# 云南大学数学与统计学院 上机实践报告

课程名称:数值计算实验	<b>年级:</b> 2015 级	上机实践成绩:
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上机实践名称:线性方程组的直接法	学号: 20151910042	上机实践日期: 2017-10-16
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## 一、实验目的

- 1. 通过对所学的线性方程组直接求解的理论方法进行编程,提升程序编写水平;
- 2. 通过对理论方法的编程实验,进一步掌握理论方法的每一个细节;
- 3. 通过数值法求解,发现数值方法与符号方法的区别,并形成专业思维。

## 二、实验内容

- 1. 编程实现高斯-若尔当列主元消元法;
- 2. 编程实现高斯-若尔当全主元消元法;
- 3. 任选一种方案, Doolittle 分解或者 Crout 分解, 编程实现矩阵的 LU 分解;
- 4. 编程实现三对角线矩阵的稀疏方式存储, 然后对其进行 LU 分解。

# 三、实验平台

Windows 10 1703 Enterprise 中文版;

Python 3.6.0;

Wing IDE Professional 6.0.5-1 集成开发环境;

MATLAB R2017b win64:

AxMath 公式编辑器:

EndNote X8 文献管理。

## 四、实验记录与实验结果分析

### 1题

编程实现:用高斯-若尔当列主元消元法求下列方程的解[1]:

$$\begin{cases}
 x_1 + 2x_2 + x_3 = 2 \\
 -2x_1 - 2x_2 - x_3 = -3 \\
 2x_1 - 3x_2 - 2x_3 = -1
\end{cases}$$
(1.1)

