If $p(x)$ is a unit, then the ideal generated by m and x is just the whole $R[X]$. In particular, there exists $s(x)$ and $t(x) \in R[X]$ s.t.

$$e = m \cdot s(x) + x \cdot t(x).$$

Similarly, consider the degree, we conclude that $t(x) = 0$ and $s(x)$ ~~has~~ $\in R$. However, m does not have an inverse, which means $s(x)$ does not exist.

If $q(x)$ is a unit, then the ideal generated by m and x is just $mR[x]$. So there exists $s(x) \in R[x]$ s.t. $x = m\, s(x)$. Consider the degree, $s(x)$ has degree one, Write $s(x) = kx + b$. Then $e = mk$, which is impossible. ~~Thus~~ Thus we arrive at a contradiction.

Above all, $R[x]$ is not a PID. $\blacksquare$

(9), Sol. Let I be an arbitrary ideal of $\mathbb{Z}/n\mathbb{Z}$. Since $\mathbb{Z}/n\mathbb{Z}$ is finite, I is finite. Write $I = \{\bar{a}_1, \bar{a}_2, \cdots, \bar{a}_m\}$. Let d be the greatest common divisor of $a_1, a_2, \cdots, a_n$. Then there exists c_i s.t.

$$c_1\bar{a}_1 + c_2\bar{a}_2 + \cdots + c_m\bar{a}_m = d$$

by Bezout's Lemma. There fore $\langle d \rangle \subset I$. Also, $I \subset \langle d \rangle$ since $d \mid a_i$ for every i. Thus $I = \langle d \rangle$. Note that I is a normal subgroup of $\mathbb{Z}/n\mathbb{Z}$ under addition, which means the order of I must divide n. So $d \mid n$. Therefore all the ideals of $\mathbb{Z}/n\mathbb{Z}$ is of the form $\langle d \rangle$ with $d \mid n$. ~~are also all the prime ide~~

From above, we conclude that $\mathbb{Z}/n\mathbb{Z}$ is a PID. In particular, an ideal is prime if and only if it is maximal. Let $P = \{p_1, \cdots, p_s\}$ be all the prime numbers that divide n and less than n. If $P = \emptyset$, n is itself a prime. So $\mathbb{Z}/n\mathbb{Z}$ has no prime (maximal) ideals.

If $P \neq \emptyset$, then $\langle p_i \rangle$ $(i = 1, \cdots, s)$ are all the prime (maximal) ideals. Assume that $\langle p \rangle$ is an ideal s.t. $\langle p_i \rangle \subset \langle p \rangle \subset \langle 1 \rangle$. Since $p \mid p_i$, $p = 1$ or p_i, which means $\langle p \rangle = \langle p_i \rangle$ or $\langle p \rangle = \langle 1 \rangle$. For any other ideals, it must be $\langle 1 \rangle$ or generated by a number with at least two prime factors. Then the ideal generated by one of its factors is an ideal that contain the original ideal and be contained in $\langle 1 \rangle$.