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CSC 349A
Assignment 6

Q1(a)

$x_0 = 0, x_1 = 2h, x_2 = 3h, h > 0$

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-2h)(x-3h)}{(0-2h)(0-3h)} = \frac{x^2-5hx+6h^2}{6h^2}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-0)(x-3h)}{(2h-0)(2h-3h)} = \frac{x^2-3hx}{-2h^2}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-0)(x-2h)}{(3h-0)(3h-2h)} = \frac{x^2-2hx}{3h}$$

Using formula
$P(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2)$ we get:

$$P(x) = \frac{x^2-5hx+6h^2}{6h^2}f(0) - \frac{x^2-3hx}{2h^2}f(2h) + \frac{x^2-2hx}{3h}f(3h)$$

Q1(b)

$$\int_0^{3h} P(x)dx =$$

$$\frac{f(0)}{6h^2}\int_0^{3h} x^2-5hx+6h^2\,dx - \frac{f(2h)}{2h^2}\int_0^{3h} x^2-3hx\,dx + \frac{f(3h)}{3h}\int_0^{3h} x^2-2hx\,dx$$

$$\frac{f(0)}{6h^2}\left[\frac{x^3}{3}-\frac{5hx^2}{2}+6h^2x\right]_0^{3h} - \frac{f(2h)}{2h^2}\left[\frac{x^3}{3}-\frac{3hx^2}{2}\right]_0^{3h} + \frac{f(3h)}{3h}\left[\frac{x^3}{3}-hx^2\right]_0^{3h}$$

$$\frac{f(0)}{6h^2}\left[9h^3-\frac{45h^3}{2}+18h^3\right] - \frac{f(2h)}{2h^2}\left[9h^3-\frac{27h^3}{2}\right] + \frac{f(3h)}{3h}[9h^3-9h^3]$$

$$= \frac{f(0)}{6h^2}\left[\frac{9h^3}{2}\right] + \frac{f(2h)}{2h^2}\left[\frac{9h^3}{2}\right] + 0 = \frac{3h}{4}f(0) + \frac{9h}{4}f(2h)$$

Q1(c)

We have:

$x_0 = 0, x_1 = 0.24, x_2 = 0.36, f(0) = 0.5, f(2h) = 0.50727, f(3h) = 0.51656, h = 0.12$

$$\int_0^{0.36} f(x)dx \approx \frac{3h}{4}(0.5) + \frac{9h}{4}(0.50727) = 0.1819629$$

Q2(a)

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Let $f(x) = 1$: (degree 0)

$$\int_{-1}^{1} 1\, dx = [x]_{-1}^{1} = 2$$

$$\frac{5}{9}(1) + \frac{8}{9}(1) + \frac{5}{9}(1) = 2$$

Let $f(x) = x$: (degree 1)

$$\int_{-1}^{1} x\, dx = \left[\frac{x^2}{2}\right]_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$$

$$\frac{5}{9}\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}(0) + \frac{5}{9}\left(\sqrt{\frac{3}{5}}\right) = 0$$

Let $f(x) = x^2$: (degree 2)

$$\int_{-1}^{1} x^2\, dx = \left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$\frac{5}{9}\left(-\sqrt{\frac{3}{5}}\right)^2 + \frac{8}{9}(0)^2 + \frac{5}{9}\left(\sqrt{\frac{3}{5}}\right)^2 = \frac{15}{45} + \frac{15}{45} = \frac{2}{3}$$

Let $f(x) = x^3$: (degree 3)

$$\int_{-1}^{1} x^3\, dx = \left[\frac{x^4}{4}\right]_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0$$

$$\frac{5}{9}\left(-\sqrt{\frac{3}{5}}\right)^3 + \frac{8}{9}(0)^3 + \frac{5}{9}\left(\sqrt{\frac{3}{5}}\right)^3 = \frac{-15}{45}\left(\sqrt{\frac{3}{5}}\right) + \frac{15}{45}\left(\sqrt{\frac{3}{5}}\right) = 0$$

