Algebra II: Tutorial 4

March 2, 2022

Problem 1 (Recap on irreducibility)**.** Let $f(x) = x^4 - 2x^2 + 9 \in \mathbb{Q}[x]$.

1. Show that $\pi_p(f(x))$ is reducible for $p = 2, 3, 5, 7$.
2. Show that $f(x)$ is irreducible over $\mathbb{Q}$.

Problem 2. Suppose that $a, b \in \mathbb{C}$ have same minimal polynomial over $\mathbb{Q}[x]$. Show that $\mathbb{Q}(a) \cong \mathbb{Q}(b)$.

Problem 3. Let p, q be two distinct prime numbers. Calculate $[\mathbb{Q}(\sqrt{p}, \sqrt{q}) : \mathbb{Q}]$.

Problem 4. Suppose that α, β are transcendental over $\mathbb{Q}$. Show that either $\alpha + \beta$ or $\alpha\beta$ is transcendental over $\mathbb{Q}$.