Signal Processing Lab Report -10

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1 Low-pass FIR filter design using windows

To design the low-pass FIR filter using the window method, we derive the impulse responses $h_{LPF}[n]$ and $h_d[n]$ based on the ideal frequency responses $H_{LPF}(e^{j\omega})$ and $H_d(e^{j\omega})$.
1. Ideal Low-Pass Filter Impulse Response $h_{LPF}[n]$ The ideal frequency response $H_{LPF}(e^{j\omega})$ is defined as:

$$H_{LPF}(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{7} \\ 0, & \frac{\pi}{7} < |\omega| < \pi \end{cases}$$

To obtain the impulse response $h_{LPF}[n]$, we perform the inverse Discrete-Time Fourier Transform (DTFT):

$$h_{LPF}[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{\text{LPF}}(e^{j\omega}) e^{j\omega n} \, d\omega$$

. Since $H_{LPF}(e^{j\omega})$ is nonzero only within $|\omega| \leq \frac{\pi}{7}$, we simplify this to:

$$h_{LPF}[n] = \frac{1}{2\pi} \int_{-\frac{\pi}{7}}^{\frac{\pi}{7}} e^{j\omega n} \, d\omega$$

Solving this integral, we find:

$$h_{LPF}[n] = \frac{\sin\left(\frac{\pi}{7}n\right)}{\pi n}, \quad for n \neq 0$$

and

$$h_{LPF}[0] = \frac{1}{7}$$

Thus, the impulse response $h_{LPF}[n]$ is:

$$h_{LPF}[n] = \begin{cases} \frac{\sin\left(\frac{\pi}{7}n\right)}{\pi n}, & n \neq 0 \\ \frac{1}{7}, & n = 0 \end{cases}$$

2. Low-Pass Filter with Linear Phase Impulse Response $h_d[n]$
The modified frequency response $H_d(e^{j\omega})$ with linear phase is given by:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega n_c}, & |\omega| \leq \frac{\pi}{7} \\ 0, & \frac{\pi}{7} < |\omega| < \pi \end{cases}$$

where $n_c = \frac{N-1}{2}$.
To find $h_d[n]$, we perform the inverse DTFT:

$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega$$

Substituting $H_d(e^{j\omega}) = e^{-j\omega n_c}$ for $|\omega| \leq \frac{\pi}{7}$, we get:

$$h_d[n] = \frac{1}{2\pi} \int_{-\frac{\pi}{7}}^{\frac{\pi}{7}} e^{j\omega(n-n_c)}\, d\omega$$

This integral evaluates to:

$$h_d[n] = \frac{\sin\left(\frac{\pi}{7}(n-n_c)\right)}{\pi(n-n_c)}, \quad forn \neq n_c$$

and

$$h_d[n_c] = \frac{1}{7}$$

Thus, the impulse response $h_d[n]$ is:

$$h_d[n] = \begin{cases} \frac{\sin\left(\frac{\pi}{7}(n-n_c)\right)}{\pi(n-n_c)}, & n \neq n_c \\ \frac{1}{7}, & n = n_c \end{cases}$$

The impulse responses are:

- **Ideal LPF Impulse Response:**

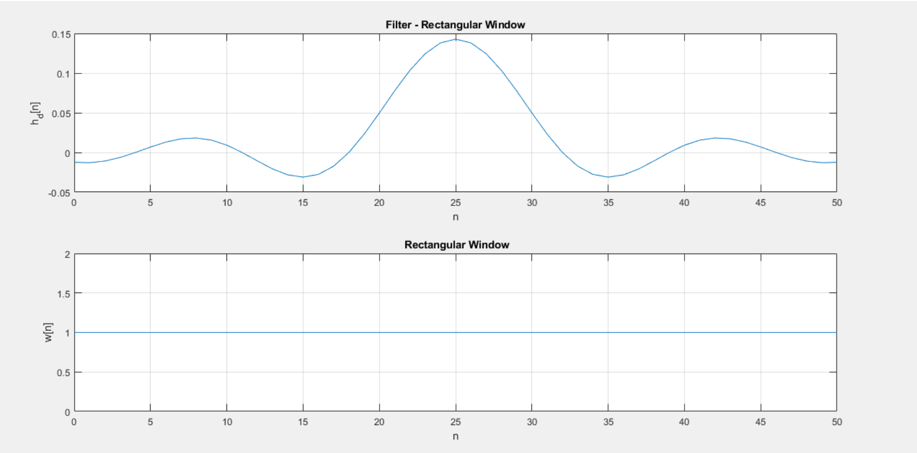
$$h_{LPF}[n] = \begin{cases} \frac{\sin\left(\frac{\pi}{7}n\right)}{\pi n}, & n \neq 0 \\ \frac{1}{7}, & n = 0 \end{cases}$$

