P48.

a b c

4. (1). $f(x_1, x_2, x_3, x_4) = (x_1x_2 + x_3x_4)(x_1x_3 + x_2x_4)(x_1x_4 + x_2x_3)$

$$= x_1^3x_2x_3x_4 + x_1^2x_2^2x_3^2 + x_1^2x_2^2x_4^2 + x_1x_2^3x_3x_4$$
$$+ x_1^2x_3^2x_4^2 + x_1x_2x_3^3x_4 + x_1x_2x_3x_4^3 + x_2^2x_3^2x_4^2.$$

$$= x_1^3x_2x_3x_4 + x_1^2x_2^2x_3^2 + x_1^2x_2^2x_4^2 + x_1^2x_3^2x_4^2$$
$$+ x_1x_2^3x_3x_4 + x_1x_2x_3^3x_4 + x_1x_2x_3x_4^3 + x_2^2x_3^2x_4^2.$$

$$= (x_1^2 + x_2^2 + x_3^2 + x_4^2)(x_1x_2x_3x_4)$$
$$+ x_1^2x_2^2x_3^2 + x_1^2x_2^2x_4^2 + x_1^2x_3^2x_4^2 + x_2^2x_3^2x_4^2$$

$$= (\sigma_1^2 - 2\sigma_2)\sigma_4 + \sigma_3^2 - 2\sigma_2\sigma_4$$

$$= \sigma_1^2 - 4\sigma_2\sigma_4 + \sigma_3^2$$

5. (2) $\sigma_2\sigma_3 = \sum x_1x_2x_3x_4x_5 + \sum x_1^2x_2x_3x_4 + \sum x_1^2x_2^2x_3$

$\sigma_1\sigma_4 = \sum x_1x_2x_3x_4x_5 + \sum x_1^2x_2x_3x_4$

$\sigma_5 = \sum x_1x_2x_3x_4x_5$

$\therefore \sum x_1^2x_2^2x_3 = \sigma_2\sigma_3 - \sigma_1\sigma_4.$

7. 证明：设4个根，$\alpha_1, \alpha_2, \alpha_3, \alpha_4$.

其中 $\alpha_3+\alpha_4=0$.

$$-\frac{a_1}{a_0} = \alpha_1+\alpha_2+\alpha_3+\alpha_4 = \alpha_1+\alpha_2$$

$$\frac{a_2}{a_0} = \alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_2\alpha_4+\alpha_3\alpha_4$$

$$-\frac{a_3}{a_0} = \alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_4+\alpha_1\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4$$

$$\frac{a_4}{a_0} = \alpha_1\alpha_2\alpha_3\alpha_4$$

$$\frac{a_2}{a_0} = \alpha_1\alpha_2+\alpha_3\alpha_4$$

$$-\frac{a_3}{a_0} = \alpha_1\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4$$

$$a_1^2a_4 = a_0^3(\alpha_1+\alpha_2)^2\alpha_1\alpha_2\alpha_3\alpha_4$$

$$a_0a_3^2 = a_0^3(\alpha_1\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4)^2$$

$$a_1a_2a_3 = a_0^3(\alpha_1+\alpha_2)(\alpha_1\alpha_2+\alpha_3\alpha_4)(\alpha_1\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4)$$

$$(\alpha_1+\alpha_2)^2\alpha_1\alpha_2\alpha_3\alpha_4 = (\alpha_1^2+2\alpha_1\alpha_2+\alpha_2^2)\alpha_1\alpha_2\alpha_3\alpha_4$$

$$(\alpha_1\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4)^2 = \alpha_3^2\alpha_4^2(\alpha_1^2+2\alpha_1\alpha_2+\alpha_2^2)$$

$$a_1^2a_4+a_0a_3^2 = (\alpha_1+\alpha_2)^2(\alpha_1\alpha_2+\alpha_3\alpha_4)\alpha_3\alpha_4$$

$$=(\alpha_1+\alpha_2)(\alpha_1\alpha_2+\alpha_3\alpha_4)(\alpha_1\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4)$$

$$=a_1a_2a_3$$，即 $a_1^2a_4+a_0a_3^2-a_1a_2a_3=0$.

必要性得证.

