Structures of vortices in underwater hill wakes

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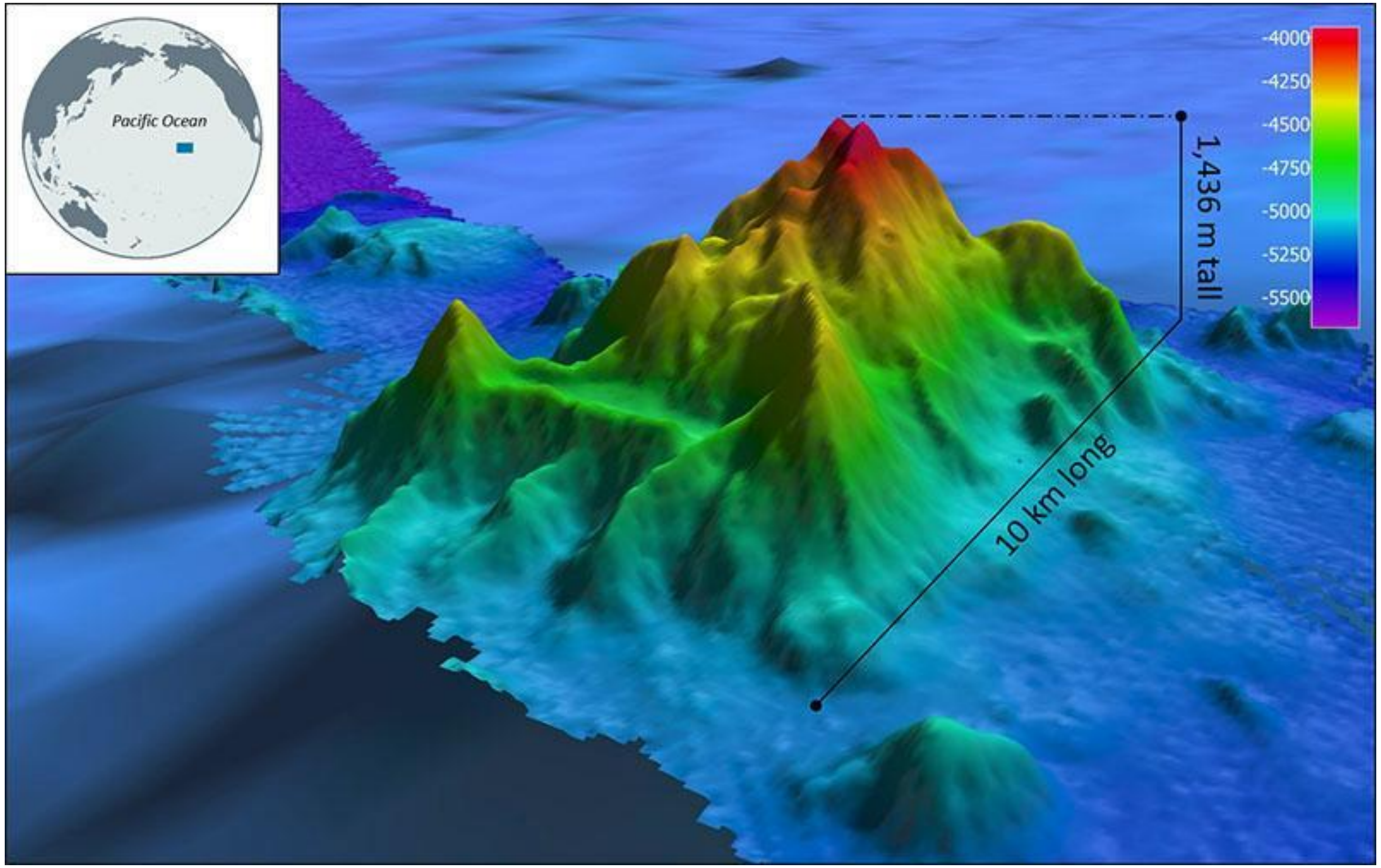
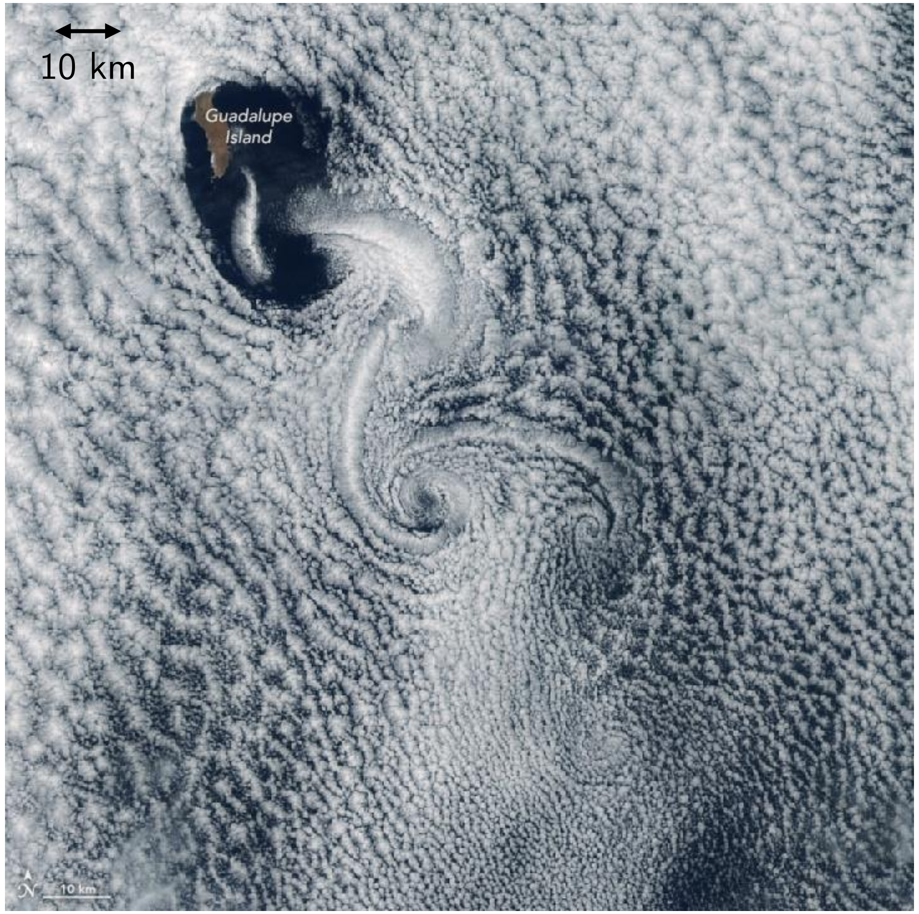
Introduction and motivation

Underwater



Okeanos Explorer Seamount, 2016 (NOAA)

Above water



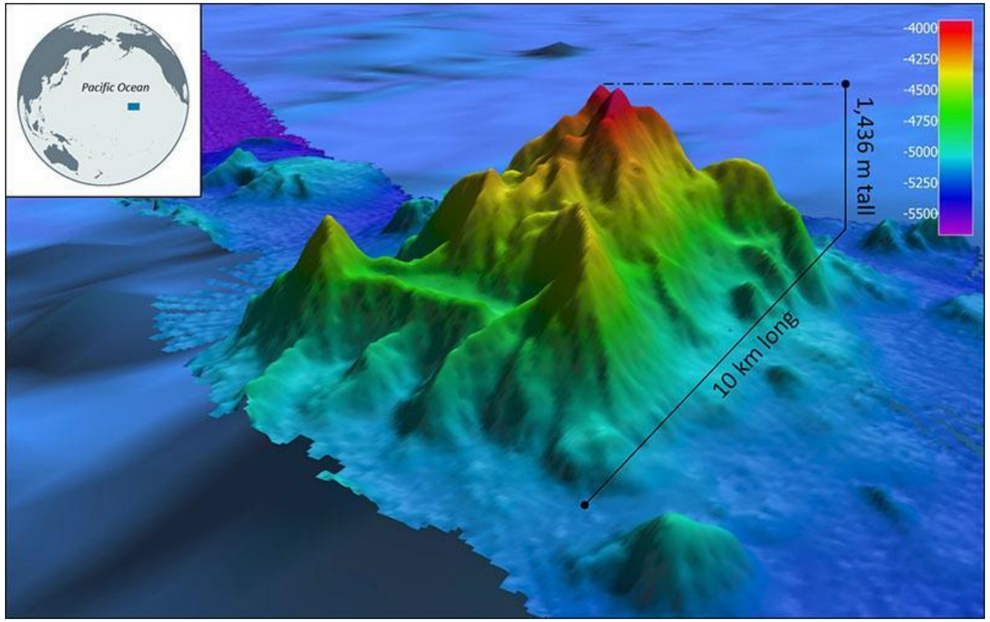
Guadalupe Island, 2017 (NASA)

Introduction and motivation

- Large spatial and temporal scales
  - impact on marine lives, weather system
- Many key factors come into play
  - **stratification**
  - **rotation - part I**
  - **tide - part II**
- Richness in vortex dymanics
- Unbalanced amount of literature in observation and in numerical simulation

inertial scale
L~O(km)
T~O(week)

viscous scale
L~O(mm)
T~O(sec)



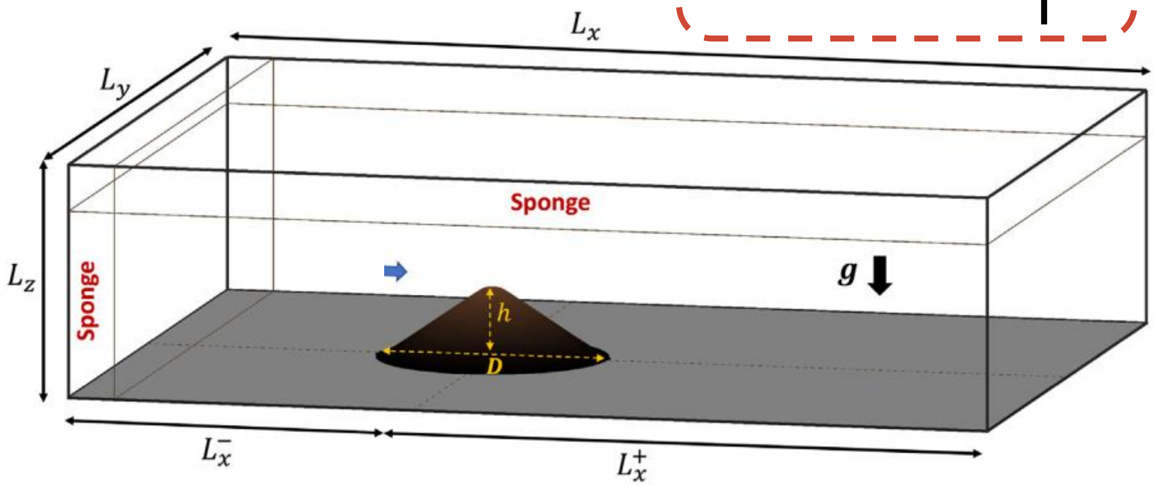
Okeanos Explorer Seamount, 2016 (NOAA)

