Karl Weierstrass

Karl Theodor Wilhelm Weierstrass (German: *Weierstraß* [ˈvaɪɐʃtʁaːs];[1] 31 October 1815 – 19 February 1897) was a German mathematician often cited as the "father of modern analysis". Despite leaving university without a degree, he studied mathematics and trained as a teacher, eventually teaching mathematics, physics, botany and gymnastics.[2]

Weierstrass formalized the definition of the continuity of a function, proved the intermediate value theorem and the Bolzano–Weierstrass theorem, and used the latter to study the properties of continuous functions on closed bounded intervals.

Contents

Karl Weierstrass

Born	31 October 1815 Ostenfelde, Province of Westphalia, Kingdom of Prussia
Died	19 February 1897 (aged 81) Berlin, Province of Brandenburg, Kingdom of Prussia
Nationality	German
Alma mater	University of Bonn Münster Academy
Known for	Weierstrass function Weierstrass product inequality (ε, δ)-definition of limit Weierstrass–Erdmann condition Weierstrass theorems Bolzano–Weierstrass theorem
Awards	PhD (Hon): University of Königsberg (1854) Copley Medal (1895)

Scientific career

Biography

Weierstrass was born in Ostenfelde, part of Ennigerloh, Province of Westphalia.[3]

Weierstrass was the son of Wilhelm Weierstrass, a government official, and Theodora Vonderforst. His interest in mathematics began while he was a gymnasium student at the Theodorianum in Paderborn. He was sent to the University of Bonn upon graduation to prepare for a government position. Because his studies were to be in the fields of law, economics, and finance, he was immediately in conflict with his hopes to study mathematics. He resolved the conflict by paying little heed to his planned course of study, but continued private study in mathematics. The outcome was to leave the university without a degree. After that he studied mathematics at the Münster Academy (which was even at this time very famous for mathematics) and his father was able to obtain a place for him in a teacher training school in Münster. Later he was certified as a teacher in that city. During this period of study, Weierstrass attended the lectures of Christoph Gudermann and became interested in elliptic functions.

In 1843 he taught in Deutsch Krone in West Prussia and since 1848 he taught at the Lyceum Hosianum in Braunsberg. Besides mathematics he also taught physics, botany, and gymnastics.[3]

Weierstrass may have had an illegitimate child named Franz with the widow of his friend Carl Wilhelm Borchardt.[4]

After 1850 Weierstrass suffered from a long period of illness, but was able to publish papers that brought him fame and distinction. The University of Königsberg conferred an honorary doctor's degree on him on 31 March 1854. In 1856 he took a chair at the Gewerbeinstitut, which later became the Technical University of Berlin. In 1864 he became professor at the Friedrich-Wilhelms-Universität Berlin, which later became the Humboldt Universität zu Berlin.

At the age of fifty-five, Weierstrass met Sofia Kovalevsky whom he tutored privately after failing to secure her admission to the University. They had a fruitful intellectual, but troubled personal relationship that "far transcended the usual teacher-student relationship". The misinterpretation of this relationship and Kovalevsky's early death in 1891 was said to have contributed to Weierstrass' later ill-health. He was immobile for the last three years of his life, and died in Berlin from pneumonia.[5]

Fields	Mathematics
Institutions	Gewerbeinstitut
Academic advisors	Christoph Gudermann
Doctoral students	Nikolai Bugaev Georg Cantor Georg Frobenius Lazarus Fuchs Wilhelm Killing Leo Königsberger Sofia Kovalevskaya Mathias Lerch Hans von Mangoldt Eugen Netto Adolf Piltz Carl Runge Arthur Schoenflies Friedrich Schottky Hermann Schwarz Ludwig Stickelberger Ernst Kötter

Mathematical contributions

Soundness of calculus

Weierstrass was interested in the soundness of calculus, and at the time, there were somewhat ambiguous definitions regarding the foundations of calculus, and hence important theorems could not be proven with sufficient rigour. While Bolzano had developed a reasonably rigorous definition of a limit as early as 1817 (and possibly even earlier) his work remained unknown to most of the mathematical community until years later, and many mathematicians had only vague definitions of limits and continuity of functions.

Delta-epsilon proofs are first found in the works of Cauchy in the 1820s.[6][7] Cauchy did not clearly distinguish between continuity and uniform continuity on an interval. Notably, in his 1821 *Cours d'analyse,* Cauchy argued that the (pointwise) limit of (pointwise) continuous functions was itself (pointwise) continuous, a statement interpreted as being incorrect by many scholars. The correct statement is rather that the *uniform* limit of continuous functions is continuous (also, the uniform limit of uniformly continuous functions is uniformly continuous). This required the concept of uniform convergence, which was first observed by Weierstrass's advisor, Christoph Gudermann, in an 1838 paper, where Gudermann noted the phenomenon but did not define it or elaborate on it. Weierstrass saw the importance of the concept, and both formalized it and applied it widely throughout the foundations of calculus.

