

Turbulence in stratified rotating topographic wakes

Jinyuan Liu^a, Pranav Puthan^a, and Sutanu Sarkar^{a,b}

^a*Mechanical and Aerospace Engineering, University of California San Diego, La Jolla, CA 92093, USA*

^b*Scripps Institution of Oceanography, La Jolla, CA 92037, USA*

Corresponding author: Sutanu Sarkar, ssarkar@ucsd.edu

ABSTRACT: Turbulence generation mechanisms in stratified, rotating flows past three-dimensional (3D) topography remain underexplored, particularly in submesoscale (SMS) regimes critical to geophysical applications. Using turbulence-resolving large-eddy simulations, we systematically dissect the interplay of stratification and rotation in governing the dynamics of wake turbulence. Our parametric study reveals that turbulent dissipation in the near wake is dominated by two distinct instabilities: (1) vertical shear-driven Kelvin-Helmholtz instability (KHI), amplified by oblique dislocation of Kármán vortices under strong stratification, and (2) centrifugal/inertial instability (CI), which peaks at intermediate rotation rates (Rossby number order unity, SMS regime) and relatively weaker stratification. Notably, strong rotation dampens vertical shear and weakens KHI-driven turbulence, while strong stratification imposes smaller vertical length scales that restricts CI-driven turbulence. By quantifying dissipation across a broad parameter space of stratification and rotation, predictive relationships between the environmental parameters and instability dominance is established. These findings highlight the regime dependence of instability mechanisms and may inform targeted observational campaigns and numerical models of oceanic and atmospheric wakes.

1. Introduction

Observations of the upper ocean have revealed complex turbulent wakes and shed vortices, e.g., from headlands and island chains ([Chang et al. 2013, 2019; MacKinnon et al. 2019; Zeiden et al. 2021; Merrifield et al. 2019; Wynne-Cattanach et al. 2022](#)). Submerged topography in the deep ocean is also replete with three-dimensional features. Individual seamounts in a chain and three-dimensional hills are associated with wake eddies, internal waves and turbulence. Even a ridge that is two-dimensional at the large scale has three-dimensional features at the submesoscale (1 to 10 km in the horizontal). It has been hypothesized that steep seamounts and hills act as stirring rods and constitute an important route to mixing since the energy of the incident current is a continuous reservoir of kinetic energy and the stratified fluid carried by it is a continuous reservoir of potential energy. But, our knowledge of turbulence and turbulent mixing by wakes in the deep ocean is severely limited by the scarcity of direct observations and the absence of non-hydrostatic large eddy simulations (LES) that can resolve these aspects of the flow. The overarching goal of high-fidelity numerical studies is to improve understanding and modeling of topographic wakes in the context of small-scale ocean turbulence and mixing, and the transport of water masses by large-scale coherent wake eddies.

Recent studies ([Johnston et al. 2019; MacKinnon et al. 2019; Rudnick et al. 2019](#)) during the Flow Encountering Abrupt Topography (FLEAT) initiative have shed light on the eddy field surrounding Palau Island which lies in the path of the North Equatorial Current (NEC). Cyclonic and anti-cyclonic eddies spanning a wide range of Rossby numbers ($Ro \approx 0.3\text{--}30$) are observed in the lee of the island with diameters ranging from 1–10 km (submesoscale, SMS) to several hundred km (mesoscale, MS), which are comparable to the width of the island.

Direct measurements of microstructure in the Palau wake from a glider survey ([St. Laurent et al. 2019](#)) show that turbulent dissipation is enhanced in two different bands within the 200 m upper ocean which includes strongly stratified thermocline waters. The elevated dissipation rate can be up to several orders of magnitude larger than the typical values observed in the stratified upper ocean layer ([St. Laurent et al. 2019; MacKinnon et al. 2019; Wijesekera et al. 2020; Wynne-Cattanach et al. 2022](#)), signifying the influence of the stirring role of the seamounts. Other notable examples of topographic wakes arise from interactions of the Kuroshio Current and the Gulf Stream with submerged topographies ([Chang et al. 2013; Gula et al. 2016](#)).

It has been suggested in past theoretical work that the elevated dissipation can be driven by different types of instabilities (Thomas et al. 2013), e.g., vertical shear instability (Kelvin-Helmholtz instability, KHI), SMS centrifugal/inertial instability (CI), and symmetric instability (SI). Examination of these instabilities has so far mainly considered the SMS eddies that are generated by baroclinically-unstable fronts. The role of these mechanisms in the dissipation of topographically-induced SMS eddies is poorly understood.

Numerical studies of topographic wakes include the utilization of the hydrostatic regional oceanic modeling system (ROMS), such as idealized, isolated topography (Dong et al. 2007; Perfect et al. 2018; Srinivasan et al. 2021) and realistic complex topography (Gula et al. 2016; Simmons et al. 2019), and the hydrostatic version of the MIT-GCM such as Liu and Chang (2018); Nagai et al. (2021); Inoue et al. (2024). While hydrostatic simulations of topographic wakes capture well the general dynamics and provide parameterized turbulence and mixing, the details of instabilities and turbulence are not resolved. Recently, LES has been used to study the dynamics of topographic wakes, and has led to findings such as tidal synchronization of eddy shedding frequency (Puthan et al. 2021), elevated drag coefficient due to tidally modulated vortices (Puthan et al. 2022) and coherent global modes in steady-current wakes (Liu et al. 2024). By applying such modeling to idealized three-dimensional topography whose non-dimensional parameters match the oceanic sites of interest, quantitative links between various metrics (turbulent dissipation, mixing, coherent vortex structures) and the governing non-dimensional parameters can be established, and further physical insights can be established, enabling applications to a broad range of three-dimensional topographic features in the ocean.

Motivated by the need to understand and model topographic wake turbulence, we employ a suite of high-resolution LES with the objective being the characterization of turbulence and mixing. The physical model is a three-dimensional (3D) conical topography sitting on the ocean bottom, as shown in figure 1 of Liu et al. (2024). The topography has a base diameter $D = 150\text{m}$, height $h = 150\text{m}$ and a slope approximately 30° . This setting represents a model of a steep topography, such as seamounts. The fluid is linearly stratified and the coordinate system rotates at a constant rate (f -plane). The flow impinging on the obstacle is a steady current and the focus of this work will be the comprehensive effects of stratification, rotation and 3D topography, and how they affect the

vortex dynamics and turbulent dissipation. Such simplifications allow a focus on wake turbulence without the loss of generality.

The rest of this paper is organized as follows: Section 2 describes the LES numerical setting and the parameter selection, whereby the strengths of stratification and rotation are systematically varied. Section 3 examines the characteristics of the KHI and the CI and the resulting turbulent dissipation. In Section 4, the combined effects of stratification and rotation are considered, and turbulent dissipation is parameterized. Section 5 provides a holistic summary and discussion of the results.

2. Numerical modeling

a. Large-eddy simulations

The flow is governed by the incompressible Navier–Stokes equations under Boussinesq approximation:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{f}_c \times \mathbf{u} = -\frac{1}{\rho_0} \nabla p^* + \nabla \cdot \boldsymbol{\tau} + b \mathbf{e}_z \quad (2)$$

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = \nabla \cdot \mathbf{J}_\rho \quad (3)$$

where the viscous stress and the scalar flux are

$$\boldsymbol{\tau} = (\nu + \nu_{\text{sgs}})(\nabla \mathbf{u} + (\nabla \mathbf{u})^T), \quad \mathbf{J}_\rho = (\kappa + \kappa_{\text{sgs}})\nabla \rho, \quad (4)$$

the buoyancy force is $b = -\rho^* g / \rho_0$, and the Coriolis force is $\mathbf{f}_c = f_c \mathbf{e}_z$ (f_c is a constant; f -plane). Here p^* and ρ^* are the deviations from the hydrostatic and geostrophic balances, and ν and κ are the molecular viscosity and diffusivity. The subscript ‘sgs’ denotes the sub-grid scale contributions to the momentum and scalar transport, and are modeled with the WALE (wall-adapted local eddy-viscosity; Nicoud and Ducros (1999)) method in an LES approach.

The non-dimensional Froude, Rossby, and Reynolds numbers,

$$Fr = \frac{U_\infty}{Nh}, \quad Ro = \frac{U_\infty}{f_c D}, \quad Re = \frac{U_\infty D}{\nu}, \quad (5)$$

are the key controlling parameters that will be systematically varied in this study. We opt to use a Southern Hemisphere rotation and f_c represents the absolute value of a negative Coriolis frequency.

The governing equations are solved with a finite-difference solver that has an immersed boundary formulation (Balaras 2004; Yang and Balaras 2006) to deal with topography. Second-order central differences on a staggered grid are used to discretize the spatial derivatives, and a third-order Runge–Kutta scheme is used for time advancement. A fractional step method is used to obtain time-accurate divergence-free velocity fields and the resulting three-dimensional pressure Poisson equation is solved with a direct method. This solver is generally validated in the simulation of unstratified and stratified turbulent wakes (Pal et al. 2017; Chongsiripinyo and Sarkar 2020). The setting is very similar to that of Liu et al. (2024), but the parameter space explored in the present work is substantially larger, and the focus is on turbulence and its generation mechanisms. More details of the numerical setup can be found in Appendix A.

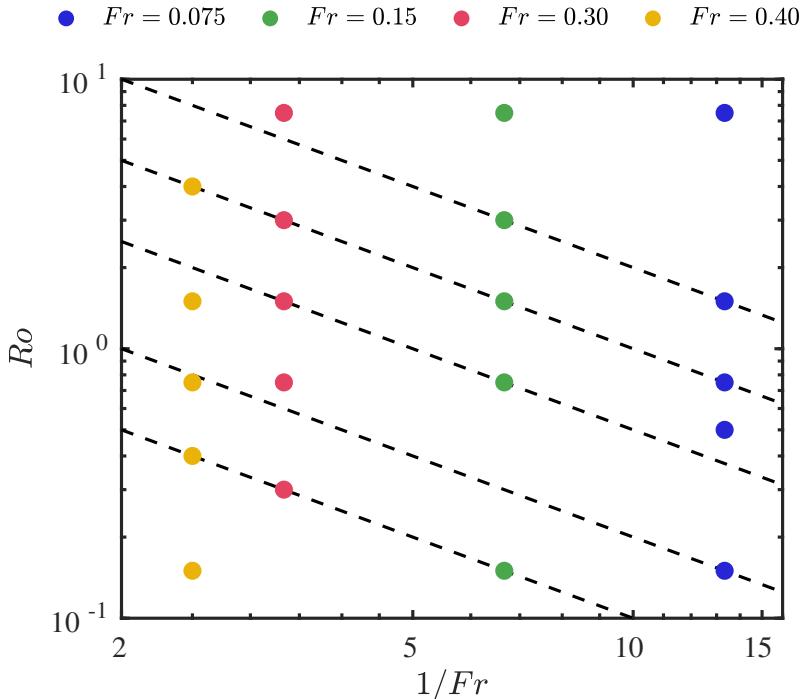


FIG. 1: Parameter space spanned by (Fr, Ro) . The horizontal axis is the inverse Froude number, Fr^{-1} , and the vertical axis is the Rossby number, Ro . Dashed lines represent constant Burger numbers, $Bu = (Ro/Fr)^2 = 1, 4, 25, 100, 400$, increasing from the lower left to the upper right. For each Fr , the four cases enclosed by $1 \leq Bu \leq 400$, will be analyzed in detail.

