UNIVERSITY OF VICTORIA
MIDTERM EXAM FEBRUARY 22 2018
COMPUTER SCIENCE 349A

NAME: ______________________________ STUDENT NO. ________________________
INSTRUCTOR: Rich Little
. DURATION: 50 minutes

TO BE ANSWERED ON THE PAPER

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO ME.

PLEASE PUT YOUR NAME ON THE VERY BACK SHEET AS WELL.

THIS QUESTION PAPER HAS 4, SINGLE-SIDED PAGES. YOU MAY USE THE BACK PAGES.

NOTES:
(0) CLOSED BOOK EXAM; ONLY BASIC CALCULATORS ARE ALLOWED,
(1) ANSWER ALL QUESTIONS,
(2) THERE ARE A TOTAL OF 30 MARKS,
(3) THE BACK PAGE OF EACH QUESTION MAY BE USED FOR YOUR ANSWERS.
(4) YOU ARE ALLOWED ONE 8.5-by-11 INCH CHEAT SHEET — DOUBLE-SIDED.
(5) SHOW YOUR WORK / SUPPORT YOUR ANSWERS!

Question	Possible marks	Actual marks
1	10	
2	10	
3	10	
Total	30	

1. (a) [2 points] Consider a hypothetical binary computer with an 8-bit normalized, floating-point representation with precision $k = 4$. If we store the lead 1, what is the largest decimal value we can store?

   (b) [2 points] Consider the following function

   $$f(x) = \frac{-x - \sqrt{x^2 - 0.1}}{2}$$

   For what values of x may the evaluation of $f(x)$ suffer from subtractive cancellation? Why?

   (c) [2 points] How many iterations of the bisection method must run to gaurantee that the absolute error of your approximation to the root is less than 10^{-6} if the initial interval is $[0, 3]$?

   (d) [4 points] Newton's method with the initial approximation $x_0 = 3$ will converge to the zero at $x = 2$ of $f(x) = e^{x-2}(14x - 12) - 7x^3 + 20x^2 - 26x + 12$. Is the order of convergence of this computation quadratic? Justify. (DO NOT USE MULTIPLICITY)

2. (a) [5 points] Determine the second order ($n = 2$) Taylor polynomial approximation for $f(x) = \sqrt{x-2}$ expanded about $a = 3$. Do not include the remainder term. Show all of your work. It is not necessary to algebraically simplify the form of this polynomial.

   (b) [2 points] Determine the remainder term for the polynomial approximation in (a).

   (c) [3 points] Determine a good upper bound for the truncation error of the Taylor polynomial approximation in (a) when $2.9 \leq x \leq 3.15$ by bounding the remainder term.

3. (a) [3 points] Use 4 decimal digit, idealized, chopping floating-point arithmetic, to show that $fl(g(0.011))$ gives the value 7.513, where

$$g(x) = \frac{x - \sin x}{x^3}.$$

(b) [3 points] The fifth order Taylor polynomial approximation of $f(x) = \sin x$, expanded about $a = 0$, is

$$\sin(x) \approx x - \frac{x^3}{6} + \frac{x^5}{120}.$$

Use this to get an accurate approximation to $g(x)$ in part (a) when x is close to 0.

(c) [4 points] To 6 significant digits the correct value of $g(0.011)$ is 0.166666. Is the function $g(x)$ given in (a) ill-conditioned or well-conditioned? Fully justify your answer using the notation and definition of condition given in class. Do NOT use the condition number.