

# 云南大学数学与统计学院

## 上机实践报告

|                  |                |                   |
|------------------|----------------|-------------------|
| 课程名称：数值计算实验      | 年级：2015 级      | 上机实践成绩：           |
| 指导教师：朱娟萍         | 姓名：刘鹏          |                   |
| 上机实践名称：线性方程组的直接法 | 学号：20151910042 | 上机实践日期：2017-10-16 |
| 上机实践编号：No.01     | 组号：            | 上机实践时间：13:22      |

### 一、实验目的

1. 通过对所学的线性方程组直接求解的理论方法进行编程，提升程序编写水平；
2. 通过对理论方法的编程实验，进一步掌握理论方法的每一个细节；
3. 通过数值法求解，发现数值方法与符号方法的区别，并形成专业思维。

### 二、实验内容

1. 编程实现高斯-若尔当列主元消元法；
2. 编程实现高斯-若尔当全主元消元法；
3. 任选一种方案，Doolittle 分解或者 Crout 分解，编程实现矩阵的 LU 分解；
4. 编程实现三对角线矩阵的稀疏方式存储，然后对其进行 LU 分解。

### 三、实验平台

Windows 10 1703 Enterprise 中文版；  
Python 3.6.0；  
Wing IDE Professional 6.0.5-1 集成开发环境；  
MATLAB R2017b win64；  
AxMath 公式编辑器（辅助）；  
EndNote X8 文献管理。

### 四、实验记录与实验结果分析

#### 4.1 1 题

编程实现：用高斯-若尔当列主元消元法求下列方程的解<sup>[1]</sup>：

$$\begin{cases} x_1 + 2x_2 + x_3 = 2 \\ -2x_1 - 2x_2 - x_3 = -3 \\ 2x_1 - 3x_2 - 2x_3 = -1 \end{cases}$$

解答：

#### 4.1.1 程序代码

```
1 # filename: 3.1 ColumnPivotMethod.py
2
```

```

3  class Matrix:
4      """Abatrct class representing a Matrix.
5
6      The internal structure is two dimension list.
7      """
8      def __init__(self,m,n,mainCol):
9          """
10         self.row:          the row of the Matrix
11         self.col:          the collumn of the Matrix
12         self.CheckedRow:   if one row has been checked,
13                             it should not be checked again
14         self.body:         the internal storing structure
15         self.mainCol:      the coefficient matrix
16         """
17         self.row = m
18         self.col = n
19         self.CheckedRow = set(range(self.row))
20         self.body = [[0 for i in range(n)] for i in range(m)]
21         self.mainCol = mainCol
22
23     def getVal(self):
24         """Giving value to each element of the matrix.
25
26         Overwrite the original value zeros.
27         """
28         for i in range(self.row):
29             for j in range(self.col):
30                 self.body[i][j] = int(input())
31
32     def valid(self,e):
33         """If the two matrix are not in the same form, return false,
34         else return true.
35
36         It is useful in the next functions.
37         """
38         if self.row != e.row or self.col != e.col:
39             return False
40         else:
41             return True
42
43     def add(self,e):
44         """A methon does not used in the pivot PCA algorithm.
45
46         Maybe it will be used in other programs.
47         """
48         self.valid(e)
49         tmp = Matrix(self.row,self.col,self.mainCol)
50         for i in range(self.row): # deep copy
51             for j in range(self.col):

```

```

52         tmp.body[i][j] = self.body[i][j]
53
54     for i in range(self.row):
55         for j in range(self.col):
56             tmp.body[i][j] += e.body[i][j]
57     return tmp
58
59     def constMulti(self,const):
60         """A constant number multiple a matrix."""
61         tmp = Matrix(self.row,self.col,self.mainCol)
62         for i in range(self.row): # deep copy
63             for j in range(self.col):
64                 tmp.body[i][j] = self.body[i][j]
65
66         for i in range(self.row):
67             for j in range(self.col):
68                 tmp.body[i][j] *= const
69     return tmp
70
71     def matrixTransform(self,target_row_Number,source_row_Number,times=None):
72         """There is a big problem, every decimal number we see is stored in the
73         RAM with the binary platform.
74
75         I can import the decimal lib to solve this problem, but I did not!
76         """
77         if times == None: # special case of matrixTransform(TarRow,times)
78
79             times_tmp = source_row_Number
80             for j in range(self.col):
81                 self.body[target_row_Number][j] *= times_tmp
82             return
83         elif times == 'exchange':
84             for i in range(self.col):
85                 self.body[target_row_Number][i],\
86                 self.body[source_row_Number][i] \
87                 = self.body[source_row_Number][i],\
88                 self.body[target_row_Number][i]
89             return
90         else:
91             for i in range(self.col):
92                 self.body[target_row_Number][i] += \
93                 times * self.body[source_row_Number][i]
94
95     def Peel(self,layer):
96         """Getting part of the matrix."""
97         B = Matrix(self.row-layer,self.col-layer,self.mainCol)
98         for i in range(self.row-layer):
99             for j in range(self.col-layer):
100                 B.body[i][j] = A.body[i + layer][j + layer]