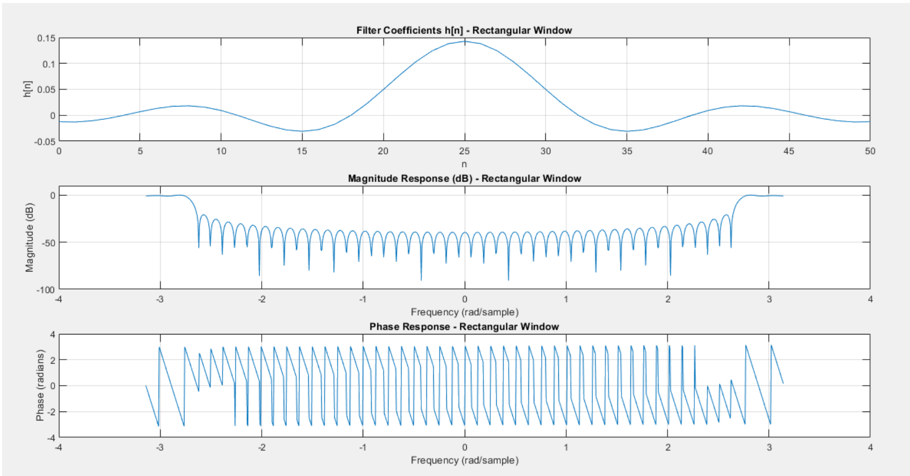
- **Linear-Phase LPF Impulse Response:**

$$h_d[n] = \begin{cases} \frac{\sin\left(\frac{\pi}{7}(n-n_c)\right)}{\pi(n-n_c)}, & n \neq n_c \\ \frac{1}{7}, & n = n_c \end{cases}$$

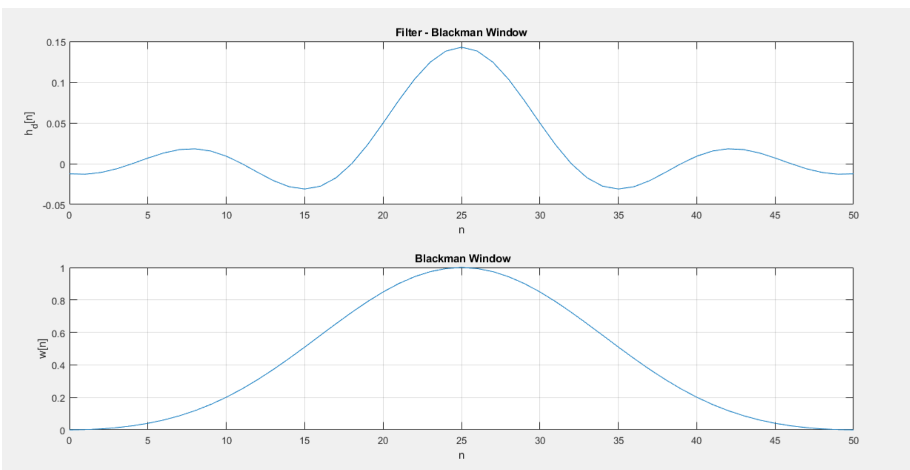
  where $n_c = \frac{N-1}{2}$.

