6	MATH3301 Assignment Part2
6	
6	(1) Gis an Abelian group.
	(1) Gis an Abelian group. (1) doesn't imply (2), as Zis an infinite Abelian group under +.
0	(1) doesn't impry (3), (15 / Contains no nonidentity exement a +0
0	such that $2\alpha = \alpha + \alpha = 0$, and \mathbb{Z} is certainly Abelian.
	such that $2\alpha = \alpha + \alpha = 0$, and \mathbb{Z} is certainly Abelian. (1) doesn't imply(4), as \mathbb{Z} contains a monidentity element $ +0\rangle$
	such that $2\cdot = + \neq 0$, and \mathbb{Z} is certainly Abelian.
	(2) G is a finite group of even order.
	(2) doesn't imply u), as $D_6 = \{e, r, r^2, rog \}$ 6, $ro6$, r^2o6^2 is a finite group of even order, but $60r = r^2o6 + ro6$ for some $r_16 \in D_6$, so D_6 is not Abelian.
	6, ro6, ro693 a finite group of even /
	order, but 60r = ro6 + ro6 for some
	7,6 ED6,50 V6 is mot Abelian.
	(2) implies (3). Assume to the contrary
	that Vae G- fe3, a2+e, so for each a, a=a iff a=e.
	we pair up each a with a which gives the following partition of G:
	$\Theta_{1}/N = \frac{5}{5} \{e^{3}, \frac{5}{5} \alpha_{1}, \alpha_{1}^{-12}, \frac{5}{5} \alpha_{2}, \alpha_{2}^{-12}, \frac{5}{5}, \frac{6}{5}, \frac{6}{5} \alpha_{1}, \frac{6}{5}, \frac{7}{5} \alpha_{1}, \frac{6}{5}, \frac{7}{5} \alpha_{1}, \frac{6}{5}, $
	This implies $ G = \{e\} + \{a_1, a_1, a_1, a_2, a_2, a_3, a_n, a_n, a_n, a_n, a_n, a_n, a_n, a_n$
	= + 2 + 2 + + 2 = 2n +
	which 3 odd, 50 -1(3) implies -1(2)
•	(2) doesn't împly(4), as $D_6 = \{e, r, r^2, 6, r_06, r^206\}$ is a finite group of even order, some monidentity element $rgives r^2 + e$.
	even order, some monidentity element rgives r = e.

(3) There exists a mon-identity element a in B satisfies a=e (3) doesn't imply (1), as $D_6 = \{e, r, r^2, 6, r^2, 6, r^2, 6\}$ contains a mon-identity
(3) doesn't imply (1), as $D_6 = 7e, r, r^3, 6, r^6, r^6, r^6, r^6, r^6, r^6, r^6, $
element 6, satisfying 62=e, but 60r=206+706 for some 6,r
CG, so Do is mot Abelian.
(3) doesn't simply(2), as $\mathbb{Q} = \mathbb{Q} \setminus \{0\}$ contains a non-sidentity element—1, satisfying (-1)=1, but \mathbb{Q}^{\times} , i.e., the multiplicative group of monzero
rational numbers, is infinite.
(3) doesn't simply (4), as Do= fe, r, r2, 6, 806, 2003 contains a mon-identity
(3) doesn't simply (4), as $D_6 = \{e, r, r^2, 6, 806, r^208\}$ contains a mon-identity element 6, satisfying $6 = e$, but $r^2 + e$ for some non-identity
element γ .
(4) Every mon-identity element a in G satisfies $a^2 = e$
(4) implies (1). For all a, bGG:
$a^2 = eardb^2 = eard(ab)^2 = 7$ $abab = e = au = aea = abba = 3$ $ab = ba$
(4) doesn't simply (2), as mentioned in Tutorial 2, for all set S,
if we equip $P(S)$ with a binary operation $\Delta:(P(S)\times(P(S)\rightarrow P(S)), A B = (A \setminus B) \cup (B \setminus A)$, then $P(S)$ is a group satisfying.
$A \triangle B = (A \setminus B) \cup (B \setminus A)$, then $P(S)$ is a group satisfying
VAEP(S), ALA=(AVA)U(AVA)=ØUØ=Ø(identity)
$\forall A \in P(S)$, $A \triangle A = (A \lor A) \lor (A \lor A) = \emptyset \lor \emptyset = \emptyset \text{ (identity)}$. However, of we take $S = N(an infinite set)$, then $P(S)$ infinite.
For the amplication (4) =(3) it is necessary to assume that G has at
least one monitority element a, otherwise the discussion here carries no meaning
(4) jmplies (3) If all $a \in G - \{e\}(\neq \emptyset)$ does the job, then certainly,
least one monitority element α , otherwise the discussion here carries no meaning (4) simplies (3). If all $\alpha \in G - \{e\}(\neq \emptyset)$ does the job, then certainly, some $\alpha \in G - \{e\}$ closs the job.
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	2.(a) HUK 13 not always a subgroup of G.
	Consider the additive group of real vectors IR2 both IRXfo3 and fo3xIR are subgroups of IR2,
•	but (IRXf03) U(f03xIR) is not a subgroup of IRas it is not closed underaddition.
•	HOK is always a subgroup of 6
	11) e∈Hande∈K=>e∈HOK;
	(2) \ a,a,eHnK ⇒ a,a,eHnK = and a,a,eK ⇒ a,a,eHnK ⇒ a,a,eHnK
	⇒ a1&eHand a1&eK ⇒ a1&eHNK (3) Ya∈G, a∈HNK ⇒ a∈Hand a∈K⇒a=eHanda=eK⇒ a=eHNK Hence, HNK is a subgroup of G
	(b) HUK is a subgroup of B of HSKorKSH
	u of "direction: If HSK or KSH, then HUK=K or HUK=H, so HUK is a subgroup of G.
	Conly of direction: Assume to the contrary that HEK and KEH. HEK implies the existence of hEG, such that hEH and hEK; KEH implies the existence of kEG, such that kEK and kEH;
•	As hkeH=> k= h(hk) eH=> 1F, itshould be true that hkeH;
	As $hk \in K \Rightarrow h = (hk)k^T \in K \Rightarrow R, its hauld be true that hk \in K;Hence, for some h, k \in HUK, hk \notin HUK, HUK is not$
	a subgroup of B1 as it is not closed under group operation in B
	And the second of the second o

