Introduction to Statistical Signal Processing (EC5.206) – Project

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Problem 1

You are given two ECG signals corrupted with noise (Signal1 & Signal2). A secondary noise reference signal is also available in each case (Noise1 & Noise2). You can assume that the original ECG signals (which are not directly available) are noise-free and stationary.

a) Apply the following adaptive filters to perform noise removal – SDAF, LMS, RLS. Set the filter order to be p = 4. Compute statistics from the data where required.

b) Comment on the relative performance of the filters in terms of noise removal and convergence.

c) If one set (Signal & Noise) out of the two set of signals is non-stationary, identify which one based on the filter performance.

Problem 2

Siva is driving a car in the x-y plane with initial position set to [1000, 0] meters. He is following a peculiar trajectory in which he drives the car with constant radial velocity (v_r) and constant angular velocity (v_θ).

a) Plot the trajectory in which Siva drives his car. Assume $v_r = 10\ m/s$ and $v_\theta = 3\ deg/s$. Total duration for which he drives is $T = 200\ seconds$. You can plot the position at every second to get the full trajectory.

Madhuri, who is located at the origin, is tracking Siva's car using sensors at her disposal. Specifically, she has a range sensor and an angle sensor. Both sensors give erroneous estimates with the errors having zero-mean and are uncorrelated across time and sensors. Range sensor has a variance of 2500 and the angle sensor has a variance of 16. Madhuri wants to track the following parameters – range, angle, radial velocity and angular velocity. Assume that measurements are available at every second in the interval $[0, T]$ seconds.

b) Write down the state equation and the measurement equation that should be used by Madhuri. Clearly identify the state transition matrix and the measurement matrix. Madhuri is aware that Siva is driving with constant radial and angular velocities (but exact velocities are unknown).

c) Write down the Kalman Filter equations that Madhuri should use for tracking.

d) The files range1 and angle1 contain measurements recorded by the two sensors. Apply the Kalman Filter and sequentially estimate the state over time. Assume initial state to be $[500, 0, 0, 0]$ and the initial state covariance matrix to be $diag([1000,\ 10,\ 50,\ 10])$. Plot the trajectory estimated by Madhuri. Also, plot the estimated velocities as a function of time. Investigate effect of changing the initial state covariance matrix on the results.

e) To improve her estimation accuracy, Madhuri has purchased another set of range and angle sensors with same error characteristics but independent of existing sensors. How does the measurement equation change if both set of sensor measurements are simultaneously used by Madhuri?

f) In addition to the files range1 and angle1, use the files range2 and angle2 which correspond to the new set of sensors and perform sequential state estimation using the Kalman Filter. Plot the estimated trajectory and the velocities. Empirically justify if the additional set of sensors have resulted in improved tracking accuracy.