Probability (part 2)

Gaston Sanchez

Multiplication Rule

Multiplication Rule

P(A and B)

chance of two events both happen

Multiplication Rule

The chance of two events both happen is the chance of one of them, times the chance of the other, given that the first happens.

To be used when calculating the probability of two events happening.

Multiplication Rule

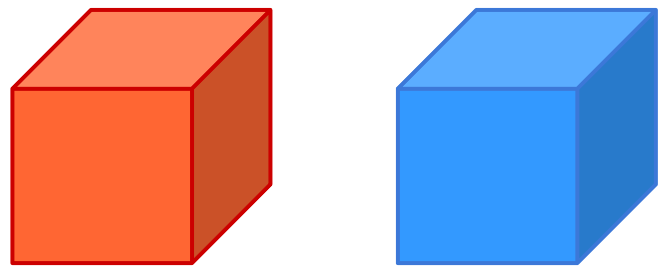
$$P(A \text{ and } B) = P(A)\, P(B \mid A)$$

$$P(B \text{ and } A) = P(B)\, P(A \mid B)$$

$$P(A \text{ and } B) = P(B \text{ and } A)$$

Conditional Probability

2 dice



A = red die is 1

B = sum of dice is 2

P(A and B) = ?

Multiplication Rule

$$P(A \text{ and } B) = P(A)\, P(B \mid A)$$

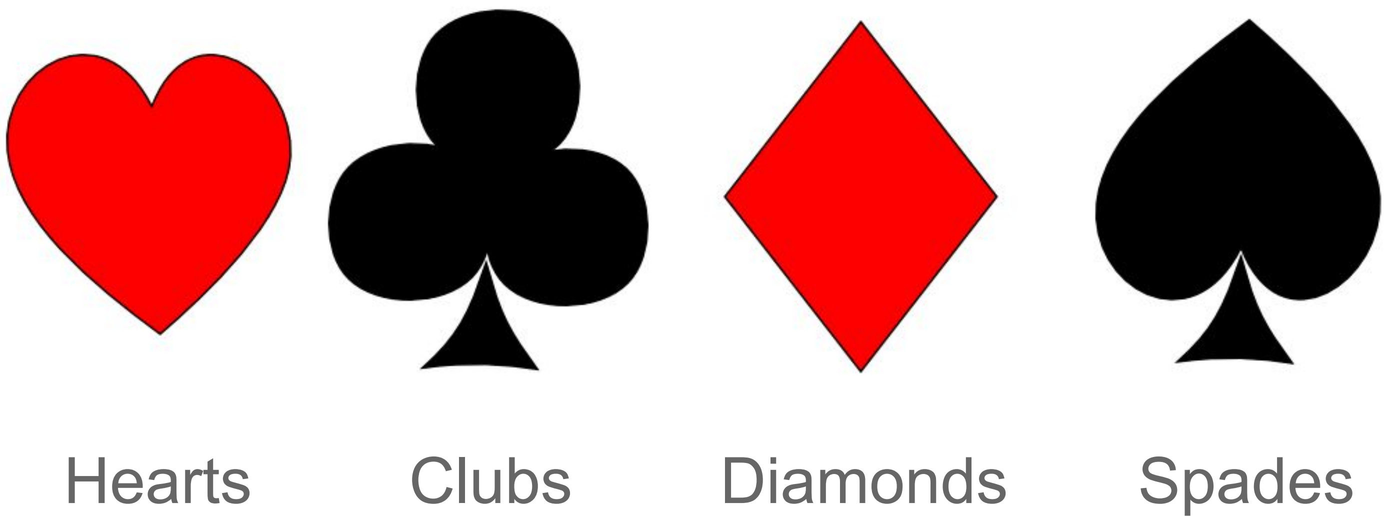
$$P(A) = 1/6$$

$$P(B \mid A) = P(\text{blue die is 1}) = 1/6$$

$$P(A \text{ and } B) = (1/6)(1/6) = 1/36$$