Physical modelling[1,2] and numerical simulations

$\Omega = 2\pi f_c$

- Topography: conical hill
  - slope angle 30 deg

$U_b = U_c + U_t \cos(2\pi f_t t)$



- Stratified, rotating, tidally modulated flow

- Governing equations: incompressible Navier-Stokes

$$\frac{\partial u_i}{\partial x_i} = 0$$

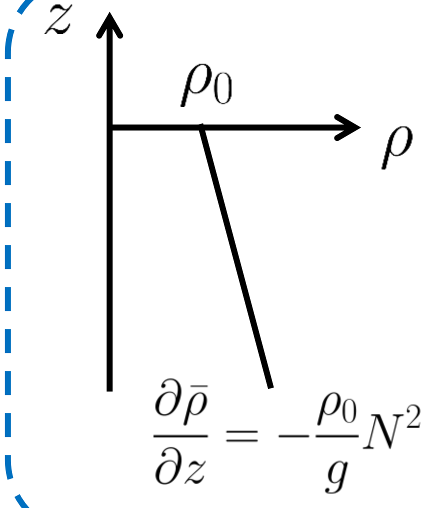
$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} - 2\pi f_c \epsilon_{ij3}(u_j - U_b(t)\delta_{i1}) = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} - \frac{\rho' g}{\rho_0}\delta_{i3} + F_{\mathrm{b}}(t)\delta_{i1}$$

$$\frac{\partial \rho}{\partial x_i} + \frac{\partial \rho u_i}{\partial x_i} = \frac{\partial \phi_i}{\partial x_i}$$

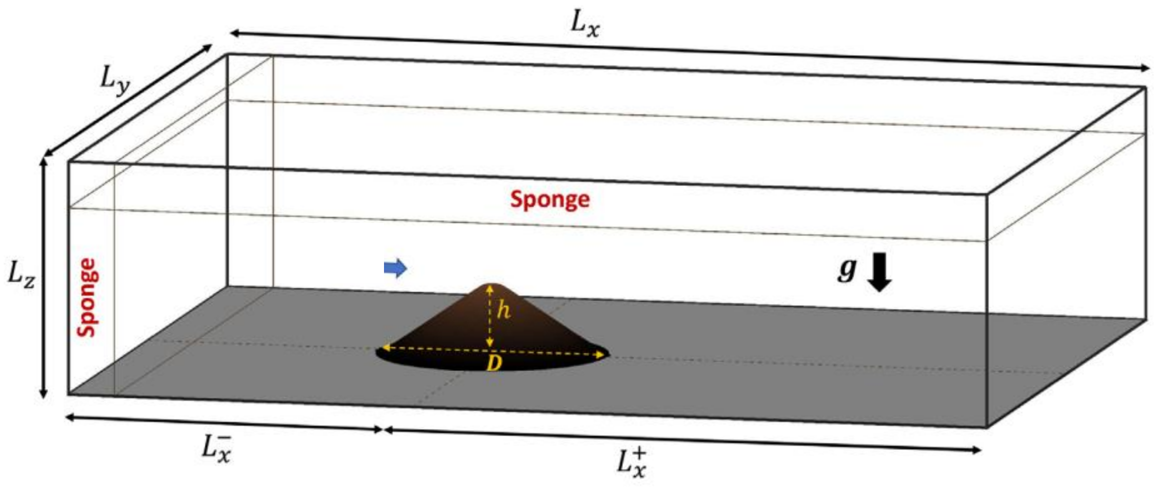
$$\tau_{ij} = (\nu + \nu_{\mathrm{sgs}})\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),\ \phi_i = (\kappa + \kappa_{\mathrm{sgs}})\frac{\partial \rho}{\partial x_i}$$

Body force:

$$F_{\mathrm{b}} = 2\pi f_{\mathrm{t}} U_{\mathrm{t}} \cos(2\pi f_{\mathrm{t}} t)$$



Physical modelling and numerical simulations[1,2]



- Large Eddy Simulation (LES)
  - [3]subgrid-scale model: WALE
  - Immersed Boundary Method[4] (IBM) formulation for the topography
  - $\mathrm{Re_D} = 20{,}000$
- Controlling nondimensional parameters

$$Fr = \frac{U}{Nh},\ Ro = \frac{U}{2\pi f_\mathrm{i} D},\ Bu = (\frac{Ro}{Fr})^2,\ Ex = \frac{U_\mathrm{t}}{2\pi f_\mathrm{t} D}$$

Froude number (stratification level)

Rossby Number (rotating rate)

Burger number (geostrophy)

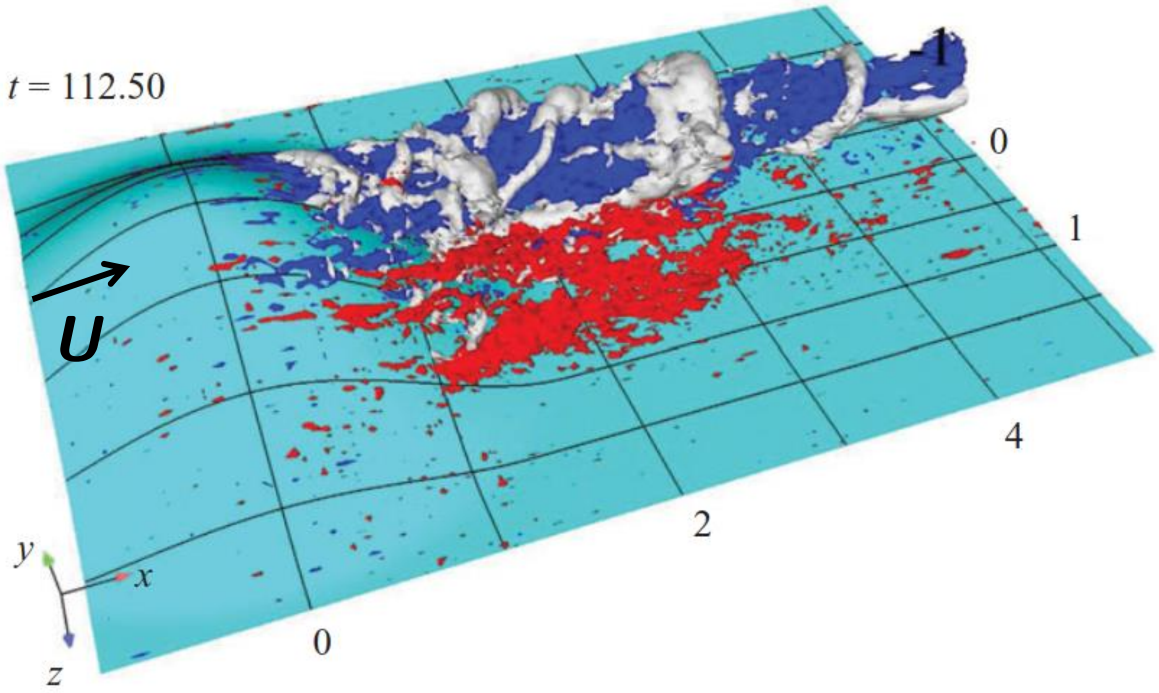
Excursion number (tidal stength)

Case	Bu	Ex	Ro	ΔT	Fr	(N_x, N_y, N_z)	N
Bu1	1		0.15				
BuK	1300	0	5.5	$310D/U_\mathrm{c}$			
BuInf	∞		∞		0.15	(1536,1280,322)	4000
Ex025	1300	0.25	5.5	$128T_\mathrm{t}$			
Ex050	1300	0.50					

Parameter table of simulations.

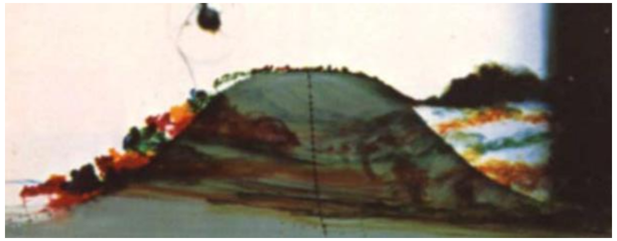
Lee vortices: a distinctive flow feature in stratified wakes

LES of an **unstratified** flow **over** a 3-D hill



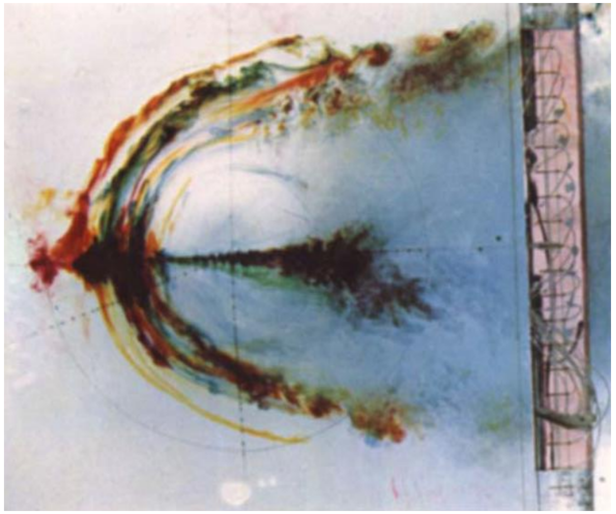
Garcia-Villalba et al. *JFM*, 2009

Experiment of a **stratified** flow **around** a 3-D hill



side

$\overrightarrow{U}$



top

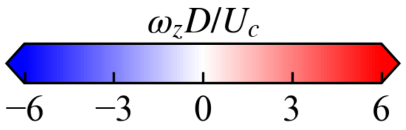
Hunt & Snyder, *JFM*, 1980

Strong rotation

no rotation

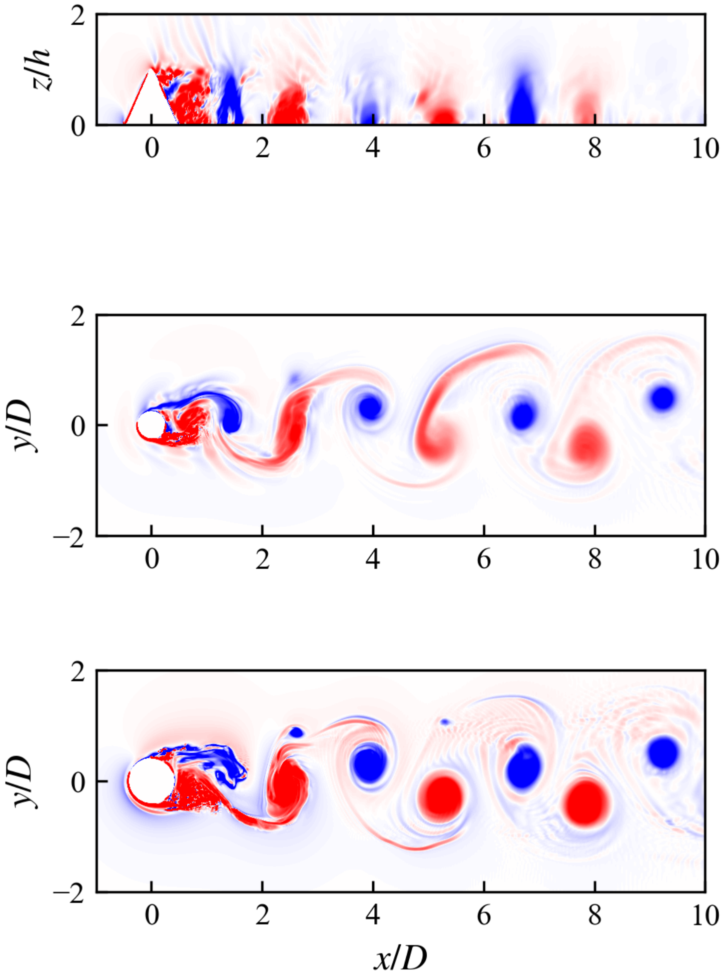
Case	Bu	Ex	Ro	ΔT	Fr	(N_x, N_y, N_z)	N
Bu1	1		0.15				
BuK	1300	0	5.5	$310D/U_c$	0.15	(1536,1280,322)	4000
BuInf	∞		∞				
Ex025	1300	0.25	5.5	$128T_t$			
Ex050	1300	0.50					