case	<i>Fr</i>	<i>Ro</i>	<i>Bu</i>	<i>Re</i>	<i>N</i> (s ⁻¹)	<i>f_c</i> (s ⁻¹)	<i>U_∞</i> (m s ⁻¹)	color
Fr007Ro015		0.15	4		9.33×10^{-4}	1.40×10^{-4}	1.05×10^{-2}	
Fr007Ro050		0.5	44		9.33×10^{-4}	4.20×10^{-5}	1.05×10^{-2}	
Fr007Ro075	0.075	0.75	100	40 000	9.33×10^{-4}	2.80×10^{-5}	1.05×10^{-2}	blue
Fr007Ro1p5		1.5	400		1.87×10^{-3}	2.80×10^{-5}	2.10×10^{-2}	
Fr007Ro7p5		7.5	1×10^4		N/A	N/A	N/A	
Fr015Ro015		0.15	1		4.67×10^{-4}	1.40×10^{-4}	1.05×10^{-2}	
Fr015Ro075		0.75	25		4.67×10^{-4}	2.80×10^{-5}	1.05×10^{-2}	
Fr015Ro1p5	0.15	1.5	100	20 000	9.33×10^{-4}	2.80×10^{-5}	2.10×10^{-2}	green
Fr015Ro3		3	400		1.87×10^{-3}	2.80×10^{-5}	4.20×10^{-2}	
Fr015Ro7p5		7.5	2.5×10^3		N/A	N/A	N/A	
Fr030Ro030		0.30	1		4.67×10^{-4}	1.40×10^{-4}	2.10×10^{-2}	
Fr030Ro075		0.75	6.25		1.17×10^{-3}	1.40×10^{-4}	5.25×10^{-2}	
Fr030Ro1p5	0.3	1.5	25	10 000	1.17×10^{-3}	7.00×10^{-5}	5.25×10^{-2}	red
Fr030Ro3		3	100		1.17×10^{-3}	3.50×10^{-5}	5.25×10^{-2}	
Fr030Ro7p5		7.5	625		N/A	N/A	N/A	
Fr040Ro015		0.15	0.14		N/A	N/A	N/A	
Fr040Ro040		0.40	1		4.67×10^{-4}	1.40×10^{-4}	2.80×10^{-2}	
Fr040Ro075	0.4	0.75	3.5	7 500	8.75×10^{-4}	1.40×10^{-4}	5.25×10^{-2}	yellow
Fr040Ro1p5		1.5	14		1.75×10^{-3}	1.40×10^{-4}	1.05×10^{-1}	
Fr040Ro4		4	100		1.46×10^{-3}	4.38×10^{-5}	8.75×10^{-2}	

TABLE 1: Parameters of the computational study which has 4 series corresponding to $Fr = 0.075$, 0.15, 0.30 and 0.40 with varying Ro within a series. Each series has a fixed combination of Fr and Re such that the stratification scale based Reynolds number, $Re_N = U_\infty^2 / \nu N = Fr Re(h/D)$, is a constant at $Re_N = 900$. Dimensional reference values for N , f_c and U_∞ are given for cases with $1 \leq Bu \leq 400$. The ranges of the buoyancy frequency and the Coriolis frequency are $0.5 \times 10^{-3} \text{ s}^{-1} \leq N \leq 0.5 \times 10^{-2} \text{ s}^{-1}$ and $2.5 \times 10^{-5} \leq f_c \leq 1.4 \times 10^{-4}$ (latitudes between 10° and 75°). The hill has base diameter $D = 500$ m and height $h = 150$ m.

b. Parameter selection

In order to cover a wide range of combinations of stratification and rotation, a parameter sweep is conducted in the Fr – Ro space, as shown in Table 1 and Fig. 1. The non-dimensional parameters are selected as follows.

Although the stratification in the deep ocean has seasonal and geographical variability, a buoyancy frequency of $N = 0.5 \times 10^{-3} \text{ s}^{-1}$ can be taken as representative. A mountain height of 1 km and a current speed between 0.2 m s^{-1} and 0.02 m s^{-1} , leads to Fr between 0.4 and 0.04. Accordingly, four values of Fr , from 0.075 to 0.40, that span half an order of magnitude, are selected. All cases lie in the flow-around regime (Hunt and Snyder 1980; Chomaz et al. 1993), and they represent

moderate to relatively strong stratification. The focus is on steep underwater topography that are known sites of elevated mixing and, accordingly, $h/D = 0.3$.

Regarding rotation, the SMS regime ($Ro = O(1)$ where rotation influences the flow but is not sufficiently strong to dominate the dynamics) has attracted much recent interest. For each Fr , we select an overall range of Ro from 0.15 to 7.5, which spans more than an order of magnitude, and is centered on the SMS while also including the limits of small MS and weakly rotating regimes.

The Fr - Ro parameter space also extends the scope of previous work employing ROMS. The work of [Perfect et al. \(2020\)](#) focused on large topographies in the MS with $Fr, Ro \sim O(0.01 - 0.1)$. Similarly, [Srinivasan et al. \(2021\)](#) studied strong rotation and strong stratification ($Fr = 0.02, 0.025 \leq Ro \leq 1$, in the present definition). In the limits of strongly rotating and/or stratified flows, the hydrostatic assumption in ROMS generally works well for the large-scale motions and both stratification and rotation act as stabilizing factors for smaller-scale motions ([Perfect et al. 2020](#)). When hydrodynamic instabilities and turbulence are of interest as is the case here, non-hydrostatic simulations are required to resolve them and, thus, elucidate their qualitative and quantitative dependence on (Fr, Ro) .

There is an additional (but not independent) non-dimensional number, the Burger number ($Bu = (Ro/Fr)^2 = (Nh/f_c D)^2$), commonly used to describe the relative importance of rotation to stratification in geophysical flows. When $Bu \leq O(1)$, rotation dominates and when $Bu > O(10)$, the effect of rotation is relatively small compared to the dominance of stratification. The range of Burger number of $1 \leq Bu \leq 400$, shown by the four diagonal dashed lines (each has a constant value of Bu) in Fig. 1, will be examined in detail.

The Reynolds number simulated should be sufficiently high so that the flow becomes turbulent and as many instabilities are triggered as possible. At $Fr = O(0.1)$, the regime studied here, the wake is in the flow-around vortex shedding regime, where the horizontal components dominate the turbulent kinetic energy (TKE). However, there are vertical structures and (oblique) dislocations (see Fig. 2) of these horizontal motions, leading to strong vertical shear and the potential for KHI. This scenario is a generic feature of vertically stratified horizontal shear flows ([Billant and Chomaz 2000; Basak and Sarkar 2006](#)) and such a turbulence generation mechanism is referred to as a shortcut in the transition ([Deloncle et al. 2008; Waite and Smolarkiewicz 2008](#)), since the flow is originally sheared in the horizontal directions.

The normal-mode KHI requires at least one point in the flow to have a gradient Richardson number less than 1/4 (Miles 1961; Howard 1961). Using the viscous length scale between dislocated layers measured by Basak and Sarkar (2006), approximately $l_d = 15\sqrt{\nu/N}$, and a velocity difference $2U_\infty$ between oppositely flapping vortex shedding, the resulting maximum vertical shear is $S_v = 2U_\infty/l_d$. In order to reach a marginal gradient Richardson number, $Ri_g = N^2/S_v^2 = 1/4$, a minimum Reynolds number of $Re_N = U_\infty^2/\nu N = 225$ is required.

Thus, $Re_N = Fr Re(h/D)$, instead of Re or Fr alone, serves as the *a priori* indicator of transition to turbulence through the KHI in the present wakes. It can also be interpreted as the Reynolds number of dislocated layers of vertical extent $\sim U_\infty/N$. For example, for $Fr = 0.15$, the constraint of $Re_N > 225$ requires a molecular Reynolds number of $Re > 5000$, which is higher than what many laboratory experiments and numerical simulations of the large-eddy and direct numerical simulation (LES and DNS) class have reached. We base the selection of the Reynolds number on such an estimate and choose a constant value of $Re_N = 900$, four times the estimated critical value to ensure turbulence. It will be shown *a posteriori* that KHI is active in all cases. The highest Reynolds number is up to $Re = 40000$ at $Fr = 0.075$. It is also noted that stronger stratification requires a larger Reynolds number to enable KH turbulence, and one would need to consider matching a lower Fr with a higher Re when designing laboratory or numerical experiments, if a similar dynamic range of turbulence is desired. A Reynolds number sensitivity study is provided in Appendix B to show that turbulent dissipation statistics approach Re -independence in the present simulations.

Each case in Table 1 is run for more than two flow-throughs to eliminate transient effects before data collection, which spans approximately five flow-throughs ($100D/U_\infty$ or 25 vortex shedding cycles).

3. Dissipation features and their link to flow instabilities

Topographic wakes are enriched by multi-scale and multi-physics interactions. Eddies whose sizes range from planetary scale, for example, MS of $O(100\text{km})$ to SMS of $O(0.1\text{-km})$ to turbulent eddies that are only a few centimeters in size evolve concurrently in the near and far wakes of the topography. The flow exhibits a Kármán shedding pattern in horizontal cross-sections with some vertical structures (Liu et al. 2024). In this work, we are concerned with wake turbulence, where

there are several physical mechanisms that can potentially lead to the destabilization of the flow and the breakdown into turbulence in the wake. In the vertical direction, intense shear (compared to the strength of stratification) can lead to KHI, while the source of shear varies from the dislocated vortex shedding at different elevations to the undulating lee wave near the apex of the obstacle. For the horizontal motions, anticyclonic vortices and shear are both subject to CI. These two key mechanisms are determined to be operative in wake turbulence later on and their parametric dependence is investigated.

a. Turbulence statistics and quantification

Definitions and notations that are used in the statistical analysis are as follows. An instantaneous signal is decomposed into time-averaged and fluctuation parts as

$$\varphi = \langle \varphi \rangle + \varphi', \quad (6)$$

with the bracket $\langle \cdot \rangle$ denoting time average over all available snapshots ($N_t \approx 375$ for 2D planes; $N_s \approx 30$ for 3D boxes), unless otherwise specified. The turbulent kinetic and potential energy (TKE and TPE) are

$$k = \frac{1}{2}(\langle u'^2 \rangle + \langle v'^2 \rangle + \langle w'^2 \rangle); k_\rho = \frac{1}{2N^2} \langle b'^2 \rangle, \quad (7)$$

and the instantaneous dissipation rates are

$$\varepsilon = (\nu + \nu_{\text{sgs}}) \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_i}{\partial x_j}; \varepsilon_\rho = \frac{1}{N^2} (\kappa + \kappa_{\text{sgs}}) \frac{\partial b'}{\partial x_j} \frac{\partial b'}{\partial x_j}, \quad (8)$$

whose time averages are the TKE and TPE dissipation, $\langle \varepsilon \rangle$ and $\langle \varepsilon_\rho \rangle$, respectively. Similarly, the instantaneous dissipation rates of the total kinetic and potential energy are denoted with tildes to distinguish from (8):

$$\tilde{\varepsilon} = (\nu + \nu_{\text{sgs}}) \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}; \tilde{\varepsilon}_\rho = \frac{1}{N^2} (\kappa + \kappa_{\text{sgs}}) \frac{\partial b}{\partial x_j} \frac{\partial b}{\partial x_j}. \quad (9)$$

