Binomial Distribution

Gaston Sanchez

Binomial Probability

Many processes have only **2** possible outcomes

Process with 2 outcomes

Flipping a coin

Process with 2 outcomes

Effectiveness of a drug:

Effective -vs- Not effective

Process with 2 outcomes

Financial Balance

Process with 2 outcomes

Success

Failure

Many processes can be broken down into
2 complementary events

Process with 2 outcomes

Rolling a die

Obtaining 1

1

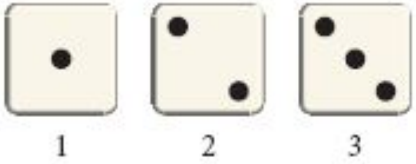
Not 1

2 3 4 5 6

Process with 2 outcomes

Rolling a die 

1, 2, or 3



4, 5, 6

Binomial Experiment

Binomial Experiment

Fixed number of trials

Repeated trials under identical conditions

Independent trials

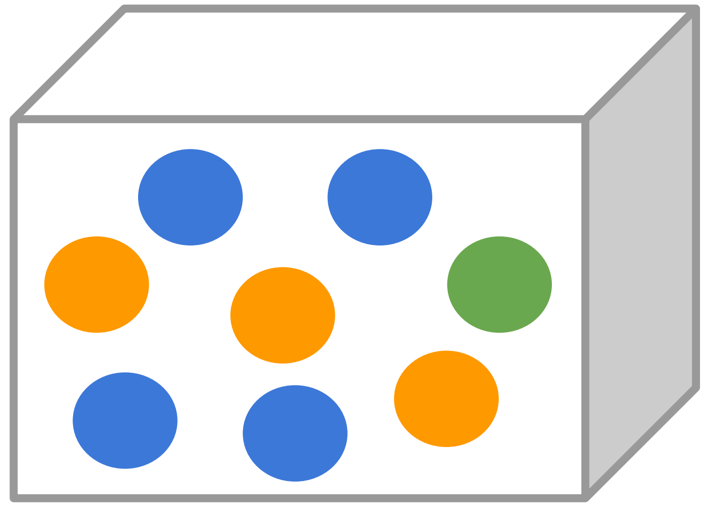
Probability of success is the same in each trial

Goal: Probability of ***k*** successes out of ***n*** trials

Binomial Experiment

Flipping a coin 5 times
Identical conditions
Independent trials
Constant probability of heads
Probability of 3 heads

2 Experiments



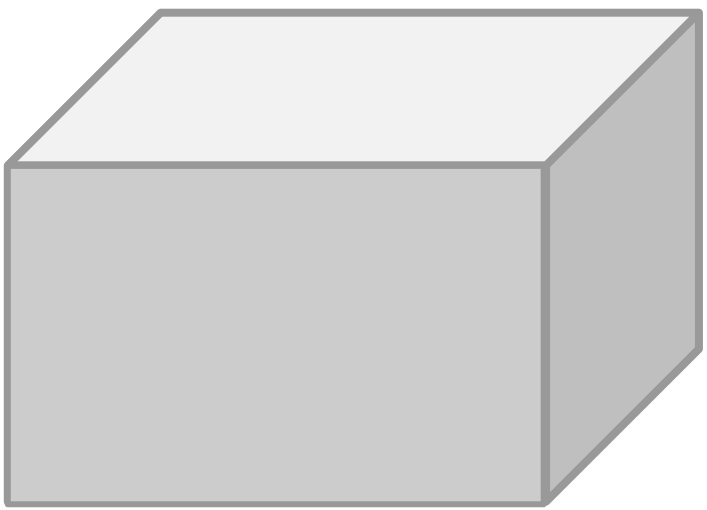
Experiment **A**: withdraw a ball, **replace it**, withdraw a 2nd ball, and counting # of orange balls

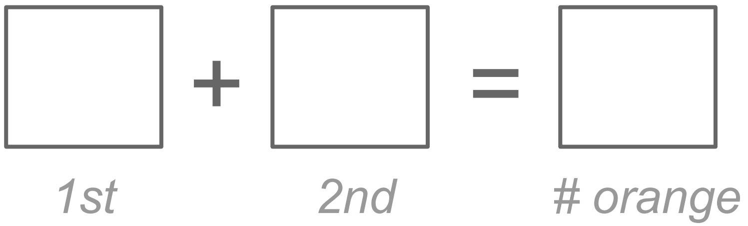
Experiment **B**: withdraw a ball, **no replacement**, withdraw a 2nd ball, and counting # of orange balls

Which experiment is Binomial?

