Advanced Graph Problem

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You should know before this practice

- Directed / Undirected Graph
- DAG / Topological Sort
- BFS / DFS / Connectivity
- Strong Connected Component

You should know after this practice

- (Weighted) Bipartite Max Matching
- Max Flow
- Min Cost Max Flow

Hungary Algorithm

- Augment((left side node) u):
  - For (right side node) v is adjacent to u
    - If v is NOT visited
      - Mark v as visited
      - If v is matched to a left side node w
        - If Augment(w) == True
          - Return True
      - Else
        - Let's match u and v
        - Return True
  - Return False
- $O(E)$ for a single augment, $O(VE)$ to compute the maximum bipartite matching

Max Flow Problem

- A source and a sink node
- Arcs/Edges with capacity
- Find the maximum possible flow from source to sink
- (Special) Integer Linear Programming Problem
- Max Flow = Min Cut (strong duality)
- Residue Network
  - How many flows can we still push on the arc/edge.
  - When the flow on u→v is increased by 1, in the residue network, the capacity of u→v is decreased by 1 while that of v→u is increased by 1.

Drainage Ditches

- N ditches and M intersections
- 1 is the pond (Source), M is the stream (Sink)
- For i-th ditch, it flows from S_i to E_i with the capacity C_i
- The maximum rate at which water may emptied from the pond

How to compute the max flow?

- BFS on the residue network using those non-zero capacity arcs/edges
- If there is a augment path from source to sink, then push as many flows as we can
- Repeat this procedure until convergence
- $O(E^2)$

Min Cost Max Flow

- Use **bellman-ford** algorithm to replace the **bfs**
- Why not Dijkstra?
  - Dijkstra algorithm assumes the costs of edges are all non-negative
  - There are negative costs in the network

Ants

- $2n$ ants in total in the plane
- n black and n white
- We are going to assign n disjoint pairs of ants, such that
  - Each pair consists of a black and a white
  - The segments generated by these pairs have no intersections
- Give a possible assignment.

Ants - Hints

- Triangle Inequality
- Minimum Weighted Bipartite Matching should have no intersections

Q&A

Thanks!