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A developed Tabu Search algorithm for heterogeneous fleet vehicle routing problem

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Abstract: This paper deals with the vehicle routing problems for urban good distribution. The mix fleet size problem is a variant of the vehicle routing problem called Heterogeneous Fleet Vehicle Routing Problem (HFVRP). The objective is to design a set of routes in order to minimize the sum of the costs. The purpose of this study is to develop a Tabu search (TS) heuristic to solve the HFVRP. The initial solution is obtained by a modified Clarke & Wright saving algorithm than treated by some fundamental and others new concepts of the TS algorithm. Our Tabu search algorithm uses a new procedure called Fusion in parallel with the split procedure in order to explore new search spaces. In addition, a number of neighborhood structure are combined together in a process for intensifying the local search. Besides, the proposed algorithm is boosted with an adaptive memory algorithm, known as probabilistic diversification and intensification. On several benchmark instances, the TS produces high-quality solutions, including two new best solutions for small sized instances. The results obtained illustrate the effectiveness of the approach and its applicability to routing problems.

Keywords: HVRP, FSM, Tabu search, Adaptive Memory Programming, local search, Last-mile.

1. INTRODUCTION

In urban contexts, due to industrial strategies, urbanization and city design; local authorities, logisticians and citizens face many challenges related to freight transport externalities, accessibility, space management and delivery delays (Rao et al. (2015), Cattaruzza et al. (2017)). Therefore, to achieve socio-economic and environmental sustainability in city transportation, a major intervention on the last mile logistics is needed. Previous research has shown how routing optimization can lead to significant economic savings (estimated between 5 to 30% by Hasle and Kloster (2007)). Moreover, mutualization and consolidation strategies improve as well urban good distribution sustainability. Since they impact the network topology of last mile delivery by means of consolidation platforms (Cardenas et al. (2017)). Furthermore, these strategies are influenced by supply chain 4.0 that consists in Physical Internet (PI) and data exchange, relatively novel notions to developing nations, in order to improve the current unsustainable freight transportation (Ambra et al. (2018)).

The present work is part of an urban logistics project that deals with the case of the city of Casablanca -Morocco-. According to a study conducted by the Moroccan Agency for Logistics Development (AMDL) in (2015) the city knows several problems related to the transport of goods and doesn't have an actual Urban Consolidation Center (UCC), which directs the project towards a strategic level of decision. Such as, defining the size and the composition of the fleet to be located in a future UCC by solving the HFVRP where the fleet is considered unlimited, with a fixed and a variable cost of each type of vehicle. Therefore, in this study, the HFVRP is adapted to an urban setting that consists: (i) in considering the depot as an Urban Consolidation Center (UCC), which is a transshipment points that involves physical resources pooling (warehouse/ platforms, vehicles) Choi and Tcha (2017) Zhou et al. (2011); and (ii) in promoting the use of small vehicles.

The Heterogenous Fleet Vehicle Routing Problem (HFVRP) is one of the several variants of the well-known Vehicle Routing Problem (VRP), which is considered as an NP-Hard combinatorial problem, hence difficult to solve. It can be defined mathematically as follows. Let the graph G= {A, E} where A is the vertex set containing n clients in addition to the depot (vertex 0), and E= {(i, j): i, j ϵ A; i ≠ j} is the set of edges

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(arcs). A fleet of m heterogeneous vehicles is located in the depot. Each vehicle k has a maximum capacity D_k, a fixed cost F_k, a unit travel cost M_k and a three-dimensional loading space of length L_k, width W_k and height H_k such that $S_k = L_k. W_k. H_k$. The variable cost of moving the vehicle $k = (1, ..., m)$ from i to j is given by $C_{ij}^k = M_k \; x \; R_{ij}$ such that R_{ij} is the distance between the customer i and j. The demand of each client $i = (1, ..., n)$ is expressed in terms of the total weight of his order. The objective of the HFVRP is to build a set of routes (one per vehicle) so that the cost is minimized. The cost consists of a fixed cost related to the use of the vehicle and a delivery cost. A feasible route must meet the following conditions: (1) Each customer should be served by exactly one vehicle and visited only once (Eq. 2); (2) Each vehicle route should start and end at the depot (Eq. 3); (3) The total weight capacity of the lots in each vehicle must not exceed the load capacity of the vehicle (Eq. 4).

