云南大学数学与统计学院
上机实践报告

课程名称： 数值计算实验	**年级：** 2015 级	**上机实践成绩：**
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上机实践名称： 解线性方程组的迭代法	**学号：** 20151910042	**上机实践日期：** 2017-11-28
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一、实验目的

1. 通过对所学的线性方程组迭代求解的理论方法进行编程，提升程序编写水平；
2. 通过对理论方法的编程实验，进一步掌握理论方法的每一个细节；
3. 通过数值法求解，掌握判断循环终结的条件，理解矩阵范数的存在意义。

二、实验内容

1. 编制求矩阵的各种范数的程序；
2. 编程实现用雅可比迭代法求线性方程组的数值解；
3. 编程实现用高斯-塞德尔方法求线性方程组的数值解；
4. 编程实现用 SOR 方法求线性方程组的数值解。

三、实验平台

Windows 10 1709 Enterprise 中文版；
Python 3.6.0；
Wing IDE Professional 6.0.5-1 集成开发环境；
MATLAB R2017b win64；
AxMath 公式编辑器；
EndNote X8 文献管理。

四、实验记录与实验结果分析

1 题

对下列矩阵计算$\|\cdot\|_\infty$，$\|\cdot\|_1$，$\|\cdot\|_2$：[1]

（1）$A=\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}$　　（2）$B=\begin{bmatrix}10 & 15\\ 0 & 1\end{bmatrix}$　　（3）$C=\begin{bmatrix}0.6 & -0.5\\ -0.1 & 0.3\end{bmatrix}$

解答：

在一般的无大型模块导入的情况下，对于行范数与列范数而言，求解都是比较简单的，但是对于谱范数却并不是很容易，首先一点，难以做到的是特征方程的求解，这个多项式方程，当阶数特别大的时候几乎不能通过公式法求解，而且写一个字符串识别程序也不见得容易。如果能写成，那么在无公式的情况下，用数值方法求解也是不容易的，数值方法的计算量本身就很大，而且得到的一般不是精确解。所以综合来看，利用特征方程求解的这一做法基本放弃。

经过阅读有关文献[2]，我决定采用一种新的可行的方案。

定义 把矩阵的下列三种变换称为列行互逆变换：

1. 互换i, j两列（$C_i \leftrightarrow C_j$），之后互换i, j两行（$R_i \leftrightarrow R_j$）；

2. 第i列乘以非零常数k（$k\cdot C_i$），之后第i行乘以非零常数$\frac{1}{k}$（$\frac{1}{k}C_i$）；

3. 第i列的k倍加到第j列（$C_j+k\cdot C_i$），之后把第j行的$-k$倍加到第i行（$R_i-k\cdot R_j$）

遗憾的是这种方法有局限性，所以这里放置一个 hook，以备以后改变内模式，而不改变接口。

程序代码：

```
"""filename: 4.1 Normal Value.py"""

"""This is a universally usable code to get different
kind of normal value of an matrix. And the matrix is
a instance of Class Matrix.

Word is clean, this is the code!
"""

class Matrix:
    """Abatrct class representing a Matrix.

    The internal structure is a two dimension list.
    """
    def __init__(self,m,n,mainCol):
        """
        self.row:           the row of the Matrix
        self.col:           the collumn of the Matrix
        self.CheckedRow:     if one row has been checked,
                            it should not be checked again
        self.body:          the internal storing structure
        self.mainCol:       the coefficient matrix
        """
        self.row = m
        self.col = n
        self.CheckedRow = set(range(self.row))
        self.body = [[0 for i in range(n)] for i in range(m)]
        self.mainCol = mainCol

    def getVal(self):
        """Giving value to each element of the matrix.

        Overwrite the original value zeros.
        """
        for i in range(self.row):
            for j in range(self.col):
                self.body[i][j] = int(input())

    def valid(self,e,kind = None):
        """If these two matrix are not in the same form, return false,
        else return true.

        It is useful in the next functions.
```

```
44          """
45          if kind == 'multi':
46              # SELF * E
47              return self.col == e.row
48
49          if self.row != e.row or self.col != e.col:
50              return False
51          else:
52              return True
53
54      def matrixAdd(self,e):
55          """A methon does not used in the pivot PCA algorithm.
56
57          Maybe it will be used in other programs.
58          """
59          self.valid(e)
60          tmp = Matrix(self.row,self.col,self.mainCol)
61          for i in range(self.row):   # deep copy
62              for j in range(self.col):
63                  tmp.body[i][j] = self.body[i][j]
64
65          for i in range(self.row):
66              for j in range(self.col):
67                  tmp.body[i][j] += e.body[i][j]
68          return tmp
69
70      def matrixConstMulti(self,const):
71          """A constant number multiple a matrix."""
72          tmp = Matrix(self.row,self.col,self.mainCol)
73          for i in range(self.row):   # deep copy
74              for j in range(self.col):
75                  tmp.body[i][j] = self.body[i][j]
76
77          for i in range(self.row):
78              for j in range(self.col):
79                  tmp.body[i][j] *= const
80          return tmp
81
82      def matrixMulti(self,e):
83          """Return the multiplication of two matrix."""
84          self.valid(e,'multi')
85
86          ans = Matrix(self.row,e.col,e.col)  # e.col has no meaning
87
88          for i  in range(self.row):
89              for j in range(e.col):
90                  tmp = 0
91                  for k in range(self.col):
92                      tmp += self.body[i][k] * e.body[k][j]
```

```
93                  ans.body[i][j] = tmp
94
95          return ans
96
97      def matrixTransform(self,target_row_Number,source_row_Number,times=None):
98          """There is a big problem, every decimal number we see is stored in the
99          RAM with the binary platform.
100
101         I can import the decimal lib to solve this problem, but I did not!
102         """
103         if times == None:    # special case of matrixTransform(TarRow,times)
104
105             times_tmp = source_row_Number
106             for j in range(self.col):
107                 self.body[target_row_Number][j] *= times_tmp
108             return
109         elif times == 'exchange':
110             for i in range(self.col):
111                 self.body[target_row_Number][i],\
112                     self.body[source_row_Number][i] \
113                     = self.body[source_row_Number][i],\
114                     self.body[target_row_Number][i]
115             return
116         else:
117             for i in range(self.col):
118                 self.body[target_row_Number][i] += \
119                 times * self.body[source_row_Number][i]
120
121     def matrixTranspose(self):
122         ans = Matrix(self.col,self.row,self.row)
123         # main column of ans is meaningless
124         for i in range(self.row):
125             for j in range(self.col):
126                 ans.body[j][i] = self.body[i][j]
127         return ans
128
129 def Quick_Sort(L):
130     """A simple code to order a list with quicksort algorithm."""
131     if len(L) <= 1:
132         return L
133
134     else:
135         less = []
136         equal = []
137         bigger = []
138
139         pivot = (L[0] + L[-1] + L[len(L)//2]) / 3
140         # pivot should be taken seriously!
141
```

```
142         for i in range(len(L)):
143             if L[i] < pivot:
144                 less.append(L[i])
145             elif(L[i] == pivot):
146                 equal.append(L(i))
147             else:
148                 bigger.append(L[i])
149         less = Quick_Sort(less)
150         bigger = Quick_Sort(bigger)
151 
152         return less + equal + bigger
153 
154 def GetNormalValue(M,kind=1):
155     """M is a matrix.
156 
157     Return the Normal value of M.
158     """
159     if kind == 'inf':
160         """NorVal is the maximum value among the sums of every row. """
161         sums = list()
162         tmp = 0
163         for i in range(M.row):
164             for j in range(M.col):
165                 tmp += M.body[i][j]
166             sums.append(tmp)
167             tmp = 0
168 
169         sums = Quick_Sort(sums)
170         return sums[-1]
171 
172     elif kind == 1:
173         """NorVal is the maximum value among the sums of every column. """
174         sums = list()
175         tmp = 0
176         for i in range(M.col):
177             for j in range(M.row):
178                 tmp += M.body[j][i]
179             sums.append(tmp)
180             tmp = 0
181 
182         sums = Quick_Sort(sums)
183         return sums[-1]
184 
185     elif kind == 2:
186         Multi = M.matrixMulti(M.matrixTranspose())
187         """The solve of elg is a little bit difficult,
188         I will change the code after learning.
189 
190         Now I use the numpy package to solve it.
```

```
191         """
192         import numpy as np
193 
194         MUL = M.matrixMulti(M.matrixTranspose())
195         tmp = np.array(MUL.body)
196 
197         a,b = np.linalg.eig(tmp)
198 
199         return np.sqrt(max(a))
200 
201     else:
202         raise ValueError("""Bad input!\n""")
203 
204 """-------------------------my Main Function-------------------------"""
205 
206 a = Matrix(2,2,2)
207 print('+----------------------+----------------------+')
208 a.body = [[1,-1],[2,1]]
209 b = GetNormalValue(a,'inf')
210 print('| infinity normal value |  ',b,'                |')
211 b = GetNormalValue(a,1)
212 print('| 1 normal value        |  ',b,'                |')
213 b = GetNormalValue(a,2)
214 print('| 2 normal value        |  ',b,'      |')
215 print('+----------------------+----------------------+')
216 
217 a.body = [[10,15],[0,1]]
218 b = GetNormalValue(a,'inf')
219 print('| infinity normal value |  ',b,'                |')
220 b = GetNormalValue(a,1)
221 print('| 1 normal value        |  ',b,'                |')
222 b = GetNormalValue(a,2)
223 print('| 2 normal value        |  ',b,'      |')
224 print('+----------------------+----------------------+')
225 
226 a.body = [[0.6,-0.5],[-0.1,0.3]]
227 b = GetNormalValue(a,'inf')
228 print('| infinity normal value |  ',b,'|')
229 b = GetNormalValue(a,1)
230 print('| 1 normal value        |  ',b,'                |')
231 b = GetNormalValue(a,2)
232 print('| 2 normal value        |  ',b,'     |')
233 print('+----------------------+----------------------+')
```

