Law of Averages

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Suppose I toss a (fair) coin 100 times, and I get 99 heads

What do you think should happen in the next toss?

Law of Averages Idea

“What should happen in the long run”

(chapter 16)

Law of Averages for coin tosses

Chapter 15
































Law of Averages for coin tosses

# of heads we should observe ≈ ½ # of tosses + **chance error**

Law of Averages for coin tosses

With a large # of tosses the chance error will tend to be **large in absolute terms** (although small compared to the # of tosses)

Demo:
Coin Tosses
Chance Error

Law of Averages for coin tosses

% of heads should be ≈ 50% + **percent error**

(chance error in percentage terms)

Law of Averages for coin tosses

With a large # of tosses the percent error will **tend to be small** (although unlikely to be zero)

Demo:
Coin Tosses
Percent Error

Kerrich's Coin Tosses (p 274)

# tosses	# heads	# expected heads	Chance error	% heads	% expected heads	Percent error
10	4					
50	25					
100	44					
500	255					
1000	502					
5000	2533					
10000	5067					

Kerrich's Coin Tosses (p 274)

# tosses	# heads	# expected heads	Chance error	% heads	% expected heads	Percent error
10	4	5				
50	25	25				
100	44	50				
500	255	250				
1000	502	500				
5000	2533	2500				
10000	5067	5000				

Kerrich's Coin Tosses (p 274)

# tosses	# heads	# expected heads	Chance error	% heads	% expected heads	Percent error
10	4	5	-1			
50	25	25	0			
100	44	50	-6			
500	255	250	5			
1000	502	500	2			
5000	2533	2500	33			
10000	5067	5000	67			

Chance error increases
in absolute terms

Kerrich's Coin Tosses (p 274)

# tosses	# heads	# expected heads	Chance error	% heads	% expected heads	Percent error
10	4	5	-1	40%	50%	
50	25	25	0	25%	50%	
100	44	50	-6	44%	50%	
500	255	250	5	51%	50%	
1000	502	500	2	50.2%	50%	
5000	2533	2500	33	50.66%	50%	
10000	5067	5000	67	50.67%	50%	

Kerrich’s Coin Tosses (p 274)

# tosses	# heads	# expected heads	Chance error	% heads	% expected heads	Percent error
10	4	5	-1	40%	50%	-10%
50	25	25	0	25%	50%	0%
100	44	50	-6	44%	50%	-6%
500	255	250	5	51%	50%	1%
1000	502	500	2	50.2%	50%	0.2%
5000	2533	2500	33	50.66%	50%	0.66%
10000	5067	5000	67	50.67%	50%	0.67%

Percent error should be close to 0%

True / False questions

Is the event more likely with 10 tosses or 1,000,000 tosses?

1) % of heads > 55%

10 tosses

More than 55% heads is really hard to do in a million tosses

True / False questions

Is the event more likely with 10 tosses or 1,000,000 tosses?

2) % of heads = 50%

10 tosses

This is easier to get with 10 tosses

Exactly 50% heads is very unlikely in a million tosses

True / False questions

Is the event more likely with 10 tosses or 1,000,000 tosses?

3) % of heads is more than 45%

1,000,000 tosses

In a million tosses you should expect to be very close to 50% heads

True / False questions

Is the event more likely with 10 tosses or 1,000,000 tosses?

4) chance error is 2 or more

1,000,000 tosses

7 or more heads in 10 tosses, versus 500,002 or more heads in 1,000,000

A Box Model

True / False questions

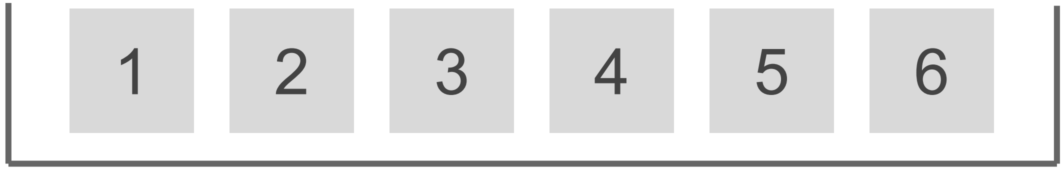
Box Models provide an analogy for many chance processes which help to analyze chance variability

A chance problem is like drawing (with replacement) from a box with numbered tickets and looking at the sum of the draws

Example

Roll a die 5 times, and add up the points

What to put in a box that will mimic the behavior of rolling a die?



Draw one ticket 5 times

Example

Toss a coin 5 times, and count # heads

T H T T H
0 1 0 0 1

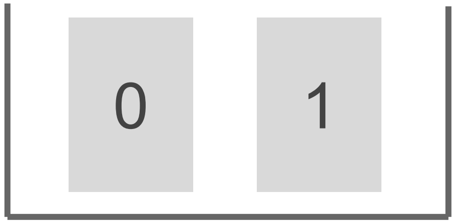
sum(heads) = 2

H T H H T
1 0 1 1 0

sum(heads) = 3

T T T H T
0 0 0 1 0

sum(heads) = 1



Draw one ticket 5 times

Making a Box Model

What numbers go in the box?

What is the quantity of interest?

What could happen to that quantity on each draw?

How many tickets of each number?

What are the chances for each draw?

How many draws?