Deck of cards

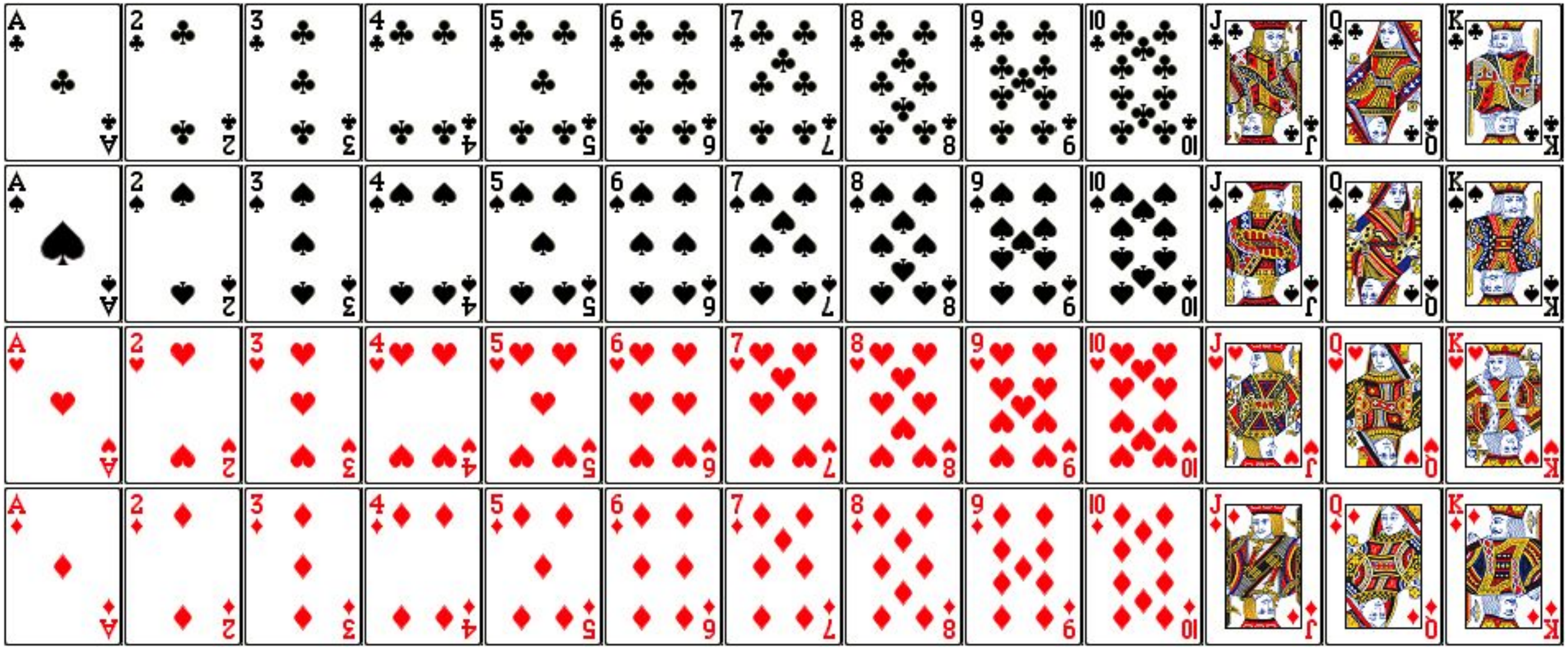
Deck of cards: 4 suits



Deck of cards

4 suits: clubs, spades, hearts, diamonds

13 ranks: Ace, 2, 3, …, 10, Jack, Queen, King



Multiplication Rule

A: 1st card is a spade

B: 2nd card is a spade

P(A and B) = P(1st spade and 2nd spade) =

P(1st spade) P(2nd spade | 1st spade)

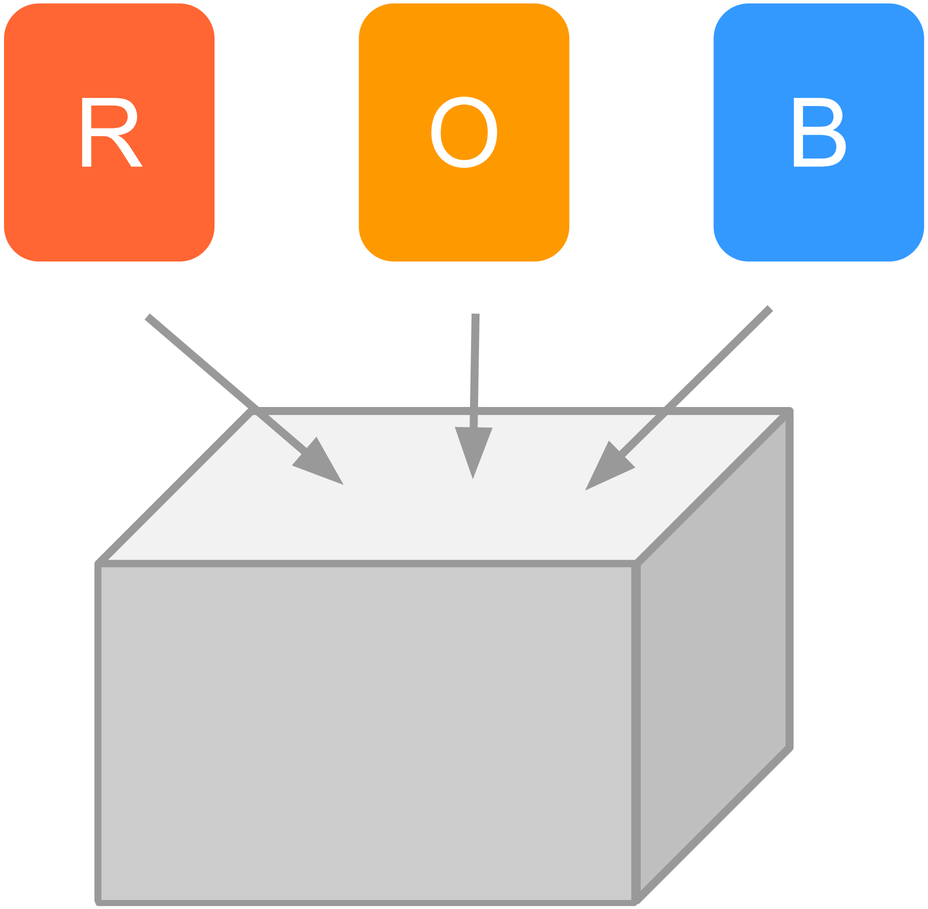
Multiplication Rule

P(1st spade) =
13 / 52

P(2nd spade | 1st spade) =
12 / 51

P(1st spade) P(2nd spade | 1st spade) =
(13 / 52) (12 / 51) = 1 / 17

Multiplication Rule



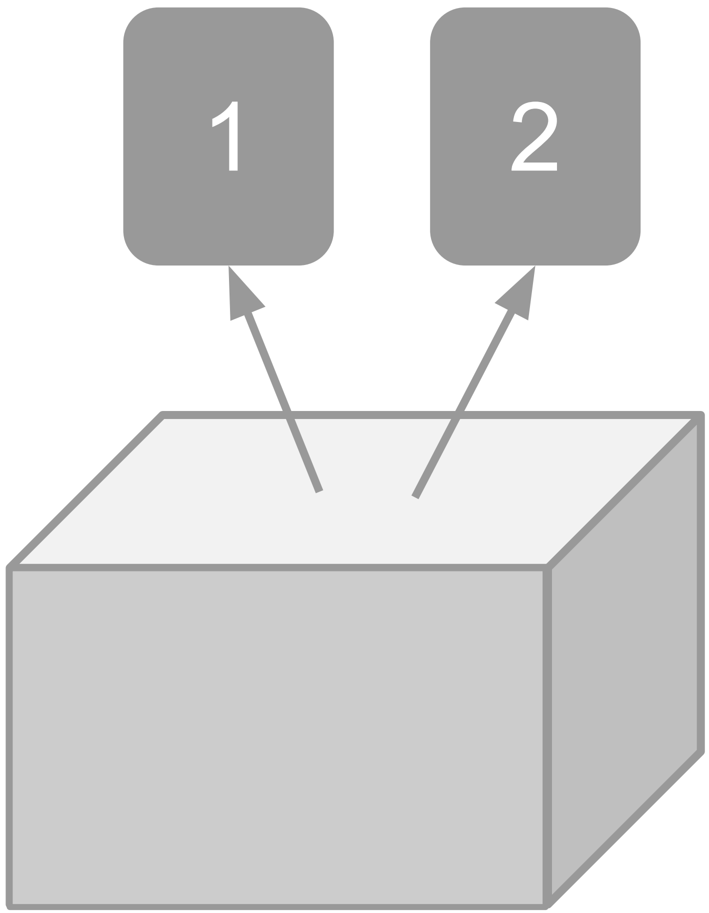
Experiment

3 cards in a box (mix them)

1st: Pick one card at random and set it aside

2nd: Out of remaining two, pick another card

Multiplication Rule



P(1st card is R) = ?

P(2nd card is B) = ?

P(1st R and 2nd B) = ?

