Graph Theory — Exercise 3

Due Wednesday, March 28th 2012

[p]1. Find the smallest 3-regular simple graph having connectivity 1.

2. Prove that if v is a cut-vertex of a graph G then v is not a cut-vertex of $\overline{G}$.

3. Prove that a connected graph G with at least two vertices is nonseparable if and only if any two adjacent edges lie on a common cycle.

4. Prove that if G is a graph with n vertices such that $\delta(G) \geq (n-1)/2$ then $\lambda(G) = \delta(G)$.

5. Let n, k be positive integers with n even, k odd and $n > k > 1$. Let G be the k-regular simple graph formed by placing n vertices on a circle and making each vertex adjacent to the opposite vertex and to the $(k-1)/2$ nearest vertices in each direction. Prove thst $\kappa(G) = k$.

[p]6. (a) Find the minimum positive integer r for which there exists an r-regular graph G such that $\kappa(G) \neq \lambda(G)$. Verify your claim.

   (b) Find the minimum positive integer r for which there exists an r-regular graph G such that $\lambda(G) \geq \kappa(G) + 2$.

7. Prove Eulaer's theorem: A nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree.

8. Show that the Petersen graph is not Hamiltonian.

[p]9. Show that the Petersen graph is 3-connected.