Probability (part 1)

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Probability Basics

Flip a fair coin

What are the chances it lands heads?

Possible outcomes when flipping a coin

50%

50%

How do you know it is 50-50?

If I flip the coin repeatedly, I expect (in the long run) the proportion of heads to be (close to) 1/2

Frequentist Approach

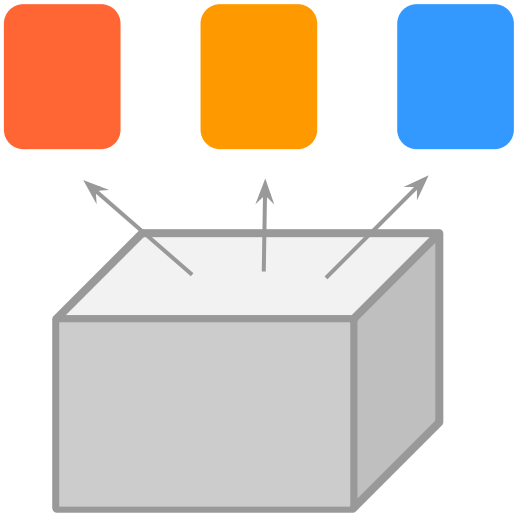

In the long run ...

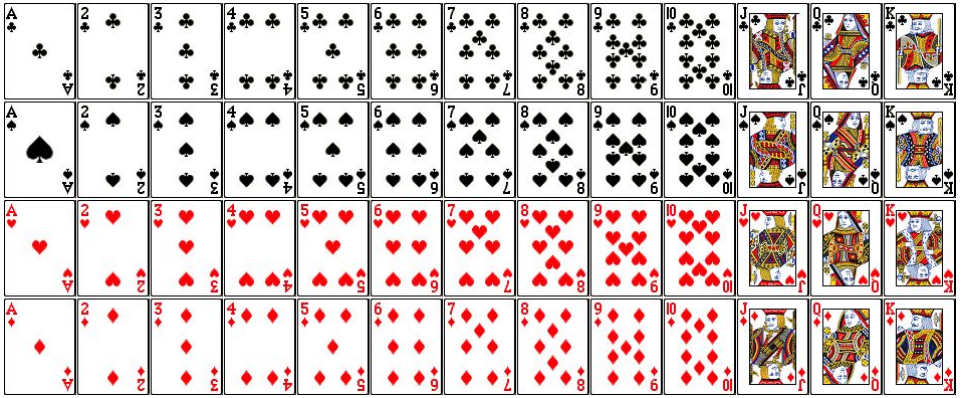
Frequency Theory

Processes which can be repeated ...

- over and over again
- independently
- under the same conditions

Frequency Theory: typical examples





60 °F | °C

Precipitation: 40%
Humidity: 72%
Wind: 16 mph

Temperature | Precipitation | Wind

Irrepeatable phenomenon

10 AM	1 PM	4 PM	7 PM	10 PM	1 AM	4 AM	7 AM
0%				1%	1%	1%	1%

Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri
60° 44°	65° 46°	68° 49°	73° 51°	72° 50°	72° 51°	72° 51°	69° 52°

50% chance a coin lands heads

≠

Different types of probability

50% chance it will rain tomorrow

If I flip the coin repeatedly, I expect (in the long run) the proportion of heads to be (close to) 1/2

Frequency Theory

The chance of something gives the percentage of time is expected to happen, when the basic process is done over and over again, independently and under the same conditions

Preliminary concepts

Events

We designate **events** with capital letters:
A, B, C, …

A: *number of new emails in your inbox*

Events

Rolling 2 dice and counting the numbers of sides facing up

A: *sum of the dice is seven*

A = { (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) }

What is probability?

Probability is a numerical measure that describes the likelihood that an event will occur.