Multiplication Rule

P(1st card **R**) = 1 / 3

P(2nd card **B**) = 1 / 2

P(1st **R** and 2nd **B**) = P(1st **R**) P(2nd **B** | 1st **R**)

= (1/3) (1/2) = 1/6

Independent Events

Independence (of two events)

A and B are independent if

$$P(B) = P(B \mid A)$$

$$P(A) = P(A \mid B)$$

Independence (of two events)

A and B are independent if

$$P(B) = P(B \mid A)$$

Equivalently,

$$P(A \text{ and } B) = P(A)\, P(B)$$

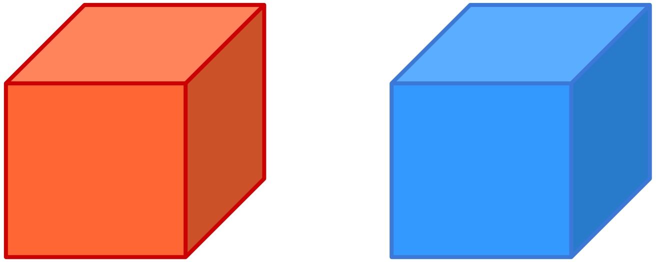
Special case of multiplication rule

Independence

$$P(B \mid A) = P(B)$$

The chance of an event B, is not affected by whether or not another event A happens

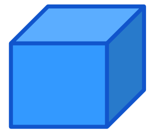
Independence (of two events)

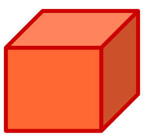


Roll red die and blue die

Find the chance of rolling at least one six

Independence (of two events)





	1	2	3	4	5	6
1						x
2						x
3						x
4						x
5						x
6	x	x	x	x	x	x

P(at least one six) = 11/36

Independence for two events

Complement of “at least one six” = “No six”

“No six” = “red not six” and “blue not six”

P(red not six) = 5 / 6

P(blue not six) = 5 / 6

P(R not six and B not six) = 25 / 36

Addition Rule

Addition Rule

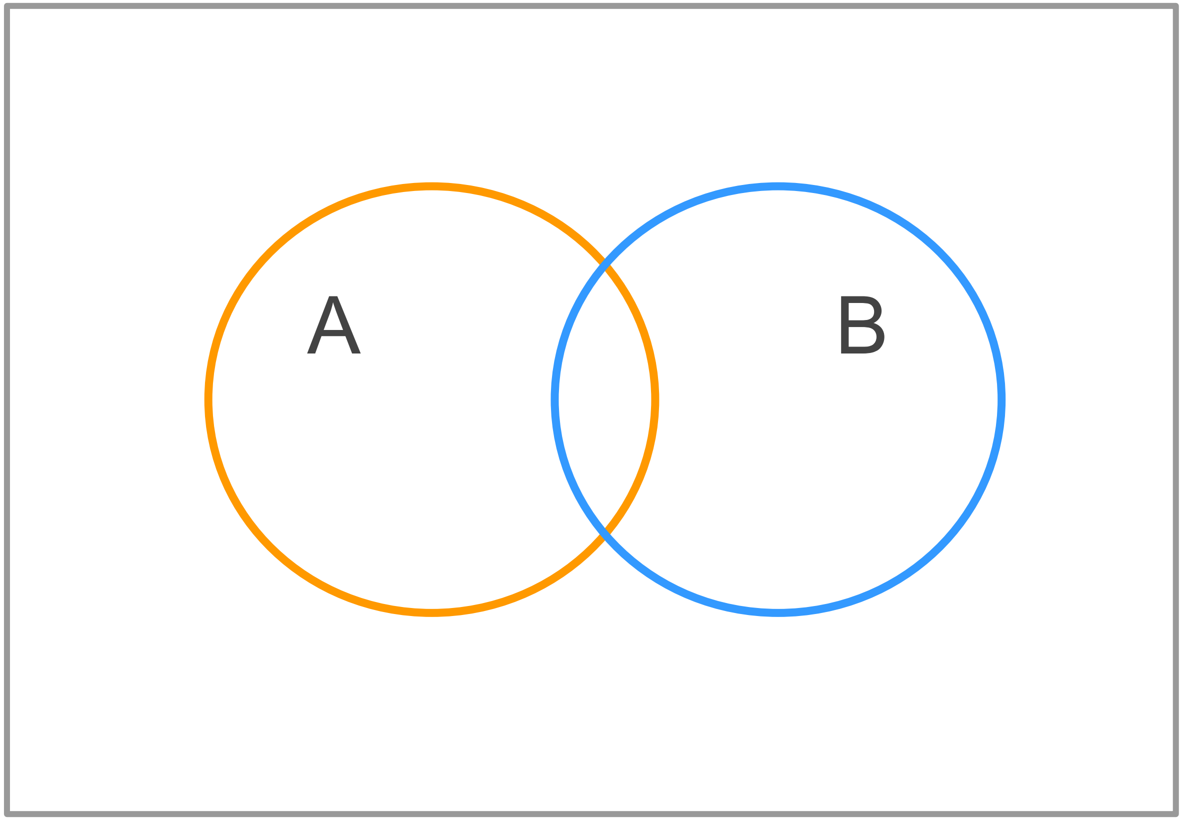
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

