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CSC 349A
Assignment 2

**Q1. (a)**

Using the representation $02003004_5$:

$02003004_5 = 0.2003 \times 5^4$ (in base 5)

Convert to decimal:

$(2*5^{-1} + 3*5^{-4})*5^4 = 253_{10}$

**Q1. (b)**

Using the representation $11004003_5$:

$11004003_5 = -0.1004 \times 5^3$(in base 5)

Convert to decimal:

$-(1*5^{-1} + 4*5^{-4})*5^3 = -25.8_{10}$

**Q1. (c)**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **0** | **1** | **0** | **0** | **0** | **1** | **4** | **4** |

It is in floating-point number system so we have:

$0.1000*5^{-24}$ because base 5 can have values (0-4)

Convert to decimal:

$(1*5^{-1})*5^{-24} = 3.3554432 \times 10^{-18}$

**Q1. (d)**

$[25, 125) \Rightarrow [5^2, 5^3)$ with b=5 and k=2 then t=3

Using gap = $b^{t-k}$ we have $5^{3-2} = 5_{10}$

**Q2. (a)**

$P(x) = 1.2x^2 - 78.99x + 1.234$

For the first formula:

$fl(b^2) = fl(78.99^2) = fl(6239.4201) = 6239$

$fl(4a) = fl(4*1.2) = 4.8$

$fl(4ac) = fl(4.8*1.234) = fl(5.9232) = 5.923$

$fl(b^2-4ac) = fl(6239-5.923) = 6233.077 = 6233$

$fl(\sqrt{b^2 - 4ac}) = fl(\sqrt{6233}) = fl(78.94935085) = 78.94$

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fl(b-$\sqrt{b^2 - 4ac}$) = fl(-157.93) = -157.9

fl(-2c) = fl(-2*1.234) = -2.468

fl($\frac{-2c}{b-\sqrt{b^2-4ac}}$) = fl($\frac{-2.468}{-157.9}$) = 0.015630145 = 0.015

For the second formula:

fl(-b-$\sqrt{b^2 - 4ac}$) = fl(78.99-78.94) = 0.05

fl(2a) = (2*1.2) = 2.4

fl($\frac{-b-\sqrt{b^2-4ac}}{2a}$) = fl($\frac{0.05}{2.4}$) = fl(0.20833333) = 0.020

**Q2. (b)**

For first formula:

$|\varepsilon_t|$ = |(0.01562594 - 0.015)/0.01562594| = 0.04005775 = $0.400\times10^{-1}$

For second formula:

$|\varepsilon_t|$ = |(0.01562594 - 0.020)/0.01562594| = 0.279922999 = 0.279

**Q2. (c)**

| Polynomial | (i) is more accurate | (ii) is more accurate |
|---|---|---|
| $0.01x^2 - 125x + 0.05$ | X | |
| $-0.3x^2 + 125x + 0.025$ | | X |

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**Q3. (a)**

f(x) = $\sqrt{x+3}$ with a = 1 and n = 2

f(x) = $\sqrt{x+3}$ f(1) = $\sqrt{4}$ = 2

f'(x) = $\sqrt{x+3}$

= $(x+3)^{1/2}$

= $\frac{1}{2}(x+3)^{-1/2}$

= $\frac{1}{2\sqrt{x+3}}$ f'(1) = $\frac{1}{4}$

f''(x) = $\frac{1}{2}*\frac{-1}{2}(x+3)^{-3/2}$

= $\frac{-1}{4}(x+3)^{-3/2}$

= $\frac{-1}{4\sqrt{(x+3)^3}}$ f''(1) = $\frac{-1}{32}$

The truncation error:

f'''(x) = $\frac{-1}{4}*\frac{-3}{2}(x+3)^{-5/2}$

= $\frac{3}{8}(x+3)^{-5/2}$

= $\frac{3}{8\sqrt{(x+3)^5}}$

= $\frac{3}{8\sqrt{(\xi+3)^5}}$

The Taylor Series expansion with truncation error term:

f(x) ≈ f(1) + f'(1)(x-1) + $\frac{f''(1)}{2!}(x-1)^2$ + $\frac{f'''(\xi)}{3!}(x-1)^3$

f(x) = $\sqrt{x+3}$ ≈ 2 + $\frac{(x-1)}{4}$ - $\frac{(x-1)^2}{64}$ + $\frac{(x-1)^3}{16\sqrt{(\xi+3)^5}}$

**Q3. (b)**

```
>> format long
>> x = 1.12;
>> 2 + (x-1)/4 - (x-1)^2/64

ans =

   2.029775000000000
```

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**Q3. (c)**

R = $\frac{(x-1)^3}{16\sqrt{(\xi+3)^5}}$ = $\frac{(1.2-1)^3}{16\sqrt{(\xi+3)^5}}$ = $\frac{1}{2000\sqrt{(\xi+3)^5}}$ where $\xi \in [1, 1.2]$

When $\xi$ is 1

R = $\frac{1}{64000}$ = 0.000015625…

It is in its worst case because as $\xi$ gets smaller, the error increases. Therefore, an upper bound for the truncation error is 1.