Binomial Experiment

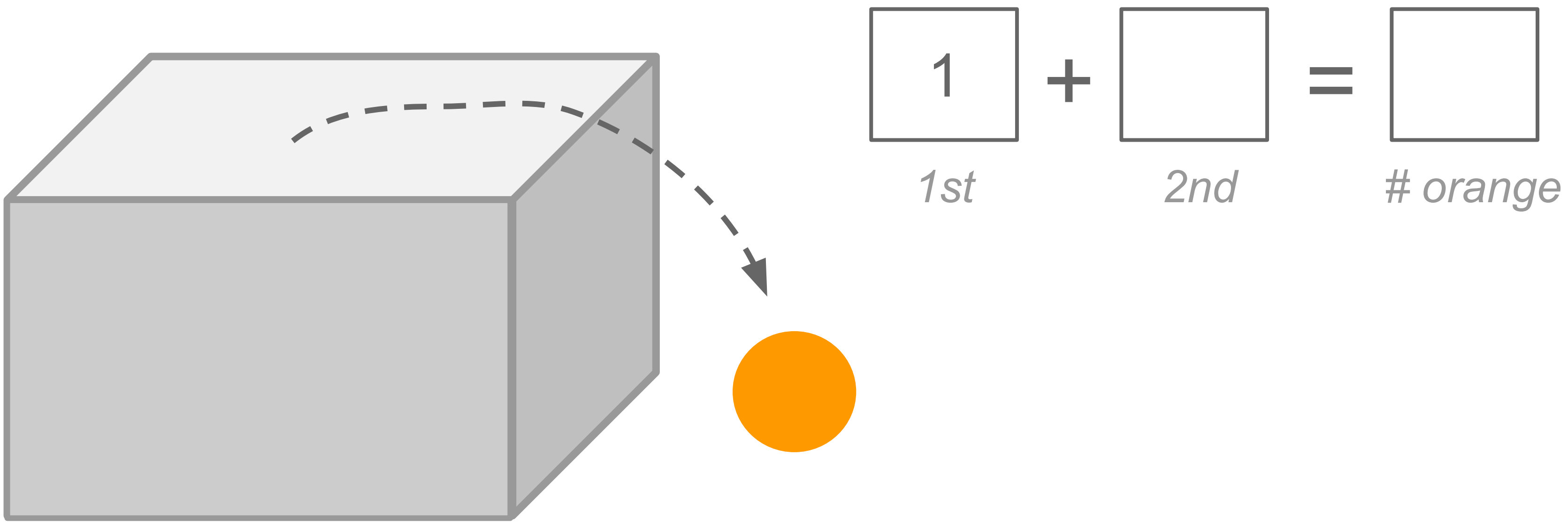
Experiment: withdraw a ball from the box, **replace it**, withdraw a 2nd ball, and counting # of orange balls





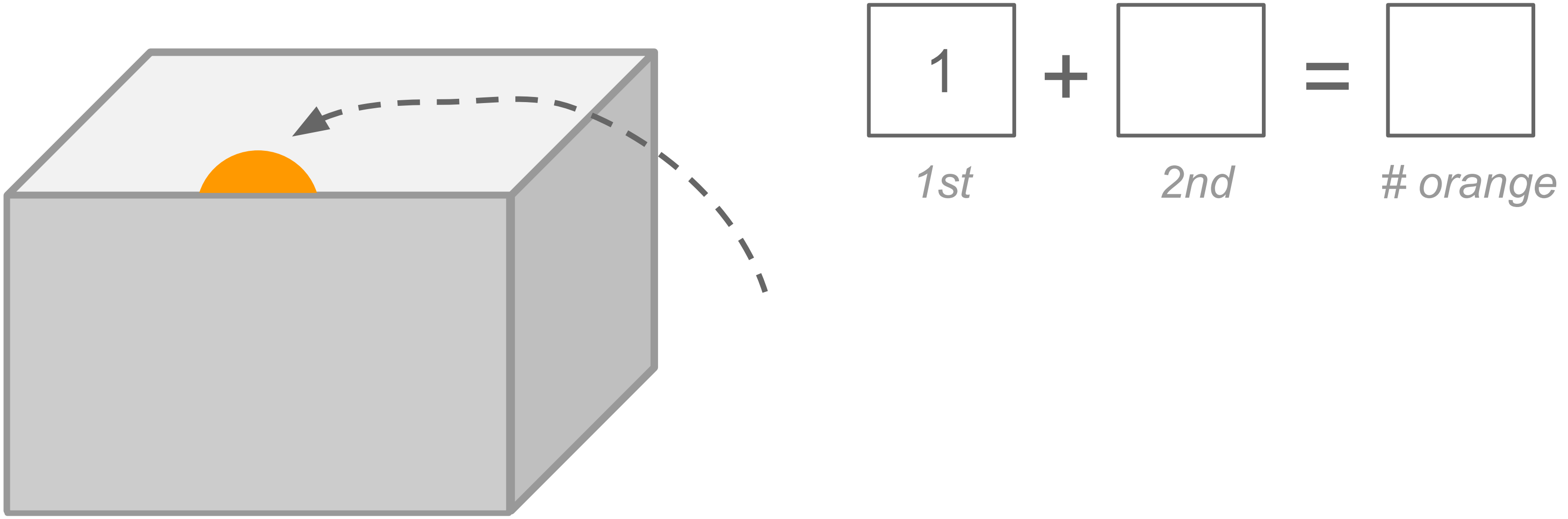
Binomial Experiment

Experiment: withdraw a ball from the box, **replace it**, withdraw a 2nd ball, and counting # of orange balls



Binomial Experiment

Experiment: withdraw a ball from the box, **replace it**, withdraw a 2nd ball, and counting # of orange balls



Binomial Experiment

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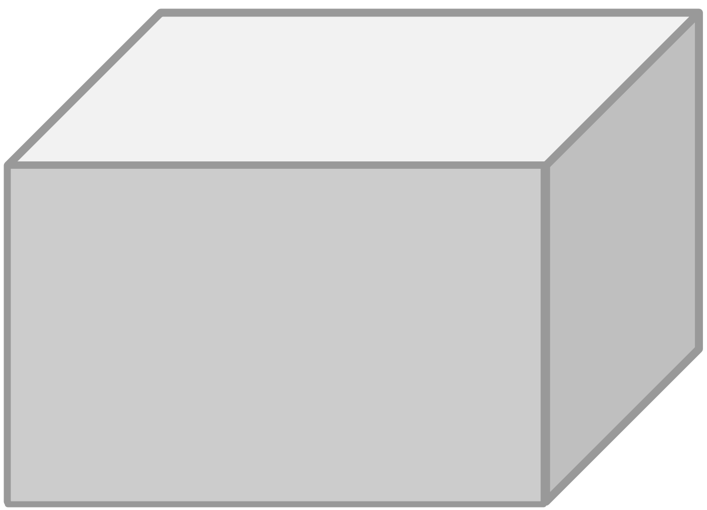

