Signal Processing Lab Report -8

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Contents

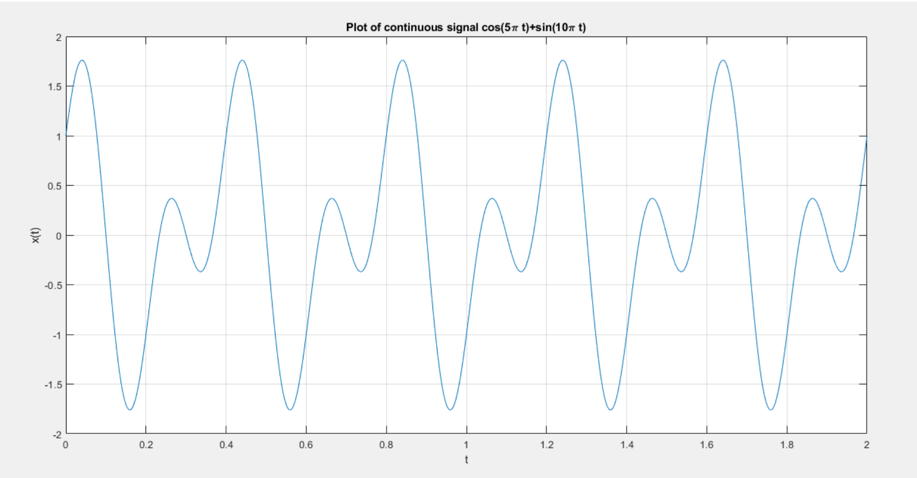
1 A signal and its samples

Given continuous-time signal

$$x(t) = cos(5\pi t) + sin(10\pi t)$$

1.1 Continuous Time signal

Plot of the continuous-time Signal :



Properties :

- The first term, $\cos(5\pi t)$, represents a cosine wave with an angular frequency of 5π radians per second. The corresponding frequency is $\frac{5\pi}{2\pi} = 2.5$ Hz.
- The second term, $\sin(10\pi t)$, represents a sine wave with an angular frequency of 10π radians per second, corresponding to a frequency of $\frac{10\pi}{2\pi} = 5$ Hz.
- This continuous signal is a combination of two sinusoidal components with different frequencies.
- The overall signal $x(t)$ is periodic, with a period of 0.4 sec.

1.2 Sampled Signal

Sampling of the continuous-time Signal :

- Sampling involves taking measurements of $x(t)$ at discrete time intervals, called the *sampling period* T_s. These intervals are typically evenly spaced, so the sample times are $t = nT_s$, where n is an integer.

- The resulting sequence is a discrete-time signal:

$$x[n] = x(nT_s)$$

where $x[n]$ represents the sampled value of $x(t)$ at time nT_s.

- The ***sampling frequency*** or *sampling rate* f_s is the rate at which samples are taken, defined by:

$$f_s = \frac{1}{T_s}$$

and is measured in samples per second (Hz).

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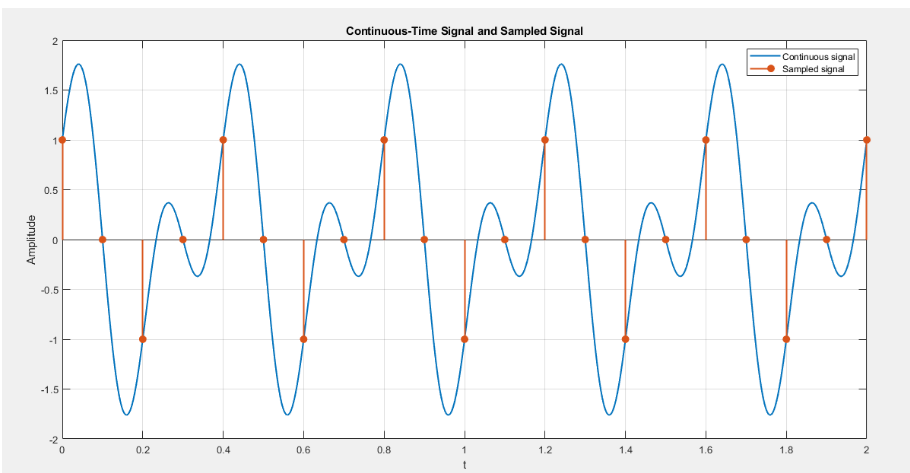
Sampling is essential for converting analog signals, such as audio or video, into digital formats for processing, transmission, and storage in electronic devices. It is widely used in telecommunications, audio processing, and data acquisition systems.
Plot of the Sampled Signal :



The Sampled Signal is periodic with Period 0.4 sec.

