21-630 Assignment #10 Due Wed., April 10, 2013

1. Consider the system

$$\frac{dX}{dt} = (1 - X^2 - Y^2)(X - Y)$$

$$\frac{dY}{dt} = (1 - X^2 - Y^2)(X + Y).$$

Put the system into polar coordinates, that is let $X = r\cos(\theta)$ and $Y = r\sin(\theta)$ and find equations for $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$. Use this to identify the positive limit set for every solution of the original system. Proof is not required, but some explanation is. Also be sure to address all solutions.

2. Assume that $f : \mathbb{R}^N \to \mathbb{R}^N$ is continuously differentiable and that $X(t)$ is a periodic solution of

$$\frac{dX}{dt} = f(X(t)),$$

that is, there is $T > 0$ such that $X(t + T) = X(t)$ for every ≥ 0. Prove that

$$\Omega(X(0)) = C^+(X(0)).$$

3. Consider the system

$$\frac{dX}{dt} = Yg(X, Y) - X(X - Y)^2$$

$$\frac{dY}{dt} = -Xg(X, Y) - Y(X - Y)^2$$

where g is continuously differentiable and $g(x, x) \neq 0$ for all $x \neq 0$. Let $w(x, y) = x^2+y^2$ and show that $D_*w \leq 0$. Use theorem 5.8 on page 132 of the notes to prove that all solutions approach $(0, 0)$ as t tends to infinity.

4. Consider the system

$$\frac{d^2X}{dt^2} + \frac{dX}{dt} + \frac{dY}{dt} + \frac{\partial P}{\partial x}(X,Y) = 0$$

$$\frac{d^2Y}{dt^2} + \frac{dX}{dt} + \frac{dY}{dt} + \frac{\partial P}{\partial y}(X,Y) = 0$$

where P is C^1.

A) Using the notation $V = \frac{dX}{dt}$ and $U = \frac{dY}{dt}$, compute $D_* w(x,y,v,u)$ where

$$w(x,y,v,u) = \frac{1}{2}(v^2 + u^2) + P(x,y).$$

B) Consider the case $P(x,y) = x^4 + y^2$. The origin is a critical point, is it asymptotically stable? Justify your answer.

C) Consider the case $P(x,y) = x^4 + y^4$. The origin is a critical point, is it asymptotically stable? Justify your answer.