Let $f(x) = x^4$: (degree 4)

$$\int_{-1}^{1} x^4\, dx = \left[\frac{x^5}{5}\right]_{-1}^{1} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

$$\frac{5}{9}\left(-\sqrt{\frac{3}{5}}\right)^4 + \frac{8}{9}(0)^4 + \frac{5}{9}\left(\sqrt{\frac{3}{5}}\right)^4 = \frac{5}{9}\left(\frac{9}{25}\right) + \frac{5}{9}\left(\frac{9}{25}\right) = \frac{2}{5}$$

Let $f(x) = x^5$: (degree 5)

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$$\int_{-1}^{1} x^5\,dx = \left[\frac{x^6}{6}\right]_{-1}^{1} = \frac{1}{6} - \frac{1}{6} = 0$$

$$\frac{5}{9}\left(-\sqrt{\frac{3}{5}}\right)^5 + \frac{8}{9}(0)^5 + \frac{5}{9}\left(\sqrt{\frac{3}{5}}\right)^5 = 0$$

$\underline{\boldsymbol{Let\ f(x) = x^6: (degree\ 6)}}$

$$\int_{-1}^{1} x^6\,dx = \left[\frac{x^7}{7}\right]_{-1}^{1} = \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$$

$$\frac{5}{9}\left(-\sqrt{\frac{3}{5}}\right)^6 + \frac{8}{9}(0)^6 + \frac{5}{9}\left(\sqrt{\frac{3}{5}}\right)^6 = \frac{3}{25} + \frac{3}{25} = \frac{6}{25}$$

$Since\ \frac{2}{7} \neq \frac{6}{25}, the\ quadrature\ formula\ has\ a\ degree\ precision\ of\ 5.$

Q2(b)

$$We\ know: x_0 = -\sqrt{\frac{3}{5}}, x_1 = 0, x_2 = \sqrt{\frac{3}{5}}, f(x_0) = e^{\sqrt{\frac{3}{5}}}\sqrt{-\sqrt{\frac{3}{5}}+2}, f(x_1) = \sqrt{2}, f(x_2) = e^{-\sqrt{\frac{3}{5}}}\sqrt{\sqrt{\frac{3}{5}}+2}$$

$$\int_{-1}^{1} e^{-x}\sqrt{x+2}\,dx \approx \frac{5}{9}\big(f(x_0)\big) + \frac{8}{9}\big(f(x_1)\big) + \frac{5}{9}\big(f(x_2)\big)$$

$$= 1.33435099 + 1.257078722 + 0.426505234 = 3.017934946$$

Q3(a)

```
function trap(a, b, maxiter, tol)
m = 1;
x = linspace(a, b, m+1);
y = f(x);
approx = trapz(x, y);
disp(' m integral approximation');
fprintf(' %5.0f %16.10f \n ', m, approx);

for i = 1 : maxiter
    m = 2*m;
    oldapprox = approx;
    x = linspace (a, b, m+1) ;
    y = f(x);
    approx = trapz(x, y);
    fprintf(' %5.0f %16.10f \n ', m, approx);
    if abs(1-oldapprox/approx) < tol
        return
    end
```

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```
end
fprintf('Did not converge in %g iterations', maxiter)
end
```

Q3(b)

<u>func1.m</u>

```
function y = func1(x)
    y = sin(1./(x));
end
```

```
>> trap(0.1, 3, 20, 1e-6, @func1)
 m integral approximation
     1    -0.3143983004
      2     0.7147254605
      4     1.3447434609
      8     1.5589483255
     16     1.4776583126
     32     1.4679626280
     64     1.5197926883
    128     1.5355585774
    256     1.5386514853
    512     1.5393496800
   1024     1.5395196356
   2048     1.5395618423
   4096     1.5395723764
   8192     1.5395750089
  16384     1.5395756669
```

<u>func2.m</u>

```
function y = func2(x)
    y = exp(3.*(x))./(sqrt((x.^3)+1));
end
```

```
>> trap(0, 1, 20, 1e-10, @func2);
 m integral approximation
     1     7.6013096811
      2     5.9133433291
      4     5.4710046573
      8     5.3585418274
     16     5.3303053079
     32     5.3232385483
     64     5.3214713803
    128     5.3210295584
    256     5.3209191010
    512     5.3208914866
   1024     5.3208845830
   2048     5.3208828571
```

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4096	5.3208824256
8192	5.3208823177
16384	5.3208822908
32768	5.3208822840
65536	5.3208822823
131072	5.3208822819