A happens or

B happens or

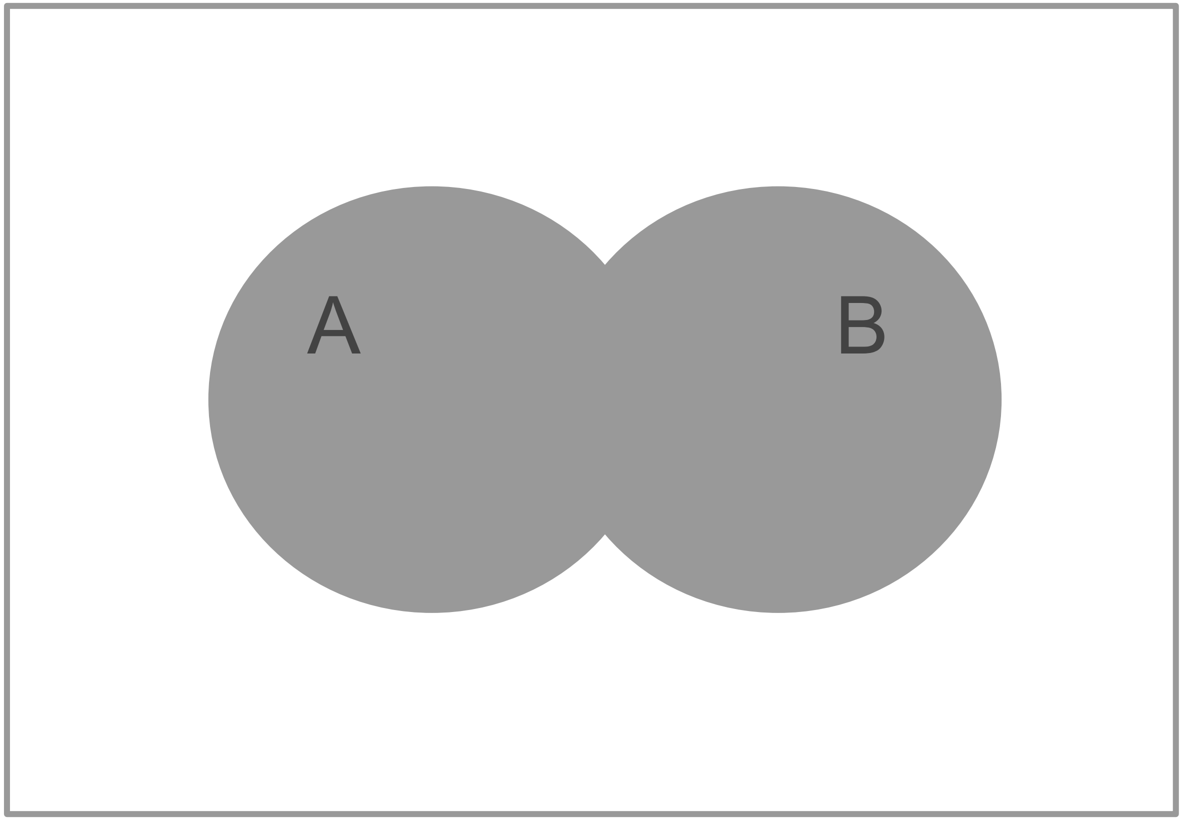
Both A, B happen

Venn Diagrams

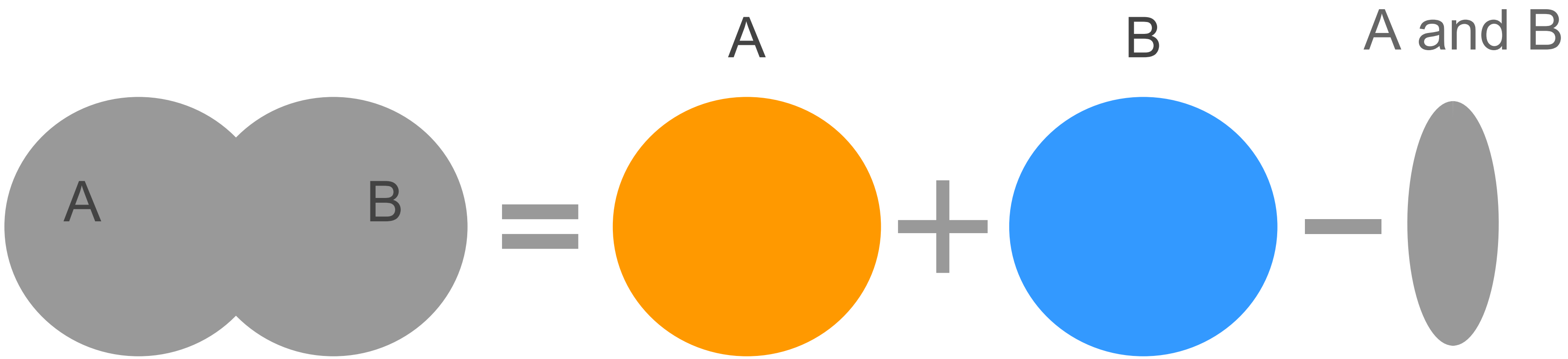


Venn Diagrams

A or B



Venn Diagrams

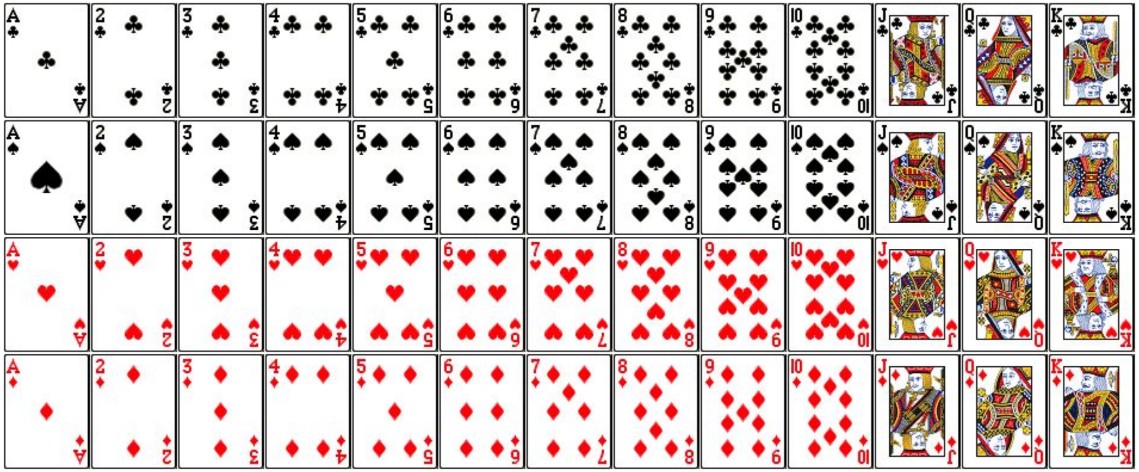


Addition rule

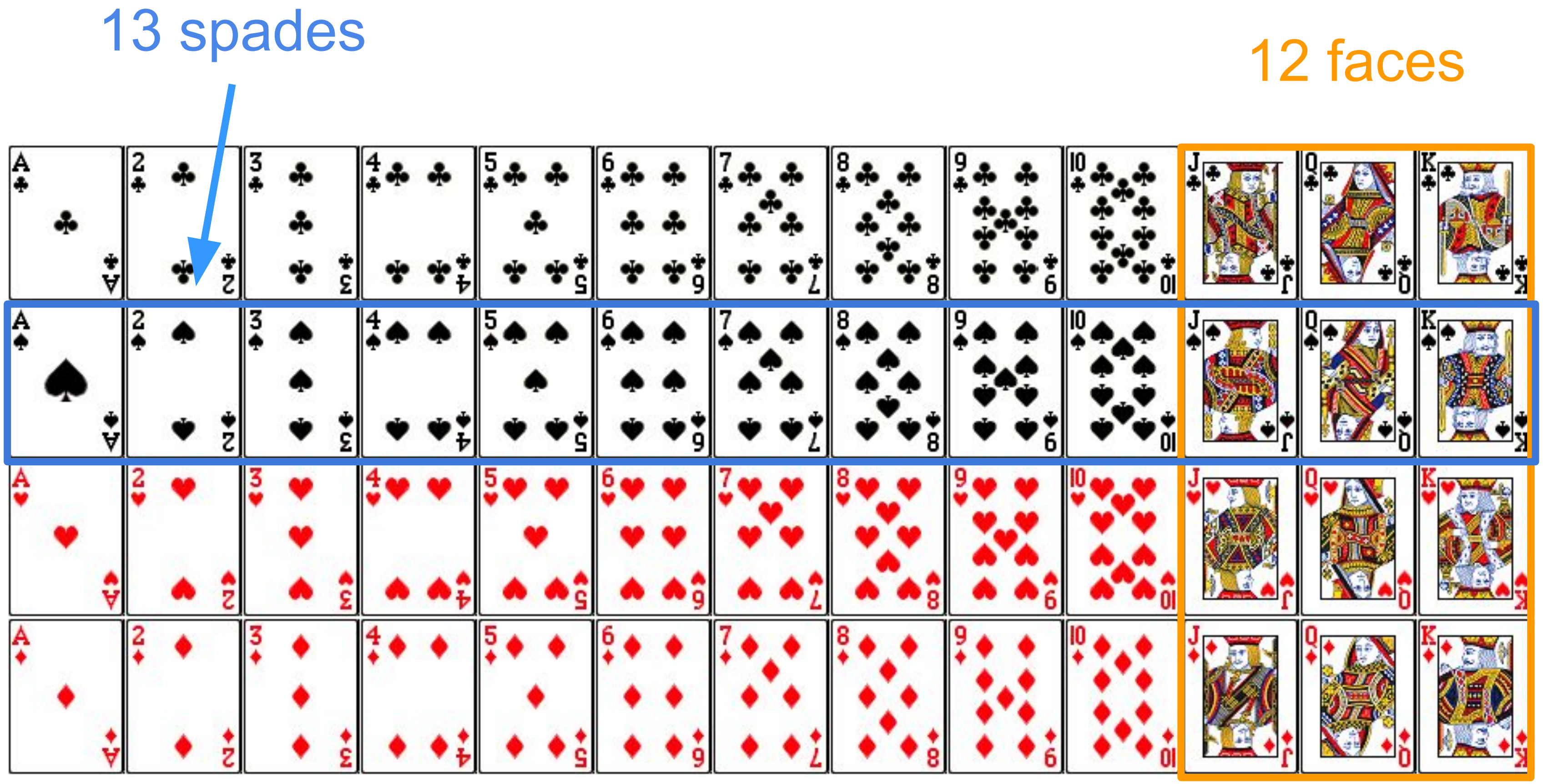
A = a spade card is selected

B = a face card is selected

P(A *or* B) = ?



Spade *or* Face



Gaston Sanchez

Addition rule

A = event a spade card is selected

B = event a face card is selected

$P(A \textit{ or } B) = P(A) + P(B) - P(A \textit{ and } B)$

$P(A) = 13 / 52$

$P(B) = 12 / 52$

$P(A \textit{ and } B) = 3 / 52$