Notation

$$P(A)$$

"P of A"

Probability of event A

Probability Rules

Equally likely outcomes

Equally Likely Outcomes

If all outcomes are **equally likely**, the chance an event happens is the number it can happen, divided by the total number of possible outcomes

Chance of an event

$$\frac{\text{\# of ways it can happen}}{\text{total \# possible outcomes}}$$

Flip a fair coin

Chance coin lands heads

one possible heads

$$P(\text{heads}) = \frac{\text{  }}{\text{  } + \text{  }} = 1/2$$

two possible outcomes

Chance coin lands tails

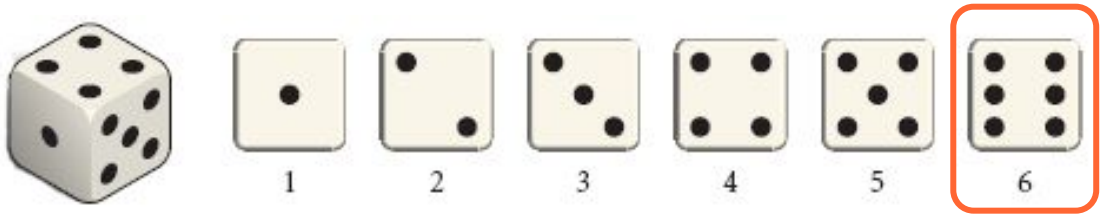
one possible tails

$$P(\text{tails}) = \frac{\text{🏺}}{\text{👩} + \text{🏺}} = 1/2$$

two possible outcomes

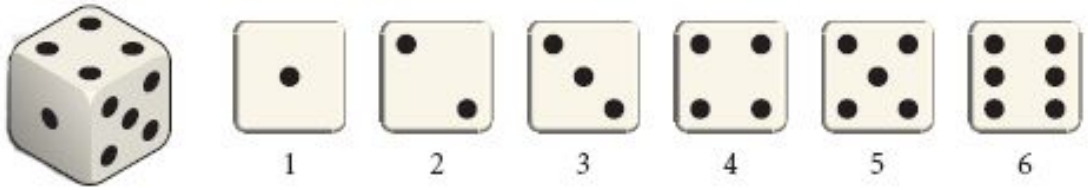
roll a *fair* die, what's the chance it lands 6?

Chance a die lands six



$$P(\text{six}) = \frac{\text{1 possible six}}{\text{6 possible outcomes}} = 1/6$$

