2024/016 MATHERO 1 Tatorial 7 [W) Proof: For all 3-cycle (15,1) & Sn. (15,1) = (1,1) (1,1) & Sthe product of 2 transpositions, so (15,1) & Am as 2 & even. (a) Proof: For all product (a,b)(c,d) of 2 transpositions: Casel: If (a,b) = (c,d), then (a,b)(c,d) = e is generated by S. Case2: If a = c and b = d, then (a,b)(c,d) = (c,a,d)(a,b) is generated by S. Case3: If (a,b)(c,d) aredisjoint, them (a,b)(c,d) = (c,a,d)(a,bc) is generated by S. Case3: If (a,b)(c,d) aredisjoint, them (a,b)(c,d) = (c,a,d)(a,bc) is generated by S. Case3: If (a,b)(c,d) aredisjoint, them (a,b)(c,d) = (c,a,d)(a,bc) is generated by S. Case3: If (a,b)(c,d) aredisjoint, them (a,b)(c,d) = (c,a,d)(a,bc) is generated by R. Case1: If r=a and s=b, then (a,b,c) = (r,s,c)(r,s,b)(r,s,c) is generated by R. Case2: If r=a and s=b, then (a,b,c) = (r,s,c)(r,s,b)(r,s,c) is generated by R. Case3: If r=a, b,c and s=b,c, then (a,b,c) = (r,s,c)(r,s,b)(r,s,c) is generated by R. Case3: If r=a, b,c and s=b,c, then (a,b,c) = (r,s,c)(r,s,b)(r,s,c) is generated by R. Case3: If r=a, b,c and s=b,c, then (a,b,c) = (r,s,c)(r,s,b)(r,s,c)(r,s,b)(r,s,c) is generated by R. This implies a generator S of An & generated by R., so An & generated by R. Case1: If r=j, then (r,s,r) = [(r,s,r)](r,s,r)[(r,s,r)](r,s		
(r,s,i)=(r,i)(r,s) is the product of 2 transpositions, so (r,s,i)=(r,i)(r,s) is the product of 2 transpositions, so (r,s,i)=(r,i)(r,s) is the product of 2 transpositions. Casel: If (a,b)=(c,d), then (a,b)(c,d)=e is generated by S. Case2: If a=c and b=d, then (a,b)(a,d)=(a,d,b) is generated by S. Case3: If (a,b)(c,d) are disjoint, then (a,b)(c,d)=(c,a,d)(a,b,c) is generated by S. This implies a generator T= f(a,b)(c,d)=An (a,b),(c,d) are transpositions is generated by S. so An is generated by S. (c) Proof For all 3=dycle: (a,b,c)=An, (a,b,c)=(r,s,c) is generated by R. Case1: If r=a and s=b, then (a,b,c)=(r,s,c) is generated by R. Case2: If r=a and s=b, then (a,b,c)=(r,s,c)(r,s,b)(r,s,c) is generated by R. Case3: If r=a, b, c and s=a,b, c, then (a,b,c)=(r,s,c)(r,s,b)(r,s,c) is generated by R. This implies a generator S of An & generated by R., so An & generated by R. Torall 3-cycle (r,s,i) eAn: Case1: If r=j, then (r,s,i)=(r,s,j) eN. Case2: If r=j, then (r,s,i)=(r,s,j) eN. Case2: If r=j, then (r,s,i)=(r,s,j)(r,s,j)(crs)(r		2024/0/6 MATH330 Tutorial 7
(r,s,i)=(r,i)(r,s) is the product of 2 transpositions, so (r,s,i)=(r,i)(r,s) is the product of 2 transpositions, so (r,s,i)=(r,i)(r,s) is the product of 2 transpositions. Casel: If (a,b)=(c,d), then (a,b)(c,d)=e is generated by S. Case2: If a=c and b=d, then (a,b)(a,d)=(a,d,b) is generated by S. Case3: If (a,b)(c,d) are disjoint, then (a,b)(c,d)=(c,a,d)(a,b,c) is generated by S. This implies a generator T= f(a,b)(c,d)=An (a,b),(c,d) are transpositions is generated by S. so An is generated by S. (c) Proof For all 3=dycle: (a,b,c)=An, (a,b,c)=(r,s,c) is generated by R. Case1: If r=a and s=b, then (a,b,c)=(r,s,c) is generated by R. Case2: If r=a and s=b, then (a,b,c)=(r,s,c)(r,s,b)(r,s,c) is generated by R. Case3: If r=a, b, c and s=a,b, c, then (a,b,c)=(r,s,c)(r,s,b)(r,s,c) is generated by R. This implies a generator S of An & generated by R., so An & generated by R. Torall 3-cycle (r,s,i) eAn: Case1: If r=j, then (r,s,i)=(r,s,j) eN. Case2: If r=j, then (r,s,i)=(r,s,j) eN. Case2: If r=j, then (r,s,i)=(r,s,j)(r,s,j)(crs)(r		(a) Proof: For all 3-cycle (7,5,1) & Sn,
So (risi) Am as 2 B even. (a) Proof For all product (a,b)(c,d) of 2 transpositions: (ase): If (a,b)=(c,d), then (a,b)(c,d)= e is generated by S (ase2: If a=c and b+d, then (a,b)(a,d)=(a,d,b) is generated by S (ase3: If (a,b)(c,d) are disjoint, then (a,b)(c,d)=(c,a,d)(a,b,c) is generated This implies a generator T= f(a,b)(c,d)=An: (a,b),(c,d) are transpositions is generated by S, so An is generated by S: (c) Proof: For all 3-cycle: (a,b,c) = An, (ase1: If r=a and s=b, then (a,b,c)=(r,s,c) (risib)(risc) is generated by R (ase2: If r=a and s+b,c, then (a,b,c)=(r,s,c)(risib)(risc) is generated by R (ase3: If r+a,b,c and s+a,b,c,then = (a,b,c)=(r,a,b)(r,b,c)=(r,s,b)(r,s,a)(risb)(risc)(risc)(risc) is generated by R This implies a generator S of An & generated by R, so An & generated by R (d) Proof: Assume that N contains a 3-cycle (r;s,j) EAn. For all 3-cycle (r;s,i) = An; (ase2: If i=j, then = (r;s,i)=(r;s,j) = N. (ase2: If i=j, then = (r;s,i)=(r;s,j)=(r;s,j)(crs)(r;s,i)] = N.		
(d) Proof: For all product (a,b)(c,d) of 2 transpositions: (ase): If (a,b)=(c,d), then (a,b)(c,d)=e is generated by S (ase2: If (a,b)(c,d), aredisjoint, then (a,b)(c,d)=(c,a,d)(a,b,c) is generated by S (ase3: If (a,b)(c,d), aredisjoint, then (a,b)(c,d)=(c,a,d)(a,b,c) is generated This implies a generator T= \{(a,b)(c,d)\) An (a,b), (c,d) are transpositions: B generated by S, so An is generated by S: (c) Proof: For all 3=bycle (a,b,c) \(\frac{1}{2}\) An, (ase): If r=a and s=b, then (a,b,c)=(r,s,c) \(\frac{1}{2}\) is generated by R (ase2: If r=a and s+b,c, then (a,b,c)=(r,s,c) \((r,s,c)\) is generated by R (ase3: If r=a and s+b,c, then (a,b,c)=(r,s,c)(r,s,b)(r,s,c) \(\frac{1}{2}\) is generated by R (a,b,c)=(r,a,b)(r,b,c)=(r,s,b)(r,s,a)(r,s,b)(r,s,c)(r,s,b)(r,s,c) is generated by R This simplies a generator S of An & generated by R=so An & generated by R (d) Proof: Assume that N contains a 3-cycle (r,s,j) \((r,s,j)\) (r,s,c) Finall 3-cycle (r,s,i) \((r,s,i)\) = [(r,s,i)\((r,s,j)\)](r,s,j) [(r,s,j)\((r,s,j)\)](exs(i,j)] = N. (ase2: If i=j, then (r,s,i) = [(r,s)(i)\((r,s,j)\)](r,s,j) [(r,s,c)(i,j)] = N.		