The purpose of this paper is to introduce a developed tabu search heuristic for the HFVRP presented earlier. Due to the fact that the HFVRP is NP-Hard since it's a variant of the basic VRP; The adoption of a metaheuristic turns out to be necessary for obtaining optimal solutions in a reasonable amount of computation time. Here the basic tabu search is boosted with the Adaptive Memory Programming (AMP). In addition to, a new neighborhood structure of a set of local searches, including Clark & Wright Saving Algorithm as a refinement procedure and a new fusion operation along with the split operation allowing a well diversification scheme neighborhood.

In the following section of the remainder, literature is reviewed. The proposed tabu search heuristic is described in section 3. In Section 4, the parameter tuning is discussed and experimental results are explained. Final conclusions and discussion are found in Section 5.

2. LITERATURE REVIEW

The heterogeneous fleet vehicle routing problem has different variants in the literature, depending on whether the fleet size is limited or unlimited and whether the costs are considered fixed or variable or both Escobar et al. (2017). The HFVRP with unlimited fleet was introduced by Golden et al. (1984) as The Fleet Size and Mix VRP (FSMVRP). On the other hand, Taillard (1999) proposed the HFVRP with fixed fleet size called Heterogeneous VRP (HVRP). Although the similarity, each one is applied in a specific situation: the FSMVRP help determining the best fleet composition and its optimal routing scheme. Whereas, the HVRP help optimizing the uses of vehicles among an existing fleet.

Golden et al. (1984) developed two heuristics to solve the HFVRP based on the Clarke & Wright Saving algorithm (1964) and the giant tour algorithms. Besides, they proposed a set of benchmark data sets to validate the heuristics. In the same field, Gendreau et al. (1999) introduced a new tabu search approach, which makes use of the GENUIS algorithm, initially developed by Gendreau et al. (1992) for the Travel Salesman Problem (TSP), and the adaptive memory programming (AMP) technique developed by Rochat and Taillard (1995). Taillard (1999) presented a column generation method to solve initially the HVRP then was adapted to solve limited fleet problem. In addition, he proposed a set of 8 instances with only variable costs. Wassan and Osman (2002) proposed as well an algorithm based on tabu search, which consists of many variants of TS such as variable neighborhoods and hashing functions. In (2007), Choi and Cha outperformed the pervious algorithms by means of a column generation algorithm based on a linear programming model and a branch and bound procedure. Moreover, Li et al. (2007) put forward a record-to-record travel algorithm. A combined approach between TS and set partitioning (SP) was developed by Lee et al. (2008) in addition to a new set of large instances for the HFVRP.

More recently, Brandão (2009) has proposed a heuristic algorithm based on TS that makes use of three different procedures to obtain initial solutions. Besides, a Variable Neighborhood Search (VNS) algorithm was developed by Imran et al. (2009). The heuristic makes use of Dijkstra's and sweep algorithms for obtaining an initial solution and several neighborhood structures in the local search phase. In the same year, Prins (2009) developed two memetic algorithms based on a hybridization of genetic algorithms with local search for all FSM variants. Furthermore, Penna et al. (2013) is the most recent work treating the same FSM variant discussed in this paper. They proposed an algorithm based on the Iterated Local Search (ILS) metaheuristic which uses a Variable Neighborhood procedure, with a random neighborhood ordering (RVND), in the local search aspect.

Although all the works listed previously treats the same problematic, only four of them considered all FSM variants. The rest of papers have considered the fixed costs and the variable costs separately when testing the performances of their heuristics, whereas the other four considered both of them.

3. THE PROPOSED TABU SEARCH ALGORITHM

In this study, a developed Tabu Search (TS) algorithm is proposed to solve the HFVRP presented earlier. The proposed algorithm (Fig. 1), not only combine the ability of the TS to better explore the neighborhood of a local optimum with the basic AMP algorithm that consists in feeding the TS with a new solution in each iteration so that new promising regions are explored. But also adopts a perturbation strategy called Split procedure by Lee et al. (2008) in addition to a new proposed strategy called Fusion as a local diversification phase within the TS in order to diversify the search to other promising area. Furthermore, a robust neighborhood structure is proposed and enhanced with a new route refinement method. Towards the end of the search the best solution found is shacked with an improved shaking approach.

