Lecture 1: Norm and Hilbert Space

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1 Complex number

Definition 1 (Complex number). *Complex number [ ], which is quivalent to:*

1. *Set of opints in* $\mathbb{R}^2$*;*
2. *Set of free vectors on a plane;*
3. *Set of ordered pairs of real numbers* (x, y).

How do identify a specific complex number? Use below:

$$z = x \cdot +y \cdot i = x + yi = x + yj = x + jy = re^{j\theta}$$

Euler form for a complex number:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

where $r = \sqrt{x^2 + y^2}$, $\sin\theta = y/r$, $\cos\theta = x/r$.

Definition 2 (Complex conjugate).

$$z^* = \overline{z} = (x + jy)^* = \overline{x + jy} = x - jy$$

Definition 3 (Complex field). *A field has the following foure operations defined:*

1. *addtiion:* $z_1 + z_2 = x_1 + x_2 + j(y_{1+y_2})$
2. *substraction:* $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. *multiply:* $z_1 z_2 = (x_1 x_2 - y_1 y_2 + j(x_1 y_2 + x_2 y_1$

4. division:

Example 1.

$$j \cdot j = j^2 = -1$$

Example 2.

$$\sqrt{3+4j} = \sqrt{5e^{j\theta}} = \sqrt{5}e^{j\theta/2},$$

where $\sin\theta = 4/5$, $\cos\theta = 3/5$.

Example 3.

$$\sqrt[3]{3+4j} = (5e^{j\theta})^{1/3} = \sqrt[3]{5}e^{(j\theta+2\pi k)/3},$$

where $\sin\theta = 4/5$, $\cos\theta = 3/5$.