云南大学数学与统计学院实验教学中心
实验报告

课程名称： 数学建模实验	**学期：** 2016~2017 学年下学期	
指导教师： 李朝迁		
学生： 刘鹏 20151910042 信计	**学生：** 王泽坤 20151910011 应数	**学生：** 段奕臣 20151910002 应数
实验名称： 无约束优化 & MATLAB 实现		**成绩：**
实验编号： NO.6	**实验日期：** 2017 年 6 月 26 日	**实验学时：** 2
学院： 数学与统计学院	**专业：** 信息与计算科学	**年级：** 2015 级

一、实验目的

1. 学习优化的基本步骤；
2. 初步学会 MATLAB 的优化操作。

二、实验内容

1. 完成课后布置的习题；
2. 调试书上的经典代码，以掌握 MATLAB 命令。

三、实验平台

Windows10 Enterprise 1703 中文版操作系统；
MATLAB R2017a 中文版。

四、实验记录与实验结果分析

1 题

利用 help 或者 document 学习 fminbnd，fminunc，fminsearch 命令。

Solution:

（1）

fminbnd

Find minimum of single-variable function on fixed interval

*fminbnd 函数能找出单变量函数在给定区间上的最小值。

`fminbnd` is a one-dimensional minimizer that finds a minimum for a problem specified by

*fminbnd 是一个一维的分析函数，能够找出如下形式的函数的最小值

$$\min_x f(x) \text{ such that } x_1 < x < x_2.$$

x, x_1, and x_2 are finite scalars, and $f(x)$ is a function that returns a scalar.

*x, x_1, x_2 都是有限量，而 $f(x)$是一个返回标量的函数。

Syntax

```
x = fminbnd (fun, x1, x2)
x = fminbnd (fun, x1, x2, options)
x = fminbnd (problem)
[x, fval] = fminbnd (___)
[x, fval, exitflag] = fminbnd (___)
[x, fval, exitflag, output] = fminbnd (___)
```

Description

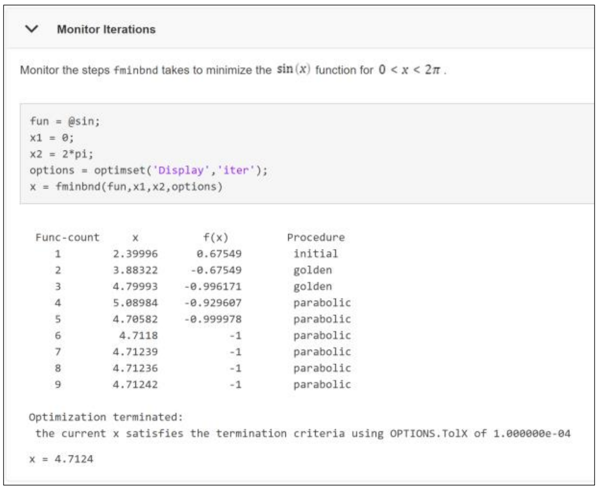
`x = fminbnd (fun, x1, x2)` returns a value `x` that is a local minimizer of the scalar valued function that is described in `fun` in the interval `x1 < x < x2`.

*`x = fminbnd (fun, x1, x2)`语句返回一个

标量数值 x，它是函数在 x1，x2 之间的最小值点。

`x = fminbnd (fun, x1, x2, options)` minimizes with the optimization options specified in `options`. Use `optimset` to set these options.

*带参数的 fminbnd，可以通过 optimset 设置参数。



Monitor Iterations

Monitor the steps `fminbnd` takes to minimize the $\sin(x)$ function for $0 < x < 2\pi$.

```
fun = @sin;
x1 = 0;
x2 = 2*pi;
options = optimset('Display','iter');
x = fminbnd(fun,x1,x2,options)
```

```
 Func-count     x          f(x)         Procedure
    1        2.39996      0.67549        initial
    2        3.88322     -0.67549        golden
    3        4.79993    -0.996171        golden
    4        5.08984    -0.929607        parabolic
    5        4.70582    -0.999978        parabolic
    6         4.7118           -1        parabolic
    7        4.71239           -1        parabolic
    8        4.71236           -1        parabolic
    9        4.71242           -1        parabolic

Optimization terminated:
 the current x satisfies the termination criteria using OPTIONS.TolX of 1.000000e-04

x = 4.7124
```

`x = fminbnd (problem)` finds the minimum for `problem`, where `problem` is a structure.

*problem 是一个结构，此语句返回 problem 的最小值。

`[x, fval] = fminbnd (___)`, for any input arguments, returns the value of the objective function computed in `fun` at the solution x.

*通过给定的参数，此语句返回最小值点 x 与其对应的最小值 $fval$。

`[x, fval, exitflag] = fminbnd (___)` additionally returns a value `exitflag` that describes the exit condition.

*在上一个函数的基础上，增加了退出条件。

`[x, fval, exitflag, output] = fminbnd (___)` additionally returns a structure `output` that contains information about the optimization.

*返回值中增加了一个 output 结构，它存数了参数的信息。

程序代码：

```
% example 1
x = fminbnd(@square,1,2);

% example 2
[a,b] = fminbnd(@square,0.5,2);

% example 3
[a,b,flag] = fminbnd(@square,0.3,2);

% example 4
[a,b,flag,output] = fminbnd(@square,0.3,2);
output

function y = square(x)
    y = x^2;
end
```

程序代码 1

运行结果：

```
命令行窗口
>> Test_fminbnd

output =

  包含以下字段的 struct:

    iterations: 21
     funcCount: 22
     algorithm: 'golden section search, parabolic interpolation'
       message: '优化已终止:   当前的 x 满足使用 1.000000e-04 的 OPTIONS.TolX 的终止条件  '
```

运行结果 1

代码分析：

通过调试 fminbnd 的几种可能用法，查看如何使用这个函数。

（2）

fminunc

Find minimum of unconstrained multivariable function

＊这个函数会返回无约束的多变量函数的最小值。

Nonlinear programming solver.