### 解答:

```
1  # filename: 3.1 ColumnPivotMethod.py
2
3  class Matrix:
4   """Abatrct class representing a Matrix.
5
6   The internal structure is two dimension list.
7   """
8   def __init__(self,m,n,mainCol):
9   """
```

```
10
             self.row:
                                 the row of the Matrix
11
             self.col:
                                 the collumn of the Matrix
12
             self.CheckedRow:
                                 if one row has been checked,
13
                                it should not be checked again
14
             self.body:
                                 the internal storing structure
15
             self.mainCol:
                                 the coefficient matrix
16
             self.row = m
17
18
             self.col = n
19
             self.CheckedRow = set(range(self.row))
20
             self.body = [[0 for i in range(n)] for i in range(m)]
21
             self.mainCol = mainCol
22
23
         def getVal(self):
             """Giving value to each element of the matrix.
24
25
26
             Overwrite the original value zeros.
27
28
             for i in range(self.row):
29
                 for j in range(self.col):
30
                    self.body[i][j] = int(input())
31
32
         def valid(self,e):
             """If the two matrix are not in the same form, return false,
33
34
             else return true.
35
            It is useful in the next functions.
36
37
             if self.row != e.row or self.col != e.col:
38
39
                 return False
40
             else:
41
                return True
42
43
         def add(self,e):
             """A methon does not used in the pivot PCA algorithm.
44
45
46
             Maybe it will be used in other programs.
47
48
             self.valid(e)
49
             tmp = Matrix(self.row, self.col, self.mainCol)
50
             for i in range(self.row): # deep copy
51
                for j in range(self.col):
52
                    tmp.body[i][j] = self.body[i][j]
53
54
             for i in range(self.row):
55
                 for j in range(self.col):
56
                    tmp.body[i][j] += e.body[i][j]
57
             return tmp
58
59
         def constMulti(self,const):
             """A constant number multiple a matrix."""
60
61
             tmp = Matrix(self.row, self.col, self.mainCol)
62
             for i in range(self.row): # deep copy
63
                 for j in range(self.col):
64
                    tmp.body[i][j] = self.body[i][j]
65
66
             for i in range(self.row):
67
                 for j in range(self.col):
```

```
68
                    tmp.body[i][j] *= const
69
             return tmp
70
71
         def matrixTransform(self,target_row_Number,source_row_Number,times=None):
72
             """There is a big problem, every decimal number we see is stored in the
73
             RAM with the binary platform.
74
75
             I can import the decimal lib to solve this problem, but I did not!
76
77
             if times == None:
                                  # special case of matrixTransform(TarRow,times)
78
79
                times_tmp = source_row_Number
80
                 for j in range(self.col):
81
                    self.body[target_row_Number][j] *= times_tmp
82
                 return
83
             elif times == 'exchange':
84
                for i in range(self.col):
85
                    self.body[target_row_Number][i],\
86
                        self.body[source row Number][i] \
87
                        = self.body[source_row_Number][i],\
88
                        self.body[target_row_Number][i]
89
                 return
90
             else:
91
                 for i in range(self.col):
92
                    self.body[target_row_Number][i] += \
93
                    times * self.body[source row Number][i]
94
95
         def Peel(self,layer):
96
             """Getting part of the matrix."""
97
             B = Matrix(self.row-layer, self.col-layer, self.mainCol)
98
             for i in range(self.row-layer):
99
                 for j in range(self.col-layer):
100
                    B.body[i][j] = A.body[i + layer][j + layer]
101
             return B
102
103
         def combine(self,B,layer):
104
             """A long with method Peel."""
105
             for i in range(self.row-layer):
106
                 for j in range(self.col-layer):
107
                    A.body[i + layer][j + layer] = B.body[i][j]
108
109
     def GaussJordanCol_col(A):
                                        # initialize the variable
110
         Max = 0
111
         position = 0
                                        # the row number of the maximum value
112
         for i in range(min(A.row, A.mainCol)):
113
114
             for j in A.CheckedRow:
                                         # getting the max and position by a flow
115
                 if abs(Max) <= abs(A.body[j][i]):</pre>
116
                    Max = A.body[j][i]
117
                    position = j
118
119
             A.CheckedRow.remove(position) # this row should not be checked again
120
121
             A.matrixTransform(position, 1 / Max)
122
123
             for j in range(A.row):
124
                 if j != position:
125
                    A.matrixTransform(j,position,-1 * A.body[j][i])
```

```
126
                           # reinitialize the variables
127
            Max = 0
128
            position = 0
129
130
        begin = 0
131
        for j in range(A.mainCol):
            for i in range(A.row):
132
133
               if A.body[i][j] == 1:
134
                   A.matrixTransform(begin,i,'exchange')
135
                   begin += 1
136
        return A
137
     """-----"""
138
139
140 A = Matrix(3,6,3)
141 A.body[0] = [1,2,3,1,0,0]
142 A.body[1] = [2,4,5,0,1,0]
143 A.body[2] = [3,5,6,0,0,1]
144
145 print('The original matrix is shown below:\n')
146
    for i in range(A.row):
147
        for j in range(A.col):
148
            print(int(A.body[i][j]), '\t', end='')
149
        print('')
150
151
    A = GaussJordanCol col(A)
152
153
     print('\nThe matrix after primary transformation is shown below:\n')
154
     for i in range(A.row):
155
        for j in range(A.col):
156
            if A.body[i][j] == 0.0:
157
               A.body[i][j] = 0
158
            if A.body[i][j] == -0.0:
               A.body[i][j] = 0
159
160
            if A.body[i][j] - round(A.body[i][j]) == 0:
               A.body[i][j] = round(A.body[i][j])
161
162
            print(round(A.body[i][j],4),'\t',end='')
163
        print('')
```

Code Box 1 列主元消元法求线性方程组的解

### 输出结果:

```
ColumnPivotMethod.| • Debug I/O (stdin, stdout, stderr) appears below
The original matrix is shown below:
                    1
-2
          -2
                    -1
                              -3
                              -1
2
          -3
                    -2
The matrix after primary transformation is shown below:
          0
                              1.0
0
          1
                    0
                              -1.0
0
          0
                              3.0
                    1
```

输出结果 1

# 代码分析:

本 Python 代码采用面向对象<sup>[2]</sup>的方式写成,首先是定义了一个 Matrix 类,用来存储矩阵,该类中的所有元素都是 public 的,这样设置是为了以后的编程调用方便。

Matrix 的实例 A 是一个三行四列的矩阵,其中最后一列是方程组中等号右边的元素,所以这个增广矩阵的计算应该遵循线性代数的法则,即在寻找列主元的时候,不能越界去增广列寻找;同样,一个已经找出过列主元的行,不应该再次存在列主元,所以要在以后的查找中剔除这一行。