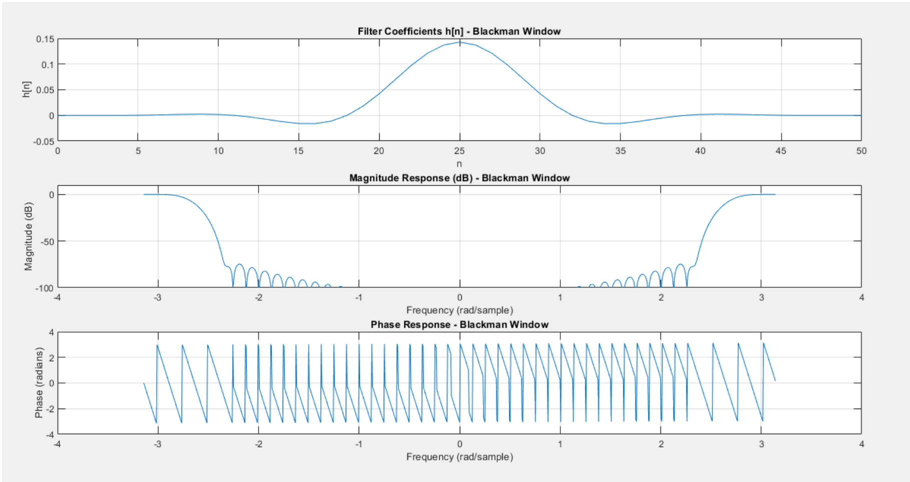
1.1 Using Rectangular Window





1.2 Using Blackmann Window





1.3 Comparison of Transition Bands and Side-Lobe Levels

1. **Transition Band:**

   - The **Rectangular Window** results in a narrower transition band because it applies no tapering to the frequency response. This leads to sharper cutoffs between the pass band and the stop band. However, this sharpness comes at the cost of more prominent side-lobes in the frequency spectrum.

   - The **Blackman Window** applies significant tapering to the filter coefficients, which broadens the transition band. As a result, the filter exhibits a smoother and more gradual cutoff between the pass band and the stop band.

2. **Side-Lobe Levels:**

   - The **Rectangular Window** has higher side-lobe levels, with the main side-lobe approximately 13 dB below the main lobe. These high side-lobe levels lead to more spectral leakage, making the filter less effective in attenuating frequencies in the stop band.

   - The **Blackman Window** significantly reduces side-lobe levels, with the first side-lobe approximately 58 dB below the main lobe. This suppression minimizes spectral leakage, making the filter more effective at attenuating unwanted frequencies.

3. **Key Trade-off:**

   - The **Rectangular Window** offers a sharper transition band but suffers from higher side-lobe levels, resulting in poor stop band attenuation.

   - The **Blackman Window** offers much lower side-lobe levels, providing better stop band attenuation, but at the cost of a wider transition band.

Practical Implication:

- If a sharp transition is crucial and stop band attenuation is less critical, the **Rectangular Window** is preferred.
- If minimizing spectral leakage and achieving better stop band attenuation is important, the **Blackman Window** is a better choice.

1.4 Passing Signals

Comparison of Filtered Signals with Original Signal

1. **Original Signal $x[n]$:** The original signal is defined as:

$$x[n] = \cos\left(\frac{\pi n}{16}\right) + 0.25\sin\left(\frac{\pi n}{2}\right).$$

This signal consists of two frequency components:

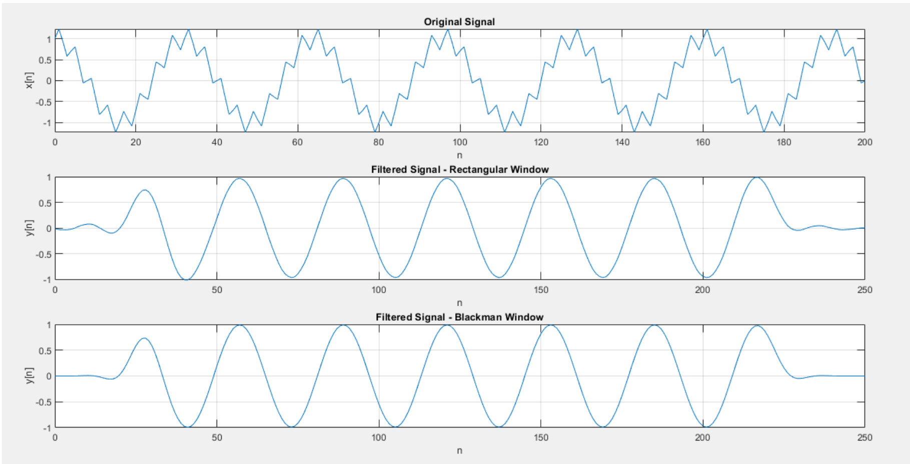
- A low-frequency component at $\omega = \frac{\pi}{16}$, corresponding to the cosine term.
- A higher-frequency component at $\omega = \frac{\pi}{2}$, corresponding to the sine term.