＊非线性规划的解决器

Finds the minimum of a problem specified by

＊这个命令会确定如下形式的函数的最小值

$$\min_x f(x)$$

where *f*(*x*) is a function that returns a scalar.

＊然后会返回一个标量。

x is a vector or a matrix; see Matrix Arguments.

＊x 可以是一个向量，也可以是一个矩阵；参见矩阵参数。

Syntax

```
x = fminunc (fun, x0)
x = fminunc (fun, x0, options)
x = fminunc(problem)
[x, fval] = fminunc (___)
[x, fval, exitflag, output] = fminunc (___)
[x, fval, exitflag, output, grad, hessian] = fminunc
(___)
```

Description

`x = fminunc (fun, x0)` starts at the point `x0` and attempts to find a local minimum `x` of the function described in `fun`. The point `x0` can be a scalar, vector, or matrix.

＊以 x0 为初始值，尝试性地去找附近的最小值点 x，x0 可以是标量向量或者矩阵。

Note: Passing Extra Parameters explains how to pass extra parameters to the objective function and nonlinear constraint functions, if necessary.

＊注意：额外参数传递解释了如何把额外的参数传递到对象函数以及非线性约束函数。

`fminunc` is for nonlinear problems without constraints. If your problem has constraints, generally use `fmincon`. See Optimization Decision Table.

＊fminunc 函数是为了解决非线性无约束问题的，如果你想要解决有约束的问题，那就需要 fmincon 函数了。

`x = fminunc (fun, x0, options)` minimizes `fun` with the optimization options specified in `options`. Use `optimoptions` to set these options.

＊带参数。

`x = fminunc(problem)` finds the minimum for `problem`, where `problem` is a structure described in Input Arguments. Create the `problem` structure by exporting a problem from Optimization app, as described in Exporting Your Work.

＊problem 是一个结构体，详见 input arguments。可以通过最优化工具箱来创建一个

`[x, fval] = fminunc (___)`, for any syntax, returns the value of the objective function `fun` at the solution x.

＊返回的不仅仅是最优解 x，而且还有 x 处的目标函数值。

`[x, fval, exitflag, output] = fminunc (___)` additionally returns a value `exitflag` that describes the exit condition of `fminunc`, and a structure `output` with information about the optimization process.

＊在上面的基础上，又增加了退出条件 – exitflag、以及关于优化进行的信息的一个结构体 output。

`[x, fval, exitflag, output, grad, hessian] = fminunc (___)` additionally returns:

＊在上面的基础上，又增加了以下两个输出：

- `grad` — Gradient of `fun` at the solution x.
  ＊在解 x 处的梯度 grad。
- `hessian` — Hessian of `fun` at the solution x. See fminunc Hessian.
  ＊在 x 处的黑森矩阵。详见 fminunc hessian。

程序代码：

```
function [f,g] = rosenbrockwithgrad(x)
% Calculate objective f
f = 100*(x(2) - x(1)^2)^2 + (1-x(1))^2;

if nargout > 1 % gradient required
    g = [-400*(x(2)-x(1)^2)*x(1)-2*(1-x(1));
        200*(x(2)-x(1)^2)];
end
```

程序代码 2

```
 % filename: Test_fminunc

options = optimoptions('fminunc','Algorithm',...
    'trust-region','SpecifyObjectiveGradient',true);

problem.options = options;
problem.x0 = [-1,2];
problem.objective = @rosenbrockwithgrad;
problem.solver = 'fminunc';

[x, fval] = fminunc(problem)
```

程序代码 3

运行结果：

```
命令行窗口
>> Test_fminunc

Local minimum found.

Optimization completed because the size of the gradient is less than
the default value of the optimality tolerance.

<stopping criteria details>


x =

    1.0000    1.0000


fval =

   1.9886e-17
```

运行结果 2

代码分析：

（3）

fminsearch

Find minimum of unconstrained multivariable function using derivative-free method

＊使用单纯形方法找无约束多变量函数的最小值。

Nonlinear programming solver that searches for the minimum of a problem specified by

＊这个函数时非线性规划问题的解决器，可以搜索问题的最小值，问题由以下式子界定：

$$\min_x f(x)$$

f(*x*) is a function that returns a scalar, and *x* is a vector or a matrix.

＊其中 f(x)是一个返回标量的函数，而 x 可以是向量，也可以是矩阵。

Syntax

```
x = fminsearch (fun, x0)
x = fminsearch (fun, x0, options)
x = fminsearch(problem)
[x, fval] = fminsearch (___)
[x, fval, exitflag] = fminsearch (___)
[x, fval, exitflag, output] = fminsearch (___)
```

Description

`x = fminsearch (fun, x0)` starts at the point `x0` and attempts to find a local minimum `x` of the function described in `fun`.

＊x = fminsearch(fun, x0)从初值 x0 开始，尝试找函数 fun 的极值。

`x = fminsearch (fun, x0, options)` minimizes with the optimization options specified in the structure `options`. Use `optimset` to set these options.

＊x = fminsearch(fun, x0, options)，指定了优化选项。使用 optimset 设置这些选项。

`x = fminsearch(problem)` finds the minimum for `problem`, where `problem` is a structure.

＊x = fminsearch(problem)，其中 problem 是一个结构，这个结构中把需要的信息都以成员的形式写出了。

`[x, fval] = fminsearch (___)`, for any previous input syntax, returns in `fval` the value of the objective function `fun` at the solution x.

＊[x, fval] = fminsearch(___)，对于任何上述的输入语法，都会 x 处的函数值。

`[x, fval, exitflag] = fminsearch (___)` additionally returns a value `exitflag` that describes the exit condition.

