21-484 Notes

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- 3 out of 4 exams 70%
- few quizzes 10%
- excercises 9 20%

→ Book: Graph Theory Tom Bohman CMU
John Mackey 21-484

Introduction to Graph Theory

Chartrand + Zhang

Definitions and terminology

A graph G is an ordered pair (V, E) where:

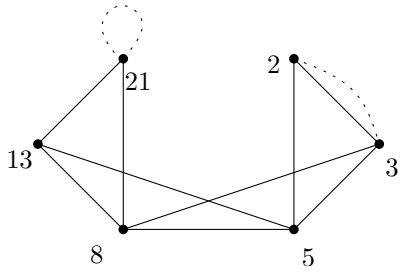
- V if a nonempty (finite) set
- E is a set of subsets of size 2 of elements of V.

V is called the vertex set and its elements are vertices.

E is called the edge set and its elements are edges.

Given a graph G, we use $V(G)$ to denote its vertes set and $E(G)$ for the edge set.

Example 1.2: $V = \{2, 3, 5, 8, 13, 21\}$ $E = \{\{2,3\}, \{2,5\}, \{3,5\}, \{3,8\}, \{5,8\}, \{5,13\}, \{8,13\}, \{8,21\}, \{13,21\}\}$



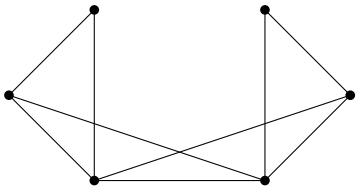
→ Remark: There are no parallel edges an no loops in "our" graphs

"our" graphs are sometimes called simple graphs

→ Hence: A graph is an irreflexive symmetric binary relation over a ground set.

→ what do we mean by $G = H$ (for graphs G and H)?

→ Sometimes we only care about the structure of the graph



unlabeled
graph

- for brevity, we may denote the edge $\{u, v\}$ as uv
- If uv is an edge of G then the verticies u and v are said to be adjacent. Also, u is a neighbor of v. If $e = uv \in E$ then u and v are called the endpoints of e.
- THe set of all neighbors of a vertex u is called the neighborhood of u and is denoted by $N(u) = \{v \in V \mid uv \in E\}$
- The degree of a verte u is denoted $d(u) = |N(u)|$.

claim: For any (multi) graph $G = (V, E)$,

$$\sum_{v \in V} d(v) = 2|E|$$

Proof: double counting.

Corollary: The sum of the degrees is always even

Remark: The claim holds also for multigraphs with loops if a loop contributes 2 to the degree.