```

```

101         return B
102
103     def combine(self,B,layer):
104         """A long with method Peel."""
105         for i in range(self.row-layer):
106             for j in range(self.col-layer):
107                 A.body[i + layer][j + layer] = B.body[i][j]
108
109 def GaussJordanCol_col(A):
110     Max = 0 # initialize the variable
111     position = 0 # the row number of the maximum value
112     for i in range(min(A.row,A.mainCol)):
113
114         for j in A.CheckedRow: # getting the max and position by a flow
115             if abs(Max) <= abs(A.body[j][i]):
116                 Max = A.body[j][i]
117                 position = j
118
119         A.CheckedRow.remove(position) # this row should not be checked again
120
121         A.matrixTransform(position, 1 / Max)
122
123         for j in range(A.row):
124             if j != position:
125                 A.matrixTransform(j,position,-1 * A.body[j][i])
126
127         Max = 0 # reinitialize the variables
128         position = 0
129
130     begin = 0
131     for j in range(A.mainCol):
132         for i in range(A.row):
133             if A.body[i][j] == 1:
134                 A.matrixTransform(begin,i,'exchange')
135                 begin += 1
136     return A
137
138 """-----my Main Function-----"""
139
140 A = Matrix(3,6,3)
141 A.body[0] = [1,2,3,1,0,0]
142 A.body[1] = [2,4,5,0,1,0]
143 A.body[2] = [3,5,6,0,0,1]
144
145 print('The original matrix is shown below:\n')
146 for i in range(A.row):
147     for j in range(A.col):
148         print(int(A.body[i][j]),'\t',end='')
149     print('')

```

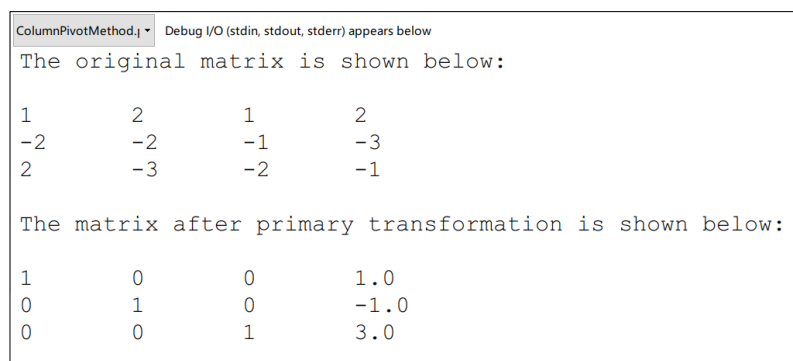
```

150
151 A = GaussJordanCol_col(A)
152
153 print('\nThe matrix after primary transformation is shown below:\n')
154 for i in range(A.row):
155     for j in range(A.col):
156         if A.body[i][j] == 0.0:
157             A.body[i][j] = 0
158         if A.body[i][j] == -0.0:
159             A.body[i][j] = 0
160         if A.body[i][j] - round(A.body[i][j]) == 0:
161             A.body[i][j] = round(A.body[i][j])
162         print(round(A.body[i][j],4), '\t', end= ' ')
163     print(' ')

```

Code Box 1 列主元消元法求线性方程组的解

#### 4.1.2 输出结果



```

ColumnPivotMethod.j Debug I/O (stdin, stdout, stderr) appears below
The original matrix is shown below:

1      2      1      2
-2     -2     -1     -3
2      -3     -2     -1

The matrix after primary transformation is shown below:

1      0      0      1.0
0      1      0     -1.0
0      0      1      3.0

```

输出结果 1

#### 4.1.3 代码分析

本 Python 代码采用面向对象<sup>[2]</sup>的方式写成，首先是定义了一个 **Matrix** 类，用来存储矩阵，该类中的所有元素都是 **public** 的，这样设置是为了以后的编程调用方便。

