云南大学数学与统计学院实验教学中心
实验报告

课程名称： 数学建模实验	**学期：** 2016~2017 学年下学期	
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实验名称： 非线性方程求解 & MATLAB 实现		**成绩：**
实验编号： NO.5	**实验日期：** 2017 年 7 月 1 日	**实验学时：** 2
学院： 数学与统计学院	**专业：** 信息与计算科学	**年级：** 2015 级

一、实验目的

1. 学习有关非线性方程求解的数学方法；
2. 学会用 MATLAB 进行非线性方程的求解；

二、实验内容

1. 对课后题进行编程求解；
2. 学会相关的 MATLAB 命令函数。

三、实验平台

Windows10 Enterprise 1703 中文版操作系统；
MATLAB R2017a 中文版。

四、实验记录与实验结果分析

1 题

利用 help 或者 document 学习 solve 命令。

Solution:

solve
Equations and systems solver

Syntax

```
S = solve (eqn, var)
S = solve (eqn, var, Name, Value)
Y = solve (eqns, vars)
Y = solve (eqns, vars, Name, Value)
[y1, ..., yN] = solve (eqns, vars)
[y1, ..., yN] = solve(eqns, vars, Name, Value)
[y1, ..., yN, parameters, conditions] = solve
(eqns, vars, 'ReturnConditions', true)
```

Description

`S = solve (eqn, var)` solves the equation `eqn` for the variable `var`. If you do not specify `var`, the `symvar` function determines the variable to solve for. For example, `solve (x + 1 == 2, x)` solves the equation $x + 1 = 2$ for x.

`S = solve(eqn, var, Name, Value)` uses additional options specified by one or more `Name,Value` pair arguments.

`Y = solve(eqns, vars)` solves the system of equations `eqns` for the variables `vars` and returns a structure that contains the solutions. If you do not specify `vars`, `solve` uses `symvar` to find the variables to solve for. In this case, the number of variables that `symvar` finds is equal to the number of equations `eqns`.

`Y = solve (eqns, vars, Name, Value)` uses additional options specified by one or more `Name,Value` pair arguments.

`[y1, ... , yN] = solve (eqns, vars)` solves the system of equations `eqns` for the variables `vars`. The solutions are assigned to the variables `y1, ... ,yN`. If you do not specify the variables, `solve` uses `symvar` to find the variables to solve for. In this case, the number of variables that `symvar` finds is equal to the number of output arguments `N`.

`[y1, ... , yN] = solve (eqns, vars, Name, Value)` uses additional options specified by one or more `Name,Value` pair arguments.

`[y1 , ..., yN, parameters, conditions] = solve (eqns, vars, 'ReturnConditions', true)` returns the additional arguments `parameters` and `conditions` that specify the parameters in the solution and the conditions on the solution.

2 题

对方程$x^2+x-14=0$求根，步骤如下：

Step 1: 利用图形法确定根所在的位置；

Step 2: 构造迭代公式

$$x_{k+1}=\frac{14}{x_k+1}$$

初值由 step1 给出；

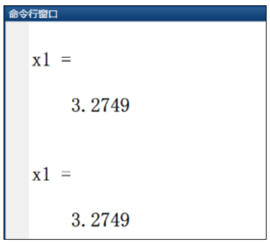
Step 3: 分别用迭代次数和$|x_{k+1}-x_k|\leq\varepsilon$作为终止条件，给出根的近似值

Solution:

程序代码：

```
% filename: Test_iteration

%- 方程求根 -%
clc;clear all;

x0 = 3;
x1 = 14 / (x0 + 1);
%- 迭代次数限制 -%
while i < 50
    x0 = x1;
    x1 = 14 / (x0 + 1);
    i = i + 1;
end
x1

%- 差的绝对值限制 -%
clear all;
x0 = 3;
x1 = 14 / (x0 + 1);
while abs(x0 - x1) > 1e-6
    x0 = x1;
    x1 = 14 / (x0 + 1);
end
x1
```

运行结果：



```
x1 =

    3.2749


x1 =

    3.2749
```

代码分析：

根据迭代公式给出代码。比较简单的循环。

3 题

请将 Step 2 中公式换为

$$x_{k+1} = x_k - \frac{x_k^2 + x_k - 14}{2x_k + 1}$$

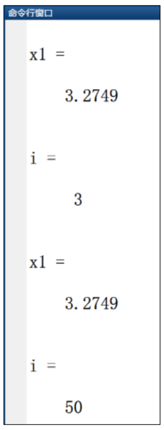
以及牛顿公式。然后再次计算。

Solution:

程序代码:

```
% filename: Test_iteration_optim

%- 方程求根 -%
clc;clear all;

x0 = 3;
x1 = x0 - (x0^2 + x0 -14) / (2 * x0 + 1);
%- 迭代次数限制 -%
i = 0;
while abs(x0 - x1) > 1e-6
    x0 = x1;
    x1 = x0 - (x0^2 + x0 -14) / (2 * x0 + 1);
    i = i + 1;
end
x1
i

%- 差的绝对值限制 -%
clear all;
i = 0;
x0 = 3;
x1 = 14 / (x0 + 1);
while abs(x0 - x1) > 1e-6
    x0 = x1;
    x1 = 14 / (x0 + 1);
    i = i + 1;
end
x1
i
```

运行结果:



```
x1 =

    3.2749


i =

     3


x1 =

    3.2749


i =

    50
```

代码分析:

可以看到，新的迭代公式下，只用了三次就很棒了，而老的方法却用了 50 次。差距相当大。

4 题

分别用 fzero 和 fsolve 程序求方程$\sin x - \frac{x^2}{2} = 0$的所有根，精确到$10^{-10}$，取不同的初值计算，输出**初值**、**根的近似值**和**迭代次数**，分析不同根的收敛域；自己构造某个迭代公式（如$x = (2\sin x)^{1/2}$等），用迭代法求解，并自己编写牛顿法的程序进行求解和比较。

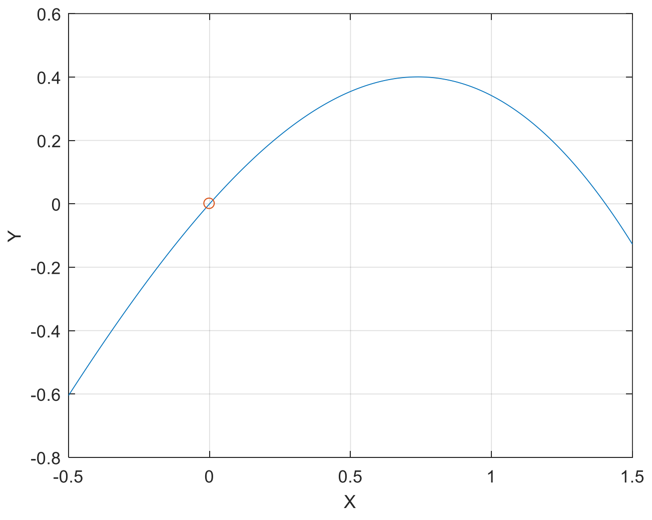
程序代码:

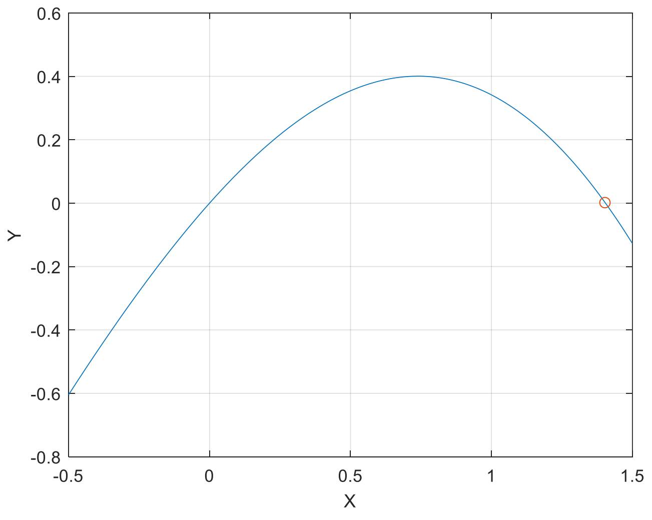
```
% filename: Function_test

clc;clear all;
fun =  @(x) sin(x) - x.^2 / 2;
Diff = @(x)cos(x) - x;
x = -0.5 : 0.01 : 1.5;
plot(x,fun(x));

%- 可以发现，在（-0.5，0.5）之内函数有零点 -%

x0 = x(1);
x1 =  x0 - fun(x0) / Diff(x0);
while abs(x0 - x1) > 1e-10
    x0 = x1;
    x1 = x0 - fun(x0) / Diff(x0);
end
fprintf('Newton Method: x = %3.18f\n\n',x0);
hold on
plot(x0,fun(x0),'o')
grid on
xlabel('X');
ylabel('Y');

%- 自行构造迭代公式 -%

clear all
Phi = @(x) sqrt(2 * sin(x));
x0 = -1;
x1 = Phi(x0);
while abs(x0 - x1) > 1e-10
    x0 = x1;
    x1 = Phi(x0);
end
fprintf('My Method: x = %3.18f\n\n',x0);
figure;
fun =  @(x) sin(x) - x.^2 / 2;
x = -0.5 : 0.01 : 1.5;
plot(x,fun(x));
hold on
plot(x0,fun(x0),'o')
grid on
xlabel('X');
```

```
ylabel('Y');

%- fsolve: x0 = -1 -%

clear all
fun =  @(x) sin(x) - x.^2 / 2;
x0 = fsolve(fun,-1);
fprintf('fsolve Method: x = %3.18f\n\n',x0);
figure
x = -0.5 : 0.01 : 1.5;
plot(x,fun(x));
hold on
plot(x0,fun(x0),'o')
grid on
xlabel('X');
ylabel('Y');

%- fsolve: x0 = 1.5 -%

clear all
fun =  @(x) sin(x) - x.^2 / 2;
x0 = fsolve(fun,1.5);
fprintf('fsolve Method: x = %3.18f\n\n',x0);
figure
x = -0.5 : 0.01 : 1.5;
plot(x,fun(x));
hold on
plot(x0,fun(x0),'o')
grid on
xlabel('X');
ylabel('Y');

%- fzero: x0 = -1 -%

clear all
fun =  @(x) sin(x) - x.^2 / 2;
x0 = fzero(fun,-1);
fprintf('fzero Method: x = %3.18f\n\n',x0);
figure
x = -0.5 : 0.01 : 1.5;
plot(x,fun(x));
hold on
plot(x0,fun(x0),'o')
grid on
xlabel('X');
ylabel('Y');

%- fsolve: x0 = 1.5 -%