Code Box 1 矩阵的三种范数

输出结果：

```
4.1 Normal Value.py (  Debug process terminated
+-----------------------+-----------------------+
| infinity normal value |   3                   |
| 1 normal value        |   3                   |
| 2 normal value        |   2.30277563773       |
+-----------------------+-----------------------+
| infinity normal value |   25                  |
| 1 normal value        |   16                  |
| 2 normal value        |   18.0469654611       |
+-----------------------+-----------------------+
| infinity normal value |   0.19999999999999998 |
| 1 normal value        |   0.5                 |
| 2 normal value        |   0.827853086715      |
+-----------------------+-----------------------+
```

输出结果 1

代码分析：

矩阵范数是迭代过程的核心，是判断迭代精度的标尺。

行、列范数都比较简单，但是谱范数比较复杂，参考了书后的 Jacobi 方法之类的迭代法，也没有给出很详细的稳定算法，MATLAB 的代码不开源，只能借用 Python3 的 numpy 进行计算，把 numpy 的方法嵌入到了 GetNormalValue 函数里面做了一个简单的封装。等到以后写出优质的稳定算法，再保留接口替换一下就可以了。

在排序过程中，自建了一个快速排序算法，其中每次的比较数值是三平均数，稳定性比较可靠。无论是随机序列还是等差序列，都可以比较好地进行递归。

2 题

设$A=\begin{bmatrix}100 & 99\\ 99 & 98\end{bmatrix}$，计算$A$的条件数$\mathrm{cond}(A)_\infty$及$\mathrm{cond}(A)_2$。

解答：

程序代码：

```
"""filename: 4.2 Conditional Value.py"""

"""This is a universally usable code to get different
kind of normal value of an matrix. And the matrix is
a instance of Class Matrix.

Word is clean, this is the code!
"""

class Matrix:
    """Abatrct class representing a Matrix.

    The internal structure is a two dimension list.
    """
    def __init__(self,m,n,mainCol):
        """
        self.row:           the row of the Matrix
        self.col:           the collumn of the Matrix
        self.CheckedRow:     if one row has been checked,
                            it should not be checked again
        self.body:          the internal storing structure
        self.mainCol:       the coefficient matrix
        """
        self.row = m
        self.col = n
        self.CheckedRow = set(range(self.row))
        self.body = [[0 for i in range(n)] for i in range(m)]
        self.mainCol = mainCol

    def getVal(self):
        """Giving value to each element of the matrix.

        Overwrite the original value zeros.
        """
        for i in range(self.row):
            for j in range(self.col):
                self.body[i][j] = int(input())

    def valid(self,e,kind = None):
        """If these two matrix are not in the same form, return false,
        else return true.

```

```
43          It is useful in the next functions.
44          """
45          if kind == 'multi':
46              # SELF * E
47              return self.col == e.row
48
49          if self.row != e.row or self.col != e.col:
50              return False
51          else:
52              return True
53
54      def matrixAdd(self,e):
55          """A methon does not used in the pivot PCA algorithm.
56
57          Maybe it will be used in other programs.
58          """
59          self.valid(e)
60          tmp = Matrix(self.row,self.col,self.mainCol)
61          for i in range(self.row):   # deep copy
62              for j in range(self.col):
63                  tmp.body[i][j] = self.body[i][j]
64
65          for i in range(self.row):
66              for j in range(self.col):
67                  tmp.body[i][j] += e.body[i][j]
68          return tmp
69
70      def matrixConstMulti(self,const):
71          """A constant number multiple a matrix."""
72          tmp = Matrix(self.row,self.col,self.mainCol)
73          for i in range(self.row):   # deep copy
74              for j in range(self.col):
75                  tmp.body[i][j] = self.body[i][j]
76
77          for i in range(self.row):
78              for j in range(self.col):
79                  tmp.body[i][j] *= const
80          return tmp
81
82      def matrixMulti(self,e):
83          """Return the multiplication of two matrix."""
84          self.valid(e,'multi')
85
86          ans = Matrix(self.row,e.col,e.col)  # e.col has no meaning
87
88          for i  in range(self.row):
89              for j in range(e.col):
90                  tmp = 0
91                  for k in range(self.col):
```

```
92                      tmp += self.body[i][k] * e.body[k][j]
93                  ans.body[i][j] = tmp
94
95          return ans
96
97      def matrixTransform(self,target_row_Number,source_row_Number,times=None):
98          """There is a big problem, every decimal number we see is stored in the
99          RAM with the binary platform.
100
101         I can import the decimal lib to solve this problem, but I did not!
102         """
103         if times == None:    # special case of matrixTransform(TarRow,times)
104
105             times_tmp = source_row_Number
106             for j in range(self.col):
107                 self.body[target_row_Number][j] *= times_tmp
108             return
109         elif times == 'exchange':
110             for i in range(self.col):
111                 self.body[target_row_Number][i],\
112                     self.body[source_row_Number][i] \
113                     = self.body[source_row_Number][i],\
114                     self.body[target_row_Number][i]
115             return
116         else:
117             for i in range(self.col):
118                 self.body[target_row_Number][i] += \
119                 times * self.body[source_row_Number][i]
120
121     def matrixTranspose(self):
122         ans = Matrix(self.col,self.row,self.row)
123         # main column of ans is meaningless
124         for i in range(self.row):
125             for j in range(self.col):
126                 ans.body[j][i] = self.body[i][j]
127         return ans
128
129 def Quick_Sort(L):
130     """A simple code to order a list with quicksort algorithm."""
131     if len(L) <= 1:
132         return L
133
134     else:
135         less = []
136         equal = []
137         bigger = []
138
139         pivot = (L[0] + L[-1] + L[len(L)//2]) / 3
140         # pivot should be taken seriously!
```

```
141
142         for i in range(len(L)):
143             if L[i] < pivot:
144                 less.append(L[i])
145             elif(L[i] == pivot):
146                 equal.append(L(i))
147             else:
148                 bigger.append(L[i])
149         less = Quick_Sort(less)
150         bigger = Quick_Sort(bigger)
151 
152         return less + equal + bigger
153 
154 def GetNormalValue(M,kind=2):
155     """M is a matrix.
156 
157     Return the Normal value of M.
158     """
159     if kind == 'inf':
160         """NorVal is the maximum value among the sums of every row. """
161         sums = list()
162         tmp = 0
163         for i in range(M.row):
164             for j in range(M.col):
165                 tmp += M.body[i][j]
166             sums.append(tmp)
167             tmp = 0
168 
169         sums = Quick_Sort(sums)
170         return sums[-1]
171 
172     elif kind == 1:
173         """NorVal is the maximum value among the sums of every column. """
174         sums = list()
175         tmp = 0
176         for i in range(M.col):
177             for j in range(M.row):
178                 tmp += M.body[j][i]
179             sums.append(tmp)
180             tmp = 0
181 
182         sums = Quick_Sort(sums)
183         return sums[-1]
184 
185     elif kind == 2:
186         Multi = M.matrixMulti(M.matrixTranspose())
187         """
188         The solving of eig is a little difficult for me,
189         I will change the code after learning more later.
```

```
190
191         Now I use the numpy package to solve it.
192         """
193         import numpy as np
194
195         MUL = M.matrixMulti(M.matrixTranspose())
196         tmp = np.array(MUL.body)
197
198         a,b = np.linalg.eig(tmp)
199
200         return np.sqrt(max(a))
201
202     else:
203         raise ValueError("""Bad input!\n""")
204
205 def GetCondValue(M,kind=2):
206     if kind == 'inf':
207         return GetNormalValue(M,'inf') * GetNormalValue(M.matrixTranspose(),'inf')
208     if kind == 1:
209         return GetNormalValue(M,1) * GetNormalValue(M.matrixTranspose(),1)
210     if kind == 2:
211         return GetNormalValue(M,2) * GetNormalValue(M.matrixTranspose(),2)
212
213
214 """------------------------my Main Function-------------------------"""
215
216 A = Matrix(2,2,2)
217 A.body = [[100,99],[99,98]]
218
219 ans_inf = GetCondValue(A,'inf')
220 ans_2 = GetCondValue(A,2)
221 print('+----------------------+----------------------+')
222 print('| inf Conditional Value\t|  ',ans_inf,'\t\t|')
223 print('| 2 Conditional Value\t|  ',ans_2,'\t|')
224 print('+----------------------+----------------------+')
```