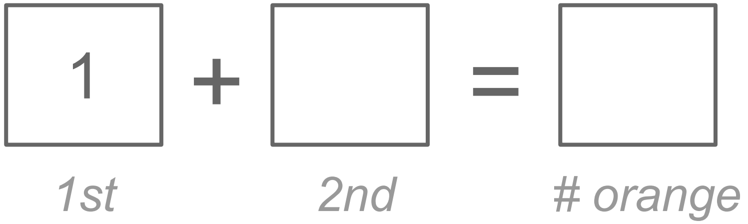

Binomial Experiment

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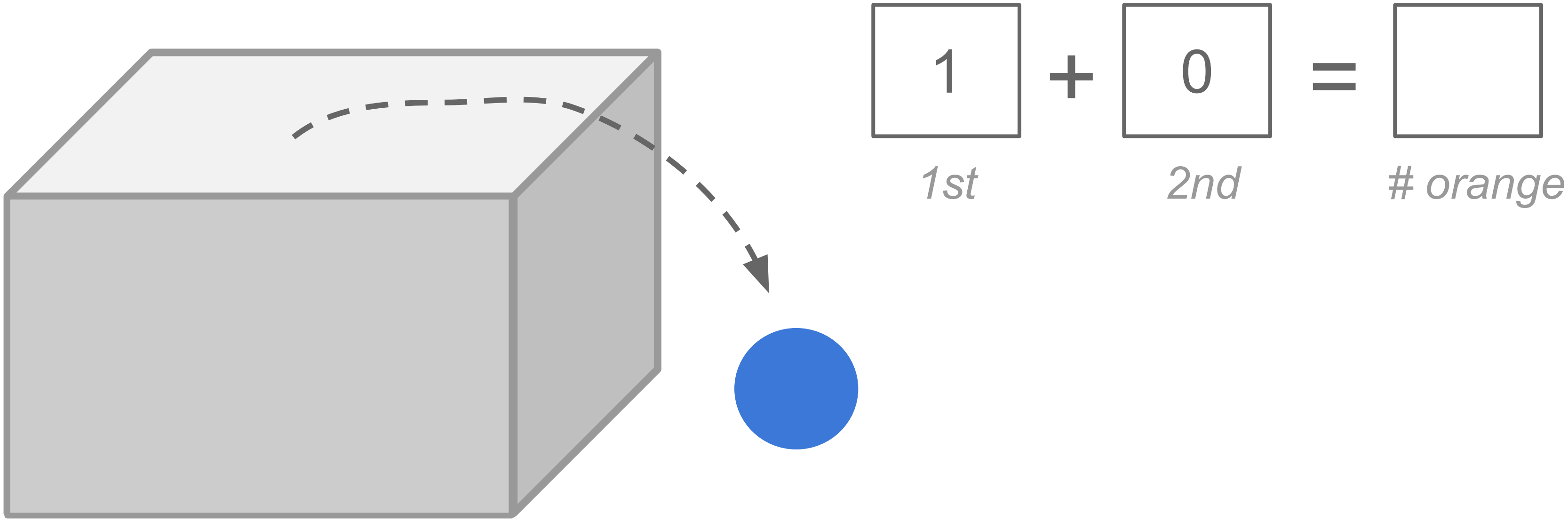

Binomial Experiment

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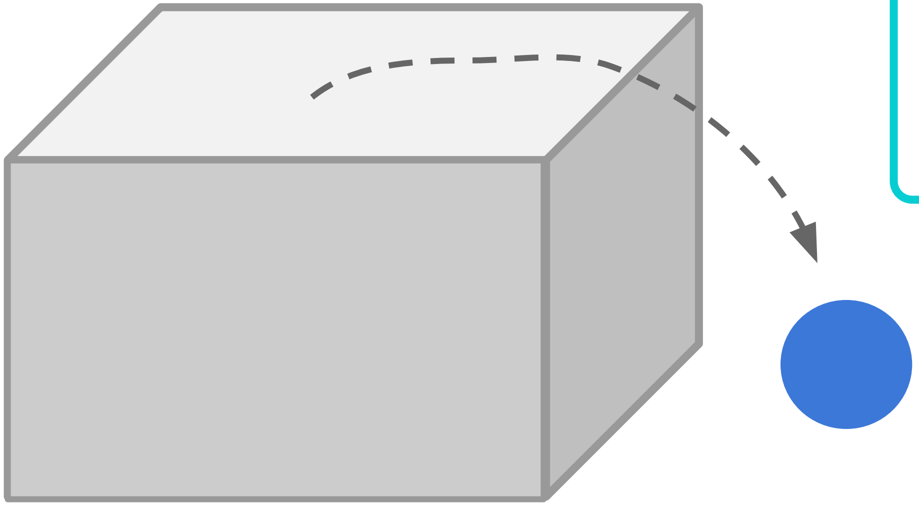
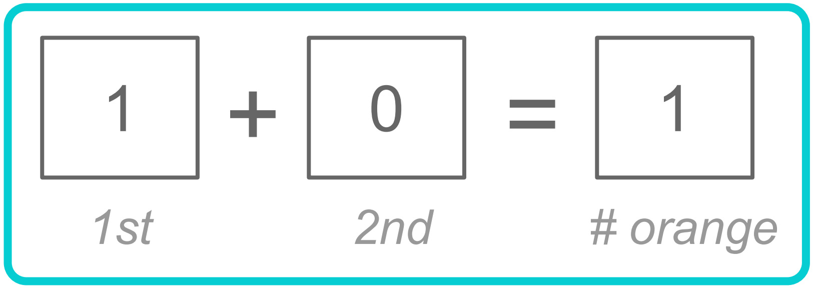




