patients between these ages produced a mean and standard deviation equal to 5.4 and 3.1 days, respectively. Construct a 95% confidence interval for the mean length of stay for the population of patients from which the sample was drawn.

8.59 When it comes to advertising, "'tweens" are not ready for the hard-line messages that advertisers often use to reach teenagers. The Geppeto Group study[6] found that 78% of 'tweens understand and enjoy ads that are silly in nature. Suppose that the study involved $n = 1030$ 'tweens.

a Construct a 90% confidence interval for the proportion of 'tweens who understand and enjoy ads that are silly in nature.

b Do you think that "more than 75%" of all 'tweens enjoy ads that are silly in nature? Why?

8.60 What *is* the normal body temperature for healthy humans? A random sample of 130 healthy human body temperatures provided by Allen Shoemaker[7] yielded 98.25 degrees and standard deviation 0.73 degrees.

a Give a 99% confidence interval for the average body temperature of healthy people.

b Does the confidence interval obtained in part (a) contain the value 98.6 degrees, the accepted average temperature cited by physicians and others? What conclusions can you draw?

8.61 A small amount of the trace element selenium, from 50 to 200 micrograms (μg) per day, is considered essential to good health. Suppose that independent random samples of $n_1 = n_2 = 30$ adults were selected from two regions of the United States, and a day's intake of selenium, from both liquids and solids, was recorded for each person. The mean and standard deviation of the selenium daily intakes for the 30 adults from region 1 were $\overline{y}_1 = 167.1$ μg and $s_1 = 24.3$ μg, respectively. The corresponding statistics for the 30 adults from region 2 were $\overline{y}_2 = 140.9$ μg and $s_2 = 17.6$ μg. Find a 95% confidence interval for the difference in the mean selenium intake for the two regions.

8.62 The following statistics are the result of an experiment conducted by P. I. Ward to investigate a theory concerning the molting behavior of the male *Gammarus pulex*, a small crustacean.[8] If a male needs to molt while paired with a female, he must release her, and so loses her. The theory is that the male *G. pulex* is able to postpone molting, thereby reducing the possibility of losing his mate. Ward randomly assigned 100 pairs of males and females to two groups of 50 each. Pairs in the first group were maintained together (normal); those in the second group were separated (split). The length of time to molt was recorded for both males and females, and the means, standard deviations, and sample sizes are shown in the accompanying table. (The number of crustaceans in each of the four samples is less than 50 because some in each group did not survive until molting time.)

	Time to Molt (days)		
	Mean	s	n
Males			
Normal	24.8	7.1	34
Split	21.3	8.1	41
Females			
Normal	8.6	4.8	45
Split	11.6	5.6	48

6. *Source:* "Caught in the Middle," *American Demographics*, July 2001, pp. 14–15.

7. *Source:* Allen L. Shoemaker, "What's Normal? Temperature, Gender and Heart Rate," *Journal of Statistics Education* (1996).

8. *Source:* "*Gammarus pulex* Control Their Moult Timing to Secure Mates," *Animal Behaviour* 32 (1984).

a Find a 99% confidence interval for the difference in mean molt time for "normal" males versus those "split" from their mates.

b Interpret the interval.

8.63 Most Americans love participating in or at least watching sporting events. Some feel that sports have more than just entertainment value. In a survey of 1000 adults, conducted by KRC Research & Consulting , 78% felt that spectator sports have a positive effect on society.[9]

a Find a 95% confidence interval for the percentage of the public that feel that sports have a positive effect on society.

b The poll reported a margin of error of "plus or minus 3.1%." Does this agree with your answer to part (a)? What value of p produces the margin of error given by the poll?

8.64 In a CNN/*USA Today*/Gallup Poll, 1000 Americans were asked how well the term *patriotic* described themselves.[10] Some results from the poll are contained in the following summary table.

	Age Group		
	All	18–34	60+
Very well	.53	.35	.77
Somewhat well	.31	.41	.17
Not Very well	.10	.16	.04
Not well at all	.06	.08	.02

a If the 18–34 and 60+ age groups consisted of 340 and 150 individuals, respectively, find a 98% confidence interval for the difference in proportions of those in these age groups who agreed that *patriotic* described them very well.

b Based on the interval that you obtained in part (a), do you think that the difference in proportions of those who view themselves as patriotic is as large as 0.6? Explain.

