

# Algebra II: Tutorial 8

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**Problem 1.** Let  $K$  be field, and  $a_1, a_2, \dots, a_n$  be algebraic over  $K$ . Let  $L = K(a_1, a_2, \dots, a_n)$ , and suppose that  $K \subset M \subset L$ .

1. Show that  $M(a_1, a_2, \dots, a_n) = L$ . Deduce that if  $L$  is a splitting field of  $f$  over  $K$ , then  $L$  is a splitting field of  $f$  over  $M$ .
2. Give an example of polynomial  $f$  over  $K$  and a field extension  $M$  of  $K$  such that the splitting fields of  $f$  over  $K$  and  $M$  are not isomorphic.

**Problem 2.** Give an example of a normal algebraic extension of  $\mathbb{Q}$  which is not finite.

**Problem 3.** Show that an algebraic extension  $K \subset L$  is normal if and only if for every  $\alpha \in L$ , the minimal polynomial of  $\alpha$  over  $K$  splits completely over  $L$ .

**Problem 4.** Show that any quadratic extension is normal.

**Problem 5.** Let  $K \subset L \subset M$  with  $K \subset L$  normal and  $L \subset M$  normal. Does this imply that  $K \subset M$  is normal?