最后,输出的结果应该保持美观性,所以按照系数矩阵经过变换之后为单位矩阵的样式进行了重新排序。

值得指出的是,Matrix 的 matrix Transform 成员函数是用多态的方式写的,可以满足行数乘、行交换、行倍加等所有的矩阵初等变换方式。

另外,由于本线性方程组一定存在唯一解,所以在排序与寻找主元的过程中,并没有加以判断,所以这并不是一个通 用的程序。

## 2题

编程实现:请用高斯-若尔当全主元消元法求下列矩阵的逆矩阵[1]:

$$B = \begin{bmatrix} 2 & 1 & -3 & -1 \\ 3 & 1 & 0 & 7 \\ -1 & 2 & 4 & -2 \\ 1 & 0 & -1 & 5 \end{bmatrix}$$
 (2.1)

```
1
     # filename: 3.2 CompletePivotMethod.py
2
3
     class Matrix:
        """Abatrct class representing a Matrix.
4
5
6
         The internal structure is two dimension list.
7
8
         def __init__(self,m,n,mainCol):
9
10
            self.row:
                                the row of the Matrix
11
             self.col:
                                the collumn of the Matrix
12
            self.CheckedRow:
                                if one row has been checked,
13
                               it should not be checked again
14
            self.body:
                                the internal storing structure
15
            self.mainCol:
                                the coefficient matrix
16
17
             self.row = m
18
             self.col = n
19
             self.CheckedRow = set(range(self.row))
20
             self.body = [[0 for i in range(n)] for i in range(m)]
             self.mainCol = mainCol
21
22
23
         def getVal(self):
24
            """Giving value to each element of the matrix.
25
26
            Overwrite the original value zeros.
27
28
            for i in range(self.row):
29
                for j in range(self.col):
30
                    self.body[i][j] = int(input())
31
32
         def valid(self,e):
            """If the two matrix are not in the same form, return false,
33
            else return true.
34
35
36
            It is useful in the next functions.
37
38
             if self.row != e.row or self.col != e.col:
39
                return False
40
             else:
```

```
41
                return True
42
43
         def add(self,e):
             """A methon does not used in the pivot PCA algorithm.
44
45
46
            Maybe it will be used in other programs.
            0.00
47
48
            self.valid(e)
49
            tmp = Matrix(self.row,self.col,self.mainCol)
50
            for i in range(self.row): # deep copy
51
                for j in range(self.col):
52
                    tmp.body[i][j] = self.body[i][j]
53
54
            for i in range(self.row):
55
                for j in range(self.col):
56
                    tmp.body[i][j] += e.body[i][j]
57
            return tmp
58
         def constMulti(self,const):
59
             """A constant number multiple a matrix."""
60
61
            tmp = Matrix(self.row,self.col,self.mainCol)
62
            for i in range(self.row): # deep copy
63
                for j in range(self.col):
64
                    tmp.body[i][j] = self.body[i][j]
65
            for i in range(self.row):
66
67
                for j in range(self.col):
68
                    tmp.body[i][j] *= const
69
            return tmp
70
71
         def matrixTransform(self,target_row_Number,source_row_Number,times=None):
72
             """There is a big problem, every decimal number we see is stored in the
73
            RAM with the binary platform.
74
75
            I can import the decimal lib to solve this problem, but I did not!
76
77
            if times == None:
                                  # special case of matrixTransform(TarRow,times)
78
79
                times tmp = source row Number
80
                for j in range(self.col):
81
                    self.body[target_row_Number][j] *= times_tmp
82
                return
83
            elif times == 'exchange':
84
                for i in range(self.col):
85
                    self.body[target row Number][i],\
86
                        self.body[source row Number][i] \
87
                        = self.body[source_row_Number][i],\
88
                        self.body[target_row_Number][i]
89
                return
```

```
90
             else:
91
                 for i in range(self.col):
92
                    self.body[target_row_Number][i] += \
93
                    times * self.body[source_row_Number][i]
94
95
         def Peel(self,layer):
96
             """Getting part of the matrix."""
97
             B = Matrix(self.row-layer,self.col-layer,self.mainCol)
98
             for i in range(self.row-layer):
99
                for j in range(self.col-layer):
100
                    B.body[i][j] = A.body[i + layer][j + layer]
101
             return B
102
         def combine(self,B,layer):
103
             """A long with method Peel."""
104
105
            for i in range(self.row-layer):
106
                for j in range(self.col-layer):
107
                    A.body[i + layer][j + layer] = B.body[i][j]
108
     def GaussJordanCol complete(A):
109
110
         Max = 0
                                        # initialize the variable
                                        # the row number of the maximum value
111
         position_row = 0
112
         position_col = 0
113
114
         for i in range(A.row):
115
             for j in A.CheckedRow:
116
117
                 for k in range(A.mainCol): # not in all the columns
118
                    if abs(Max) <= abs(A.body[j][k]):</pre>
119
                        Max = A.body[j][k]
120
                        position_row = j
121
                        position_col = k
122
123
             A.matrixTransform(position_row, 1 / Max)
124
125
             A.CheckedRow.remove(position row)
126
127
             for j in range(A.row):
128
                if j != position_row:
129
                    A.matrixTransform(j,position_row,-1 * A.body[j][position_col])
130
131
             Max = 0
132
             position row = 0
133
             position_col = 0
134
135
         begin = 0
136
         for j in range(A.mainCol):
137
             for i in range(A.row):
138
                 if A.body[i][j] == 1:
```

```
139
                   A.matrixTransform(begin,i,'exchange')
140
                   begin += 1
141
        return A
142
     """-----my Main Function-----"""
143
144
145 A = Matrix(4,8,4)
146 A.body[0] = [2,1,-3,1,1,0,0,0]
147 A.body[1] = [3,1,0,7,0,1,0,0]
148 A.body[2] = [-1,2,4,-2,0,0,1,0]
149 A.body[3] = [1,0,-1,5,0,0,0,1]
150
151 print('The original matrix is shown below:\n')
152 for i in range(A.row):
153
        for j in range(A.col):
154
            print(int(A.body[i][j]),'\t',end='')
155
        print('')
156
157
    A = GaussJordanCol_complete(A)
158
159
    print('\nThe matrix after primary transformation is shown below:\n')
160
   for i in range(4):
161
        for j in range(8):
162
            if A.body[i][j] == 0.0:
163
               A.body[i][j] = 0
164
            if A.body[i][j] == -0.0:
165
               A.body[i][j] = 0
166
            if A.body[i][j] - round(A.body[i][j]) == 0:
167
               A.body[i][j] = round(A.body[i][j])
168
            print(round(A.body[i][j],4),' ',end='')
        print('')
169
```