$P(A \textit{ or } B) = 13 / 52 + 12 / 52 - 3 / 52 = 22 / 52$

Recap

Equally likely outcomes:

P = # ways it happens / total # outcomes

Complement rule:

$P(A) = 1 - P(A^c)$

Multiplication rule:

$P(A \text{ and } B) = P(A)\, P(B|A)$

Addition rule:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Mutually Exclusive Events

Mutually Exclusive

A and B are mutually exclusive if the occurrence of one of them stops the occurrence of the other

(they cannot happen together)

Addition Rule with mutually exclusive events

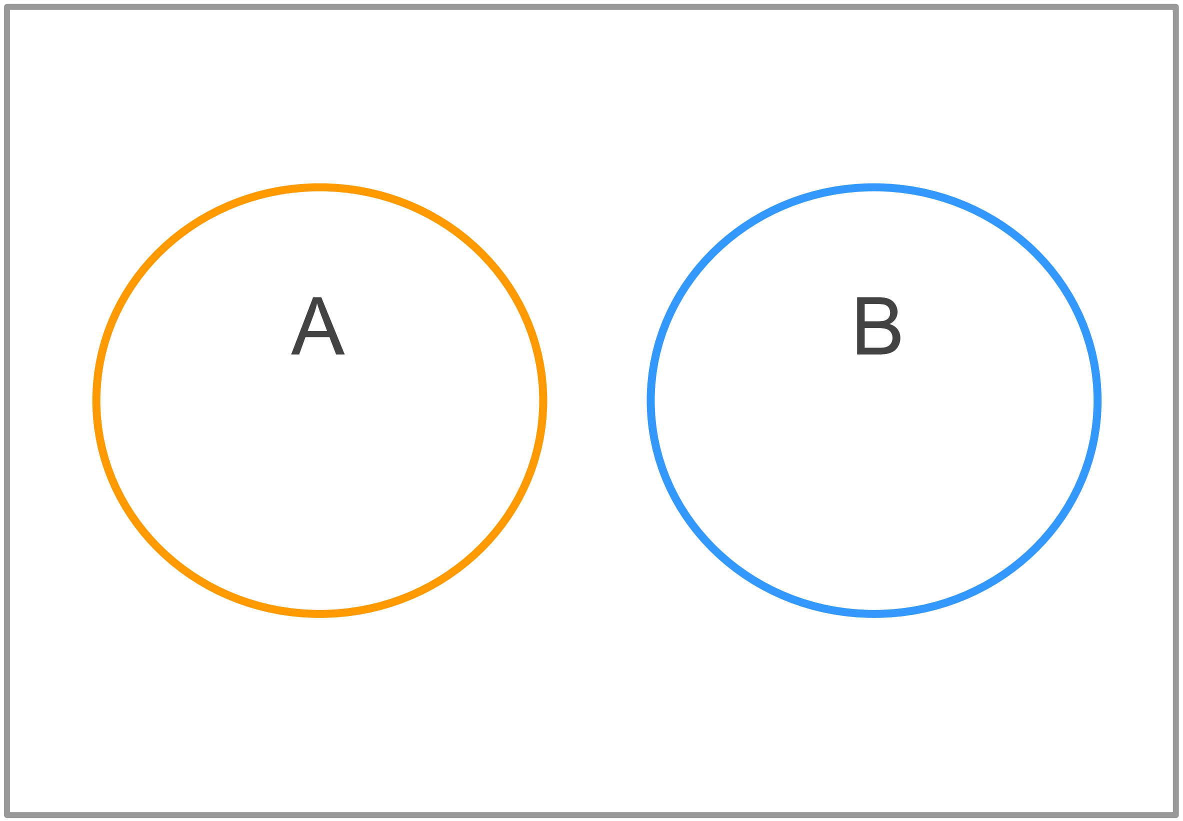
A, B mutually exclusive events:

