
Degrees of freedom

$$SD^{+} = \sqrt{24 / 23} \times 0.3586 = 0.366$$

$$SE\ avg = 0.366 / \sqrt{24} = 0.0747$$

$$t = \frac{Obs - EV}{SE} = \frac{6.806 - 7.0}{0.0747} = -2.59$$

Water purification



Conclusion

- We reject the null hypothesis: the purifying process changes the pH of water < 7.0
- Observed differences are not just chance
- Something else is going on

One more example

Avg MSAT in a given county = 420

SRS of 5 students from one school with a sample average = 550, SD sample = 110

MSAT scores roughly follow normal curve

Does this school have higher avg MSAT than the county, or could this just be chance? Use a significance level of 0.05.

$\boxed{\text{Avg MSAT in a given county = 420}}$ expected

SRS of 5 students from one school with a sample
$\boxed{\text{average = 550, SD sample = 110}}$ observed
MSAT scores roughly follow normal curve

Does this school have higher avg MSAT than the county, or could this just be chance?

MSAT scores example

Null:

a) Results can be exaplined by chance
b) Avg MSAT of all school students = **420**

Alternative:

a) Not just chance
b) Avg MSAT of all school students is **greater than 420**

MSAT scores example

$$t = \frac{\text{Obs} - \text{EV}}{\text{SE}} = \frac{550 - 420}{\text{SE}}$$

$$\text{SE avg} = \text{SD}^{+} / \sqrt{\#\ \text{draws}}$$

$$\text{SD}^{+} = \ ?$$

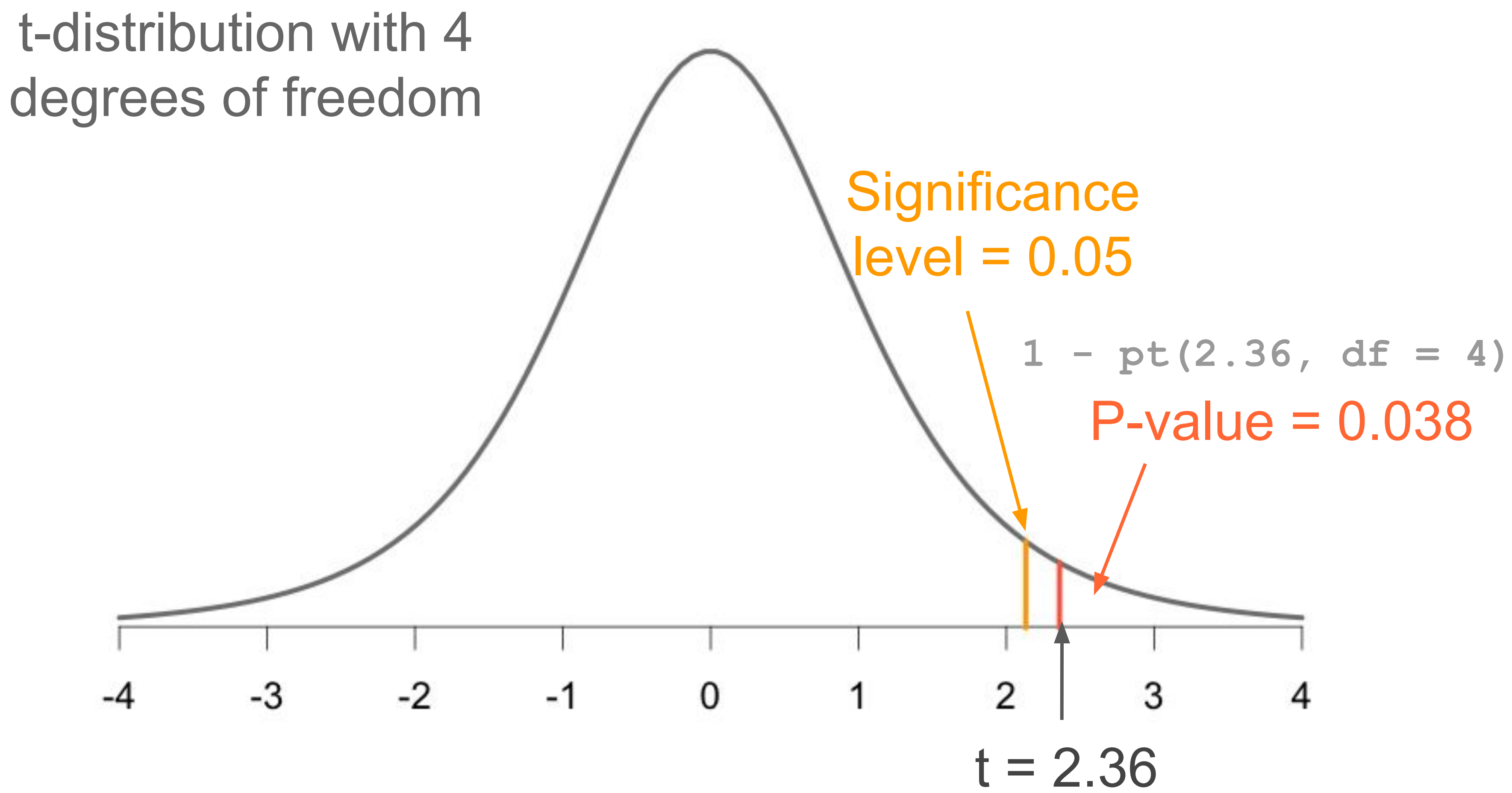
$$SD^{+} = \sqrt{5 / 4} \times 110 = 123$$

