Algebra II: Tutorial 11

April 27, 2022

Problem 1 (Galois extensions)**.** Determine whether the following extensions $L : K$ are Galois:

1. $K = \mathbb{Q}, L = \mathbb{Q}(\sqrt[3]{2})$.
2. $K = \mathbb{Q}, L = \mathbb{Q}(\sqrt[4]{2})$.
3. $K = \mathbb{Q}(\sqrt{2}), L = \mathbb{Q}(\sqrt[4]{2})$.
4. $K = \mathbb{Q}(i), L = \mathbb{Q}(i, \sqrt[4]{2})$.
5. $K = \mathbb{Q}(t^2), L = \mathbb{Q}(t)$.
6. $K = \mathbb{F}_2(t^2 + t), L = \mathbb{F}_2(t)$.

Problem 2 (Computing Galois groups)**.** Compute $G(L) = \mathrm{Aut}_K(L)$, list all subgroups H of $G(L)$ and determine the corresponding intermediate field L^H for each of the following field extensions L over K:

1. $K = \mathbb{Q}$ and $L = \mathbb{Q}(i + \sqrt{2})$.
2. $K = \mathbb{Q}(i)$ and $L = K(\sqrt[4]{2})$.