$$P(A \text{ or } B) = P(A) + P(B) + \cancel{P(A \text{ and } B)}$$

A happens or B happens

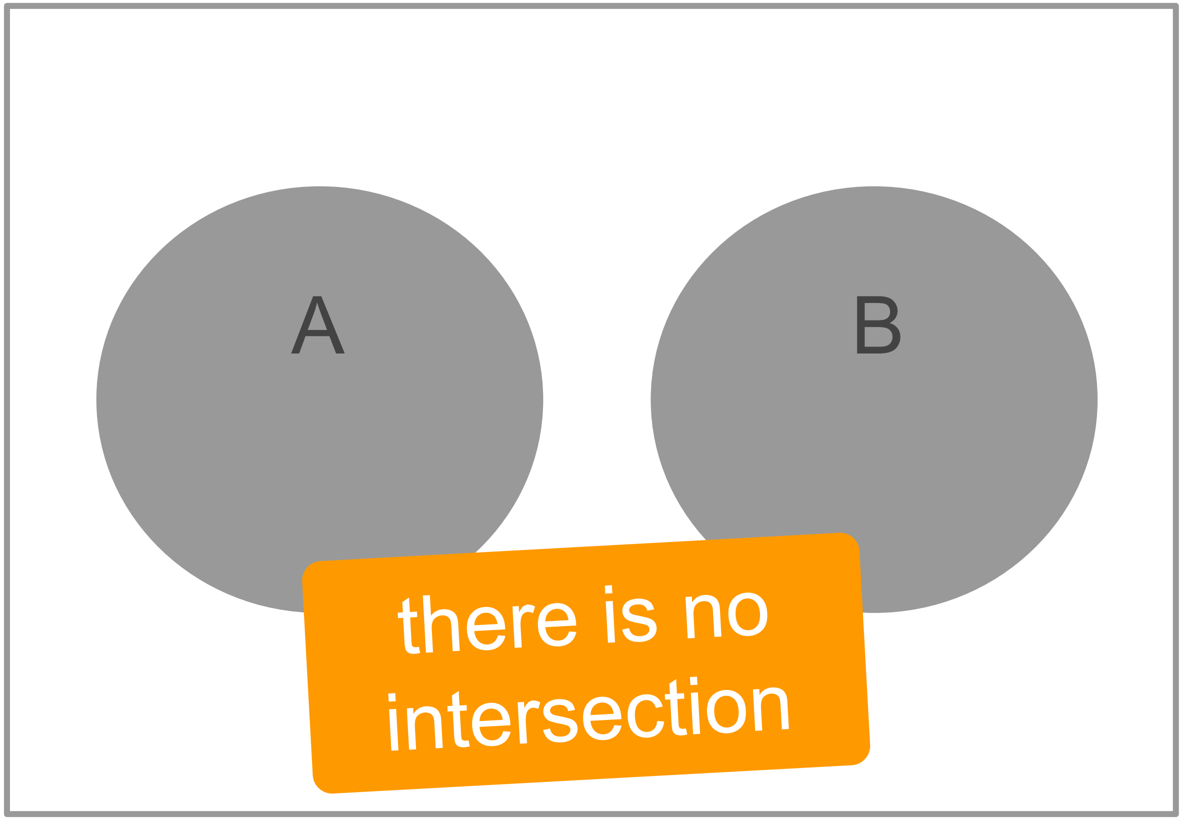
(but both A, B cannot happen)

Venn Diagrams: mutually exclusive events



Venn Diagrams: mutually exclusive events

A and B



Multiplication Rule with mutually exclusive events

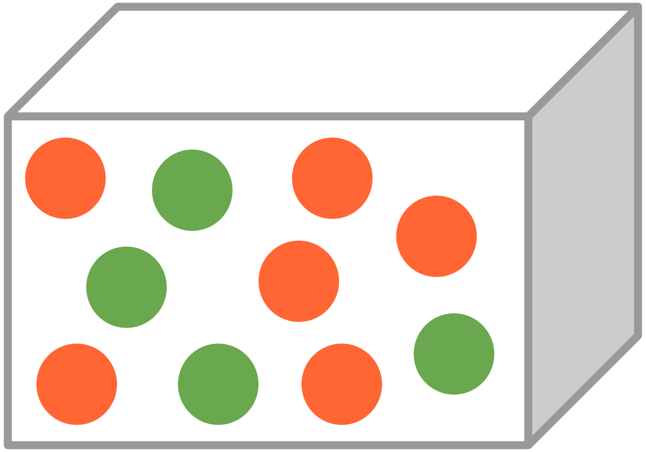
$$P(A \text{ and } B) = 0$$

both A, B cannot happen

Example: 2 balls from a box

Example

A box contain 10 balls: **4 green**, **6 red**.



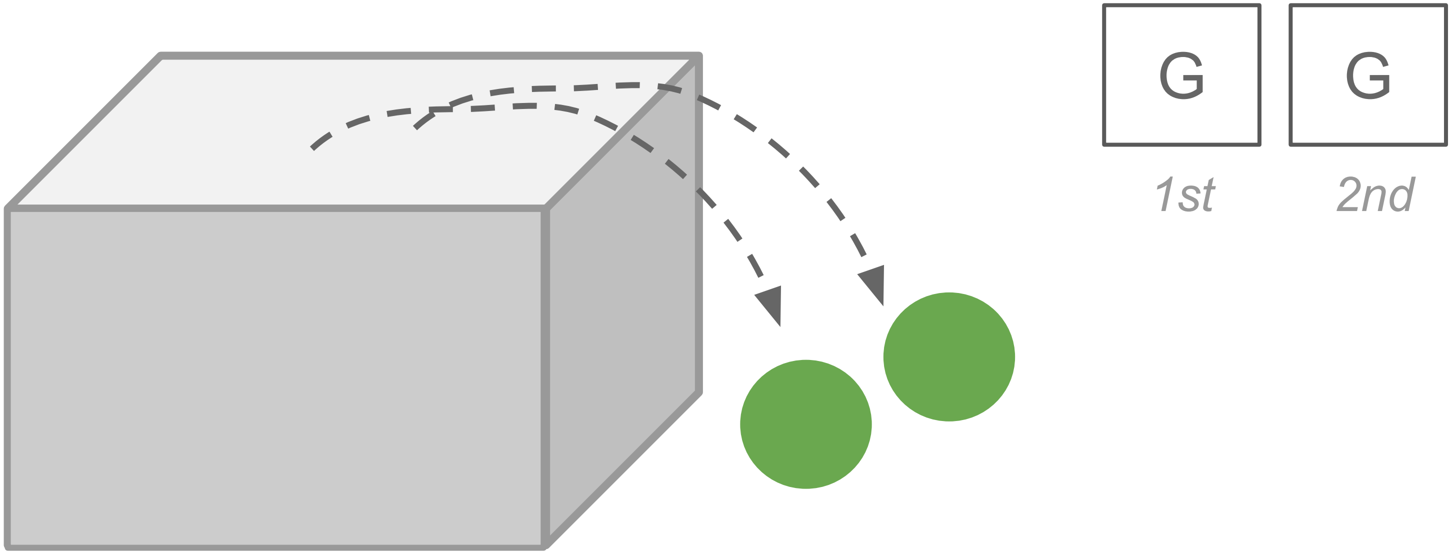
Pick a 1st ball, then a 2nd ball.