An initial solution is constructed with the modified Saving algorithm. Subsequently, the AMP memory is updated with a simple TS method using a relaxed local search procedure. Afterwards, at each iteration, the AMP generates an initial solution before starting the phase of TS optimization. Besides, towards the end of the TS phase the best solution found is updated to the AMP memory. The search is repeated until the max iteration is reached.

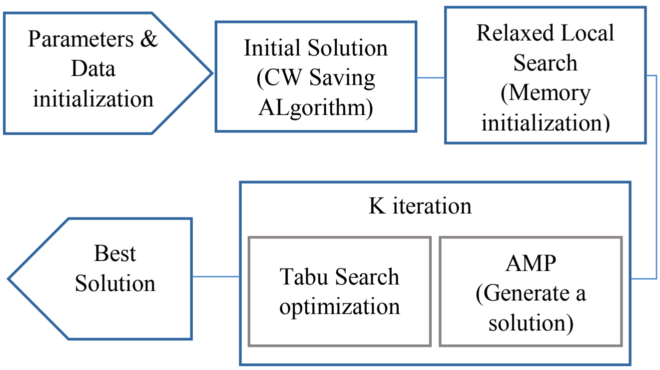

Fig. 1. Proposed Algorithm.

3.1 The modified Clarke and Wright (CW) Saving Algorithm:

The Saving algorithm of Clarke and Wright (1964) was chosen due to its simplicity to be implemented and flexibility to be adapted to new practical considerations. In addition, to being used in so many works related to routing problems and considered as an excellent method for constructing the initial solution (Zhang et al. (2015), Wei et al. (2014)…). The saving algorithm consists in allocating for each customer a route that can contain its demand. The constructive heuristic proceeds by merging two routes in one with a better saving cost until no more feasible merging exist. The saving cost is defined as the total cost of two old routes minus the cost of the new one. Besides, the new merged route needs to respect all the constraints; otherwise it is not considered.

Each time a route is modified, either by adding a new customer or deleting an existing one, the vehicle allocation method is performed. The method keeps the vehicle already affected to the route if the capacity occupied by the orders of the current route is near to the capacity supported by the vehicle. Otherwise, the most suitable vehicle with just sufficient capacity to load the orders of the concerned route is assigned. The same procedure is adopted to a new route.

3.2. Neighborhood Structure:

The performance of any Tabu search algorithm is related to its neighborhood structure (Local search procedure), by which the quality of promoting regions to be visited is determined. The proposed neighborhood structure is composed of six inter-routes operations axed on two basic but effective moves according to Brandão (2011): insertion and swap. The six operations are performed in every iteration so the algorithm doesn't miss any promoting region to explore; in contrary, if they are performed separately. The details of the neighborhood moves are presented below:

1) Customer-insertion: in which a customer c_k randomly chosen is deleted from its current route r_i and inserted in another route r_j. The destination route r_j should contain at least one of the μ-nearest neighbors of c_k. the value of μ will be defined later on. The μ-neighborhood restriction introduced by Gendreau et al. (1994) contributes to avoid unnecessary moves and to find better solutions. Therefore, the computational time will be reduced.
2) Swap-operation: consists in exchanging a customer c_k belonging to r_i with its neighbor c_l from a route r_j. The swap is feasible if and only if the destination route r_j contain at least one of the μ-nearest neighbors of c_k and one of the routes has more than one customer.
3) Double-insertion: A two consecutive customers from a random route are deleted and inserted in another route. The destination route should contain one of the μ-nearest neighbors of one customer among the two selected
4) Triple-insertion: in which three neighbor customers from the same route are selected instead of two.
5) 2-1 swap: consists in exchanging two consecutive customers belonging to the same route r_i with one neighbor of the two customers. The 2-1 swap is considered only if the neighbor belongs to a different route than r_i.
6) 3-2 swap: Three consecutive customers are selected and exchanged with 2 other consecutive neighbors. The same swap conditions are kept.

After each operation, the vehicle allocation method is applied to the new generated routes. For example, after a successful insertion move, a smaller vehicle is assigned to r_i and a larger one to r_j if the new order doesn't fit in the existed vehicle.

