Pavel Samuilovich Urysohn

Born: 3 February 1898 in Odessa, Ukraine
Died: 17 August 1924 in Batz-sur-Mer, France

Pavel Urysohn is also known as **Pavel Uryson.** His father was a financier in Odessa, the town in which Pavel Samuilovich was born. He came from a family descended from the sixteenth century Rabbi M Jaffe. It was a well-off family and Urysohn received his secondary education in Moscow at a private school there.

In 1915 Urysohn entered the University of Moscow to study physics and in fact he published his first paper in this year. Being interested in physics at this time it is not surprising that this first paper was on a physics topic, and indeed it was, being on Coolidge tube radiation. However his interest in physics soon took second place for after attending lectures by Luzin and Egorov at the University of Moscow he began to concentrate on mathematics.

Urysohn graduated in 1919 and continued his studies there working towards his doctorate. The authors of [8] write:-

> *Luzin was a dynamic mathematician and it was he who persuaded Urysohn to stay on in order to study for a doctorate during* 1919-21.

At this stage Urysohn was interested in analysis, in particular integral equations, and this was the topic of his habilitation. He was awarded his habilitation in June 1921 and, following this, became an assistant professor at the University of Moscow.

Urysohn soon turned to topology. He was asked two questions by Egorov and it was these which occupied him during the summer of 1921. The first question that Egorov posed was to find a general intrinsic topological definition of a curve which when restricted to the plane became Cantor's notion of a continuum which is nowhere dense in the plane. The second of Egorov's questions was a similar one but applied to surfaces, again asking for an intrinsic topological definition.

These were difficult questions which had been around for some time. It was not that Egorov had come up with new questions, rather he was giving the bright young mathematician Urysohn two really difficult problems in the hope that he might come up with new ideas. Egorov was not to be disappointed, for Urysohn attacked the questions with great determination. He did not sit still waiting for inspiration to strike, rather he tried one idea after another to see if it would give him the topological definition of

dimension that he was looking for.

A holiday with other young Moscow mathematicians to the village of Burkov, on the banks of the river Kalyazmy near to the town of Bolshev, did not stop him trying to find the "right" definition of dimension. Quite the opposite, it was a good chance for him to think in congenial surroundings, and one morning near the end of August he woke up with an idea in his mind which he felt, even before working through the details, was right. Immediately he told his friend Aleksandrov about his inspiration.

Of course there was a lot of hard work after the moment of inspiration. During the following year Urysohn worked through the consequences building a whole new area of dimension theory in topology. It was an exciting time for the topologists in Moscow for Urysohn lectured on the topology of continua and often his latest results were presented in the course shortly after he had proved them. He published a series of short notes on this topic during 1922. The complete theory was presented in an article which Lebesgue accepted for publication in the *Comptes rendus* of the Academy of Sciences in Paris. This gave Urysohn an international platform for his ideas which immediately attracted the interest of mathematicians such as Hilbert.

Urysohn published a full version of his dimension theory in *Fundamenta mathematicae*. He wrote a major paper in two parts in 1923 but they did not appear in print until 1925 and 1926. Sadly Urysohn had died before even the first part was published. The paper begins with Urysohn stating his aim which was:-

> *To indicate the most general sets that still merit being called "lines" and "surfaces" ...*

In fact Urysohn set out to do far more in this paper than to answer the two questions that Egorov had posed to him. As Crilly and Johnson write [8]:-

> *Not only did he seek definitions of curve and surface, but also definitions of n-dimensional Cantorian manifold and hence of dimension itself. The dimension concept was, in fact, the centre of his attention.*

Although Urysohn did not know of Brouwer's contribution when he worked out the details of his theory of topological dimension, Brouwer had in fact published on that topic in 1913. He had given a global definition, however, and this was in contrast to Urysohn's local definition of dimension. Another important aspect of Urysohn's ideas was the fact that he presented them in the context of compact metric spaces. After Urysohn's death, Aleksandrov argued that although Urysohn's definition of dimension was given for a metric space, it is, nevertheless, completely equivalent to the definition given by Menger for general topological spaces.

Urysohn visited Göttingen in 1923. His reports to the Mathematical Society of Göttingen interested Hilbert and while in Göttingen he learnt of Brouwer's contributions to the area made in the paper of 1913 to which we referred above. Urysohn spotted an error in Brouwer's paper regarding a definition of dimension while he was studying it in Göttingen and easily constructed a counter-example. He met Brouwer at the annual meeting of the German Mathematical Society in Marburg where both gave lectures and Urysohn mentioned Brouwer's error, and his counter-example, in his talk. It was an occasion which made Brouwer begin to think about topology again, for his interests had turned to intuitionism, the subject of his talk at Marburg.

In the summer of 1924 Urysohn set off again with Aleksandrov on a European trip through Germany, Holland and France. Again the two mathematicians visited Hilbert and, by 7 May, they must have left since Hilbert wrote to Urysohn on that day telling him his paper with Aleksandrov was accepted for publication in *Mathematische Annalen* (see below). This letter, given in [11], also thanks Urysohn for caviar he had given Hilbert, and expresses the hope that Urysohn will visit again the following summer.

They then met Hausdorff who was impressed with Urysohn's results. He also wrote a letter to Urysohn which was dated 11 August 1924 (see [11]). The letter discusses Urysohn's metrization theorem and his construction of a universal separable metric space. The construction of a universal metric space, containing an isometric image of any metric space, was one of Urysohn's last results. Like Hilbert, Hausdorff expressed the hope that Urysohn would visit again the following summer. Van Dalen writes in [13] about their final mathematical visit which was to Brouwer:-

> *This time [Urysohn and Aleksandrov] visited Brouwer, who was most favourably impressed by the two Russians. He was particularly taken with Urysohn, for whom he developed something like the attachment to a lost son.*

After this visit the two mathematicians continued their holiday to Brittany where they rented a cottage. Urysohn drowned in rough seas while on one of their regular swims off the coast.

Urysohn was not only an "inseparable friend" to Aleksandrov but the two collaborated on important publications such as *Zur Theorie der topologischen Räume* Ⓣ published in *Mathematische Annalen* in 1924. Urysohn's main contributions, in addition to the theory of dimension discussed above, are the introduction and investigation of a class of normal surfaces, metrization theorems, and an important existence theorem concerning mapping an arbitrary normed space into a Hilbert space with countable basis. He is remembered particularly for 'Urysohn's lemma' which proves the existence of a certain continuous function taking values 0 and 1 on particular closed subsets.

After Urysohn's death Brouwer and Aleksandrov made sure that the mathematics he left was properly dealt with. As van Dalen writes [13]:-

> *Brouwer was broken hearted. He decided to look after the scientific estate of Urysohn as a tribute to the genius of the deceased. Together with Aleksandrov he acquitted himself of this task.*

Crilly and Johnson write [8]:-

> *Considering that he only had three years to devote to topology, he made his mark in his chosen field with brilliance and passion. He transformed the subject into a rich domain of modern mathematics. How much more might he there have been, had he not died so young?*

Article by: *J J O'Connor* and *E F Robertson*

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List of References (14 books/articles)

Mathematicians born in the same country

Other Web sites

1. Dictionary of Scientific Biography
2. D E Cameron (More pictures)
3. D E Cameron (Urysohn's death)
4. Mathematical Genealogy Project
5. MathSciNet Author profile
6. zbMATH entry