Turbulent Kármán wakes in nature: a parametric study

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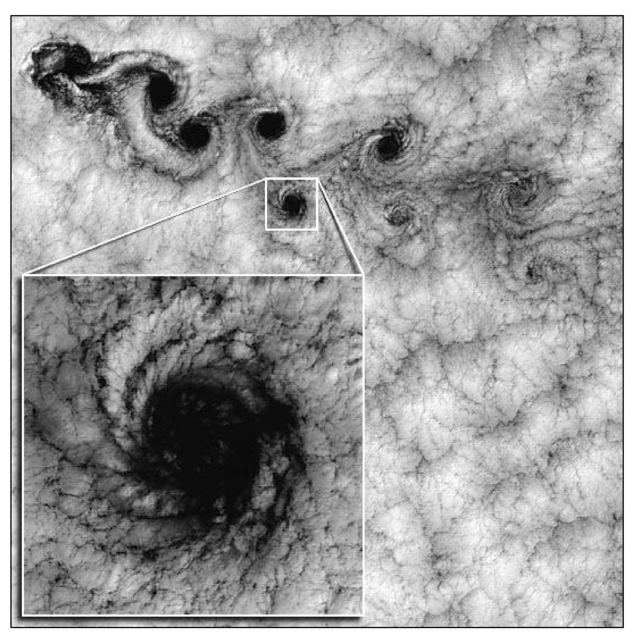
Scientific background

- Turbulent wakes are ubiquitous in nature, such as flow past mountains, islands, and seamounts. Eddies are shed from the topographies, often forming a Kármán vortex street pattern. These wakes are made unique by stable density stratification and Earth's rotation.
- Enhanced turbulent dissipation and mixing in the near wake at high Reynolds numbers are of tremendous relevance to geophysical applications, but their parametric dependence is still unclear, prohibiting accurate parameterizations of topographies in global or regional ocean models.
- High-resolution numerical simulations are employed over a reasonably large parameter space to answer the following questions:
  - -- What are the routes to turbulence and what are their parametric dependence
  - -- What is the scaling of turbulent dissipation rate in the wake

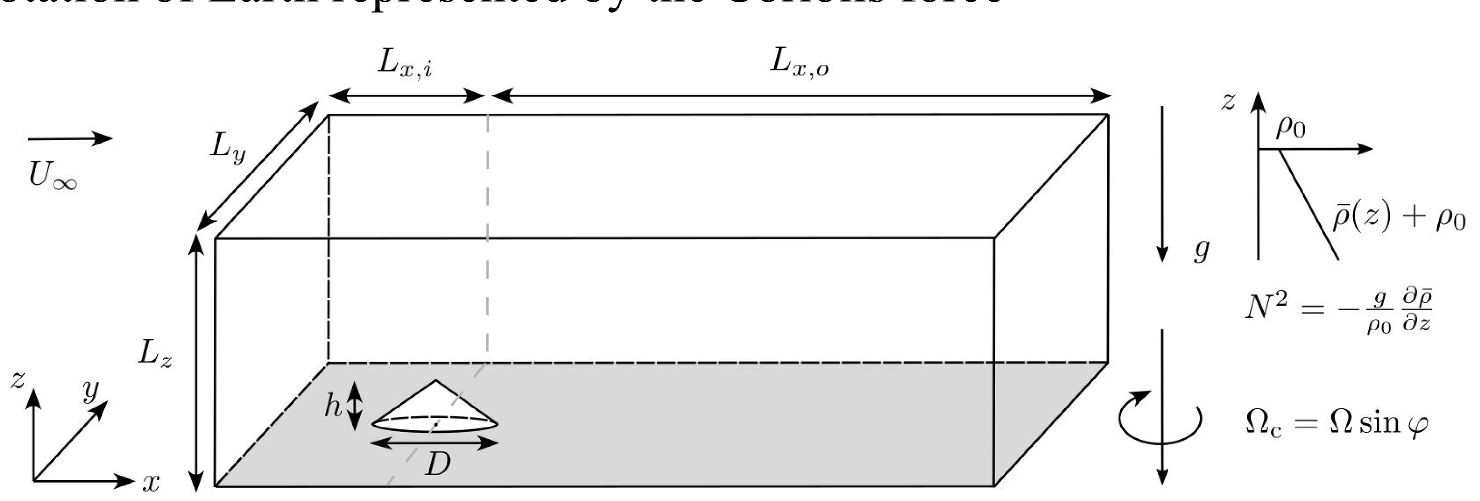


(Figure: Juan Fernanderz Island; NASA Landsat 7)