The formal definition of continuity of a function, as formulated by Weierstrass, is as follows:

$f(x)$ is continuous at $x = x_0$ if $\forall \, \varepsilon > 0 \, \exists \, \delta > 0$ such that for every x in the domain of f, $|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$. In simple English, $f(x)$ is continuous at a point $x = x_0$ if for each x close enough to x_0, the function value $f(x)$ is very close to $f(x_0)$, where the "close enough" restriction typically depends on the desired closeness of $f(x_0)$ to $f(x)$. Using this definition, he proved the Intermediate Value Theorem. He also proved the Bolzano–Weierstrass theorem and used it to study the properties of continuous functions on closed and bounded intervals.

Calculus of variations

Weierstrass also made significant advancements in the field of calculus of variations. Using the apparatus of analysis that he helped to develop, Weierstrass was able to give a complete reformulation of the theory which paved the way for the modern study of the calculus of variations. Among the several significant axioms, Weierstrass established a necessary condition for the existence of strong extrema of variational problems. He also helped devise the Weierstrass–Erdmann condition, which gives sufficient conditions for an extremal to have a corner along a given extrema, and allows one to find a minimizing curve for a given integral.

Other analytical theorems

- Stone–Weierstrass theorem
- Casorati–Weierstrass–Sokhotski theorem
- Weierstrass's elliptic functions
- Weierstrass function
- Weierstrass M-test
- Weierstrass preparation theorem
- Lindemann–Weierstrass theorem
- Weierstrass factorization theorem
- Enneper–Weierstrass parameterization

Students

- Edmund Husserl
- Sofia Kovalevskaya
- Gösta Mittag-Leffler
- Hermann Schwarz
- Carl Johannes Thomae
- Georg Cantor

Honours and awards

The lunar crater Weierstrass and the asteroid 14100 Weierstrass are named after him. Also, there is the Weierstrass Institute for Applied Analysis and Stochastics in Berlin.

Selected works

- *Zur Theorie der Abelschen Funktionen* (1854)
- *Theorie der Abelschen Funktionen* (1856)
- *Abhandlungen-1 (http://name.umdl.umich.edu/AAN8481.0001.001)*, Math. Werke. Bd. 1. Berlin, 1894

- *Abhandlungen-2 (http://name.umdl.umich.edu/AAN8481.0002.001)*, Math. Werke. Bd. 2. Berlin, 1895
- *Abhandlungen-3 (http://name.umdl.umich.edu/AAN8481.0003.001)*, Math. Werke. Bd. 3. Berlin, 1903
- *Vorl. ueber die Theorie der Abelschen Transcendenten (http://name.umdl.umich.edu/AAN8481.0004.001)*, Math. Werke. Bd. 4. Berlin, 1902
- *Vorl. ueber Variationsrechnung (http://name.umdl.umich.edu/AAN8481.0007.001)*, Math. Werke. Bd. 7. Leipzig, 1927

See also

- List of things named after Karl Weierstrass

References

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2. Weierstrass, Karl Theodor Wilhelm. (2018). In Helicon (Ed.), *The Hutchinson unabridged encyclopedia with atlas and weather guide*. [Online]. Abington: Helicon. Available from: http://libezproxy.open.ac.uk/login?url=https://search.credoreference.com/content/entry/heliconhe/weierstrass_karl_theodor_wilhelm/0?institutionId=292 [Accessed 8 July 2018].
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4. Biermann, Kurt-R.; Schubring, Gert (1996). "Einige Nachträge zur Biographie von Karl Weierstraß. (German) [Some postscripts to the biography of Karl Weierstrass]" (http://www.ams.org/mathscinet-getitem?mr=1388786). *History of mathematics*. San Diego, CA: Academic Press. pp. 65–91.
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6. Grabiner, Judith V. (March 1983), "Who Gave You the Epsilon? Cauchy and the Origins of Rigorous Calculus" (http://www.maa.org/sites/default/files/pdf/upload_library/22/Ford/Grabiner185-194.pdf) (PDF), *The American Mathematical Monthly*, **90** (3): 185–194, doi:10.2307/2975545 (https://doi.org/10.2307%2F2975545), JSTOR 2975545 (https://www.jstor.org/stable/2975545)
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External links

- O'Connor, John J.; Robertson, Edmund F., "Karl Weierstrass" (http://www-history.mcs.st-andrews.ac.uk/Biographies/Weierstrass.html), *MacTutor History of Mathematics archive*, University of St Andrews.
- Digitalized versions of Weierstrass's original publications (http://bibliothek.bbaw.de/bibliothek-digital/digitalequellen/schriften/autoren/weierstr/) are freely available online from the library of the *Berlin Brandenburgische Akademie der Wissenschaften (http://bibliothek.bbaw.de/bibliothek-digital)*.

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