

# Measures of Center (part 1)

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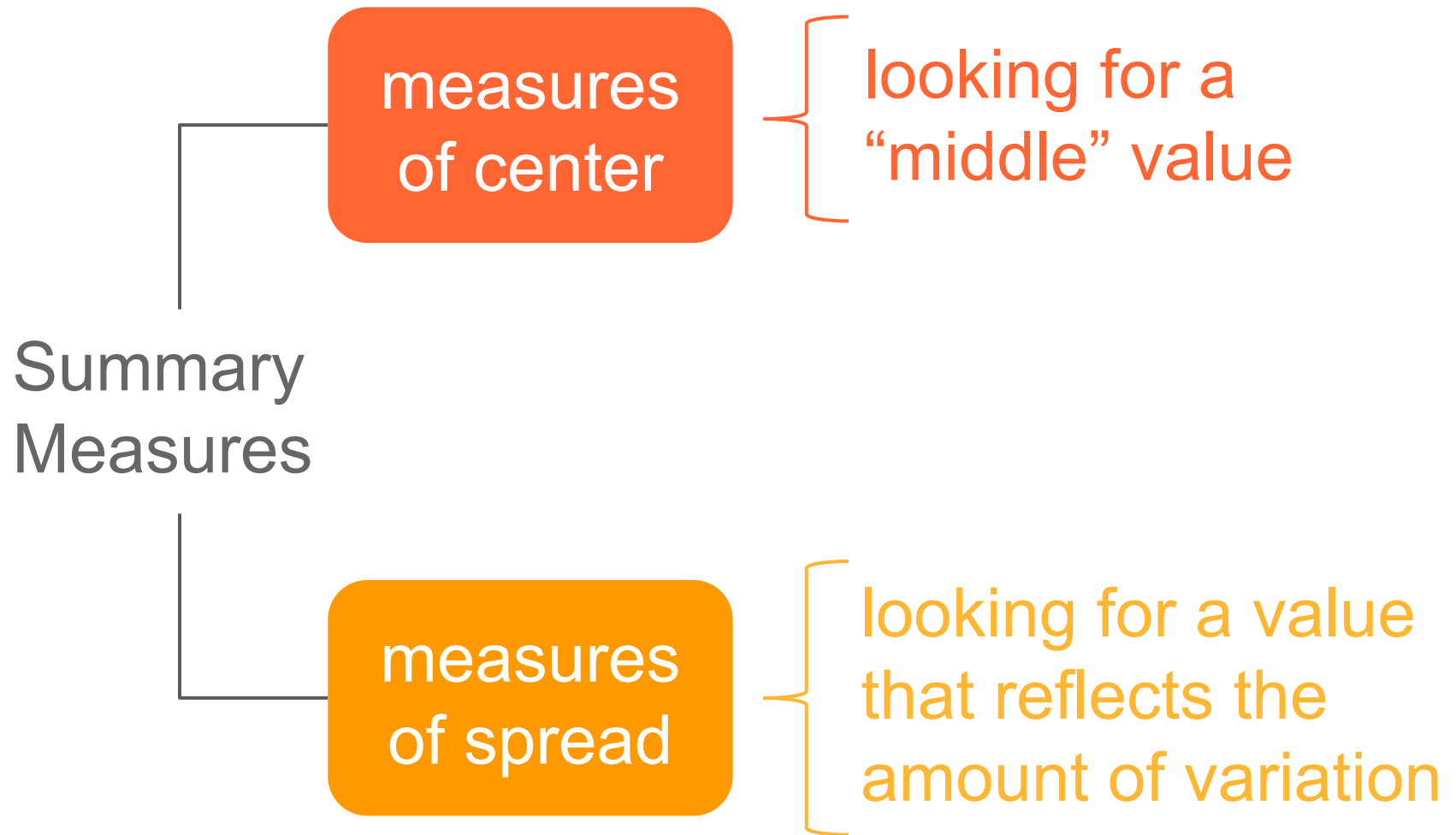
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# Descriptive Statistics

1 Frequency Tables

2 Charts & Graphics

3 Numeric Summaries



# Measures of Center

Looking for a  
“middle value”

## Measures of center

### Middle or Central Value

Is there a “**representative**” value around which all values concentrate?

