P151.

1. $$\begin{cases} f(\xi_1)-2f(\xi_2)+f(\xi_3)=4 \\ f(\xi_1)+f(\xi_2)=4 \\ -f(\xi_1)+f(\xi_2)+f(\xi_3)=-2 \end{cases} \Rightarrow \begin{cases} f(\xi_1)=\frac{18}{5} \\ f(\xi_2)=\frac{2}{5} \\ f(\xi_3)=\frac{6}{5} \end{cases}$$

$$f(x_1\xi_1+x_2\xi_2+x_3\xi_3)=\frac{18}{5}x_1+\frac{2}{5}x_2+\frac{6}{5}x_3$$

2. $$\begin{cases} f(\xi_1)+f(\xi_3)=1 \\ f(\xi_2)-2f(\xi_3)=-1 \\ f(\xi_1)+f(\xi_2)=-3 \end{cases} \Rightarrow \begin{cases} f(\xi_1)=4 \\ f(\xi_2)=-7 \\ f(\xi_3)=-3 \end{cases}$$

$$f(x_1\xi_1+x_2\xi_2+x_3\xi_3)=4x_1-7x_2-3x_3$$

3. $$\begin{cases} f(\xi_1)+f(\xi_3)=0 \\ f(\xi_2)-2f(\xi_3)=0 \\ f(\xi_1)+f(\xi_3)=0 \end{cases} \Rightarrow \begin{cases} f(\xi_1)=-1 \\ f(\xi_2)=2 \\ f(\xi_3)=1 \end{cases}$$

$$f(x_1\xi_1+x_2\xi_2+x_3\xi_3)=-x_1+2x_2+x_3$$

7 (1). 设 $v\in V$ 且 $v=k_1\alpha_1+k_2\alpha_2+k_3\alpha_3$.

$\alpha_3=\beta_3$, $\alpha_2=\beta_2-\beta_3$, $\alpha_1=\beta_1-\beta_2$.

$v=k_1\beta_1+(k_2-k_1)\beta_2+(k_3-k_2)\beta_3$

V中任意元素均可用 β_1,β_2,β_3 表示，且 β_1,β_2,β_3 线性无关

于 $\alpha_1,\alpha_2,\alpha_3$ 个数相同，故 β_1,β_2,β_3 是V的一组基.

(2) $f_3'(\beta_3) = f_3'(\alpha_3) = 1$

$f_3'(\beta_2) = f_3'(\alpha_2+\alpha_3) = 0$. $f_3'(\alpha_2) = -1$

$f_3'(\beta_1) = f_3'(\alpha_1+\alpha_2+\alpha_3) = 0$. $f_3'(\alpha_1) = 0$

$f_3' = f_3 - f_2$

$f_2'(\beta_3) = f_2'(\alpha_3) = 0$.

$f_2'(\beta_2) = f_2'(\alpha_2+\alpha_3) = 1$ $f_2'(\alpha_2) = 1$

$f_2'(\beta_1) = f_2'(\alpha_1+\alpha_2+\alpha_3) = 0$. $f_2'(\alpha_1) = -1$

$f_2' = f_2 - f_1$

$f_1'(\beta_3) = f_1'(\alpha_3) = 0$.

$f_1'(\beta_2) = f_1'(\alpha_2+\alpha_3) = 0$. $f_1'(\alpha_2) = 0$.

$f_1'(\beta_1) = f_1'(\alpha_1+\alpha_2+\alpha_3) = 1$. $f_1'(\alpha_1) = 1$

$f_1' = f_1$

$\beta_1, \beta_2, \beta_3$ 都对偶基.

$$\begin{cases} f_1' = f_1' \\ f_2' = f_2 - f_1 \\ f_3' = f_3 - f_2 \end{cases}$$