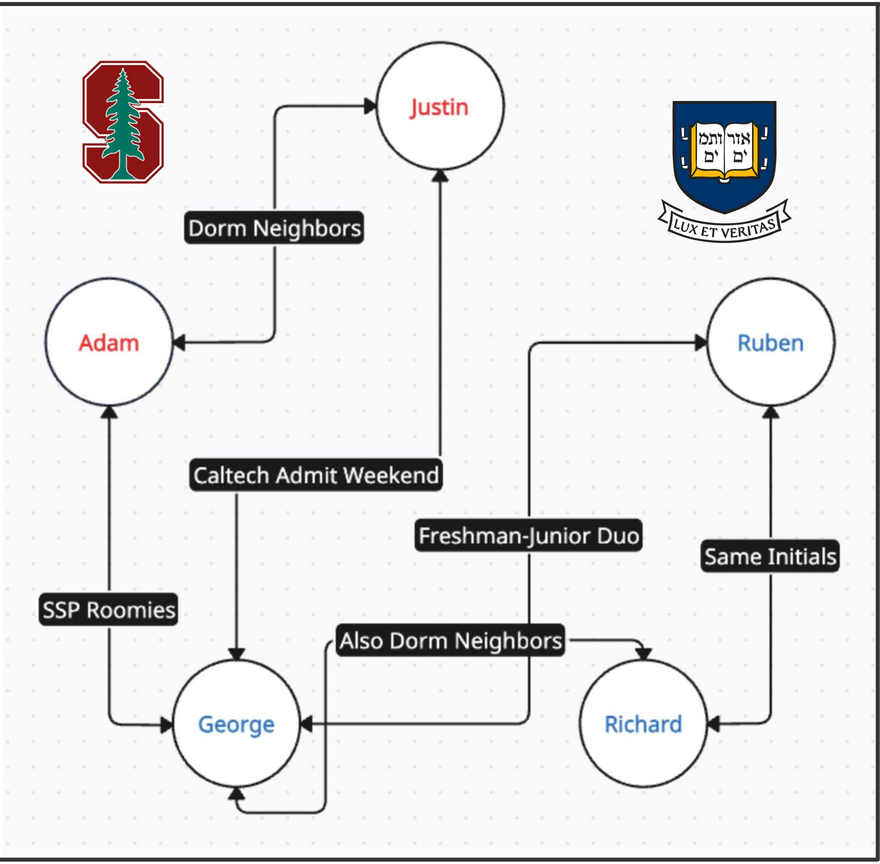
67 Qubits

⟨ *iQuHACK 2026* | ***Superquantum*** ⟩

Who are we?



Justin
Dorm Neighbors
Adam
Ruben
Caltech Admit Weekend
Freshman-Junior Duo
Same Initials
SSP Roomies
Also Dorm Neighbors
George
Richard
LUX ET VERITAS

Overview

Quantum circuits are highly expressive in theory, but fragile and costly in practice.

In particular, the T gate <u>enables universal expression</u> of quantum operations but introduce fragile π/4 phase shifts, which <u>make circuits error-sensitive</u>.

We studied this tradeoff over the past 24 hours at iQuHACK.

Goal: Implement or approximate 12 target unitary operators.

Constraint: Balance <u>accuracy</u> (operator norm distance) against <u>cost</u> (T-count).

$$H \quad \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$T \quad \begin{bmatrix}1 & 0\\ 0 & e^{i\cdot\frac{\pi}{4}}\end{bmatrix}$$

$$T^t \quad \begin{bmatrix}1 & 0\\ 0 & e^{-i\cdot\frac{\pi}{4}}\end{bmatrix}$$

$$S \quad \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

$$S^t \quad \begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$$

Types of Problems

Finding an Exact Solution

8. Structured Unitary 1.

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

Finding an Approximate Solution

2. Controlled-$R_y(\pi/7)$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sin(\pi/14) & -\cos(\pi/14) \\ 0 & 0 & \cos(\pi/14) & \sin(\pi/14) \end{bmatrix}$$

In this case, one can find a "clever" construction that exactly realizes the gate!