Code Box 2

输出结果：

```
4.2 Conditional Value.  Debug I/O (stdin, stdout, stderr) appears below
+----------------------+----------------------+
| inf Conditional Value |   39601              |
| 2 Conditional Value   |   39205.9999745      |
+----------------------+----------------------+
```

输出结果 2

代码分析：

条件数是矩阵的范数与矩阵的转置的范数的乘积。

3 题

用高斯-赛德尔迭代法解下列线性方程组，要求当$\|x^{(K+1)}-x^{(K)}\|\leqslant 10^{-5}$时迭代终止。

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 6 \\ -2 \\ 6 \end{bmatrix} \tag{3.1}$$

解答：

程序代码：

```
"""filename: 4.3 Gauss-Seidel Method.py"""

"""filename: 4.4 Plot.py"""

import matplotlib.pyplot as pl
import numpy as np

class Matrix:
    """Abatrct class representing a Matrix.

    The internal structure is a two dimension list.
    """
    def __init__(self,m,n,mainCol):
        """
        self.row:           the row of the Matrix
        self.col:           the collumn of the Matrix
        self.CheckedRow:     if one row has been checked,
                            it should not be checked again
        self.body:          the internal storing structure
        self.mainCol:       the coefficient matrix
        """
        self.row = m
        self.col = n
        self.CheckedRow = set(range(self.row))
        self.body = [[0 for i in range(n)] for i in range(m)]
        self.mainCol = mainCol

    def getVal(self):
        """Giving value to each element of the matrix.

        Overwrite the original value zeros.
        """
        for i in range(self.row):
            for j in range(self.col):
                self.body[i][j] = int(input())

```

```
37      def valid(self,e,kind = None):
38          """If these two matrix are not in the same form, return false,
39          else return true.
40
41          It is useful in the next functions.
42          """
43          if kind == 'multi':
44              # SELF * E
45              return self.col == e.row
46
47          if self.row != e.row or self.col != e.col:
48              return False
49          else:
50              return True
51
52      def matrixAdd(self,e):
53          """A methon does not used in the pivot PCA algorithm.
54
55          Maybe it will be used in other programs.
56          """
57          self.valid(e)
58          tmp = Matrix(self.row,self.col,self.mainCol)
59          for i in range(self.row):   # deep copy
60              for j in range(self.col):
61                  tmp.body[i][j] = self.body[i][j]
62
63          for i in range(self.row):
64              for j in range(self.col):
65                  tmp.body[i][j] += e.body[i][j]
66          return tmp
67
68      def matrixConstMulti(self,const):
69          """A constant number multiple a matrix."""
70          tmp = Matrix(self.row,self.col,self.mainCol)
71          for i in range(self.row):   # deep copy
72              for j in range(self.col):
73                  tmp.body[i][j] = self.body[i][j]
74
75          for i in range(self.row):
76              for j in range(self.col):
77                  tmp.body[i][j] *= const
78          return tmp
79
80      def matrixMulti(self,e):
81          """Return the multiplication of two matrix."""
82          self.valid(e,'multi')
83
84          ans = Matrix(self.row,e.col,e.col)  # e.col has no meaning
85
```

```
86          for i  in range(self.row):
87              for j in range(e.col):
88                  tmp = 0
89                  for k in range(self.col):
90                      tmp += self.body[i][k] * e.body[k][j]
91                  ans.body[i][j] = tmp
92
93          return ans
94
95      def matrixTransform(self,target_row_Number,source_row_Number,times=None):
96          """There is a big problem, every decimal number we see is stored in the
97          RAM with the binary platform.
98
99          I can import the decimal lib to solve this problem, but I did not!
100         """
101         if times == None:    # special case of matrixTransform(TarRow,times)
102
103             times_tmp = source_row_Number
104             for j in range(self.col):
105                 self.body[target_row_Number][j] *= times_tmp
106             return
107         elif times == 'exchange':
108             for i in range(self.col):
109                 self.body[target_row_Number][i],\
110                     self.body[source_row_Number][i] \
111                     = self.body[source_row_Number][i],\
112                     self.body[target_row_Number][i]
113             return
114         else:
115             for i in range(self.col):
116                 self.body[target_row_Number][i] += \
117                 times * self.body[source_row_Number][i]
118
119     def matrixTranspose(self):
120         ans = Matrix(self.col,self.row,self.row)
121         # main column of ans is meaningless
122         for i in range(self.row):
123             for j in range(self.col):
124                 ans.body[j][i] = self.body[i][j]
125         return ans
126
127 def Quick_Sort(L):
128     """A simple code to order a list with quicksort algorithm."""
129     if len(L) <= 1:
130         return L
131
132     else:
133         less = []
134         equal = []
```

```
135         bigger = []
136 
137         pivot = (L[0] + L[-1] + L[len(L)//2]) / 3
138         # pivot should be taken seriously!
139 
140         for i in range(len(L)):
141             if L[i] < pivot:
142                 less.append(L[i])
143             elif(L[i] == pivot):
144                 equal.append(L[i])
145             else:
146                 bigger.append(L[i])
147         less = Quick_Sort(less)
148         bigger = Quick_Sort(bigger)
149 
150         return less + equal + bigger
151 
152 def GetNormalValue(M,kind=2):
153     """M is a matrix.
154 
155     Return the Normal value of M.
156     """
157     if kind == 'inf':
158         """NorVal is the maximum value among the sums of every row. """
159         sums = list()
160         tmp = 0
161         for i in range(M.row):
162             for j in range(M.col):
163                 tmp += M.body[i][j]
164             sums.append(tmp)
165             tmp = 0
166 
167         sums = Quick_Sort(sums)
168         return sums[-1]
169 
170     elif kind == 1:
171         """NorVal is the maximum value among the sums of every column. """
172         sums = list()
173         tmp = 0
174         for i in range(M.col):
175             for j in range(M.row):
176                 tmp += M.body[j][i]
177             sums.append(tmp)
178             tmp = 0
179 
180         sums = Quick_Sort(sums)
181         return sums[-1]
182 
183     elif kind == 2:
```