若 $a_1^2a_4+a_0a_3^2-a_1a_2a_3=0$. 显然 $a_0\neq 0$.

$$\left(\frac{a_1}{a_0}\right)^2\frac{a_4}{a_0}+\left(\frac{a_3}{a_0}\right)^2-\frac{a_1}{a_0}\cdot\frac{a_2}{a_0}\cdot\frac{a_3}{a_0}=0$$

设四个根 $\alpha_1,\alpha_2,\alpha_3,\alpha_4$.

$$(\alpha_1+\alpha_2+\alpha_3+\alpha_4)^2\alpha_1\alpha_2\alpha_3\alpha_4+(\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_4+\alpha_1\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4)^2=(\alpha_1+\alpha_2+\alpha_3+\alpha_4)(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_2\alpha_4+\alpha_3\alpha_4)(\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_4+\alpha_1\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4)$$

$$[\alpha_1^2+\alpha_2^2+\alpha_3^2+\alpha_4^2+2(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_2\alpha_4+\alpha_3\alpha_4)]\alpha_1\alpha_2\alpha_3\alpha_4+[\alpha_1^2\alpha_2^2\alpha_3^2+\alpha_1^2\alpha_2^2\alpha_4^2+\alpha_1^2\alpha_3^2\alpha_4^2+\alpha_2^2\alpha_3^2\alpha_4^2+2(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_2\alpha_4+\alpha_3\alpha_4)\alpha_1\alpha_2\alpha_3\alpha_4]=(\alpha_1+\alpha_2+\alpha_3+\alpha_4)(\alpha_1\alpha_2+\cdots+\alpha_3\alpha_4)(\alpha_1\alpha_2\alpha_3+\cdots)$$

$$\alpha_1\alpha_2\alpha_3\alpha_4\sum\alpha_1^2+\sum\alpha_1^2\alpha_2^2\alpha_3^2+4\alpha_1\alpha_2\alpha_3\alpha_4\sum\alpha_1\alpha_2=\left(\sum\alpha_1\right)\left(\sum\alpha_1\alpha_2\right)\left(\sum\alpha_1\alpha_2\alpha_3\right)$$

$$\alpha_1\alpha_2\alpha_3\alpha_4\sum\alpha_1^2+\sum\alpha_1^2\alpha_2^2\alpha_3^2=\left(\sum\alpha_1\alpha_2\right)(\alpha_1^2\alpha_2\alpha_3+\alpha_1^2\alpha_2\alpha_4+\alpha_1^2\alpha_3\alpha_4+\alpha_1\alpha_2^2\alpha_3+\alpha_1\alpha_2^2\alpha_4+\alpha_2^2\alpha_3\alpha_4+\alpha_1\alpha_2\alpha_3^2+\alpha_1\alpha_3^2\alpha_4+\alpha_2\alpha_3^2\alpha_4+\alpha_1\alpha_2\alpha_4^2+\alpha_1\alpha_3\alpha_4^2+\alpha_2\alpha_3\alpha_4^2)$$

$$\sum\alpha_1^3\alpha_2\alpha_3\alpha_4+\sum\alpha_1^2\alpha_2^2\alpha_3^2$$
$$=3\sum\alpha_1^2\alpha_2^2\alpha_3^2+3\sum\alpha_1^3\alpha_2\alpha_3\alpha_4+4\sum\alpha_1^2\alpha_2^2\alpha_3\alpha_4+\sum\alpha_1^3\alpha_2^2\alpha_3$$

$$2\sum\alpha_1^2\alpha_2^2\alpha_3^2+2\sum\alpha_1^3\alpha_2\alpha_3\alpha_4+4\sum\alpha_1^2\alpha_2^2\alpha_3\alpha_4+\sum\alpha_1^3\alpha_2^2\alpha_3=0$$

$$(\alpha_1+\alpha_2)(\alpha_1+\alpha_3)(\alpha_1+\alpha_4)(\alpha_2+\alpha_3)(\alpha_2+\alpha_4)(\alpha_3+\alpha_4)=0.$$

$\alpha_1,\alpha_2,\alpha_3,\alpha_4$ 中有两个和为0. 充分性得证.

8. (1)

$$R(f,g)=\begin{vmatrix} 1 & -3 & 2 & - & - & - & - \\ 0 & 1 & -3 & 2 & - & - & - \\ \cdots & & & & & & \\ 0 & 0 & \cdots & 1 & -3 & 2 & \\ 1 & \cdots & 0 & 0 & 1 & 0 & \\ 0 & 1 & \cdots & 0 & 0 & 1 & \end{vmatrix} = \prod_{i=1}^{n}(1-\alpha_i)\prod_{i=1}^{n}(2-\alpha_i)$$

$$=\left(\sum_{i=0}^{n}\sigma_i(-1)^i\right)\left(\sum_{i=0}^{n}2^{n-i}\sigma_i(-1)^i\right)$$

$$=\left(\sum_{i=0}^{n}a_i\right)\left(\sum_{i=0}^{n}2^{n-i}a_i\right)$$

$$=2^{n+1}+2$$

(3)

