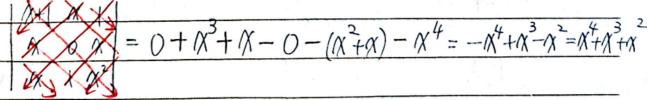
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20250525 MATH 4302 Assignment 3	
(1) Solution: For the following 15[x]-entry mutrix:	
$A = \begin{pmatrix} \alpha + 1 & \alpha & 1 \\ \alpha & O & \alpha \\ \alpha & 1 & \alpha^2 \end{pmatrix}$	
Step : All x minors of A are of, W, V, W, o, W, W, W, W	
Hence, the ordeal I(A) generated by them has a single gener	
$m_1(A) = \gcd(x+1, x, 1, 0, x^2) = 1$	
Step 2: All 2x2 minors of A are:	= -
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
	$\left \frac{1}{2} \right = \sqrt{3}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	X = V.
Hence, the oderal Io(A) generated by them has a single of	Lenerator
$M_{2}(A) = \gcd(\chi^{2}, \underline{\chi^{2}} + \underline{\chi^{3}} + \underline{\chi^{2}} + \underline{\chi}, \underline{\chi^{3}} + \underline{\chi^{2}} + + \chi^{$	= .
inedacible in $[f_2[x]] - \infty < [eq(x) < 2]$	1

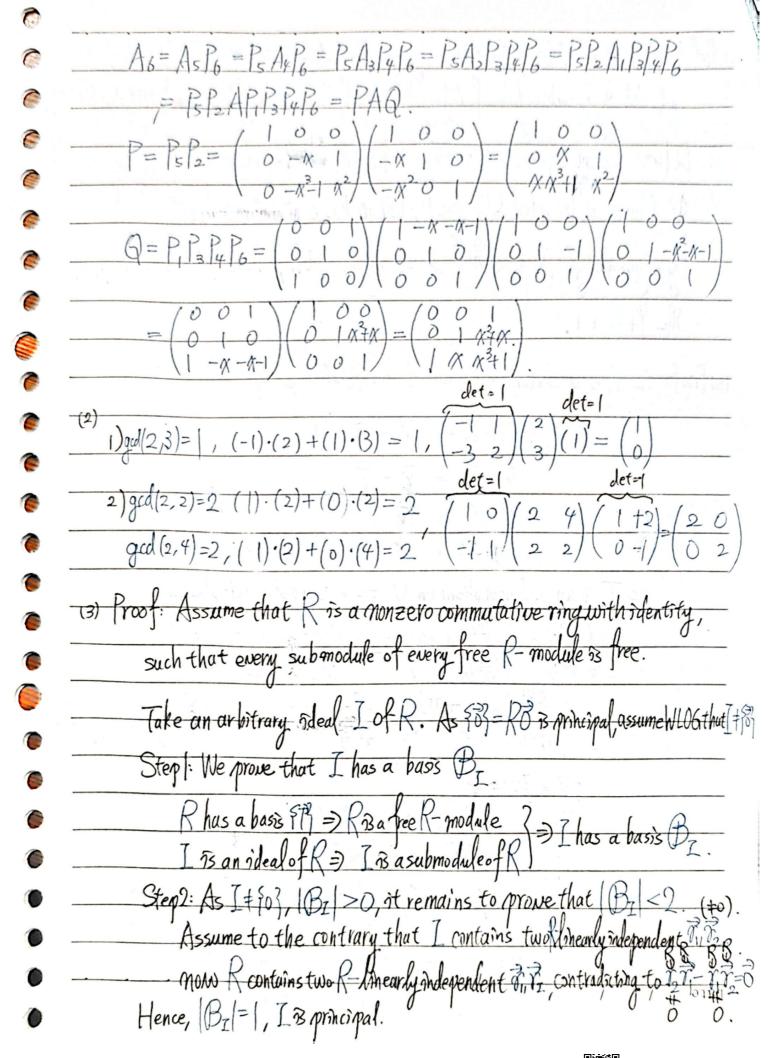
Step3: All 3×3minors of A are:

