Algebra II: Tutorial 1

January 27, 2022

1. If R is an integral domain, then either $\mathrm{char}(R)$ is equal to zero or a prime number.
2. If R is an integral domain and a, b are non-zero, then $(a) = (b)$ if and only if $a = ub$ where $u \in R^{\times}$.
3. Every Euclidean domain is a PID.

Problem 1 (Ideal correspondence)**.** Let R, S be commutative rings, with additive identities 0_R and 0_S respectively. Suppose that $f : R \to S$ is a surjective ring homomorphism.

1. Show that if I is an ideal of R, then $f(I) = \{f(r) \mid r \in I\}$ is an ideal of S.
2. Show that if J is an ideal of S, then $f^{-1}(J) = \{r \in R \mid f(r) \in J\}$ is an ideal of R containing $\mathrm{Ker}(f)$, the kernel of f.
3. Deduce that there is a one-to-one correspondence between ideals of S and ideals of R containing $\mathrm{Ker}(f)$.
4. Show that this correspondence descends to a bijection between prime (resp. maximal) ideal of S and prime (resp, maximal) ideals of R containing $\mathrm{Ker}(f)$.