THE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH4406
Introduction to Partial Differential Equations
Tutorial 1

Problem 1. Evaluate the following integrals.

(i) $\displaystyle\int_{-\pi}^{\pi} \cos(x)\cos(4x)\,dx$

(ii) $\displaystyle\int_{0}^{2\pi} x^2 \sin(mx)\,dx$, $m \in \mathbb{Z}$.

Problem 2. For any $t > 0$ and $x > \mathbb{R}$, verify that

$$v(t,x) := t^{-\frac{1}{2}} \exp\left(\frac{2(x-2)^2}{t}\right)$$

satisies the heat equation

$$\partial_t v + \frac{1}{8}\partial_{xx} v = 0.$$

Problem 3. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Suppose that $f(0) > 0$. Show that there exists $\delta > 0$ such that

$$\int_{-\delta}^{\delta} f(x)\,dx > 0.$$

Problem 4. Let $u := u(t)$ be a solution to the initial-value problem (IVP) of the ordinary differential equation (ODE)

$$\begin{cases} \dfrac{du}{dt} = 4u^3 \quad \text{for } t > 0, \\ u(0) = 2. \end{cases}$$

Show that there exists a constant $T > 0$ such that

$$\lim_{t \to T^-} u(t) = \infty.$$

Problem 5. Let

$$A := \begin{pmatrix} 0 & -1 \\ 4 & -4 \end{pmatrix},$$

and complete the following parts.

(i) Show that -2 is the only eigenvalue of A and find the corresponding eigenvectors.

(ii) Find an invertible 2×2 real matrix P such that

$$P^{-1}AP = J_2(2) := \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix}.$$

(iii) Solve the following system of linear ordinary differential equations (ODE):

$$\frac{d}{dt}\mathbf{x}(t) = A\mathbf{x}(t)$$

with the initial condition

$$\mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$