

Structures of vortices in underwater hill wakes

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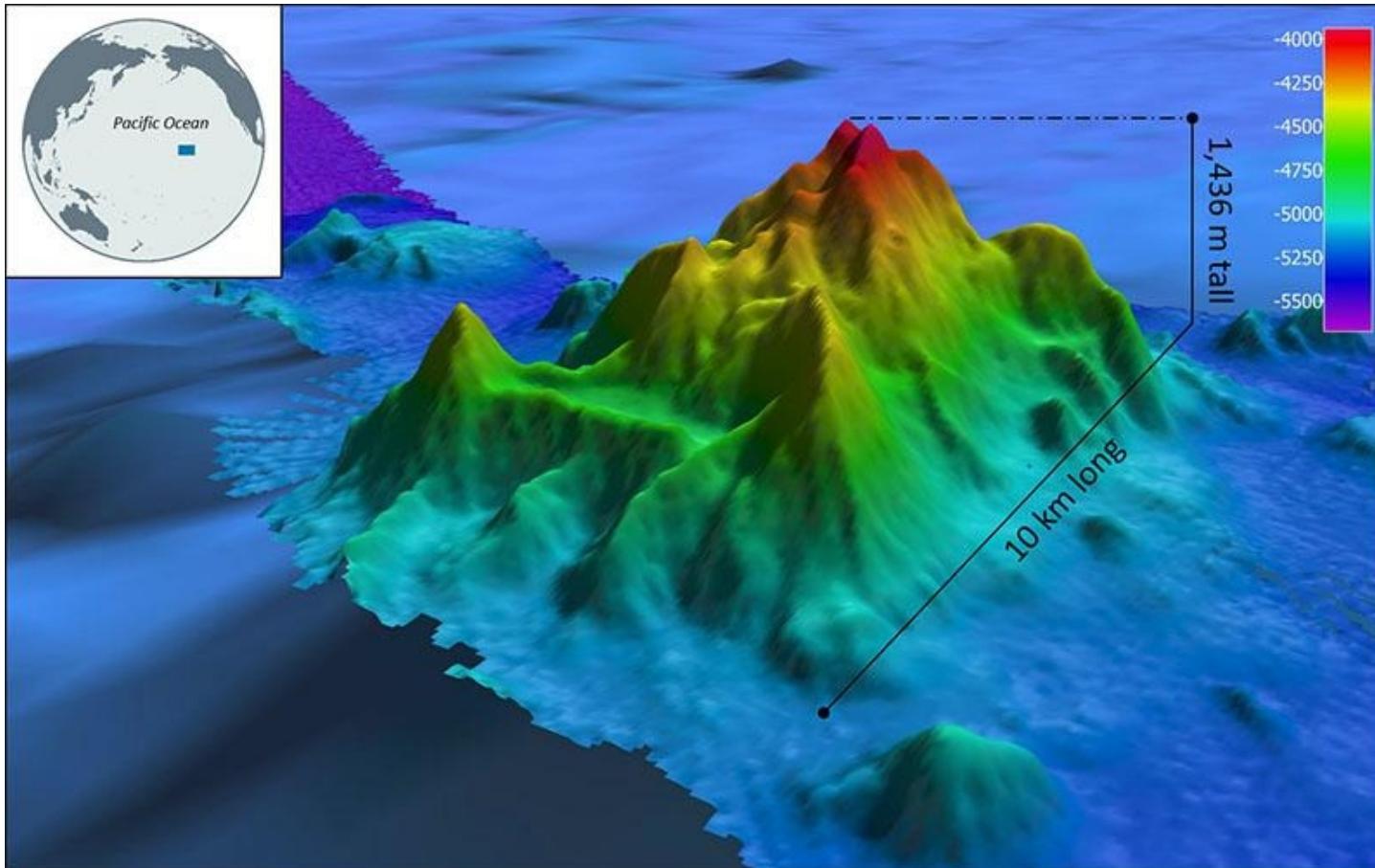
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SoCal XV, UCLA, '22

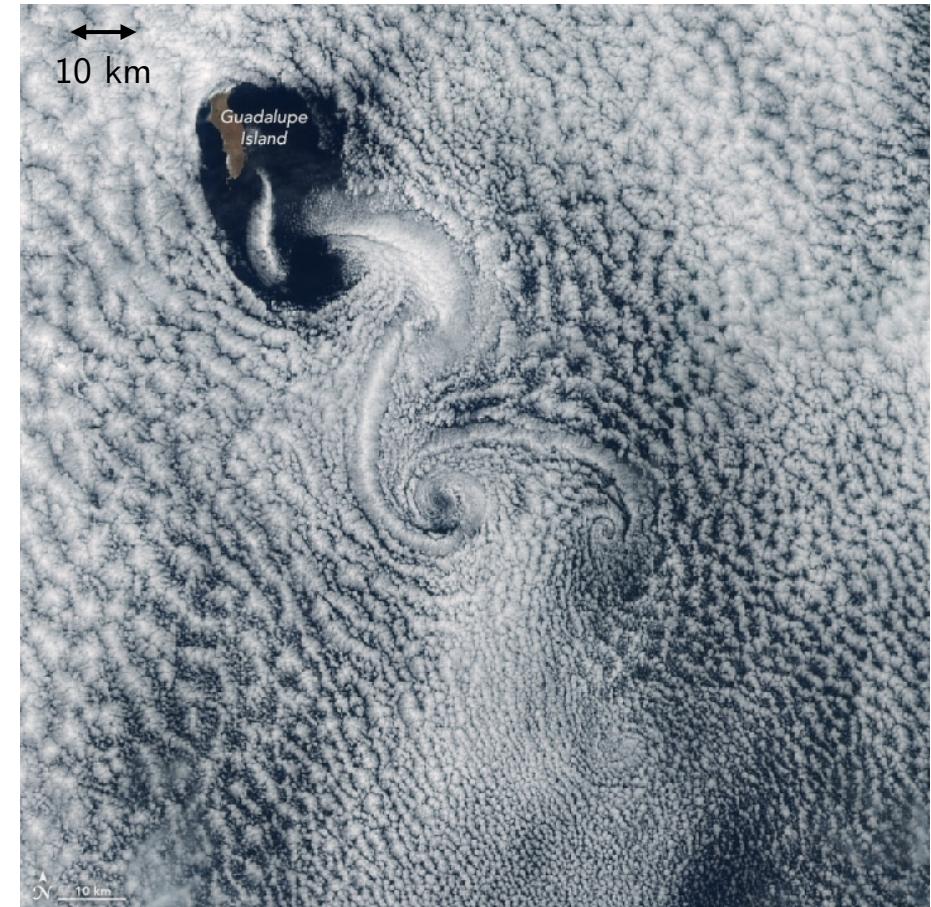
Introduction and motivation

Underwater



Okeanos Explorer Seamount, 2016 (NOAA)

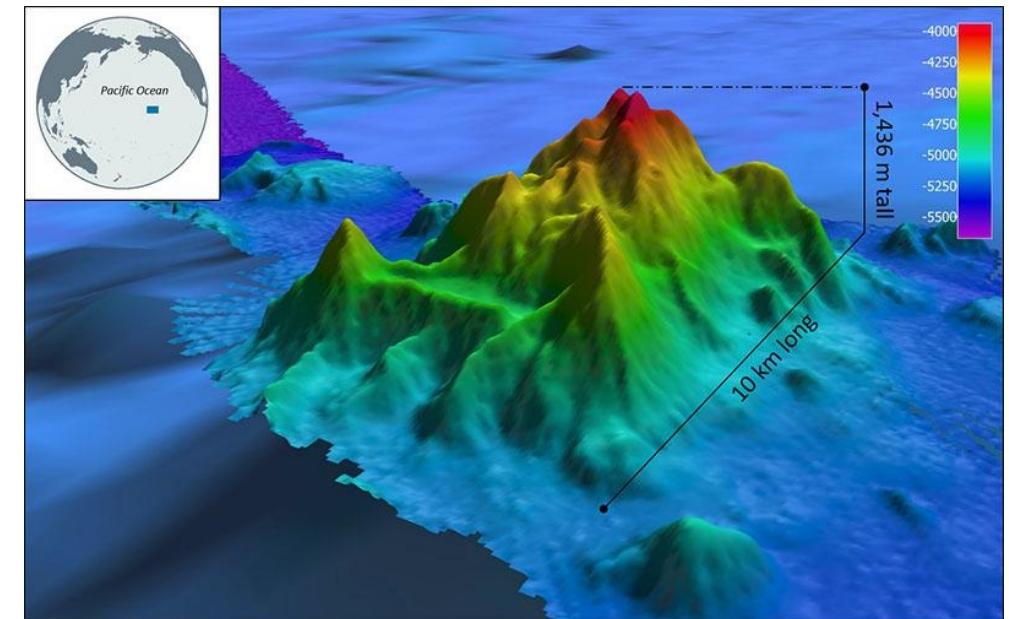
Above water



Guadalupe Island, 2017 (NASA)