8. Structured Unitary 1.

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

2D QFT

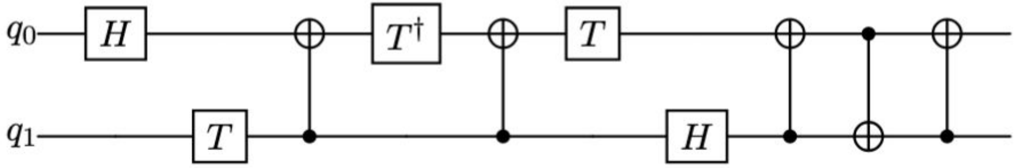


FIGURE 3. Quantum Circuit for Challenge 8

Unfortunately, this is not possible for higher dimensional quantum Fourier transforms.

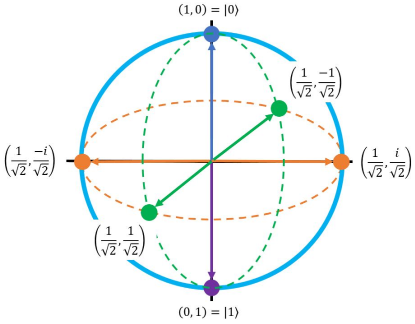
In this case, there is (provably) no exact solution. One must approximate it!

2. Controlled-$R_y(\pi/7)$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sin(\pi/14) & -\cos(\pi/14) \\ 0 & 0 & \cos(\pi/14) & \sin(\pi/14) \end{bmatrix}$$

More T-gates → better approximation

# of T-gates	Operator Norm Distance
104	$1.44 * 10^{-5}$
380	$5.5 * 10^{-14}$



[RS16] N. J. Ross and P. Selinger, Optimal ancilla-free Clifford+T approximation of z-rotations. *Quantum Inf. Comput.* **16** (2016), no. 11&12, 901–953.

Exact Example

11. 4-Qubit Diagonal Unitary. Consider U acting on 4 qubits such that $U|x\rangle = e^{i\phi(x)}|x\rangle$ and $x \in \{0,1\}^4$. The phase $\varphi(x)$ is the following:

$$
\begin{array}{llll}
\varphi(0000) = 0, & \varphi(0001) = \pi, & \varphi(0010) = \frac{5}{4}\pi, & \varphi(0011) = \frac{7}{4}\pi, \\
\varphi(0100) = \frac{5}{4}\pi, & \varphi(0101) = \frac{7}{4}\pi, & \varphi(0110) = \frac{3}{2}\pi, & \varphi(0111) = \frac{3}{2}\pi \\
\varphi(1000) = \frac{5}{4}\pi, & \varphi(1001) = \frac{7}{4}\pi, & \varphi(1010) = \frac{3}{2}\pi, & \varphi(1011) = \frac{3}{2}\pi, \\
\varphi(1100) = \frac{3}{2}\pi, & \varphi(1101) = \frac{3}{2}\pi, & \varphi(1110) = \frac{7}{4}\pi, & \varphi(1111) = \frac{5}{4}\pi,
\end{array}
$$

Compile this circuit with as few T and CNOT gates as possible.

Following the framework of Amy and Mosca (and the workshop!), we:

1. Find a polynomial P(x) representing the phase.
2. Use P(x) to construct a quantum circuit.

1.

$$\varphi(x_3 x_2 x_1 x_0) = \frac{\pi}{4} P(x_0, x_1, x_2, x_3)$$

$$P(x_0, x_1, x_2, x_3) = \sum_{S \subseteq \{0,1,2,3\}} a_S \bigoplus_{i \in S} x_i \in \mathbb{Z}_8[x_0, x_1, x_2, x_3]$$

There are $8^{2^4-1} \approx 3.5 \cdot 10^{13}$ candidate polynomials. How do we find the right one?

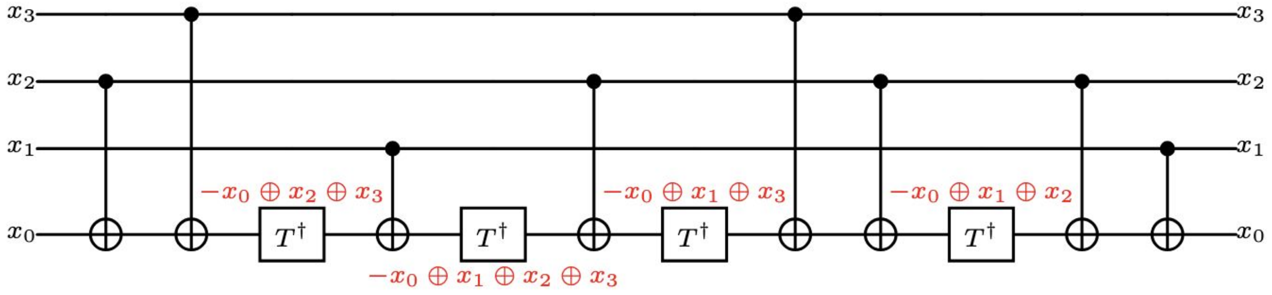
M. Amy, M. Mosca, T-Count Optimization and Reed–Muller Codes. *IEEE Trans. Inform. Theory* **65** (2019), 4771–4784.