2 Reconstruction methods

2.1 Zero-order Hold Reconstruction

- Zero-Order Hold reconstruction is a method used to reconstruct a continuous-time signal from its discrete-time samples by holding each sample value constant until the next sample is available.

- Given a discrete-time signal $x[n]$, sampled at a regular interval T_s, reconstruction produces a continuous-time signal $x_{\mathrm{r}}(t)$ that holds each sample value $x[n]$ constant over each interval $t \in [nT_s, (n+1)T_s)$.

- The Zero-order Hold reconstructed signal can be mathematically expressed as:

$$x_{\mathrm{r}}(t) = x[n], \quad \text{for } t \in [nT_s, (n+1)T_s)$$

  where n is an integer representing the sample index.

- The resulting signal $x_{\mathrm{r}}(t)$ is piecewise constant, with each sample value held until the next sample point. This approximation makes Zero-order Hold suitable when the signal does not vary significantly between samples.

2.2 Linear Interpolation

- Linear interpolation is a method used to reconstruct a continuous-time signal from discrete samples by connecting each sample point with a straight line. Unlike Zero-Order Hold , which holds the sample value constant until the next sample, **Linear interpolation** provides a smooth, linearly varying approximation between sample points.

- Suppose we have a discrete-time signal $x[n]$, sampled at regular intervals T_s. Linear interpolation reconstructs a continuous-time signal $x_{\mathrm{lin}}(t)$ by connecting each pair of adjacent samples $x[n]$ and $x[n+1]$ with a straight line.

- The linearly interpolated signal between $t = nT_s$ and $t = (n+1)T_s$ is given by:

$$x_{\mathrm{lin}}(t) = x[n] + \frac{x[n+1] - x[n]}{T_s}(t - nT_s), \quad \text{for } t \in [nT_s, (n+1)T_s)$$

  This expression represents the equation of a line passing through the points $(nT_s, x[n])$ and $((n+1)T_s, x[n+1])$.

2.3 Sinc interpolation

- Sinc interpolation is a method used to perfectly reconstruct a continuous-time signal from its discrete-time samples, assuming the signal is band-limited and sampled at or above the Nyquist rate. This interpolation method uses the sinc function as a reconstruction filter, which is the ideal interpolation function for band-limited signals due to its frequency-domain properties.

- Given a discrete-time signal $x[n]$ sampled from a continuous-time band-limited signal $x(t)$ at a sampling interval T_s, Sinc interpolation reconstructs the original signal $x_{\mathrm{sinc}}(t)$ as a weighted sum of sinc functions centered at each sample point.

- The reconstructed signal is defined by:

$$x_{\text{sinc}}(t) = \sum_{n=-\infty}^{\infty} x[n]\,\text{sinc}\left(\frac{t - nT_s}{T_s}\right)$$

  where the sinc function is given by:

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

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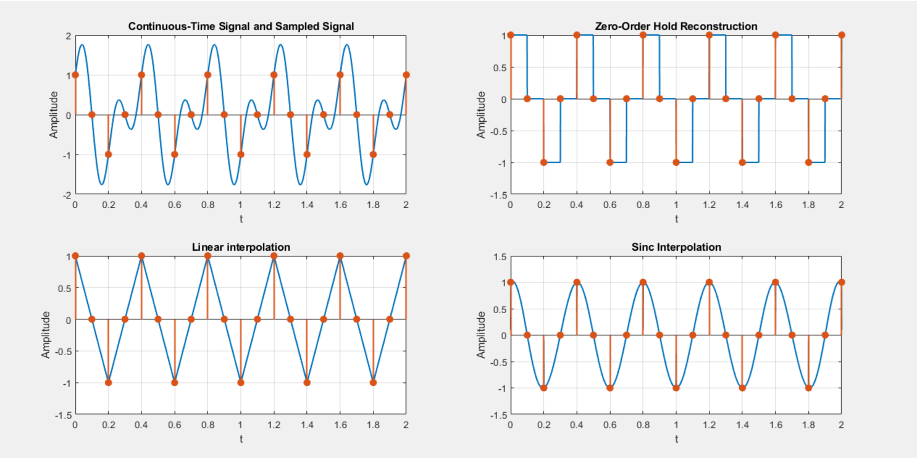
2.4 Observation

Plot of the Reconstructed Signal :



It is observed Sinc Interpolation has good reconstruction properties.