Problem formulation

Flow configuration:

- Uniform inflow past an isolated 3D topography on a flat surface
- Density linearly stratified in the vertical direction
- Rotation of Earth represented by the Coriolis force



Governing equations and numerical procedures:

- Incompressible Navier-Stokes equations under Boussinesq approximation
- Finite difference with an immersed boundary formulation
- Large-eddy simulation (LES) with WALE model

$$\frac{\partial u_i}{\partial x_i}=0,$$

$$\frac{\partial u_i}{\partial t}+\frac{\partial u_i u_j}{\partial x_j}-f_c\epsilon_{ij3}(u_j-U_\infty\delta_{j1})=-\frac{1}{\rho_0}\frac{\partial p^*}{\partial x_i}+\frac{\partial \tau_{ij}}{\partial x_j}-\frac{\rho^* g}{\rho_0}\delta_{i3},$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u_i}{\partial x_i}=\frac{\partial J_{\rho,i}}{\partial x_i},$$

$$\tau_{ij}=(\nu+\nu_{\mathrm{sgs}})\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\ J_{\rho,i}=(\kappa+\kappa_{\mathrm{sgs}})\frac{\partial \rho}{\partial x_i}$$

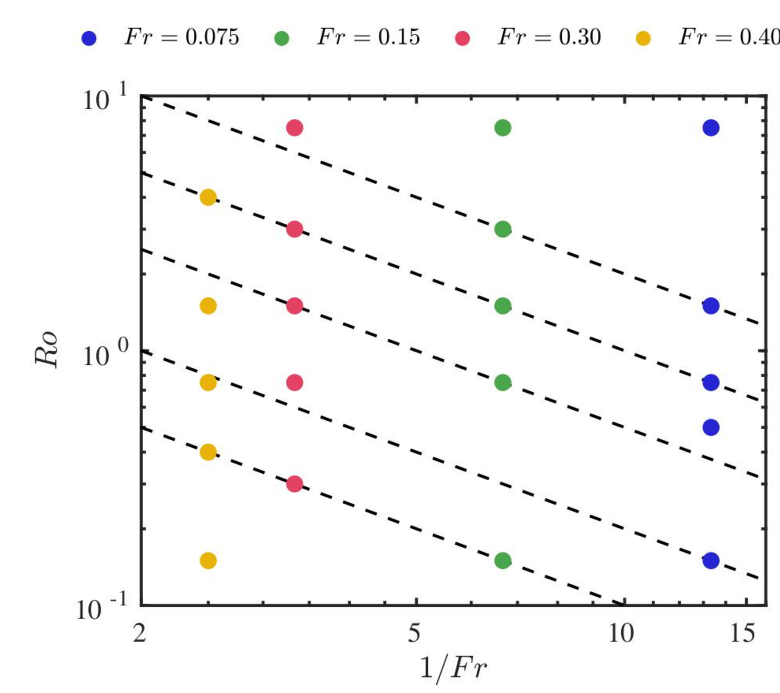
(*: deviation from the geostrophic and hydrostatic balances)

Parameter regimes

Non-dimensional parameters:

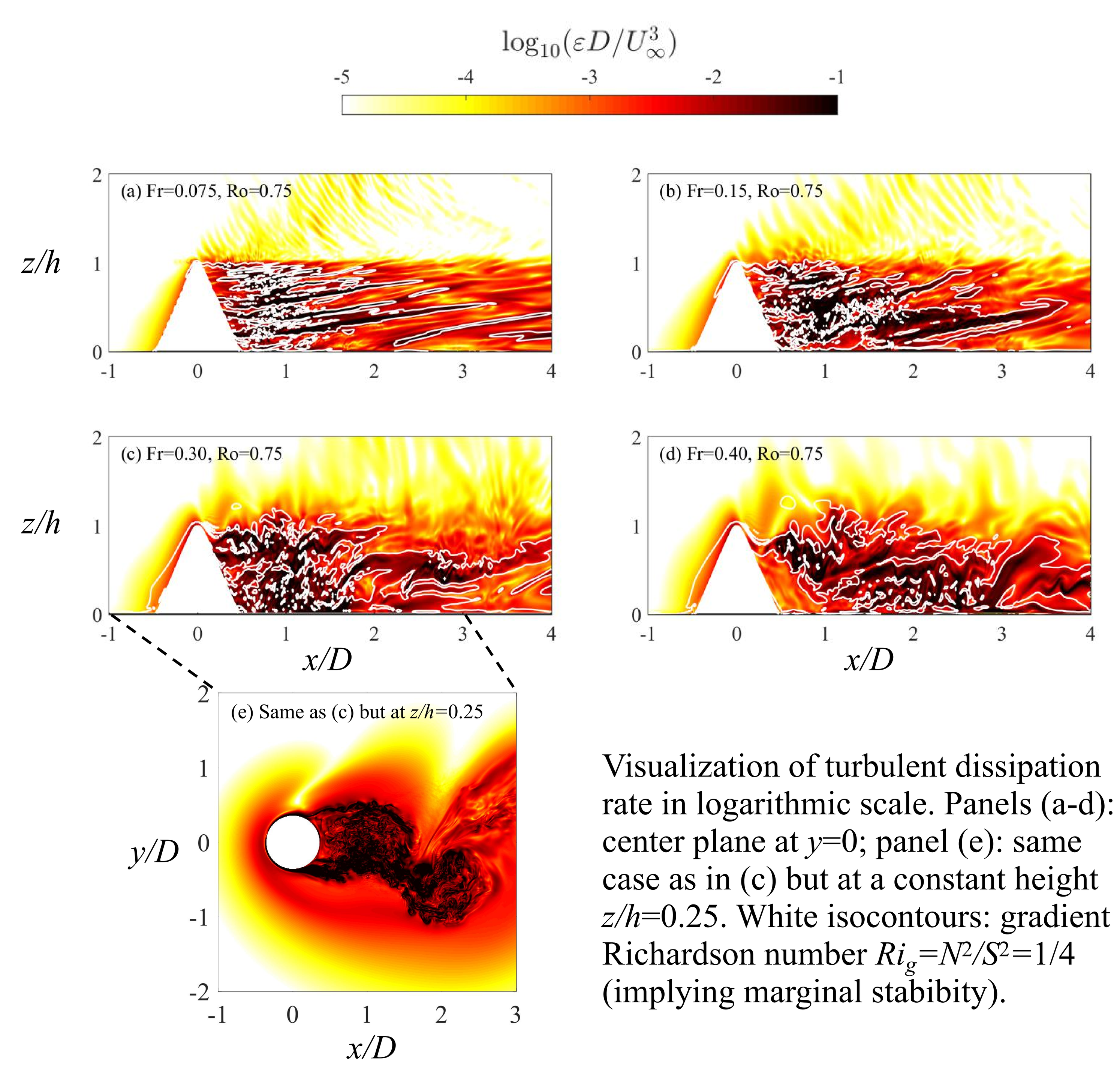
- Reynolds number: $Re = U_\infty D/\nu$
- Froude number: $Fr = U_\infty/Nh$
- Rossby number: $Ro = U_\infty/f_c D$
- Burger number: $Bu = (Ro/Fr)^2$

(Dashed lines: Bu = 1, 4, 25, 100, 400)



- A parameter sweep is conducted in the (Fr, Ro) space
  - -- Four levels of stratification ($0.075<Fr<0.4$) are selected, representative of the flow-around, vortex-shedding regime.
  - -- At each Fr, five rotation strengths from $Ro = O(0.1)$-$O(5)$ are chosen, covering mesoscale (10-100 km) to small submesoscale (≤100 m-10km) regimes.
- The Reynolds number varies among cases and is up to 40 000, sufficient to trigger relevant instabilities. The buoyancy-length-scale-based Reynolds number, $Re_N = U_\infty^2/\nu N$, is kept constant at 900.