**Matrix** 的实例 **A** 是一个三行四列的矩阵，其中最后一列是方程组中等号右边的元素，所以这个增广矩阵的计算应该遵循线性代数的法则，即在寻找列主元的时候，不能越界去增广列寻找；同样，一个已经找出过列主元的行，不应该再次存在列主元，所以要在以后的查找中剔除这一行。

最后，输出的结果应该保持美观性，所以按照系数矩阵经过变换之后为单位矩阵的样式进行了重新排序。

值得指出的是，**Matrix** 的 **matrixTransform** 成员函数是用多态的方式写的，可以满足行数乘、行交换、

行倍加等所有的矩阵初等变换方式。

另外，由于本线性方程组一定存在唯一解，所以在排序与寻找主元的过程中，并没有加以判断，所以这并不是一个通用的程序。

## 4.2 2 题

编程实现：请用高斯-若尔当全主元消元法求下列矩阵的逆矩阵<sup>[1]</sup>：

$$B = \begin{bmatrix} 2 & 1 & -3 & -1 \\ 3 & 1 & 0 & 7 \\ -1 & 2 & 4 & -2 \\ 1 & 0 & -1 & 5 \end{bmatrix}$$

### 4.2.1 程序代码

```

1  # filename: 3.2 CompletePivotMethod.py
2
3  class Matrix:
4      """Abatrct class representing a Matrix.
5
6      The internal structure is two dimension list.
7      """
8      def __init__(self,m,n,mainCol):
9          """
10             self.row:          the row of the Matrix
11             self.col:          the collumn of the Matrix
12             self.CheckedRow:    if one row has been checked,
13                                 it should not be checked again
14             self.body:          the internal storing structure
15             self.mainCol:       the coefficient matrix
16             """
17             self.row = m
18             self.col = n
19             self.CheckedRow = set(range(self.row))
20             self.body = [[0 for i in range(n)] for i in range(m)]
21             self.mainCol = mainCol
22
23     def getVal(self):
24         """Giving value to each element of the matrix.
25
26         Overwrite the original value zeros.
27         """
28         for i in range(self.row):
29             for j in range(self.col):
30                 self.body[i][j] = int(input())
31
32     def valid(self,e):
33         """If the two matrix are not in the same form, return false,
34         else return true.
35

```

```

36         It is useful in the next functions.
37         """
38         if self.row != e.row or self.col != e.col:
39             return False
40         else:
41             return True
42
43     def add(self,e):
44         """A methon does not used in the pivot PCA algorithm.
45
46         Maybe it will be used in other programs.
47         """
48         self.valid(e)
49         tmp = Matrix(self.row,self.col,self.mainCol)
50         for i in range(self.row): # deep copy
51             for j in range(self.col):
52                 tmp.body[i][j] = self.body[i][j]
53
54         for i in range(self.row):
55             for j in range(self.col):
56                 tmp.body[i][j] += e.body[i][j]
57         return tmp
58
59     def constMulti(self,const):
60         """A constant number multiple a matrix."""
61         tmp = Matrix(self.row,self.col,self.mainCol)
62         for i in range(self.row): # deep copy
63             for j in range(self.col):
64                 tmp.body[i][j] = self.body[i][j]
65
66         for i in range(self.row):
67             for j in range(self.col):
68                 tmp.body[i][j] *= const
69         return tmp
70
71     def matrixTransform(self,target_row_Number,source_row_Number,times=None):
72         """There is a big problem, every decimal number we see is stored in the
73         RAM with the binary platform.
74
75         I can import the decimal lib to solve this problem, but I did not!
76         """
77         if times == None: # special case of matrixTransform(TarRow,times)
78
79             times_tmp = source_row_Number
80             for j in range(self.col):
81                 self.body[target_row_Number][j] *= times_tmp
82             return
83         elif times == 'exchange':
84             for i in range(self.col):