# Meaning of Middle Value

Mean: Average

Median: Middle point

Mode: Most common

*We'll focus on  
these 2*



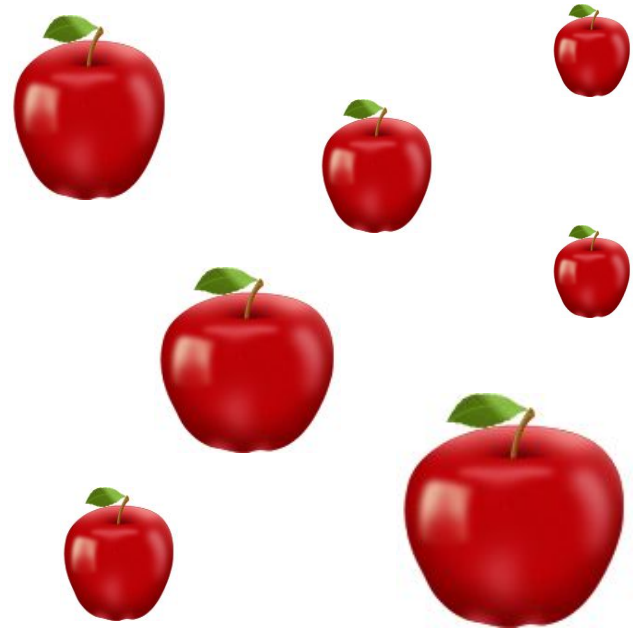
# Average or Mean

# Average

## Average or mean value

When we talk about the Average we refer to the  
**arithmetic mean**

# Toy Example



<i>num</i>	Weight oz	Carbs	Acidity	Shape
1	5	20.0	medium	round
2	6	24.3	high	oval
3	7	25.0	medium	round
4	7	25.5	low	square
5	6	24.7	medium	round
6	8	26.1	low	round
7	6	25.2	high	square
8	9	23.7	high	oval
9	10	21.0	low	round
10	8	27.4	medium	oval

## Finding the Average

weights: 5, 6, 6, 6, 7, 7, 8, 8, 9, 10

$$\text{Avg} = \frac{5 + 6 + 6 + 6 + 7 + 7 + 8 + 8 + 9 + 10}{10}$$

$$\text{Avg} = 7.2$$

# Average Formula

$$\text{Avg} = \frac{\text{sum of all values}}{n}$$
$$\text{Avg} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

*sum of all values*

*number of values*

## Average Formula: “X-bar”

$$\bar{X} = \frac{\text{sum of entries}}{n}$$

$x_1 + x_2 + x_3 + \dots + x_n$

*number of entries*

## Average Formula (summation notation)

$$\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$$

*number of entries*



## Average (summation notation)

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n x_i$$

equivalently

$$\bar{X} = \sum_{i=1}^n \frac{x_i}{n}$$

Very Important  
Property (VIP)

# Average Formula

$$\text{Avg} = \frac{\text{sum of entries}}{n}$$

$x_1 + x_2 + x_3 + \dots + x_n$

$n$

*number of entries*

## Very Important Property (VIP)

$$\sum_{i=1}^n (x_i - ?) = 0$$

## Very Important Property (VIP)

$$\sum_{i=1}^n (x_i - \bar{x}) = 0$$

The average causes the sum of all deviations to be exactly zero

## Very Important Property (VIP)

$$\sum_{i=1}^n (x_i - \bar{x}) = 0$$

So what?

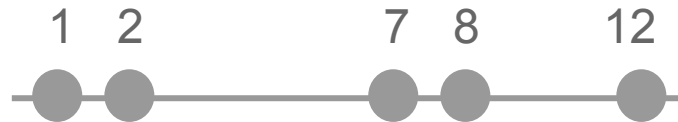
# About the average

some data values



# About the average

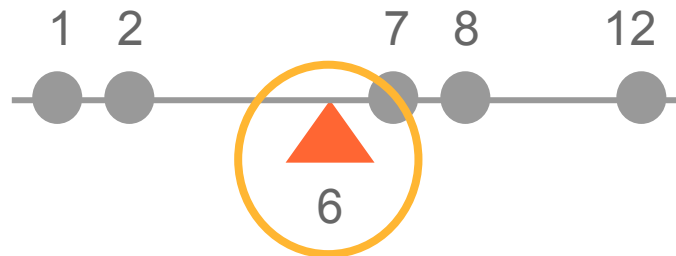
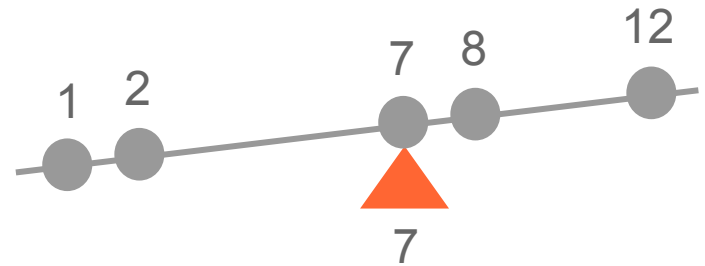
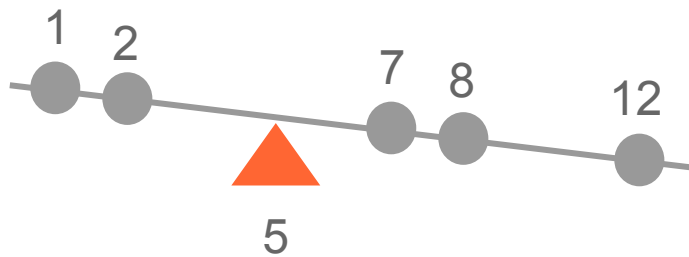
some data values



where is the  
balancing point?



