MATH3301 Tutorial 2

1. Let G be a group and let a, b, c be any elements in G.
   (a) Using the axioms of group theory, demonstrate how $(ab)^{-1}$ can be expressed as a product of the inverses of a and b.
   (b) Show that the equation $ax = b$ (with unknown x) has a unique solution.
   (c) Show that the equation $xa = b$ (with unknown x) has a unique solution.
   (d) Develop **Cancellation Laws**.
   [Hint: In $\mathbb{Z}$, one part of the cancellation law yields that $3m = 3n$ implies $m = n$.]

2. Let X be a set. For any subsets U, V of X, we define

$$U - V = \{x \in U : x \notin V\} \qquad \text{and} \qquad U \ominus V = (U - V) \cup (V - U).$$

   Let $P(X)$ be the **power set of** X (i.e. the set of all subsets of X). Show that
   (i) $\ominus$ is a well-defined operation, and
   (ii) $(P(X), \ominus)$ is a group.

3. Let $\varnothing \neq G$ be a set and $*$ be an associative binary operation on G (i.e. $a*(b*c) = (a*b)*c$, $\forall\ a, b, c \in G$). Consider the following properties:

   (RId) $\exists\ e \in G$ such that $a * e = a$, $\forall\ a \in G$.
   (LInv) $\forall\ a \in G$, $\exists\ b \in G$ such that $b * a = e$.
   (RInv) $\forall\ a \in G$, $\exists\ b \in G$ such that $a * b = e$.
   (LC) $a * b = a * c \Rightarrow b = c$,
   (RC) $b * a = c * a \Rightarrow b = c$.

   (a) Take G to be the set of all constant functions from the set $\{\pm 1\} := \{-1, +1\}$ to $\{\pm 1\}$, and the operation $*$ to be the function composition.
   (i) Show that G has the property (RId). Is the element e in Condition (RId) unique?
   (ii) Show that G has the property (LInv). Is $(G, *)$ a group?
   (b) Show that G is a group if G satisfies (RId) and (RInv)
   (c) Show that G is a group if G is finite and G satisfies both (LC) and (RC).

End