Binomial Probability

n independent trials

k successes

p probability of success

$$P(k \text{ successes}) = {}_{n}C_{k}\, p^{k} (1-p)^{n-k}$$

Binomial Probability

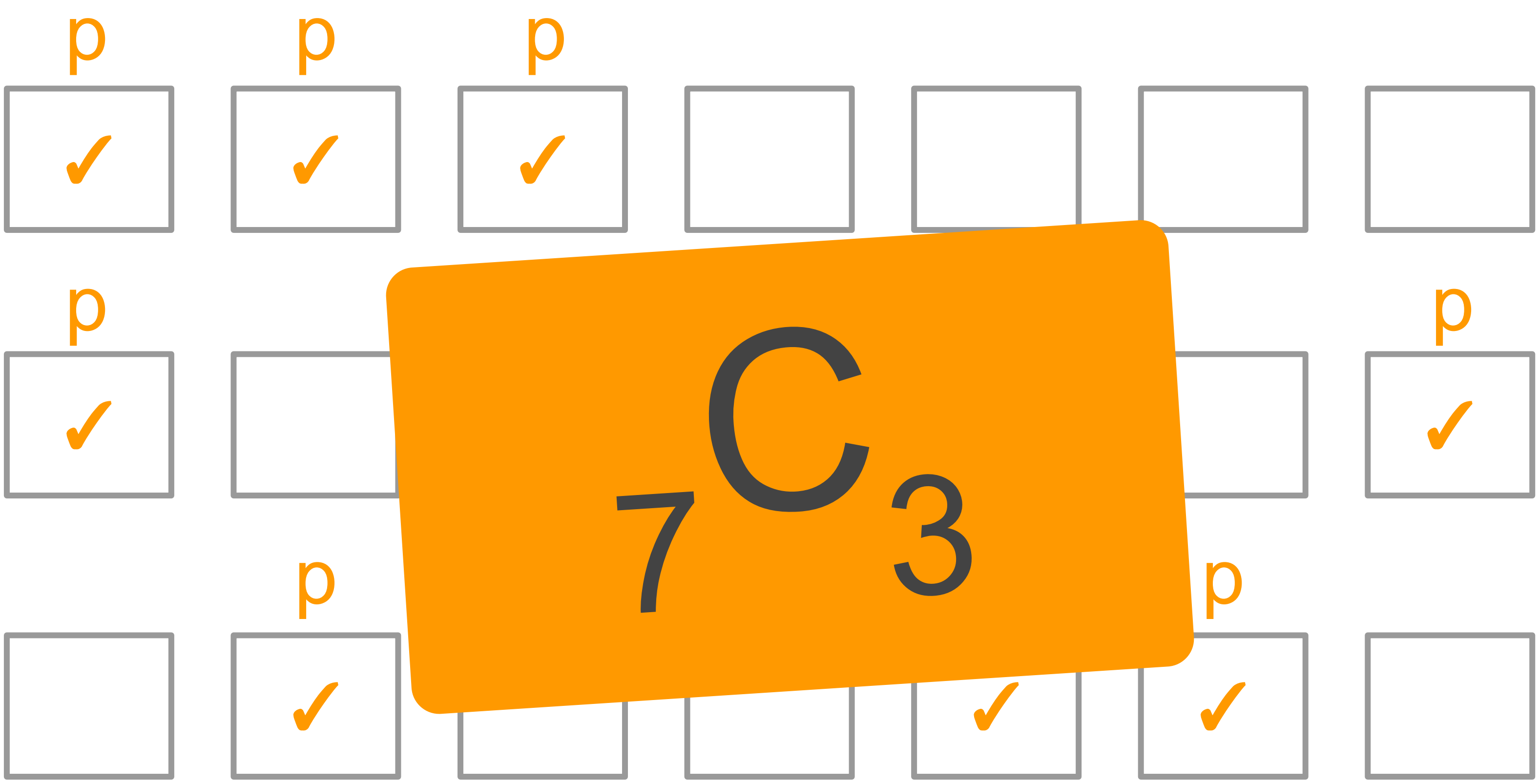
$$P(k \text{ successes}) = {}_nC_k \, p^k \, (1-p)^{n-k}$$

equivalently

$$P(k \text{ successes}) = \binom{n}{k} p^k \, (1-p)^{n-k}$$

k=3 successes in n=7 trials

k=3 successes in n=7 trials



How many different ways to get 3 successes in 7 trials?

Binomial Probability Formula

probability *k* success

$$P(k_{\text{successes}}) = \boxed{\binom{n}{k}}\ \boxed{p^k}\ \boxed{(1-p)^{n-k}}$$

k successes
in *n* trials

probability of
n-k failures

Gaston Sanchez

Example

3 coins flipped

Probabilities of number of heads ?