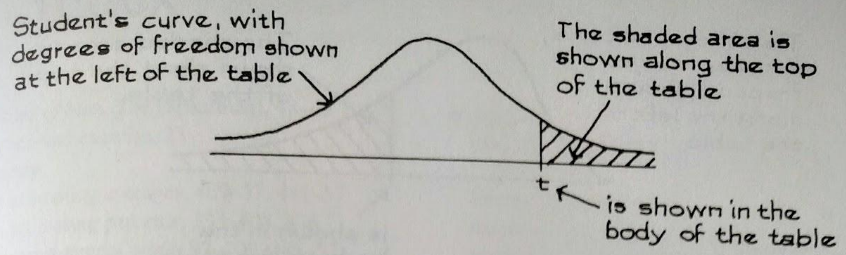
$$SE\ avg = SD^{+} / \sqrt{\#\ draws}$$

$$SE\ avg = 123 / \sqrt{5} = 55$$

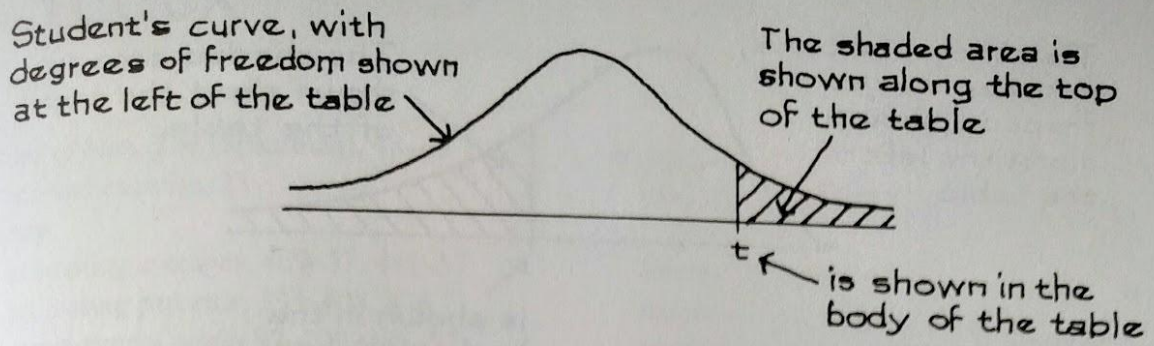
$$t = \frac{Obs - EV}{SE} = \frac{550 - 420}{55} = 2.36$$

t-distribution





Degrees of freedom	*25%*	*10%*	*5%*	*2.5%*	*1%*	*0.5%*
1	1.00	3.08	6.31	12.71	31.82	63.66
2	0.82	1.89	2.92	4.30	6.96	9.92
3	0.76	1.64	2.35	3.18	4.54	5.84
4	0.74	1.53	2.13	2.78	3.75	4.60
5	0.73	1.48	2.02	2.57	3.36	4.03
6	0.72	1.44	1.94	2.45	3.14	3.71
7	0.71	1.41	1.89	2.36	3.00	3.50
8	0.71	1.40	1.86	2.31	2.90	3.36
9	0.70	1.38	1.83	2.26	2.82	3.25
10	0.70	1.37	1.81	2.23	2.76	3.17
11	0.70	1.36	1.80	2.20	2.72	3.11
12	0.70	1.36	1.78	2.18	2.68	3.05
13	0.69	1.35	1.77	2.16	2.65	3.01
14	0.69	1.35	1.76	2.14	2.62	2.98
15	0.69	1.34	1.75	2.13	2.60	2.95
16	0.69	1.34	1.75	2.12	2.58	2.92
17	0.69	1.33	1.74	2.11	2.57	2.90
18	0.69	1.33	1.73	2.10	2.55	2.88
19	0.69	1.33	1.73	2.09	2.54	2.86
20	0.69	1.33	1.72	2.09	2.53	2.85
21	0.69	1.32	1.72	2.08	2.52	2.83
22	0.69	1.32	1.72	2.07	2.51	2.82
23	0.69	1.32	1.71	2.07	2.50	2.80
24	0.68	1.32	1.71	2.06	2.49	2.80
25	0.68	1.32	1.71	2.06	2.49	2.79



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9	0.70	1.38	1.83	2.26	2.82	3.25
10	0.70	1.37	1.81	2.23	2.76	3.17

$$t = 2.36 > 2.13$$

MSAT scores example

Conclusion:

- At a 5% significance level, we reject the null hypothesis.
- The difference (550 -vs- 420) doesn’t seem to be explained by chance
- Looks like students in school have avg MSAT greater than the county