EE602: ALGORITHMS

HOMEWORK #2 **Due: 9/19**

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(1) Exercises 3.1-2

$(n + a)^b = \binom{n}{0} n^b \binom{n}{1} n^{b-1}$ b + ⋯ + $\binom{n}{0} a^b$ The most significant term is n^b, so it is $O(n^b)$. Also, it is obviously greater than n^b, so it is $\Omega(n^b)$. Therefore, it is $\Theta(n^b)$.

(2) Exercises 3.1-4

a. $2^{n+1} = 2 * 2^n$ we choose c = 2 in the definition of Big O notation, we get it is $O(2^n)$.

b. $2^{2n} = 2^n*2^n$. There is no constant c such that for n > c, $2^n*2^n < c*2^n$.

(3) Exercises 4.3.-1

We have to prove there is a constant c such that T(n) <= $c*n^2$.

T(n) = T(n-1) + n <= $c(n-1)^2 = cn^2$-2cn+c+n, if I pick c = 1; then T(n) <= n^2-n+1 <=n^2 for n >=1. So, T(n) is $O(n^2)$.

(4) Exercises 4.4-9

We can assume that $\alpha \leq \frac{1}{2}$, Otherwise 1-α would <= 1/2. Thus, it is easy to see that the depth of recursion tree is logn, and each level is cn. We can guess the leave is $\theta(n)$.

So, T(n) = cn * logn + $\theta(n) = \theta(nlogn)$.

(5) Give asymptotic upper and lower bounds for T(n). Assume T(n) is constant for n <=2.

a. T(n) = 2T(n/2) + n^3

a = 2, b = 2. $n^{\log_b a}$ = n. So, f(n) = $\Omega(n^3)$, $2(n/2)^3 <= 2 n^3$, we can apply Master theory 3, So, T(n) = $\theta(n^3)$.

b. T(n) = T(9n/10) + n

$a = 1$, $b = 10/9$. $n^{\log_b a} = 1$, $f(n) = n$, so $f(n)$ is polynomially larger than $n^{\log_b a}$. $T(n) = \theta(n)$.

c. $T(n) = 16T(n/4) + n^2$

$A = 16$, $b= 4$, $n^{\log_b a} = n^2$, $f(n) = n^2 = \theta(n^2)$. Apply Master theory 2, $T(n) = \theta(n^2 lgn)$.

d. $T(n) = 7T(n/3) + n^2$

$A = 7$, $b = 3$, $n^{\log_b a}=n^{1.5}$ $f(n) = n^2$, polynomially larger than $n^{1.5}$, $T(n) = \theta(n^2)$

e. $T(n) = 7T(n/2) + n^2$

$a = 7$, $b = 2$, $n^{\log_b a}=n^{2.6}$ $f(n) = n^2$ polynomially smaller than $n^{\log_b a}$, $T(n) = \theta(n^{log 2^7})$

f. $T(n) = 2T(n/4) + n^{1/2}$

$A = 2$, $b = 4$, $n^{\log_b a}=n^{0.5}$ $f(n) = n^{0.5}= \theta(n^{0.5})$, so, Apply master theory 2, $T(n) = \theta(n^{0.5} lgn)$

g. $T(n) = T(n-1) + n$

$T(n) = T(n-2)+n-1+n = T(n-3) + n-2 + n-1 + n = T(0) + 1 + 2 +\ldots+n = (1+n)n/2$, This is $\theta(n^2)$

h. $T(n) = T(n^{1/2}) + 1$

$T(n) = T(n^{1/4}) + 1 + 1 = T(n^{1/m}) + logm$ so, $T(n) = \theta\ (logn)$