＊[x, fval, exitflag] = fminsearch(___)另外返回一个值 exitflag，该值描述退出条件。

`[x, fval, exitflag, output] = fminsearch (___)` additionally returns a structure `output` with information about the optimization process.

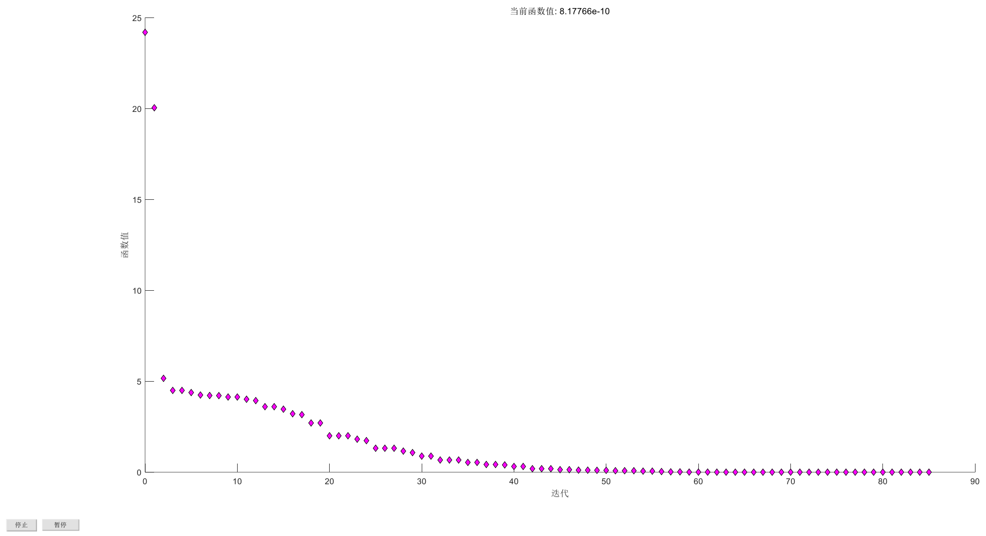
＊[x, fval, exitflag, output] = fminsearch(___)，额外地返回一个包含优化过程信息的结构输出。

程序代码：

```
% filename: Test_fminsearch

fun = @(x)100*(x(2) - x(1)^2)^2 + (1 - x(1))^2;

options = optimset('PlotFcns',@optimplotfval);
x0 = [-1.2,1];
x = fminsearch(fun,x0,options)
```

程序代码 4

运行结果：



代码分析：

options 的所有写入内容都是建立在优化工具箱的基础之上的。

2 题

利用 help 或者 document 学习 lsqnonlin，lsqcurvefit 命令。

Solution:

（1）

lsqnonlin

Solve nonlinear least-squares (nonlinear data-fitting) problems

＊解决非线性最小二乘（非线性数据拟合）问题。

Nonlinear least-squares solver

＊非线性最小二乘解决器。

Solves nonlinear least-squares curve fitting problems of the form

＊解决如下形式的非线性最小二乘曲线拟合问题：

$$\min_x \|f(x)\|_2^2 = \min_x \left(f_1(x)^2 + f_2(x)^2 + \dots + f_n(x)^2\right)$$

with optional lower and upper bounds *lb* and *ub* on the components of *x*.

＊可以选择参数 lb 和 ub，它们分别是 x 的上下界。

x, *lb*, and *ub* can be vectors or matrices; see Matrix Arguments.

＊x，lb，ub 可以是向量，也可以是矩阵，详见矩阵参数。

Rather than compute the value $\|f(x)\|_2^2$ (the sum of squares), `lsqnonlin` requires the user-defined function to compute the *vector*-valued function

＊除了进行最小二乘计算之外，这个函数还可以计算用户自定义的函数以及向量值函数。

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}.$$

Syntax

```
x = lsqnonlin (fun, x0)
x = lsqnonlin (fun, x0, lb, ub)
x = lsqnonlin (fun, x0, lb, ub, options)
x = lsqnonlin (problem)
[x, resnorm] = lsqnonlin (___)
[x, resnorm, residual, exitflag, output] = lsqnonlin (___)
[x, resnorm, residual, exitflag, output, lambda, jacobian] = lsqnonlin (___)
```

Description

`x = lsqnonlin (fun, x0)` starts at the point `x0` and finds a minimum of the sum of squares of the functions described in `fun`. The function `fun` should return a vector of values and not the sum of squares of the values. (The algorithm implicitly computes the sum of squares of the components of `fun(x)`.)

＊从 x0 处开始，找到由 fun 描述的最小二乘的值。函数 fun 返回的是一个向量，而不是平方的和。

Note: Passing Extra Parameters explains how to pass extra parameters to the vector function fun(x), if necessary.

＊注意：额外参数传递规则解释了在必要情况下，如何将额外的参数传递给向量值函数 fun(x)。

`x = lsqnonlin (fun, x0, lb, ub)` defines a set of lower and upper bounds on the design variables in x, so that the solution is always in the range `lb ≤ x ≤ ub`. You can fix the solution component `x(i)` by specifying `lb(i) = ub(i)`.

＊x 的上下界被 lb 与 ub 约束，所以输出结果就在区间[lb, ub]之内。可以通过指定 lb(i)= ub(i)来确定 x(i)的取值。

Note: If the specified input bounds for a problem are inconsistent, the output x is x0 and the outputs resnorm and residual are [].