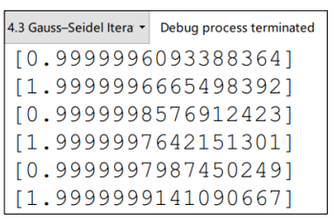
```
184         Multi = M.matrixMulti(M.matrixTranspose())
185         """
186         The solving of eig is a little difficult for me,
187         I will change the code after learning more later.
188 
189         Now I use the numpy package to solve it.
190         """
191         import numpy as np
192 
193         MUL = M.matrixMulti(M.matrixTranspose())
194         tmp = np.array(MUL.body)
195 
196         a,b = np.linalg.eig(tmp)
197 
198         return np.sqrt(max(a))
199 
200     else:
201         raise ValueError("""Bad input!\n""")
202 
203 def vectorNormalValue(a,kind=2):
204     """Get the normal value of a vector."""
205     if kind == 1:
206         ans = 0
207         for i in a:
208             ans += abs(i)
209         return ans
210 
211     if kind == 2:
212         from math import sqrt as sq
213         ans = 0
214         for i in a:
215             ans += i * i
216         return sq(ans)
217 
218     if kind == 'inf':
219         tmp = list()
220         for i in a:
221             tmp.append(abs(i))
222         Q = Quick_Sort(tmp)
223         return Q[-1]
224 
225 def iterFormat(A,b):
226     """Get the iter matrix."""
227     if b.col != 1:
228         return
229     for i in range(A.row):
230         b.body[i][0] = b.body[i][0] / A.body[i][i]
231 
232     for i in range(A.row):
```

```
233         A.matrixTransform(i,1 / A.body[i][i])
234     B = Matrix(A.row,A.col,A.mainCol)
235     for i in range(B.row):
236         for j in range(B.col):
237             if i == j:
238                 B.body[i][j] = 0
239             else:
240                 B.body[i][j] = -1 * A.body[i][j]
241     ans = (B,b)
242     return ans
243 
244 def GS_op1(B,b,x):
245     """simple function."""
246     for i in range(x.row):
247         tmp = 0
248         for j in range(B.col):
249             tmp += B.body[i][j] * x.body[j][0]
250         x.body[i][0] = tmp + b.body[i][0]
251     return x
252 
253 def GS(times):
254     for i in range(times-1):
255         GS_op1(B,b,x)
256 
257     tmp = Matrix(6,1,1)
258     for i in range(x.row):
259         tmp.body[i][0] = x.body[i][0]  # deep copy
260 
261     GS_op1(B,b,x)       # do it once more
262 
263     step3 = x.matrixConstMulti(-1)
264     there = tmp.matrixAdd(step3)
265 
266     c = list()
267 
268     for i in range(there.row):
269         c.append(there.body[i][0])
270 
271     return vectorNormalValue(c)
272 
273 """------------------------my Main Function-------------------------"""
274 
275 A = Matrix(6,6,6)
276 A.body = [[4,-1, 0,-1, 0, 0]\
277         ,[-1, 4,-1, 0,-1, 0]\
278         ,[ 0,-1, 4, 0, 0,-1]\
279         ,[-1, 0, 0, 4,-1, 0]\
280         ,[ 0,-1, 0,-1, 4,-1]\
281         ,[ 0, 0,-1, 0,-1, 4]]
```

```
282
283  b = Matrix(6,1,1)
284  b.body = [[0],[5],[0],[6],[-2],[6]]
285
286  x = Matrix(6,1,1)
287  x.body = [[1],[1],[1],[1],[1],[1]]
288
289  tmp = iterFormat(A,b)
290  B,b = tmp[0],tmp[1]
291
292  guide = 10
293  times = 5
294  while guide > 1e-5:
295      guide = GS(times)
296
297  for i in x.body:
298      print(i)
```

Code Box 3

输出结果：



输出结果 3

代码分析：

高斯-塞德尔方法，无非是在计算过程中，使实时的结果及时反馈，从而提高迭代效率。

4 题

用雅可比迭代法、高斯-赛德尔迭代法解下列线性方程组，比较在不同迭代深度下，两种迭代法的差异。

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 6 \\ -2 \\ 6 \end{bmatrix} \tag{4.1}$$

解答：

程序代码：

```
"""filename: 4.4 Plot.py"""

import matplotlib.pyplot as pl
import numpy as np

class Matrix:
    """Abatrct class representing a Matrix.

    The internal structure is a two dimension list.
    """
    def __init__(self,m,n,mainCol):
        """
        self.row:           the row of the Matrix
        self.col:           the collumn of the Matrix
        self.CheckedRow:     if one row has been checked,
                            it should not be checked again
        self.body:          the internal storing structure
        self.mainCol:       the coefficient matrix
        """
        self.row = m
        self.col = n
        self.CheckedRow = set(range(self.row))
        self.body = [[0 for i in range(n)] for i in range(m)]
        self.mainCol = mainCol

    def getVal(self):
        """Giving value to each element of the matrix.

        Overwrite the original value zeros.
        """
        for i in range(self.row):
            for j in range(self.col):
                self.body[i][j] = int(input())

    def valid(self,e,kind = None):
        """If these two matrix are not in the same form, return false,
```

```
37          else return true.
38
39          It is useful in the next functions.
40          """
41          if kind == 'multi':
42              # SELF * E
43              return self.col == e.row
44
45          if self.row != e.row or self.col != e.col:
46              return False
47          else:
48              return True
49
50      def matrixAdd(self,e):
51          """A methon does not used in the pivot PCA algorithm.
52
53          Maybe it will be used in other programs.
54          """
55          self.valid(e)
56          tmp = Matrix(self.row,self.col,self.mainCol)
57          for i in range(self.row):   # deep copy
58              for j in range(self.col):
59                  tmp.body[i][j] = self.body[i][j]
60
61          for i in range(self.row):
62              for j in range(self.col):
63                  tmp.body[i][j] += e.body[i][j]
64          return tmp
65
66      def matrixConstMulti(self,const):
67          """A constant number multiple a matrix."""
68          tmp = Matrix(self.row,self.col,self.mainCol)
69          for i in range(self.row):   # deep copy
70              for j in range(self.col):
71                  tmp.body[i][j] = self.body[i][j]
72
73          for i in range(self.row):
74              for j in range(self.col):
75                  tmp.body[i][j] *= const
76          return tmp
77
78      def matrixMulti(self,e):
79          """Return the multiplication of two matrix."""
80          self.valid(e,'multi')
81
82          ans = Matrix(self.row,e.col,e.col)  # e.col has no meaning
83
84          for i  in range(self.row):
85              for j in range(e.col):
```

```
86                  tmp = 0
87                  for k in range(self.col):
88                      tmp += self.body[i][k] * e.body[k][j]
89                  ans.body[i][j] = tmp
90
91          return ans
92
93      def matrixTransform(self,target_row_Number,source_row_Number,times=None):
94          """There is a big problem, every decimal number we see is stored in the
95          RAM with the binary platform.
96
97          I can import the decimal lib to solve this problem, but I did not!
98          """
99          if times == None:    # special case of matrixTransform(TarRow,times)
100
101             times_tmp = source_row_Number
102             for j in range(self.col):
103                 self.body[target_row_Number][j] *= times_tmp
104             return
105         elif times == 'exchange':
106             for i in range(self.col):
107                 self.body[target_row_Number][i],\
108                     self.body[source_row_Number][i] \
109                     = self.body[source_row_Number][i],\
110                     self.body[target_row_Number][i]
111             return
112         else:
113             for i in range(self.col):
114                 self.body[target_row_Number][i] += \
115                 times * self.body[source_row_Number][i]
116
117     def matrixTranspose(self):
118         ans = Matrix(self.col,self.row,self.row)
119         # main column of ans is meaningless
120         for i in range(self.row):
121             for j in range(self.col):
122                 ans.body[j][i] = self.body[i][j]
123         return ans
124
125 def Quick_Sort(L):
126     """A simple code to order a list with quicksort algorithm."""
127     if len(L) <= 1:
128         return L
129
130     else:
131         less = []
132         equal = []
133         bigger = []
134
```

```
135         pivot = (L[0] + L[-1] + L[len(L)//2]) / 3
136         # pivot should be taken seriously!
137 
138         for i in range(len(L)):
139             if L[i] < pivot:
140                 less.append(L[i])
141             elif(L[i] == pivot):
142                 equal.append(L[i])
143             else:
144                 bigger.append(L[i])
145         less = Quick_Sort(less)
146         bigger = Quick_Sort(bigger)
147 
148         return less + equal + bigger
149 
150 def GetNormalValue(M,kind=2):
151     """M is a matrix.
152 
153     Return the Normal value of M.
154     """
155     if kind == 'inf':
156         """NorVal is the maximum value among the sums of every row. """
157         sums = list()
158         tmp = 0
159         for i in range(M.row):
160             for j in range(M.col):
161                 tmp += M.body[i][j]
162             sums.append(tmp)
163             tmp = 0
164 
165         sums = Quick_Sort(sums)
166         return sums[-1]
167 
168     elif kind == 1:
169         """NorVal is the maximum value among the sums of every column. """
170         sums = list()
171         tmp = 0
172         for i in range(M.col):
173             for j in range(M.row):
174                 tmp += M.body[j][i]
175             sums.append(tmp)
176             tmp = 0
177 
178         sums = Quick_Sort(sums)
179         return sums[-1]
180 
181     elif kind == 2:
182         Multi = M.matrixMulti(M.matrixTranspose())
183         """
```

```
184         The solving of eig is a little difficult for me,
185         I will change the code after learning more later.
186 
187         Now I use the numpy package to solve it.
188         """
189         import numpy as np
190 
191         MUL = M.matrixMulti(M.matrixTranspose())
192         tmp = np.array(MUL.body)
193 
194         a,b = np.linalg.eig(tmp)
195 
196         return np.sqrt(max(a))
197 
198     else:
199         raise ValueError("""Bad input!\n""")
200 
201 def vectorNormalValue(a,kind=2):
202     """Get the normal value of a vector."""
203     if kind == 1:
204         ans = 0
205         for i in a:
206             ans += abs(i)
207         return ans
208 
209     if kind == 2:
210         from math import sqrt as sq
211         ans = 0
212         for i in a:
213             ans += i * i
214         return sq(ans)
215 
216     if kind == 'inf':
217         tmp = list()
218         for i in a:
219             tmp.append(abs(i))
220         Q = Quick_Sort(tmp)
221         return Q[-1]
222 
223 def iterFormat(A,b):
224     """Get the iter matrix."""
225     if b.col != 1:
226         return
227     for i in range(A.row):
228         b.body[i][0] = b.body[i][0] / A.body[i][i]
229 
230     for i in range(A.row):
231         A.matrixTransform(i,1 / A.body[i][i])
232     B = Matrix(A.row,A.col,A.mainCol)
```