Introduction and motivation

- Large spatial and temporal scales
 - impact on marine lives, weather system
- Many key factors come into play
 - **stratification**
 - **rotation - part I**
 - **tide - part II**
- Richness in vortex dynamics
- Unbalanced amount of literature in observation and in numerical simulation

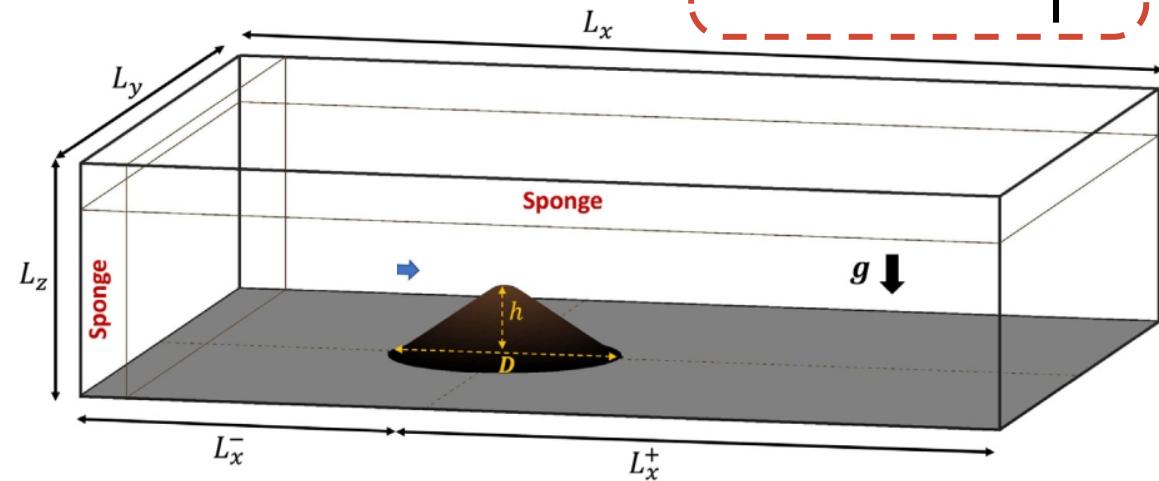


Okeanos Explorer Seamount, 2016 (NOAA)

Physical modelling^{1,2} and numerical simulations

- Topography: conical hill
 - slope angle 30 deg
- Stratified, rotating, tidally modulated flow
- Governing equations: incompressible Navier-Stokes

$$U_b = U_c + U_t \cos(2\pi f_t t)$$



$$\frac{\partial u_i}{\partial x_i} = 0$$

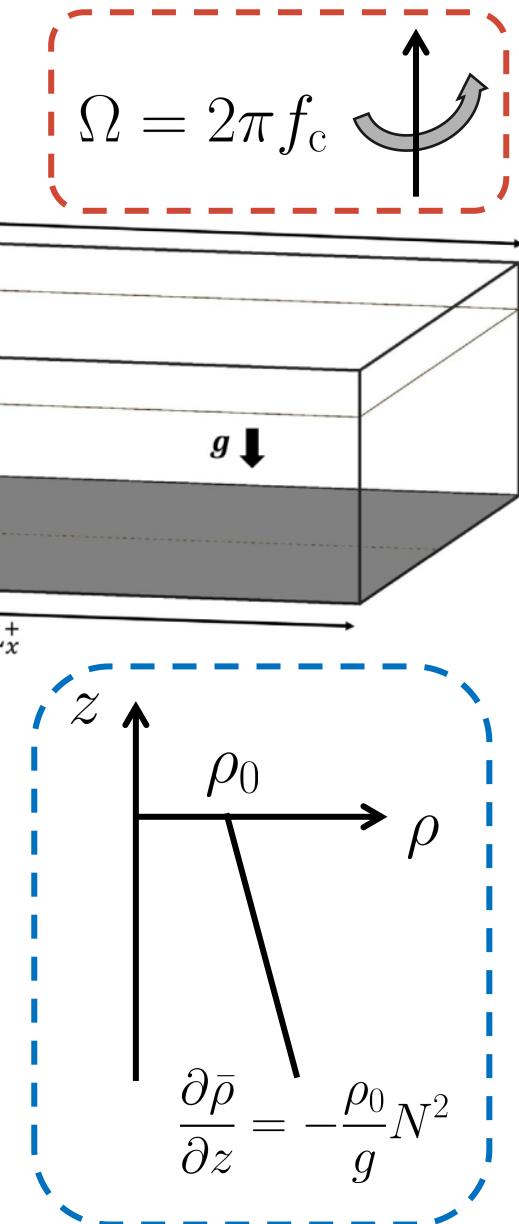
$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} - 2\pi f_c \epsilon_{ij3} (u_j - U_b(t) \delta_{i1}) = -\frac{1}{\rho_0} \frac{\partial p'}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} - \frac{\rho' g}{\rho_0} \delta_{i3} + F_b(t) \delta_{i1}$$

$$\frac{\partial \rho}{\partial x_i} + \frac{\partial \rho u_i}{\partial x_i} = \frac{\partial \phi_i}{\partial x_i}$$

$$\tau_{ij} = (\nu + \nu_{sgs}) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad \phi_i = (\kappa + \kappa_{sgs}) \frac{\partial \rho}{\partial x_i}$$

Body force:

$$F_b = 2\pi f_t U_t \cos(2\pi f_t t)$$

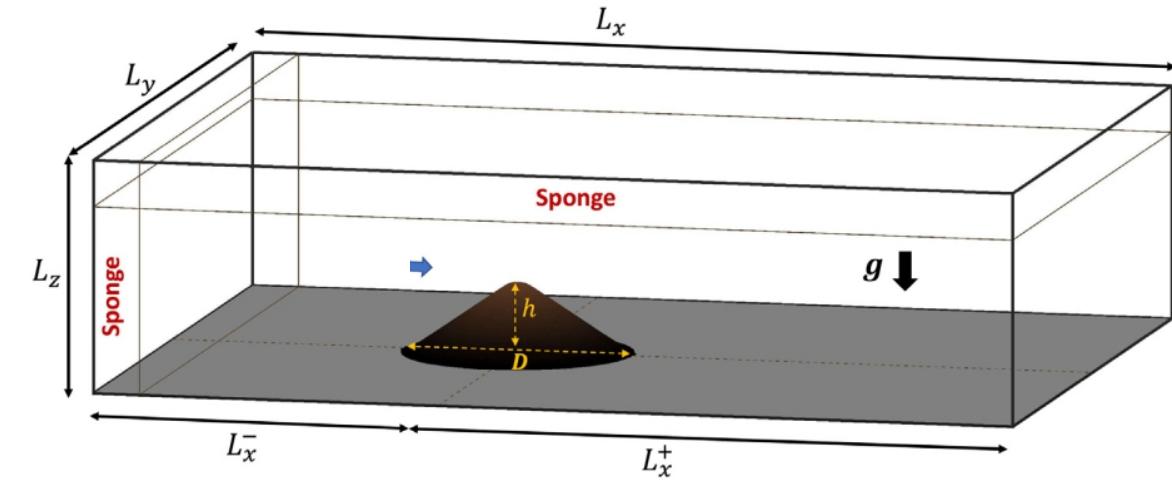


Physical modelling and numerical simulations^{1,2}

- Large Eddy Simulation (LES)
 - ³subgrid-scale model: WALE
 - Immersed Boundary Method⁴ (IBM) formulation for the topography
 - $Re_D = 20,000$
- Controlling nondimensional parameters

$$Fr = \frac{U}{Nh}, \quad Ro = \frac{U}{2\pi f_i D}, \quad Bu = \left(\frac{Ro}{Fr}\right)^2, \quad Ex = \frac{U_t}{2\pi f_t D}$$

Froude number Rossby Number Burger number Excursion number
(stratification level) (rotating rate) (geostrophy) (tidal strength)