Binomial example

3 coins are flipped

X = number of heads

$$P(X = 0) = {}_3C_0 \, (1/2)^0 \, (1/2)^3$$

Binomial example

3 coins are flipped X = number of heads

$$P(X = 0) = {}_3C_0 \, (½)^0 \, (½)^3$$

$$P(X = 1) = {}_3C_1 \, (½)^1 \, (½)^2$$

$$P(X = 2) = {}_3C_2 \, (½)^2 \, (½)^1$$

Binomial example

3 coins are flipped

X = number of heads

$$P(X = 0) = {}_3C_0 \, (½)^0 \, (½)^3$$

$$P(X = 1) = {}_3C_1 \, (½)^1 \, (½)^2$$

$$P(X = 2) = {}_3C_2 \, (½)^2 \, (½)^1$$

$$P(X = 3) = {}_3C_3 \, (½)^3 \, (½)^0$$

Binomial example

3 coins are flipped X = number of heads

$$P(X = 0) = {}_3C_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^3 = 1/8$$

$$P(X = 1) = {}_3C_1 \left(\tfrac{1}{2}\right)^1 \left(\tfrac{1}{2}\right)^2 = 3/8$$

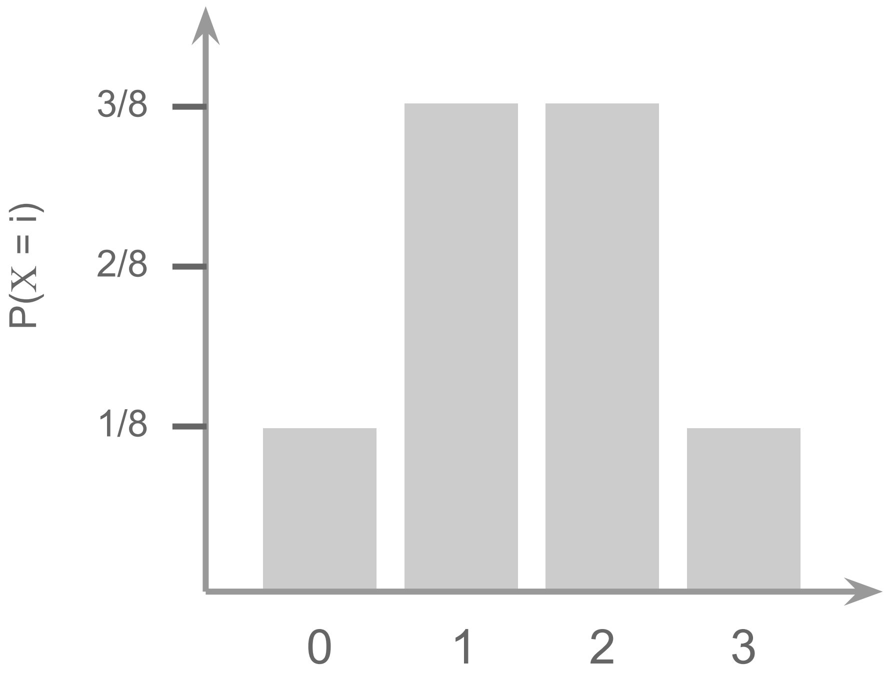
$$P(X = 2) = {}_3C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^1 = 3/8$$

$$P(X = 3) = {}_3C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^0 = 1/8$$

Binomial example

3 coins are flipped

X = number of heads



Some Expressions

Inequalities Expressions

Notation	Expression
$k = 4$	Four successes
$k \geq 4$ $(k = 4, 5, 6, \ldots, n)$	Four or more successes At least four successes No fewer than four successes
$k \leq 4$ $(k = 0, 1, 2, 3, 4)$	Four or fewer successes At most four successes No more than four successes
$k > 4$ $(k = 5, 6, \ldots, n)$	More than four successes
$k < 4$ $(k = 0, 1, 2, 3)$	Fewer than four successes


kipsta

Graphing binomial distributions

Jim makes about 50% of the field goals he attempts

Draw the distribution probability that Jim will make 0, 1, 2, 3, 4, 5, or 6 shots out of six attempts.

$n = ?$ $p = ?$ $k = ?$

Graphing binomial distributions

$n = 6$

$p = 0.5$

$k = 0, 1, 2, 3, 4, 5, 6$

$$P(X = k) = \binom{6}{k} 0.5^{k} (1-0.5)^{6-k}$$

Graphing binomial distributions

k	P(k)
0	
1	
2	
3	
4	
5	
6	

Graphing binomial distributions

k	P(k)
0	0.016
1	0.094
2	0.234
3	0.312
4	0.234
5	0.094
6	0.016

Graphing binomial distributions

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0	0.016
1	0.094
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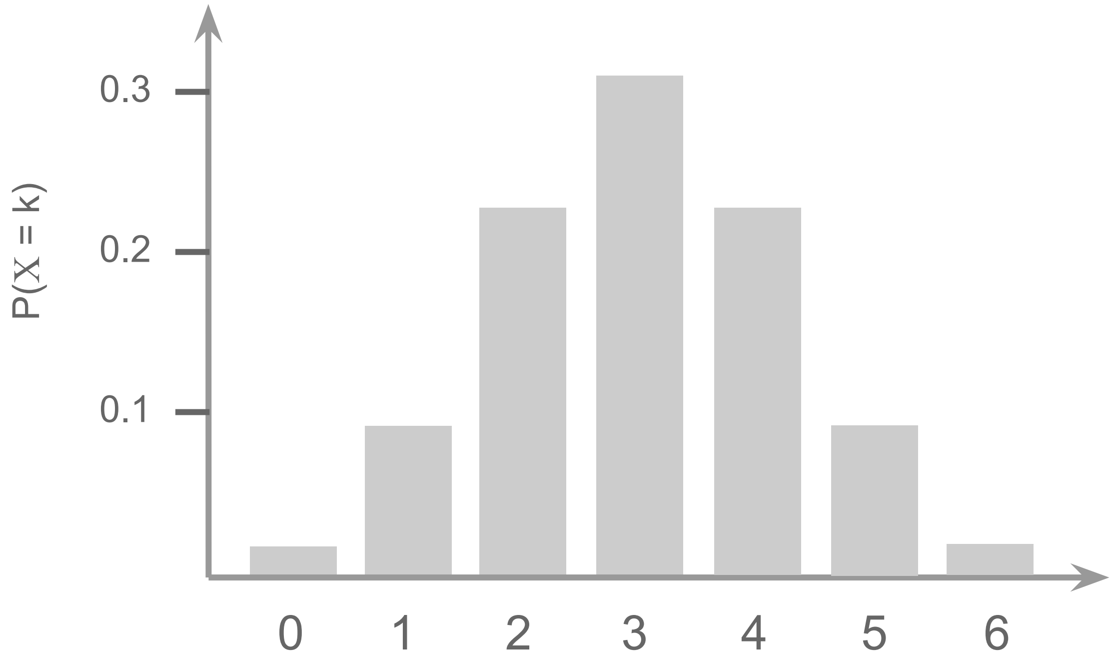