```

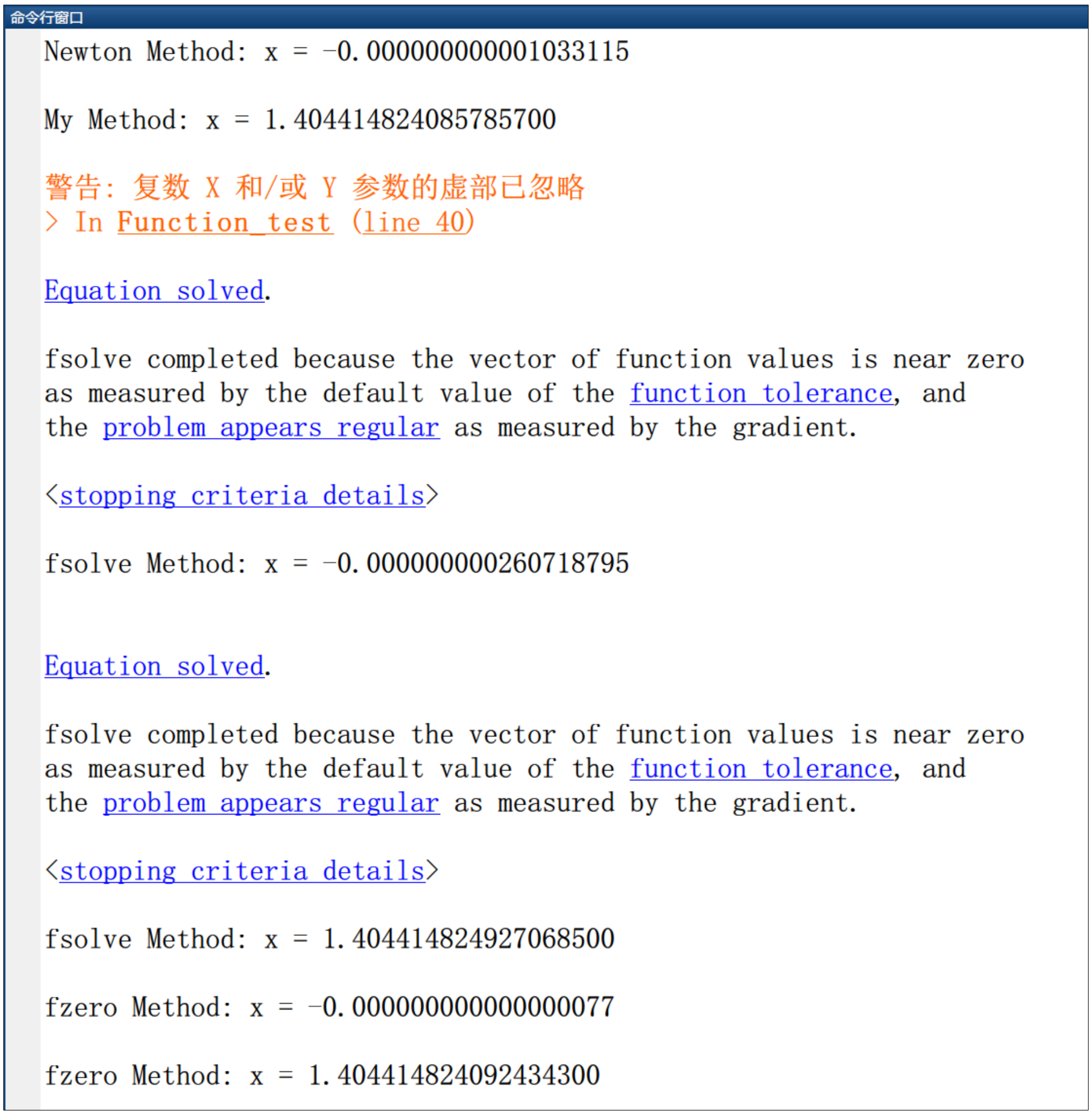
```
clear all
fun =  @(x) sin(x) - x.^2 / 2;
x0 = fzero(fun,1.5);
fprintf('fzero Method: x = %3.18f\n\n',x0);
figure
x = -0.5 : 0.01 : 1.5;
plot(x,fun(x));
hold on
plot(x0,fun(x0),'o')
grid on
xlabel('X');
ylabel('Y');
```