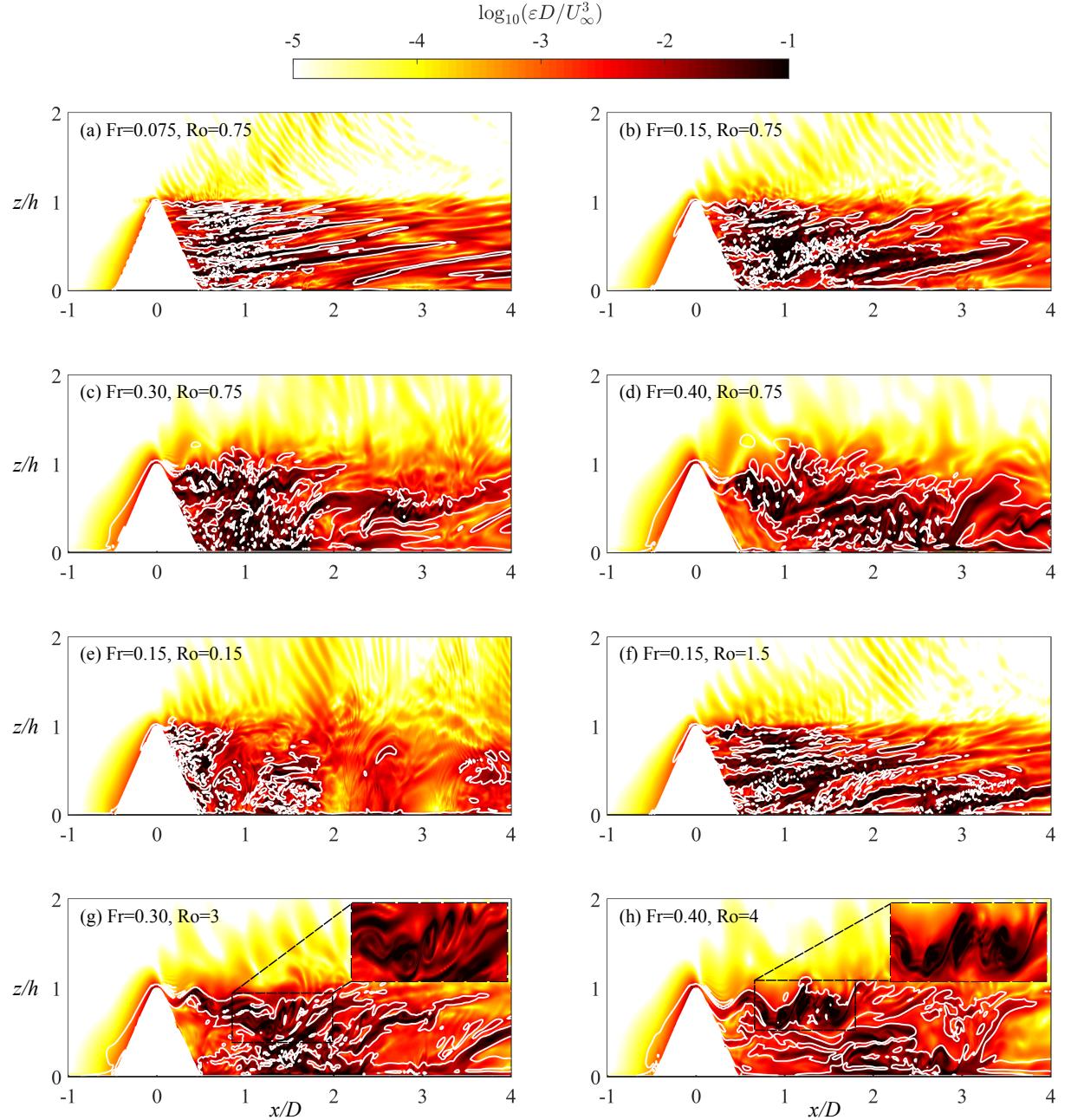


FIG. 2: Instantaneous dissipation rate ε in the center plane ($y = 0$), with white isolines of instantaneous values of $Ri_g = 1/4$ overlaid on the top. (a-d) $Fr = 0.075, 0.15, 0.30, 0.40$ and $Ro = 0.75$. (e,f) $Fr = 0.15$, and $Ro = 0.15, 1.5$, respectively. (g) $Fr = 0.30, Ro = 3$. (h) $Fr = 0.40, Ro = 4$. The insets of (g,h) are enlarged views of the KH rollups due to the hydraulic jet.

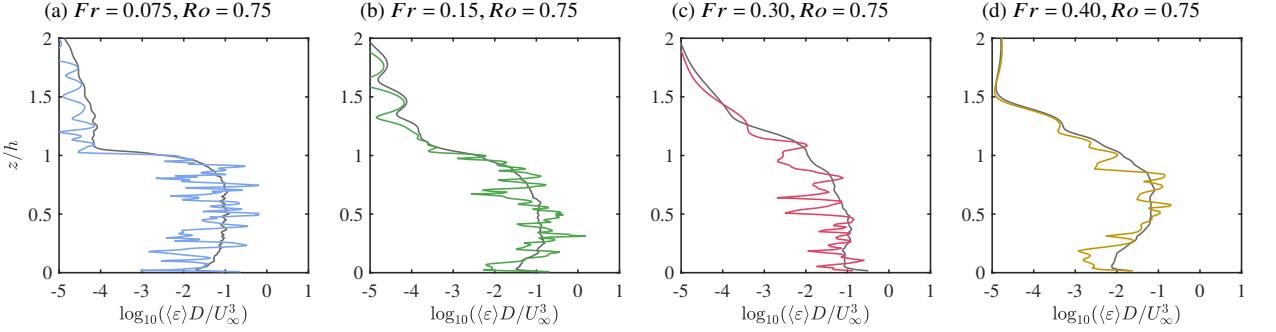


FIG. 3: Instantaneous dissipation rate ε (colored) and its time-averaged $\langle \varepsilon \rangle$ (TKE dissipation, gray), probed on the line $y = 0, x = 1$. Time instances are the same as those in Fig. 2(a-d).

b. Contribution of the vertical shear instability (KHI)

Figure 2 shows the normalized instantaneous TKE dissipation rate, $\varepsilon D / U_\infty^3$, enclosed by the white contours of the local gradient Richardson number indicator, $Ri_g = 1/4$, defined as

$$Ri_g = \frac{(\partial_z b)^2}{(\partial_z u)^2 + (\partial_z v)^2}. \quad (10)$$

Here u, v and b are instantaneous quantities. A value of $Ri_g = 1/4$ typically indicates a marginal instability state due to vertical shear. It is clear that the shear-unstable regions coincide with the strongest dissipation, suggesting that the KHI is active.

Comparing Figs. 2(a-d), which are at the same $Ro = 0.75$, it can be seen that strong localized dissipation has a similar magnitude at different Fr . Furthermore, as stratification increases (Fr decreases), the number of oblique layers increases and the thickness of the layers also reduces. These spatial structures of dissipation align well with the vortex structures in these wakes when rotation is not dominantly strong (Liu et al. 2024), which are indeed slanted 3D coherent structures instead of dislocated stacks of pancake vortices as previously suggested. Without the dominance of rotation, each individual vortex is better described by the tilted vortex model in Boulanger et al. (2007); Canals et al. (2009) instead of pancake vortices. Although tilted and pancake-shaped vortices are qualitatively different and each has distinct vorticity-density structures (Beckers et al. 2001; Basak and Sarkar 2006), they are both associated with intensified vertical shear due to the flow layering – a direct consequence of stratification.

Comparing Figs. 2(b,e,f), at $Fr = 0.15$ and various Ro , it can be seen that the SMS cases $Ro = 0.75, 1.5$ are similar while the MS case $Ro = 0.15$ shows fewer large- ε patches. At $Ro = 0.75, 1.5$, layers of tilted coherent vortex structures are shed and the shear instability serves as the main contributor to turbulence. When rotation is strong ($Ro = O(0.1)$), vertical gradients are significantly reduced and columnar vortices emerge which, further downstream, advect as stratified Taylor columns (Liu et al. 2024). Figure 2(e) shows turbulent dissipation associated with these columnar vortices, which is significantly weaker than the dissipation associated with the slanted layers, shown in (b,f). The contrast indicates that the form of the coherent structures, in turn, influenced by rotation, can also influence turbulence intensity. This might be regarded as the indirect effect of rotation on turbulence.

Another contributor to turbulence is the hydraulic jet that plunges below the mountain crest and also sets up a near-field internal wave response. The topmost portion of the mountain, from its crest to U_f/N below (Winters and Armi 2012), participates in the jet and the lee wave, whose dominant vertical wavelength is $2\pi U_m/N$ (Klymak et al. 2010). Here U_f and U_m are the freestream and mean velocity, respectively. The undulating jet and the adjacent wave region are found to be a dissipation hotspot that becomes more pronounced with increasing Ro and Fr of the examined cases. The flow in the hotspots breaks down through the KHI and forms rolled-up billows that are evident in the insets of Figs. 2(g-h), corresponding to Fr030Ro3 and Fr040Ro4.

Moreover, in all cases, there is turbulent dissipation due to the unsteady internal waves propagating into the background, as shown by the yellowish colors above the top of the hill, albeit smaller than the dissipation in the wake (note the logarithmic color scale in Fig. 2). While the wave dissipation represents a different process that is also important, wake dissipation will be the focus of this work. In the vertical center plane, TPE dissipation (not shown) has a spatial distribution that aligns well with TKE dissipation. Accompanying movies of ε for vertical center planes as shown in panels (a-d) can be found in the supplementary materials.

Figure 3 shows the vertical profile of instantaneous (at the same instances as in Fig. 2(a-d)) and time-averaged TKE dissipation at $x/D = 1, y = 0$. As stratification increases, the vertical length scale of the instantaneous dissipation patches (approximately $\propto U_\infty/N$) decreases, the spikes in their profiles increase in sharpness, and the number of spikes also increases. Quantitatively, the magnitude of time-averaged dissipation in the wake reaches $\langle \varepsilon \rangle \sim 10^{-1} U_\infty^3/D$, while the

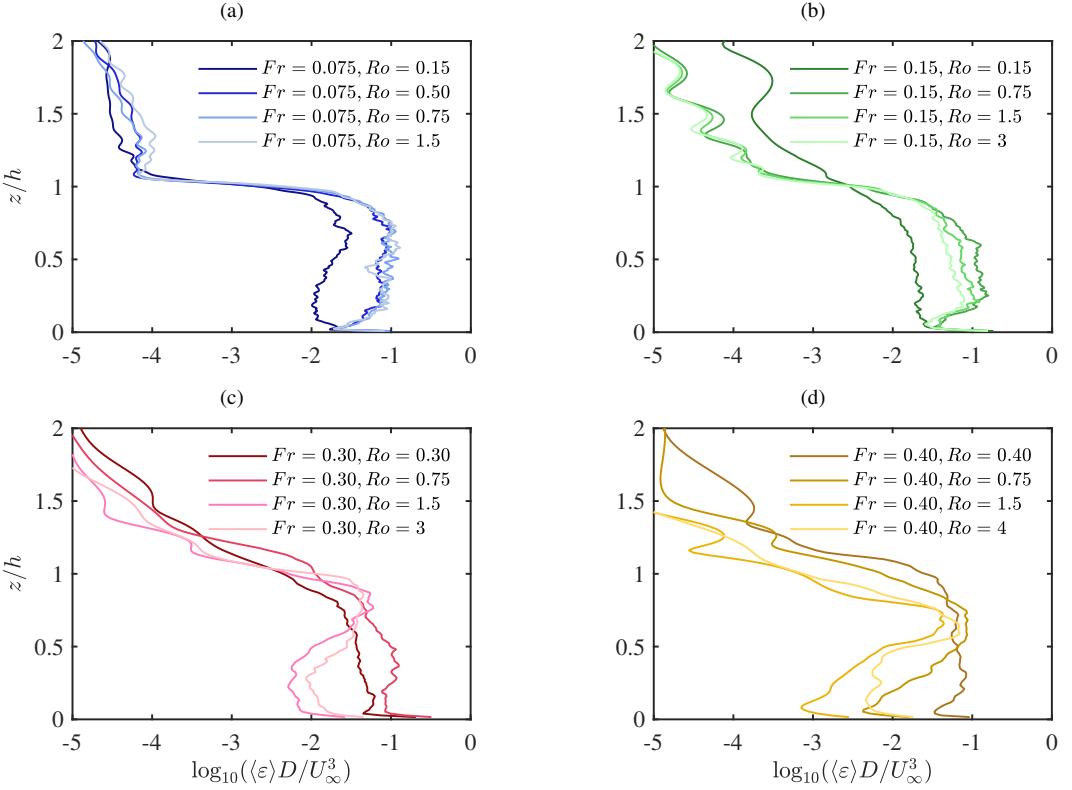


FIG. 4: TKE dissipation $\langle \varepsilon \rangle$ measured at $y = 0, x = 1$. (a) $Fr = 0.075$, (b) $Fr = 0.15$, (c) $Fr = 0.30$, and (d) $Fr = 0.40$.

instantaneous peak value could be an order of magnitude higher, as shown in Fig. 3 (a,b). Here, dissipation is represented in the inertial units, U_∞^3/D , which can be scaled up or down for varying current speeds and seamount dimensions, and is equivalent to $2 \times 10^{-6} \text{ W kg}^{-1}$ for $U_\infty = 0.1 \text{ m s}^{-1}$ and $D = 500 \text{ m}$. The magnitude $O(10^{-1}) U_\infty^3/D$ is consistent in different cases, while the dissipation in the ambient is around $10^{-5} \sim 10^{-4} (U_\infty^3/D)$, more than 1000 times lower. Appendix B shows that at the present Reynolds numbers, the dissipation is relatively independent of Re .