Code Box 2 全主元消元法求矩阵的逆

# 输出结果:

```
CompletePivotMetho • Debug I/O (stdin, stdout, stderr) appears below
The original matrix is shown below:
2
               -3
       1
                       1
                               1
                                      Ω
                                              0
                                                       0
       1
               0
                               0
                                       1
                                               0
                                                       0
-1
                       -2
       2
               4
                               0
                                      0
                                                       0
1
       0
               -1
                      5
                               0
The matrix after primary transformation is shown below:
                   -0.0506
                            0.5823
1
         0
              0
                                       -0.2658
                                                  -0.9114
    0
                                       0.443 0.519
-0.0127 -0.3291
    1
         0
              0
                   0.4177
                             -0.3038
       1
                   -0.2405 0.2658
            0
0
    0
                            -0.0633 0.0506 0.3165
0
    0 0
            1
                   -0.038
```

输出结果 2

# 代码分析:

此段代码是在 Code Box 1 的基础上略微修改而得的,由于全主元的位置没有固定性,所以要把每一次找到的全主元的行列坐标都存储下来。

其余部分与 Code Box 1 相同。

# 3 题

编程实现:利用 LU 分解法 (Doolittle),  $\bar{x}[x_1, x_2, x_3, x_4]^T$ 的数值解。

$$\begin{bmatrix} 12 & -3 & 3 & 4 \\ -18 & 3 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 15 \\ -15 \\ 6 \\ 2 \end{bmatrix}$$
 (3.1)

```
1
     # filename: 3.3 LU Decomposition.py
2
3
     class Matrix:
        """Abatrct class representing a Matrix.
4
5
6
         The internal structure is two dimension list.
         .....
7
        def __init__(self,m,n,mainCol):
8
9
10
            self.row:
                                the row of the Matrix
11
            self.col:
                                the collumn of the Matrix
12
            self.CheckedRow:
                                if one row has been checked,
13
                               it should not be checked again
14
                                the internal storing structure
            self.body:
15
                                the coefficient matrix
            self.mainCol:
16
17
            self.row = m
18
            self.col = n
            self.CheckedRow = set(range(self.row))
19
20
            self.body = [[0 for i in range(n)] for i in range(m)]
21
            self.mainCol = mainCol
22
23
        def getVal(self):
24
            """Giving value to each element of the matrix.
25
26
            Overwrite the original value zeros.
27
28
            for i in range(self.row):
29
                for j in range(self.col):
                    self.body[i][j] = int(input())
30
31
32
        def valid(self,e):
            """If the two matrix are not in the same form, return false,
33
34
            else return true.
35
36
            It is useful in the next functions.
37
38
            if self.row != e.row or self.col != e.col:
39
                return False
40
            else:
```

```
41
                return True
42
43
        def add(self,e):
            """A methon does not used in the pivot PCA algorithm.
44
45
46
            Maybe it will be used in other programs.
            0.00
47
48
            self.valid(e)
49
            tmp = Matrix(self.row,self.col,self.mainCol)
50
            for i in range(self.row): # deep copy
51
                for j in range(self.col):
52
                    tmp.body[i][j] = self.body[i][j]
53
54
            for i in range(self.row):
55
                for j in range(self.col):
56
                    tmp.body[i][j] += e.body[i][j]
57
            return tmp
58
        def constMulti(self,const):
59
            """A constant number multiple a matrix."""
60
61
            tmp = Matrix(self.row,self.col,self.mainCol)
62
            for i in range(self.row): # deep copy
63
                for j in range(self.col):
64
                    tmp.body[i][j] = self.body[i][j]
65
            for i in range(self.row):
66
67
                for j in range(self.col):
68
                    tmp.body[i][j] *= const
69
            return tmp
70
71
        def matrixTransform(self,target_row_Number,source_row_Number,times=None):
72
            """There is a big problem, every decimal number we see is stored in the
73
            RAM with the binary platform.
74
75