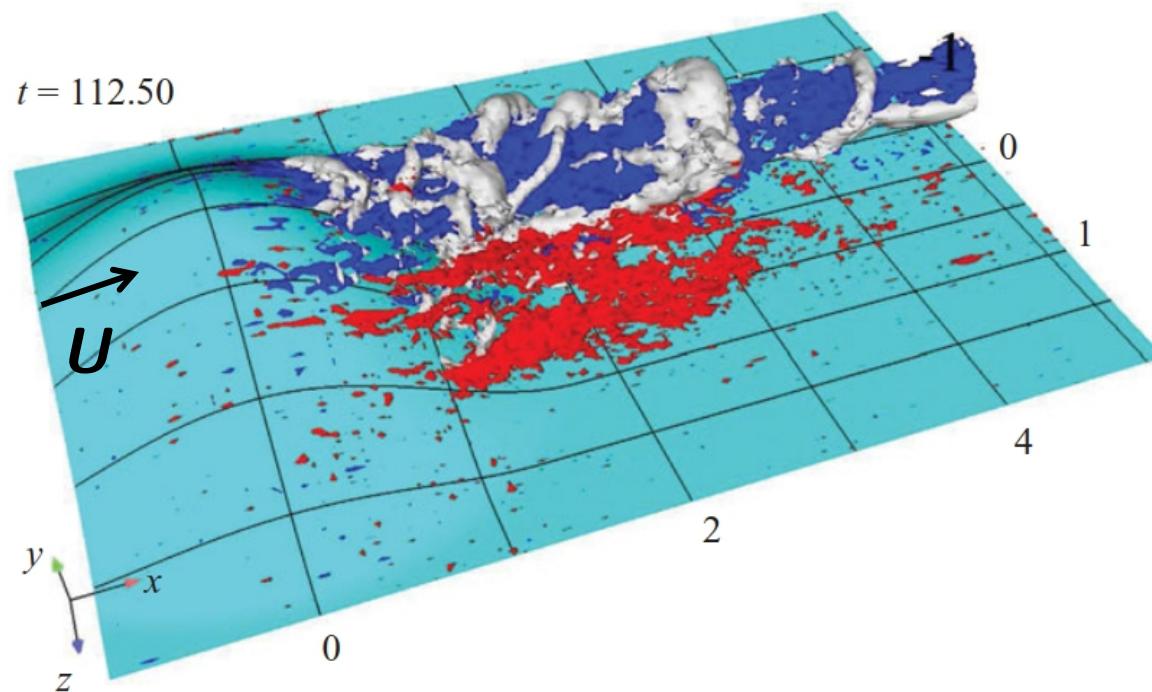


Case	Bu	Ex	Ro	ΔT	Fr	(N_x, N_y, N_z)	N
Bu1	1		0.15				
BuK	1300	0	5.5	$310D/U_c$			
BuInf	∞		∞			0.15 (1536,1280,322)	4000
Ex025	1300	0.25					
Ex050	1300	0.50	5.5	$128T_t$			

Parameter table of simulations.

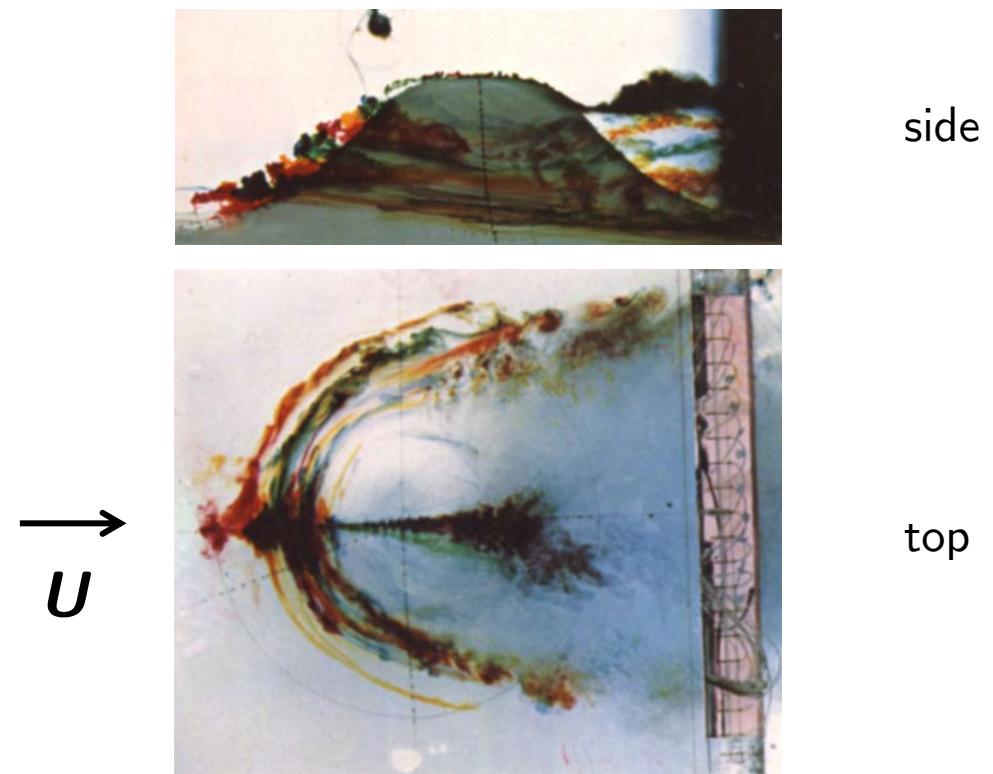
Lee vortices: a distinctive flow feature in stratified wakes

LES of an **unstratified** flow **over** a 3-D hill



Garcia-Villalba et al. *JFM*, 2009

Experiment of a **stratified** flow **around** a 3-D hill



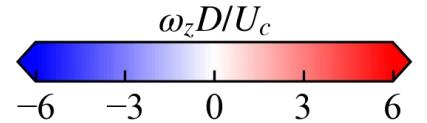
Hunt & Snyder, *JFM*, 1980

Strong rotation

no rotation

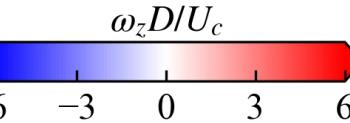
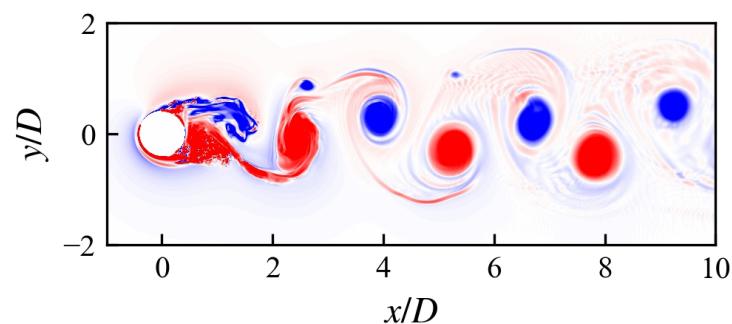
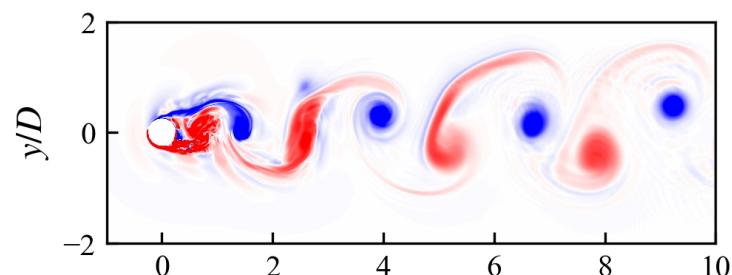
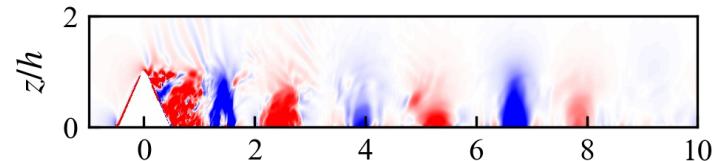
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Ex025	1300	0.25	5.5		$128T_t$		
Ex050	1300	0.50					