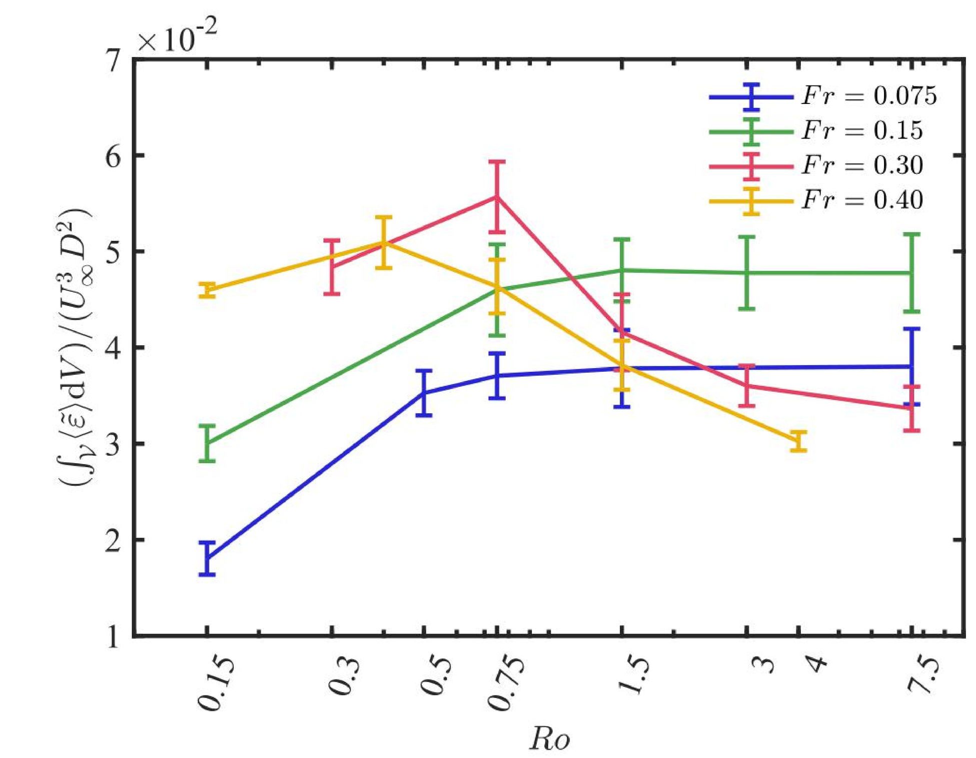
Flow phenomenology



Visualization of turbulent dissipation rate in logarithmic scale. Panels (a-d): center plane at y=0; panel (e): same case as in (c) but at a constant height z/h=0.25. White isocontours: gradient Richardson number $Ri_g=N^2/S^2=1/4$ (implying marginal stabibity).

- In horizontal directions, the flow exhibits Kármán vortex street pattern at the large scales, and strong turbulent dissipation at the fine scales. The Kármán vortices are subject to the centrifugal/inertial instability (CI).
- In the vertical direction, the flow is dislocated into oblique layers, whose vertical length scale and structural complexity depend on the strength of stratification. The dislocations are subject to the vertical shear instability (Kelvin-Helmholtz instability, KHI).

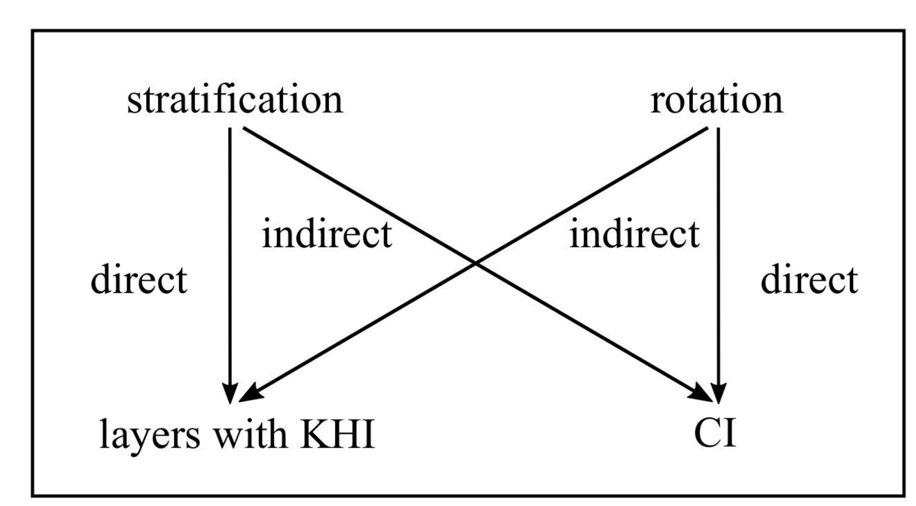
Turbulent dissipation: parametric dependence



- The volume-integrated turbulent dissipation rate scales as U_∞^3/D, the inertial scaling, and has different dependence on Ro at different Fr.
  - (1) When Fr = 0.075, 0.15, dissipation increases monotonically with Ro. In this case, CI is suppressed, and the role of rotation is stabilizing.
  - (2) When Fr = 0.30, 0.40, dissipation peaks at $Ro = O(0.5)$, corresponding to the strongest contribution of CI in the submesoscale regime.

Summary

- Stratification and rotation both influence turbulence directly and indirectly, through corresponding instabilities. Their effects are intertwined.
  - (1) Direct effects. Stratification leads to flow layering. The shear between the layers triggers KHI. Rotation leads to CI, which is itself the strongest at intermediate rotation rates (Ro=O(0.5)).
  - (2) Indirect effects. Strong rotation dampens vertical shear and weakens KHI-driven turbulence. Strong stratification imposes smaller vertical length scales that restrict CI-driven turbulence.



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