2.

$$P(x_3, x_2, x_1, x_0) = 7 \cdot x_0 \oplus x_1 \oplus x_2 + 7 \cdot x_0 \oplus x_1 \oplus x_3 + 7 \cdot x_0 \oplus x_2 \oplus x_3 + 7 \cdot x_0 \oplus x_1 \oplus x_2 \oplus x_3$$

4 odd coefficients → **T-Count = 4**

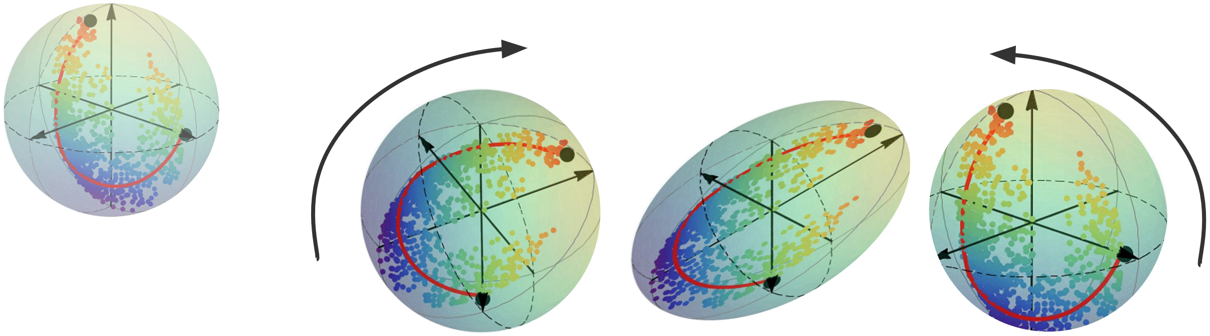
And this is the limit!