＊注意：如果上下界前后矛盾，那么输出就是初值 x0，而其他的也都是空的。

Components of x0 that violate the bounds lb ⩽ x ⩽ ub are reset to the interior of the box defined by the bounds. Components that respect the bounds are not changed.
*超过限制的 x0，将会被抹掉，而合理的就不会变。

x = lsqnonlin (fun, x0, lb, ub, options) minimizes with the optimization options specified in options. Use optimoptions to set these options. Pass empty matrices for lb and ub if no bounds exist.
*带了参数。

x = lsqnonlin(problem) finds the minimum for problem, where problem is a structure described in Input Arguments. Create the problem structure by exporting a problem from Optimization app, as described in Exporting Your Work.
*结构体参数。

[x, resnorm] = lsqnonlin (___), for any input arguments, returns the value of the squared 2-norm of the residual at x : sum (fun(x).^2).
*额外输出 x 平方的和。

[x, resnorm, residual, exitflag, output] = lsqnonlin (___) additionally returns the value of the residual fun(x) at the solution x, a value exitflag that describes the exit condition, and a structure output that contains information about the optimization process.
*额外返回 x 处的残差值，描述退出条件的值 exitflag，以及包含有关优化过程的结构体 output。

[x, resnorm, residual, exitflag, output, lambda, jacobian] = lsqnonlin (___) additionally returns a structure lambda whose fields contain the Lagrange multipliers at the solution x, and the Jacobian of fun at the solution x.
*额外另外返回一个结构 lambda，它包含解 x 的拉格朗日乘数，返回函数在解处的雅可比矩阵。

（2）

lsqcurvefit

Solve nonlinear curve-fitting (data-fitting) problems in least-squares sense

＊利用最小二乘，解决非线性曲线拟合问题。

Nonlinear least-squares solver

＊非线性最小二乘解决器。

Find coefficients *x* that solve the problem

＊找到能解决这个问题的最小系数 x

$$\min_x \|F(x, xdata) - ydata\|_2^2 = \min_x \sum_i (F(x, xdata_i) - ydata_i)^2,$$

given input data *xdata*, and the observed output *ydata*, where *xdata* and *ydata* are matrices or vectors, and *F* (*x*, *xdata*) is a matrix-valued or vector-valued function of the same size as *ydata*. Optionally, the components of *x* can have lower and upper bounds *lb*, and *ub*. The arguments *x*, *lb*, and *ub* can be vectors or matrices; see Matrix Arguments.

＊xdata 和 ydata 可以是矩阵或者向量，F(x, xdata)是矩阵值或者向量值的函数，与 ydata 同型。x 也可以有上下界 lb 和 ub，x，lb，ub 可以是矩阵或者向量。

The `lsqcurvefit` function uses the same algorithm as `lsqnonlin`. `lsqcurvefit` simply provides a convenient interface for data-fitting problems.

＊lsqnonlin 与 lsqcurvefit 的算法是一致的，后者为数据拟合问题提供了好的用户接口。

Rather than compute the sum of squares, `lsqcurvefit` requires the user-defined function to compute the *vector*-valued function

＊除了计算平方和之外，lsqcurvefit 要求用户定义的函数是向量值函数。

$$F(x, xdata) = \begin{bmatrix} F(x, xdata(1)) \\ F(x, xdata(2)) \\ \vdots \\ F(x, xdata(k)) \end{bmatrix}.$$

Syntax

```
x = lsqcurvefit (fun, x0, xdata, ydata)
x = lsqcurvefit (fun, x0, xdata, ydata, lb, ub)
x = lsqcurvefit (fun, x0, xdata, ydata, lb, ub, options)
x = lsqcurvefit (problem)
[x, resnorm] = lsqcurvefit (___)
[x, resnorm, residual, exitflag, output] = lsqcurvefit (___)
[x, resnorm, residual, exitflag, output, lambda, jacobian] = lsqcurvefit (___)
```

Description

`x = lsqcurvefit (fun, x0, xdata, ydata)` starts at `x0` and finds coefficients x to best fit the nonlinear function `fun (x, xdata)` to the data `ydata` (in the least-squares sense). `ydata` must be the same size as the vector (or matrix) `F` returned by `fun`.

＊从 x0 开始，找到能最好地拟合非线性函数 fun(x, xdata)系数 x，ydata 必须与 fun 的返回值同型。

Note: Passing Extra Parameters explains how to pass extra parameters to the vector function fun(x), if necessary.

＊＊注意：额外参数传递规则解释了在必要情况下，如何将额外的参数传递给向量值函数 fun(x)。

`x = lsqcurvefit (fun, x0, xdata, ydata, lb, ub)` defines a set of lower and upper bounds on the design variables in x, so that the solution is always in the range `lb ≤ x ≤ ub`. You can fix the solution component `x(i)` by specifying `lb(i) = ub(i)`.

＊加入了参数 x 的上下限。

Note: If the specified input bounds for a problem are inconsistent, the output x is x0 and the outputs resnorm and residual are [].

＊＊注意：如果上下界前后矛盾，那么输出就是初值 x0，而其他的也都是空的。

Components of x0 that violate the bounds lb ≤ x ≤ ub are reset to the interior of the box defined by the bounds. Components that respect the bounds are not changed.

＊超过限制的 x0，将会被抹掉，而合理的就不会变。

x = lsqcurvefit (fun, x0, xdata, ydata, lb, ub, options) minimizes with the optimization options specified in options. Use optimoptions to set these options. Pass empty matrices for lb and ub if no bounds exist.
*加入了 options 选项。如果 lb 与 ub 都不存在，那么需要补上两个空矩阵。

x = lsqcurvefit(problem) finds the minimum for problem, where problem is a structure described in Input Arguments. Create the problem structure by exporting a problem from Optimization app, as described in Exporting Your Work.
*结构体。

[x, resnorm] = lsqcurvefit(___), for any input arguments, returns the value of the squared 2-norm of the residual at x: sum((fun(x, xdata)-ydata).^2).
*多了残差输出。

[x, resnorm, residual, exitflag, output] = lsqcurvefit (___) additionally returns the value of the residual fun(x, xdata)-ydata at the solution x, a value exitflag that describes the exit condition, and a structure output that contains information about the optimization process.
*额外返回 x 处的残差值，描述退出条件的值 exitflag，以及包含有关优化过程的结构体 output。