3 Sampling non-band-limited signal

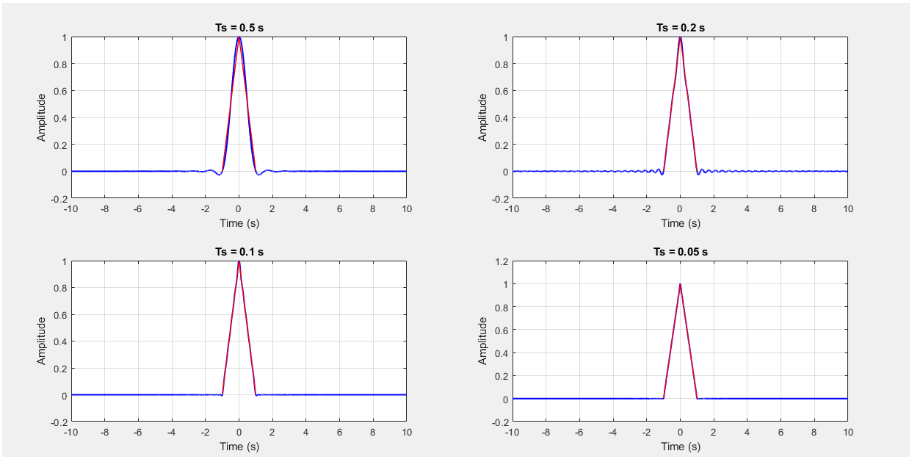


Figure 3.1: Obtained plots

Observations as Sampling Interval is Changed

As the sampling interval T_s decreases (from 0.5s to 0.05s), here are the observations:

1. **Resolution and Reconstruction Quality**: With a larger T_s (like 0.5s), the samples are sparse, and the reconstructed signal shows significant distortions due to insufficient data points. As T_s decreases, more samples capture the details of the original signal, resulting in a smoother and more accurate reconstruction.

2. **Aliasing Effects**: Since the signal is not band-limited, aliasing is present in all cases. However, with finer sampling intervals (smaller T_s), aliasing effects become less pronounced, leading to a reconstruction that more closely resembles the original triangular pulse shape.

3. **Interpolation Smoothness**: Sinc interpolation at smaller T_s values will yield a smoother reconstructed signal, as the interpolation function has more sample points to work with. This reduces the "ringing" effect typically seen in reconstructions with larger sampling intervals.

4. **Approximation to the Original Signal**: When T_s is very small (like 0.05s), the reconstructed signal will closely approximate the original triangular pulse. However, as T_s increases, the reconstruction becomes less faithful to the original shape, with potential overshooting or undershooting.

```
>> q1
Maximum Absolute Error (MAE) in the interval [0.25, 1.75]:
Zero-Order Hold Reconstruction: 1.7601
Linear Interpolation: 1.2076
Sinc Interpolation: 1.7793
```

Figure 3.2: Obtained MAE

4 Aliasing

4.1 Nyquist Theorem and Aliasing :

- The *Nyquist-Shannon sampling theorem*states that to accurately capture a signal without losing information, the sampling frequency f_s must be at least twice the highest frequency $f_{\max}$ present in $x(t)$:

$$f_s \geq 2f_{\max}$$

- If f_s is lower than $2f_{\max}$, *aliasing* occurs. Aliasing causes higher frequency components of $x(t)$ to be indistinguishably "folded" into lower frequencies in the sampled signal, leading to distortion.
- If the Nyquist criterion is met, the original continuous signal $x(t)$ can theoretically be reconstructed from $x[n]$ using an ideal low-pass filter or sinc interpolation.
- This reconstruction assumes $x(t)$ is *band-limited*, meaning it contains no frequency components higher than $f_{\max}$.

