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CSC 349A
Assignment 3

Q1(a)

fl(cosx) = fl(cos(3.154)) = fl(-0.999923029) = -0.9999

fl(1+cosx) = fl(1+cos(3.154)) = fl(1-0.9999) = fl(0.0001) = 0.0001

fl(π)= 3.142

fl(x- π) = fl(3.154-3.142) = fl(0.012) = 0.012

fl(x-π)^2 = fl(0.012^2) = fl(0.000144) = 1.440×10^{-4}

fl($\frac{1+\cos x}{(x-\pi)^2}$) = fl($\frac{0.0001}{0.000144}$) = fl(0.694444444) = 0.6945

$|E_t|$ = $\left|\frac{0.49999359-0.6945}{0.49999359}\right|$ = 0.389017807 ≈ 38%

Q1(b)

$cosx$	
$f(x) = cosx$	$f(\pi) = cos(\pi) = -1$
$f'(x) = -sinx$	$f'(\pi) = -sin(\pi) = 0$
$f''(x) = -cosx$	$f''(\pi) = -cos(\pi) = 1$
$f'''(x) = sinx$	$f'''(\pi) = sin(\pi) = 0$
$f''''(x) = cosx$	$f''''(\pi) = cos(x) = -1$

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f''''(a)}{4!}(x-a)^4$$

cosx ≈ -1 + 0 + $\frac{(x-\pi)^2}{2!}$ + 0 - $\frac{(x-\pi)^4}{4!}$

cosx ≈ -1 + $\frac{(x-\pi)^2}{2!}$ - $\frac{(x-\pi)^4}{4!}$

Q1(c)

f(x) ≈ $\frac{1+\left(-1+\frac{(x-\pi)^2}{2}-\frac{(x-\pi)^4}{24}\right)}{(x-\pi)^2}$ = $\frac{\left(\frac{(x-\pi)^2}{2}-\frac{(x-\pi)^4}{24}\right)}{(x-\pi)^2}$ = $\frac{1}{2}-\frac{(x-\pi)^2}{24}$

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Q1(d)

Find exact value when x = 3.154 using polynomial approximation from (c).

$f(3.154) \approx \frac{1}{2} - \frac{(3.154-\pi)^2}{24} = 0.499993585$

Consider perturb with $\hat{x} = 3.154 + \varepsilon$ with $\varepsilon = 0.01$

$f(3.155) \approx \frac{1}{2} - \frac{(3.155-\pi)^2}{24} = 0.49999251$

Relative error input:

$|E_t| = \left|\frac{3.154-3.155}{3.154}\right| = 0.003170577 \approx 0.03\%$

Relative error output:

$|E_t| = \left|\frac{0.499993585-0.49999251}{0.499993585}\right| = 0.00000215 \approx 0.0002\%$

Therefore, f(3.154) is well-conditioned since the exact value of f(3.154+ε) is approximately equal to the exact value of f(3.154) whenever $\left|\frac{\varepsilon}{3.154}\right|$ is small.

Q1(e)

From (a), we found that our floating-point computation with rounding:

$x = 3.154 \longrightarrow 1$

Consider perturbed exact computation:

$\hat{x} = 3.154 + \varepsilon \longrightarrow \frac{1}{2} - \frac{(3.154+\varepsilon-\pi)^2}{24}$

with $\left|\frac{\varepsilon}{3.154}\right|$ small

$0.5 - \frac{(3.154+\varepsilon-\pi)^2}{24}$

$0.5 - \frac{(3.154+\varepsilon-\pi)(3.154+\varepsilon-\pi)}{24}$

$0.5 - \frac{(9.94716 + 6.308\varepsilon - 6.308\,\pi - 2\varepsilon\pi + \pi^2)}{24}$

$0.5 - \frac{(-0.000402057 + 6.308\varepsilon - 2\varepsilon\pi)}{24}$

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We see that 0.5 - $\frac{(-0.000402057 + 6.308\varepsilon - 2\varepsilon\pi)}{24}$ is approximately equal to 0.4989… for all values of ε such that $\left|\frac{\varepsilon}{3.154}\right|$ is small. This value is not close to our floating-point approximation 0.6945. Thus, the computation is unstable.