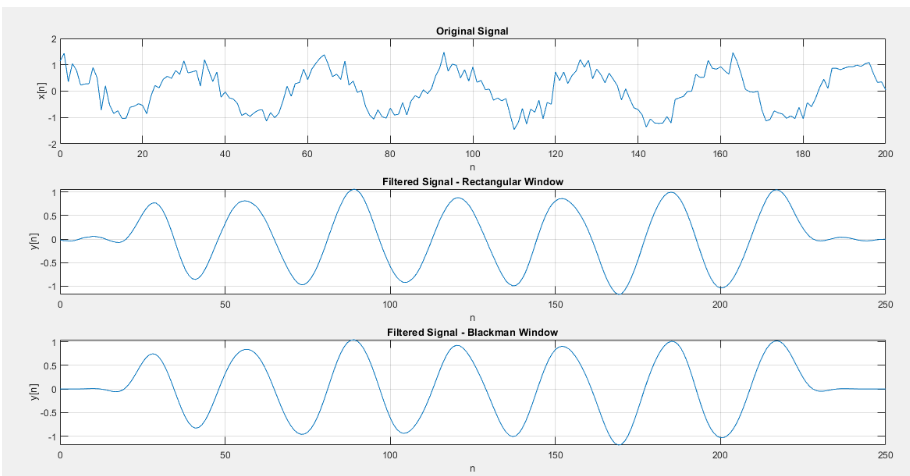
2. **Filtered Signal using Rectangular Window Filter:**

- The rectangular window filter preserves the lower frequency component $\cos\left(\frac{\pi n}{16}\right)$ effectively, as it falls within the pass band.
- The higher frequency component $\sin\left(\frac{\pi n}{2}\right)$ is attenuated due to its proximity to the stop band, but some leakage occurs due to the higher side-lobe levels of the rectangular window.
- The filtered signal retains the general shape of the original signal but exhibits slight ripples due to side-lobe leakage.

3. **Filtered Signal using Blackman Window Filter:**

- The Blackman window filter also preserves the lower frequency component $\cos\left(\frac{\pi n}{16}\right)$ effectively, with less distortion compared to the rectangular window filter.
- The higher frequency component $\sin\left(\frac{\pi n}{2}\right)$ is attenuated more effectively due to the lower side-lobe levels of the Blackman window.
- The filtered signal is smoother, with reduced ripples compared to the rectangular window filter, indicating better stop band attenuation.