$$R(f,g)=\begin{vmatrix} 1 & 0 & \cdots & 1 & 1 & 0 \\ 0 & 1 & 0 & \cdots & 1 & 1 \\ 1 & -3 & 2 & 0 & \cdots & 0 \\ 0 & 1 & -3 & 2 & \cdots & 0 \\ \cdots & & & & & \\ 0 & \cdots & 0 & 1 & -3 & 2 \end{vmatrix} = \prod_{i=0}^{n}(\alpha_i-1)\prod_{i=0}^{n}(\alpha_i-2)$$

$$=\prod_{i=0}^{n}(1-\alpha_i)\prod_{i=0}^{n}(2-\alpha_i)$$

$$=\left(\sum_{i=0}^{n}a_i\right)\left(\sum_{i=0}^{n}2^{n-i}a_i\right)$$

$$=3(2^n+2+1)$$

$$=3\cdot 2^n+9$$

9. (1). $F_x(y)=5y^2-6xy+5x^2-16$

$G_x(y)=y^2-(x+1)y+2x^2-x-4.$

$$R_y(f,g)=\begin{vmatrix} 5 & -6x & 5x^2-16 & 0 \\ 0 & 5 & -6x & 5x^2-16 \\ 1 & -(x+1) & 2x^2-x-4 & 0 \\ 0 & 1 & -(x+1) & 2x^2-x-4 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 5-x & -5x^2+5x+4 & 0 \\ 0 & 0 & 5-x & -5x^2+5x+4 \\ 1 & -(x+1) & 2x^2-x-4 & 0 \\ 0 & 1 & -(x+1) & 2x^2-x-4 \end{vmatrix}$$

$$= \begin{vmatrix} 5-x & -5x^2+5x+4 & 0 \\ 0 & 5-x & -5x^2+5x+4 \\ 1 & -(x+1) & 2x^2-x-4 \end{vmatrix}$$

$$= \begin{vmatrix} -6x^2+9x+9 & (2x^2-x-4)(x-5) \\ 5-x & -5x^2+5x+4 \end{vmatrix}$$

$$= 32x^4-96x^3+32x^2+96x-64$$

$$= 32(x^4-3x^3+x^2+3x-2)$$

$$= 32(x-1)^2(x+1)(x-2)$$

$x=1$. $\begin{cases} 5y^2-6y-11=0, & (5y-11)(y+1)=0 \\ y^2-2y-3=0, & (y-3)(y+1)=0 \end{cases}$ $y=-1$

$x=-1$ $\begin{cases} 5y^2+6y-11=0, & (5y+11)(y-1)=0 \\ y^2-1=0, & (y+1)(y-1)=0 \end{cases}$ $y=1$

$x=2$, $\begin{cases} 5y^2-12y+4=0, & (5y-2)(y-2)=0. \\ y^2-3y+2=0 & (y-2)(y-1)=0. \end{cases}$ $y=2$.

方程组的解为 $(1,-1)$, $(-1,1)$, $(2,2)$.

16.
$$\begin{cases} x = t^2 - t + 1 \\ y = 2t^2 + t - 3 \end{cases}$$

$y - 2x = 3t - 5$, $t = \frac{1}{3}(y - 2x + 5)$

$$x = \frac{1}{9}(y-2x+5)^2 - \frac{1}{3}(y-2x+5) + 1$$

$$9x = (y-2x)^2 + 10(y-2x) + 25 - 3(y-2x+5) + 9$$

$$9x = y^2 - 4xy + 4x^2 + 10y - 20x + 25 - 3y + 6x - 6$$

$4x^2 + y^2 - 4xy - 23x + 7y + 19 = 0$，为所求直角坐标方程

17.
$$\begin{cases} x = \frac{2(t+1)}{t^2+1} \\ y = \frac{t^2}{2t-1} \end{cases}$$

$(t^2+1)x = 2(t+1)$　　$t^2 = \frac{2}{x}(t+1) - 1$

$t^2 = y(2t-1)$.　　$y(2t-1) = \frac{2}{x}(t+1) - 1$

$(2y - \frac{2}{x})t = y + \frac{2}{x} - 1$，$t = \frac{y + \frac{2}{x} - 1}{2y - \frac{2}{x}} = \frac{xy - x + 2}{2xy - 2}$

$$y = \frac{\frac{(x(y-1)+2)^2}{4(xy-1)^2}}{\frac{xy-x+2}{xy-1} - 1} = \frac{x^2(y^2-2y+1) + 4x(y-1) + 4}{4(3-x)(xy-1)}$$

$$= \frac{x^2y^2 - 2x^2y + x^2 + 4xy - 4x + 4}{12xy - 4x^2y - 12 + 4x}$$

$$12xy^2 - 4x^2y^2 - 12y + 4xy$$

$$= x^2y^2 - 2x^2y + x^2 + 4xy - 4x + 4$$

$$5x^2y^2 - 2x^2y + x^2 - 12xy^2 - 4x + 12y + 4 = 0,$$

为所求直角坐方程