Q2(a)

```
function root = Bisect ( xl , xu , eps , imax, f, enablePlot )
i = 1;
fl = feval(f, xl);

fprintf ( ' iteration approximation \n')
while (i <= imax)
    xr = (xl+xu/2);
    fprintf ( ' %6.0f %18.8f \n', i, xr )
    fr = feval(f, xr);

    %when enablePlot is 1 it shows each iteration of the bisection method
    if (enablePlot && (i == 1) || (i == 2) || (i == 4) || (i == 6))
        hold on;
        x = [xl:0.001:xu];
        fx = feval(f, x);

        y = [xl:xr:xu];
        fy = feval(f, y);
        plot(x, fx)
        plot(y, fy)
        hold off;
    end


    if (fr == 0 || (xu-xl/abs(xu+xl)) < eps)
        root = xr;
        return;
    end
    i = i + 1;
    if (fl*fr < 0)
        xu = xr;
    else
        xl = xr;
        fl = fr;
    end
end
    fprintf ( ' failed to converge in %g iterations\n', imax )
end
```

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Q2(b)

$$0 = 1 - \frac{Q^2}{gA_c^3}B \qquad A_c = 3y + \frac{y^2}{2} \quad B = 3 + y$$

$$\frac{Q^2}{gA_c^3}B = 1 \ \rightarrow \ \left(\frac{Q^2}{g}\right)B = A_c^3$$

$$\left(\frac{Q^2}{g}\right)(3+y) = \left(3y + \frac{y^2}{2}\right)^3$$

$$\left(\frac{Q^2}{g}\right)(3+y) = \left(3y + \frac{y^2}{2}\right)\left(3y + \frac{y^2}{2}\right)\left(3y + \frac{y^2}{2}\right)$$

$$\left(\frac{Q^2}{g}\right)(3+y) = 27y^3 + \frac{27}{2}y^4 + \frac{9y^5}{4} + \frac{y^6}{8}$$

$$\left(\frac{20^2}{9.81}\right)(3+y) = \frac{y^6}{8} + \frac{9}{4}y^5 + \frac{27}{2}y^4 + 27y^3$$

$$\frac{400}{9.81}(3+y) = \frac{y^6}{8} + \frac{9}{4}y^5 + \frac{27}{2}y^4 + 27y^3$$

$$\frac{1200}{9.81} + \frac{400}{9.81}y = \frac{y^6}{8} + \frac{9}{4}y^5 + \frac{27}{2}y^4 + 27y^3$$

$$\frac{1}{8}y^6 + \frac{9}{4}y^5 + \frac{27}{2}y^4 + 27y^3 - \frac{400}{9.81}y - \frac{1200}{9.81} = 0$$

$$\mathbf{f(y) = \frac{1}{8}y^6 + \frac{9}{4}y^5 + \frac{27}{2}y^4 + 27y^3 - \frac{400}{9.81}y - \frac{1200}{9.81}}$$

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Q2(c)

```
%critical depth function
function [ fy ] = depth( y )

fy = 1 - 20.^2./(9.81.*(3.*y+y.^2./2).^3).*(3+y);

end


>> xl = 0.5;
>> xu = 2.5;
>> eps = 0.01;
>> imax = 10;
>> Bisect(xl, xu, eps, imax, @depth, 1);
 iteration    approximation
      1          1.50000000
      2          2.00000000
      3          1.75000000
      4          1.62500000
      5          1.56250000
      6          1.53125000
      7          1.51562500
      8          1.50781250

root =

    1.5078
```

