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CSC 349A
Assignment 4

Q1(a)

$$\left[\begin{array}{cccc|c} 0 & -2 & -2 & -4 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 2 & 4 & -2 & 0 & 0 \\ 1 & 1 & -1 & 0.5 & 1 \end{array}\right]$$

Swap row 1 and row 3 because 2 > all values in the column

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 1 & 1 & -1 & 0.5 & 1 \end{array}\right]$$

Pivot on a_{11}:

$\underline{E_2 = E_2 - m_{21}E_1}$

$m_{21} = -1/2$

$a_{21} = -1 - (-1/2)(2) = 0$
$a_{22} = -1 - (-1/2)(4) = 1$
$a_{23} = 1 - (-1/2)(-2) = 0$
$a_{24} = 0 - (-1/2)(0) = 0$
$b_2 = 0 - (-1/2)(0) = 0$

$\underline{E_4 = E_4 - m_{41}E_1}$

$m_{41} = 1/2$

$a_{41} = 1 - (1/2)(2) = 0$
$a_{42} = 1 - (1/2)(4) = 1$
$a_{43} = -1 - (1/2)(-2) = 0$
$a_{44} = 1/2 - (1/2)(0) = 1/2$
$b_4 = 1 - (1/2)(0) = 1$

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 0 & 1 & 0 & 0.5 & 1 \end{array}\right]$$

Swap rows 2 and 3 because the absolute value of -2 is > than all values in column 2

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.5 & 1 \end{array}\right]$$

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Pivot on a_{22}:

$\underline{E_3 = E_3 - m_{32}E_2}$

$m_{21} = -1/2$

$a_{32} = 1 - (-1/2)(-2) = 0$
$a_{33} = 0 - (-1/2)(-2) = -1$
$a_{34} = 0 - (-1/2)(-4) = -2$
$b_3 = 0 - (-1/2)(0) = 0$

$\underline{E_4 = E_4 - m_{42}E_2}$

$m_{21} = -1/2$

$a_{42} = 1 - (-1/2)(-2) = 0$
$a_{43} = 0 - (-1/2)(-2) = -1$
$a_{44} = 1/2 - (-1/2)(-4) = -1.5$
$b_4 = 1 - (-1/2)(0) = 1$

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & -1 & -1.5 & 1 \end{array}\right]$$

Pivot on a_{32}:

$\underline{E_4 = E_4 - m_{42}E_2}$

$m_{43} = 1$

$a_{43} = -1 - (1)(-1) = 0$
$a_{44} = -1.5 - (1)(-2) = 0.5$
$b_4 = 1-(1)(0) = 1$

We get the upper triangular matrix:

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 0 & 0 \\ 0 & -2 & -2 & -4 & 0 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0.5 & 1 \end{array}\right]$$

Back substitution:

$0.5x_4 = 1$
$x_4 = 2$

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$-x_3 + (-2)(2) = 0$
$x_3 = -4$

$-2x_2 - 2x_3 - 4x_4 = 0$
$-2x_2 - 2(-4) - 4(2) = 0$
$x_2 = 0$

$2x_1 + 0 - 2x_3 = 0$
$2x_1 + 0 - 2(-4) = 0$
$x_1 = -4$

The fourth column of A^{-1} is $\begin{bmatrix} -4 \\ 0 \\ -4 \\ 2 \end{bmatrix}$

Q1 (b)

The determinant of A is the product of the diagonal values of the matrix in upper triangular form. From (a) we get:

(2)(-2)(-1)(0.5) = 2

Since we did 2 row interchanges which is an even number, the sign of the determinant is positive.

Q2 (a)

$x_1 \leftarrow b_1/a_{1,1}$
$x_2 \leftarrow (b_2 - a_{2,1} * x_1)/a_{2,2}$
$for\ i = 3,4, \dots, n$
    $x_i \leftarrow (b_i - a_{i,i-2:i-1} * x_{i-2:i-1})/a_{i,i}$
end

Q2 (b)

Line 1: division, +1
Line 2: Subtract, multiply, division once, +1
Line 4: a multiplies x, 3 times, and subtract and divide n times

Total number of floating-points: 1 + 1 + n * 3 = 3n + 2

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Q2 (c)

```
function x = forwardsub(a, b)
%Input: a = n x n non-singular lower triangle matrix
%       b = n x 1 matrix
%Output: solution to x in Ax=b

%get the size of of b
%initialize x with a n x 1 matrix filled with 0's
n = length(b);
x = zeros(n,1);

x(1) = b(1)/a(1,1);
x(2) = (b(2)-a(2,1)*x(1))/a(2,2);
for i = 3:n
    x(i) = (b(i)-a(i,i-2:i-1)*x(i-2:i-1))/a(i,i);
end
```

Calling the function and its results from the given matrix A and b:

```
>> A = [1, 0, 0, 0; 2, 3, 0, 0; 4, 5, 6, 0; 0, 7, 8, 9;]

A =

     1     0     0     0
     2     3     0     0
     4     5     6     0
     0     7     8     9

>> b = [1;5;15;24]

b =

     1
     5
    15
    24

>> forwardsub(A, b)

ans =

     1
     1
     1
     1
```

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Q3 (a)

$$\left[\begin{array}{cc|c} -0.2345 & 2.107 & -2.345 \\ 0.1234 & -1.115 & 1.001 \end{array}\right]$$

$m_{21} = fl\left(\frac{0.1234}{-0.2345}\right) = fl(-0.526226012) = -0.5262$
$a_{22} = fl\big(-1.115 - (-0.5262)(2.107)\big)$
$= fl\,(-1.115 - fl(-1.1087034))$
$= fl\,(-1.115 - (-1.109))$
$= -0.006$

$b_2 = fl\,(1.001 - (-0.5262)(-2.345))$
$= fl\,(1.001 - fl(1.233939))$
$= fl\,(1.001 - 1.234)$
$= -0.233$

$$\left[\begin{array}{cc|c} -0.2345 & 2.107 & -2.345 \\ 0 & -0.006 & -0.233 \end{array}\right]$$

$x_2 = fl(\frac{-0.233}{-0.006})$
$x_2 = fl(38.83333333)$
$x_2 = 38.83$

$-0.2345x_1 + 2.107(38.83) = -2.345$
$-0.2345x_1 + fl(81.81481) = -2.345$
$-0.2345x_1 + 81.81 = -2.345$
$-0.2345x_1 = fl(-84.155)$
$-0.2345x_1 = -84.16$
$x_1 = fl(358.891258)$
$x_1 = 358.9$

Q3 (b)

```
>> A = [-0.2345, 2.107;0.1234, -1.115;]

A =

   -0.2345    2.1070
    0.1234   -1.1150

>> cond(A)

ans =

   3.9304e+03
```

Since the condition number is not between 1 and 10 and is greater than 1000, A is ill-conditioned.