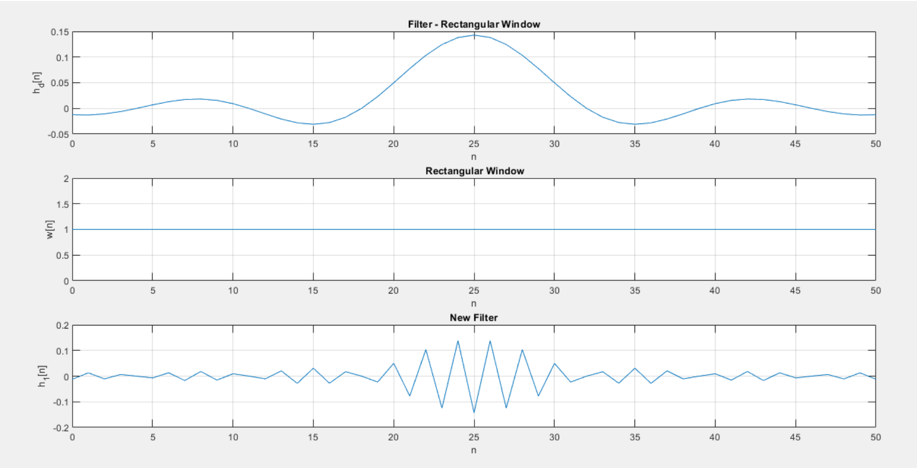


4. **Overall Comparison:**

   - The rectangular window filter exhibits a sharper transition band but higher side-lobe leakage, leading to minor distortions in the stop band.
   - The Blackman window filter provides smoother results with better stop band attenuation but a slightly broader transition band, resulting in reduced distortion in the filtered signal.

Conclusion The Blackman window filter outperforms the rectangular window filter in terms of stop band attenuation and smoothness of the filtered signal, making it a better choice for applications requiring minimal spectral leakage.

1.5 New Filter



Description of the New Filter $h_1[n] = (-1)^n h[n]$
The new filter is obtained by modifying the impulse response $h[n]$, where:

$$h_1[n] = (-1)^n \cdot h[n].$$

Here:

- $h[n]$ is the impulse response of a low-pass filter designed using the rectangular window method.
- The term $(-1)^n$ alternates the sign of each coefficient of $h[n]$ for every successive n.

Effect on Frequency Response
1. **Frequency Shifting:**

- Multiplying $h[n]$ by $(-1)^n$ in the time domain corresponds to a frequency shift of π in the frequency domain.
- The frequency response $H(e^{j\omega})$ of the original filter is shifted by π, resulting in the new frequency response:

$$H_1(e^{j\omega}) = H(e^{j(\omega-\pi)}).$$

2. **Transformation to a High-pass Filter:**

- The original filter $h[n]$ is a low-pass filter with a pass band centered at $\omega = 0$.
- The frequency shift by π moves the pass band to $\omega = \pi$, transforming the filter into a high-pass filter.

Impulse Response Characteristics

- The alternation of signs in $h_1[n]$ results in oscillations in the impulse response.
- The filter retains the same length and symmetry properties as the original low-pass filter, ensuring it remains linear-phase.

Applications of the New Filter

- The new filter can effectively attenuate low-frequency components while preserving high-frequency components.
- It is useful in applications requiring high-pass filtering, such as noise removal in signals dominated by low-frequency noise.

Conclusion The new filter $h_1[n]$ is a high-pass filter obtained by frequency shifting the low-pass filter designed using the rectangular window. It retains linear-phase characteristics and is well-suited for applications where high-frequency signal components are of interest.

2 Digital Band Limited differentiator

2.1 Theory

DIGITAL BANDLIMITED DIFFERENTIATOR

$f_s = 1 \backslash T$

$H_{diff}(e^{j\omega}) = j\frac{\omega}{T}$, $\omega \in [-\pi, \pi]$

T: sampling interval

a) $h_{Diff}[n] = \begin{cases} 0 & , n = 0 \\ \frac{\cos \pi n}{nT} = \frac{(-1)^n}{nT} & , n \neq 0. \end{cases}$

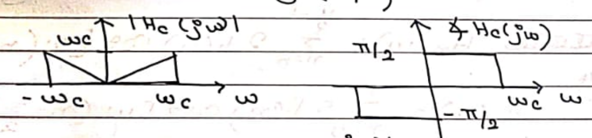
Frequency response of continuous time differentiating filter: $H_c(j\omega) = j\omega$