```

```

85         self.body[target_row_Number][i],\
86         self.body[source_row_Number][i] \
87         = self.body[source_row_Number][i],\
88         self.body[target_row_Number][i]
89     return
90 else:
91     for i in range(self.col):
92         self.body[target_row_Number][i] += \
93         times * self.body[source_row_Number][i]
94
95 def Peel(self,layer):
96     """Getting part of the matrix."""
97     B = Matrix(self.row-layer,self.col-layer,self.mainCol)
98     for i in range(self.row-layer):
99         for j in range(self.col-layer):
100             B.body[i][j] = A.body[i + layer][j + layer]
101     return B
102
103 def combine(self,B,layer):
104     """A long with method Peel."""
105     for i in range(self.row-layer):
106         for j in range(self.col-layer):
107             A.body[i + layer][j + layer] = B.body[i][j]
108
109 def GaussJordanCol_complete(A):
110     Max = 0 # initialize the variable
111     position_row = 0 # the row number of the maximum value
112     position_col = 0
113
114     for i in range(A.row):
115
116         for j in A.CheckedRow:
117             for k in range(A.mainCol): # not in all the columns
118                 if abs(Max) <= abs(A.body[j][k]):
119                     Max = A.body[j][k]
120                     position_row = j
121                     position_col = k
122
123     A.matrixTransform(position_row, 1 / Max)
124
125     A.CheckedRow.remove(position_row)
126
127     for j in range(A.row):
128         if j != position_row:
129             A.matrixTransform(j,position_row,-1 * A.body[j][position_col])
130
131     Max = 0
132     position_row = 0
133     position_col = 0

```



```

134
135     begin = 0
136     for j in range(A.mainCol):
137         for i in range(A.row):
138             if A.body[i][j] == 1:
139                 A.matrixTransform(begin,i,'exchange')
140                 begin += 1
141     return A
142
143     """-----my Main Function-----"""
144
145     A = Matrix(4,8,4)
146     A.body[0] = [2,1,-3,1,1,0,0,0]
147     A.body[1] = [3,1,0,7,0,1,0,0]
148     A.body[2] = [-1,2,4,-2,0,0,1,0]
149     A.body[3] = [1,0,-1,5,0,0,0,1]
150
151     print('The original matrix is shown below:\n')
152     for i in range(A.row):
153         for j in range(A.col):
154             print(int(A.body[i][j]),'\t',end='')
155         print('')
156
157     A = GaussJordanCol_complete(A)
158
159     print('\nThe matrix after primary transformation is shown below:\n')
160     for i in range(4):
161         for j in range(8):
162             if A.body[i][j] == 0.0:
163                 A.body[i][j] = 0
164             if A.body[i][j] == -0.0:
165                 A.body[i][j] = 0
166             if A.body[i][j] - round(A.body[i][j]) == 0:
167                 A.body[i][j] = round(A.body[i][j])
168             print(round(A.body[i][j],4),'\t',end='')
169     print('')

```

Code Box 2 全主元消元法求矩阵的逆

#### 4.2.2 输出结果

```

CompletePivotMetho Debug I/O (stdin, stdout, stderr) appears below
The original matrix is shown below:

2      1      -3      1      1      0      0      0
3      1      0      7      0      1      0      0
-1     2      4      -2     0      0      1      0
1      0     -1      5      0      0      0      1

The matrix after primary transformation is shown below:

1      0      0      0     -0.0506     0.5823     -0.2658     -0.9114
0      1      0      0     0.4177     -0.3038     0.443      0.519
0      0      1      0     -0.2405     0.2658     -0.0127     -0.3291
0      0      0      1     -0.038     -0.0633     0.0506     0.3165

```

输出结果 2

### 4.2.3 代码分析

此段代码是在 Code Box 1 的基础上略微修改而得的，由于全主元的位置没有固定性，所以要把每一次找到的全主元的行列坐标都存储下来。其余部分与 Code Box 1 相同。

### 4.3 3 题

编程实现：利用 LU 分解法（Doolittle），求  $[x_1, x_2, x_3, x_4]^T$  的数值解。

$$\begin{bmatrix} 12 & -3 & 3 & 4 \\ -18 & 3 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 15 \\ -15 \\ 6 \\ 2 \end{bmatrix}$$

#### 4.3.1 程序代码

```

1  # filename: 3.3 LU Decomposition.py
2
3  class Matrix:
4      """Abatrct class representing a Matrix.
5
6      The internal structure is two dimension list.
7      """
8      def __init__(self,m,n,mainCol):
9          """
10             self.row:          the row of the Matrix
11             self.col:          the collumn of the Matrix
12             self.CheckedRow:    if one row has been checked,
13                                it should not be checked again
14             self.body:          the internal storing structure
15             self.mainCol:       the coefficient matrix
16             """
17             self.row = m
18             self.col = n
19             self.CheckedRow = set(range(self.row))