The dissipation obtained in the present simulations compares qualitatively and quantitatively well with observational data, in the wakes of Palau (MacKinnon et al. 2019; St. Laurent et al. 2019; Wijesekera et al. 2020; Wynne-Cattanach et al. 2022), the Green Island (Chang et al. 2013), and seamounts in the Tokara Strait (Nagai et al. 2021). Spatially localized dissipation sites and spiky vertical dissipation profiles were also observed in MacKinnon et al. (2019); Wynne-Cattanach et al. (2022). The *in situ* measured dissipation magnitude lies between $10^{-7} \sim 10^{-5} \text{ W kg}^{-1}$ in Chang et al. (2013), $10^{-6} \sim 10^{-4} \text{ W kg}^{-1}$ in Wijesekera et al. (2020), and $10^{-7} \sim 10^{-6} \text{ W kg}^{-1}$ in Nagai

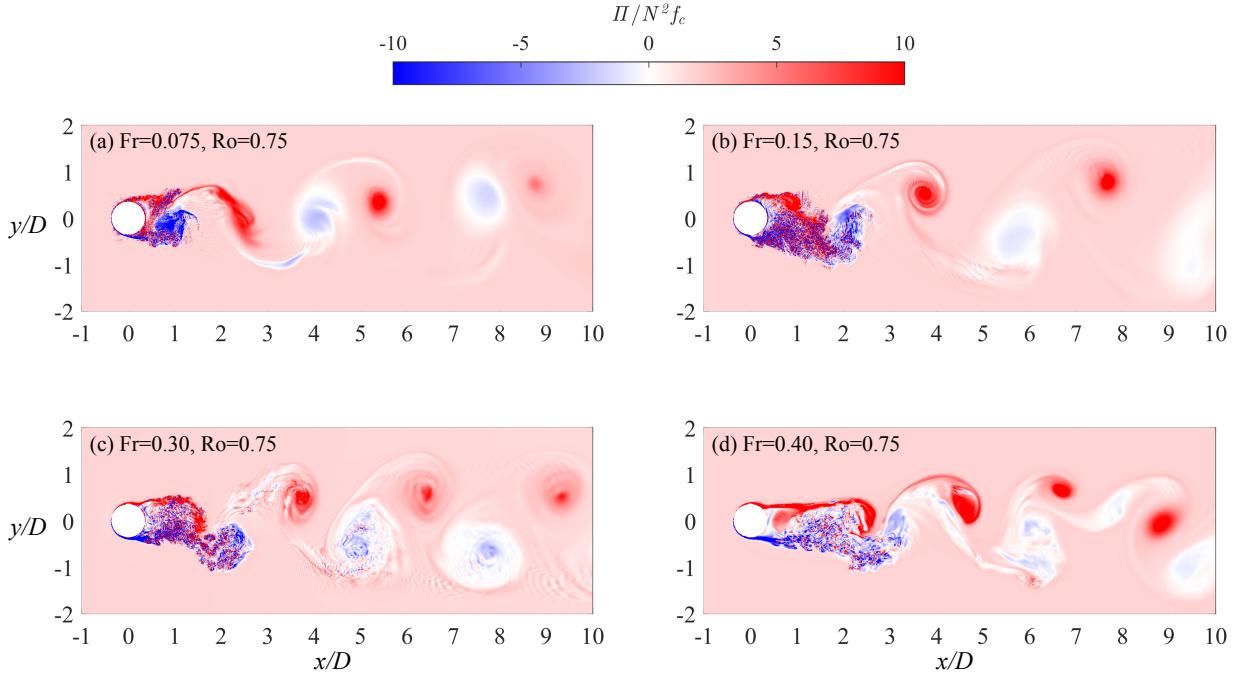


FIG. 5: Contours of the potential vorticity, $\Pi/N^2 f_c$, in the horizontal plane at $z/h = 0.25$. Panels (a-d): $Fr = 0.075, 0.15, 0.30, 0.40$, respectively, and $Ro = 0.75$ for all panels. The background value is unity for all panels, corresponding to normalized background PV (unity) due to the system rotation.

et al. (2021), all in general agreement with the present results. This range of dissipation values in the observations could be due to a number of factors, for example, differences in topography size, in strengths of the mean current and tidal flows, and in the background N , but an overall agreement within measurements and between measurements and the present simulations is reached. It is also noted that the tidal component, which is another destabilizing factor, has not been included here and is the subject of separate study for situations where its magnitude is as strong as or even stronger than the current.

Figure 4 compares the dissipation profiles at $x/D = 1, y = 0$ for different (Fr, Ro) cases. The dual roles of rotation can be seen. On one hand, as shown in Fig. 4(c), case Fr030Ro075 with rotation at SMS has the largest dissipation at the centerline relative to Ro values that are higher or lower. The peak at SMS Ro , which will also be shown in volume-integrated ε later, is linked to CI. On the other hand, in Fig. 4(a-b), very strong rotation ($Ro = 0.15$) significantly reduces ε by almost an order of magnitude compared to $Ro \geq O(1)$, through the reduction of the vertical velocity gradients and the consequent vertical shear instability. Moving from Fig. 4(b) to (c), the

effect of rotation appears to be non-monotonic due to the reasons mentioned above, but a more comprehensive measure of the dissipation in the 3D domain is required and the examination of the CI instabilities is needed. These investigations are reserved for the next sections.

c. Contribution of the centrifugal/inertial instability (CI)

Planetary rotation substantially changes the spatial organization of flow structures that are large enough to feel it ($Ro < O(10)$), and alters the dissipation in various ways, both direct and indirect. The direct effect of rotation can be sensed by the CI, whose intensity has a parametric dependence on rotation and is stronger at $Ro = O(1)$ (see Appendix C). The indirect effect, although still associated with CI, comes from the change of the mean/base flow through the modification of the coherent structures. For the latter, eddies that rotate in the same direction as the system rotation (cyclonic vortices, CVs) and those rotate in the opposite direction (anticyclonic vortices, AVs) behave differently. Among flows with $|\omega_z| = O(f)$, anticyclonic vortices and shear are often subject to CI, leading to an appreciable asymmetry between the two sides of the wake. We note that in the present wake, rotation is Southern Hemisphere and AVs are primarily on the right-hand-side of the flow ($y < 0$ and $\omega_z > 0$).

We move on to the potential vorticity (PV),

$$\Pi = (\mathbf{f}_c + \boldsymbol{\omega}) \cdot \nabla \tilde{b}, \quad (11)$$

where $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ is the relative vorticity and $\tilde{b} = b + N^2 z$ is the ‘total buoyancy’. PV is a useful diagnostic in large-scale flow analyses since it is conserved along isopycnal surfaces in the absence of friction or mixing. On the other hand, the sign of PV has implications for scales smaller than the balanced motions. Negative PV serves as an indicator of several hydrodynamic instabilities (Thomas et al. 2013), that are precursors to turbulence. The horizontal and vertical components of PV,

$$\Pi_h = \omega_x \frac{\partial \tilde{b}}{\partial x} + \omega_y \frac{\partial \tilde{b}}{\partial y}; \quad \Pi_v = (f_c + \omega_z) \frac{\partial \tilde{b}}{\partial z}, \quad (12)$$

are indicative of the symmetric and centrifugal/inertial instabilities, respectively (Thomas et al. 2013).

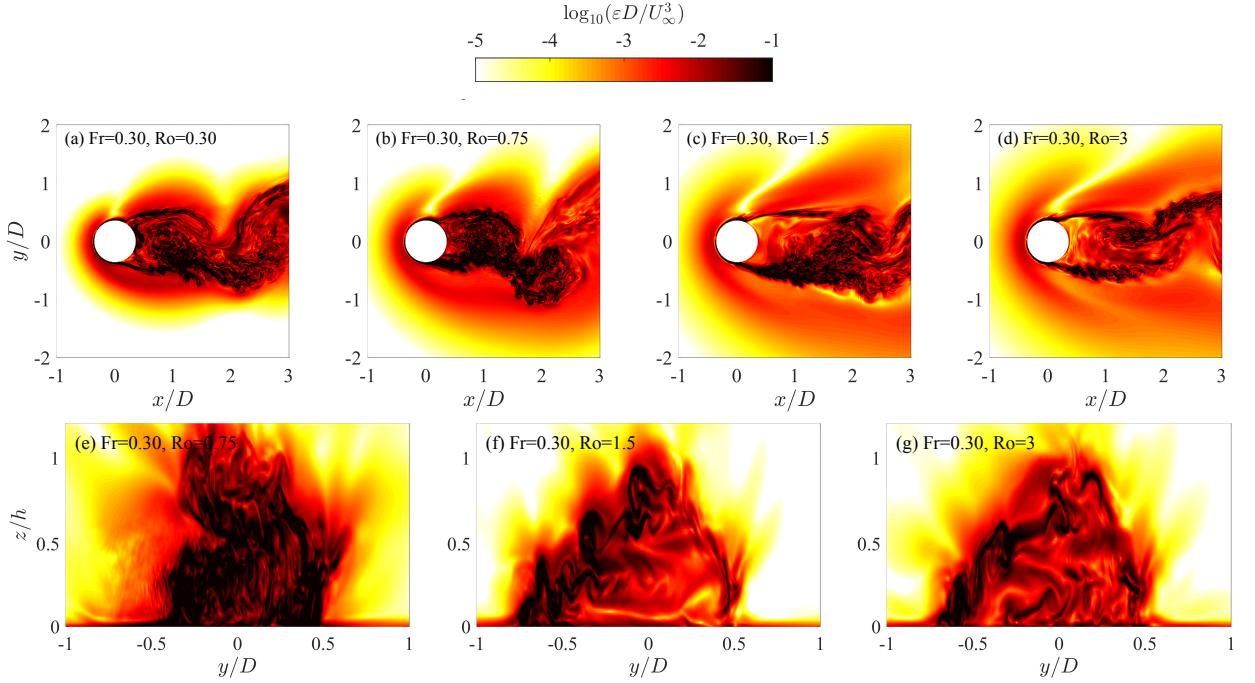


FIG. 6: Instantaneous dissipation rate ε in (a-d) the $z/h = 0.25$ plane and (e-g) the $x/D = 1$ plane. Panels (b,e), (c,f), and (d,g) correspond to the same time instances, respectively.

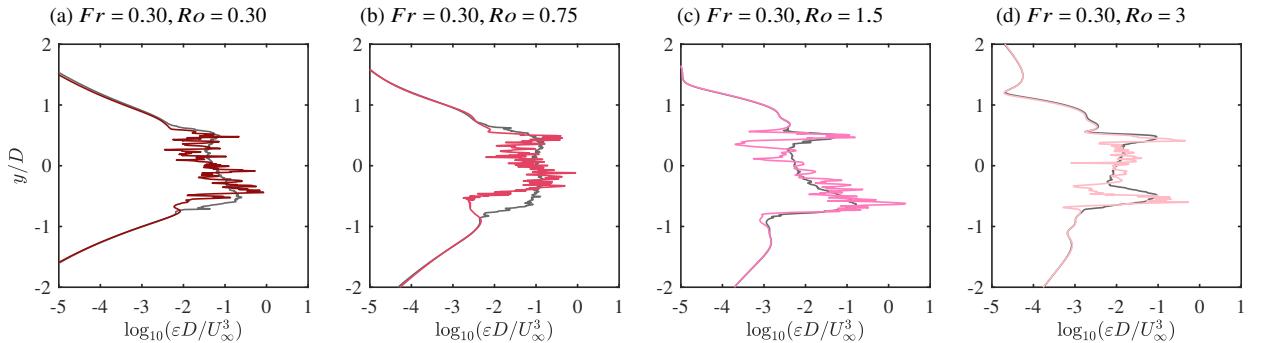


FIG. 7: Instantaneous dissipation rate ε (colored) and its time-average $\langle \varepsilon \rangle$ (TKE dissipation, gray), probed on the horizontal line $z/h = 0.25, x = 1$. The time instances are the same as those in Fig. 6(a-d).