Case: If (a,b)=(c,d), then (a,b)(c,d)=e is generated by S Case: If a=c and b=d, then (a,b)(c,d)=(a,d,b) is generated by S Case: If (a,b)(c,d) are disjoint, then (a,b)(c,d)=(c,a,d)(a,b,c) is generated This implies a generator T= \{(a,b)(c,d)\)eAn: (a,b), (c,d) are transpositions Begenerated by S, so An is generated by S: (c) Proof: For all 3-bycle: (a,b,c) & An, is generated by S: (ase: If r=a and s=b, then (a,b,c)=(r,s,c) is generated by R Case: If r=a and s=b, then (a,b,c)=(r,s,c) (r,s,b)(r,s,c) is generated by R Case: If r=a and s=b, c, then (a,b,c)=(r,s,c)(r,s,b)(r,s,c) is generated by R This implies a generator S of An & generated by R=so An & generated by R This implies a generator S of An & generated by R=so An & generated by R Torall 3-cycle (r,s,i) eAn: Case: If i=a, then (r,s,i)=(r,s,j) eN. Case: If i=a, then (r,s,i)=(r,s,j)(r,s,j)(cr,s)(r,s,		
Case 3: If (a,b)(c,d) are disjoint, then (a,b)(c,d)=(a,d,b) is generated by S Case 3: If (a,b)(c,d) are disjoint, then (a,b)(c,d)=(c,a,d)(a,b,c) is generated This implies a generator T= \{(a,b)(c,d) \in An\}: (a,b)(c,d) \in An\}: (a,b), (c,d) are transpositions By generated by S, so An is generated by S; (c) Proof: For all 3-cycles (a,b,c) \in An\}; generated by S; (ase 1: If r=a and s=b, then (a,b,c)=(r,s,c) is generated by R. Case 2: If r=a and s+b,c, then (a,b,c)=(r,s,c)(r,s,b)(r,s,c) is generated by R. Case 3: If r=a, b, c and s+a,b,c, then (a,b,c)=(r,a,b)(r,b,c)=(r,s,b)(r,s,a)(r,s,b)(r,s,c)(r,s,b)(r,s,c) is generated by R. This implies a generator S of An & generated by R=so An & generated by R This implies a generator S of An & generated by R=so An & generated by R Torall 3-cycle (r,s,i) eAm; Case 1: If r=j, then (r,s,i)=(r,s,j) eN. Case 2: If r=j, then (r,s,i)=(r,s,j) eN.		
This implies a generator $T = \{(a,b)(c,d) \in A_n: (a,b), (c,d) \text{ are transpositions}\}$ By generated by S , so A_n is generated by S : (c) Proof: For all 3 -dycle $(a,b,c) \in A_n$ Case 1 : If $Y = a$ and $S = b$, then $(a,b,c) = (Y,S,c)$ is generated by R . Case 2 : If $Y = a$ and $Y = b$, then $(a,b,c) = (Y,S,c)(Y,S,c$,	Case 2: If a=c and b=d, then (a,b)(a,d)=(a,d,b) is generated by S
This implies a generator $T = \{(a,b)(c,d) \in A_n: (a,b), (c,d) \text{ are transpositions}\}$ By generated by S , so A_n is generated by S : (c) Proof: For all 3 -dycle $(a,b,c) \in A_n$ Case 1 : If $Y = a$ and $S = b$, then $(a,b,c) = (Y,S,c)$ is generated by R . Case 2 : If $Y = a$ and $Y = b$, then $(a,b,c) = (Y,S,c)(Y,S,c$		Case 3: If (a,b)(c,d) aredisjoint, then (a,b)(c,d)=(c,a,d)(a,b,c) is generated
By generated by S, so An is generated by S' (c) Proof: For all 3-dycler (a,b,c) & An, (ase): If r=a and s=b, then (a,b,c)=(r,s,c) is generated by R. (ase): If r=a and s+b,c, then (a,b,c)=(r,s,c)(r,s,b)(r,s,c) is generated by R. (a,b,c)=(r,a,b)(r,b,c)=(r,s,b)(r,s,a)(r,s,b)(r,s,c)(r,s,b)(r,s,c) is generated by R. This ômphes a generator S of An is generated by R=so An is generated by Roof: Assume that N contains a 3-cycle (r,s,j) & An. For all 3-cycle (r,s,i) & Am; (ase): If i=j, then (r,s,i)=(r,s,j) & N. (ase): If i=j, then (r,s,i)=[(r,s)(i,j)](r,s,j)[(r,s)(i,j)] & N.		This simplies a generator $T = \{(a,b)(c,d) \in A_n: (a,b), (c,d) \text{ are transpositions}\}$
(c) Proof: For all 3-dycle (a,b,c) & An, (a,b,c) - (r,s,c) is generated by R. Case 2: If r=a and s=b, then (a,b,c) = (r,s,c) (r,s,b) (r,s,c) is generated by R. Case 3: If r=a and s+b,c, then (a,b,c) = (r,s,c) (r,s,b) (r,s,c) is generated by R. (a,b,c) = (r,a,b) (r,b,c) = (r,s,b) (r,s,a) (r,s,b) (r,s,c) (r,s,b) (r,s,c) is generated by R. This implies a generator S of An & generated by R, so An & generated by R of the contains a 3-cycle (r,s,j) & An. Torall 3-cycle (r,s,i) & An; (ase 1: If i=j, then (r,s,i) = (r,s,j) & N. Case 2: If i+j, then (r,s,i) = [(r,s)(i,j)](r,s,j)[(r,s)(i,j)] & N.		