# About the average



the average!!!

# Median

# Median

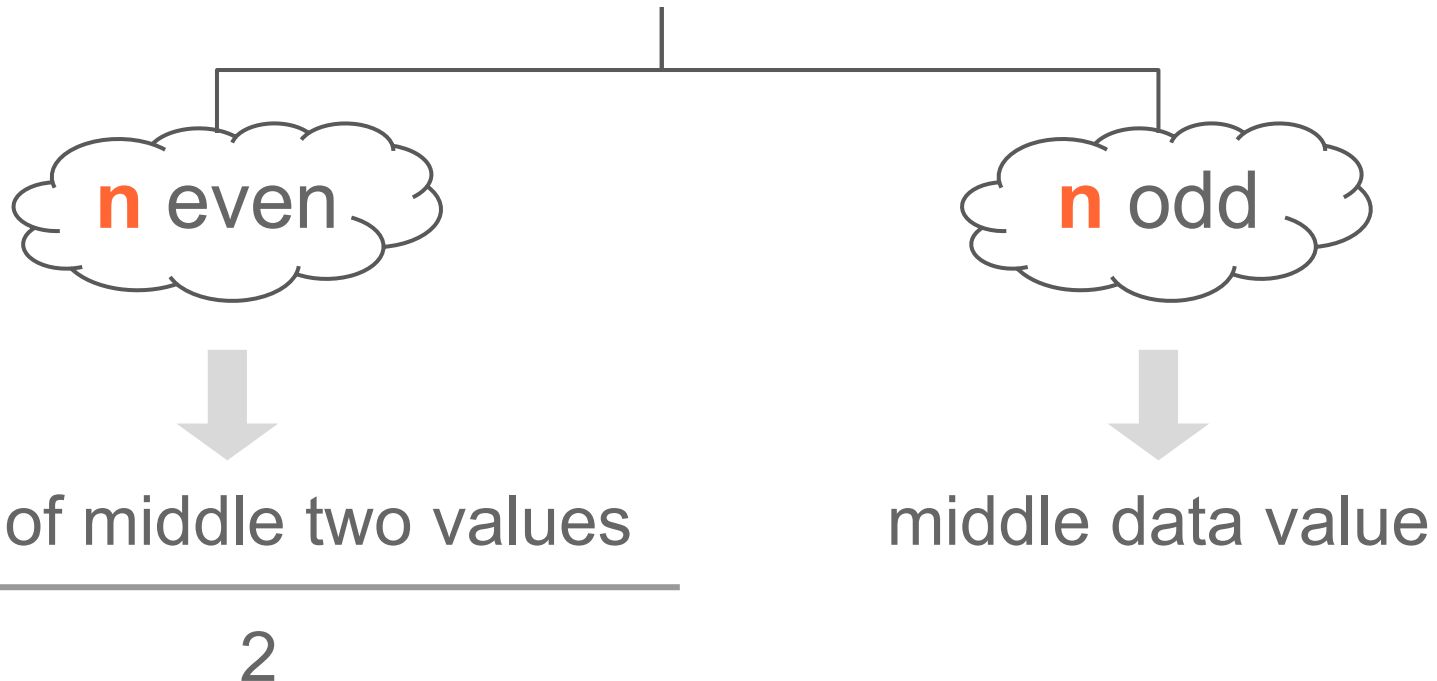
The median is the **midpoint** or central value of an **ordered** distribution

At least half of the values must be less than or equal to the median, the other half must be greater than or equal to the median

## How to find the Median?

Order values from smallest to largest

Determine number of values: **n**



<i>num</i>	Weight oz	Carbs	Acidity	Shape
1	5	20.0	medium	round
2	6	24.3	high	oval
3	7	25.0	medium	round
4	7	25.5	low	square
5	6	24.7	medium	round
6	8	26.1	low	round
7	6	25.2	high	square
8	9	23.7	high	oval
9	10	21.0	low	round
10	8	27.4	medium	oval

## Finding the Median

Apple weight values

5, 6, 7, 7, 6, 8, 6, 9, 10, 8

Order values from smallest to largest

5, 6, 6, 6, 7, 7, 8, 8, 9, 10

How many values?

