P47.

1. $f(x_1, x_2, x_3, x_4)$

$= 2x_1^3 x_2 x_3^4 - \frac{1}{2}x_1^3 x_2 x_3^2 + 3x_2^6 x_4^3 + 5x_2^3 x_4$

$-6x_2 x_4^2 + 7x_3^2 - 8$

$g(x_1, x_2, x_3, x_4) = x_1^2 x_2 + x_1 x_2^2 + x_3^2 x_4 + x_3 x_4^2.$

$f(x_1, x_2, x_3, x_4)$ 与 $g(x_1, x_2, x_3, x_4)$ 的乘积首项

为 $2x_1^5 x_2^2 x_3^4$

2. 设3个根 $\alpha_1, \alpha_2, \alpha_3$,

$\frac{5}{2} = \alpha_1 + \alpha_2 + \alpha_3.$

$-2 = \alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_1\alpha_3$

$-6 = \alpha_1\alpha_2\alpha_3$

不妨令 $\alpha_3 = \alpha_2$, 则有

$\frac{5}{2} = \alpha_1 + 2\alpha_2$, $\alpha_1 = \frac{5}{2} - 2\alpha_2.$

$-2 = 2\alpha_1\alpha_2 + \alpha_2^2$, $-2 = 5\alpha_2 - 3\alpha_2^2$

$-6 = \alpha_1\alpha_2^2$, $\frac{5}{2}\alpha_2^2 - 2\alpha_2^3 = -6$

得 $\alpha_2 = 2$, $\alpha_1 = -\frac{3}{2}$

原方程的解为, $x_1 = -\frac{3}{2}$, $x_2 = x_3 = 2.$

3. 证：设三个根为 a, ad, ad^2.

由韦达定理，

$$-p = a + ad + ad^2.$$

$$q = a^2d + a^2d^2 + a^2d^3$$

$$-r = a^3d^3$$

$$q^3 = -a^6d^3(1+d+d^2)^3$$

$$= -a^3(1+d+d^2)^3 \cdot a^3d^3$$

$$= p^3 r.$$

证毕.