[x, resnorm, residual, exitflag, output, lambda, jacobian] = lsqcurvefit(___) additionally returns a structure lambda whose fields contain the Lagrange multipliers at the solution x, and the Jacobian of fun at the solution x.
*额外另外返回一个结构 lambda，它包含解 x 的拉格朗日乘数，返回函数在解处的雅可比矩阵。

3 题

对 Page 151 例 1、2 进行验证。

Solution:

程序代码：

```
% filename: Data_fit

clc
cd ..;cd ..;cd ./#Data/06
i = xlsread('experiment_6.xlsx','A2:A34');
y = xlsread('experiment_6.xlsx','B2:B34');

fun = @(x,t) x(1) + x(2) * exp(-x(4) * t) + x(3) * exp(-x(5) * t);
x0 = [0.5,1.5,-1,0.01,0.02];


opt = optimset('Display','iter');
x = lsqcurvefit(fun,x0,i,y,[],[],opt)
y = y + rand()/2;
x = lsqcurvefit(fun,x0,i,y,[],[],opt)
```

程序代码 5

运行结果：

Iteration	Func-count	f(x)	Norm of step	First-order optimality
0	6	7.03048		346
1	12	7.03048	10	346
2	18	0.972321	2.5	58.9
3	24	0.972321	5	58.9
4	30	0.0639713	1.25	1.83
5	36	0.0639713	2.5	1.83
6	42	0.0639713	0.625	1.83
7	48	0.0634801	0.15625	0.279
8	54	0.0629471	0.3125	1.47
9	60	0.0629471	0.625	1.47
10	66	0.0627967	0.15625	0.546
11	72	0.0627825	0.3125	0.618
12	78	0.0627632	0.3125	0.572
13	84	0.0627445	0.3125	0.489
14	90	0.0627253	0.3125	0.373
15	96	0.0627253	0.625	0.373
16	102	0.0627003	0.15625	0.66
17	108	0.0627003	0.3125	0.66
18	114	0.0626835	0.078125	0.44
19	120	0.0626667	0.15625	0.482
20	126	0.0626667	0.3125	0.482
21	132	0.0626543	0.078125	0.539
22	138	0.06264	0.15625	0.361
23	144	0.06264	0.3125	0.361
24	150	0.0626255	0.078125	0.723
25	156	0.0626083	0.15625	0.27
26	162	0.0626052	0.3125	1.6

```
    27          168       0.0625375       0.078125          0.242
    28          174       0.0624883        0.15625            1.7
    29          180       0.0624204         0.3125          0.389
    30          186       0.0624189          0.625          0.941
    31          192       0.0624046        0.15625          0.251
    32          198       0.0623886         0.3125           1.85
    33          204       0.0623391         0.3125          0.336
    34          210       0.0623253          0.625          0.465
    35          216       0.0623253           1.25          0.465
    36          222       0.0623201         0.3125          0.323
    37          228       0.0623201          0.625          0.323
    38          234       0.0623142        0.15625          0.934
    39          240       0.0623062         0.3125          0.277
    40          246       0.0622759          0.625          0.829
    41          252       0.0622759           1.25          0.829
    42          258       0.0622721         0.3125          0.571
    43          264       0.0622708          0.625          0.691
    44          270       0.0622692          0.625          0.698
    45          276       0.0622675          0.625          0.659
    46          282        0.062266          0.625          0.609
    47          288       0.0622645          0.625          0.554
    48          294       0.0622645           1.25          0.554
    49          300       0.0622627         0.3125           0.32
    50          306       0.0622627          0.625           0.32
    51          312       0.0622601        0.15625          0.944
    52          318       0.0622569         0.3125          0.205
    53          324       0.0588124          0.625           24.7
    54          330       0.0570094           1.25          0.813
    55          336       0.0570094            2.5          0.813
    56          342       0.0570088          0.625          0.757
    57          348       0.0570083          0.625          0.721
    58          354       0.0570078          0.625          0.689
    59          360       0.0570074          0.625          0.658
    60          366        0.057007          0.625          0.628
    61          372       0.0570066          0.625          0.598
    62          378       0.0570062          0.625          0.568
    63          384       0.0570058          0.625          0.537
    64          390       0.0570055          0.625          0.518
    65          396       0.0570051          0.625          0.523
    66          402       0.0570048          0.625          0.529

Local minimum possible.

lsqcurvefit stopped because the final change in the sum of squares relative to
its initial value is less than the default value of the function tolerance.

<stopping criteria details>


x =

    0.2217  -13.3675   14.1518    0.0270    0.0278


                                         Norm of      First-order
 Iteration  Func-count     f(x)          step          optimality
     0           6       5.42566                           304
     1          12       5.42566             10            304
     2          18      0.384073            2.5           32.6
```

