**Problem 1.** Consider the following initial value problem

$$
\begin{cases}
\partial_t u - \partial_{xx} u = f & \text{for } -\infty < x < \infty \text{ and } t > 0 \\
u|_{t=0} = \phi,
\end{cases}
\tag{1}
$$

$f$ and $\phi$ will be given differently in different parts below. Solve the initial value problem (1) in the following cases:

(i) $f(x,t) := 2t$ and $\phi(x) := x$;

(ii) $f(x,t) := -4xt$ and $\phi(x) := x^3$;

(iii) $f(x,t) := 3xe^t$ and $\phi(x) := \cos x$.

**Problem 2.** Consider the following initial value problem

$$
\begin{cases}
\partial_t u - k\partial_{xx} u = f & \text{for } -\infty < x < \infty \text{ and } t > 0 \\
u|_{t=0} = \phi,
\end{cases}
\tag{2}
$$

where $k > 0$ is a given constant, $f$ and $\phi$ will be given differently in different parts below. Solve the initial value problem (2) in the following cases:

(i) $f(t,x) := 5$ and $\phi(x) := x$;

(ii) $f(t,x) := \sin t$ and $\phi(x) := x^4$;

(iii) $f(t,x) := x\ln(1+t)$ and $\phi(x) := \begin{cases} 4 & \text{if } |x| \le 1 \\ 0 & \text{if } |x| > 1; \end{cases}$

**Problem 3.** Consider the Cauchy problems for non-homogeneous wave equations in the whole line below, and complete the following parts.

(i) Solve
$$\begin{cases} \partial_{tt}u - \partial_{xx}u = \sin x & \text{for } -\infty < x < \infty \text{ and } t > 0 \\ u|_{t=0} = \cos x \\ \partial_t u|_{t=0} = x^2. \end{cases}$$

(ii) Let $u := u(t,x)$ be the solution to
$$\begin{cases} \partial_{tt}u - 4\partial_{xx}u = t(t+1) & \text{for } -\infty < x < \infty \text{ and } t > 0 \\ u|_{t=0} = \partial_t u|_{t=0} = x^2. \end{cases}$$

Prove or disprove
$$u(x,t) \geq 0 \qquad \text{for all } (x,t) \in (-\infty, \infty) \times [0, \infty).$$