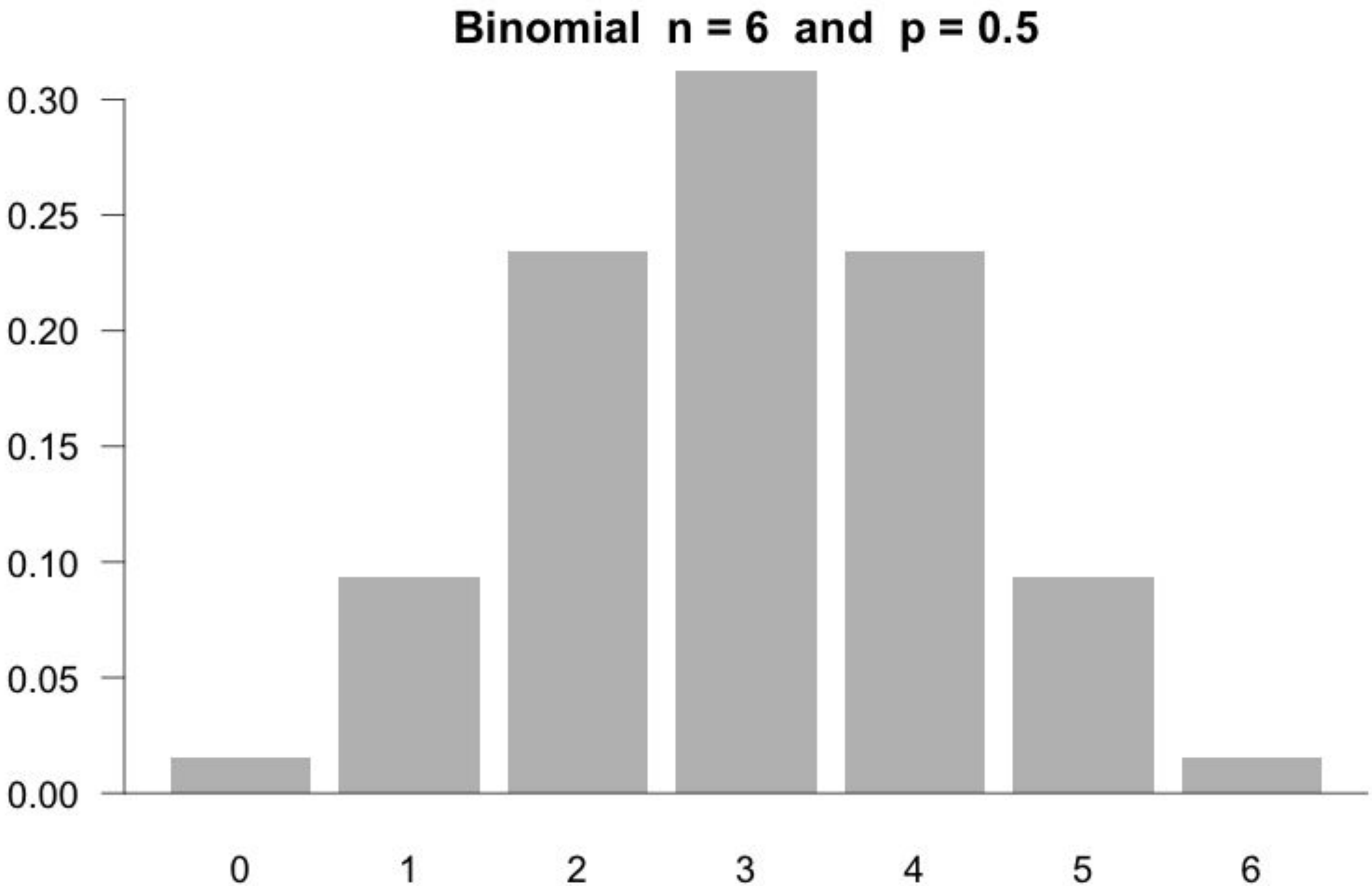
Graphics of Binomial Distribs.