3	24	0.384073	5	32.6
4	30	0.384073	1.25	32.6
5	36	0.0959295	0.3125	0.6
6	42	0.0959295	0.625	0.6
7	48	0.09525	0.15625	0.476
8	54	0.09525	0.3125	0.476
9	60	0.0946324	0.078125	1.96
10	66	0.09369	0.15625	0.456
11	72	0.0550464	0.3125	6.44
12	78	0.0550464	0.625	6.44
13	84	0.0515626	0.15625	2.05
14	90	0.0515626	0.3125	2.05
15	96	0.0512174	0.078125	0.204
16	102	0.0512174	0.15625	0.204
17	108	0.0512123	0.0390625	0.0827
18	114	0.0512123	0.078125	0.0827
19	120	0.0511951	0.0195312	0.446
20	126	0.0511761	0.0390625	0.098
21	132	0.0511289	0.078125	0.57
22	138	0.0511289	0.15625	0.57
23	144	0.0511109	0.0390625	0.14
24	150	0.0511075	0.078125	0.115
25	156	0.0511075	0.15625	0.115
26	162	0.0511019	0.0390625	0.246
27	168	0.0511019	0.078125	0.246
28	174	0.0510983	0.0195312	0.131
29	180	0.0510948	0.0390625	0.168
30	186	0.0510948	0.078125	0.168
31	192	0.0510924	0.0195312	0.157
32	198	0.0510897	0.0390625	0.115
33	204	0.0510897	0.078125	0.115
34	210	0.0510869	0.0195312	0.214
35	216	0.0510836	0.0390625	0.0727
36	222	0.0510804	0.078125	0.628
37	228	0.0510582	0.0195312	0.0654
38	234	0.0510247	0.0390625	0.866
39	240	0.050988	0.078125	0.152
40	246	0.050988	0.15625	0.152
41	252	0.0509863	0.0390625	0.0974
42	258	0.0509863	0.078125	0.0974
43	264	0.0509842	0.0195312	0.212
44	270	0.0509817	0.0390625	0.063
45	276	0.0509764	0.078125	0.65
46	282	0.0509582	0.078125	0.14
47	288	0.0509582	0.15625	0.14
48	294	0.0509568	0.0390625	0.105
49	300	0.0509568	0.078125	0.105
50	306	0.0509554	0.0195312	0.182
51	312	0.0509538	0.0390625	0.0741
52	318	0.0509534	0.078125	0.396
53	324	0.0509467	0.0195312	0.0606
54	330	0.0509376	0.0390625	0.542
55	336	0.0509268	0.078125	0.128
56	342	0.0509268	0.15625	0.128
57	348	0.0509257	0.0390625	0.115
58	354	0.0509257	0.078125	0.115
59	360	0.0509248	0.0195312	0.157

Local minimum possible.

```
lsqcurvefit stopped because the final change in the sum of squares relative to
its initial value is less than the default value of the function tolerance.

<stopping criteria details>


x =

    0.1812   -1.8910    2.7765    0.0156    0.0211

>>
```

运行结果 3

代码分析：

扰动比较大。可以发现，迭代收敛变快了。

4 题

Page 168 的 1 题：

取不同的初值计算下列平方和形式的非线性规划，尽可能求出所有局部极小点，进而找出全局极小点，并对不同算法（搜索方向、搜索步长、数值梯度与分析梯度等）的结果进行分析、比较。

(1) $\min\,(x_1^2+x_2-11)^2+(x_1+x_2^2-7)^2$

(2) $\min\,(x_1^2+12x_2-1)^2+(49x_1^2+49x_2^2+84x_1+2324x_2-681)^2$

(3) $\min\,(x_1+10x_2)^2+5(x_3-x_4)^2+(x_2-2x_3)^4+10(x_1-x_4)^4$

(4) $\min\ 100\{[x_3-10\theta(x_1,x_2)]^2+[(x_1^2+x_2^2)^{1/2}-1]^2\}+x_3^2$

$$\theta(x_1,x_2)=\begin{cases}\dfrac{1}{2\pi}\arctan(x_2/x_1), & x_1>0\\ \dfrac{1}{2\pi}\arctan(x_2/x_1)+\dfrac{1}{2}, & x_1<0\end{cases}$$

Solution:

（1）

程序代码：

```
clc;clear all
fun1 = @(x)min((x(1)^2 + x(2) - 11)^2 + (x(1) + x(2)^2 + 7)^2);

x0 = [0,0];
[x_search,fval_search] = fminsearch(fun1,x0);
[x_unc,fval_unc] = fminunc(fun1,x0);
fprintf('fminsearch meathod: \nx = (%3.8f,%3.8f), \ny = %3.8f\n\n',...
    x_search(1),...
    x_search(2),...
    fval_search);
fprintf('fminunc method: \nx = (%3.8f,%3.8f), \ny = %3.8f\n',...
    x_unc(1),...
    x_unc(2),...
    fval_unc);
```

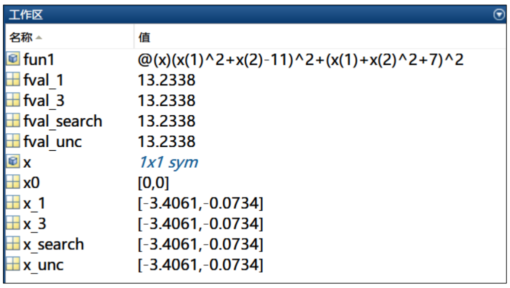
程序代码 6

运行结果：

```
命令行窗口

Local minimum found.

Optimization completed because the size of the gradient is less than
the default value of the optimality tolerance.

<stopping criteria details>

fminsearch meathod:
x = (-3.40613579,-0.07344468),
y = 13.23377869

fminunc method:
x = (-3.40613354,-0.07339699),
y = 13.23377867
```

运行结果 4

代码分析：



工作区

名称	值
fun1	@(x)(x(1)^2+x(2)-11)^2+(x(1)+x(2)^2+7)^2
fval_1	13.2338
fval_3	13.2338
fval_search	13.2338
fval_unc	13.2338
x	*1x1 sym*
x0	[0,0]
x_1	[-3.4061,-0.0734]
x_3	[-3.4061,-0.0734]
x_search	[-3.4061,-0.0734]
x_unc	[-3.4061,-0.0734]

