Algebra II: Tutorial 6

March 22, 2022

Problem 1. Factorise the polynomial $x^{24} - 1$ over $\mathbb{Q}$. Hence, find the minimal polynomial of $\exp(\frac{2\pi i}{24})$.

Problem 2 (Algebraically closed fields are infinite)**.** Show that every algebraically closed field is infinite.

Problem 3 (Extensions of algebraically closed fields)**.** Suppose that F is algebraically closed, and let $F \subset L$ be an algebraic extension. Show that $F = L$.

Problem 4 (Degree of splitting fields: upper bound)**.** Let K be any field, and suppose that $f \in K[x]$ is a polynomial of degree n. Let L be a splitting field of f over K. Show that $[L : K] \leq n!$ (Hint: use induction on n).