**n = 10** (even)

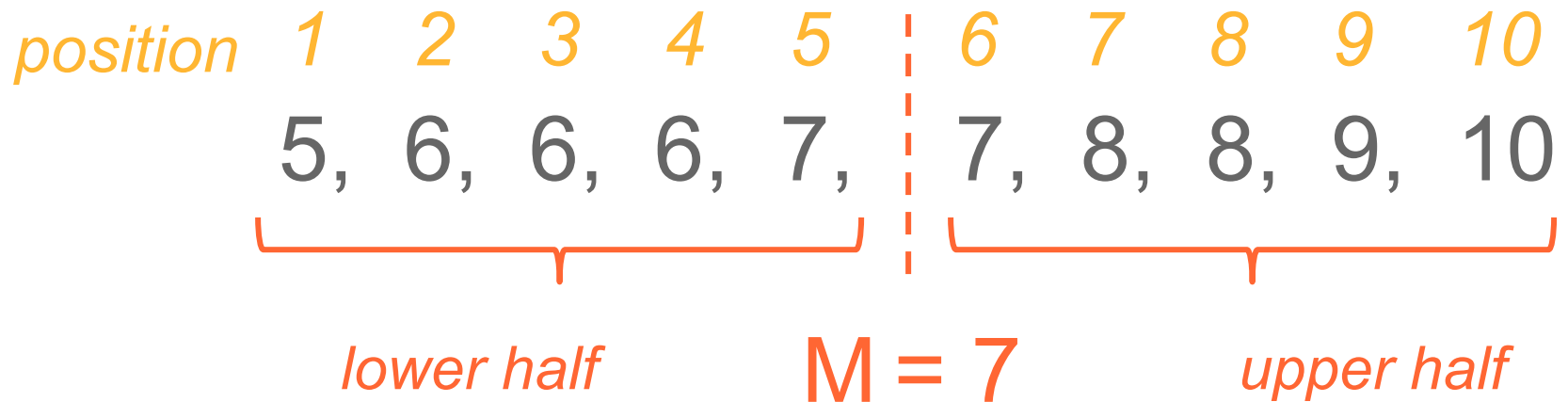
## Finding the Median (even number of values)

$$M_{0.5} = \frac{\text{sum of middle two values}}{2}$$

<i>position</i>	1	2	3	4	5	6	7	8	9	10
	5	6	6	6	7	7	8	8	9	10
<i>middle two values</i>					↑	↑				

$$\text{Median} = (7 + 7) / 2 = 7$$

## What the median does





## Finding the Median (odd number of values)

Let's remove one observation

Order values from smallest to largest


5, 6, 6, 6, 7, 7, 8, 9, 10

How many values?

**n = 9** (odd)

## Finding the Median (odd number of values)

<i>position</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>
	5,	6,	6,	6,	7,	7,	8,	9,	10
					<i>middle value</i>				



**Median = 7**

# Algebra Reminder

# Notation Review

Symbol	Meaning
$n$	number of individuals
$x_i$	value of individual <b>i</b>
$\Sigma$	summation
$x^2$	square value
$ x $	absolute value

# Summation reminder

$\Sigma$  “sigma”  
(upper-case)

# Summation reminder, example 1

5 observations  $x_1, x_2, x_3, x_4, x_5$

$$x_1 + x_2 + x_3 + x_4 + x_5$$

sum of all values

$$\sum_{i=1}^5 x_i = x_1 + x_2 + x_3 + x_4 + x_5$$

## Summation reminder, example 2

$$\sum_{i=1}^5 |x_i| = |x_1| + |x_2| + |x_3| + |x_4| + |x_5|$$

sum of absolute values

## Summation reminder, example 3

$$\sum_{i=1}^5 x_i^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$$


sum of square values



# Summation properties

$$\sum_{i=1}^5 a = a + a + a + a + a = 5a$$

constant




In general:

$$\sum_{i=1}^n a = na$$

# Summation properties

$$\sum_{i=1}^5 3 = 3 + 3 + 3 + 3 + 3 = 5(3)$$

constant



$$\sum_{i=1}^5 3 = 15$$

# Summation properties

$$\sum_{i=1}^n (x_i + a) = \sum_{i=1}^n x_i + \sum_{i=1}^n a$$

## Summation reminder, example 4

$$\sum_{i=1}^n x_i^2 \quad ? \quad \left( \sum_{i=1}^n x_i \right)^2$$

equal or different?

## Summation reminder, example 4

$$\left( \sum_{i=1}^5 x_i \right)^2 = \left( x_1 + x_2 + x_3 + x_4 + x_5 \right)^2$$

square sum of values

## Summation reminder, example 4

$$\sum_{i=1}^5 x_i^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$$

$\neq$

$$\left( \sum_{i=1}^5 x_i \right)^2 = \left( x_1 + x_2 + x_3 + x_4 + x_5 \right)^2$$