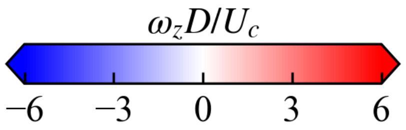
Part-I: The effect of rotation



Strong rotation

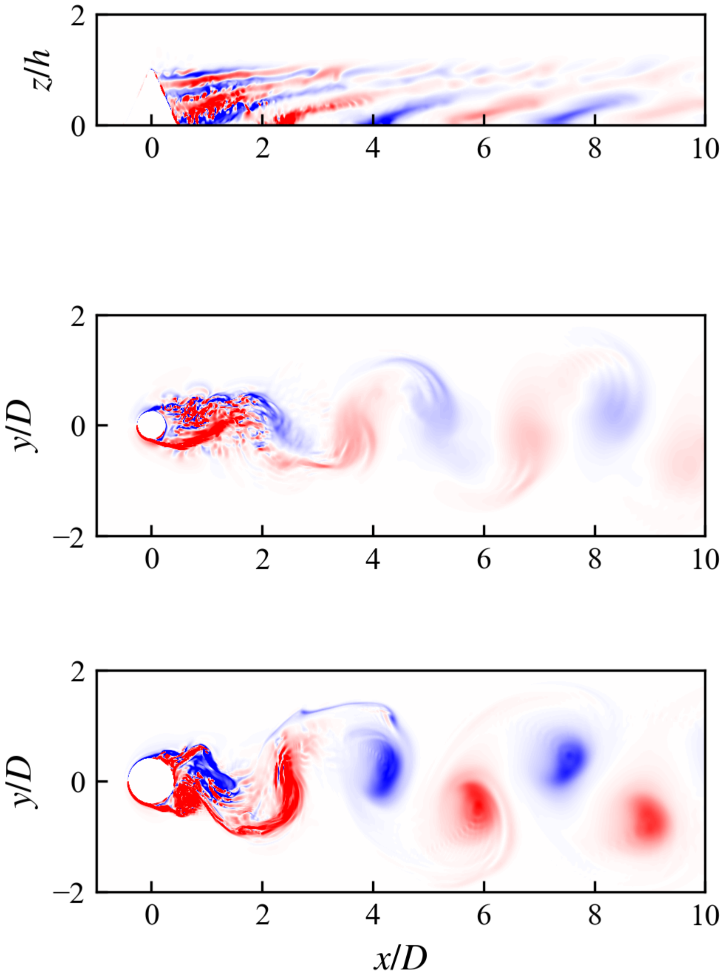
$T = 120.8\ U_c/D$





no rotation

$T = 72.6\ U_c/D$



vertical: y=0

horizontal: z/h=0.25

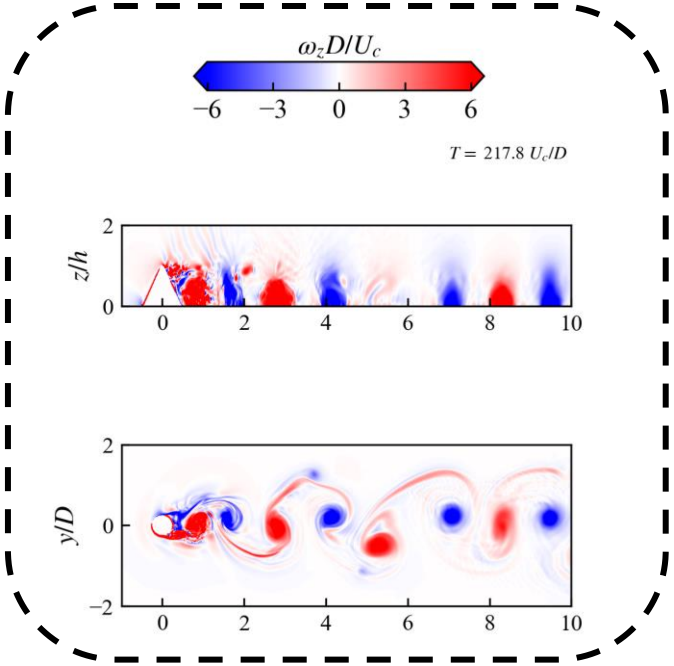
horizontal: z/h=0.50

$Fr = 0.15,\ Ro = 0.15, Bu = 1$

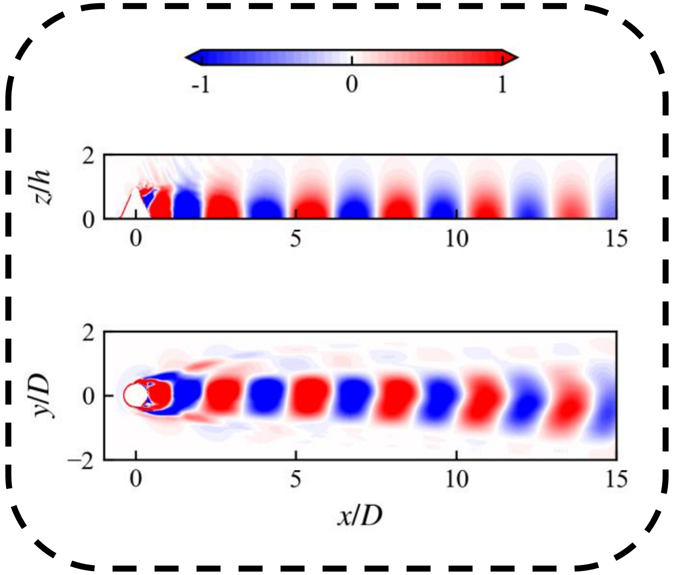
$Fr = 0.15,\ Ro = \infty, Bu = \infty$

An overview of SPOD

- Motivations of doing SPOD on ω_z
  - Extract the features of the vortical motion **in a statistical way**
  - Identify characteristic length/time scales



Case: Bu1



- SPOD problem formulation[1]

$$\mathbf{C}(\mathbf{x},\mathbf{x}',\tau) = \langle \mathbf{q}(\mathbf{x},\mathbf{t})\mathbf{q}^*(\mathbf{x}',\mathbf{t}+\tau)\rangle$$
$$\mathbf{S}(\mathbf{x},\mathbf{x}',f) = \int_{-\infty}^{\infty} \mathbf{C}(\mathbf{x},\mathbf{x}',\tau)\exp(i2\pi f\tau)\,\mathrm{d}\tau$$
$$\int_\Omega \mathbf{S}(\mathbf{x},\mathbf{x}',f)\boldsymbol{\psi}(\mathbf{x}',f)\,\mathrm{d}\mathbf{x}' = \lambda(f')\boldsymbol{\psi}(\mathbf{x},f')$$
$$\mathbf{S}(\mathbf{x},\mathbf{x}',f) = \sum_{j=1}^{\infty} \lambda_j(f)\boldsymbol{\psi}(\mathbf{x},f)\boldsymbol{\psi}^*(\mathbf{x}',f)$$

- Numerical implementation[2]

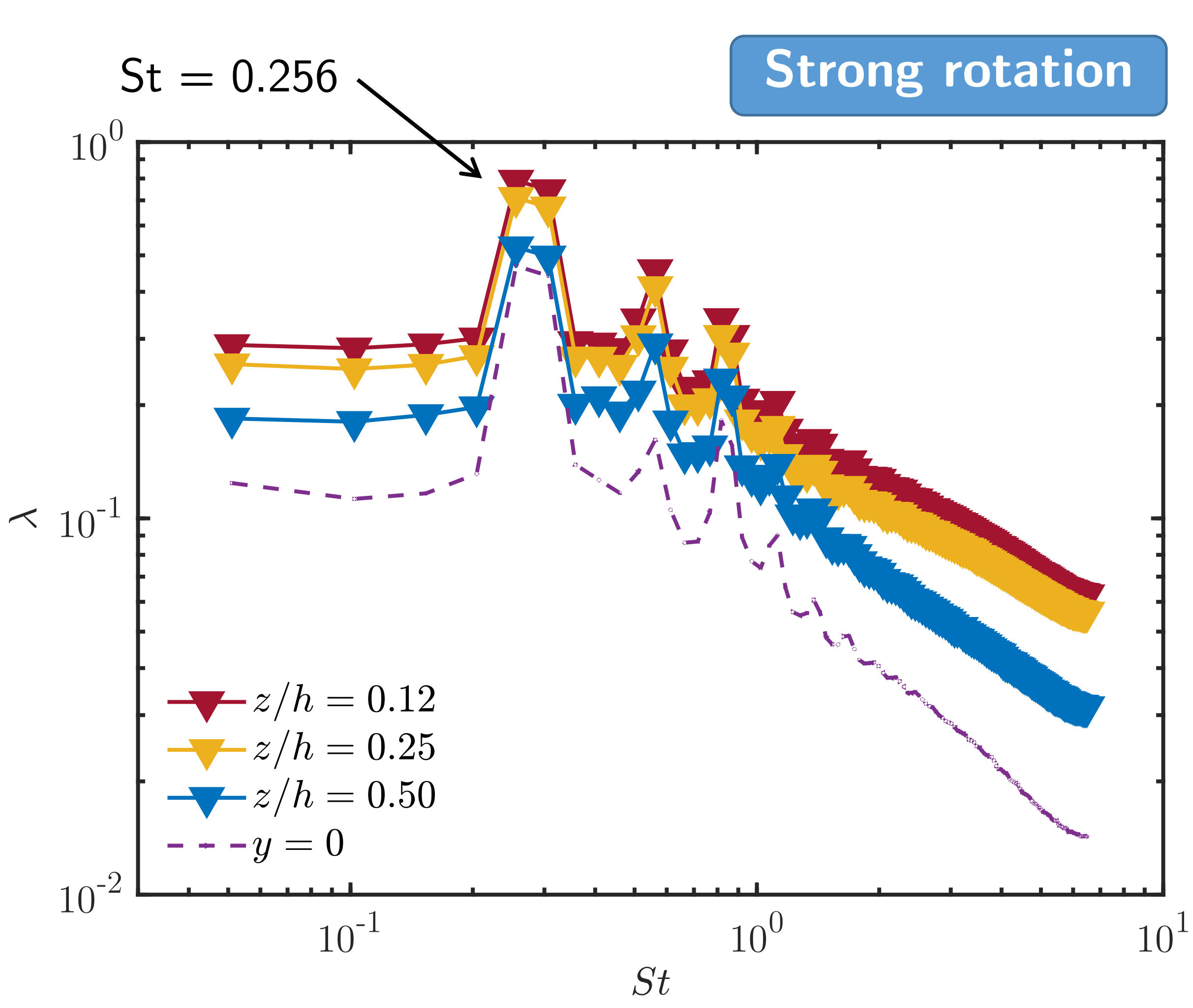
$$\hat{C} = \langle \hat{Q}\hat{Q}^H\rangle,\ \hat{Q} \in \mathbb{C}^{m\times n}$$
$$\hat{C}\hat{\Phi} = \hat{\Phi}\hat{\Sigma}$$
$$\hat{\Phi} = \hat{Q}\hat{\Psi}$$
$$\hat{\Phi}^H\hat{\Phi} = I$$