Another particularity of the proposed neighborhood structure remains in sweeping all the μ-nearest neighbors. Since the best movement of a customer may not be the one with it closest neighbor. Consequently, the concerned neighborhood operation performs trial moves of all the μ candidates and consider only the best one.

3.3. Mathematical formulation:

For each solution, R, an objective function, called score, is calculated and denoted as *S(R)*. A solution, S, might contain many routes and each route is served by only one vehicle. However, the score of a solution is the sum of the scores of all the routes of a solution S. The decision variables considered in this formulation are explained below:

x_{ij}^k Equal to 1 if the road (i, j) is served by the vehicle k, 0 otherwise.

y_k Binary, 1 if the vehicle k is used, 0 otherwise.

z_i^k Equal to 1 if the order of customer i is delivered by vehicle k.

The developed TS algorithm seeks to minimize the S(R) defined by the following equation:

$$S(R) = \sum_{k=0}^{r} \left(F_k . y_k + \sum_i \sum_j C_{ij}^k . x_{ij}^k \right) \qquad (Eq.\ 1)$$

Subject to:

$$\sum_{k=1}^{m} \sum_{j=0}^{n} x_{ij}^k = 1, \ i \epsilon [0, n] \, / \, i \neq j \qquad (Eq.\ 2)$$

$$\sum_{i=1}^{n} x_{i0}^k = 1 \quad and \quad \sum_{j=1}^{n} x_{0j}^k = 1 \quad \forall\, k = 1, \dots, m \qquad (Eq.\ 3)$$

$$\sum_{i=1}^{n} d_i \le D_k z_i^k \quad \sum_{i=1}^{n} s_i \le S_k z_i^k \quad , \ \forall\, k \in [1, m] \qquad (Eq.\ 4)$$

Where, r is the number of routes in S, F_k the fixed cost of the vehicle k, C_{ij}^k the variable cost of the vehicle k from customer i to j, D_k the capacity of the vehicle k and $S_k = W_k . L_k . H_k$ to be the volume of the vehicle k.

3.4. Route improvement procedure:

Clarke and Wright saving algorithm, besides being used to generate the initial solution, is as well applied as a refinement procedure. Here, after performing an operation, the new generated routes are improved by the CW procedure. The CW algorithm has been modified and adapted to generate a new client sequence of a route if there is any. However, there are some cases where the route improvement procedure doesn't obtain the best route organization. Therefore, towards the end of the search an intra-route swap procedure is applied to the best solution found.

3.5. Split & fusion procedure:

For each iteration of the TS algorithm, the split or the fusion procedure is applied after the local search phase in order to introduce a new search space. The split procedure consists in splitting the longest route by two sub routes. Therefore, two new routes are generated better in term of cost than the initial giant route. The procedure was considered by Lee et al. (2008) as a second phase of the tabu search to diversify the search and explore new wide space. In contrast with the split procedure, the fusion procedure consists in merging two routes in order to intensify the search by reexploring some search spaces.

3.7. Adaptive memory programming:

The adaptive memory programming was first introduced by Rochat and Taillard (1995) as a method to boost the TS to solve the VRP. The main idea of the AMP consists in the possibility of constructing a good solution from different components of other good solutions. As in genetic search where two offspring are created from two parents Goldberg (1989).

In contrast with the traditional AMP, where memory initialization is done in advance, the memory is initialized through a relaxed TS during the phase 1 of the developed algorithm, where good solutions are stored and sorted based on their score. Thereafter, in each iteration of the second phase a new solution is created using the data in the memory and improved by the tabu search. The best solution found is updated in the memory. The details are given below:

Step1. Memory initialization using a relaxed local search. Then all the route of the different solutions in the memory are updated and sorted in the route memory

Step2. A new solution is constructed by combining components of the route memory. The selection of a route follows a probabilistic principle which promote the route with the higher score. The process is repeated till no more routes exist in the memory. As for the remaining customers the CW algorithm is applied to complete the new solution.

Step 3. Memory update by the storage of all routes of the best solution found in the TS provided that they aren't already in the memory pool.