Binomial example 1

X binomial random variable

$n = 6$

$p = 0.5$

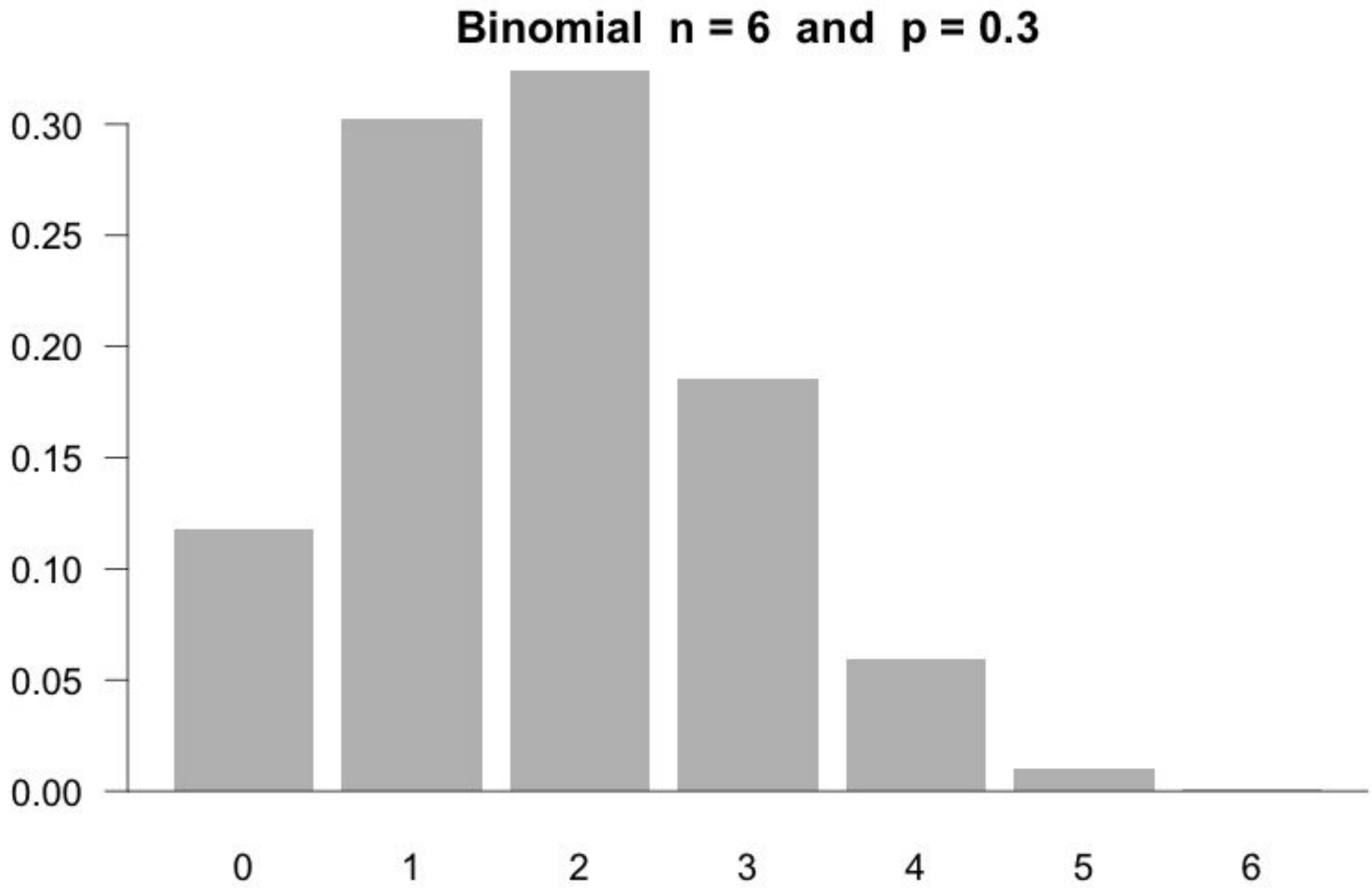

Binomial n = 6 and p = 0.5
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
6