Strong rotation

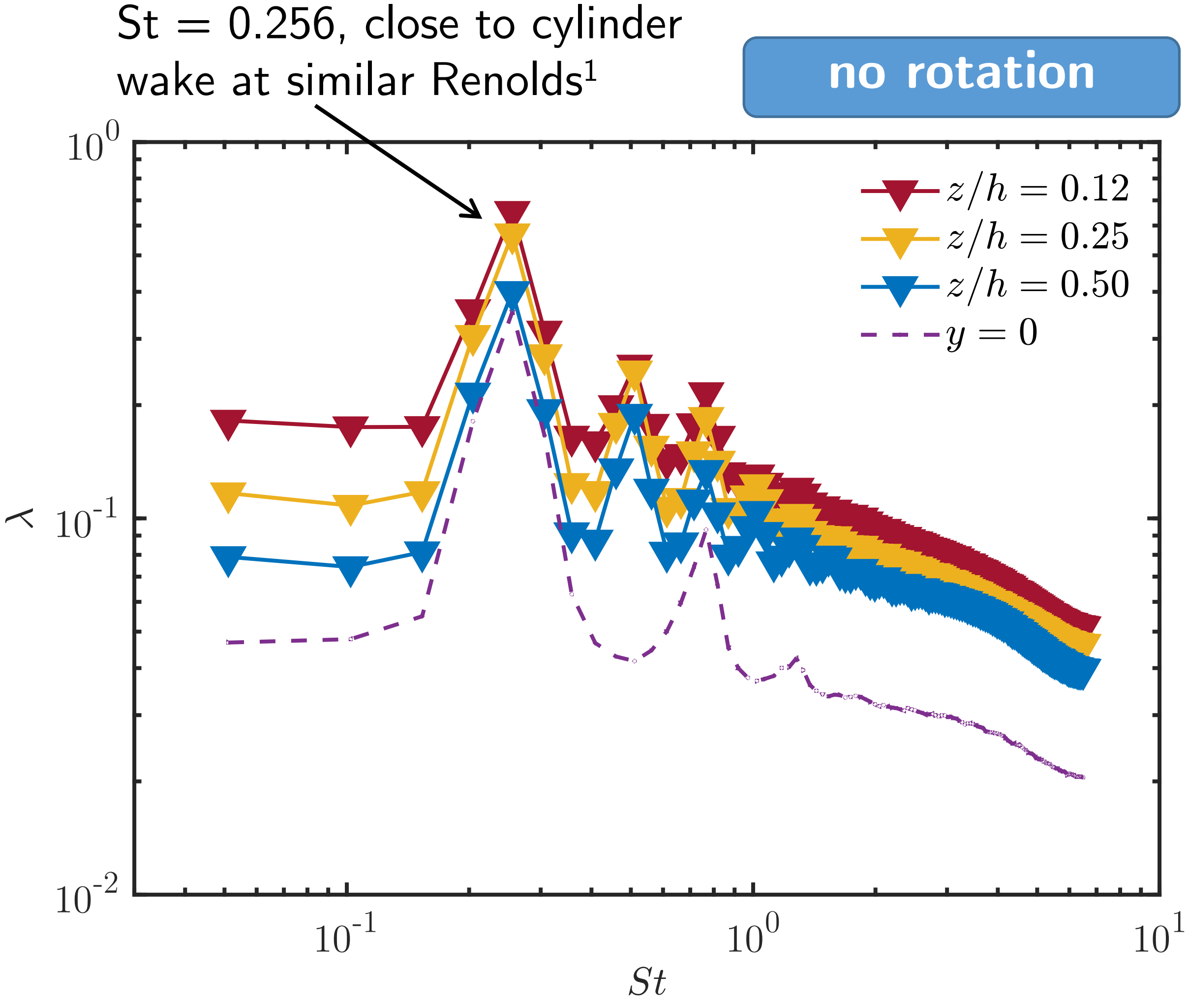
[1]Towne et al, *JFM*, 2018; [2]Schmidt & Colonius, *AIAA Journal*, 2020

SPOD eigenspectra, with a nearly global Strouhal number

$$St = \frac{fD}{U_c}$$



$Fr = 0.15,\ Ro = 0.15, Bu = 1$

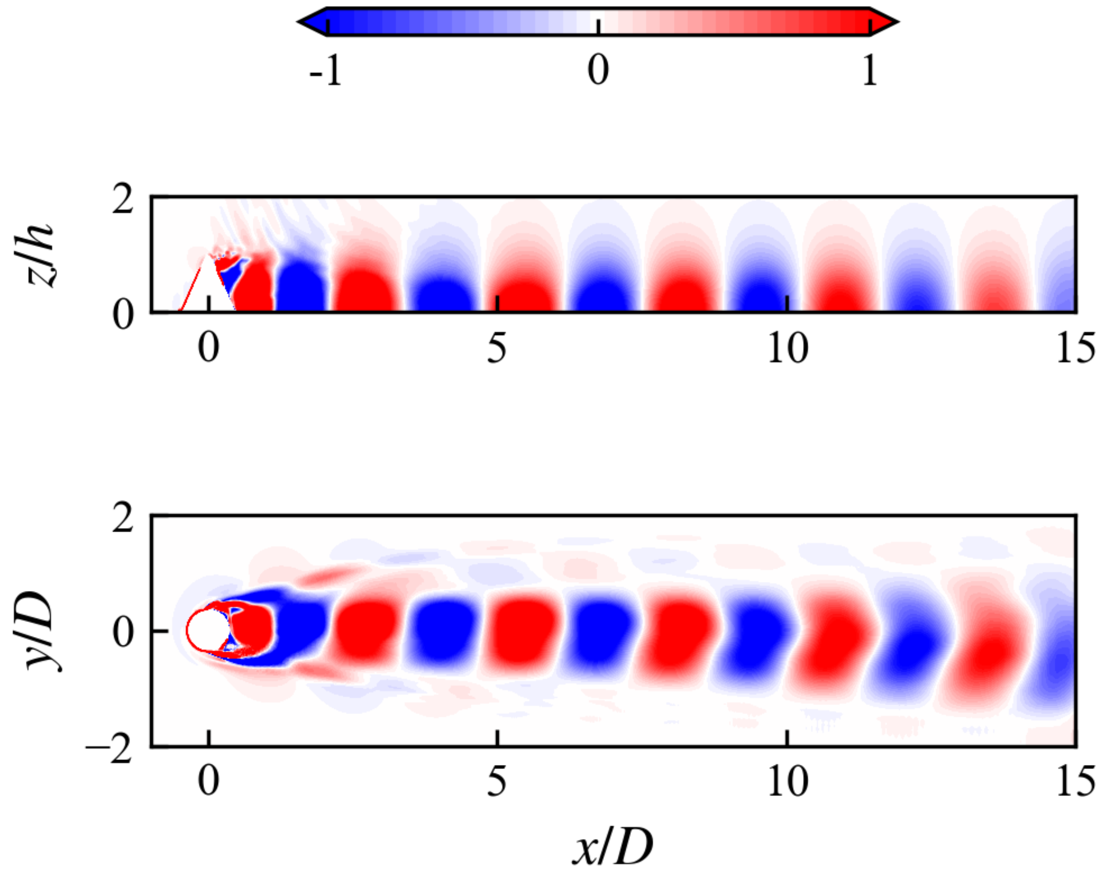


$Fr = 0.15,\ Ro = \infty, Bu = \infty, 1000$

[1]Williamson & Brown, *Journal of Fluids and Structures*, 1998

Leading SPOD eigenmodes (shedding frequency)

Strong rotation

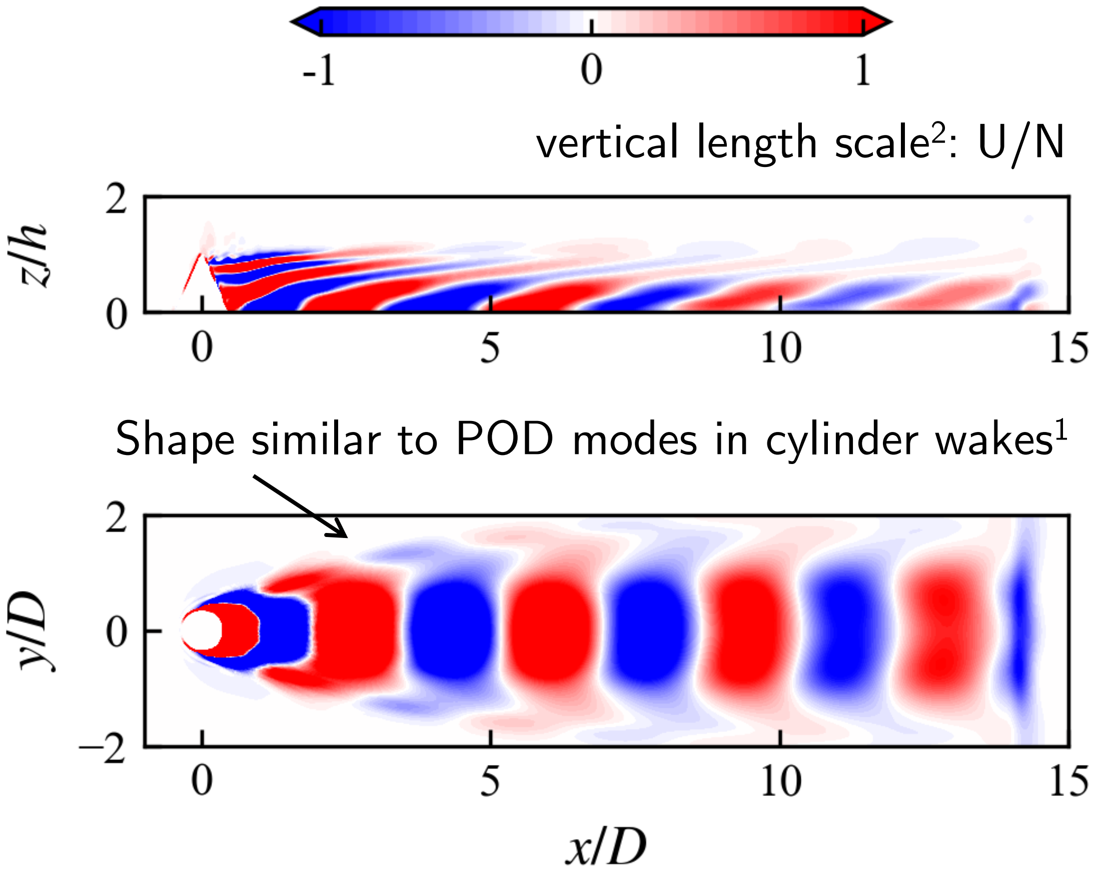


$Fr = 0.15,\ Ro = 0.15, Bu = 1$

no rotation

vertical length scale[2]: U/N

Shape similar to POD modes in cylinder wakes[1]