4. COMPUTATIONAL RESULTS

In this section the adopted parameter calibration is presented within the computational results on benchmark instances

4.1. Parameters tuning:

As any heuristics, parameters affect directly the performance of the algorithm. Thus, finding the right value for these parameters is crucial. However, there is no way of determining the most effective value of the parameters. Hence, they were set according to the literature and preliminary experiments as described by Gendreau & Potvin (2019). Although better setting could exist, the proposed one here allowed good results.

A synthesis of all the parameters used in the algorithm is presented below:

- The tabu tenure value θ;
- The μ-neighborhood restriction that contributes to avoid unnecessary moves and drive the search to explore promising region. Therefore, it helps reducing the computational time. The value of μ is changed during the search;
- Adaptive memory size limit T;
- Maximum number of iterations in the algorithm K;
- Limit of the TS phase K_{TS};
- Limit of the local search procedure inside the TS phase K_{LS};

Table 1. Computational results for HFVRP with fixed and variable costs

Inst.No.	Size	Best Known Solution	CG		SMA-U1		VNS1		ILS-RVND		Proposed algorithm		
			Choi and Tcha		Prins		Imran et al.		Penna et al.				
			Best Sol.	Time	Best Sol.	Time	Best Sol.	Time	Best Sol.	Time	Best Sol.	Time	Gap %
3	20	**1144,22**	**1144,22**	0,25	**1144,22**	0,01	**1144,22**	19	**1144,22**	3,87	**1144,22**	3,8	0,00%
4	20	**6437,33**	**6437,33**	0,45	**6437,33**	0,07	**6437,33**	17	**6437,33**	2,77	**6437,33**	4,5	0,00%
5	20	**1322,26**	**1322,26**	0,19	**1322,26**	0,02	**1322,26**	24	**1322,26**	4,57	**1322,26**	3,6	0,00%
6	20	**6516,47**	**6516,47**	0,41	**6516,47**	0,07	**6516,47**	21	**6516,47**	2,8	**6516,47**	4,4	0,00%
13	50	**2964,65**	**2964,65**	3,95	**2964,65**	0,32	**2964,65**	328	**2964,65**	27,67	**2964,65**	64,2	0,00%
14	50	**9126,90**	**9126,90**	51,7	**9126,90**	8,9	**9126,90**	250	**9126,90**	11,27	**9126,90**	79,6	0,00%
15	50	**2634,96**	**2634,96**	4,36	2635,21	1,04	**2634,96**	275	**2634,96**	13,47	**2631,05**	189	-0,15%
16	50	**3168,92**	**3168,92**	5,98	3169,14	13,05	3168,95	313	**3168,92**	17,55	**3165,61**	111,5	-0,10%
17	75	**2004,48**	2023,61	68,11	**2004,48**	23,92	**2004,48**	641	**2004,48**	43,33	2032,75	189,6	1,41%
18	75	**3147,99**	**3147,99**	18,78	3153,16	24,85	3153,67	835	3149,63	47,39	3157,05	260,8	0,29%
19	100	**8661,81**	8664,29	905,2	8664,67	163,25	8666,57	1411	**8661,81**	60,33	8668,36	981,6	0,08%
20	100	**4153,02**	4154,49	53,02	4154,49	41,25	4164,85	1460	**4153,02**	58,97	4170,57	676,1	0,42%
# Best Solution			**9**		**7**		**8**		**11**		**8**		
												Total gap value	1,94%

The two most influential parameters of the TS algorithm are the tabu tenure θ and the neighbourhood size μ. The values of both parameters are set according to the number of customers. Regarding θ, a range between 5 to 10 is used so cycling are evited as well as forbidding good movement. Regarding parameter μ, a common sense will opt to fix a large value of μ to increase the efficiency of local search. However, a balance between efficiency and effectiveness is needed, since large values of μ may result in excessive computational time consumption. A range between 8 to 15 was found to provide a good compromise. As for the memory limit, it was fixed at $T = 300$ according to Gendreau et al. (1999).