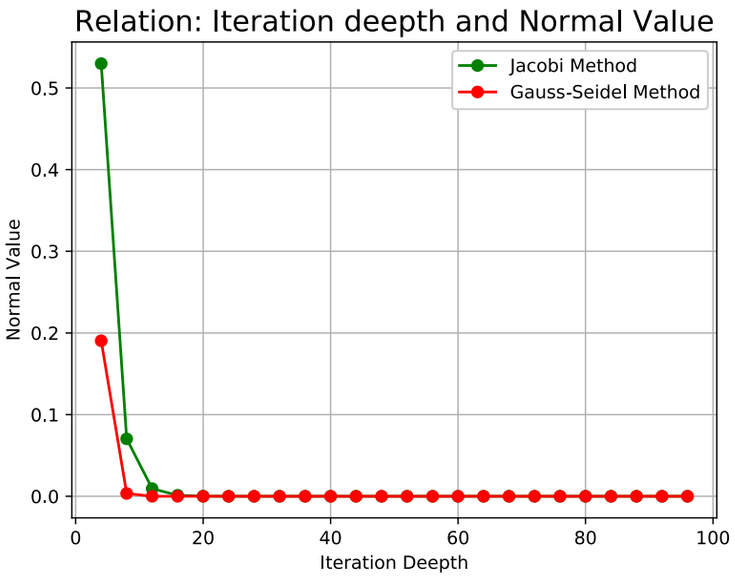
```
233     for i in range(B.row):
234         for j in range(B.col):
235             if i == j:
236                 B.body[i][j] = 0
237             else:
238                 B.body[i][j] = -1 * A.body[i][j]
239     ans = (B,b)
240     return ans
241 
242 def GS_op1(B,b,x):
243     """simple function."""
244     for i in range(x.row):
245         tmp = 0
246         for j in range(B.col):
247             tmp += B.body[i][j] * x.body[j][0]
248         x.body[i][0] = tmp + b.body[i][0]
249         # x is changed during one iteration
250     return x
251 
252 def relax_op1(B,b,x):
253     """simple function."""
254     for i in range(x.row):
255         tmp = 0
256         for j in range(B.col):
257             tmp += B.body[i][j] * x.body[j][0]
258         x.body[i][0] = tmp + b.body[i][0]
259         # x is changed during one iteration
260     return x
261 
262 class Plot:
263     """Just for testing."""
264     def __init__(self):
265         """Initialize this class.
266 
267         As you can see, this function could not be universally used.
268         """
269         A = Matrix(6,6,6)
270         A.body = [[4,-1, 0,-1, 0, 0]\
271                ,[-1, 4,-1, 0,-1, 0]\
272                ,[ 0,-1, 4, 0, 0,-1]\
273                ,[-1, 0, 0, 4,-1, 0]\
274                ,[ 0,-1, 0,-1, 4,-1]\
275                ,[ 0, 0,-1, 0,-1, 4]]
276 
277         b = Matrix(6,1,1)
278         b.body = [[0],[5],[0],[6],[-2],[6]]
279         tmp = iterFormat(A,b)
280 
281         '''
```

```
282         self.B:             the matrix to iterate
283         self.b:             the matrix with only one column
284         self.x:             the original value we give,
285                             Default set it [0,0,0,0,0,0]'
286         '''
287         self.B = tmp[0]
288         self.b = tmp[1]
289         self.x = Matrix(6,1,1)
290 
291     def Jacobi(self,times):
292         self.__init__()
293         for i in range(times-1):
294             step1 = self.B.matrixMulti(self.x)
295             step2 = step1.matrixAdd(self.b)
296             self.x = step2
297 
298         lastStep = Matrix(self.x.row,self.x.col,self.x.mainCol)
299         for i in range(self.x.row):
300             lastStep.body[i][0] = self.x.body[i][0]
301 
302         step1 = self.B.matrixMulti(lastStep)
303         lastStep = step1.matrixAdd(self.b)
304 
305         step3 = lastStep.matrixConstMulti(-1)
306         there = step2.matrixAdd(step3)
307 
308         c = list()
309         for i in range(there.row):
310             c.append(there.body[i][0])
311 
312         return vectorNormalValue(c)
313 
314     def GS(self,times):
315         self.__init__()
316 
317         for i in range(times-1):
318             GS_op1(self.B,self.b,self.x)
319 
320         tmp = Matrix(6,1,1)
321         for i in range(self.x.row):
322             tmp.body[i][0] = self.x.body[i][0]  # deep copy
323 
324         GS_op1(self.B,self.b,self.x)        # do it once more
325 
326         step3 = self.x.matrixConstMulti(-1)
327         there = tmp.matrixAdd(step3)
328 
329         c = list()
330 
```

```
331         for i in range(there.row):
332             c.append(there.body[i][0])
333 
334         return vectorNormalValue(c)
335 
336     def plot(self,n=100):
337         """Default iteration deepth is 100."""
338 
339         x_jacobi = []
340         y_jacobi = []
341         for i in range(4,n,4):
342             x_jacobi.append(i)
343             y_jacobi.append(self.Jacobi(i))
344 
345         x_gs = []
346         y_gs = []
347         for i in range(4,n,4):
348             x_gs.append(i)
349             y_gs.append(self.GS(i))
350 
351         pl.grid()
352         pl.title('Relation: Iteration deepth and Normal Value',fontsize=16)
353         pl.plot(x_jacobi,y_jacobi,'o-g',label='Jacobi Method')
354         pl.plot(x_gs,y_gs,'o-r',label='Gauss-Seidel Method')
355         pl.legend()
356         pl.xlabel('Iteration Deepth')
357         pl.ylabel('Normal Value')
358 
359         pl.show()
360 
361 M = Plot()
362 M.plot()
```

Code Box 4

输出结果



输出结果 4

代码分析：

可以发现，在相同的迭代次数下，高斯-赛德尔方法的精度更高。

5 题

用松弛法解

$$\begin{cases} 4x_1 - x_2 = 1 \\ -x_1 + 4x_2 - x_3 = 4 \\ -x_2 + 4x_3 = -3 \end{cases} \tag{5.1}$$

分别取$\omega = 1.03$，$\omega = 1$，$\omega = 1.1$。要求当$\|x^{(K)} - x^{(K-1)}\| < 5 \times 10^{-6}$时迭代终止，并对每个$\omega$值确定迭代次数（初值$x^{(0)} = (0,0,0)^{\mathrm{T}}$）。