```

```

20     self.body = [[0 for i in range(n)] for i in range(m)]
21     self.mainCol = mainCol
22
23     def getVal(self):
24         """Giving value to each element of the matrix.
25
26         Overwrite the original value zeros.
27         """
28         for i in range(self.row):
29             for j in range(self.col):
30                 self.body[i][j] = int(input())
31
32     def valid(self,e):
33         """If the two matrix are not in the same form, return false,
34         else return true.
35
36         It is useful in the next functions.
37         """
38         if self.row != e.row or self.col != e.col:
39             return False
40         else:
41             return True
42
43     def add(self,e):
44         """A methon does not used in the pivot PCA algorithm.
45
46         Maybe it will be used in other programs.
47         """
48         self.valid(e)
49         tmp = Matrix(self.row,self.col,self.mainCol)
50         for i in range(self.row): # deep copy
51             for j in range(self.col):
52                 tmp.body[i][j] = self.body[i][j]
53
54         for i in range(self.row):
55             for j in range(self.col):
56                 tmp.body[i][j] += e.body[i][j]
57         return tmp
58
59     def constMulti(self,const):
60         """A constant number multiple a matrix."""
61         tmp = Matrix(self.row,self.col,self.mainCol)
62         for i in range(self.row): # deep copy
63             for j in range(self.col):
64                 tmp.body[i][j] = self.body[i][j]
65
66         for i in range(self.row):
67             for j in range(self.col):
68                 tmp.body[i][j] *= const

```

```

69         return tmp
70
71     def matrixTransform(self, target_row_Number, source_row_Number, times=None):
72         """There is a big problem, every decimal number we see is stored in the
73         RAM with the binary platform.
74
75         I can import the decimal lib to solve this problem, but I did not!
76         """
77         if times == None:    # special case of matrixTransform(TarRow, times)
78
79             times_tmp = source_row_Number
80             for j in range(self.col):
81                 self.body[target_row_Number][j] *= times_tmp
82             return
83         elif times == 'exchange':
84             for i in range(self.col):
85                 self.body[target_row_Number][i], \
86                 self.body[source_row_Number][i] \
87                 = self.body[source_row_Number][i], \
88                 self.body[target_row_Number][i]
89             return
90         else:
91             for i in range(self.col):
92                 self.body[target_row_Number][i] += \
93                 times * self.body[source_row_Number][i]
94
95     def Peel(self, layer):
96         """Getting part of the matrix."""
97         B = Matrix(self.row-layer, self.col-layer, self.mainCol)
98         for i in range(self.row-layer):
99             for j in range(self.col-layer):
100                 B.body[i][j] = A.body[i + layer][j + layer]
101         return B
102
103     def combine(self, B, layer):
104         """A long with method Peel."""
105         for i in range(self.row-layer):
106             for j in range(self.col-layer):
107                 A.body[i + layer][j + layer] = B.body[i][j]
108
109     def LU_Decompose_Doolittle(A):
110         """Decompose matrix A with Dolittle method.
111
112         In this function, I use devide and conquer method.
113         """
114         for i in range(A.col-1):
115             if i == 0:
116                 for j in range(1, A.row):
117                     A.body[j][i] = A.body[j][i] / A.body[0][0]

```

```

118         else:
119             for j in range(A.col-i):
120                 for k in range(i):
121                     A.body[i][i+j] = A.body[i][i+j] - \
122                         A.body[i][k] * A.body[k][i+j]
123             for j in range(A.row-i-1):
124                 for k in range(i):
125                     A.body[i+j+1][i] = A.body[i+j+1][i] - \
126                         A.body[i+j+1][k] * A.body[k][i]
127                     A.body[i+j+1][i] = A.body[i+j+1][i] / A.body[i][i]
128         return A
129
130 """-----my Main Function-----"""
131
132 A = Matrix(4,5,4)
133 A.body[0] = [12,-3,3,4,15]
134 A.body[1] = [-18,3,-1,-1,-15]
135 A.body[2] = [1,1,1,1,6]
136 A.body[3] = [3,1,-1,1,2]
137
138 print('The original matrix is shown below:\n')
139 for i in range(A.row):
140     for j in range(A.col):
141         print(int(A.body[i][j]),'\t',end='')
142     print('')
143
144 A = LU_Decompose_Doolittle(A)
145
146 B = Matrix(4,4,4)
147 for i in range(A.row):
148     for j in range(A.col):
149         B.body[i][j] = A.body[i][j]
150
151 L1 = Matrix(4,4,4)
152
153 for i in range(L1.col):
154     for j in range(L1.row-i):
155         L1.body[j+i][i] = B.body[j+i][i]
156 for i in range(L1.row):
157     L1.body[i][i] = 1
158
159 U = L1.constMulti(-1).add(B)    # I am the best! professional!
160 for i in range(L1.row):
161     U.body[i][i] += 1
162
163 print('-----\nA:\n')
164 for i in range(4):
165     for j in range(4):
166         if B.body[i][j] == 0.0:

```

```

167         B.body[i][j] = 0
168     if B.body[i][j] == -0.0:
169         B.body[i][j] = 0
170     if B.body[i][j] - round(B.body[i][j]) == 0:
171         B.body[i][j] = round(B.body[i][j])
172     print(round(B.body[i][j],2), '\t', end=' ')
173     print('')
174
175 print('-----\nU:\n')
176
177 for i in range(4):
178     for j in range(4):
179         if U.body[i][j] == 0.0:
180             U.body[i][j] = 0
181         if U.body[i][j] == -0.0:
182             A.body[i][j] = 0
183         if U.body[i][j] - round(U.body[i][j]) == 0:
184             U.body[i][j] = round(U.body[i][j])
185         print(round(U.body[i][j],2), '\t', end=' ')
186     print('')
187
188 print('-----\nL:\n')
189
190 for i in range(4):
191     for j in range(4):
192         if L1.body[i][j] == 0.0:
193             L1.body[i][j] = 0
194         if L1.body[i][j] == -0.0:
195             L1.body[i][j] = 0
196         if L1.body[i][j] - round(L1.body[i][j]) == 0:
197             L1.body[i][j] = round(L1.body[i][j])
198         print(round(L1.body[i][j],2), '\t', end=' ')
199     print('')

```

Code Box 3 LU 分解

### 4.3.2 输出结果

```

3.3 LU Decomposition.py (pid 17808) (no process) - Debug process terminated
The original matrix is shown below:

12      -3      3      4      15
-18     3      -1     -1     -15
1       1       1       1       6
3       1      -1       1       2
-----
A:

12      -3      3      4
-1.5    -1.5    3.5    5
0.08    -0.83   3.67   4.83
0.25    -1.17   0.64   2.76
-----
U:

12      -3      3      4
0       -1.5    3.5    5
0        0      3.67   4.83
0        0       0      2.76
-----
L:

1       0       0       0
-1.5    1       0       0
0.08    -0.83   1       0
0.25    -1.17   0.64    1

```

输出结果 3

通过生成的 LU 矩阵，可以轻易得到， $[x_1, x_2, x_3, x_4]^T$  的数值解为  $[1, 2, 3, 0]$ ，这也是解析解。

### 4.3.3 代码分析

代码仍旧是基于面向对象技术，程序的算法是基于紧凑法求解 LU 矩阵的形式化做法。

如此编程，相对而言比较节约内存。

## 4.4 4 题

编程实现：用追赶法解下列严格对角优势的三对角线方程组，要求用稀疏格式存储矩阵，主内存占用为  $O(n)$ ，其中  $n$  为矩阵的行数。

$$\begin{bmatrix} 4 & -1 & & & \\ -1 & 4 & -1 & & \\ & -1 & 4 & -1 & \\ & & -1 & 4 & -1 \\ & & & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 200 \\ 200 \\ 100 \end{bmatrix}$$

### 4.4.1 程序代码

```
1 # filename: 3.4 Seize Method.py
```

```

2
3 class ThreeTriangleMatrix:
4     def __init__(self,a,b,c,m):
5         self.a = a
6         self.b = b
7         self.c = c
8         self.row = m
9         self.col = m
10
11     def calculate(self):
12         alpha = []
13         beta = []
14         gamma = self.a
15
16         alpha.append(self.b)
17         beta.append(self.c / alpha[0])
18
19         for i in range(1,self.row-1):
20             alpha.append(self.b - gamma * beta[i-1])
21             beta.append(self.c / alpha[i])
22
23         alpha.append(self.b - gamma * beta[-1])
24
25         return [gamma,alpha,beta]
26
27 A = ThreeTriangleMatrix(-1,4,-1,5)
28 A.calculate()
29
30 L = [[0 for i in range(A.row)] for j in range(A.col)]
31
32 for i in range(A.row-1):
33     L[i+1][i] = A.calculate()[0]
34     L[i][i] = A.calculate()[1][i]
35
36 L[A.row-1][A.row-1] = (A.calculate()[1])[A.row-1]
37
38 U1 = [[0 for i in range(A.row)] for j in range(A.col)]
39 for i in range(A.row):
40     U1[i][i] = 1
41
42 for i in range(A.row-1):
43     U1[i][i+1] = A.calculate()[2][i]
44
45 print('-----\nU1:\n')
46 for i in range(A.row):
47     for j in range(A.col):
48         print(round(U1[i][j],2),'\t',end='')
49     print(' ')
50

