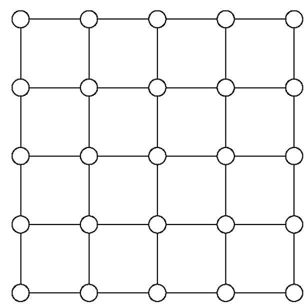
Statistical Machine Learning, Problem set 4

Jacob Goldberger



You are given a 5×5 grid binary 0-1 MRF.

$$p(x) = \frac{1}{Z} \prod_{ij \in E} \phi_{ij}(x_i, x_j) = \frac{1}{Z} \exp\left(\sum_{ij \in E} \varphi_{ij}(x_i, x_j) \right)$$

$$p(x|y) = \frac{1}{Z} \prod_{ij \in E} \phi_{ij}(x_i, x_j) \prod_i \phi_i(x_i, y_i) = \frac{1}{Z} \exp\left(\sum_{ij \in E} \varphi_{ij}(x_i, x_j) + \sum_i \varphi_i(x_i, y_i) \right)$$

$$\varphi_{ij}(x_i, x_j) = 1_{\{x_i = x_j\}}, \qquad \varphi_i(x_i, y_i) = -0.5(x_i - y_i)^2$$

- Simulate data (x, y). First create x using Gibbs sampling, then create y, note the $y_i|x_i \sim N(x_i, 1)$.
- Compute the correct marginal $p(x_i|y)$.
- Apply Gibbs sampling algorithm to compute $\hat{p}(x_i|y)$.
- Plot $\sum_i (p(x_i|y) - \hat{p}(x_i|y))^2$ as a function of the numbers of samples.
- (optional) Apply the Mean-Field approximation to compute $\hat{p}(x_i|y)$. Compute the approximation performance: $\sum_i (p(x_i|y) - \hat{p}(x_i|y))^2$.