运行结果：





由于生成的图片太多而且基本一样，这里就不全部展示了。总得来看，牛顿方法，自定义迭代方法，以及 fsolve 与 fzero，采用的核心原理基本一致，都与给定的初值有关，所以，所得根都在初值的附近。



```
Newton Method: x = -0.000000000001033115

My Method: x = 1.404414824085785700

警告: 复数 X 和/或 Y 参数的虚部已忽略
> In Function_test (line 40)

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the default value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>

fsolve Method: x = -0.000000000260718795


Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the default value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>

fsolve Method: x = 1.404414824927068500

fzero Method: x = -0.000000000000000077

fzero Method: x = 1.404414824092434300
```

代码分析：

5 题

给定 4 种物质对应的参数a_i，b_i，c_i和交互作用矩阵 $\boldsymbol{Q}$ 如下：

$a_1 = 18.607$, $a_2 = 15.841$, $a_3 = 20.443$, $a_4 = 19.293$;
$b_1 = 2643.31$, $b_2 = 2755.64$, $b_3 = 4628.96$, $b_4 = 4117.07$;
$c_1 = 239.73$, $c_2 = 219.16$, $c_3 = 252.64$, $c_4 = 227.44$;

$$Q = \begin{bmatrix} 1.0 & 0.192 & 2.169 & 1.611 \\ 0.316 & 1.0 & 0.477 & 0.524 \\ 0.377 & 0.360 & 1.0 & 0.296 \\ 0.524 & 0.282 & 2.068 & 1.0 \end{bmatrix}$$

在压强 p = 760mmHg 下，为了形成均相共沸混合物，温度和组分分别是多少？请尽量找出所有可能的解。

程序代码:

```matlab
% filename: Mixed

clc;clear all
n = 4;
P = 760;
a = [18.607, 15.841, 20.443, 19.293]';
b = [2643.31, 2755.64, 4628.96, 4117.07]';
c = [239.73, 219.16, 252.64 227.44]';
Q = [   1.0  ,0.192  ,2.169  ,1.611
      0.316  ,1.0  ,0.477  ,0.524
      0.377  ,0.360  ,1.0  ,0.296
      0.524  ,0.282  ,2.065  ,1.0];
XT0 = [0.50 0.10 0.20 30];
[XT,Y] = fsolve(@azeofun,XT0,[],n,P,a,b,c,Q)

function f = azeofun(XT,n,P,a,b,c,Q)
    x(n) = 1;
    for i = 1 : n - 1
        x(i) = XT(i);
        x(n) = x(n) - x(i);
    end
    T = XT(n);
    p = log(P);
    for i = 1 : n
        d(i) = x * Q(i,1:n)';
        dd(i) = x(i) / d(i);
    end
    for i = 1 : n
        f(i) = x(i) * (b(i) / (T + c(i)) + ...
            log(x * Q(i, 1:n)') + ...
            dd * Q(1:n,i) - a(i) - 1 + p);
    end
end
```

运行结果：

代码分析：

可以看到，第一种物质的参数都很小，所以最终结果的组分为 0。

4. 设国民经济由农业、制造业和服务业三个部门构成，已知某年它们之间的投入产出关系、外部关系、初始投入等如表 5.6 所示。

表 5.6 国民经济三个部门之间的投入产出表　　单位：亿元 o9

产出 投入	农业	制造业	服务业	外部需求	总产出
农业	15	20	30	35	100
制造业	30	10	45	115	200
服务业	20	60	0	70	150
初始投入	35	110	75		
总投入	100	200	150		

四、实验过程

设计算法，编写程序并调试。

五、实验总结

六、参考文献

[1] 大学数学实验/姜启源，谢金星，邢文训，张立平，北京：清华大学出版社，2010.12
[2] MATLAB 教程/张志涌，杨祖樱，北京：北京航空航天大学出版社，2015.1

七、教师评语