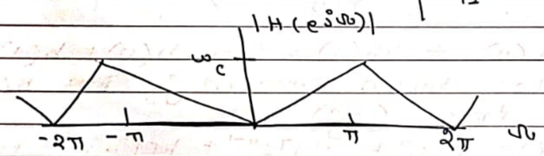
For band limited differentiator with cutoff freq. ω_c:

$$H_c(j\omega) = \begin{cases} j\omega, & |\omega| < \omega_c \\ 0 & , |\omega| > \omega_c \end{cases}$$

for $\omega_s = 2\omega_c$,

$H_d(e^{j\omega}) = j\left(\frac{\omega}{T}\right)$, $|\omega| < \pi$





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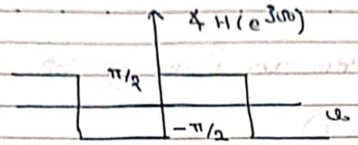

Let $x_c(t) = \frac{\sin(\pi t/T)}{\pi t}$, T= sampling period

Then,

$$X_c(j\omega) = \begin{cases} 1 & |\omega| < \pi/T \\ 0 & \text{o.w} \end{cases}$$

(Sufficient to ensure sampling $x_c(t)$ at frequency $\omega_s = 2\pi/T$ does not give rise to any aliasing).

Output of digital differentiator,

$$y_c(t) = \frac{d}{dt} x_c(t) = \frac{\cos(\pi t/T)}{Tt} - \frac{\sin(\pi t/T)}{\pi t^2}$$

$$x_d[n] = x_c(nT) = \frac{1}{T}\delta[n]$$

for $n \neq 0$, $x_c(nT) = 0$,

while $x_d[0] = x_c(0) = \frac{1}{T}$

Similarly, $y_d[n] = y_c(nT) = \begin{cases} \frac{(-1)^n}{nT^2}, & n \neq 0 \\ 0, & n = 0 \end{cases}$

Thus, $h_d[n] = \begin{cases} \frac{(-1)^n}{nT}, & n \neq 0 \\ 0, & n = 0 \end{cases}$