Find the chance they match.

Example

A box contain 10 balls: **4 green**, **6 red**.

Pick 1st ball, then 2nd ball.



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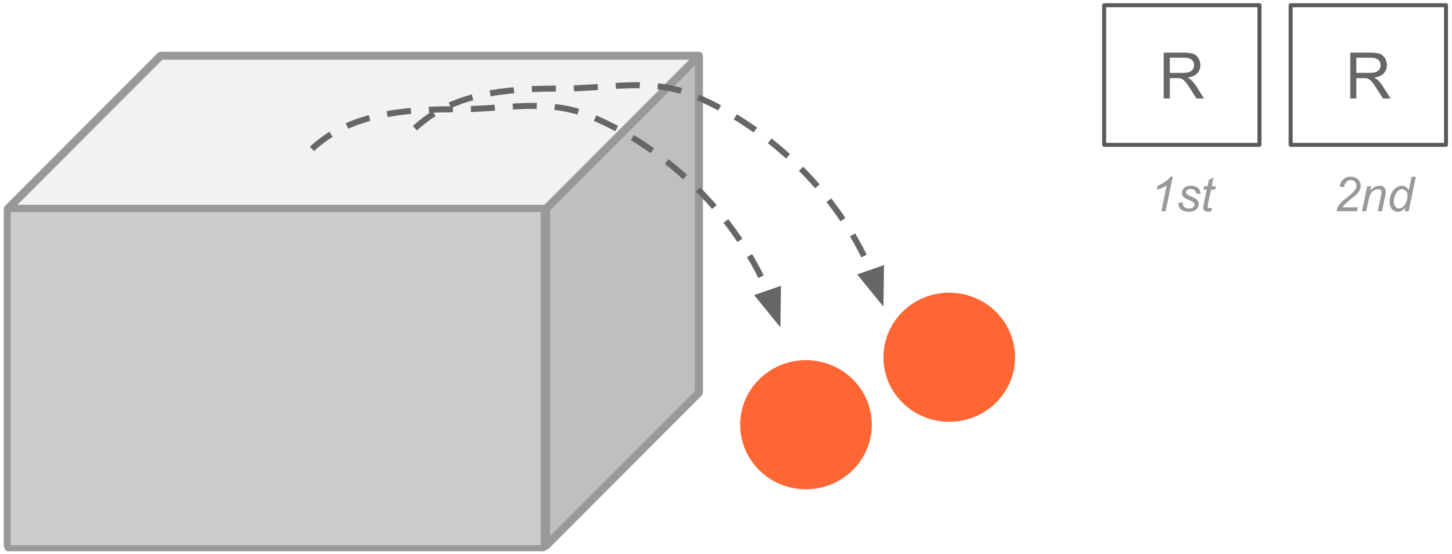

Example

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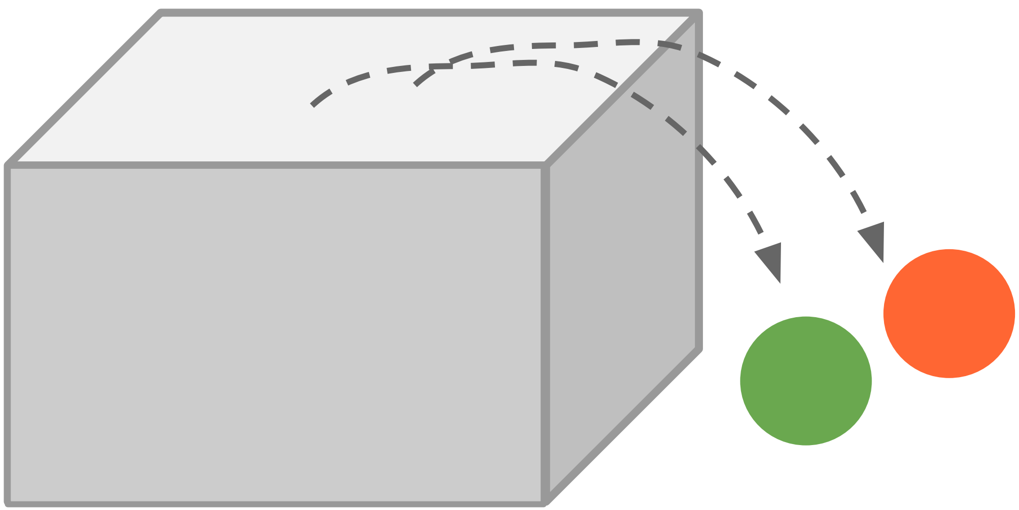

G	R
1st	*2nd*

no match

Example

A box contain 10 balls: **4 green**, **6 red**.

Pick two balls at random. Find the chance they match.

P(balls match) = P(both green OR both red)

P(**GG** or **RR**) = P(**GG**) + P(**RR**) - P(~~**GG** and **RR**~~)

Example

A box contain 10 balls: **4 green**, **6 red**.

$P(GG \text{ or } RR) = \boxed{P(GG)} + P(RR)$

$P(GG) = P(G_1 \text{ and } G_2)$

$P(G_1 \text{ and } G_2) = P(G_1)\, P(G_2 \mid G_1)$

$P(G_1) = 4 / 10$

$P(G_2 \mid G_1) = 3 / 9$

$P(G_1)\, P(G_2 \mid G_1) = (4/10)\,(3/9) = 12 / 90 = 2 / 15$

Example

A box contain 10 balls: **4 green**, **6 red**.