解答：

可以从更高的观点来看这个问题。下面编程作图以比较。

程序代码：

```
# -*- coding: utf-8 -*-
"""
Created on Tue Nov 28 18:30:53 2017

@author: Newton
"""

"""filename: 4.5 Relaxation Method.py"""

import matplotlib.pyplot as pl
import numpy as np

class Matrix:
    """Abatrct class representing a Matrix.

    The internal structure is a two dimension list.
    """
    def __init__(self,m,n,mainCol = None):
        """
        self.row:           the row of the Matrix
        self.col:           the collumn of the Matrix
        self.CheckedRow:     if one row has been checked,
                            it should not be checked again
        self.body:          the internal storing structure
        self.mainCol:       the coefficient matrix
        """
        self.row = m
        self.col = n
        self.CheckedRow = set(range(self.row))
        self.body = [[0 for i in range(n)] for i in range(m)]
        self.mainCol = mainCol

    def getVal(self):
        """Giving value to each element of the matrix.
```

```
35
36          Overwrite the original value zeros.
37          """
38          for i in range(self.row):
39              for j in range(self.col):
40                  self.body[i][j] = input()
41
42      def valid(self,e,kind = None):
43          """If these two matrices are not in the right form, return false,
44          else return true.
45
46          It is useful in the next functions.
47          """
48          if kind == 'multi':
49              # SELF * E
50              return self.col == e.row
51
52          if self.row != e.row or self.col != e.col:
53              return False
54          else:
55              return True
56
57      def matrixAdd(self,e):
58          """One matrix add to another and generate a new one.
59
60          The original one would not change.
61          """
62          self.valid(e)
63          # if e and self are not in the same form, return false.
64
65          ans = Matrix(self.row,self.col,self.mainCol)
66
67          for i in range(self.row):
68              for j in range(self.col):
69                  ans.body[i][j] = self.body[i][j] + e.body[i][j]
70
71          return ans
72
73      def matrixConstMulti(self,const):
74          """A constant number multiple a matrix."""
75
76          ans = Matrix(self.row,self.col,self.mainCol)
77
78          for i in range(self.row):
79              for j in range(self.col):
80                  ans.body[i][j] = self.body[i][j] * const
81
82          return ans
83
```

```
84      def matrixMulti(self,e):
85          """Return the multiplication of two matrices.
86
87          Attention! ans = self * e, not e * self.
88          """
89
90          self.valid(e,'multi')
91          # if e and self could not make multiplication, return false
92
93          ans = Matrix(self.row, e.col, e.col)
94
95          for i in range(self.row):
96              for j in range(e.col):
97                  tmp = 0
98                  for k in range(self.col):
99                      tmp += self.body[i][k] * e.body[k][j]
100                 ans.body[i][j] = tmp
101
102         return ans
103
104     def matrixTransform(self,target_row,source_row,times=None):
105         """
106         case 1:    two coefficient, make coe(1) row times coe(2)
107         case 2:    exchange the two rows with 'exchange' reminding
108         case 3:    coe(1) rows add the coe(3) times of coe(2) row.
109
110         """
111
112         ans = Matrix(self.row, self.col, self.mainCol)
113         for i in range(self.row):   # deep copy
114             for j in range(self.col):
115                 ans.body[i][j] = self.body[i][j]
116
117         if times == None:    # special case of matrixTransform(TarRow,times)
118             times_tmp = source_row
119             for j in range(self.col):
120                 ans.body[target_row][j] = \
121                 ans.body[target_row][j] * times_tmp
122
123         elif times == 'exchange':
124             for i in range(self.col):
125                 ans.body[target_row][i], ans.body[source_row][i] \
126                     = ans.body[source_row][i], ans.body[target_row][i]
127
128         else:
129             for i in range(self.col):
130                 ans.body[target_row][i] += \
131                 times * ans.body[source_row][i]
132
```

```
133         return ans
134 
135     def matrixTranspose(self):
136         """Generate a new matrix which is the transpose of the old one."""
137 
138         ans = Matrix(self.col,self.row,self.row)
139         # main column of ans is meaningless
140 
141         for i in range(self.row):
142             for j in range(self.col):
143                 ans.body[j][i] = self.body[i][j]
144         return ans
145 
146     def matrixNormVal(self, kind = 2):
147         """
148         Return the Normal value of this matrix.
149         """
150 
151         if kind == 'inf':
152             """NorVal is the maximum value among the sums of every row. """
153             sums = list()
154             tmp = 0
155             for i in range(self.row):
156                 for j in range(self.col):
157                     tmp += self.body[i][j]
158                 sums.append(tmp)
159                 tmp = 0
160 
161             sums = Quick_Sort(sums)     # quick sort algorithm
162             return sums[-1]
163 
164         elif kind == 1:
165             """NorVal is the maximum value among the sums of every column. """
166             sums = list()
167             tmp = 0
168             for i in range(self.col):
169                 for j in range(self.row):
170                     tmp += self.body[j][i]
171                 sums.append(tmp)
172                 tmp = 0
173 
174             sums = Quick_Sort(sums)
175             return sums[-1]
176 
177         elif kind == 2:
178             """
179             The solving of eig is a little difficult for me,
180             I will change the code after learning more later.
181 
```

```
182             Now I use the numpy package to solve it.
183             """
184             import numpy as np
185 
186             MUL = self.matrixMulti(self.matrixTranspose())
187             tmp = np.array(MUL.body)
188 
189             a,b = np.linalg.eig(tmp)
190 
191             return np.sqrt(max(a))
192 
193         else:
194             raise ValueError("""Bad input!\n""")
195 
196     def matrixInversion(self):
197         """Generate a new matrix which is the old one's inversion."""
198 
199         Max = 0                         # initialize the variable
200         position_row = 0                 # the row number of the maximum value
201         position_col = 0
202 
203 
204         ans = Matrix(self.row, self.col, self.mainCol)
205         for i in range(self.row):
206             for j in range(self.col):
207                 ans.body[i][j] = self.body[i][j]
208 
209         eyes = Matrix(self.row, self.col, self.mainCol)
210         for i in range(eyes.row):
211             eyes.body[i][i] = 1
212             ans.body[i] += eyes.body[i]
213 
214         ans.mainCol = ans.col
215         ans.col *= 2
216 
217         for i in range(ans.row):
218             for j in ans.CheckedRow:
219                 for k in range(ans.mainCol):  # not in all the columns
220                     if abs(Max) <= abs(ans.body[j][k]):
221                         Max = ans.body[j][k]
222                         position_row = j
223                         position_col = k
224 
225             ans = ans.matrixTransform(position_row, 1 / Max)
226 
227             ans.CheckedRow.remove(position_row)
228 
229             for j in range(ans.row):
230                 if j != position_row:
```

```
231                     ans = ans.matrixTransform\
232                     (j, position_row,-1 * ans.body[j][position_col])
233 
234             Max = 0
235             position_row = 0
236             position_col = 0
237 
238         begin = 0
239         for j in range(ans.mainCol):
240             for i in range(ans.row):
241                 if ans.body[i][j] == 1:
242                     ans = ans.matrixTransform(begin,i,'exchange')
243                     begin += 1
244 
245         new = Matrix(self.row, self.row, None)
246         for i in range(new.row):
247             for j in range(new.col):
248                 new.body[i][j] = ans.body[i][j+ans.row]
249         ans = new
250         return ans
251 
252 def Quick_Sort(L):
253     """A simple code to order a list with quicksort algorithm."""
254 
255     if len(L) <= 1:
256         return L
257 
258     else:
259         less = []
260         equal = []
261         bigger = []
262 
263         pivot = (L[0] + L[-1] + L[len(L)//2]) / 3
264         # pivot should be taken seriously!
265 
266         for i in range(len(L)):
267             if L[i] < pivot:
268                 less.append(L[i])
269             elif(L[i] == pivot):
270                 equal.append(L[i])
271             else:
272                 bigger.append(L[i])
273         less = Quick_Sort(less)
274         bigger = Quick_Sort(bigger)
275 
276         return less + equal + bigger
277 
278 #---------------------------new class---------------------------
279 
```

```
class MatrixIterMethods:
    """This class includes three methods which could be used in the
    solving of linear equations.

    Method 1:    Jabobi method
    Method 2:    Gauss-Seidel method
    Method 3:    Relaxation method

    All these three methods will return x and the iteration deepth.
    """

    def __init__(self,A,b,x0,omega=1):
        """Initialize this class.

        A:    coefficient matrix
        b:    where Ax = b
        x0:   the original value of x we choose

        These global variables would not change after init operation.
        """

        if omega == 1:
            if b.col != 1:
                raise ValueError('Bad inputs, please Check it!')

            tmp_b = Matrix(b.row, b.col, b.mainCol)
            for i in range(b.row):
                for j in range(b.col):
                    tmp_b.body[i][j] = b.body[i][j]

            for i in range(tmp_b.row):
                tmp_b.body[i][0] = (tmp_b.body[i][0]) / A.body[i][i]

            tmp_A = Matrix(A.row, A.col, A.mainCol)
            for i in range(A.row):
                for j in range(A.col):
                    tmp_A.body[i][j] = A.body[i][j]

            for i in range(A.row):
                tmp_A = tmp_A.matrixTransform(i,1 / A.body[i][i])

            B = Matrix(A.row, A.col, A.mainCol)
            for i in range(B.row):
                for j in range(B.col):
                    if i == j:
                        B.body[i][j] = 0
                    else:
                        B.body[i][j] = -1 * tmp_A.body[i][j]
            ans = (B,tmp_b)
```

```
329
330         else:
331             L = Matrix(A.row, A.col, A.mainCol)
332             for i in range(A.row):
333                 for j in range(A.col):
334                     L.body[i][j] = A.body[i][j]
335                     if i <= j:
336                         L.body[i][j] = 0
337
338             U = Matrix(A.row, A.col, A.mainCol)
339             for i in range(A.row):
340                 for j in range(A.col):
341                     U.body[i][j] = A.body[i][j]
342                     if i >= j:
343                         U.body[i][j] = 0
344
345             D = Matrix(A.row, A.col, A.mainCol)
346             for i in range(A.row):
347                 for j in range(A.col):
348                     D.body[i][j] = A.body[i][j]
349                     if i != j:
350                         D.body[i][j] = 0
351
352             step1 = L.matrixConstMulti(omega)
353             step2 = D.matrixAdd(step1)
354             step3 = step2.matrixInversion() # mark
355
356             step4 = D.matrixConstMulti(-1 * omega)
357             step5 = U.matrixConstMulti(-1 * omega)
358             step6 = D.matrixAdd(step4)
359             step7 = step6.matrixAdd(step5)  # mark
360
361             step8 = b.matrixConstMulti(omega)
362
363             end_B = step3.matrixMulti(step7)
364             end_b = step3.matrixMulti(step8)
365
366             ans = (end_B, end_b)
367
368         self.B = ans[0]
369         self.b = ans[1]
370         self.x = Matrix(x0.row, x0.col, x0.mainCol)
371
372         for i in range(x0.row):
373             for j in range(x0.col):
374                 self.x.body[i][j] = x0.body[i][j]
375
376     def GS_op1(self):
377         """Do one time iteration with GS method or Relaxation method.
```

```
378
379         self.x will be changed after this operation.
380         """
381         for i in range(self.x.row):
382             tmp = 0
383             for j in range(self.B.col):
384                 tmp += self.B.body[i][j] * self.x.body[j][0]
385             self.x.body[i][0] = tmp + self.b.body[i][0]
386 
387     def jacobiMethod(self,accuracy):
388         """The iteration would not stop by iter times.
389 
390         It will stop while the accuracy gets.
391         """
392         self.__init__(A,b,x0)   # omega uses the default setting
393 
394         save_tmp = self.x.matrixConstMulti(-1)
395         # save the original value for comparing.
396 
397         step1 = self.B.matrixMulti(self.x)
398         step2 = step1.matrixAdd(self.b)
399 
400         self.x = step2
401         # update the value of self.x
402 
403         p = step2.matrixAdd(save_tmp)
404         accu = p.matrixNormVal()
405 
406         iter_deepth = 1
407 
408         while(accu > accuracy):
409             save_tmp = self.x.matrixConstMulti(-1)
410             # save the original value for comparing.
411 
412             step1 = self.B.matrixMulti(self.x)
413             step2 = step1.matrixAdd(self.b)
414 
415             self.x = step2
416             # update the value of self.x
417 
418             p = step2.matrixAdd(save_tmp)
419             accu = p.matrixNormVal()
420 
421             iter_deepth += 1
422 
423         return (self.x, iter_deepth)
424 
425     def gsMethod(self, accuracy):
426         """Iteration will stop when the accuracy gets."""
```