Probability of each face

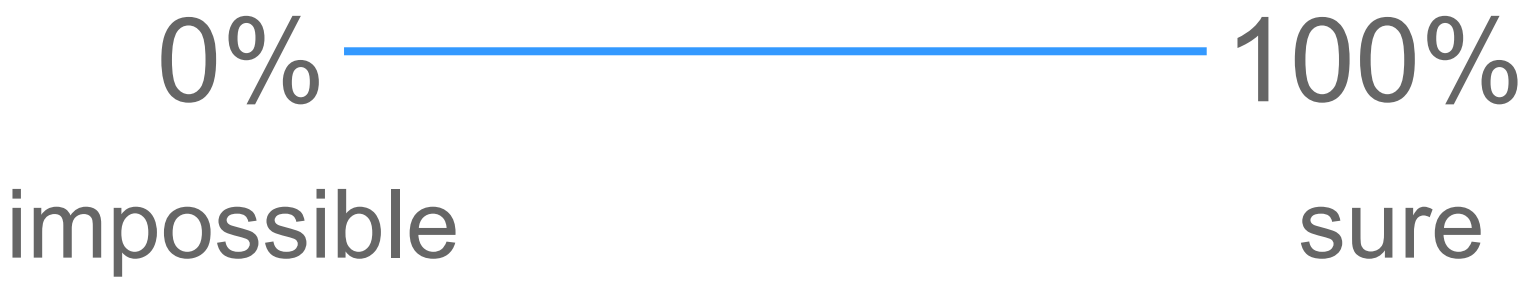


P(1) = 1/6 P(2) = 1/6 P(3) = 1/6

P(4) = 1/6 P(5) = 1/6 P(6) = 1/6

Probability Range

Chances are between 0% and 100%

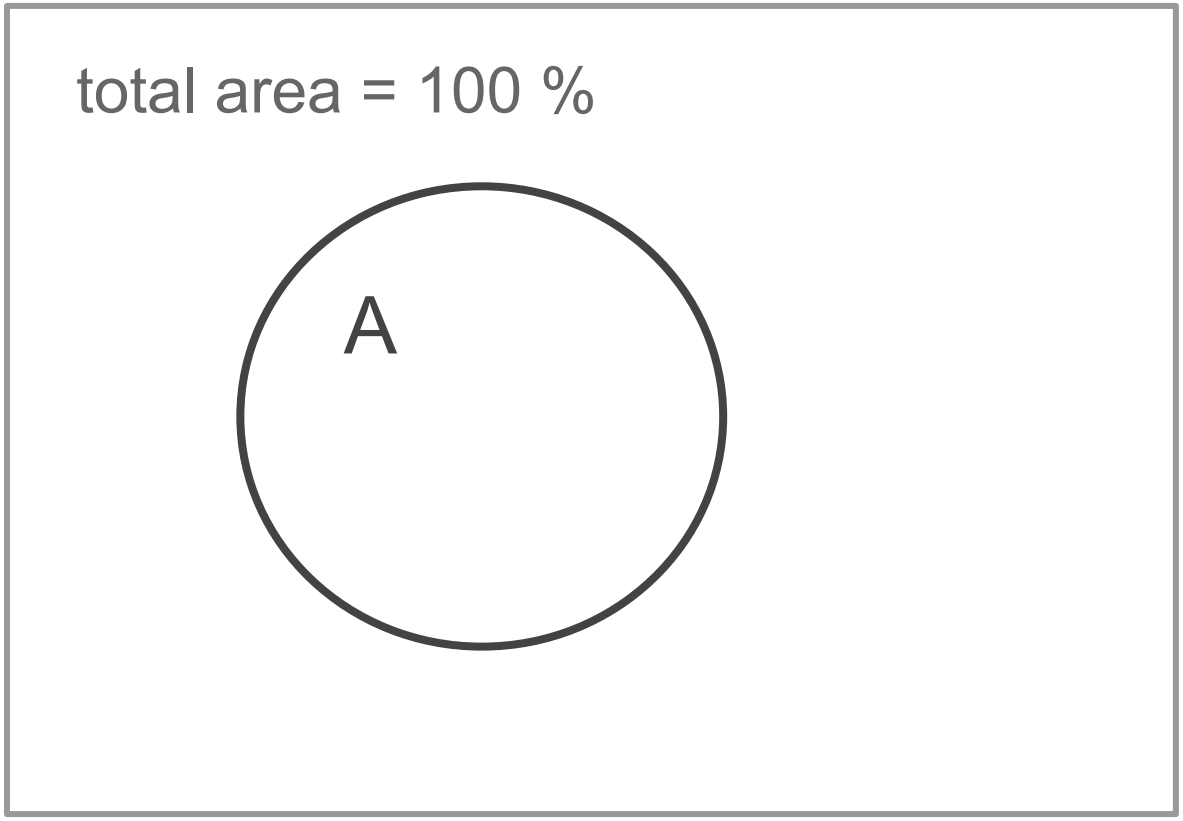


Complement Rule