Case 2: If $r=a$ and $s+b$, c , then $(a_1b,c)=(r,s,c)$ (r,s,c) is generated a_1b , a_2b , a_3b	•	
Case 3: If $r \neq a,b,c$ and $s \neq a,b,c$, then $(a,b,c) = (r,a,b)(r,b,c) = (r,s,b)(r,s,a)(r,s,b) \in r,s,c)(r,s,b)(r,s,c)$ $rose_{assume} = rose_{assume} = rose$		Case : If r=a and s=b, then (a,b,c)=(r,s,c) is generated by R.
Case 3: If $r \neq a,b,c$ and $s \neq a,b,c$, then $(a,b,c) = (r,a,b)(r,b,c) = (r,s,b)(r,s,a)(r,s,b) \in r,s,c) \cdot (r,s,b)(r,s,c)$ $r_{1} = r_{2} = r_{3} = r_{4} = r_{3} = r_{4} = r$		Case 2: If r=a and s+b,c, then (a,b,c)=(r,s,c)(r,s,b)(r,s,c) as generated
(a,b,c)=(r,a,b)(r,b,e)=(r,s,b)(r,s,a)(r,s,b)(r,s,c)(r,s,b)(r,s,c) 3 generated by R. This implies a generator S of An & generated by R=, so An & generated by Contains a 3-cycle (r,s,j) \in An. Forall 3-cycle (r,s,i) \in An; (a\in 1=j, then=(r,s,i)=(r,s,j)\in N. (a\in 2: If i=j, then=(r,s,i)=(r,s,j)\in N. (a\in 2: If i=j, then (r,s,i)=[(r,s)(i,j)](r,s,j)[(r,s)(i,j)] \in N.		
This implies a generator S of An & generated by R=, so An & generated b		(a,b,c)=(r,a,b)(r,b,c)=(r,s,b)(r,s,a)(r,s,b)(r,s,c)(r,s,b)(r,s,c)
This implies a generator S of A_n B generated by R_n so A_n B generated by R_n A_n A		73 generated by R.
In Proof: Assume that N contains a 3-cycle (r,s,j) $\in A_n$. For all 3-cycle (r,s,i) $\in A_n$; Case: If $i = j$, then $= (r,s,i) = (r,s,j) \in N$. Case2: If $i \neq j$, then $= (r,s,i) = [(r,s)(i,j)](r,s,j)[(r,s)(i,j)] \in N$.		This amplies a generator Sof An & generated by R, so An B generated by
For all 3-cycle $(r,s,i) \in A_m$; $(a \neq i) = j$, then $(r,s,i) = (r,s,j) \in N$. $(a \neq 2) = [(r,s)(i),j) = [(r,$		(d) Proof. Assume that N contains a 3-cycle (r, s, DEAn
Case: If $i = j$, then $(r,s,i) = (r,s,j) \in \mathbb{N}$. Case2: If $i \neq j$, then $(r,s,i) = [(r,s)(i)j)](r,s,j)[(r,s)(i)j)] \in \mathbb{N}$.		Firell 2-cycle (rsi) eAn.