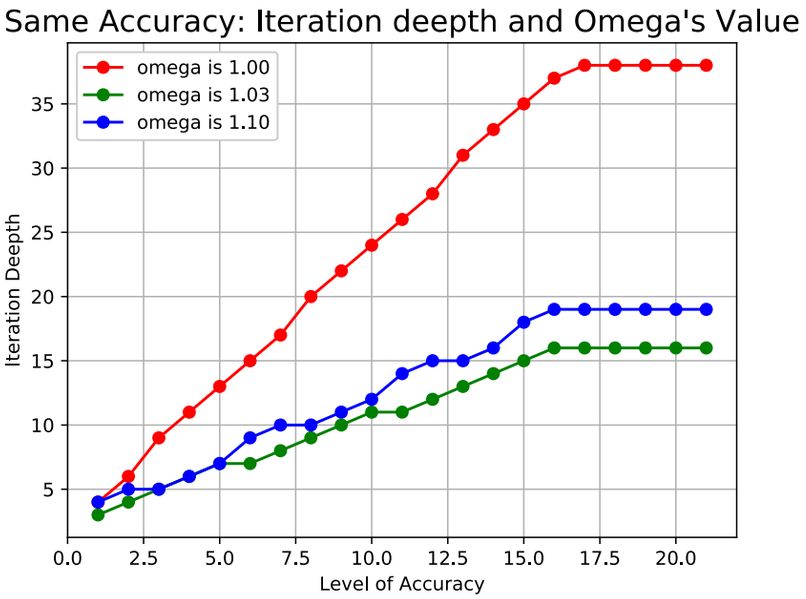
```
427
428         self.__init__(A,b,x0)
429
430         save_tmp = self.x.matrixConstMulti(-1)
431         # save the original value for comparing.
432
433         self.GS_op1()
434         # self.x has been changed auto
435
436         p = self.x.matrixAdd(save_tmp)
437         accu = p.matrixNormVal()
438
439         iter_deepth = 1
440
441         while(accu > accuracy):
442             save_tmp = self.x.matrixConstMulti(-1)
443             # save the original value for comparing.
444
445             self.GS_op1()
446             # self.x has been changed
447
448             p = self.x.matrixAdd(save_tmp)
449             accu = p.matrixNormVal()
450
451             iter_deepth += 1
452
453         return(self.x, iter_deepth)
454
455     def relaxMethod(self, omega, accuracy):
456         """Iteration will stop when the accuracy gets."""
457
458         self.__init__(A,b,x0, omega)   # omega uses the default setting
459
460         save_tmp = self.x.matrixConstMulti(-1)
461         # save the original value for comparing.
462
463         step1 = self.B.matrixMulti(self.x)
464         step2 = step1.matrixAdd(self.b)
465
466         self.x = step2
467         # update the value of self.x
468
469         p = step2.matrixAdd(save_tmp)
470         accu = p.matrixNormVal()
471
472         iter_deepth = 1
473
474         while(accu > accuracy):
475             save_tmp = self.x.matrixConstMulti(-1)
```

```
476             # save the original value for comparing.
477
478             step1 = self.B.matrixMulti(self.x)
479             step2 = step1.matrixAdd(self.b)
480
481             self.x = step2
482             # update the value of self.x
483
484             p = step2.matrixAdd(save_tmp)
485             accu = p.matrixNormVal()
486
487             iter_deepth += 1
488         return (self.x, iter_deepth)
489
490 def Plot(A, b, x0, e=1e-16):
491     """A pure function for ploting.
492
493     e is the target accuracy.
494     """
495
496     ERROR = list()
497     tmp = 1
498     while tmp > e:
499         tmp /= 10
500         ERROR.append(tmp)
501
502     deepth_1 = list()
503     for i in ERROR:
504         J = MatrixIterMethods(A, b, x0)
505         M_1 = J.relaxMethod(1.0, i)
506         # Relaxation factor is 1.0, equal to GSM
507
508         deepth_1.append(M_1[1])
509
510     deepth_2 = list()
511     for i in ERROR:
512         M_1 = J.relaxMethod(1.03, i)
513         # Relaxation factor is 1.03
514         deepth_2.append(M_1[1])
515
516     deepth_3 = list()
517     for i in ERROR:
518         M_1 = J.relaxMethod(1.1,i)
519         # Relaxation factor is 1.1
520         deepth_3.append(M_1[1])
521
522     import math
523     for i in range(len(ERROR)):
524         ERROR[i] = -1 * math.log10(ERROR[i])
```

```
525
526     pl.grid()
527     pl.title("Same Accuracy: Iteration deepth and Omega's Value",fontsize=16)
528     pl.plot(ERROR, deepth_1, 'o-r',label = 'omega is 1.00')
529     pl.plot(ERROR, deepth_2, 'o-g',label = 'omega is 1.03')
530     pl.plot(ERROR, deepth_3, 'o-b',label = 'omega is 1.10')
531     pl.legend()
532     pl.xlabel('Level of Accuracy')
533     pl.ylabel('Iteration Deepth')
534     pl.show()
535
536 if __name__ == "__main__":
537
538     A = Matrix(6, 6)
539     A.body = [[4, -1,  0, -1,  0,  0]\
540            ,[-1,  4, -1,  0, -1,  0]\
541            ,[ 0, -1,  4,  0,  0, -1]\
542            ,[-1,  0,  0,  4, -1,  0]\
543            ,[ 0, -1,  0, -1,  4, -1]\
544            ,[ 0,  0, -1,  0, -1,  4]]
545
546     b = Matrix(6, 1)
547     b.body = [[0],[5],[0],[6],[-2],[6]]
548
549     x0 = Matrix(6, 1)
550     x0.body = [[1],[1],[1],[1],[1],[1]]
551
552     Plot(A, b, x0, 1e-20)
553
554     A = Matrix(3, 3)
555     A.body = [[4, -1,  0]\
556            ,[-1,  4, -1]\
557            ,[ 0, -1,  4]]
558
559     b = Matrix(3, 1)
560     b.body = [[1],[4],[-3]]
561
562     x0 = Matrix(3, 1)
563     x0.body = [[1],[1],[1]]
564
564     Plot(A, b, x0, 1e-20)
```