$P(GG \text{ or } RR) = P(GG) + \boxed{P(RR)}$

$P(RR) = P(R_1 \text{ and } R_2)$

$P(R_1 \text{ and } R_2) = P(R_1)\, P(R_2 \mid R_1)$

$P(R_1) = 6 / 10$

$P(R_2 \mid R_1) = 5 / 9$

$P(R_1)\, P(R_2 \mid R_1) = (6/10)\,(5/9) = 30 / 90 = 1 / 3$

Example

A box contain 10 balls: **4 green**, **6 red**.

P(**GG** or **RR**) = P(**GG**) + P(**RR**) =

$P(G_1 \text{ and } G_2) + P(R_1 \text{ and } R_2) = (2/15) + (1/3) = 7/15$

Mutually Exclusive

vs

Independence

Consider two events

A: heads when tossing a coin

B: six when rolling a die

Independent?
Mutually Exclusive?
None of the above?

Independent Events

A: heads when tossing a coin

B: six when rolling a die

$\boxed{\text{Indep:} \quad P(A \mid B) = P(A)}$

M. Exc: $P(A \text{ and } B) = 0$

None: $P(A \text{ and } B) = P(A)\,P(B \mid A)$

Independent events

A: heads when tossing a coin

B: six when rolling a die

$$P(A \mid B) = P(A)$$

$$P(B \mid A) = P(B)$$

Crosstable for current enrollment in public and private schools by level of education

	Public	Private	***Total***
Elementary	20%	30%	*50%*
High School	15%	20%	*35%*
College	10%	5%	*15%*
Total	*45%*	*55%*	***100%***

Probability that a student randomly selected is enrolled in Elementary and High School?

P(enrolled in Elementary and HS) = ?

	Public	Private	***Total***
Elementary	20%	30%	*50%*
High School	15%	20%	*35%*
College	10%	5%	*15%*
Total	*45%*	*55%*	***100%***

Independent?
Mutually Exclusive?
None of the above?

P(enrolled in Elementary and HS) = ?

	Public	Private	***Total***
Elementary	20%	30%	*50%*
High School	15%	20%	*35%*
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Total	*45%*	*55%*	***100%***

mutually exclusive events

Keep in mind ...

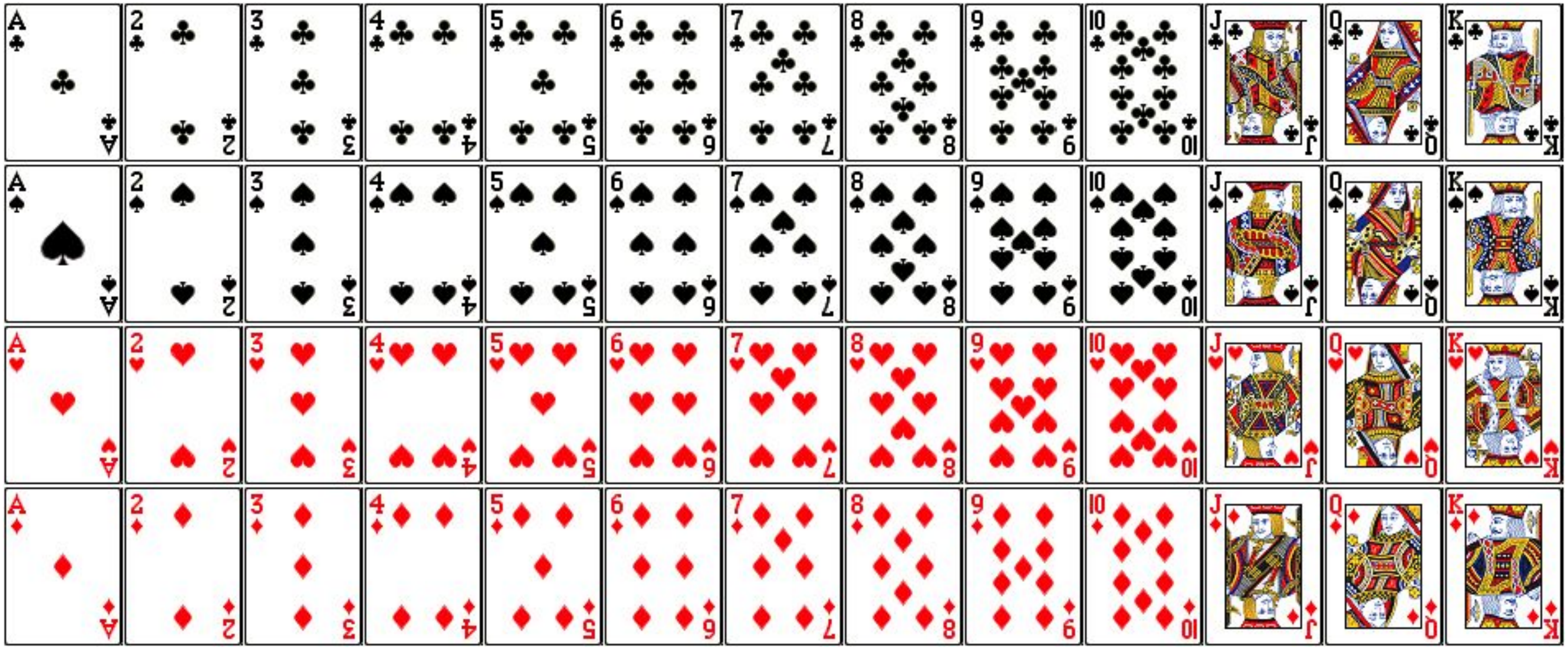
Mut. Exclusive events	$\neq$	Independent events
typically has to do with outcomes of same experiment		*typically has to do with outcomes of different experiments*

Example: 2nd card from deck

Example

Consider a standard deck of cards.

P(2nd card is a spade) = ?



P(2nd card is a Spade)

P(2nd card is a spade) = ?

P(1st **S** and 2nd **S** **OR** 1st **not-S** and 2nd **S**) =

P(1st **S** and 2nd **S**) = (13/52) (12/51)

P(1st **not-S** and 2nd **S**) = (39/52) (13/51)

P(2nd **S**) = (13/52) (12/51) + (39/52) (13/51) = 13/52

Example: one head in 3 flips

Example

You flip a fair coin 3 times. What is the chance you get exactly one head?

Example

You flip a fair coin 3 times. What is the chance you get exactly one head?

Different ways of getting one head in 3 flips:

- HTT
- THT
- TTH

P(HTT or THT or TTH) = ?

$= (½)^3 + (½)^3 + (½)^3 = 3(½)^3$