```



```

51 print('-----\nL:]\n')
52 for i in range(A.row):
53     for j in range(A.col):
54         print(round(L[i][j],2),'\t',end='')
55     print('')

```

Code Box 4 追赶公式求三对角线矩阵的 LU 分解

#### 4.4.2 输出结果

```

-----
U1:
1      -0.25  0      0      0
0       1     -0.27  0      0
0       0      1    -0.27  0
0       0      0      1    -0.27
0       0      0      0      1
-----
L:]
4       0      0      0      0
-1     3.75  0      0      0
0      -1     3.73  0      0
0       0     -1     3.73  0
0       0      0     -1     3.73

```

输出结果 4

做得了 LU 分解，将数据导入 MATLAB，可以迅速求出  $[x_1, x_2, x_3, x_4, x_5]^T$  的数值解为  $[46.15, 84.61, 92.30, 84.61, 46.15]^T$

#### 4.4.3 代码分析

此程序机械化比较高，所以比较简单，就是简单地翻译数学语言。

### 五、实验体会

在此次实验中，集中利用了 MATLAB 与 Python 3 进行程序设计。

主程序是采用 Python 3 进行编写，验算求解是利用 MATLAB 进行。当写的程序足够多之后，就会发现，真正重要的东西是数学思维，编程完全决定于数学思维的深度。如果一味追求编程的快感，很容易在这虚无的成就感中迷失自己。所以编程不如不编，能透彻理解数学思维已经很不容易了。

但是从另一个角度看，编程对数学思维又有很强检验作用，尽管在数学思维之外，高技巧度编程又是另外广阔的一片天地。通过适度的编程，选择一类题目的典型例子进行编程，就可以在一个程序的设计、实现时间里，考验自己的一批数学思维的掌握牢固程度。

MATLAB 的强大之处不在于它编程方便，而是两方面：友好的交互式控制台，众多高质量的程序包。正是这两点，使得 MATLAB 在工业中得到广泛的应用。但是在数值计算的学习上，利用 Python 或许是更好的选择。Python 的数值计算依赖于数组，所以对于循环的考验非常高。Python 不像 MATLAB 的基本运算都是基于矩阵那样地方便，而正是这不方便这一点，使得 Python 这个看起来更加拙劣的工具更适合新手。

除了工具，更重要的一点是编程。这四个程序的编写基本上没有遇到困难。如果说在理论方面已经理解得足够透彻，那么在编程上不应该存在任何思路困难。数值计算以及数学建模中的程序，很大程度上都是数学语言的直接翻译。这一点与 DSA 设计有相当大的差别。现在出现的一个问题是，编程趋向 IDD 模式，即 IDE Driven，没有了编译器就不会单步调试，进而很难发现哪里出了差错。这不能说是思路理解得不到位，因为如果单纯是这个原因，那么在纸上演算就会遇到问题。但是很显然不是这样。以后的一个突破，可能就在这里——直接用文本编辑器进行代码书写，然后用 IDE 进行检验。

最后需要说明，因为有很多东西没有或者不能调用，所以这四个程序基本上是为题目量身定做。虽然面向对象设计里的几个成员函数具有通用性，但是总得来看，一些全局函数还是缺乏相应的判断。当然，如果不能运行，就从侧面说明输入的矩阵不合格。所以，这四个程序的通用性有待进一步挖掘，数据与程序能彻底分开，是判断程序通用性强弱的主要依据之一。

## 六、参考文献

- [1] 金一庆, 陈越, 王冬梅. 数值方法 [M]. 北京: 机械工业出版社, 2000.2.
- [2] GOODRICH M T, TAMASSIA R, GOLDWASSER M H. Data Structures and Algorithms in Python [M]. U.S: Wiley & Sons, Inc., 2013.