            I can import the decimal lib to solve this problem, but I did not!
76
77
            if times == None:
                                 # special case of matrixTransform(TarRow,times)
78
79
                times tmp = source row Number
80
                for j in range(self.col):
81
                    self.body[target_row_Number][j] *= times_tmp
82
                return
83
            elif times == 'exchange':
84
                for i in range(self.col):
85
                    self.body[target row Number][i],\
86
                       self.body[source_row_Number][i] \
87
                       = self.body[source_row_Number][i],\
88
                       self.body[target_row_Number][i]
89
                return
```

```
90
            else:
91
                for i in range(self.col):
92
                    self.body[target_row_Number][i] += \
93
                    times * self.body[source_row_Number][i]
94
95
        def Peel(self,layer):
96
            """Getting part of the matrix."""
97
            B = Matrix(self.row-layer, self.col-layer, self.mainCol)
98
            for i in range(self.row-layer):
99
                for j in range(self.col-layer):
100
                   B.body[i][j] = A.body[i + layer][j + layer]
101
            return B
102
103
        def combine(self,B,layer):
104
            """A long with method Peel."""
105
            for i in range(self.row-layer):
106
                for j in range(self.col-layer):
107
                    A.body[i + layer][j + layer] = B.body[i][j]
108
    def LU_Decompose_Doolittle(A):
109
110
         """Decompose matrix A with Dolittle method.
111
112
        In this function, I use devide and conquer method.
113
114
        for i in range(A.col-1):
115
            if i == 0:
116
                for j in range(1,A.row):
117
                    A.body[j][i] = A.body[j][i] / A.body[0][0]
118
            else:
119
                for j in range(A.col-i):
120
                    for k in range(i):
121
                       A.body[i][i+j] = A.body[i][i+j] - \
122
                           A.body[i][k] * A.body[k][i+j]
                for j in range(A.row-i-1):
123
124
                    for k in range(i):
125
                       A.body[i+j+1][i] = A.body[i+j+1][i] - \setminus
126
                           A.body[i+j+1][k] * A.body[k][i]
127
                    A.body[i+j+1][i] = A.body[i+j+1][i] / A.body[i][i]
128
        return A
129
130
                      -----my Main Function-----
131
132 A = Matrix(4,5,4)
133 A.body[0] = [12, -3, 3, 4, 15]
134 A.body[1] = [-18,3,-1,-1,-15]
135 A.body[2] = [1,1,1,1,6]
136 A.body[3] = [3,1,-1,1,2]
137
138 print('The original matrix is shown below:\n')
```

```
139 for i in range(A.row):
140
        for j in range(A.col):
141
            print(int(A.body[i][j]), '\t', end='')
142
        print('')
143
144 A = LU_Decompose_Doolittle(A)
145
146 B = Matrix(4,4,4)
147 for i in range(A.row):
148
        for j in range(A.row):
149
           B.body[i][j] = A.body[i][j]
150
151 L1 = Matrix(4,4,4)
152
153
   for i in range(L1.col):
154
        for j in range(L1.row-i):
155
            L1.body[j+i][i] = B.body[j+i][i]
156 for i in range(L1.row):
157
        L1.body[i][i] = 1
158
159 U = L1.constMulti(-1).add(B) # I am the best! professional!
160 for i in range(L1.row):
161
        U.body[i][i] += 1
162
163 print('----\nA:\n')
164 for i in range(4):
165
        for j in range(4):
           if B.body[i][j] == 0.0:
166
167
               B.body[i][j] = 0
168
           if B.body[i][j] == -0.0:
169
               B.body[i][j] = 0
170
           if B.body[i][j] - round(B.body[i][j]) == 0:
               B.body[i][j] = round(B.body[i][j])
171
172
            print(round(B.body[i][j],2),'\t',end='')
173
        print('')
174
175
    print('----\nU:\n')
176
177
    for i in range(4):
178
        for j in range(4):
           if U.body[i][j] == 0.0:
179
180
               U.body[i][j] = 0
181
           if U.body[i][j] == -0.0:
182
               A.body[i][j] = 0
183
           if U.body[i][j] - round(U.body[i][j]) == 0:
184
               U.body[i][j] = round(U.body[i][j])
185
            print(round(U.body[i][j],2),'\t',end='')
186
        print('')
187
```

```
188
    print('----\nL:\n')
189
190 for i in range(4):
191
        for j in range(4):
192
           if L1.body[i][j] == 0.0:
193
              L1.body[i][j] = 0
194
           if L1.body[i][j] == -0.0:
195
              L1.body[i][j] = 0
196
           if L1.body[i][j] - round(L1.body[i][j]) == 0:
197
              L1.body[i][j] = round(L1.body[i][j])
198
           print(round(L1.body[i][j],2),'\t',end='')
199
        print('')
```