Part-I: The effect of rotation



Strong rotation

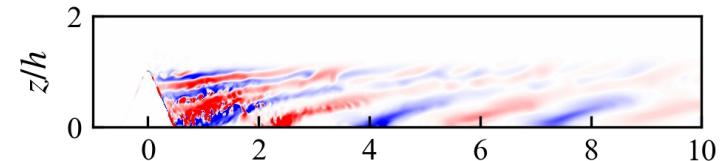
$T = 120.8 U_c/D$



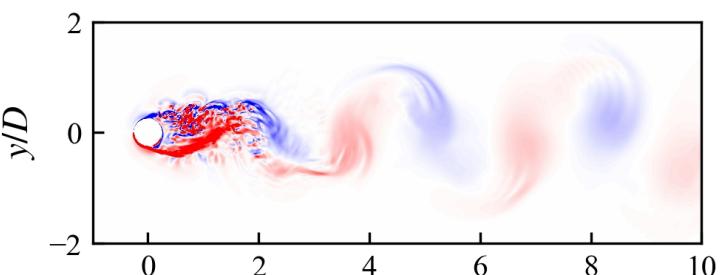
no rotation

$T = 72.6 U_c/D$

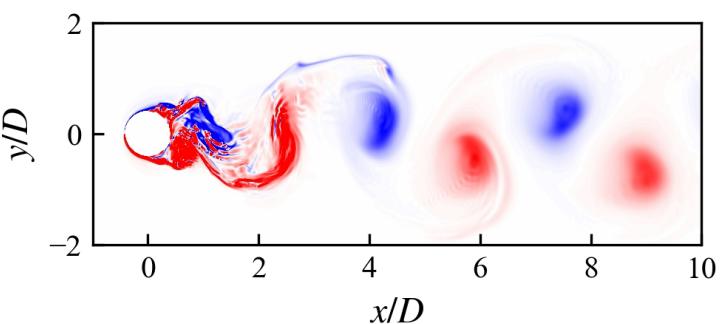
vertical: $y=0$



horizontal: $z/h=0.25$



horizontal: $z/h=0.50$

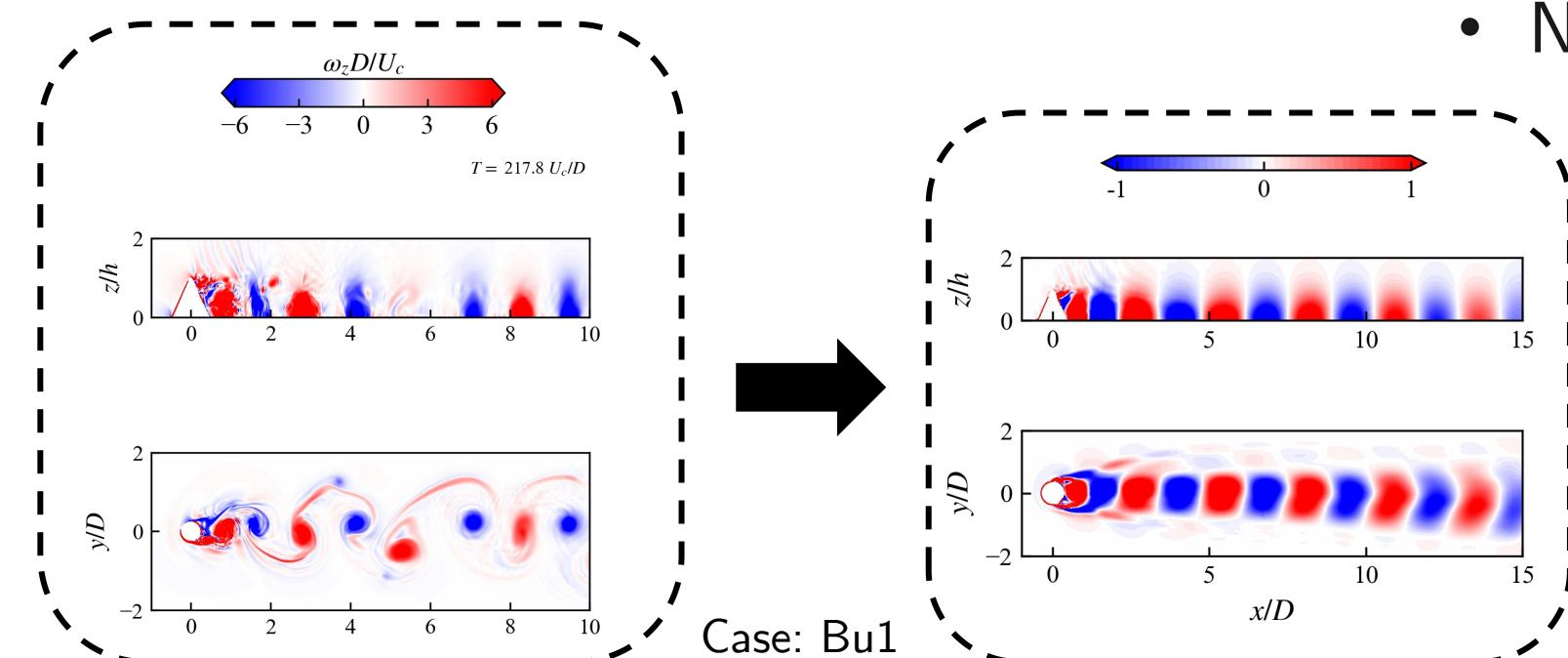


$Fr = 0.15, Ro = 0.15, Bu = 1$

$Fr = 0.15, Ro = \infty, Bu = \infty$

An overview of SPOD

- Motivations of doing SPOD on ω_z
 - Extract the features of the vortical motion **in a statistical way**
 - Identify characteristic length/time scales



- SPOD problem formulation¹

$$C(\mathbf{x}, \mathbf{x}', \tau) = \langle \mathbf{q}(\mathbf{x}, t) \mathbf{q}^*(\mathbf{x}', t + \tau) \rangle$$

$$S(\mathbf{x}, \mathbf{x}', f) = \int_{-\infty}^{\infty} C(\mathbf{x}, \mathbf{x}', \tau) \exp(i2\pi f \tau) d\tau$$

$$\int_{\Omega} S(\mathbf{x}, \mathbf{x}', f) \psi(\mathbf{x}', f) d\mathbf{x}' = \lambda(f') \psi(\mathbf{x}, f')$$

$$S(\mathbf{x}, \mathbf{x}', f) = \sum_{j=1}^{\infty} \lambda_j(f) \psi(\mathbf{x}, f) \psi^*(\mathbf{x}', f)$$

- Numerical implementation²

$$\hat{C} = \langle \hat{Q} \hat{Q}^H \rangle, \hat{Q} \in \mathbb{C}^{m \times n}$$