Figure 1

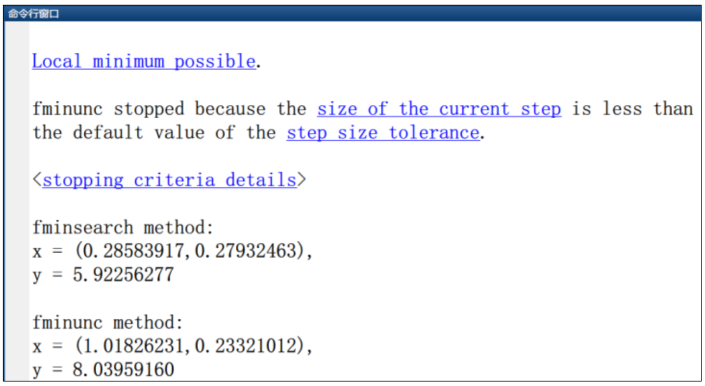
（2）

程序代码：

```
clc
fun2 = @(x)min((x(1)^2 + 12 * x(2) - 1)^2 + (49 * x(1)^2 ...
    + 49 * x(2)^2.+ 84 * x(1) + 2324 * x(2) - 681)^2);

x0 = [1,0];
[x_search,fval_search] = fminsearch(fun2,x0);
[x_unc,fval_unc] = fminunc(fun2,x0);
fprintf('fminsearch method: \nx = (%3.8f,%3.8f), \ny = %3.8f\n\n',...
    x_search(1),...
    x_search(2),...
    fval_search);
fprintf('fminunc method: \nx = (%3.8f,%3.8f), \ny = %3.8f\n',...
    x_unc(1),x_unc(2),fval_unc);
```

程序代码 7

运行结果：



运行结果 5

代码分析：

（3）

程序代码：

```
clc
fun2 = @(x)...
    min((x(1) + 10 * x(2))^2 + 5 * (x(3) - x(4))^2 ...
+ (x(2) - 2 * x(3))^4 + 10 * (x(1) - x(4))^4);

x0 = [10,10,10,10];
[x_search,fval_search] = fminsearch(fun2,x0);
[x_unc,fval_unc] = fminunc(fun2,x0);
fprintf('fminsearch method:\nx = (%3.8f,%3.8f,%3.8f,%3.8f), \ny = %3.8f\n\n',...
    x_search(1),...
    x_search(2),...
    x_search(3),...
    x_search(4),...
    fval_search);
fprintf('fminunc method:\nx = (%3.8f,%3.8f,%3.8f,%3.8f), \ny = %3.8f\n',...
    x_unc(1),...
    x_unc(2),...
    x_unc(3),...
    x_unc(4),...
    fval_unc);
```

程序代码 8

运行结果：

```
命令行窗口

Local minimum found.

Optimization completed because the size of the gradient is less than
the default value of the optimality tolerance.

<stopping criteria details>

fminsearch method:
x = (-0.00034837,0.00003484,0.00041416,0.00041408),
y = 0.00000000

fminunc method:
x = (0.05192633,-0.00519316,0.04414283,0.04408800),
y = 0.00007641
```

运行结果 6

代码分析：

（4）

程序代码：

```
clc

fun4 = @(x)min(100 * ((x(3) - 10 * Theta(x(1),x(2)))^2 ...
    + ((x(1)^2 + x(2)^2)^(1/2) - 1)^2) + x(3)^2);

x0 = [10,10,10];
[x_search,fval_search] = fminsearch(fun4,x0);
[x_unc,fval_unc] = fminunc(fun4,x0);
fprintf('fminsearch method: \nx = (%3.8f,%3.8f,%3.8f), \ny = %3.8f\n\n',...
    x_search(1),...
    x_search(2),...
    x_search(3),...
    fval_search);
fprintf('fminunc method: \nx = (%3.8f,%3.8f,%3.8f), \ny = %3.8f\n',...
    x_unc(1),...
    x_unc(2),...
    x_unc(3),...
    fval_unc);

%- Definition of function Theta -%
function y = Theta(a,b)
    if a > 0
        y = 1 / (2 * pi) * atan(a ./ b);
    else
        y = 1 / (2 * pi) * atan(a ./ b) + 1 / 2;
    end
end
```

程序代码 9

运行结果：

命令行窗口

```
Solver stopped prematurely.

fminunc stopped because it exceeded the function evaluation limit,
options.MaxFunctionEvaluations = 300 (the default value).

fminsearch method:
x = (0.00004275,-0.99998980,-0.00008034),
y = 0.00000003

fminunc method:
x = (0.00395040,-0.99939480,-0.00566065),
y = 0.00010747
```

运行结果 7

代码分析：

5 题

有一组数据(t_i,y_i) $(i=1,2,\cdots,33)$，其中$t_i=10(i-1)$，y_i由表 7.9 给出。现要用这组数据拟合函数

$$f(\boldsymbol{x},t)=x_1+\boldsymbol{x}_2 \mathrm{e}^{-x_4 t}+x_3 \mathrm{e}^{-x_5 t}$$

中的参数$\boldsymbol{x}$，初值可选为$(0.5,1.5,-1,0.01,0.02)$，用 GN 和 LM 两种方法求解。对y_i作一扰动，即y_i+e_i，e_i为$(-0.05,0.05)$内的随机数，观察并分析迭代收敛是否会变慢。

i	y_i	i	y_i	i	y_i
1	0.844	12	0.718	23	0.478
2	0.908	13	0.685	24	0.467
3	0.932	14	0.658	25	0.457
4	0.936	15	0.628	26	0.48
5	0.925	16	0.603	27	0.438
6	0.908	17	0.58	28	0.431
7	0.881	18	0.558	29	0.424
8	0.85	19	0.538	30	0.42
9	0.818	20	0.522	31	0.414
10	0.784	21	0.506	32	0.411
11	0.751	22	0.49	33	0.406

表 7.9

Solution:

程序代码：

```
% filename: Data_fit

cd ..;cd ..;cd ./#Data/06
i = xlsread('experiment_6.xlsx','A2:A34');
y = xlsread('experiment_6.xlsx','B2:B34');

fun = @(x,t) x(1) + x(2) * exp(-x(4) * t) + x(3) * exp(-x(5) * t);
x0 = [0.5,1.5,-1,0.01,0.02];

[x,xx,xxx,xxxx,xxxxx] = lsqcurvefit(fun,x0,i,y)
y = y + rand()/2;
[x,xx,xxx,xxxx,xxxxx] = lsqcurvefit(fun,x0,i,y)
```

运行结果：

代码分析：

6 题

经济学中著名的 Cobb-Douglas 生产函数的一般形式为

$$Q(K,L)=aK^{\alpha}L^{\beta},\qquad 0<\alpha,\ \beta<1$$

其中Q，K，L分别表示产值、资金、劳动力，式中α，β，a要由经济统计数据确定。现有《中国统计年鉴（2003）》给出的统计数据如表 7.10 所示，请分别用线性和非线性最小二乘拟合求出式中的α，β，a，并解释α，β的含义。

年份	总产值/万亿元	资金/万亿元	劳动力/亿人
1984	0.7171	0.0910	4.8179
1985	0.8964	0.2543	4.9873
1986	1.0202	0.3121	5.1282
1987	1.1962	0.3792	5.2783
1988	1.4928	0.4754	5.4334
1989	1.6909	0.441	5.5329
1990	1.8548	0.4517	6.4749
1991	2.1618	0.5595	6.5491
1992	2.6638	0.808	6.6152
1993	3.4634	1.3072	6.6808
1994	4.6759	1.7042	6.7455
1995	5.8478	2.0019	6.8065
1996	6.7885	2.2914	6.895
1997	7.4463	2.4941	6.982
1998	7.8345	2.8406	7.0637
1999	8.2068	2.9854	7.1394
2000	9.9468	3.2918	7.2085
2001	9.7315	3.7314	7.3025
2002	10.4791	4.35	7.374

表 7.10

Solution:

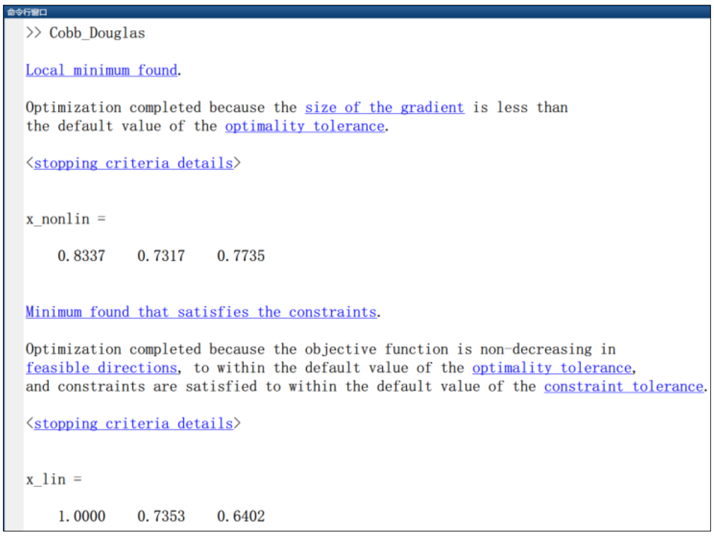
程序代码：

```
% filename: Cobb_Douglas

%% initialization
cd ..;cd ..;cd ./#Data/06

Total_product = xlsread('experiment_6.xlsx','Sheet2','B2:B20');

Fund = xlsread('experiment_6.xlsx','Sheet2','C2:C20');
Labour = xlsread('experiment_6.xlsx','Sheet2','D2:D20');

fun = @(x,xdata) x(1) .* xdata(:,1).^x(2) .* xdata(:,2).^x(3);

```

```
x0 = [0.6,0.091,4.8179];

%% 非线性解法
xdata = [Labour Fund ];
ydata = Total_product;
x_nonlin = lsqcurvefit(fun,x0,xdata,ydata)

%% 线性解法

lb = zeros(3,1);

C = [ones(19,1),log(Fund),log(Labour)];

ub = ones(3,1);
x = lsqlin(C,log(Total_product),[],[],[],[],lb,ub);
x_lin = [exp(x(1)),x(2),x(3)]
```

程序代码 10

运行结果：


```
>> Cobb_Douglas

Local minimum found.

Optimization completed because the size of the gradient is less than
the default value of the optimality tolerance.

<stopping criteria details>


x_nonlin =

    0.8337    0.7317    0.7735


Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.

<stopping criteria details>


x_lin =

    1.0000    0.7353    0.6402
```

运行结果 8

代码分析：

要想用线性方法来完成这个题目，首先需要将方程进行转化，然后求解。如下所示：

$$\boldsymbol{Q}(K,L) = a\boldsymbol{K}^{\alpha}\boldsymbol{L}^{\beta} \xrightarrow{\log} \ln\boldsymbol{Q} = \ln a + (\ln\boldsymbol{K})\cdot\alpha + (\ln\boldsymbol{L})\cdot\beta$$

这也就是$\boldsymbol{x} = \begin{bmatrix} \ln a \\ \alpha \\ \beta \end{bmatrix}$，$\boldsymbol{C} = [\boldsymbol{1} \quad \ln\boldsymbol{K} \quad \ln\boldsymbol{L}]$，$\boldsymbol{d} = \ln\boldsymbol{Q}$，这样就有等式：$\boldsymbol{C}\cdot\boldsymbol{x} - \boldsymbol{d} = \boldsymbol{0}$。对于非线性拟合，直接根据数据与函数设计函数进行就可以了，比较简单。

四、实验过程

这次试验的难度有点大。

五、实验总结

六、参考文献

[1] 大学数学实验/姜启源，谢金星，邢文训，张立平，北京：清华大学出版社，2010.12
[2] MATLAB 教程/张志涌，杨祖樱，北京：北京航空航天大学出版社，2015.1

七、教师评语