1:

kn rooks are placed onto an $n \times n$ chess board in a way such that every file (column) of the chess board has k rooks, and every rank (row) has k rooks ($1 \leq k \leq n$). Prove that we can choose a set of n of these rooks such that no rooks in the set are attacking each other. Formally, this means no two rooks of the set are in the same rank or file.

^{P}2:

Prove that, for every k-coloring of a k-chromatic graph and every color c there is a vertex x of color c which is adjacent to vertices of every other color.

3:

Prove that if $\chi(G) \geq k$, then $\|G\| \geq \binom{k}{2}$. $||G||$ denotes the size of G, which is the number of edges. You must prove any theorems you use here.

4:

Given that Brooks' theorem holds for graphs that are 2-connected, and k-regular with $k \geq 3$, prove that Brooks' theorem holds for all graphs.

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^{P}**5:**

Let G be a graph with n vertices and m edges such that n is odd and

$$m > \frac{(n-1)\Delta(G)}{2}.$$

Prove that $\chi_1(G) = 1 + \Delta(G)$.

6:

For a positive integer k, let H be a $2k$-regular graph of order $4k+1$ (that is, with $4k+1$ vertices). Let G be obtained from H by removing a set of $k-1$ independent edges from H. Prove that $\chi_1(G) = \Delta(G) + 1$.

7:

Prove that if G is a subdivision of K_5 or $K_{3,3}$, then G is not planar. [You may not apply theorems after Corollary 9.3; many of them directly imply this result.]