Binomial example 2

X binomial random variable

$n = 6$

$p = 0.3$

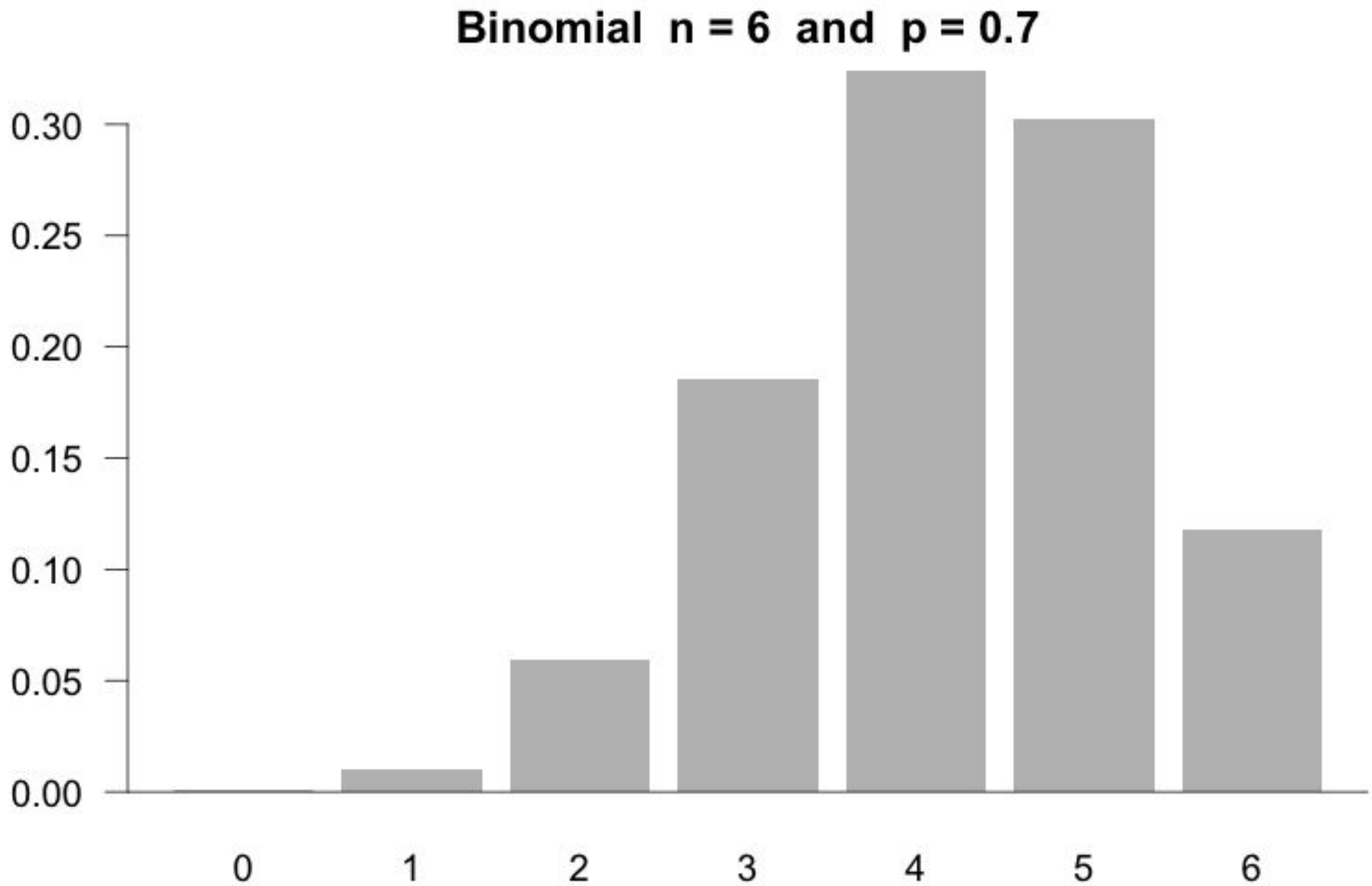

Binomial n = 6 and p = 0.3
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
6

Binomial example 3

X binomial random variable

$n = 6$

$p = 0.7$



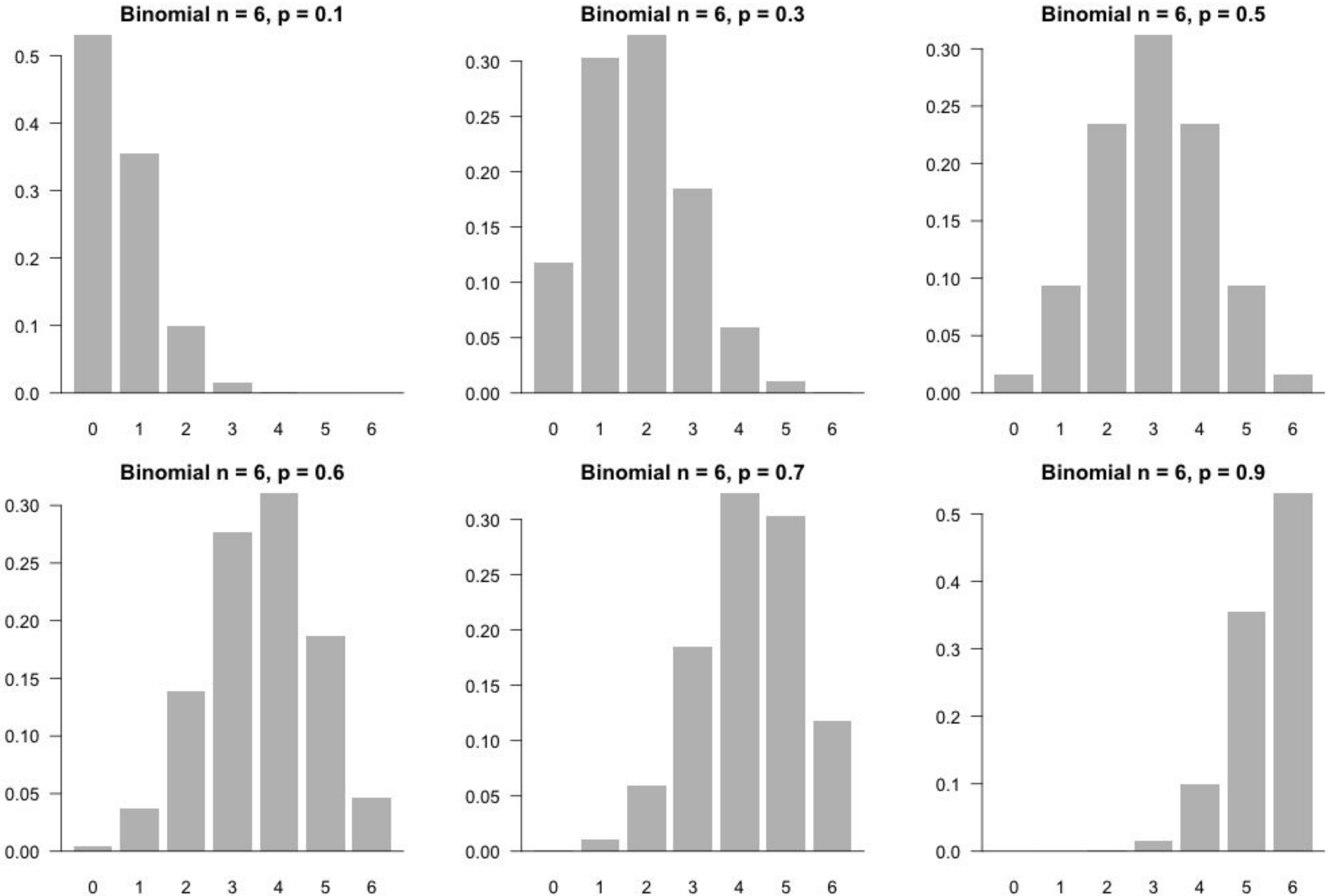
Binomial n = 6 and p = 0.7
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
6

Binomial example 4

X binomial random variable

$n = 6$

$p = 0.1, 0.3, 0.5, 0.6, 0.7, 0.9$



Binomial n = 6, p = 0.1
Binomial n = 6, p = 0.3
Binomial n = 6, p = 0.5
Binomial n = 6, p = 0.6
Binomial n = 6, p = 0.7
Binomial n = 6, p = 0.9
0 1 2 3 4 5 6