Algebra II: Tutorial 5

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Problem 1. Suppose that p and q are distinct primes. Show that $\mathbb{Q}(\sqrt{p}, \sqrt{q}) = \mathbb{Q}(\sqrt{p} + \sqrt{q})$.

Problem 2. Let L be a finite field extension of K, and consider $\alpha, \beta \in L$. If α and β have the same minimal polynomial in $K[x]$, then $K(\alpha)$ and $K(\beta)$ are isomorphic. Is the converse true?

Problem 3. Let L be a field generated over $K \subset L$ by two elements α, β. Let $p = [K(\alpha)\colon K]$ and $q = [K(\beta)\colon K]$ and assume that p and q are relatively prime.

1. Prove that $[L\colon K] = pq$.
2. If α is a fifth root of 2 and β a seventh root of 3, deduce that $[\mathbb{Q}(\alpha, \beta) : \mathbb{Q}] = 35$.

Problem 4. Let L/K be a field extension and let $f(x), g(x) \in K[x]$. Show that the greatest common divisor (with leading coefficient 1) of $f(x)$ and $g(x)$ in $L[x]$ is the same as the greatest common divisor of $f(x)$ and $g(x)$ in $K[x]$.