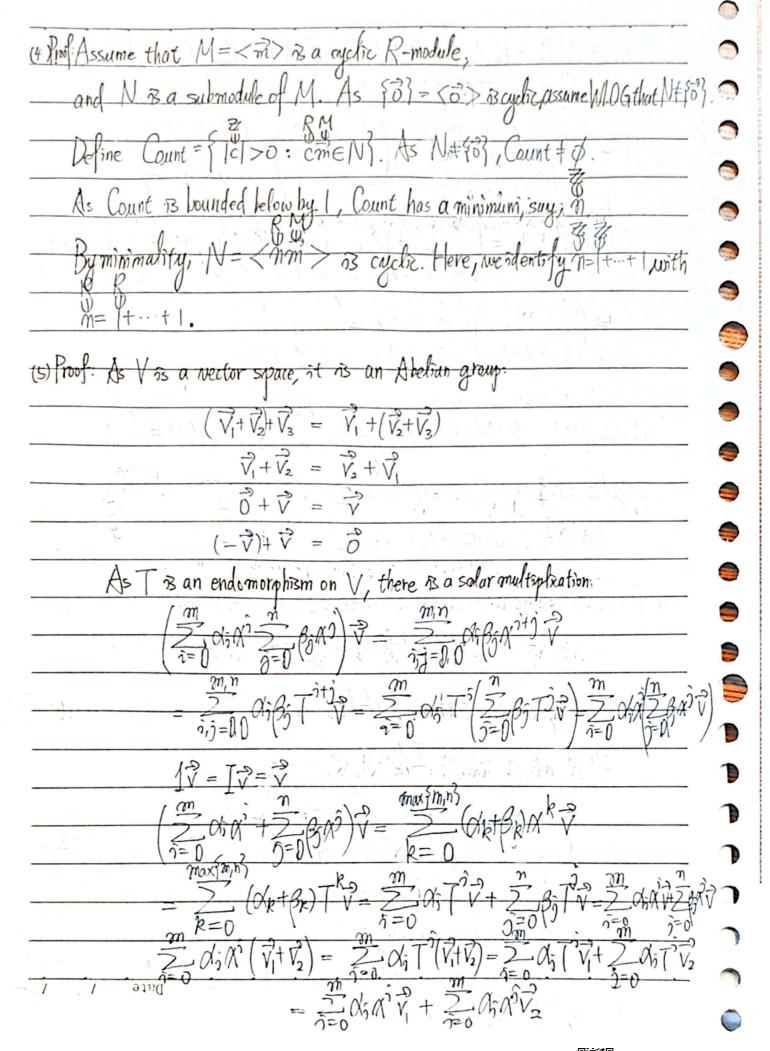


Hence, the rideal Is(A) generated by them has a single generator:

m3(A)=ged (x4+1x3+1x2)=x4+1x3+1x2

To conclude, the invariant factors of A are: $d_1 = m_1 = 1$, $d_2 = m_2/m_1 = 1/1 = 1$, $d_3 = m_3/m_2 = (\chi^4 + \chi^3 + \chi^2)/1 = \chi^4 + \chi^3 + \chi^2$ The Smith Normal Form of A is: course, we may also compute this by reduction. HAFR 0