The chance of something equals 100% minus the chance of the opposite thing

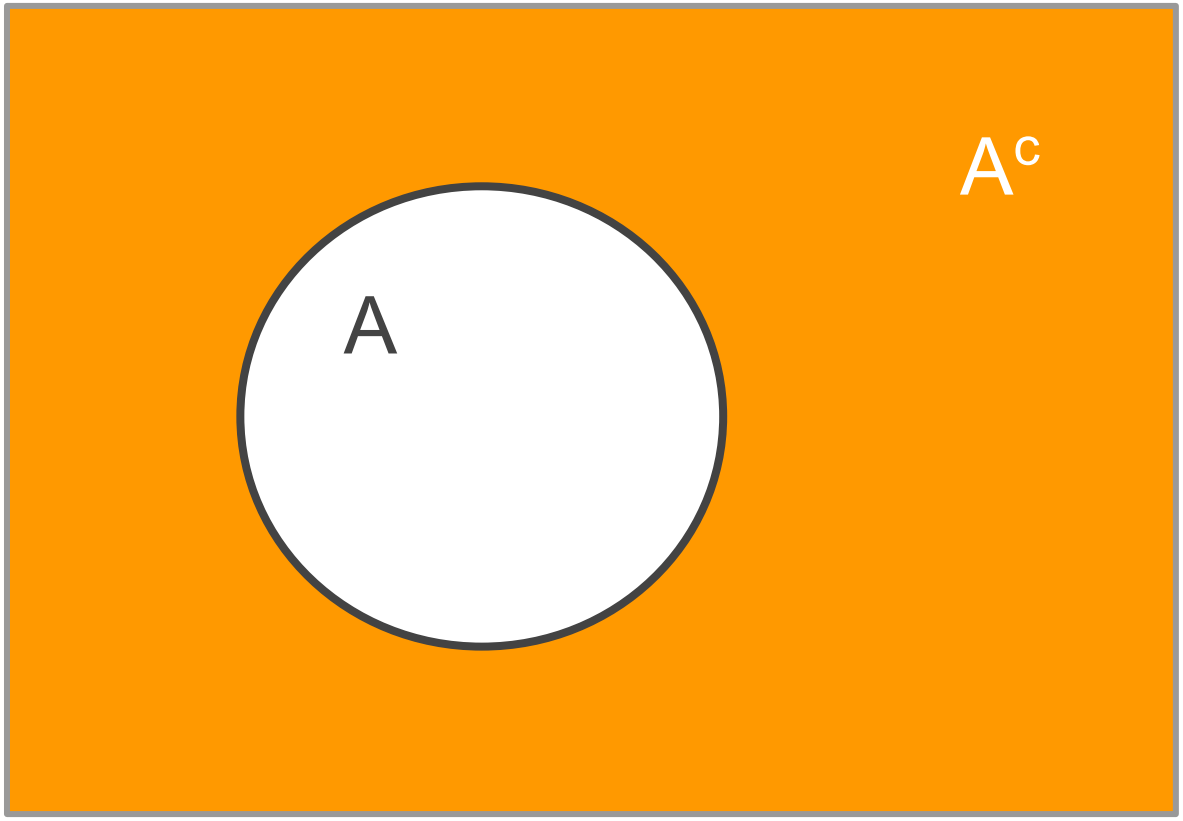
complement

Venn Diagrams



Venn Diagrams

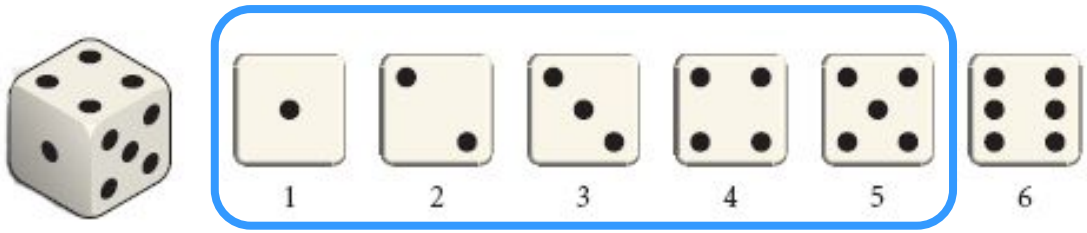
A^c = complement of A



area of A + area of A^c = 100 %

roll a fair die, what's the chance you get 5 or less?