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	3.(a) Proof: First, we prove that C(H) is a subgroup of G.
	u) eeB, and theH, eh=he, so eeC(H);
	(2) Forall gigse (CH):
	Wheth, gih=hg, and theth, gih=hgs
	HheH, g,h=hg, and theH, g,h=hg, => theH, g,g,h=g,hg,=hg,g=> g,g=eC(H)
	(3) Forall ge CCH)
	Thettigh=hg > thettigth=gthgd=gtghgt=hgt)gecc
	Combine (1) (2)(3) above, we've proven that CLH)≤61.
	Second, we prove that Zis a subgroup of G.
	As Z= {geG: gh=hy forall heG3 = C(G1) < G1 it follows that Z < G.
	Third, we prove that NCH) is a subgroup of G.
	1) es G and Yhoth, ehet=ethe=hoth, so esN(H);
	(2) For all qug_ ENCH):
	[thet, gingiett and gingiett]
	and[Yhettigshg=ettandg=hg=ett]
	=> \het, [q,g)h(q,g)=q, (g,hg-1)q-1et]
)	and [(q,q) th (q,q) = g=(q,thq) q=ett]
	= gg GNCH)
	(3) Forall geNCH):
)	Thoff, ghat Extrand of hackt
	=> \theit (g1) hg7) = g7hgeHand[g1) hg7 = ghgtet]
	=> y GNUH).
	Combine (1)(2)(3) above, we've proven that NCH)≤G.

(b) Zomust be abelian. YanazeZ,	
REZ = [YyeG, xy=yx] 2 xx=xx	
MEZ => [YNEG, MY=YN]] => MIX2=NAMI.	
CCH) is not necessarily. Abeban.	
Choose an arbitrary non Abelian group, says-GL2(1R).	
Choose H={I3, then C(\$I3)={GEGL(1R): IG=GI3=GL(1R),	
which implies CCFIZ) is non Abelian.	
N(H) recessarily Abelsan. Matrix	
Choose G= GL2CIR). Choose H=SL2(IR)={BG612(IR):det(G)=1}	
Mathx	
N(SLICR))= {GEGLIR)-GHGESLUR) and GTHGESLUR)?	
= {6,666(1R): det(6HGT)=1 and det(5TH6)=13.	
det(G) det(H) det(G) det(G) det(H) det(G)	
det(G) det(H) det(G) det(G) det(H) det(G) = G 12((R), which implies N(SL2((R))) is mon Abebran.	
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