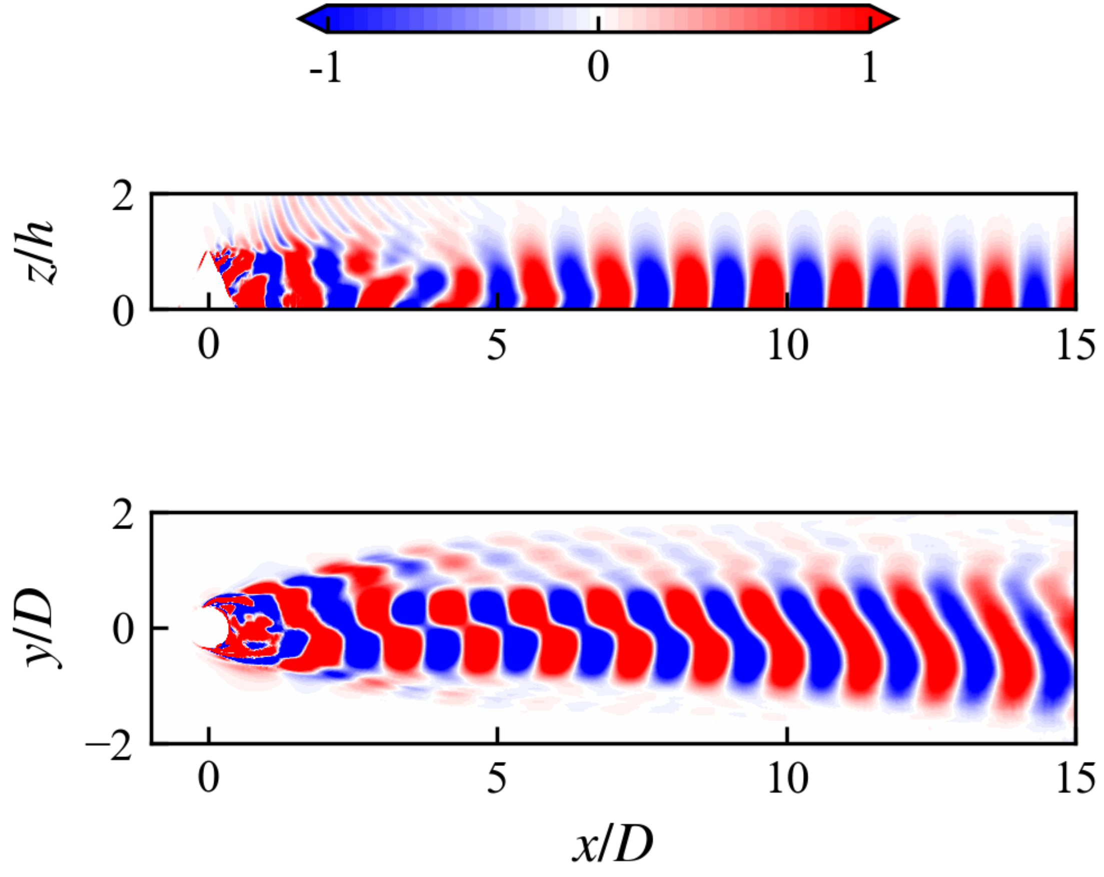


$Fr = 0.15,\ Ro = \infty, Bu = \infty, 1000$

[1]Gunes, Sirisup, and Karniadakis, *JCP*, 2016; [2]Billant & Chomaz, *PoF*, 2001

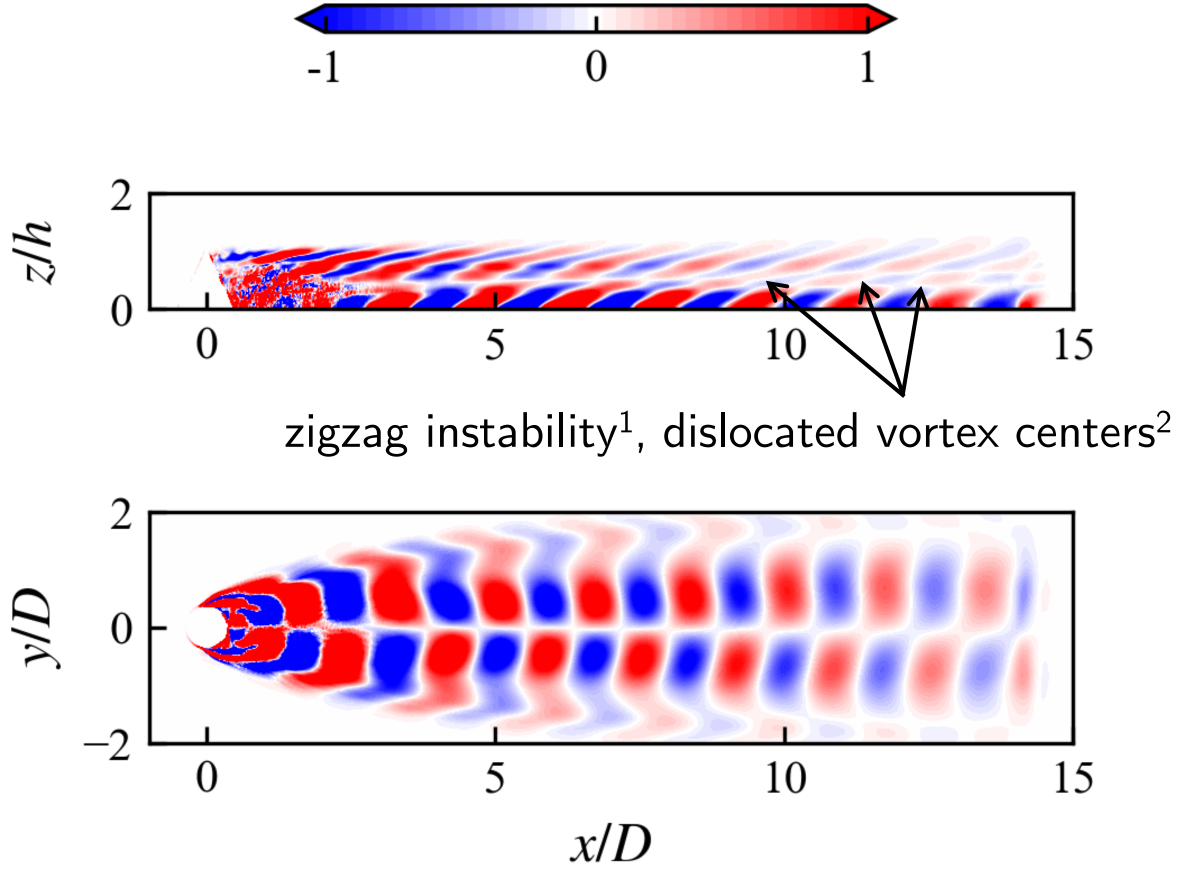
2nd leading SPOD eigenmodes (twice the shedding frequency)

Strong rotation



$Fr = 0.15,\ Ro = 0.15, Bu = 1$

no rotation



$Fr = 0.15,\ Ro = \infty, Bu = \infty, 1000$

no tide

moderate tide

Case	Ex	Ro	ΔT	Fr	(N_x, N_y, N_z)	N
Bu1		0.15				
BuK	0	5.5	$310D/U_c$			
BuInf		∞		0.15	(1536,1280,322)	4000
Ex025	0.25					
		5.5	$128T_t$			
Ex050	0.50					