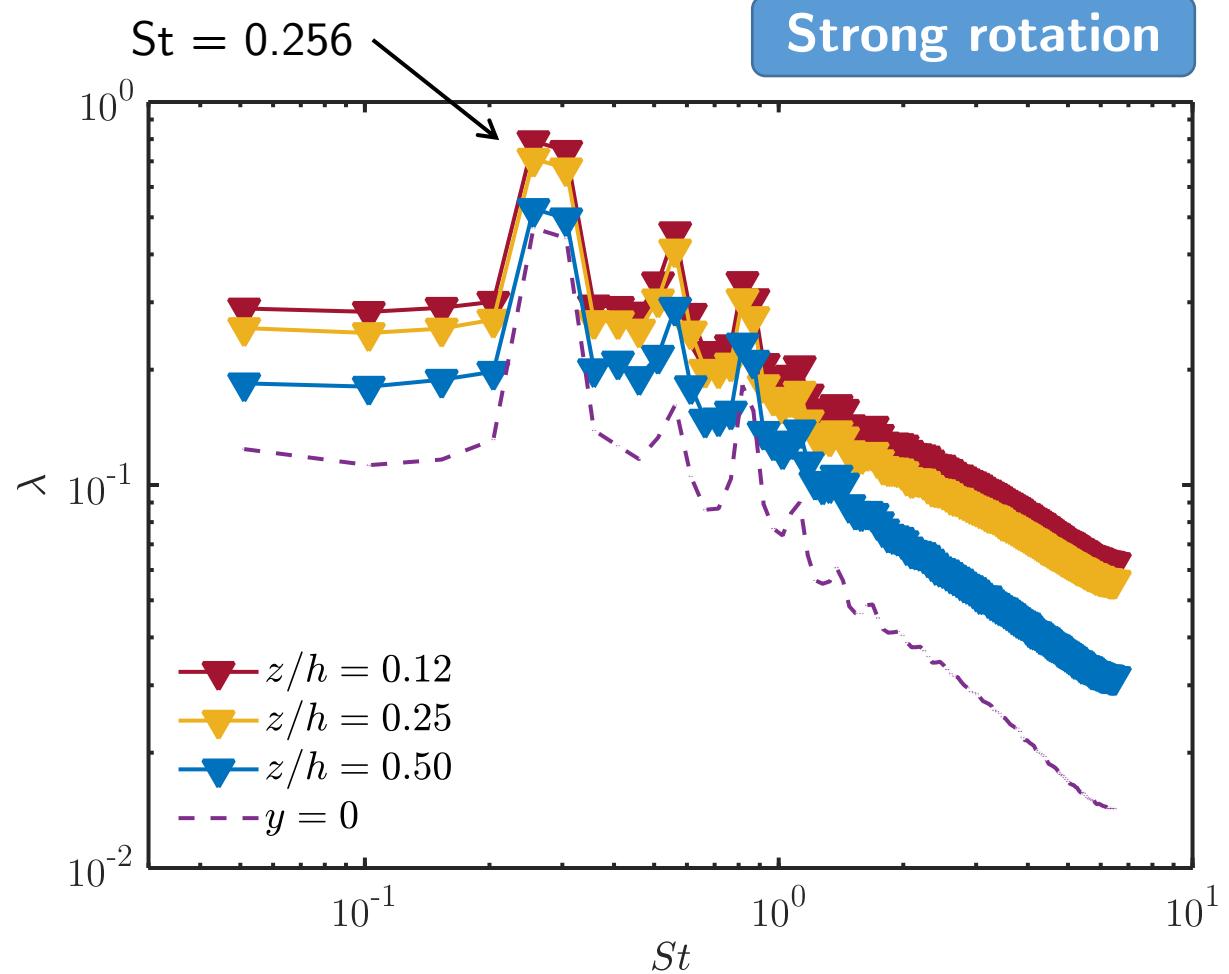
$$\hat{C} \hat{\Phi} = \hat{\Phi} \hat{\Sigma}$$