Case 2: If $i \neq j$, then $(r, s, a) = [(r, s)(i)j](r, s, j)[(r, s)(i)j] \in \mathbb{N}$.		. • //
I IIS OMALINES I ULDMULINS A GENERATOR IS OF RIS 30 17 CM		
¥		I his omarines 10 contains a generator is of the 30 11-th

2.00 Proof Since M, (a, ..., a) are disjoint, M, (a, ..., a) are disjoint, $50 \quad 6^{+}(Q_{11}Q_{21}Q_{3}) \quad 6(Q_{11}Q_{31}Q_{3})^{-} = (Q_{11}...,Q_{1r})^{-} \mu^{-}(Q_{11}Q_{21}Q_{3}) \mu^{-}(Q_{11}...,Q_{1r})^{-} \mu^{-}(Q_{11}Q_{21}Q_{3}) \mu^{-}(Q_{11}Q_{31}Q_{3}) \mu^{-}(Q_{11}Q_{31}Q_{3}) \mu^{-}(Q_{11}Q_{31}Q_$ = (a, ..., ar) / / / (a, d2, d3) (a, ..., ar) (a, d2, (3) $(a_1, a_2, a_3)(a_1, a_2, a_3) =$ orall 3<R<Y: 2) ak= (a1, ..., ar) (a1, a2, a3) (a1, ..., ar) (a1, a2, a3) ak = (a,, dr) (a, a2, a3) (a,, ..., ar) ak = (a, ..., ar) (a1, a2, a3) art = (a, ..., ar) art = ak It remains to investigate and, a, a, a; $2 Q_1 = (a_1, \dots, a_r)^{-1} (a_1, a_2, a_3) (a_1, \dots, a_r) (a_1, a_2, a_3)^{-1} Q_1$ (a, .., ar) (a, az, az) (a, .., ar) az-(a, .., dr) (u, a2, a3) ay = (a, .., dr) ay = a3 $\mathcal{L}(Q_2 = (Q_1, \dots, Q_r)^{-1}(Q_1, Q_2, Q_3)(\tilde{Q}_1, \dots, Q_r)(Q_1, Q_2, Q_3)^{-1}Q_2$ (a,, a,) (a,d2, d3) (dy,, dr) d1 = $(a_1, a_2, a_3) a_2 = (a_1, a_2, a_3) a_3 = a_2$ 21 Cl3 = (a, -, a,) (a, d2, a3) (a,, -, ar) (a, d2, a3) d3 (a, ..., ar) (a, az, az) (a, -, av) az $=(\alpha_1, \dots, \alpha_r)^{\dagger}(\alpha_1, \alpha_2, \alpha_3) \alpha_3 = (\alpha_1, \dots, \alpha_r)^{\dagger} \alpha_1 = \alpha_r$ $u_r = (a_1, ..., a_r)^{-1}(a_1, a_2, a_3)(a_1, ..., a_r)(a_1, a_2, a_3)^{-1}a_r$ $= (a_1, \dots, a_r)^{-1}(a_1, a_2, a_3)(a_1, \dots, a_r) a_r$ $= (a_1, \dots, a_r)^{-1}(a_1, a_2, a_3)a_1 = (a_1, \dots, a_r)^{-1}(a_2 = a_1)$ lence, 2 = (a, l3, ar) is a 3-cycle, it remains to prove 201: <u>(5i) (d,,d2,03) 6(a,,d2,03) ¹∈N</u>

(b) Proof: 6= M(a4, a5, a6) (a1, a2, a3), where M, (a1, a2, a3), (a4, a5, a6) are mutually dizjoint. Since 4 (a11aza), (a4az, 06) are mutually disjoint, 4,(a1, az, a4) are disjoint, 30 5 (a, a, a, a, b) 5(a, a, a, a, a) = (a, a, a, a) (a, a, a, a) 1/2 (a, a, a, a, b) 1/2 (a, a, a, a, b) (Q1,Q2,Q3) (Q1,Q2,Q4) = (Q1,Q2,Q3) (Q4,Q5,Q6) 1/10 /4 (Q1,Q5,Q4) (Q4,Q5,Q6) (Q1,Q2,Q3) Each factor is disjoint with \$1,50 21 is disjoint with \$1. According to my permutation calculator to in the contract of 21= (Q1, Q3, Q2) (Q4, Q6, Q5) (Q1, Q2, Q4) (Q4, Q5, Q6) (Q1, Q2, Q6) (Q1, Q4, Q2) = (a,, a4, a2, a6, a3) 73 a cycle of length 5 It remains toppose 2001 => 21=6 [(a, b, a) 6(a, b, a)] EN t (19) (and 204) 6(and 204) (A) (c) Proof: Assume that $\mu = (a_4, a_5)(a_6, a_7)$, (where (a, a, a), (a, a, a), (a, a, a) are mutually define 6= (U(a,a,a,a))=(a4,le)(a6,a7)(a,a,a3))=(a4,a5)2(a6,a7)2(a,a2,a3)2 = (a1, a3, a2) & a 3-cycle, and 66N > 6°6N (d) froof: 6= µ(ls, a4)(a, a2), where