4.2 Calculation

The given signal

$$x(t) = cos(5\pi t)$$

It is know that angular frequency of above signal is $\omega = 5\pi$.
From the relation :

$$\omega = 2\pi f$$

Therefore the corresponding frequency f in Hz is:

$$f = \frac{\omega}{2\pi} = \frac{5\pi}{2\pi} = 2.5\,\text{Hz}$$

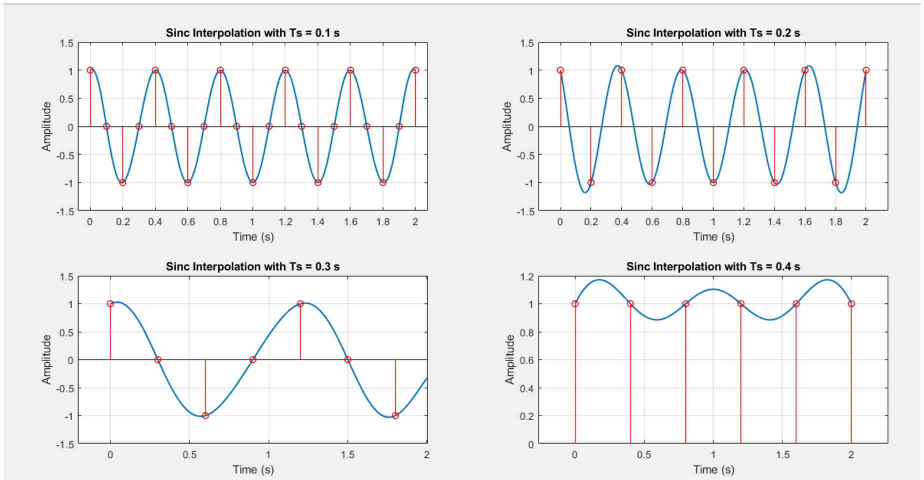
According to the sampling theorem, the Nyquist rate f_s is twice the signal's highest frequency component:

$$f_s = 2 \times f = 2 \times 2.5 = 5\,\text{Hz}$$

Thus, the Nyquist sampling interval T_s is:

$$T_s = \frac{1}{f_s} = \frac{1}{5} = 0.2\,\text{s}$$

4.3 Observation



- **For $T_s = 0.1\,\mathbf{s}$:**
  This sampling interval is smaller than the Nyquist interval. The sinc-interpolated reconstruction closely approximates the original signal $x(t)$ without any visible aliasing artifacts. The reconstructed signal maintains the correct frequency content of the original cosine wave.

- **For $T_s = 0.2\,\mathbf{s}$:**
  This interval matches the Nyquist interval. The sinc interpolation still provides an accurate reconstruction of $x(t)$ with no aliasing effects. However, this sampling rate is the threshold, so any further increase in the sampling interval may introduce aliasing.

- **For $T_s = 0.3\,\mathbf{s}$:**
  This interval is larger than the Nyquist interval, leading to under sampling. As a result, aliasing artifacts appear, causing the reconstructed signal to exhibit a frequency lower than the original signal. This phenomenon occurs because the sampling frequency is insufficient to capture the original frequency content.

- **For $T_s = 0.4\,\mathbf{s}$:**
  At this sampling interval, which is significantly above the Nyquist threshold, severe aliasing occurs. The reconstructed signal deviates substantially from the original signal, showing a much lower apparent frequency. The effect is due to strong aliasing, which misrepresents the frequency of the original signal.

From these observations, we conclude that:

- When the sampling interval T_s is less than or equal to the Nyquist interval (e.g., $T_s = 0.1\,\mathrm{s}$ and $T_s = 0.2\,\mathrm{s}$), the sinc interpolation accurately reconstructs the continuous-time signal.
- For $T_s > 0.2\,\mathrm{s}$, aliasing occurs, causing distortion in the reconstructed signal. The signal appears to have a lower frequency than the original signal due to under sampling.

Thus, to accurately reconstruct $x(t) = \cos(5\pi t)$ without aliasing, the sampling interval should be at or below the Nyquist interval $T_s = 0.2\,\mathrm{s}$.