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	As V os finite-domensional, ot os a finite K-span, thus afinite K[a]-span.
	As Tos an endomorphism on a finite-climensional space V, Thus a minimal
	molynomial f(x) + 0, so every element i annihilates when acted by f(x), it is torsioned
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	(6) Proof: OEN = DE DN;
	$\overrightarrow{V}_{i} \in \mathbb{N}_{i}$ and $\overrightarrow{V}_{i} \in \mathbb{N}_{j} \Rightarrow \overrightarrow{V}_{i} + \overrightarrow{V}_{i} \in \mathbb{N}_{mux_{1},j} \subseteq \mathbb{N}_{i}$.
	$\overrightarrow{V_i} \in \mathbb{N}_i \Rightarrow -\overrightarrow{V_i} \in \mathbb{N}_i \subseteq \overrightarrow{U} \times \mathbb{N}_i$
	Hence, $0 V_i = \{ \vec{v} \in M : \vec{n} \text{ belongs to some } V_j \} $ a sub-module of M . (7) Proof: ϕ_n is a well-defined function $(m_1, m_2 \in \mathbb{Z}, m_1 = m_2 \pmod{n}) \Rightarrow \phi_n(m_1) = \phi_n(m_2)$
•	(7) Proof: ϕ_n is a well-defined function \Rightarrow $(+m_1, m_2 \in \mathbb{Z}_{/\Delta}, m_1 = m_2 (m_0 d_n) \Rightarrow \phi_n(m_1) = \phi_n(m_2))$
•	$\Leftrightarrow (\forall m \in \mathbb{Z}, m \equiv O(m \circ d n) \Rightarrow \not p_n(m) = O) \Leftrightarrow m \circ d = \emptyset$
	Here, we assumed that on is a homomorphism offit is well-defined function.
•	This is the case, because $\varphi_n(k_1+k_2)=(k_1k_2)a=k_1a+k_2a=\varphi_n(k_1)+\varphi_n(k_2)$.
e	Let's proceed to prove that the two Z-modules Homz (Z/nZiff), An
	are somorphic. To do so, construct a map: 6: tom (2/n2,A) + In, 4/n + 14/n(1).
<u></u>	This map is well-defined, because n. 4n(1) = 4n(n.1) = 4n(0)=0, 4n(1) = An.
	The map is a homomorphism, because $6(2 + \theta_n) = (2 + \theta_n)(1) = 2 + (1) + \theta_n(1) = 6(2 + \theta_n) + 6(6 + \theta_n)$
•	$6(k^{2}n) = (k^{2}n)(1) = k^{2}n(1) = k^$
	The map is originations, because 6(4/n)=0 =) HREZ/nZ, 2/n(k)=k3/n(l)=k6/4/n=0)4/n=0
•	Trismap is surjective, betause $\forall a \in A_n$, $\exists \phi_n \in Hom_2(\mathbb{Z}/n\mathbb{Z}, A)$, $G(\phi_n) = a$
	Hence, 6 ns an isomorphism.
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(8) Proof: Define a function 6: 2- + long (2/mil., 2/mil), 6(k) = pk. Here, ok Z/mZ ->Z/mZ Bdefined by ok(1) = k.[m].[m].l, where [m,n] is an inverse of m in (Z/nZ). The map as well-defined, because $\frac{m}{(m,n)}$, n are copyine, $\left[\frac{m}{(m,n)}\right]^{-1}$ exists, and: $l_1 \equiv l_2(mod^{\frac{1}{2}}) = p(l_1) \equiv p(l_2) \pmod{n}, \quad l_1 \equiv l_2(mod^{\frac{1}{2}}) = p(l_1) \equiv p(l_2) \pmod{n}, \quad l_2 \equiv p(l_1) \equiv p(l_2) \pmod{n}, \quad l_3 \equiv p(l_1) \equiv p(l_2) \pmod{n}, \quad l_4 \equiv p(l_4) \equiv p(l_4) \pmod{n}, \quad l_4 \equiv p(l_4) \equiv p(l_4) \pmod{n}, \quad l_4 \equiv p(l_4) \equiv p(l_4) \pmod{n}, \quad l_4 \equiv p(l_4) \pmod{n}, \quad l$ $\phi_{k}(\Lambda\ell) = k \left[\frac{n}{(n_{1}n)} \right] \left[\frac{m}{(m_{1}n)} \right] (\Lambda\ell) = \Lambda \left(k \left[\frac{n}{(n_{1}n)} \right] \left[\frac{m}{(m_{1}n)} \right] \right) = \Lambda \phi_{k}(\ell) \phi_{k} \psi_{m} (2) \phi_{m} (2)$ This maps a homomorphism, because $\frac{1}{\sqrt{k_1+k_2}(\ell)} = \frac{1}{(k_1+k_2)} \left[\frac{m}{(m,n)}\right] \left[\frac{m}$ $\phi_{Ak}(\ell) = (Ak) \left[\frac{m}{(m,n)} \right] \left[\frac{m}{(m,n)} \right] \ell = A \left(k \left[\frac{n}{(m,n)} \right] \left[\frac{m}{(m,n)} \right] \ell \right) = A \phi_k(\ell)$ This map 6 has kernel (m,n) &, because $\phi_k = 0 \Leftrightarrow \phi_k(1) = k \left[\frac{n}{(m_i n)} \right] \left[\frac{m_1 - 1}{(m_i n)} \right] = 0 \Leftrightarrow k \text{ is divisible by } (m_i n).$ This map 6 03 surjective, because: 1) E Homy It mit, It mit) => The order of (1) dinides (m,n) => (m,n) 2/ (1) 3 a multiple of M => 2/= PR Hence, of follows from the first Bomorphism theorem that: $\frac{I/(m,n)}{I} \cong \text{Hom}_{\mathbb{Z}}(\overline{I}/m\overline{I},\overline{I}/n\overline{I})$ When m=30, n=2, $\text{Hom}_{\mathbb{Z}}(\overline{I}/30\overline{I},\overline{I}/21\overline{I})$ has three elements: \$6 110 0. 7.10 . l. p: 111 1.7.10 . l. p.: \$192.7 10 - 8

(9). Solution: Recall that column space of invariant under column transformations.
1-70-6\ Column D+ Column O+(-2)/-70-6\ Column O+(olumn O+(-1))
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
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(-10-6) Column (0+(-1) / 10-6) Column (0+(6)
(-10-6) Column (0+(-1)) (0 -6) Column (0+(6))
006/006/
0 3 0). Hence, we not only computed the smith mormal form of A
(006)
but also gove an explorit characterization of Col(A) = ZX3ZX6Z.
Now 73/11-(7,7,7) /(2,27,17) (7/27,4/17)
(VIW 4/1V - (4×4×4)/(8×34×04)=4/34×4/04)
(lo) Proof:
Nossfinitely generated $\Rightarrow N$ has a spanning set $S = \{\vec{s_i}, \vec{s_i}, -, \vec{s_i}\}$.
M/N of shortely generated => M/N has a spanning set T= FE+N, F2+N, -; F2+1V?
Hence, M has a spanning set (\$1,52-547,72,-53), so Miss finitely generated.
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1 21-2 12 1 2 1 2 1 2 1 2 1 2 2 1 1 1 2 2 1 1 1 2 2 1 1 1 2 2 1 1 1 2 2 1 1 1 2 2 1 1 1 2 2 1 1 1 2 2 1 1 1 2 2 1 1 1 2 2 1
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