Figure 5 shows PV on the horizontal plane $z/h = 0.25$ at $Ro = 0.75$ and different Fr . In the near wake of each case, entangled fine-scale structures of positive and negative PV can be seen, as a result of near wake turbulence. As the flow evolves into the intermediate wake, large patches of negative PV (indicated by blue color) can be seen on the anticyclonic side ($y < 0$), with the magnitude decaying in the streamwise direction as the flow gradually adjusts to rotation, eventually reaching a near-zero PV, stable state with little turbulence. On the cyclonic side ($y > 0$), strong

large-scale coherent CVs can be found with positive PV, and they remain intact from destruction during their downstream propagation. This asymmetry between AVs and CVs is a characteristic feature of the SMS.

At the $Fr \leq O(1)$ values of this case study, the flow exhibits a von Kármán shedding pattern in horizontal planes. However, there is a subtle difference in the vortex dynamics. At the lowest Fr ($Fr = 0.075$, Fig. 5(a)), dipoles are formed due to strong mutual interaction between the AVs and the CVs, where the CVs are systematically stronger and they attract AVs to the cyclonic side, leading to the veering of the wake. The increased horizontal vortex–vortex interaction that is closer to 2D dynamics is a consequence of strong stratification and limited vertical motions. This behavior is similar to the Bu25 case in Liu et al. (2024), where the vortices were tracked in time and dipole formation was statistically shown by the mean vortex trajectories and conditional vorticity distribution. At $Fr = 0.15, 0.30$ (Fig. 5(b-c)), the overall shedding pattern mimics that of a standard Kármán street while the CI of the AVs is more appreciable visually. At $Fr = 0.40$ (Fig. 5(d)), the recirculation zone is significantly longer than that in other cases (which can also be seen in Fig. 2) and the vortex shedding pattern is less regular. The former is due to the fact that the hydraulic jet reaches a lower downward distance at lower Fr , and it interacts strongly with the separation (Chomaz et al. 1993). The vertical PV (Π_v , not shown) is similar to the total PV, and the horizontal PV (Π_h , not shown) did not show evidence of the symmetric instability.

Figure 6 shows instantaneous ε at $Fr = 0.30$ and various Ro , where both the Kármán vortices and the shear layers present instabilities due to rotation. In all cases, small-scale turbulence structures can be seen in the near wake, whose spatial location coincides well with those wake eddies and the shear layer. These worm-like structures mimic the intense, randomly oriented vortex tubes in isotropic turbulence and are indicative of fully triggered turbulence as a consequence of the breakdown of the 3D instabilities and the establishment of a forward cascade. However, wake turbulence decays in the streamwise direction due to the stabilizing effect of strong stratification. Hence, these fine-scale structures don't persist long after their generation and turbulence is localized to the near wake, which can also be seen in Fig. 5. The time-evolution of ε and ε_ρ for panels (a-d) can be found in the supplementary materials (movies). The spatial structure of ε_ρ is similar to ε .

At $Fr = 0.30$, case Fr030Ro075 show the strongest turbulent dissipation (also evidenced by volumetric dissipation shown later in Fig. 8), with the turbulent worm patches fully filling the

interior of the eddies and crossing the centerline. Case Fr030Ro030 and Fr030Ro1p5 both have less dissipation, but they present the strongest lateral asymmetry, with the anticyclonic side unstable to CI that generates turbulence and the cyclonic side being much more stable. In case Fr030Ro3, the Burger number is $Bu = 100$ and the effect of rotation is relatively small. It shows weaker dissipation but both the AV and CV sides display braids of instability structures.

The aforementioned dynamics are also reflected in the statistics, shown in Fig. 7. In cases Fr030Ro030 and Fr030Ro1p5, the asymmetry between AV and CV sides is evident, with the AV dissipation having a wider spread and a higher magnitude. The dissipation asymmetry is the largest in the Fr030Ro1p5 case, where the anticyclonic peak ε is almost an order of magnitude higher than the cyclonic ε . In case Fr030Ro075, the asymmetry is less appreciable as dissipation fills more space near the centerline than in other cases. In the Fr030Ro3 case, the symmetry of the AV and CV dissipation gets close to the limit of a non-rotating case. The above scenarios are also qualitatively and quantitatively similar in the $Fr = 0.40$ cases with similar Ro (not shown).

Now we turn our attention to y - z planes in the near wake ($x/D = 1$), as shown in Fig. 6(e-g), in which the structures of CI-induced dissipation can be seen more clearly. For cases Fr030Ro1p5 and Fr030Ro3 in (b,c), rolled up dissipation structures can be seen at various heights on the anticyclonic (left) side, while the cyclonic shear layer is more stable (at this x/D location). These dissipation structures are associated with the quasi-streamwise vortices during the growth period of the CI, and were also found in other rotating flows, e.g., in [Kloosterziel et al. \(2007\)](#); [Arobone and Sarkar \(2012\)](#); [Carnevale et al. \(2013\)](#). A linear analysis of the growth of the streamwise/azimuthal vorticity during the CI is provided in Appendix C, which demonstrates its relevance to the diagnosis of CI and the relation between the growth rates and respective instability criteria.

In Fig. 6(e), the wake of case Fr030Ro075 is filled with three-dimensional turbulent vortices ('worms'), unlike the wakes in (f,g), which have a hollow core enclosed by the shear layers. These 'worms' are the results of the subsequent nonlinear interaction and rapid three-dimensionalization after the onset of CI, and the strong interaction of the anticyclonic and cyclonic vortices/shear layers. The more intense structures and more 'mature' turbulence in this case are both suggestive of stronger CI. It is also noted that CI is baroclinic/three-dimensional and, similar to KHI, it also distorts the isopycnals and enhances mixing.

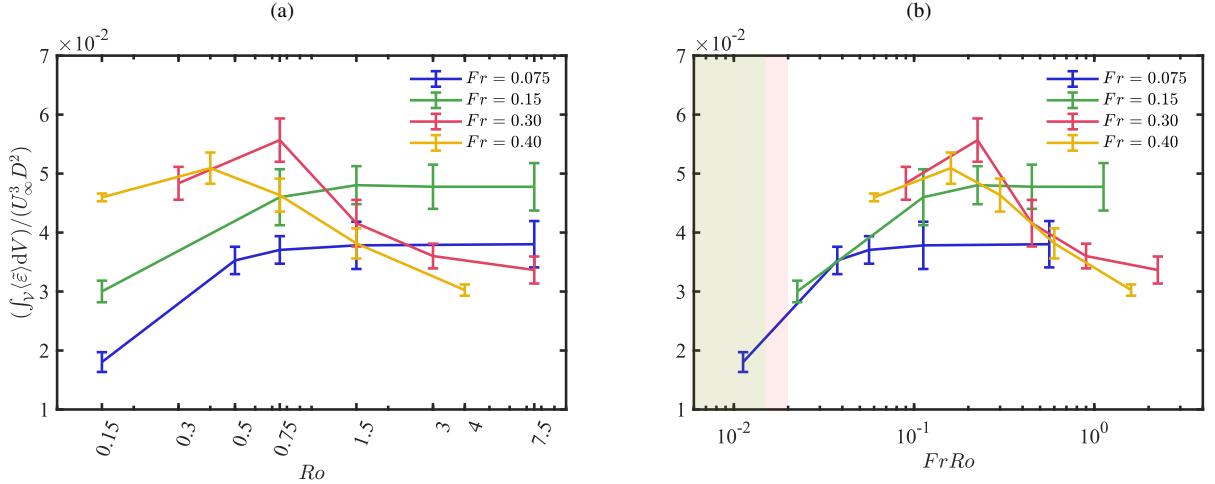


FIG. 8: Volume-integrated dissipation, $\int_V \langle \tilde{\varepsilon} \rangle dV$, as a function of (a) Ro and (b) $FrRo$. Here the integration domain is $\mathcal{V} = [-D, 4D] \times [-2D, 2D] \times [0, 2h]$. The vertical bars denote one standard variance above and below the averages. The green and red shades in (b) mark the overlap of the present parameters and those in [Perfect et al. \(2020\)](#) and [Srinivasan et al. \(2021\)](#), respectively.

4. Parametric dependence of dissipation on stratification and rotation

KHI and CI, which were discussed separately, in the previous section co-exist and their combined influence is responsible for the dependence of wake turbulence on stratification and rotation. An overall measure of turbulence is chosen for quantifying the dependence. Specifically, the time- and volume-integrated dissipation

$$\mathcal{E} = \int_{\mathcal{V}} \langle \tilde{\varepsilon} \rangle dV \quad (13)$$

is employed. The size of the integration subdomain is $\mathcal{V} = [-D, 4D] \times [-2D, 2D] \times [0, 2h]$ that encloses the turbulent near wake. Here $\langle \tilde{\varepsilon} \rangle$ is the time-averaged (over $N_t \approx 30$ for 3D snapshots) dissipation rate of the total kinetic energy (instead of TKE), for better converged statistics and greater relevance to field measurements. For case Fr040, in which the recirculation bubble is longer and separation is later downstream, (13) is accordingly computed in an elongated domain $\mathcal{V}' = [-D, 8D] \times [-2D, 2D] \times [0, 2h]$ and no significant difference is found.

Figure 8 shows the time- and volume-integrated dissipation rate \mathcal{E} for each Fr -series as a function of Ro in (a) and $FrRo$ in (b). It can be seen that the data break into two groups: (1) the lower- Fr group, Fr007 and Fr015, and (2) the higher- Fr group, Fr030 and Fr040.

In the first group with very strong stratification, the effect of rotation appears to be solely stabilizing. The dissipation increases as Ro increases, and eventually saturates at high Ro or weak rotation. The dissipation in Fr007 cases is consistently lower than that in Fr015 cases, even though the Re is twice higher in the Fr007 cases. The trends of monotonic decrease of dissipation with decreasing Ro and Fr are qualitatively consistent with the findings and the parameter ranges in previous ROMS studies of [Perfect et al. \(2020\)](#) and [Srinivasan et al. \(2021\)](#). A difference is that the weak rotation (higher Ro) saturation was not observed in the previous work since it concerned the strong rotation/large topography regime of $Fr, Ro \leq O(0.1)$. In Fig. 8(b), the green and red shaded regions mark the upper end of the parameter combinations of $FrRo$ in [Perfect et al. \(2020\)](#) and [Srinivasan et al. \(2021\)](#).

In the second group where stratification is not too strong, the effect of rotation is non-monotonic – there is an intermediate value of Ro where the volume-integrated dissipation peaks. The intermediate value is $Ro = 0.40$ for the Fr040 cases and $Ro = 0.75$ for the Fr030 cases. Both peaks fall within $Ro = O(0.5 – 1)$, corresponding to SMS topographies or eddies. The dissipation maxima at the SMS and the associated most destabilizing rotation have not been well explored in the previous parameterizations of topographic wakes and highlight the significance of CI and its Ro -dependence.

Similar non-monotonic rotation dependence was seen in the rotating horizontal shear layer with and without vertical stratification ([Yanase et al. 1993](#); [Arobone and Sarkar 2012](#)), and is characteristic of CI. The numerical stability analysis of [Yanase et al. \(1993\)](#) and [Arobone and Sarkar \(2012\)](#) revealed that, with no rotation, the three-dimensionally most unstable mode is the 2D KH mode ($k_z = 0$), which is still the case when there is vertical stratification ([Arobone and Sarkar 2012](#)). As the authors found, when there is weak rotation ($Ro = O(10)$), the most unstable mode is still the KH mode, but a nearly streamwise invariant inertial mode starts to emerge. As the rotation rate increases and Ro reaches $O(1)$, the growth rate of the inertial instability overtakes that of the KH mode. However, when rotation is further increased to $Ro \sim O(0.1)$, the inertial mode disappears. The existence of a most destabilizing rotation rate can be shown in a linear analysis of both parallel and circular flows in Appendix C, which explains the existence of an SMS dissipation peak in the Fr030 and Fr040 cases when stratification is not too strong.