Code Box 3 LU 分解

## 输出结果

2.2.111.Danaman	-i+i (-i-l 17000	\(\( \)	Dah	
	sition.py (pid 17808			
The ori	ginal ma	atrıx is	shown	below:
12	-3	3	4	15
1			_	
-18	3	-1	-1	-15
1	1	1	1	6
3	1	-1	1	2
A:				
12	-3	3	4	
1	-1.5		5	
1	-0.83			
	-1.17			
U:				
1.0	2	2	4	
12	-3 1 =	3	4	
0	-1.5		5	
0	0	3.67		
0	0	0	2.76	
L:				
1	0	0	0	
-1.5	1	0	0	
	-0.83		0	
0.25	-1.17	0.64	1	

输出结果 3

通过生成的 LU 矩阵,可以轻易得到,  $[x_1, x_2, x_3, x_4]^{\mathrm{T}}$ 的数值解为[1, 2, 3, 0],这也是解析解。

# 代码分析:

代码仍旧是基于面向对象技术,程序的算法是基于紧凑法求解 LU 矩阵的形式化做法。 如此编程,相对而言比较节约内存。

### 4题

编程实现:用追赶法解下列严格对角优势的三对角线方程组,要求用稀疏格式存储矩阵,主内存占用为O(n),其中n为矩阵的行数。

$$\begin{bmatrix} 4 & -1 & & & \\ -1 & 4 & -1 & & \\ & -1 & 4 & -1 & \\ & & -1 & 4 & -1 \\ & & & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 200 \\ 200 \\ 100 \end{bmatrix}$$