Part-II: The effect of tide

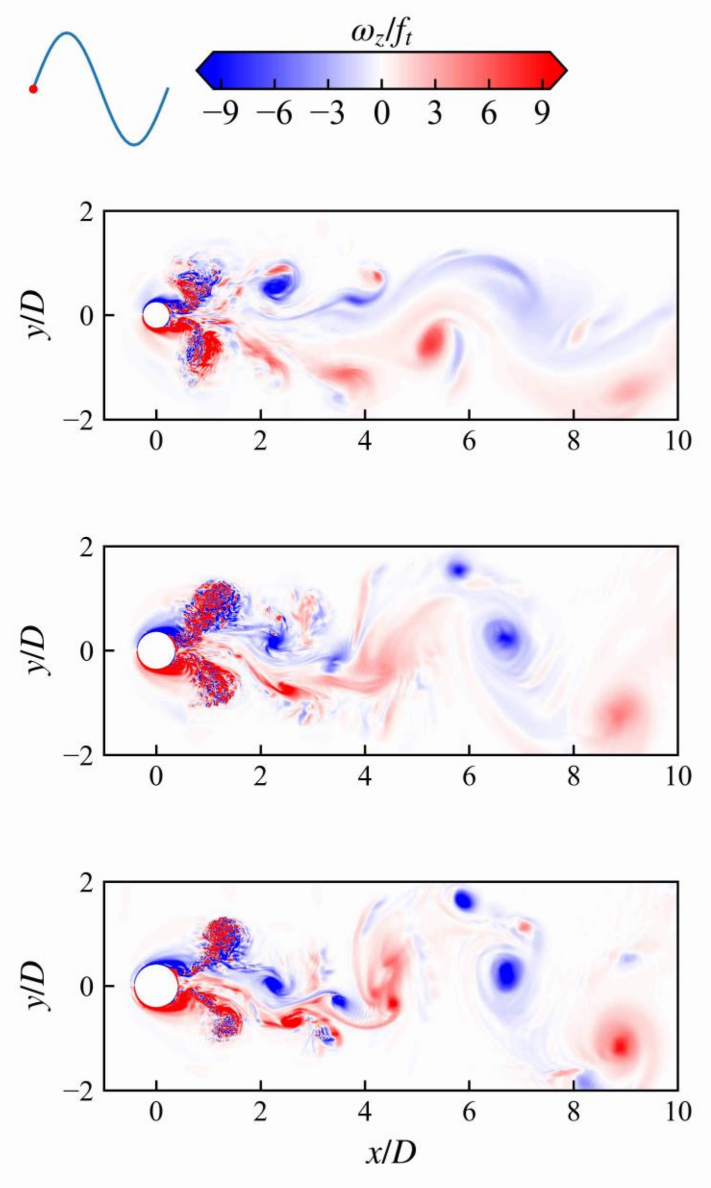
Near and far wake decomposition

Instantaneous

z/h=0.12

z/h=0.25

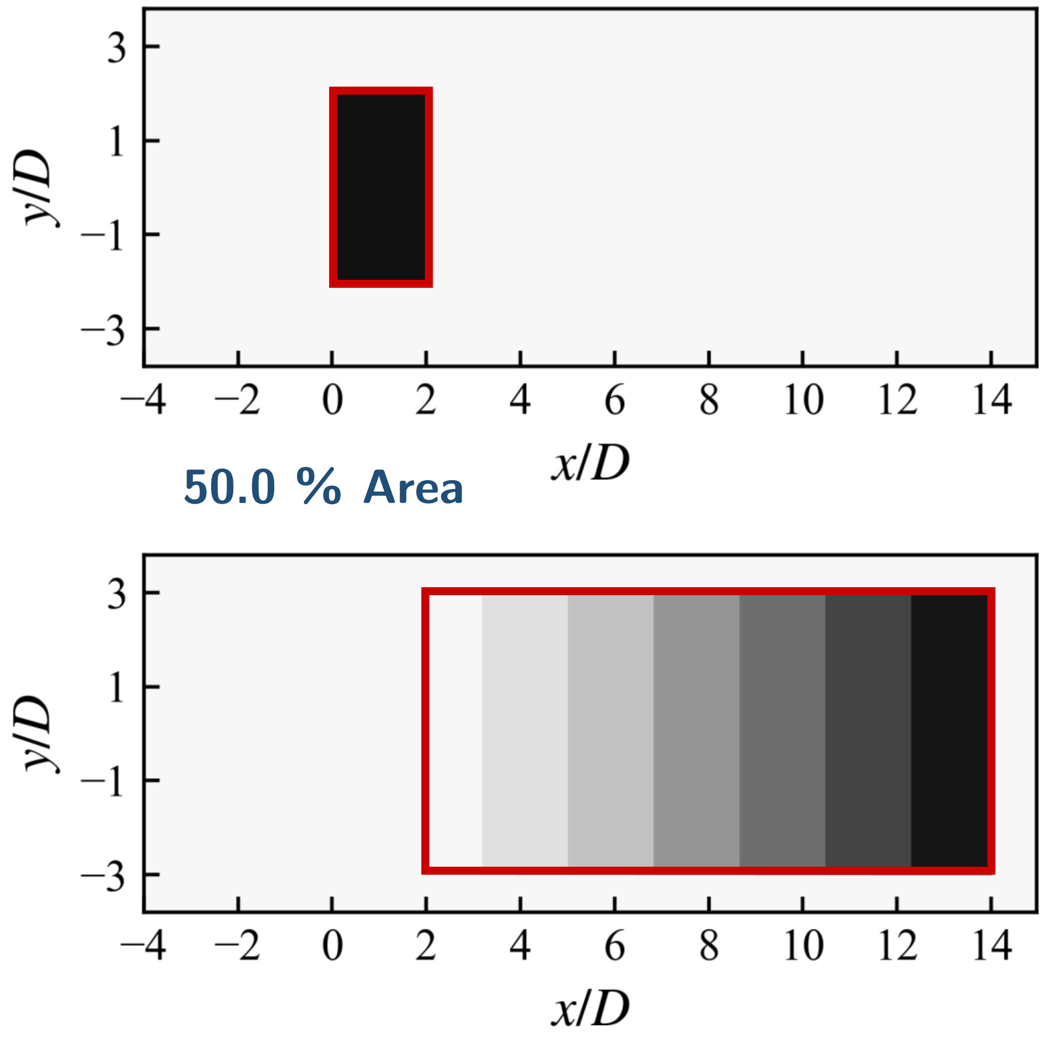
z/h=0.50



Schematic of region division

5.5 % Area

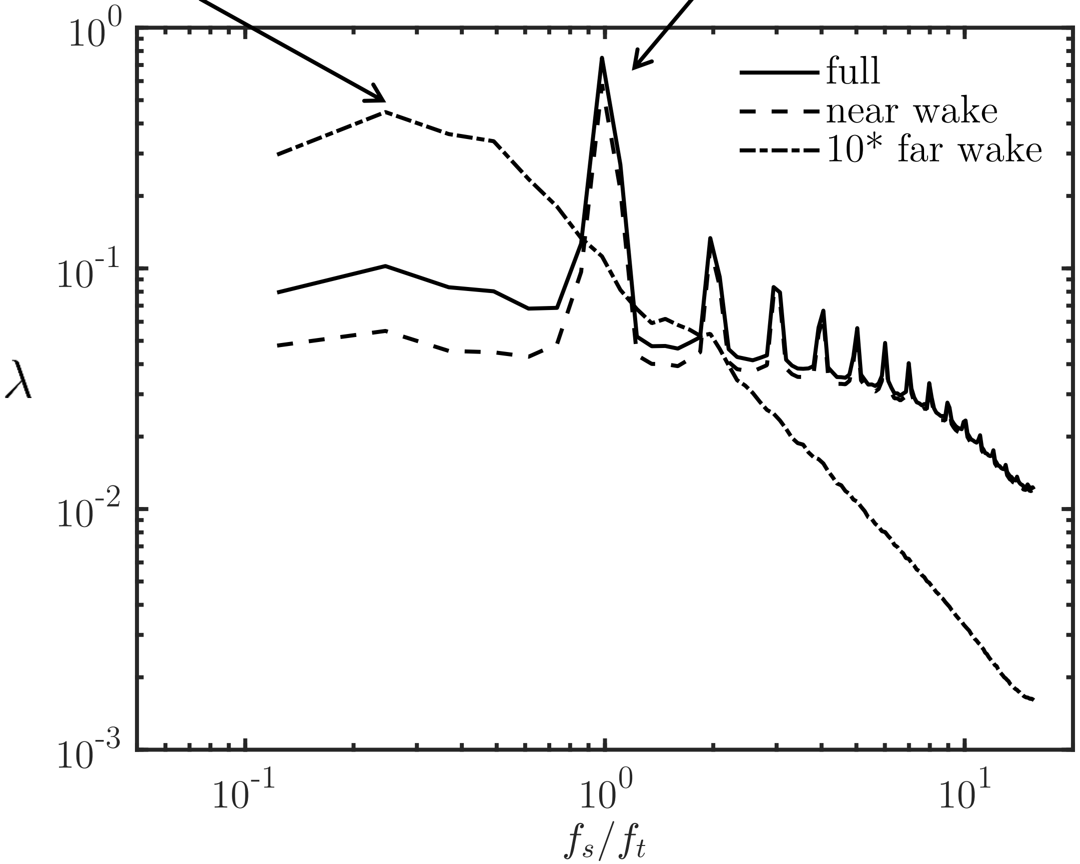
50.0 % Area



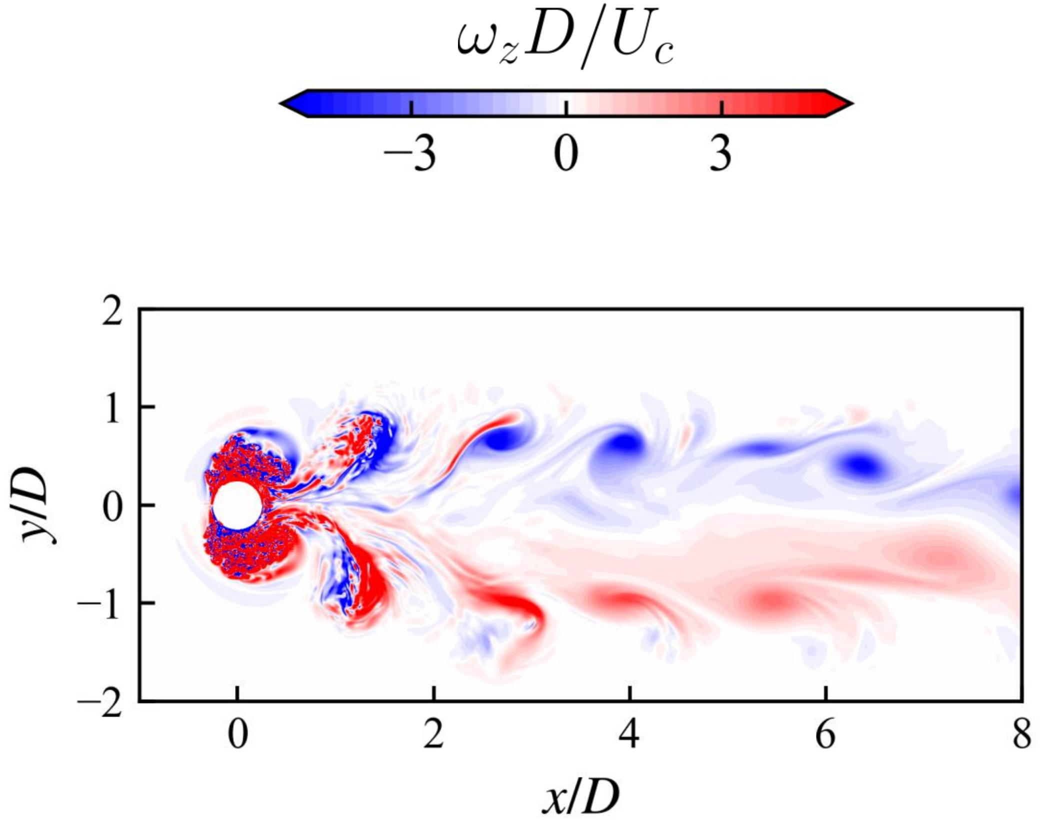
SPOD eigenspectra, influence of tide

Far wake: tidal sychronization[1] to subharmonics $f_t/4$

Near wake: tide dominates



(location z/h = 0.12)



[1]Puthan et al. *Geophysical Research Letter*, 2021.

Summary

- **Part-I: the effect of rotation**
  - alters the vertical structure of vortex shedding
    - the flow lost vertical alignment with weak rotation
  - preserves the vortex shedding frequency
- **Part-II: the effect of tide**
  - dominates the vortex shedding in the near wake
  - synchronizes the far wake into its subharmonic
- **Future work**
  - varying Fr number: vertical length scaling

Thank you!

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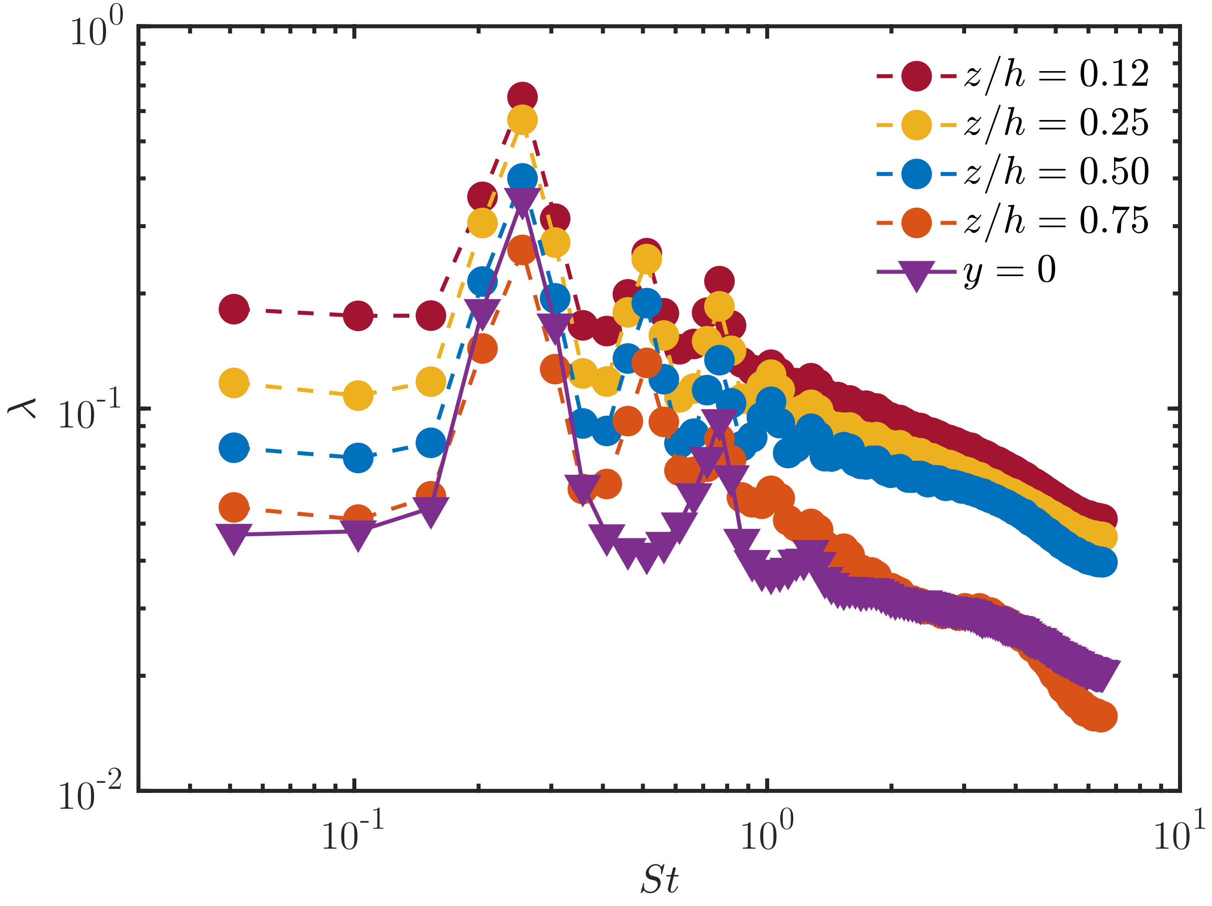
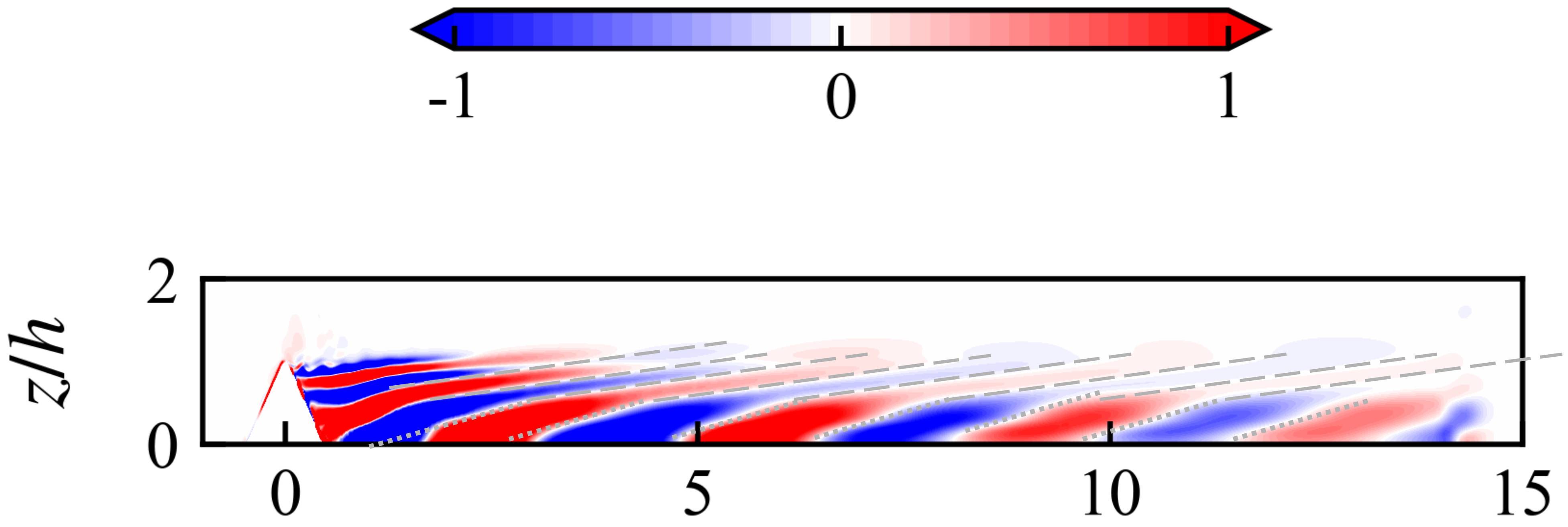
Appendix