4.2. Comparative results:

The algorithm was programmed in JAVA and executed in an Intel Core™ i7 Processor 2.50 GHz with 8 Go of RAM memory running in Windows 10. The described heuristic considers both fixed and variable cost and was tested on twelve instances taken from Golden et al. (1984) and Taillard (1999). The numbering system of these instances is kept the same as these authors, to simplify the comparison. Choi and Tcha (2007) generated the data set for both costs by taking vehicle fixed costs from Golden et al. (1984) and variable costs from Taillard (1999), since each study consider only one type of costs. For each instance, the algorithm was executed 10 times and the computational results of the best run are presented in Table 1. The best solution found is compared to: Choi and Tcha (2007), Prins (2009), Imran et al. (2009) and Penna et al. (2013). The best-known solutions are marked in bold characters and the new best one found are underlined. Besides, the new solutions are detailed in Appendix A.

According to the results presented in Table 1, the developed algorithm failed to equal the results of four instances but it was capable of finding better results for two other ones. The underperformance of the algorithm is remarkable when the number of customers is superior to 50 where the results are slightly worse than the best ones in the literature, but still optimal. As for the CPU time, although its comparison isn't obvious under different machines, it is clear that the proposed algorithm isn't the fastest due to memory initialization and the execution of all six moves in each iteration.

5. CONCLUSION

This paper treated a variant of the well-known NP-Hard VRP called the Heterogeneous Fleet Vehicle Routing Problem (HFVRP) for urban good distribution. The fleet was considered unlimited with a fixed cost and a unit variable costs for each vehicle. A developed Tabu Search algorithm has been proposed to solve the HFVRP and tested on twelve benchmark instances with up to 100 customers. However, the heuristic was outperformed when the number of customers is superior to 75, the results obtained illustrated the effectiveness of the approach and its applicability to routing problems. Furthermore, two new best solutions were introduced.

As perspective, an enhancement of the proposed algorithm is possible to improve its efficiency to solve large sized instances. In addition to adding some other variants to the HFVRP as time windows, split deliveries, multi-depots…etc.

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REFERENCES

(AMDL). (2015). Etude pour la structuration de la logistique urbaine Casablanca, état des lieux et perspective. *http://www.amdl.gov.ma/amdl/wp-content/uploads/2015/05/logismed-amdl.pdf.*

Ambra, T., Caris, A., & Macharis, C. (2018). Towards freight transport system unification: reviewing and combining the advancements in the physical internet and synchromodal transport research. *International Journal of Production Research*, 1-18.

Brandão, J. (2009). A deterministic tabu search algorithm for the fleet size and mix vehicle routing problem .

European journal of operational research, 195(3), 716-728.

Brandão, J. (2011). A tabu search algorithm for the heterogeneous fixed fleet vehicle routing problem. *Computers & Operations Research, 38(1)*, 140-151.

Cardenas, I., Borbon-Galvez, Y., Verlinden, T., Van de Voorde, E., Vanelslander, T., & Dewulf, W. (2017). City logistics, urban goods distribution and last mile delivery and collection. *Competition and Regulation in Network Industries, 18(1-2)*, 22-43.

Cattaruzza, D., Absi, N., & Feillet, D. G.-F. (2017). Vehicle routing problems for city logistics . *EURO Journal on Transportation and Logistics, 6(1)*, 51-79.

Choi, E., & Tcha, D.-W. (2007). A column generation approach to the heterogeneous. *European Journal of Operational Research 34*, 2080–2095.

Clarke, G., & Wright, J. (1964). Scheduling of vehicle from central depot to a number of delivery points . *Operations Research 12*, 568–581.

Escobar, J. W., Adarme-Jaimes, W., & Clavijo-Buriticá, N. (2017). Comparative analysis of granular neighborhoods in a Tabu Search for the vehicle routing problem with heterogeneous fleet and variable costs (HFVRP). *Facultad de Ingeniería 26(46)*, 93-104.

Gendreau, M., & Potvin, J. Y. (2019). Tabu Search. *Handbook of Metaheuristics, Springer, Cham*, 37-55.

Gendreau, M., Hertz, A., & Laporte, G. (1992). New insertion and post optimization procedures for the traveling salesman problem. *Operations Research 40*, 1086-1094.

Gendreau, M., Hertz, A., & Laporte, G. (1994). A tabu search heuristic for the vehicle routing problem. *Management science, 40(10)*, 1276-1290.

Gendreau, M., Laporte, G., Musaraganyi, C., & Taillard, E. (1999). A Tabu search heuristic for the heterogeneous fleet vehicle routing problem. *Computers and Operations Research 26*, 1153–1173.