$$\hat{\Phi} = \hat{Q} \hat{\Psi}$$

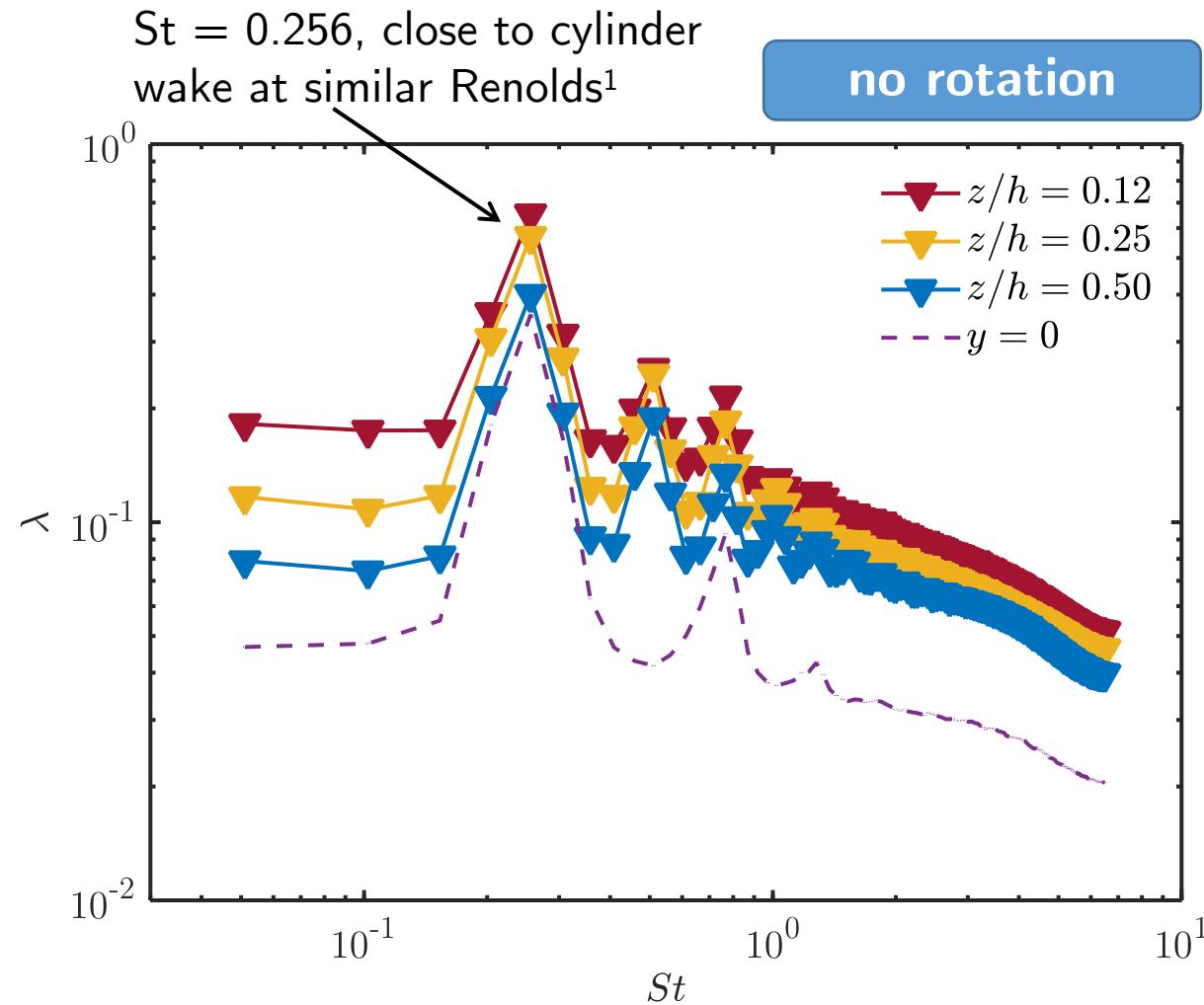
$$\hat{\Phi}^H \hat{\Phi} = I$$

SPOD eigenspectra, with a nearly global Strouhal number

$$St = \frac{fD}{U_c}$$



$Fr = 0.15, Ro = 0.15, Bu = 1$

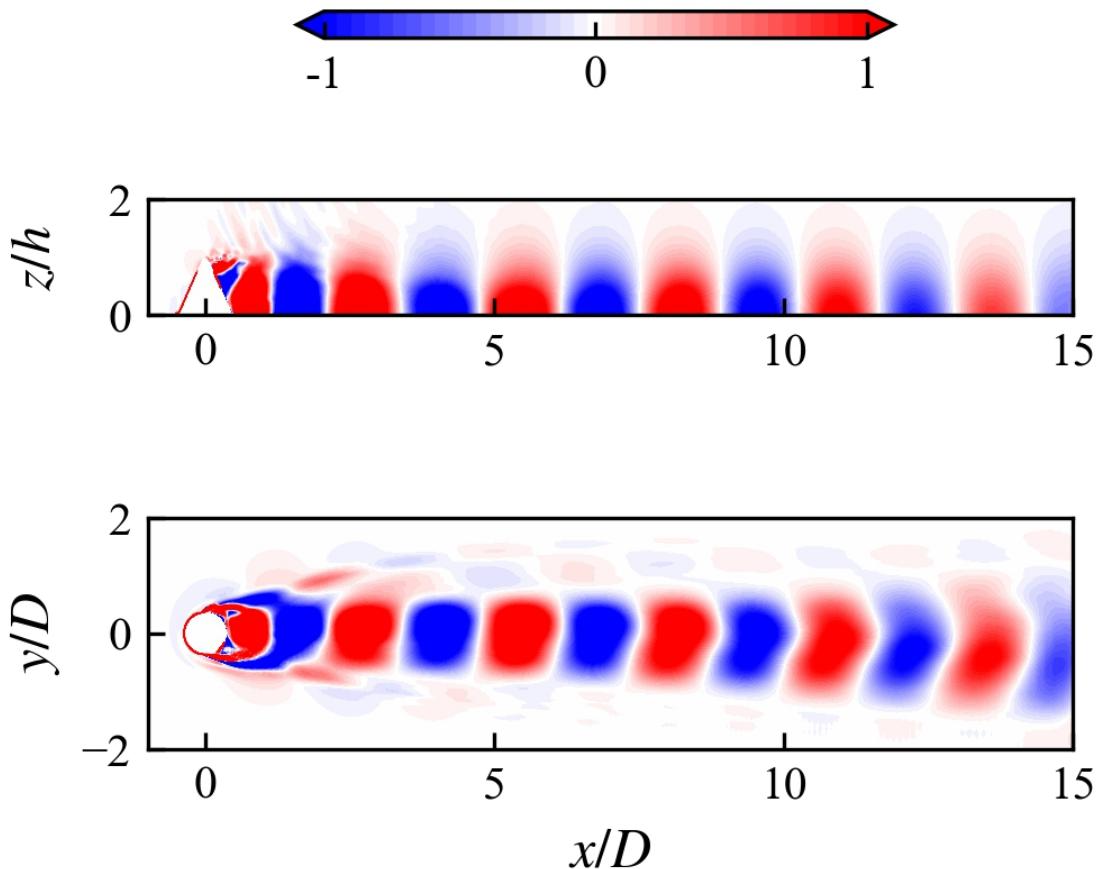


$Fr = 0.15, Ro = \infty, Bu = \infty, 1000$

¹Williamson & Brown, *Journal of Fluids and Structures*, 1998

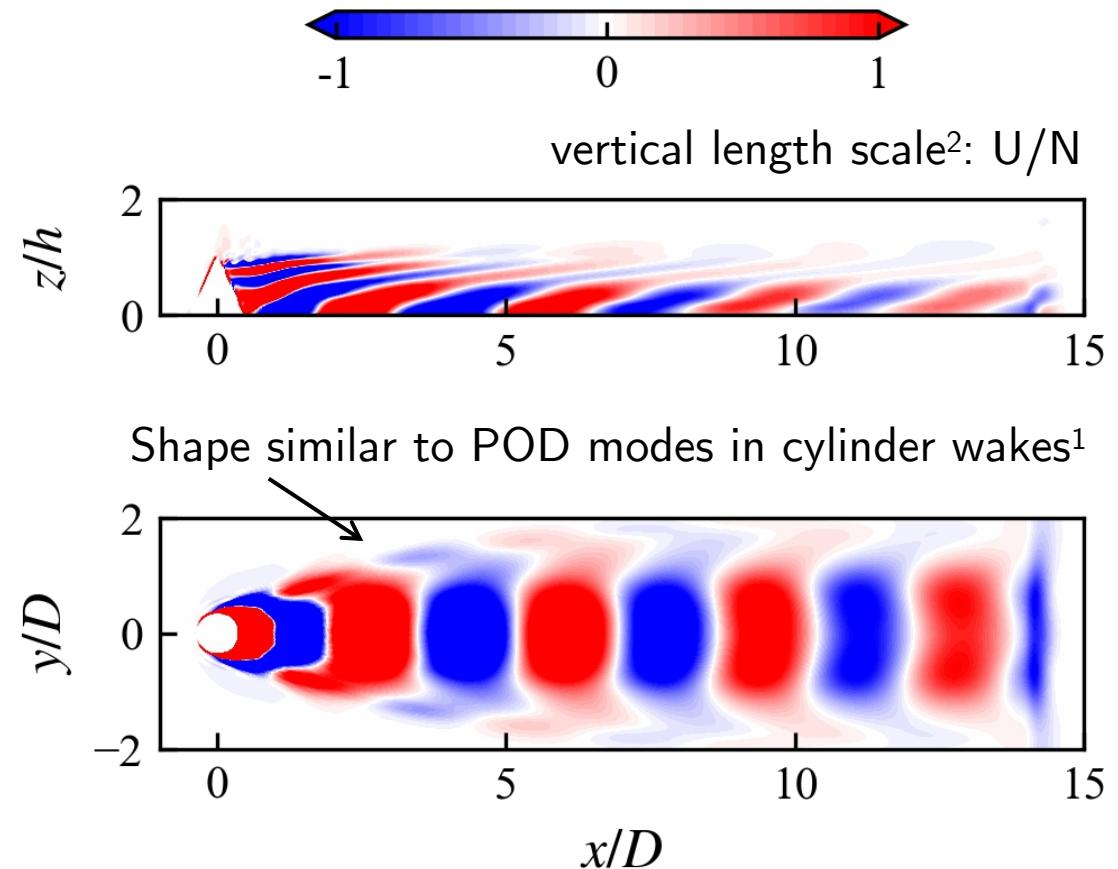
Leading SPOD eigenmodes (shedding frequency)