µ, (a3, a4), (a, a2) are mutually disjoint. Since M. (a3, a4), (a,, a2) are mutually obsioint, U, (a,, a2, a3) are disjoint SO 5 (a, a, a, a) 6(a, a, a, a) = (a, a, t, t) (a, a, a, a) \(\mu(\a, \a, \a) \(\mu(\a, \a, \a) \) (a, b) (a, b) = (0,40) (as, a4) / 1/4 (a,102,03) (03,04) (a,1,02) (a,102,03) (a, d2) (a, d4) (a, d2, a3) (a, d4) (a, d2) (a, d2, a3) =2 Each factor is disjoint with (1,50 1) is disjoint with a According to my permutation calculator: 2 = (a, a, (a, d, d, (a, d, d, d) (a, a, l) (a, a, l) (a, d) = $(a_1, a_3)(a_2, a_4)$ 73 the product of two disjoint-cycles. Date

Date

(b) Proof. Assume to the contrary that An has a montravial proper mormal subgroup N.
Take 66 N/Ee3, and factorize 65 nto dozjoint cycles.
$6 = 6 \times 6_{14} \times 6_{2} = 6_{1}$
According to 2(a) and 1(d), the decomposition contains no cycle of length 1>
According to 2(b), the decomposition contains at most one 3-cycle.
According to 2(d), the decomposition contains at most one 2-cycle.
NOW 6 = (1,2) (contradiction)
or 6 = (1,2,3) (contradiction)
or $6 = (1/2)(3,4,5)$ (contradiction)
ence, our assumption is false, and we've proven that An issimple.
= 4.00 \$\overline{\pi_8}\overline{\pi_5}, they are the same group)
(T4× t2)× to and (T2× t2)× ts.
(c) (Thex to xtls and (thex thex the) xtls
€ 5.(a) False. Consider the sample group Am(n>5);
it has a montrivial proper subgroup Se, (1/2)(3,4)?
(b) True. If a group has no nontrivial proper subgroup,
then at has no nontravial proper normal subgroup, so it is sample.
(c) True. In an Abelian group, every subgroup is mormal,
so the Abelian group has no montrivial proper normal subgroup
mylines at has no montrainal proper subgroup.
As a consequence, this group is cyclic of arrime order.
(d) True. In Zyo, [5] to has order 8
(e) True. In an Abelian group of order 40=2.5, 2,5 are two prime lactors of 40,
so there exist 92, 95 with ord (92)=2 and ord (92)=5. Now ord (93)=10=1.2.m.(2,5).

b.(a) Proof. Do prime factorization:	
G= [p, part]x[ps part] xx[ps part]	
ΔΑ	
$d = p_1^{\beta_1} \times p_2^{\beta_2} \times \cdots \times p_n^{\beta_n}$	
P ₁ ≤ α ₁ ⇒ [p ₁ part] has a subgroupHpforder p ₁ ^p	
B≤0 => [part] has a subgroup bforder Be	
βe≤αe => [pe part] has a surgroupt of order pe	
It suffices to take H=H, XH, XXHe	
(b) Consider the group Z4XZ with order 8	
(27/2) × //2 and 7/4×(0/2) are two subgroups of 7/1/2 of order 4	_
Notice that (2 they) × the doesn't have an element of order 4	•
and L4X(0Z2) has an element ([1]4,[0]2) of order 4	
50 (2ty)× 1/2 = Tyx(0t/2).	
(c) If d has a prime factor of with multiplicity. 02,	1
then take [ZnoxZnox-xZno] x [The remaining parts] and meaned one.	.
If I has no prime factor of with multiplicity <>2,	
$\frac{1}{2}$	
we can find g=q, q=-qm with order d.	
<u> </u>	
) 91td	