As stratification becomes stronger, such as in cases Fr007 and Fr015, the vertical length scale decreases (see Fig. 2(a-d)) and the space for CI existence shrinks until it is eventually suppressed. Hence, the reason for rotation appearing to be solely stabilizing at $Fr = 0.075, 0.15$ is the absence of CI at strong stratification.

The present parametric dependence has several implications. First, the inertial scaling $\varepsilon = C_\varepsilon U_\infty^3/D$, where C_ε is a scaling coefficient, works reasonably well for the wake dissipation over a wide range in the parameter space of (Fr, Ro) (the value of $FrRo$ varies by 2 orders of magnitude). It provides a means for contextualizing field measurements or modeling wake dissipation/mixing in regional or global climate models. On the other hand, the variability of C_ε over the range of Fr and Ro investigated reflects the comprehensive effects of stratification and rotation. It is evident that turbulence in seamount or hill wakes at different levels of stratification depends differently on rotation, with a transition point between $Fr = 0.15$ and $Fr = 0.30$ due to the activation of the CI. That being said, C_ε is likely not a simple function of Fr and Ro .

In previous numerical studies of topographic wakes ([Perfect et al. 2018, 2020](#); [Srinivasan et al. 2021](#)), it was suggested that the vortex dynamics and turbulent dissipation might both be categorized by a single parameter, instead of (Fr, Ro) . For example, [Perfect et al. \(2018\)](#) varied both Fr and Ro in the range of $0.014 \leq Fr^* \leq 0.14$ and $0.053 \leq Ro^* \leq 0.21$, which is equivalent to $0.014 \leq Fr \leq 0.14$ and $0.0265 \leq Ro \leq 0.105$ after a conversion to our definition ($Fr = Fr^*, Ro = Ro^*/2$). It was suggested that the vortex dynamics can be characterized as a function of the Burger number, $Bu = (Ro/Fr)^2 = (Nh/f_c D)^2$, and the dissipation can further be parameterized as a function of a positive power of $FrRo$. [Srinivasan et al. \(2021\)](#) studied topographic wakes as a function of Ro ($0.025 \leq Ro \leq 1$) when $Fr = 0.02$ is fixed, and found that the dissipation monotonically increases as a function of the Rossby number. For comparison, the same data in Fig. 8(a) is plotted in Fig. 8(b), as a function of $FrRo$. Although the data seems to collapse better, the division into two groups with different dependencies is still very clear. The parameters in [Perfect et al. \(2020\)](#) have $(FrRo)_{\max} \approx 0.015$ and those in [Srinivasan et al. \(2021\)](#) have $(FrRo)_{\max} = 0.02$. Both overlap with the lower end of the present parameters, as shown by the green and red shades in Fig. 8(b).

It was pointed out by [Liu et al. \(2024\)](#) that there is still vertical coupling of the vortex shedding at $Fr = 0.15$, $Bu = \infty$ and hence the Bu -determination of coupling/decoupling is incomplete without an additional Fr -dependence – stronger stratification than $Fr = 0.15$ is required to vertically

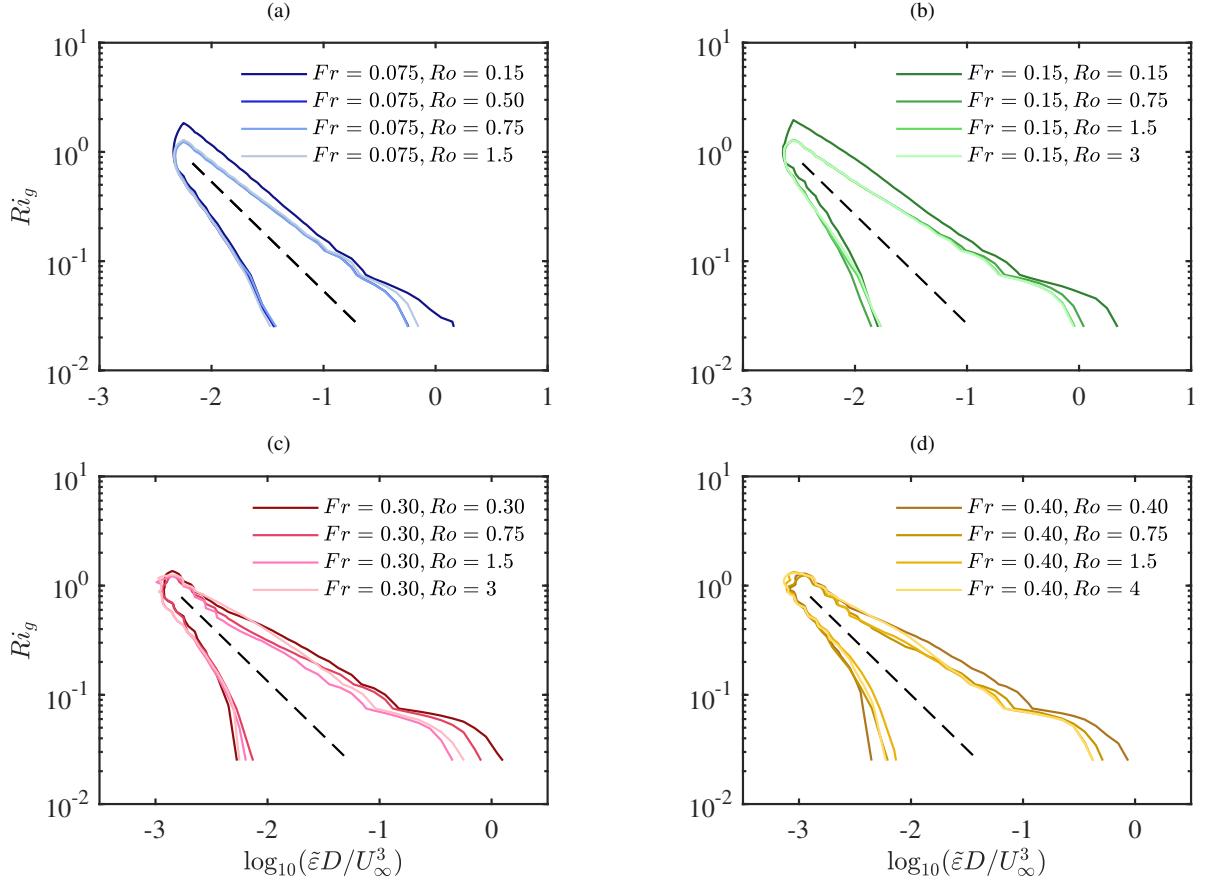


FIG. 9: Joint p.d.f (JPDF) of the total KE dissipation ($\tilde{\varepsilon}$, defined in (9)) and the local gradient Richardson number (Ri_g , defined in (10)). Contours enclose 85% of the JPDF in each case. Plane location $z/h = 0.25$. The Fr -dependent black dashed lines in each figure are given by $\tilde{\varepsilon}D/U_\infty^3 = C_R Fr^{-1} Ri_g^{-1}$, with the same fitting constant $C_R = 1/2500$.

decouple the vortex shedding in rotating and non-rotating Kármán wakes. A similar role of stratification is found in the dissipation dependence that the activation of the CI occurs between $Fr = 0.15$ and $Fr = 0.30$, which contributes greatly to dissipation and changes its Ro -dependence.

Besides integrated dissipation, the distribution of instantaneous dissipation and its point-wise correlation to the gradient Richardson is useful for correlating the shear instability to turbulent dissipation. Figure 9 shows the joint probability distribution functions (JPDFs) of the instantaneous total KE dissipation ($\tilde{\varepsilon}$) and the local gradient Richardson number (Ri_g). Here $\tilde{\varepsilon}$ and Ri_g are both based on instantaneous velocity components/buoyancy as previously defined in (9) and (10). The shear-stable background flow with $\tilde{Re}_b < 0.1$ is excluded from the JPDF calculation. As a consequence, regions of $Ri_g > O(1)$ are excluded in the JPDF, allowing a focus on the shear-

unstable states. The contour for each case approximates the boundary of the ensemble with $\tilde{Re}_b > 0.1$ in $(Ri_g, \tilde{\varepsilon})$ space and is helpful to compare the behavior among cases. Specifically, the contour encloses the JPDF-based top 85% of the ensemble, i.e., the shown contour encloses 85% of the ensemble counting downward in JPDF from its maximum value. By changing the percentage to 80% and 90% similar results are obtained.

The shape of the JPDFs presents noteworthy similarity in shape among all cases. Also, the axis of the JPDF follows

$$\tilde{\varepsilon}D/U_\infty^3 = C_R Fr^{-1} Ri_g^{-1}, \quad (14)$$

which is a consequence of the scaling analysis in the following paragraph. Equation (14) is plotted in each panel of Figure 9 with the choice of $C_R = 1/2500$ as the coefficient. Evidently, it is a reasonable approximation to the axis of the JPDF.

Assume a state of stratified layered turbulence where the dissipation is predominantly from the vertical shear of horizontal velocity components, $\tilde{\varepsilon} \sim \nu S_l^2$, where $S_l^2 = (\partial_z u)^2 + (\partial_z v)^2$ is the instantaneous, squared shear. The stratification (N_l) in the vicinity of the dissipation layers is close to the global constant N . The gradient Richardson is hence $Ri_g = N_l^2/S_l^2 \approx N^2/S_l^2$. Thus, the dissipation is $\tilde{\varepsilon}D \approx \nu S_l^2 \approx \nu N^2 Ri_g^{-1}$ and, its inertially-normalized value becomes

$$\tilde{\varepsilon}D/U_\infty^3 \approx Fr_D^{-2} Re^{-1} Ri_g^{-1}. \quad (15)$$

With $Fr_D Re = Re_N = 900$ and $h/D = 0.3$ being constant in the present study, the above relationship further simplifies to (14).

Equation (14) serves as a reference for the instantaneous dissipative regions in the context of stratified turbulence parameters. It can be seen in Fig. 9 that the JPDFs are very narrow around the dashed line given by (14) when Ri_g is close to unity and they widen up as Ri_g decreases below 1/4 (more unstable). The low- Ri_g spread of $\tilde{\varepsilon}$ is particularly evident at the larger $Fr = 0.3$ and 0.4 values, when the same Ri_g corresponds to $\tilde{\varepsilon}$ that ranges in one to two orders of magnitudes and the largest $\tilde{\varepsilon}$ deviates from (14) by an order of magnitude. This is due the intermittency of the local shear and stratification. As strong 3D turbulence is present (large $\tilde{\varepsilon}$), larger local shear and lower local stratification (due to mixing of density) are correlated, both reducing Ri_g .

5. Discussion and conclusion

Topographic features are ubiquitous on the seafloor and are hot spots of turbulence generation. Both the physical mechanisms that lead to turbulence and the accurate parametric dependence of dissipation on the overall governing parameters are crucial in the understanding and modeling of bottom ocean flows. To this end, LES of the wake of an isolated 3D topography is employed for a cross-combination of four Fr and five Ro , representing moderately strong to strong stratification and rotation rates that range from small MS to small SMS. The LES is conducted at high resolution, sufficient to resolve flow instabilities and the energy-containing scales of wake turbulence. Two instability mechanisms, the KHI and the CI, are found to be the major contributors to turbulent dissipation. Volume-integrated dissipation in the near wake is quantified and rendered in the parameter space (Fr, Ro) and, furthermore, the connection of this parametric dependence to the instability mechanisms is established.