SPOD eigenspectra for case BuInf

$$Fr = 0.15,\ Ro = \infty$$



slopes of vertical structures, for case BuInf

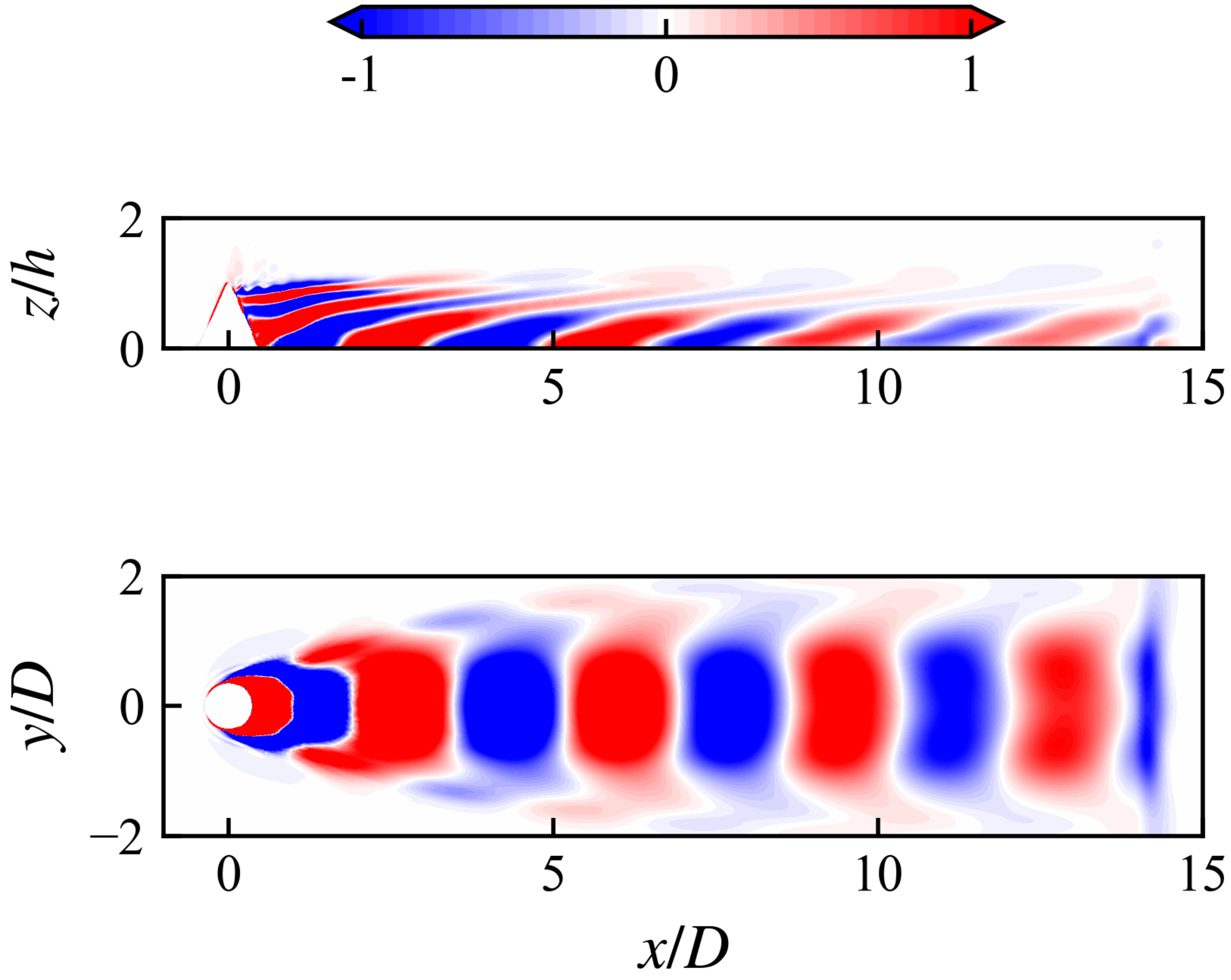


same slope as in BuK (Ro=5.5), and is not changing downstream

mode 1, case BuInf

$Fr = 0.15, \; Ro = \infty$

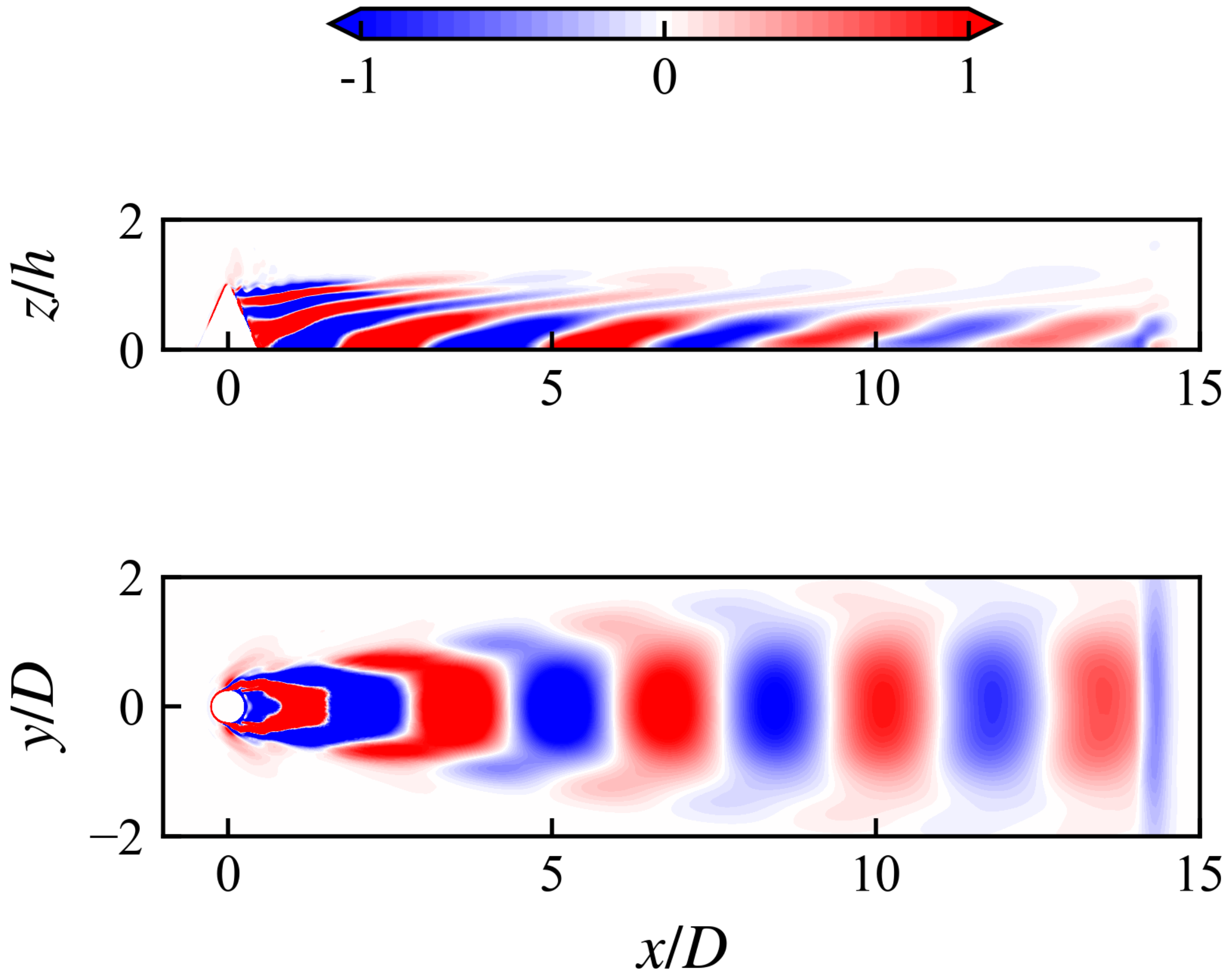
z/h = 0.25



mode 1, case BuInf

$Fr = 0.15,\ Ro = \infty$

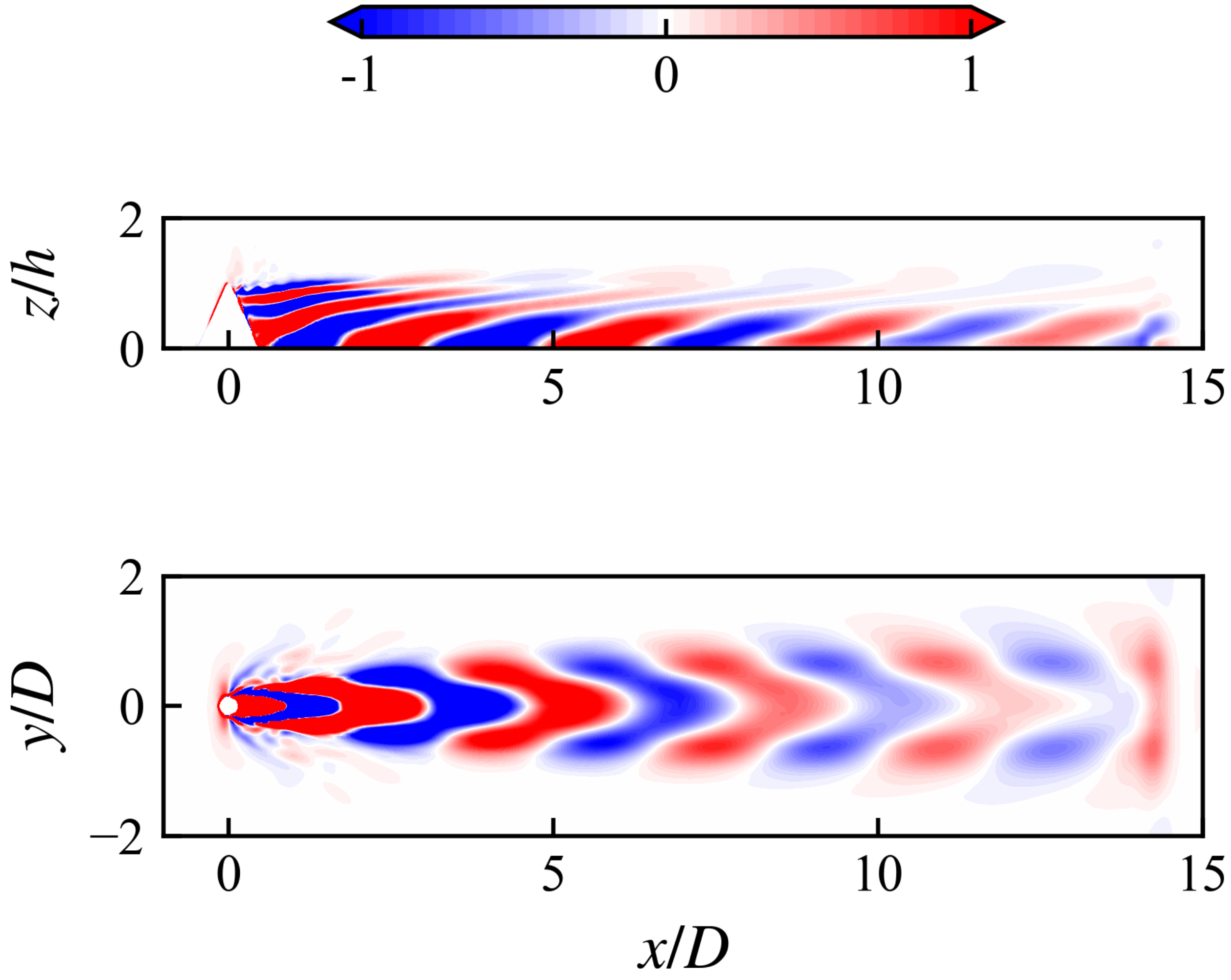
z/h = 0.50



mode 1, case BuInf

$Fr = 0.15,\ Ro = \infty$

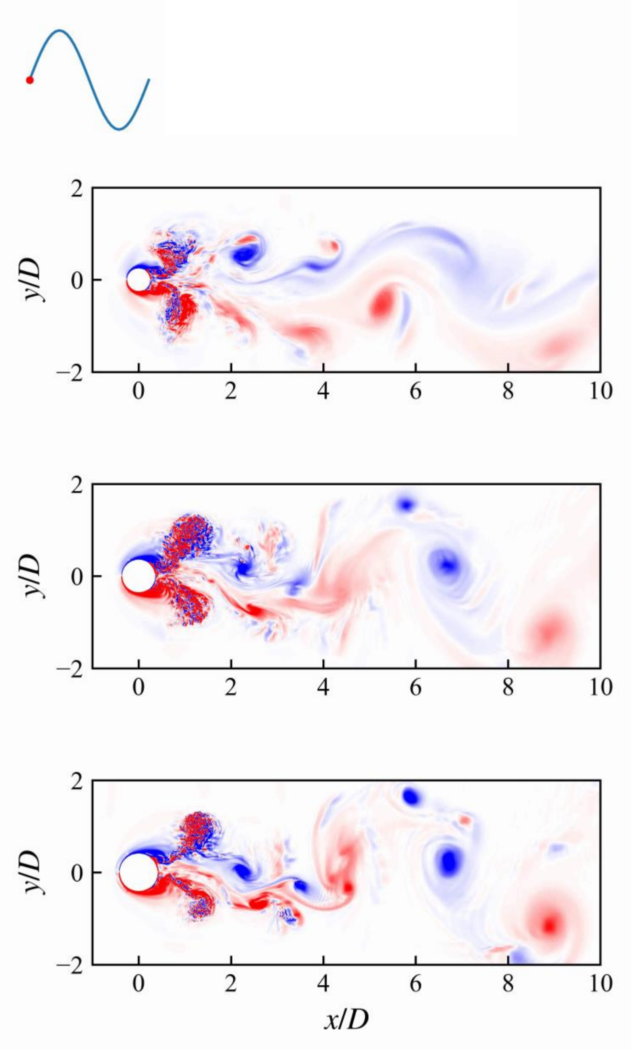
z/h = 0.75



Phase-decomposition of the tidally modulated case

Instantaneous

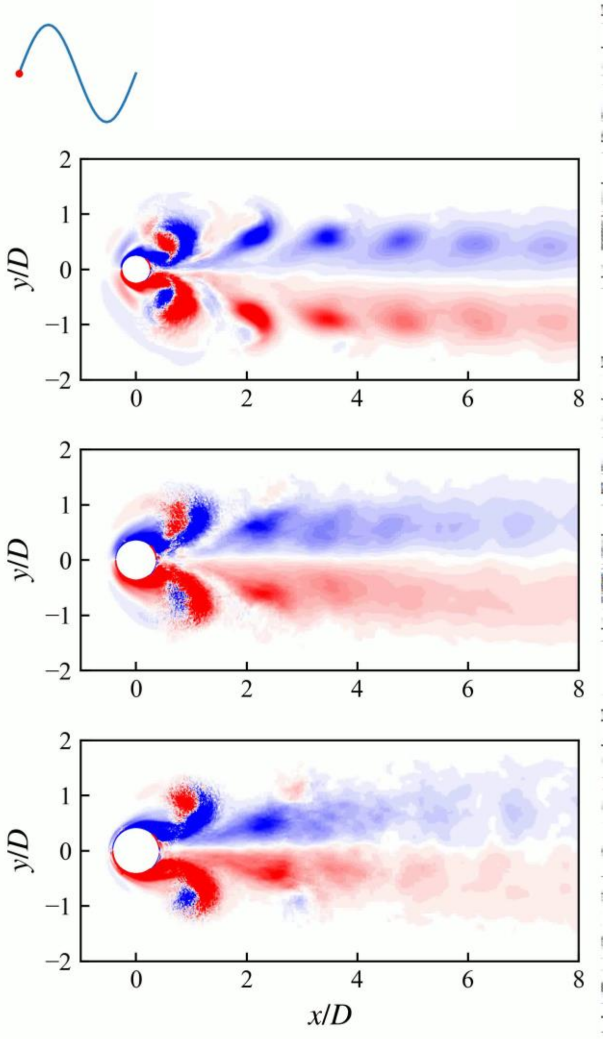
Phase-coherent (phase-average flow)



z/h=0.50

z/h=0.25

z/h=0.12



Phase-decomposition of unsteady flows