Strong rotation



$$Fr = 0.15, Ro = 0.15, Bu = 1$$

no rotation

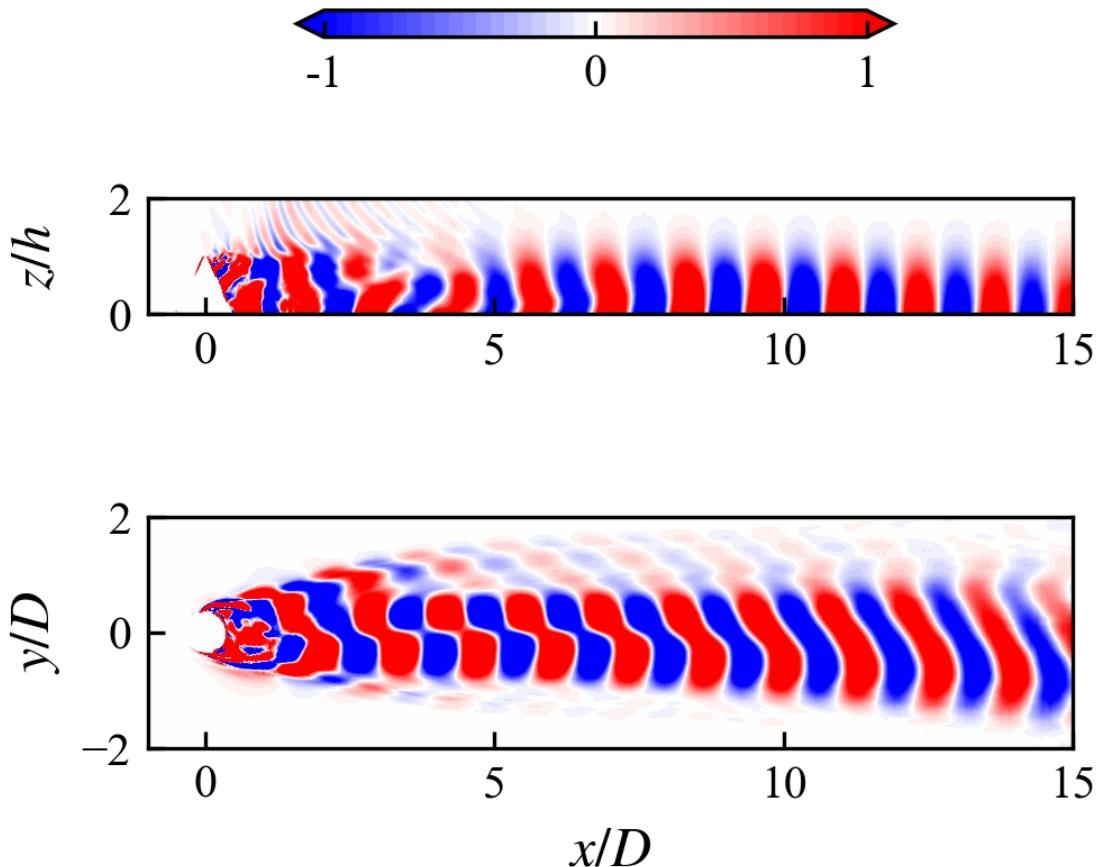


$$Fr = 0.15, Ro = \infty, Bu = \infty, 1000$$

¹Gunes, Sirisup, and Karniadakis, *JCP*, 2016; ²Billant & Chomaz, *PoF*, 2001

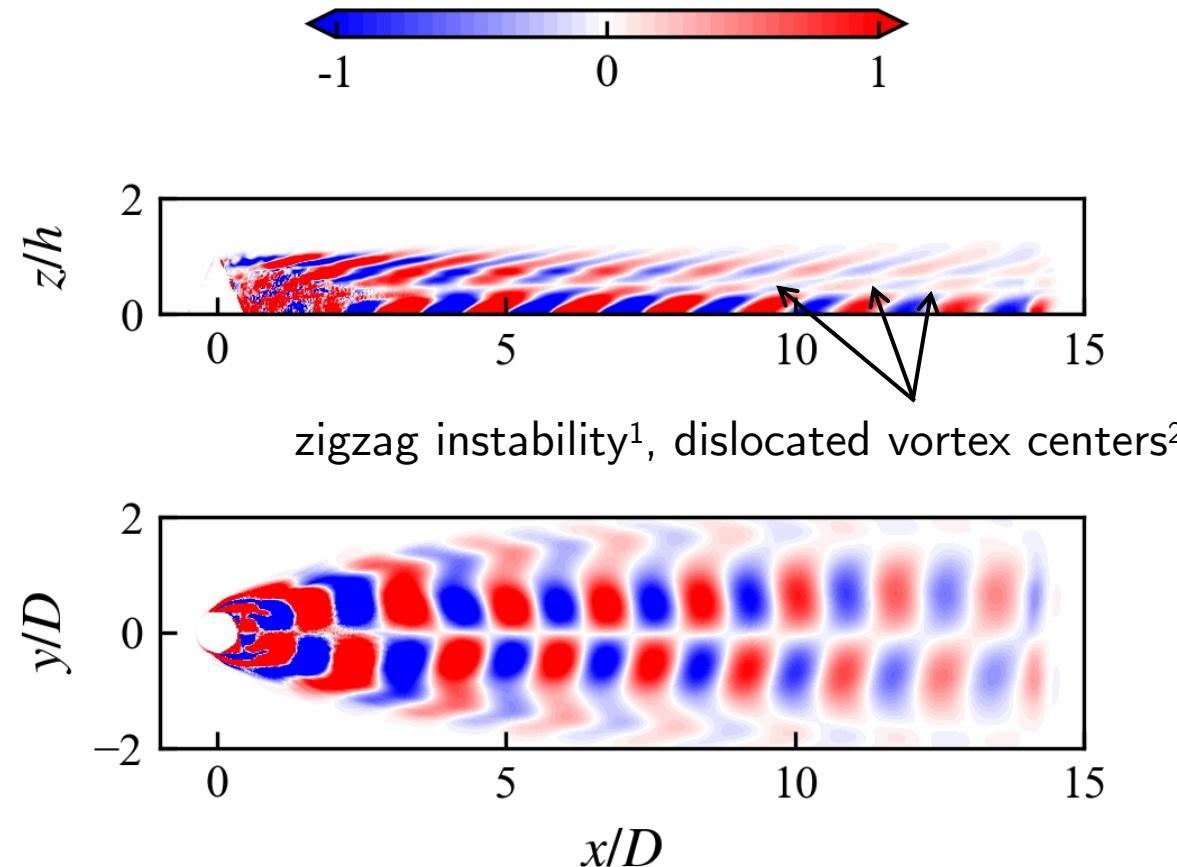
2nd leading SPOD eigenmodes (twice the shedding frequency)

Strong rotation



$$Fr = 0.15, Ro = 0.15, Bu = 1$$

no rotation



$$Fr = 0.15, Ro = \infty, Bu = \infty, 1000$$

¹Billant & Chomaz, *JFM*, 2000; ²Basak & Sarkar, *JFM*, 2006;

no tide

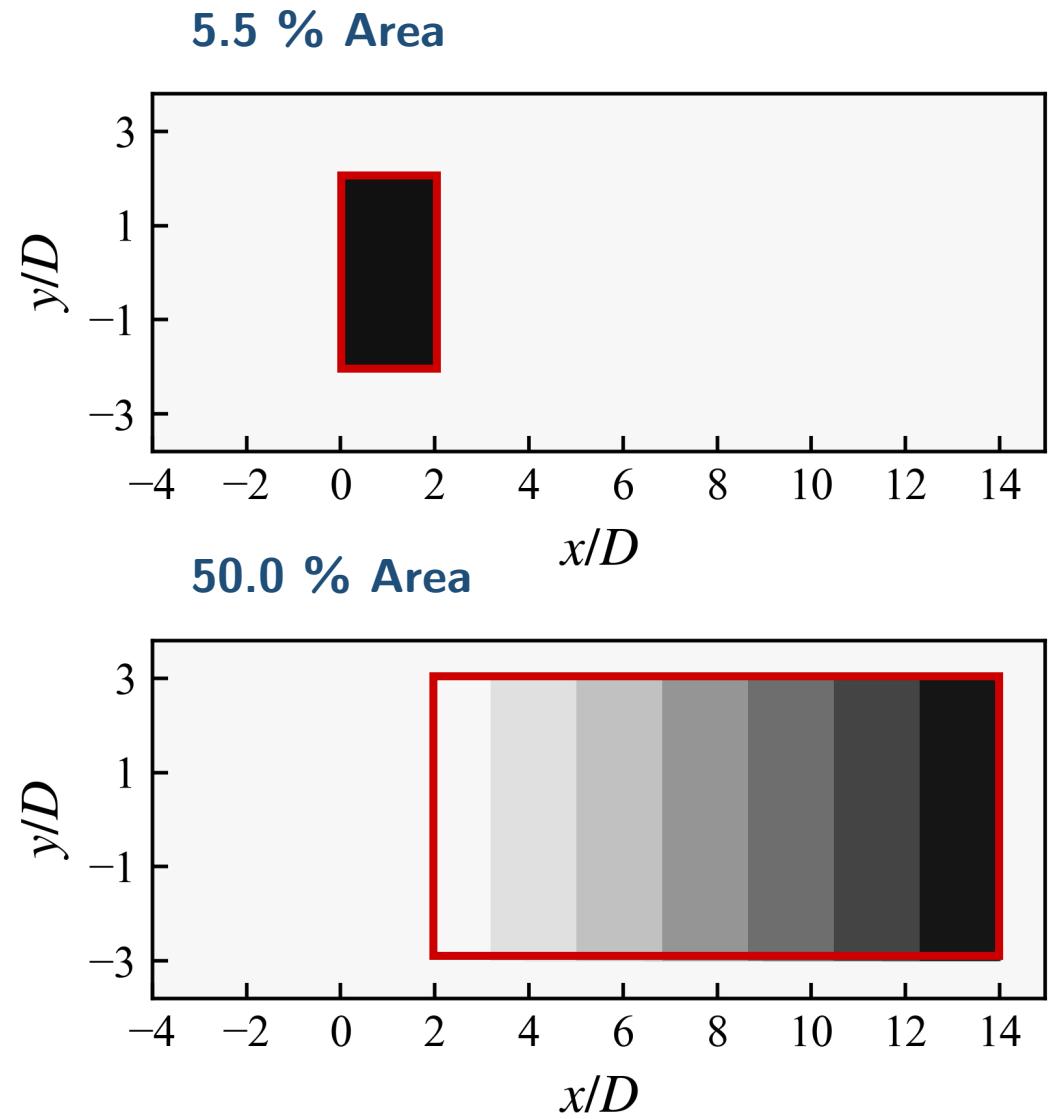
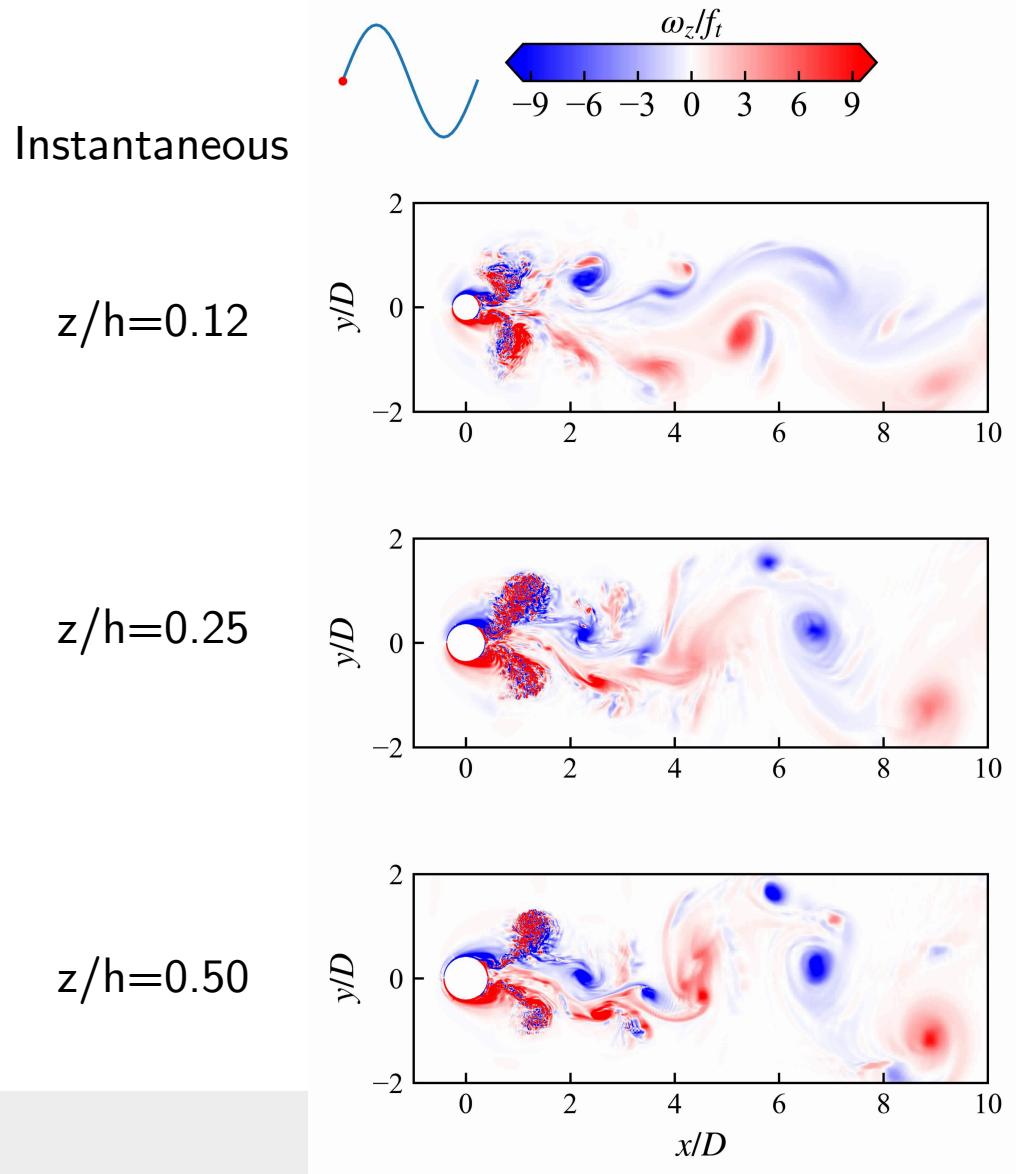
moderate tide