The primary instability in strongly stratified wakes is the KHI between the dislocated layers of velocity–buoyancy structures, albeit oblique rather than horizontal. As stratification increases, the vertical length scale U_∞/N decreases and the distance between the layers becomes thinner (see Fig. 2). The velocity variation over such a distance, largely due to the out-of-phase vortex shedding at different heights, leads to intense shear collocated with the dislocation. The dimensional values of ε as well as the spiky vertical profiles agree reasonably well with observational measurements. When stratification is relatively weaker, in cases Fr030 and Fr040, the hydraulic response becomes strong, which also breaks down to turbulence via the KHI (see Fig. 2(g,h)).

Rotation influences wake turbulence both indirectly and directly and the latter role depends on stratification. The indirect effect is through the modification of the coherent structures, which was comprehensively studied in Liu et al. (2024). When rotation is weak ($Ro > O(1)$), the vertical structures of wake vortices and dissipation are forward-slanted ‘surfboards’ with $O(U_\infty/N)$ vertical thickness, as shown in Fig. 2(a,b), which is a configuration that favors KHI. When rotation is strong ($Ro \leq O(0.1)$), upright ‘columns’ that are reminiscent of stratified Taylor columns are formed (Liu et al. 2024) and the vertical shear and KH turbulence are significantly reduced (see Fig. 2(e)). This indirect effect is also reflected in the volume-integrated dissipation rate for Fr007 and Fr015 cases, shown in Fig. 8, that rotation appears to be solely stabilizing. The direct effect of rotation enters as stratification weakens, in cases Fr030 and Fr040, through the CI. As the vertical length scale

increases with increasing Fr , its constraint on the CI is released and CI is able to destabilize the flow and change the dependence of dissipation on Ro – a dissipation peak emerges in the SMS range of $Ro = O(0.5)$. The existence of a most destabilizing rotation rate agrees with results from rotating horizontal shear layers (Yanase et al. 1993; Arobone and Sarkar 2012) and circular flows (Yim et al. 2016), and can be theoretically expected as shown by the linear analysis in Appendix C.

To summarize, the instability mechanisms, KHI and CI, and the effects of stratification and rotation, are strongly intertwined. The KHI results directly from the oblique dislocated layers due to strong stratification, but it is also indirectly influenced by rotation which alters the vertical shear. The CI has a non-monotonic dependence on rotation, while its growth is restricted by stratification. It is the co-existence of KHI and CI, their subsequent nonlinear evolution and cross-dependence/influence that enriches the flow physics, while posing challenges to simple parameterizations. The idealized interaction scheme is shown in Fig. 10.

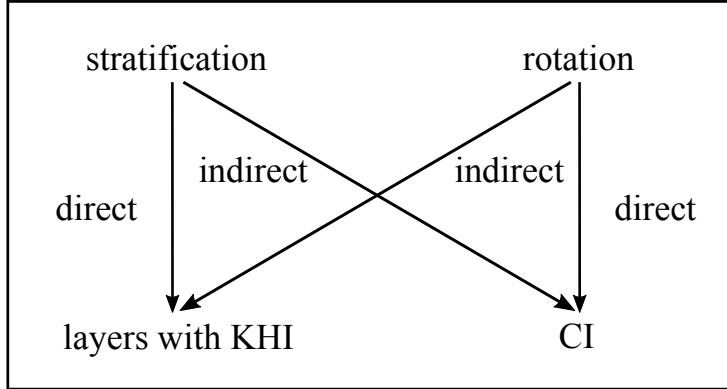


FIG. 10: Schematic showing the cross-influence of stratification and rotation on the KHI and CI in the present wakes. Arrows point to the direction of influence.

Despite the variabilities as the instability diagnostics and integrated measures reveal, wakes at different (Fr, Ro) share many characteristics in common. For example, the near wake box-integrated dissipation scales as $U_\infty^3 D^2$, and the scatter among different (Fr, Ro) is within approximately a factor of three (see Fig. 8). The JPDFs of $\tilde{\varepsilon}$ and Ri_g shown in Fig. 9 share very similar shapes in all cases. The JPDFs are centered around $\tilde{\varepsilon}D/U_\infty^3 \sim Fr^{-1} Ri_g^{-1}$ but the distributions widen when Ri_g is below its critical value 1/4, corresponding to the intermittency of the KHI.

Toward the future research advancements of wake turbulence and coherent eddies with LES studies, consideration of more realistic geometry such as multiscale topography or more complex

seamount shapes is a promising direction. Inclusion of realistic background flow that includes elements such as non-linear stratification, non-uniform currents and strong tides would likely be very useful. Topographic internal waves constitute an important ingredient of oceanic variability. Although the waves have induced dissipation that is weaker than wake turbulence (see Fig. 2) in the present problem, they propagate momentum and energy into the ambient and, through their subsequent breakdown into turbulence, present a reservoir for remote topographic mixing of the ocean interior. The characteristics and parametric dependence of those waves in the context of 3D topography, and their role in accomplishing remote mixing are subjects for future work.

Acknowledgments. This work is supported by ONR grant N00014-22-1-2024.

Data availability statement. The reported data and corresponding MATLAB scripts to read them will be made public in a repository with a DOI locator.

APPENDIX A

Detailed simulation setups

This appendix describes the details of the numerical setup, such as domain size, grid number, and resolution. Table A1 provides the numerical setting.

In the horizontal direction, the grid is dense near the hill (around $0.003 \sim 0.006D$ or $1.5 \sim 3$ m) and is mildly stretched upstream and downstream. The stretching ratio is reduced in the Fr040 cases, where the separation occurs further downstream, to resolve the gradients and control the dispersive error due to grid stretching. In the vertical direction, the resolution below $1.2h$ is kept at $15 \sim 30$ grid points per U_∞/N , in different cases, and there is also a mild stretching above. With the stratification vertical length scale U_∞/N decreases as stratification increases, the vertical grid number increases nearly inverse proportionally with Fr .

A similar resolution was found to be able to provide good-quality LES in Liu et al. (2024). The approximate computational cost per case (production run) is listed in Table A1 and the total cost is approximately 2 million CPU hours, not counting the CPU time spent in pre-production scoping simulations.

APPENDIX B

Fr	$[N_x, N_y, N_z]$	$[L_x, L_y, L_z]/D$	$[\Delta x, \Delta y, \Delta z]/D$ in the NW	CPUs	CPU hours per case
0.075	[1536, 1152, 432]	[19, 7.6, 4.8]	[0.0034, 0.0066, 0.0016]	384	120 k
0.15	[1536, 1280, 320]	[19, 7.6, 4.2]	[0.0034, 0.0059, 0.0024]	256	75 k
0.30	[1536, 1280, 216]	[19, 7.6, 4.2]	[0.0034, 0.0059, 0.0038]	256	50 k
0.40	[1920, 1536, 216]	[16, 10, 4.2]	[0.0034, 0.0065, 0.0038]	384	60 k

TABLE A1: Simulation details. Overall, the largest near-wake (NW) grid spacing is in the y -direction for all cases, which is $\Delta y = 0.0066D = 3.3$ m (with $D = 500$ m). The vertical spacing is as small as $\Delta z = 0.8$ m in the $Fr = 0.075$ series. The degree of freedom of the simulations ranges from 0.42 to 0.76 billion.

Effect of the Reynolds number

At sufficiently high Reynolds number, TKE dissipation is expected to be independent of viscosity. In equilibrium turbulence, the universal inertial scaling for dissipation $\langle \varepsilon \rangle = C_\varepsilon \mathcal{U}^3 / \mathcal{L}$ is valid at large Reynolds number limit, with the scaling coefficient being $C_\varepsilon \approx 0.1$. Here \mathcal{U} is the fluctuating velocity scale and \mathcal{L} is the integral length scale. At moderate Reynolds number, the ‘constant’ slightly decays as Re increases until the asymptote is reached (Sreenivasan 1984; Vassilicos 2015).

In order to examine the sensitivity of our results to the Reynolds number, three cases, Fr015Ro015, Fr015Ro075, Fr015Ro1p5, are selected for the Re -sensitivity study. They are run at $Re = 20000$ according to Table 1. Simulations at two additional Reynolds numbers, $Re = 10000$, $Re = 30000$, are conducted for each of the three cases.

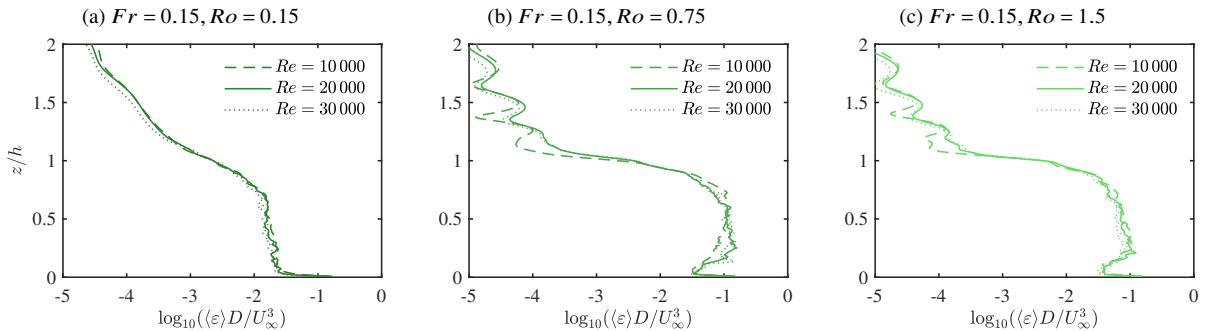


FIG. B1: TKE dissipation $\langle \varepsilon \rangle$ at various Reynolds numbers.

In Fig. B1, the time-averaged dissipation at $y = 0, x/D = 1$ is compared for three different Reynolds numbers. It can be seen that regardless of rotation, the dissipation rates at different Reynolds numbers agree reasonably well. In Fig. B1(a,c), a slight decrease in dissipation as Re

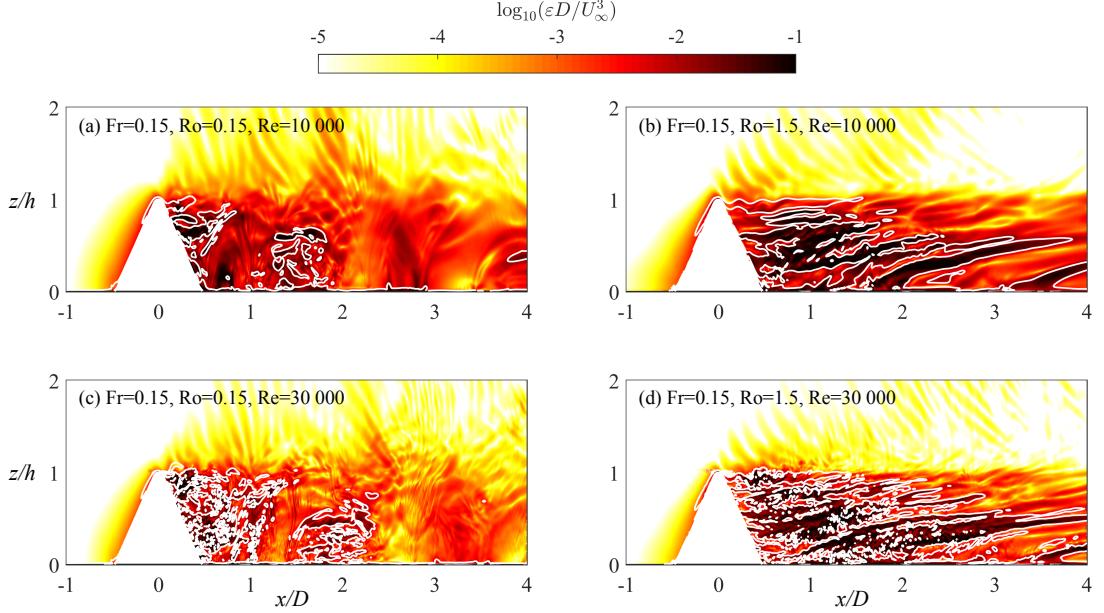


FIG. B2: Similar to Fig. 2. The Reynolds numbers are $Re = 10000$ ($Re_N = 450$) in (a,b) and $Re = 30000$ ($Re_N = 1350$) in (c,d). Panels (c,d) look very similar to Fig. 2 (c-d), which are at $Re = 20000$ ($Re_N = 900$).

increases is found, due to the moderate Reynolds number effect (Sreenivasan 1984; Vassilicos 2015), but no additional instability mechanism or qualitative difference appeared.