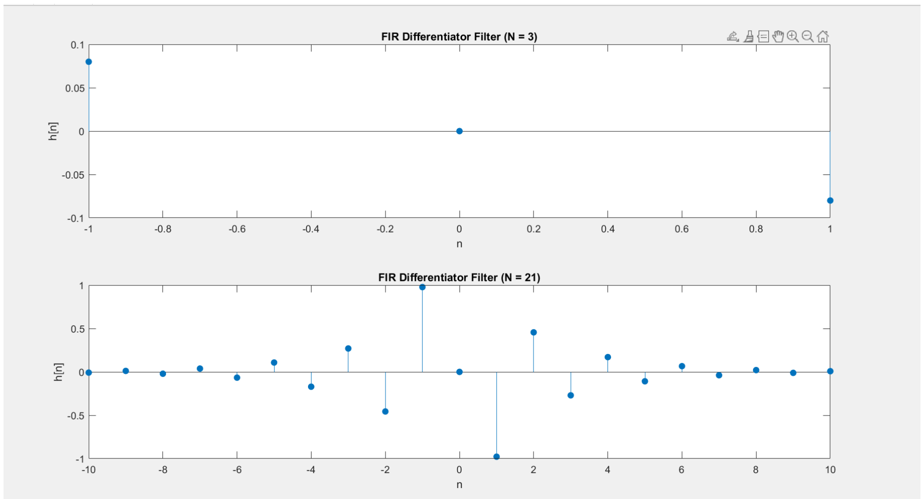

Figure 2.1: Obtained filters

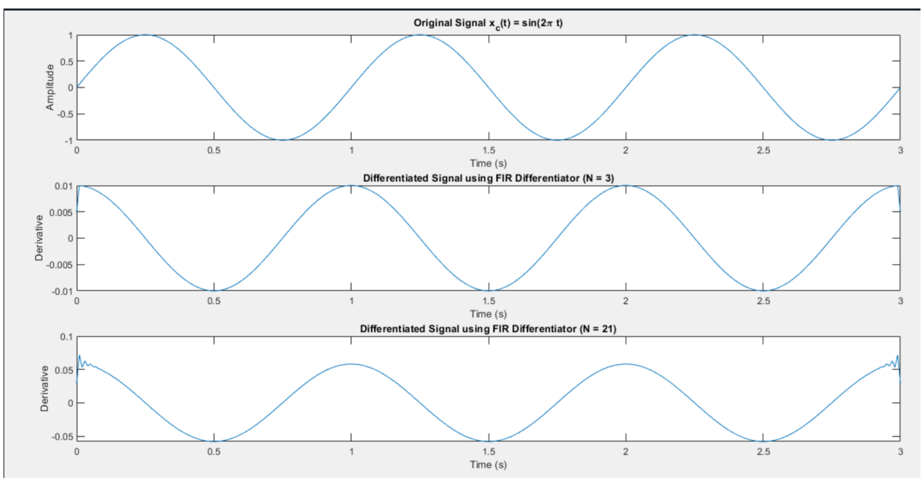

Figure 2.2: Obtained graph

3 Filter design using filter Designer GUI

3.1 Low pass FIR filter using window method

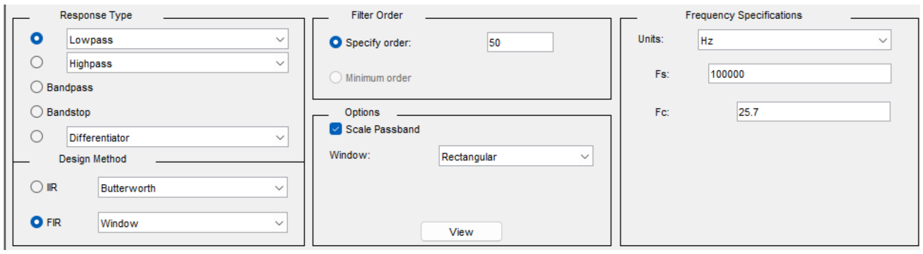

Figure 3.1: Data of Filter

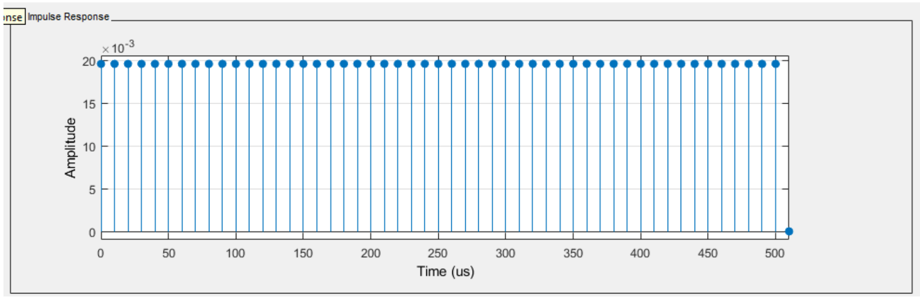

Figure 3.2: Impulse Response

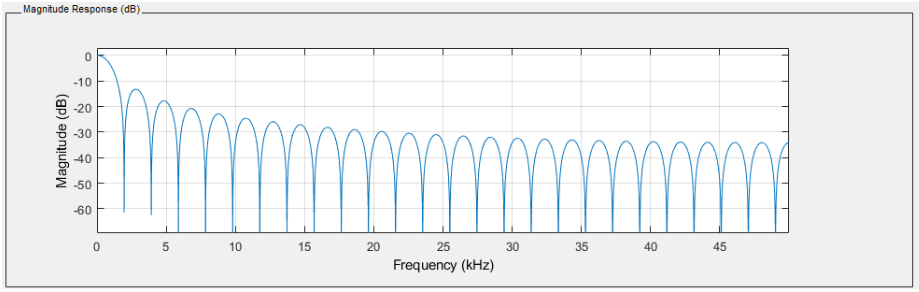