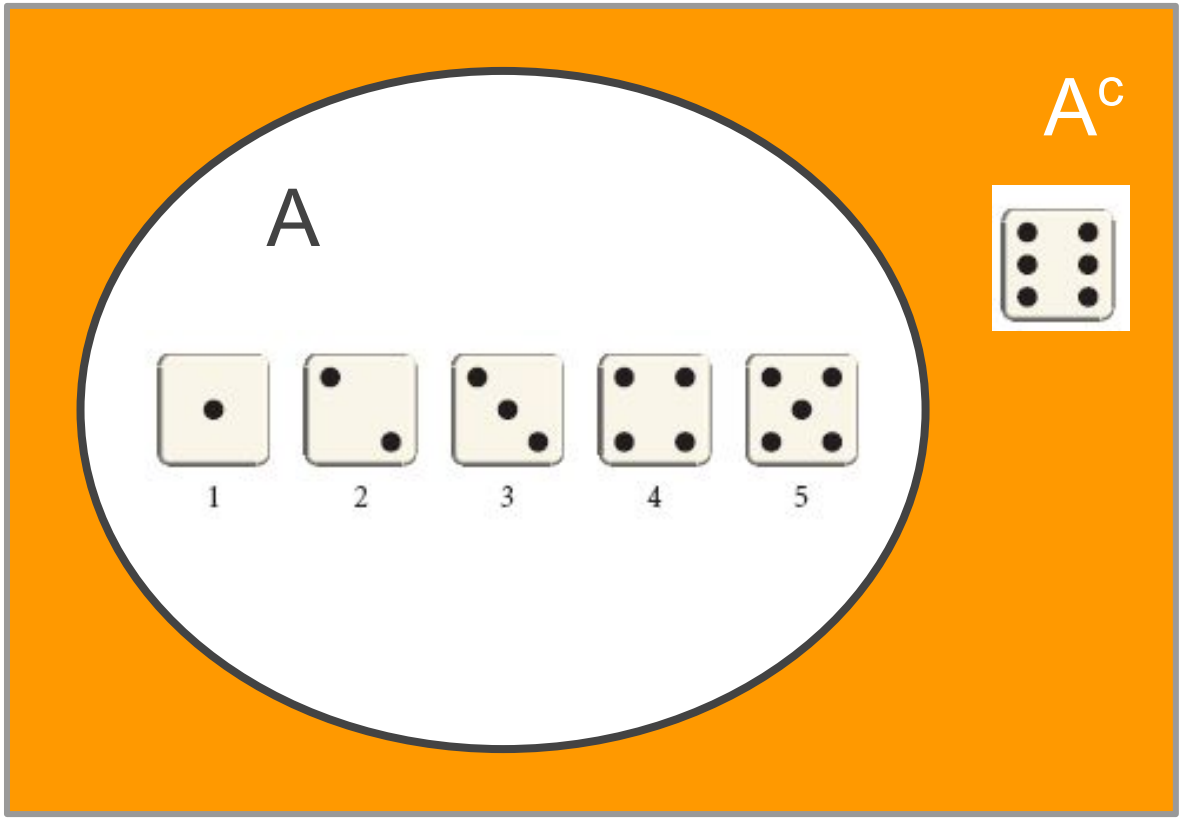
Chance of getting 5 or less



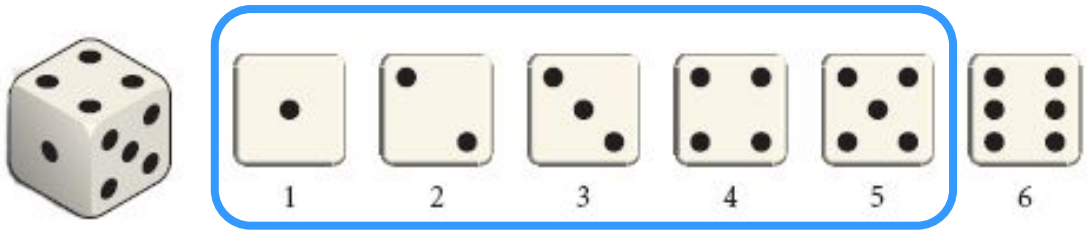
$$P(5 \text{ or less}) = P(5) + P(4) + P(3) + P(2) + P(1)$$

$$P(5 \text{ or less}) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 5/6$$

Venn Diagrams



Chance of getting 5 or less



P(5 or less) = 1 - P(six) =

P(5 or less) = 1 - 1/6 = 5/6

P(5 or less) = P(Not six)

Conditional Probability

Conditional Probability:

The chance of one event given that another event happens

Conditional Probability

$$P(B \mid A)$$

Probability of event B given that event A happens

Conditional Probability: alternative notation

$$P(A \mid B)$$

Probability of event A given that event B happens

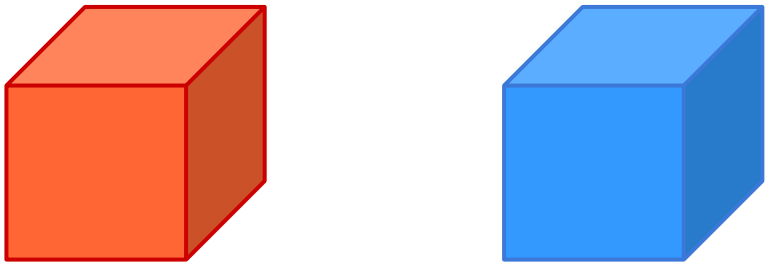
Conditional Probability

$$P(A \mid B) \neq P(B \mid A)$$

not necessarily equal

Conditional Probability

2 dice



A = red die is 1 B = sum of dice is 2

P(B | A) = P(sum is 2 | red is 1)

P(B | A) = P(blue is 1) = 1/6