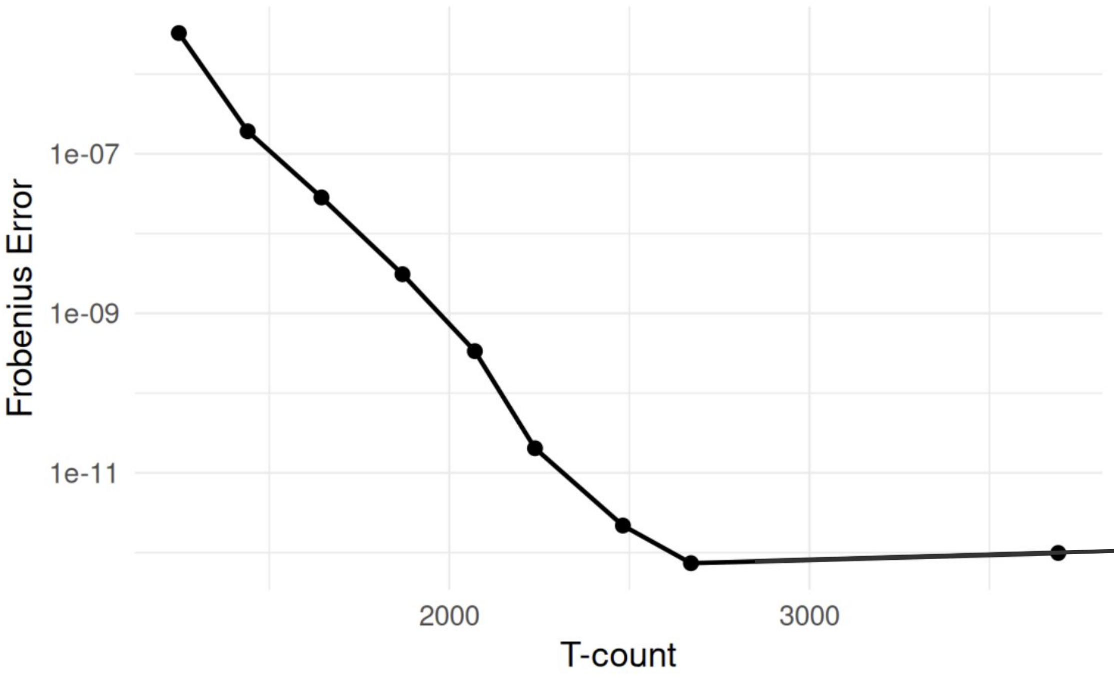
Approximation Example

10. Random Unitary.

$$\begin{bmatrix} 0.1448081895 + 0.1752383997i & -0.5189281551 - 0.5242425896i & -0.1495585824 + 0.312754999i & 0.1691348143 - 0.5053863118i \\ -0.9271743926 - 0.0878506193i & -0.1126033063 - 0.1818584963i & 0.1225587186 + 0.0964028611i & -0.2449850904 - 0.0504584131i \\ -0.0079842758 - 0.2035507051i & -0.3893205530 - 0.0518092515i & 0.2605170566 + 0.3286402481i & 0.4451730754 + 0.6558933250i \\ 0.0313792249 + 0.1961395216i & 0.4980474972 + 0.0884604926i & 0.3407886532 + 0.7506609982i & 0.0146480652 - 0.1575584270i \end{bmatrix}$$



$$(K_1 \otimes K_2)\ A\ (K_3 \otimes K_4) = U$$


Frobenius Error
1e-07
1e-09
1e-11
2000
3000
T-count

67 Qubits

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1. Controlled-Y Gate.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix}$$

2. Controlled-$R_y(\pi/7)$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sin(\pi/14) & -\cos(\pi/14) \\ 0 & 0 & \cos(\pi/14) & \sin(\pi/14) \end{bmatrix}$$

3. Exponential of a Pauli String.

$$e^{i\frac{\pi}{7}Z\otimes Z}$$

4. Exponential of a Hamiltonian.

$$e^{i\frac{\pi}{7}H_1},$$

where $H_1 = X \otimes X + Y \otimes Y$.

5. Exponential of a Hamiltonian.

$$e^{i\frac{\pi}{4}H_2},$$

where $H_2 = X \otimes X + Y \otimes Y + Z \otimes Z$.

6. Transverse Field Ising Model. Let $H_3 = XX + ZI + IZ$:

$$e^{i\frac{\pi}{7}H_3}$$

Note. This is a time evolution under a 2-qubit transverse field Ising model.

7. State Preparation. Design $U \in \mathbb{C}^{4\times 4}$ such that

$$\begin{aligned} |00\rangle \mapsto\ & (0.1061479384 - 0.679641467i)\,|00\rangle \\ & + (-0.3622775887 - 0.453613136i)\,|01\rangle \\ & + (0.2614190429 + 0.0445330969i)\,|10\rangle \\ & + (0.3276449279 - 0.1101628411i)\,|11\rangle\,. \end{aligned}$$

8. Structured Unitary 1.

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9. Structured Unitary 2.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}+\frac{i}{2} & \frac{1}{2}+\frac{i}{2} \\ 0 & i & 0 & 0 \\ 0 & 0 & \frac{-1}{2}+\frac{i}{2} & -\frac{1}{2}-\frac{i}{2} \end{bmatrix}$$

10. Random Unitary.

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\varphi(1100) = \frac{3}{2}\pi, & \varphi(1101) = \frac{3}{2}\pi, & \varphi(1110) = \frac{7}{4}\pi, & \varphi(1111) = \frac{5}{4}\pi,
\end{array}$$

Compile this circuit with as few T and CNOT gates as possible.

12. Bonus - Commuting Pauli Phase Diagram. In quantum simulation and fault-tolerant compilation, one often needs to implement products of small-angle exponentials of Pauli strings. In this task, you are given a dense list of commuting Pauli strings with fixed ($\frac{\pi}{8}$)-quantized angles, and you must compile the resulting unitary into Clifford+T with as few T gates as possible.

$$U := \prod_{j=1}^{m} \exp\left(-i\frac{\pi}{8}k_j P_j\right)$$