Case	Ex	Ro	ΔT	Fr	(N_x, N_y, N_z)	N
Bu1		0.15				
BuK	0	5.5	$310D/U_c$			
BuInf		∞		0.15	(1536,1280,322)	4000
Ex025	0.25					
Ex050	0.50	5.5	$128T_t$			

Part-II: The effect of tide

Near and far wake decomposition

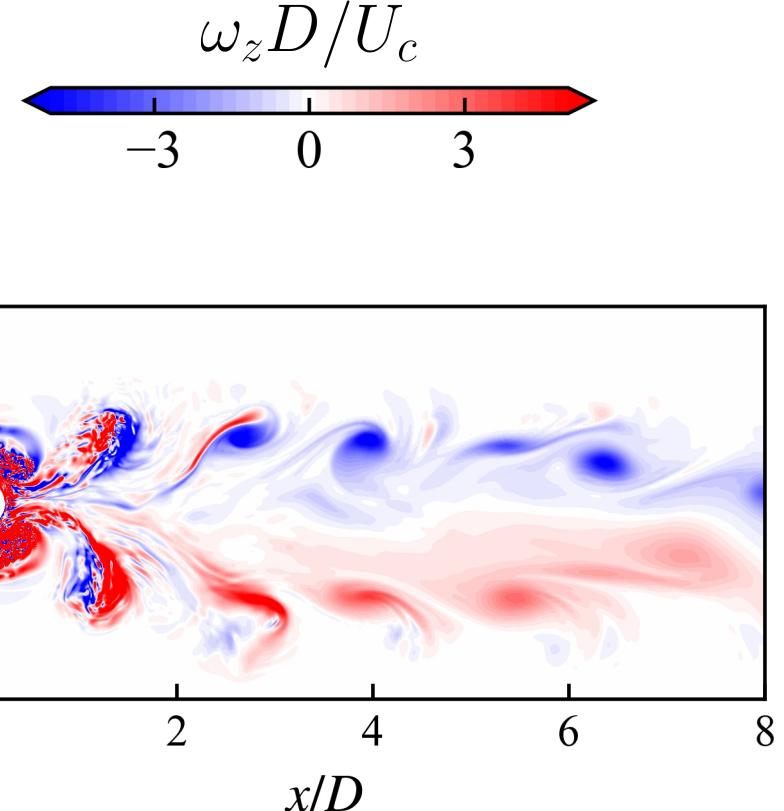
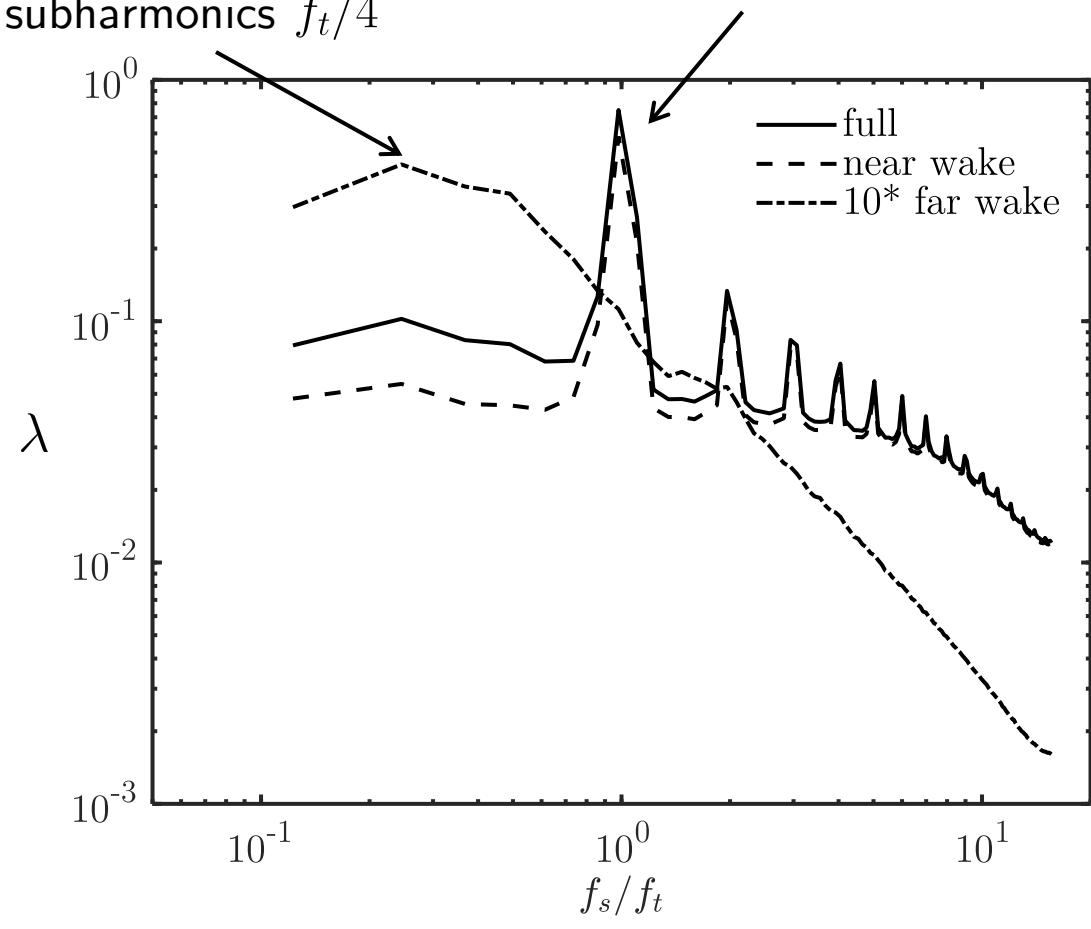
Schematic of region division



SPOD eigenspectra, influence of tide

Far wake: tidal synchronization¹ to subharmonics $f_t/4$

Near wake: tide dominates



Summary

- **Part-I: the effect of rotation**
 - alters the vertical structure of vortex shedding
 - the flow lost vertical alignment with weak rotation
 - preserves the vortex shedding frequency
- **Part-II: the effect of tide**
 - dominates the vortex shedding in the near wake
 - synchronizes the far wake into its subharmonic
- **Future work**
 - varying Fr number: vertical length scaling

Thank you!