Figure 3.3: Magnitude Spectrum

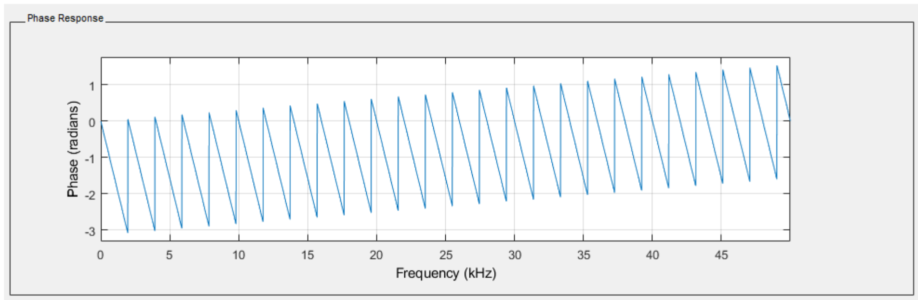

Figure 3.4: Phase Spectrum

3.2 Low- pass FIR Least square filter

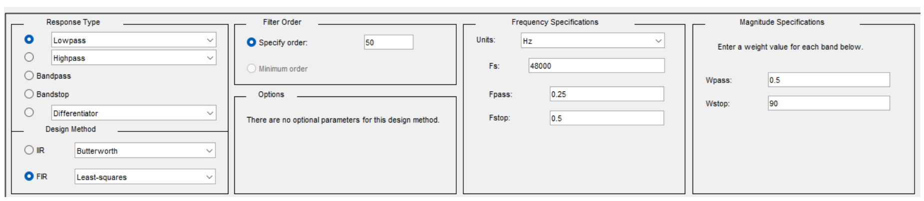

Figure 3.5: Data of Filter

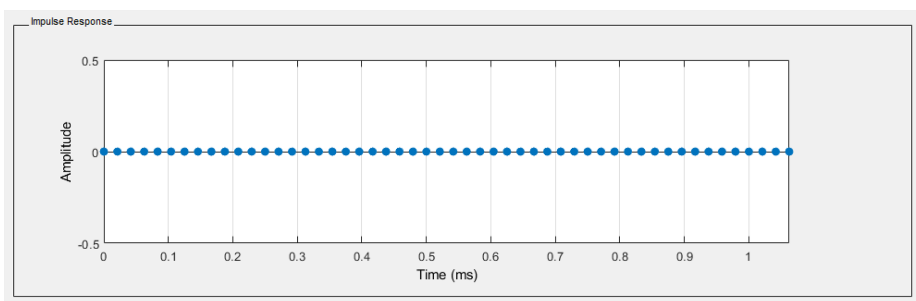

Figure 3.6: Impulse Response

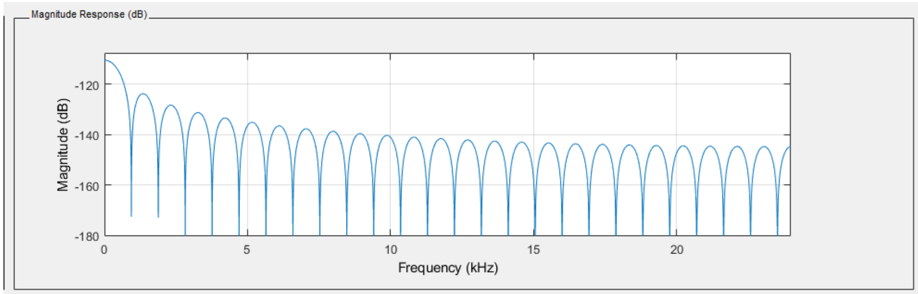

Figure 3.7: Magnitude Spectrum

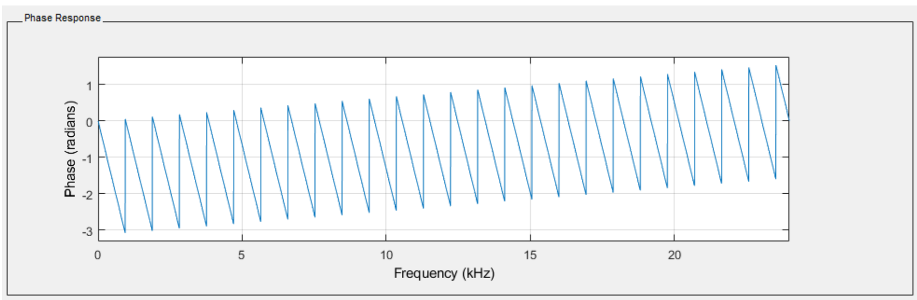

Figure 3.8: Phase Spectrum