Goldberg, D. E. (1989). Genetic algorithm in search. *Optimization and Machine Learning*, 343-349.

Golden, B., Assad, A., Levy, L., & Gheysens, F. (1984). The fleet size and mix vehicle routing. *Computers and Operations Research 11*, 49–66.

Hasle, G., & Kloster, O. (2007). Geometric modelling, numerical simulation, and optimization. *Operations Research Computer Science Interfaces* , 397–436.

Imran, A., Salhi, S., & Wassan, N. A. (2009). A variable neighborhood-based heuristic for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research, 197(2)*, 509-518.

Lee, Y., Kim, J., Kang, K., & Kim, K. (2008). A heuristic for vehicle fleet mix problem using Tabu search and set partitioning. *Journal of the Operational Research Society 59*, 833–841.

Li, F., Golden, B., & Wasil, E. (2007). A record-to record travel algorithm for solving the heterogeneous fleet vehicle routing problem. *Computers and Operations Research 34*, 2918–2930.

Makaci, M. R.-G. (2017). Pooled Warehouse Management: An Empirical Study. *Computers & Industrial Engineering*.

Penna, P. H., Subramanian, A., & Ochi, L. S. (2013). An iterated local search heuristic for the heterogeneous fleet vehicle routing problem . *Journal of Heuristics, 19(2)*, 201-232.

Prins, C. (2009). Two memetic algorithms for heterogeneous fleet vehicle routing problems. *Engineering Applications of Artificial Intelligence, 22(6)*, 916-928.

Rao, C., Goh, M., Zhao, Y., & al., e. (2015). Location Selection of City Logistics Centers Under Sustainability. *Transportation Research Part D: Transport and Environment 36(0)*, 29–44.

Rochat, Y., & Taillard, E. (1995). Probabilistic diversification and intensification in local search vehicle routing. *Journal of Heuristic 1*, 147–167.

Taillard, E. (1999). A heuristic column generation method for the heterogeneous fleet VRP. *Recherche Operationnelle 33*, 1–14.

Wassan, N., & Osman, I. (2002). Tabu Search variants for the mix fleet vehicle routing problem. *Journal of the Operational Research Society 53*, 768–782.

Wei, L., Zhang, Z., & Lim, A. (2014). An adaptive variable neighborhood search for a heterogeneous fleet vehicle routing problem with three-dimensional loading constraints. *IEEE Computational Intelligence Magazine, 9(4)*, 18-30.

Zhang, Z., Wei, L., & Lim, A. (2015). An evolutionary local search for the capacitated vehicle routing problem minimizing fuel consumption under three-dimensional loading constraints . *Transportation Research Part B: Methodological, 82* , 20-35.

Zhou, G., Van Hui, Y., & Liang, L. (2011). Strategic alliance in freight consolidation. *Transportation Research Part E: Logistics and Transportation Review, 47(1),* , 18-29.

Appendix A.

New best solution for instance n°15:			**New best solution for instance n°16:**		
Route number	Vehicle type	Route	Route number	Vehicle type	Route
1	A	0-41-13-0	1	B	0-25-14-6-27-0
2	A	0-48-23-6-0	2	A	0-38-5-0
3	A	0-20-29-3-0	3	B	0-20-35-36-28-22-1-0
4	A	0-32-35-36-22-1-0	4	A	0-46-12-0
5	A	0-37-47-27-0	5	A	0-24-43-23-0
6	A	0-34-21-16-0	6	A	0-50-30-9-0
7	A	0-33-45-15-0	7	A	0-34-21-29-0
8	A	0-14-25-0	8	B	0-49-10-39-33-45-15
9	A	0-46-49-9-38-0	9	A	0-7-26-31-0
10	A	0-28-31-8-0	10	A	0-32-37-44-17-0
11	A	0-26-7-43-24-0	11	B	0-13-41-40-19-42-0
12	A	0-18-4-0	12	B	0-18-4-47-0
13	A	0-11-2-0	13	A	0-48-8-0
14	A	0-5-12-0	14	B	0-2-3-16-11-0
15	A	0-50-30-39-10-0			
16	A	0-19-40-42-44-17-0			
Solution cost : 2631.5			**Solution cost : 3165.61**		