



**You**

Is  $V = \{f \mid f' \text{ is constant}\}$  a vector space?



**ChatGPT**

To determine if the set  $V = \{f \mid f' \text{ is constant}\}$  is a vector space, we need to check the three vector space properties:

1. **Zero vector:** The constant function  $f(x) = c$  has a derivative of  $f'(x) = 0$ , which is a constant. Therefore, the zero function is in  $V$ .
2. **Closure under addition:** Let  $f$  and  $g$  be functions in  $V$  with constant derivatives. Consider the function  $h(x) = f(x) + g(x)$ . The derivative of  $h$  is the sum of the derivatives of  $f$  and  $g$ , which are constants. So,  $h$  also has a constant derivative, and  $V$  is closed under addition.
3. **Closure under scalar multiplication:** Let  $f$  be a function in  $V$  with a constant derivative. If we multiply  $f$  by a scalar  $c$ , the derivative of  $cf$  is  $cf'$ , which is still a constant. Thus,  $V$  is closed under scalar multiplication.

Since  $V$  satisfies all three vector space properties, it is indeed a vector space.