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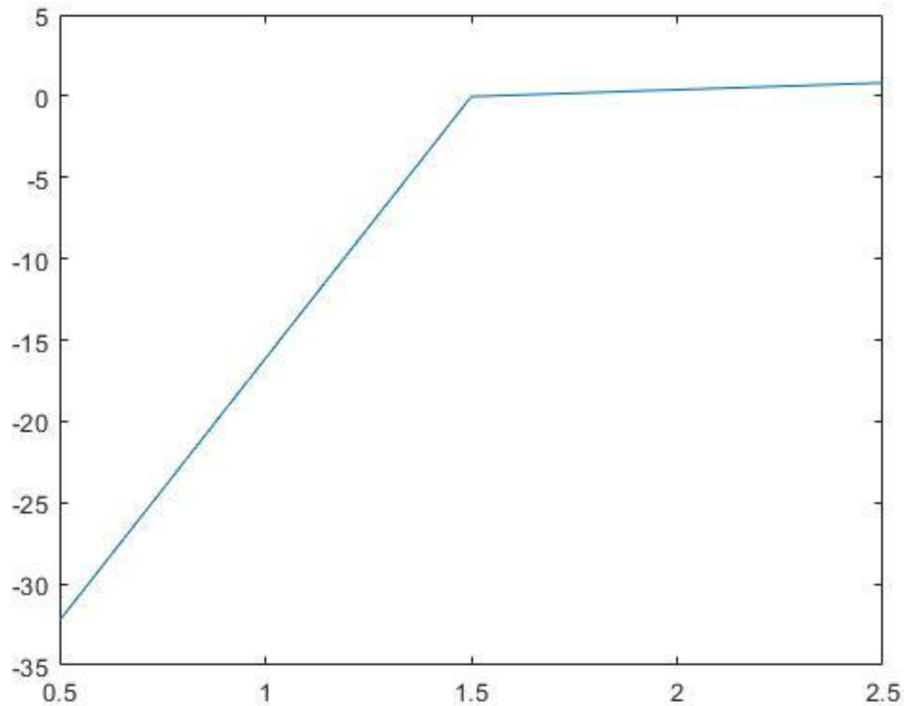
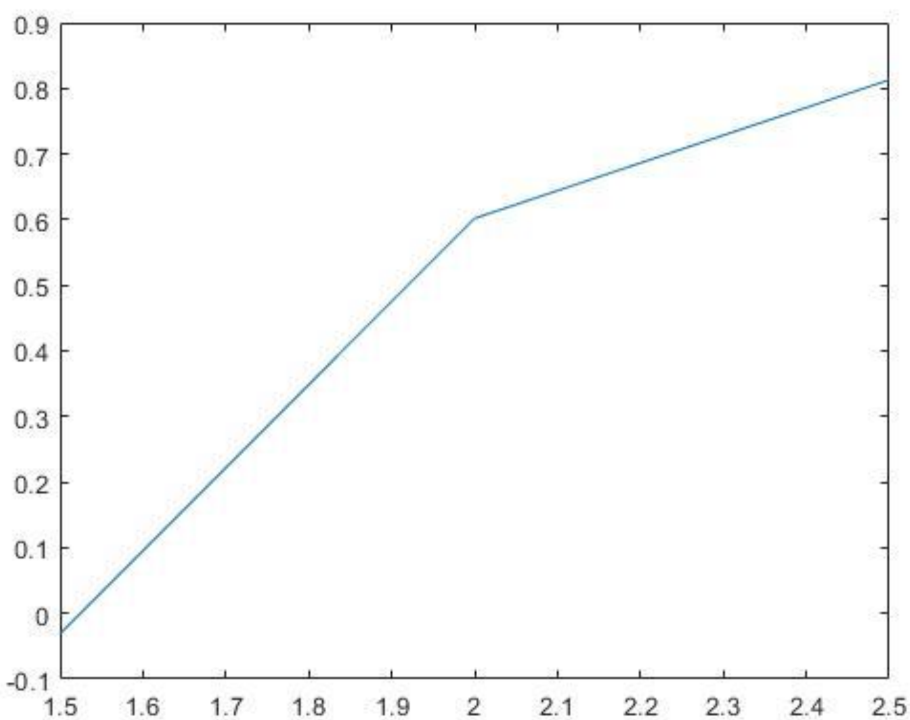
Plots for i = 1, 2, 4, 6





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