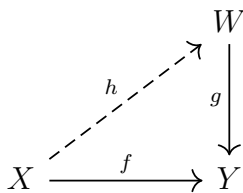


MATH3301 Tutorial 1

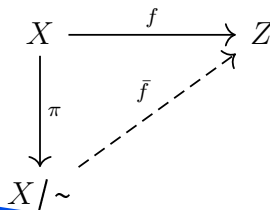
1. Explain the mathematical meaning of the underlined word(s) below: (i) a map from the set S to the set T , (ii) the composition $\psi \circ \phi$ of two maps ϕ and ψ , (iii) an injective map, (iv) a surjective map and a bijective map.
2. Let X, Y be two sets and $f : X \rightarrow Y$ be a function.
 - (a) Suppose f is surjective. Show that for any two functions $g : Y \rightarrow Z$ and $h : Y \rightarrow Z$, $g \circ f = h \circ f$ implies $g = h$.
 - (b) Suppose for any two functions $g : Y \rightarrow Z$ and $h : Y \rightarrow Z$, $g \circ f = h \circ f$ implies $g = h$. Show that f is surjective.
 - (c) Suppose for any set W and any function $g : W \rightarrow Y$, there exists a function h such that the following diagram commutes:



Must the function f be surjective? Justify your answer.

3. Let X, Y be two sets and $f : X \rightarrow Y$ be a function.
 - (a) Give an example of functions f, g, h such that f is injective, $g \circ f = h \circ f$ but $g \neq h$.
 - (b) Suppose for any two functions $g : Z \rightarrow X$ and $h : Z \rightarrow X$, $f \circ g = f \circ h$ implies $g = h$. Show that f is injective. Is the converse true?

4. (a) Explain the mathematical meaning of an equivalence relation.
- (b) Explain whether the following relations are equivalence relations.
- (i) Consider the set \mathbb{Z} of all integers and define the relation \sim on \mathbb{Z} as follows:
for any $a, b \in \mathbb{Z}$, $a \sim b$ if $ab \geq 0$.
- (ii) Consider the set C of all composite numbers (i.e. positive integers that are not primes) and define the relation \sim on C as follows:
for any $a, b \in C$, $a \sim b$ if $\gcd(a, b) > 1$.
5. Let $\{S_\alpha : \alpha \in \mathcal{A}\}$ be a family of non-empty sets, and $X = \bigcup_{\alpha \in \mathcal{A}} S_\alpha$. Assume $S_\alpha \cap S_\beta = \emptyset$ for any $\alpha \neq \beta \in \mathcal{A}$. Define the relation on X by
- for any $x, y \in X$, $x \sim y$ if both x, y are belonged to the same S_α , for some $\alpha \in \mathcal{A}$.
- (a) Show that \sim is an equivalence relation on X . Describe, in terms of S_α , the equivalence classes $[x]$ of elements $x \in X$.
- (b) Could we obtain the same result if the union is not disjoint? Justify your answer.
- (c) Write X/\sim for the quotient set of the equivalence relation, i.e. the set of all equivalence classes.
- (i) Show that $\pi : X \rightarrow X/\sim$, $x \mapsto [x]$, is a well-defined surjective function.
- (ii) If $f : X \rightarrow Z$ is a function that is constant on every S_α , i.e. for all $\alpha \in \mathcal{A}$, there exists $z_\alpha \in Z$ such that $f(x) = z_\alpha$ for all $x \in S_\alpha$, show that there is a unique function $\bar{f} : X/\sim \rightarrow Z$ such that the following diagram commutes:



- (iii) Does the converse of Part (ii) hold?

End