$$(4.1)$$

```
# filename: 3.4 Seize Method.py
2
3
    class ThreeTriangleMatrix:
4
       def __init__(self,a,b,c,m):
5
           self.a = a
           self.b = b
6
7
           self.c = c
           self.row = m
8
9
           self.col = m
10
       def calculate(self):
11
12
           alpha = []
13
           beta = []
14
           gamma = self.a
15
           alpha.append(self.b)
16
17
           beta.append(self.c / alpha[0])
18
19
           for i in range(1, self.row-1):
20
               alpha.append(self.b - gamma * beta[i-1])
21
               beta.append(self.c / alpha[i])
22
           alpha.append(self.b - gamma * beta[-1])
23
24
25
           return [gamma,alpha,beta]
26
   A = ThreeTriangleMatrix(-1,4,-1,5)
27
28
   A.calculate()
29
   L = [[0 for i in range(A.row)] for j in range(A.col)]
30
31
32
   for i in range(A.row-1):
       L[i+1][i] = A.calculate()[0]
33
       L[i][i] = A.calculate()[1][i]
34
35
36
   L[A.row-1][A.row-1] = (A.calculate()[1])[A.row-1]
37
38 U1 = [[0 for i in range(A.row)] for j in range(A.col)]
```

```
39 for i in range(A.row):
      U1[i][i] = 1
40
41
42 for i in range(A.row-1):
43
      U1[i][i+1] = A.calculate()[2][i]
44
45 print('----\nU1:\n')
46 for i in range(A.row):
47
      for j in range(A.col):
48
         print(round(U1[i][j],2),'\t',end='')
49
      print ('')
50
51 print('----\nL:]\n')
52 for i in range(A.row):
53
      for j in range(A.col):
54
         print(round(L[i][j],2),'\t',end='')
55
    print('')
```

Code Box 4 追赶公式求三对角线矩阵的 LU 分解

## 输出结果:

3.4 Seize Method.py (pid 16484) (no process) - Debug I/O (stdin, stdout, stderr) appears below						
U1:						
				_		
1	-0.25	0	0	0		
0	1	-0.27	0	0		
0	0	1	-0.27	0		
0	0	0	1	-0.27		
0	0	0	0	1		
  L:]						
,						
4	0	0	0	0		
-1	3.75	0	0	0		
0	-1	3.73	0	0		
0	0		3.73	0		
0	0	0	-1	3.73		

输出结果 4

做得了 LU 分解,将数据导入 MATLAB,可以迅速求出  $[x_1, x_2, x_3, x_4, x_5]^{\mathrm{T}}$ 的数值解为:

 $[46.15, 84.61, 92.30, 84.61, 46.15]^{T}$ 

# 代码分析:

此程序机械化比较高,所以比较简单,就是简单地翻译数学语言。

### 五、实验体会

在此次实验中,集中利用了MATLAB与 Python 3进行程序设计。

主程序是采用 Python 3 进行编写,验算求解是利用 MATLAB 进行。当写的程序足够多之后,就会发现,真正重要的东西是数学思维,编程完全决定于数学思维的深度。如果一味追求编程的快感,很容易在这虚无的成就感中迷失自己。所以编程不如不编,能透彻理解数学思维已经很不容易了。

但是从另一个角度看,编程对数学思维又有很强的检验作用,尽管在数学思维之外,高技巧度编程又是另外广阔的一片天地。通过适度的编程,选择一类题目的典型例子进行编程,就可以在一个程序的设计、实现时间里,考验自己的一批数学思维的掌握牢固程度。

MATLAB 的强大之处不在于它编程方便,而是两方面:友好的交互式控制台,众多高质量的程序包。正是这两点,使得 MATLAB 在工业中得到广泛的应用。但是在数值计算的学习上,利用 Python 或许是更好的选择。Python 的数值计算依赖于数组,所以对于循环的考验非常高。Python 不像 MATLAB 的基本运算都是基于矩阵那样地方便,而正是这不方便这一点,使得 Python 这个看起来更加拙劣的工具更适合新手。

除了工具,更重要的一点是编程。这四个程序的编写基本上没有遇到困难。如果说在理论方面已经理解得足够透彻,那么在编程上不应该存在任何思路困难。数值计算以及数学建模中的程序,很大程度上都是数学语言的直接翻译。这一点与 DSA 设计有相当大的差别。现在出现的一个问题是,编程趋向 IDD 模式,即 IDE Driven,没有了编译器就不会单步调试,进而很难发现哪里出了差错。这不能说是思路理解得不到位,因为如果单纯是这个原因,那么在纸上演算就会遇到问题。但是很显然不是这样。以后的一个突破,可能就在这里——直接用文本编辑器进行代码书写,然后用 IDE 进行检验。

最后需要说明,因为有很多东西没有或者不能调用,所以这四个程序基本上是为题目量身定做。虽然面向对象设计里的几个成员函数具有通用性,但是总得来看,一些全局函数还是缺乏相应的判断。当然,如果不能运行,就从侧面说明输入的矩阵不合格。所以,这四个程序的通用性有待进一步挖掘,数据与程序能彻底分开,是判断程序通用性强弱的主要依据之一。

# 六、参考文献

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