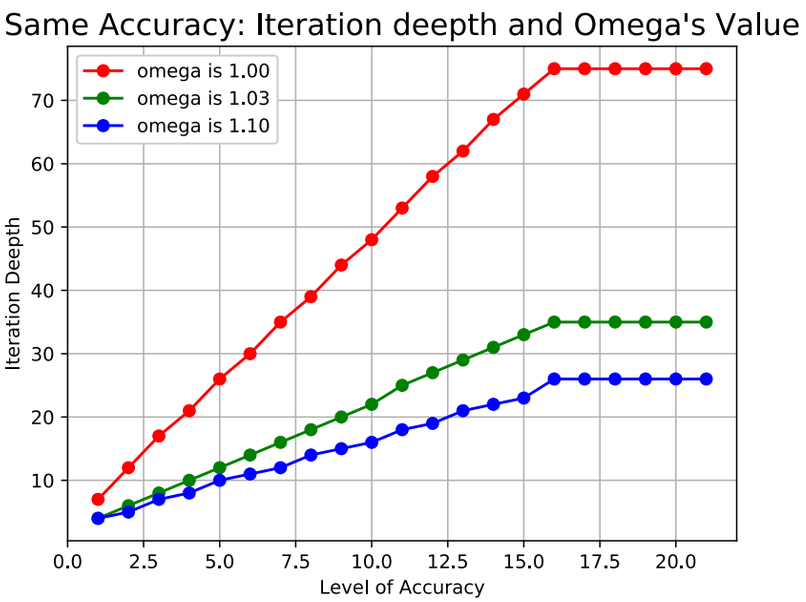
Code Box 5

输出结果：



输出结果 5 三阶矩阵

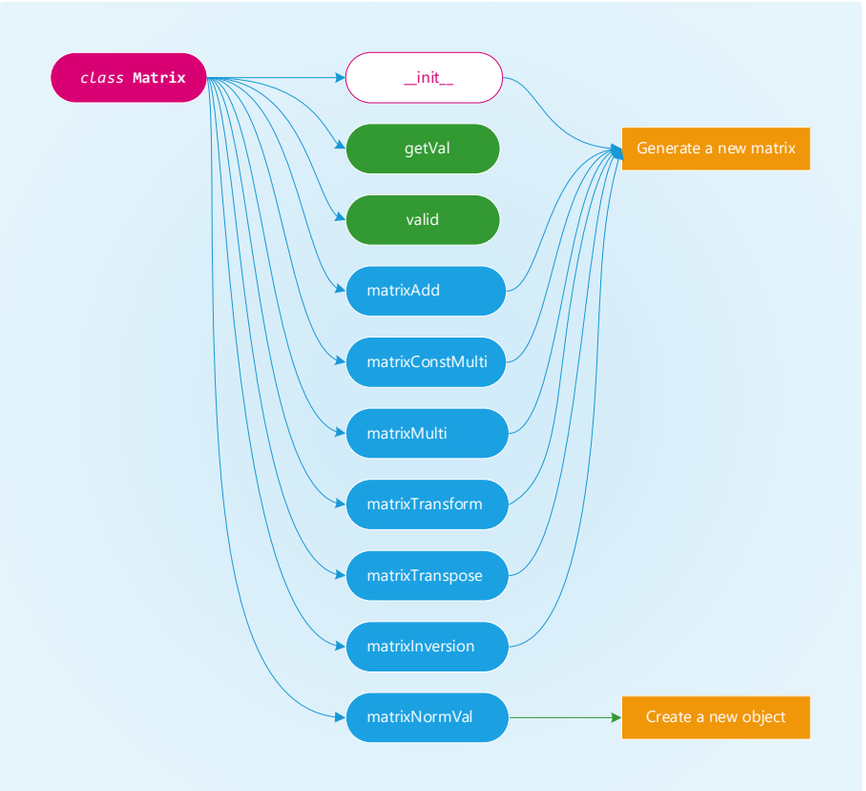
其实，还可以对第 4 题进行作图比较：



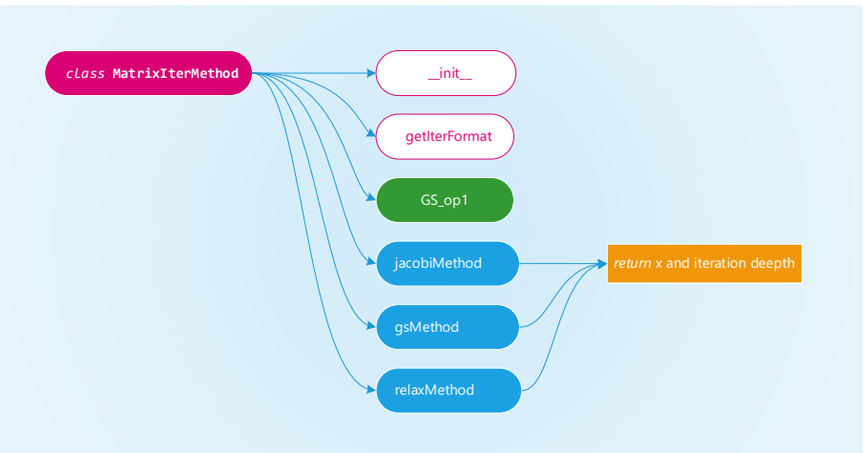
输出结果 6 六阶矩阵

可以发现，收敛速度最快的松弛因子，对于不同矩阵来说是不一样的。

代码分析：



Code Analysis 1



Code Analysis 2

本题代码行数比较多，而且涉及到了多个类的交互，所以对于参数的设计、传递，都有着巨大的考验，为了能够顺利构建出最终的图像，必须把 `class Matrix` 的对外行为设计得合乎规范，这样才能保证在调用矩阵的方法时毫无差错。可以从 Code Analysis 1 中看到，矩阵的所有方法，都没有改变该矩阵，换言之，这个类是不可更改的。这要求调用的时候，必须参考这一原则，写清楚赋值语句。而第二个类 `class MatrixIterMethod` 中，三个方法都列入了其中，虽然本题从实质上并没有调用雅可比方法，不过从先前的实验中可以看到，相同的精度下，它需要的迭代次数更多。

本段代码的更改经历了很长的时间，最终发现只有深刻理解才能写出程序。超松弛迭代法有着很深刻的数学经验在里面，而且在这里，超松弛迭代也经历了一个从一般分量形式到矩阵形式的转换。从时间统筹角度看，采用 MATLAB 进行编程会更加方便，也可以留出更多的时间来进行理论学习。

五、实验体会

从某种意义上讲，本次实验选错了语言，可能用基于矩阵的 MATLAB 会更加方便，而 Python 的 numpy 并不支持原生运算符，所以还是存在一定的局限性。

本试验报告的所有数据都经过 MATLAB 的验证，俱无问题。

如果有可能，在以后的实验报告中我将采用 MATLAB 进行编程。

六、参考文献

[1] 金一庆, 陈越, 王冬梅. 数值方法[M]. 北京: 机械工业出版社; 2000.2.
[2] 黄金伟. 矩阵的特征值与特征向量的简易求法[J]. 福建信息技术教育. 2006.