- Statistically unsteady flow:

$$U_b = U_c + U_t \cos(2\pi f_t t)$$

- Phase-average[1]: a realization of triple decomposition w.r.t. a certain phase (frequency)

$$\mathbf{u}(\mathbf{x}, t) = \underbrace{\mathbf{u}_{\mathrm{B}}(\mathbf{x})}_{\text{base flow}} + \underbrace{\mathbf{u}_{\mathrm{pc}}(\mathbf{x}, t)}_{\text{phase coherent}} + \underbrace{\mathbf{u}_{\mathrm{res}}(\mathbf{x}, t)}_{\text{residual}}$$

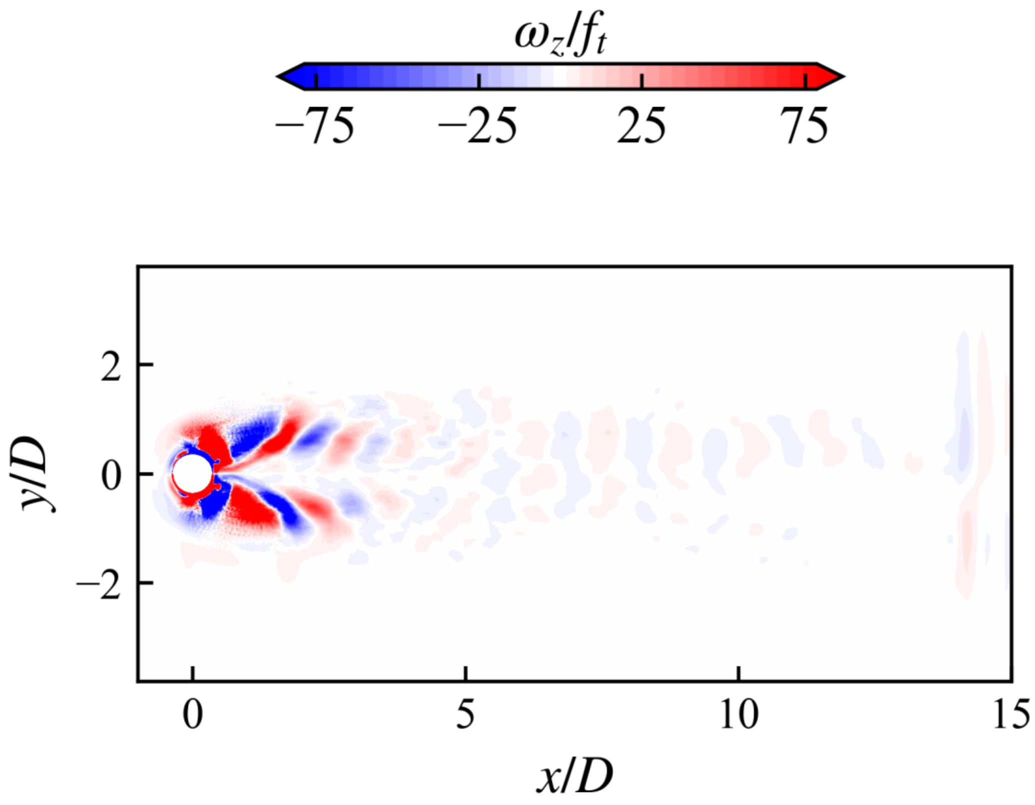
  - a kind of conditional average: $(x, y, z, t) \rightarrow (x, y, z, \phi)$

$$t \in [0, T_{\mathrm{sim}}],\ \phi \in [0, 2\pi]$$

[1]Hussain & Reynolds, *JFM*, 1970

SPOD modes for the full domain -- i0038 (z/h=0.25)

first SPOD mode for f = f_t



first SPOD mode for f = 1/4 f_t