Figure B2 presents the contours of instantaneous dissipation in the center plane for cases Fr015Ro015 and Fr015Ro1p5, similar to Fig. 2(e,f) but at higher and lower Reynolds numbers. There are similarities of the large-scale structures between the Reynolds numbers, indicating that the large-scale coherent structures in stratified rotating wakes still remain robust despite higher Reynolds numbers. Meanwhile, there is a substantial addition of small scales to the flow as Re increases from 10,000 to 30,000 as a consequence of the breakdown of the instabilities. However, the change of the topology of the ε field from $Re = 20,000$, which previously shown as Fig. 2 (c-d), to $Re = 30,000$ is small. This similarity and difference between the Reynolds numbers comply with the general picture of turbulence that the large scales are determined by the boundaries and forcing and the viscosity-dependent smallest scales become finer as the Reynolds number increases (Tennekes and Lumley 1972).

Overall, the present Reynolds number is sufficiently high to trigger instabilities and the dissipation rate is relatively independent on Re . The differences among cases with different (Fr, Ro) are much more notable than those among different Re .

APPENDIX C

The inertial/Coriolis instability: a linear analysis

When a base flow with its own primary barotropic instability in the horizontal direction (for example, the inflection-point instability in jets or shear layers, or the Kármán shedding in bluff-body wakes) is subject to unstable vertical rotation, inertial/centrifugal/Coriolis (CI) modes emerge. These modes are baroclinic in the sense that they are usually characterized as horizontal vortex rollers (Kloosterziel et al. 2007; Arobone and Sarkar 2012; Carnevale et al. 2013), which also lead to diapycnal overturns in a stratified fluid. For completeness, in this Appendix, a linear analysis of CI is performed for two types of base flows: (1) parallel shear flow and (2) axisymmetric flow. The equations of horizontal vorticity perturbations are employed as in Arobone and Sarkar (2012) for parallel shear flow and are generalized to curvilinear flow. It will be pointed out that conditions favorable for CI will lead to (initial) exponential growth of horizontal perturbation vorticities, and more specifically, at a growth rate of the square root of respective stability criteria. By examining the dependence of the initial growth rate on Ro , it is demonstrated that for both types of base flows the most destabilizing rotation rate falls roughly at $Ro = O(1)$.

The starting point is the non-dimensional, linearized vorticity equation

$$\frac{\bar{D}\omega'_i}{\bar{D}t} = \omega'_j \langle S_{ij} \rangle + (\langle \omega_j \rangle + f_c \delta_{j3}) S'_{ij} - u'_j \frac{\partial \langle \omega_i \rangle}{\partial x_j} + \frac{1}{2} \epsilon_{ij3} f_c \omega'_j + \epsilon_{ij3} Ri_b \frac{\partial \rho'}{\partial x_j}, \quad (C1)$$

where primes denote perturbations with respect to the base flow (denoted by brackets). The operator $\bar{D}/\bar{D}t = \partial_t + \langle u_i \rangle \partial_{x_i}$ represents mean convection and S_{ij} is the rate-of-strain tensor. The free indices $i = 1, 2, 3$ denote the three spatial dimensions that are (x, y, z) in Cartesian coordinates and are (r, θ, z) in cylindrical coordinates, and the corresponding velocity components are (u, v, w) . The base flow is $\langle u \rangle = U_{SL}(y)$ in the parallel shear flow case and is $\langle v \rangle = \langle v \rangle(r)$ in the axisymmetric case. The buoyancy Richardson number is $Ri_b = Fr_h^{-2}$, where Fr_h is the horizontal Froude number, and the non-dimensional Coriolis parameter is $f_c = Ro^{-1}$. Viscous effects are neglected ($Ek?$).

a. Parallel shear flow

By solving linear stability equations, [Yanase et al. \(1993\)](#) and [Arobone and Sarkar \(2012\)](#) showed that the eigenmodes of CI are typically streamwise-invariant ($\partial_x \approx 0$). With a simplifying assumption $\partial_y w' \ll \partial_z v'$, the ω'_x, ω'_y components of (C1) reduce to

$$\frac{\partial \omega'_x}{\partial t} = f_c \omega'_y - Ri_b \frac{\partial \rho'}{\partial y}, \quad (\text{C2})$$

$$\frac{\partial \omega'_y}{\partial t} = -(\langle \omega_z \rangle + f_c) \omega'_x, \quad (\text{C3})$$

which form an autonomous system when the stratification is weak ($Ri_b \ll 1$):

$$\frac{\partial}{\partial t} \begin{pmatrix} \omega'_x \\ \omega'_y \end{pmatrix} = \begin{pmatrix} 0 & f_c \\ -(\langle \omega_z \rangle + f_c) & 0 \end{pmatrix} \begin{pmatrix} \omega'_x \\ \omega'_y \end{pmatrix}. \quad (\text{C4})$$

Here, the background vorticity is $\langle \omega_z \rangle = -\partial_y \langle u \rangle = -2 \langle S_{yx} \rangle$. The closed linear system (C4) has two eigenvalues that satisfy

$$\lambda^2 = -f_c(\langle \omega_z \rangle + f_c). \quad (\text{C5})$$

When

$$-f_c(\langle \omega_z \rangle + f_c) > 0, \quad (\text{C6})$$

there is a pair of real eigenvalues of opposite signs, with the positive one $\lambda_+ = [-f_c(f_c + \omega_z)]^{1/2}$ corresponding to instability. Otherwise, a pair of purely imaginary, conjugate eigenvalues will imply inertial waves.

The condition (C6) is equivalent to the absolute vorticity criterion for inertial instability ([Holton 1972](#)). In particular, for a base flow with shear $\langle \omega_z \rangle$, the growth rate of the perturbations is the largest when $f_c = f_{c,\max} = -\langle \omega_z \rangle / 2$, equivalently $Ro_{\max} = \langle \omega_z \rangle / f_{c,\max} = -2$ (anticyclonic). In other words, the most destabilizing (anticyclonic) rotation with respect to the given shear, is in the SMS range. When rotation is very strong ($Ro \ll O(1)$), $|f_c| \gg |\langle \omega_z \rangle|$ and hence $-f_c(\langle \omega_z \rangle + f_c) \approx -f_c^2 < 0$, the CI mode is stabilized, consistent with [Yanase et al. \(1993\)](#) and [Arobone and Sarkar \(2012\)](#). When rotation is very weak ($Ro \gg O(1)$), the former analysis does not apply. We also note that

the optimal growth rate may not always be achieved, but nevertheless, it serves as a good predictor for the strength of the CI.

The stability of the system (C4) can be interpreted as the response of the shear layer to coordinate rotation. The system is the most resonant when the forcing frequency (f_c) is the closest to the intrinsic frequency of the system ($\langle \omega_z \rangle / 2$, the angular velocity of the solid-body rotation part of fluid motions). The amplification mechanism is through the linearized Navier–Stokes.

b. Axisymmetric flow

A similar linear theory can be established for axisymmetric flows, where (u, v, w) and $(\omega_r, \omega_\theta, \omega_z)$ denote the velocity and vorticity components in r, θ, z directions, respectively. The effect of stratification is similar to the parallel flow case and is not included at the outset.

Similar to the quasi-streamwise-mode assumption in parallel flows, a quasi-axisymmetric-mode assumption ($\partial_\theta \approx 0$ for dependent variables; $\partial_\theta e_r = e_\theta$ and $\partial_\theta e_\theta = -e_r$ still apply) leads to

$$\begin{aligned} \langle u \rangle \cdot \nabla \omega' &= \frac{\langle v \rangle}{r} \frac{\partial}{\partial \theta} (\omega'_r e_r + \omega'_\theta e_\theta + \omega'_z e_z) \\ &= \left(\frac{\langle v \rangle}{r} \frac{\partial \omega'_r}{\partial \theta} - \frac{\langle v \rangle \omega'_\theta}{r} \right) e_r + \left(\frac{\langle v \rangle}{r} \frac{\partial \omega'_\theta}{\partial \theta} + \frac{\langle v \rangle \omega'_r}{r} \right) e_\theta + \frac{\langle v \rangle}{r} \frac{\partial \omega'_z}{\partial \theta} e_z \\ &\approx -\frac{\langle v \rangle \omega'_\theta}{r} e_r + \frac{\langle v \rangle \omega'_r}{r} e_\theta. \end{aligned} \quad (\text{C7})$$

Equation (C1) is cast in cylindrical coordinates for ω'_r and ω'_θ as

$$\frac{\partial \omega'_r}{\partial t} - \frac{\langle v \rangle \omega'_\theta}{r} = \omega'_\theta \langle S_{r\theta} \rangle + (\langle \omega_z \rangle + f_c) S'_{rz} + \frac{f_c}{2} \omega'_r \quad (\text{C8})$$

$$\frac{\partial \omega'_\theta}{\partial t} + \frac{\langle v \rangle \omega'_r}{r} = \omega'_r \langle S_{r\theta} \rangle + (\langle \omega_z \rangle + f_c) S'_{\theta z} - \frac{f_c}{2} \omega'_r. \quad (\text{C9})$$

For axisymmetric base flow $\langle v \rangle(r)$, the mean rate-of-strain tensor and the mean vorticity are

$$\langle S_{r\theta} \rangle = \frac{1}{2} \left\langle r \frac{\partial}{\partial r} \left(\frac{v}{r} \right) \right\rangle = \frac{1}{2} \left(\frac{\partial \langle v \rangle}{\partial r} - \frac{\langle v \rangle}{r} \right); \langle \omega_z \rangle = \frac{\partial \langle v \rangle}{\partial r} + \frac{\langle v \rangle}{r}. \quad (\text{C10})$$

Furthermore, the quasi-axisymmetry also leads to $\omega'_r \approx -\partial_z v' = -2S'_{\theta z}$ and the simplifying assumption $\partial_r w' \ll \partial_z u'$ leads to $\omega'_r = 2S'_{rz}$.

The system (C8)-(C9) then takes an autonomous form

$$\frac{\partial}{\partial t} \begin{pmatrix} \omega'_r \\ \omega'_{\theta} \end{pmatrix} = \begin{pmatrix} 0 & \langle \omega_z \rangle + f_c \\ -\left(\frac{2\langle v \rangle}{r} + f_c\right) & 0 \end{pmatrix} \begin{pmatrix} \omega'_r \\ \omega'_{\theta} \end{pmatrix}, \quad (\text{C11})$$

with its eigenvalues satisfying

$$\lambda^2 = -\left(\frac{2\langle v \rangle}{r} + f_c\right)(\langle \omega_z \rangle + f_c). \quad (\text{C12})$$

The condition for instability is the eigenvalues being real, or

$$-\chi = -\left(\frac{2\langle v \rangle}{r} + f_c\right)(\langle \omega_z \rangle + f_c) > 0, \quad (\text{C13})$$

where χ is exactly the generalized Rayleigh discriminant (Kloosterziel and Van Heijst 1991; Mutabazi et al. 1992). Taking $\lambda_+(r) = \sqrt{-\chi(r)}$ as the estimated local growth rate for regions with $\chi < 0$, the most destabilizing rotation rate can be searched for any axisymmetric eddy profile. It is typical that an annular region satisfies $\chi < 0$ and becomes unstable. Taking the derivative of (C12) with respect to f_c , the locally most destabilizing rotation rate is $f_{c,\max}(r) = -(\langle v \rangle/r + \langle \omega_z \rangle/2)$. Although it should be evaluated with a global measure and it is flow-specific, the formal expression of $f_{c,\max}$ still suggests that an intermediate rotation rate at $Ro = O(1)$ will lead to the fastest three-dimensionalization.

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