8.65 For a comparison of the rates of defectives produced by two assembly lines, independent random samples of 100 items were selected from each line. Line A yielded 18 defectives in the sample, and line B yielded 12 defectives.

a Find a 98% confidence interval for the true difference in proportions of defectives for the two lines.

b Is there evidence here to suggest that one line produces a higher proportion of defectives than the other?

8.66 Historically, biology has been taught through lectures, and assessment of learning was accomplished by testing vocabulary and memorized facts. A teacher-devoloped new curriculum, Biology: A Community Content (BACC), is standards based, activity oriented, and inquiry centered. Students taught using the historical and new methods were tested in the traditional sense on biology concepts that featured biological knowledge and process skills. The results of a test on biology concepts were published in *The American Biology Teacher* and are given in the following table.[11]

9. *Source:* Mike Tharp, "Ready, Set, Go. Why We Love Our Games—Sports Crazy," *U.S. News & World Report*, 15 July 1997, p. 31.

10. *Source:* Adapted from "I'm a Yankee Doodle Dandy," Knowledge Networks: 2000, *American Demographics*, July 2001, p. 9.

11. *Source:* William Leonard, Barbara Speziale, and John Pernick, "Performance Assessment of a Standards-Based High School Biology Curriculum," *The American Biology Teacher* 63(5) (2001): 310–316.

Substituting this value for σ_1 and σ_2 in the earlier equation, we obtain

$$1.96\sqrt{\frac{(2)^2}{n} + \frac{(2)^2}{n}} = 1.$$

Solving, we obtain $n = 30.73$. Therefore, each group should contain $n = 31$ members. ■

Exercises

8.70 Let Y be a binomial random variable with parameter p. Find the sample size necessary to estimate p to within .05 with probability .95 in the following situations:

a If p is thought to be approximately .9

b If no information about p is known (use $p = .5$ in estimating the variance of $\hat{p}$).

8.71 A state wildlife service wants to estimate the mean number of days that each licensed hunter actually hunts during a given season, with a bound on the error of estimation equal to 2 hunting days. If data collected in earlier surveys have shown σ to be approximately equal to 10, how many hunters must be included in the survey?

8.72 Telephone pollsters often interview between 1000 and 1500 individuals regarding their opinions on various issues. Does the performance of colleges' athletic teams have a positive impact on the public's perception of the prestige of the institutions? A new survey is to be undertaken to see if there is a difference between the opinions of men and women on this issue.

a If 1000 men and 1000 women are to be interviewed, how accurately could you estimate the difference in the proportions who think that the performance of their athletics teams has a positive impact on the perceived prestige of the institutions? Find a bound on the error of estimation.

b Suppose that you were designing the survey and wished to estimate the difference in a pair of proportions, correct to within .02, with probability .9. How many interviewees should be included in each sample?

8.73 Refer to Exercise 8.59. How many 'tweens should have been interviewed in order to estimate the proportion of 'tweens who understand and enjoy ads that are silly in nature, correct to within .02, with probability .99? Use the proportion from the previous sample in approximating the standard error of the estimate.

8.74 Suppose that you want to estimate the mean pH of rainfalls in an area that suffers from heavy pollution due to the discharge of smoke from a power plant. Assume that σ is in the neighborhood of .5 pH and that you want your estimate to lie within .1 of μ with probability near .95. Approximately how many rainfalls must be included in your sample (one pH reading per rainfall)? Would it be valid to select all of your water specimens from a single rainfall? Explain.

8.75 Refer to Exercise 8.74. Suppose that you wish to estimate the difference between the mean acidity for rainfalls at two different locations, one in a relatively unpolluted area along the ocean and the other in an area subject to heavy air pollution. If you wish your estimate to be correct to the nearest .1 pH with probability near .90, approximately how many rainfalls (pH values) must you include in each sample? (Assume that the variance of the pH measurements is approximately .25 at both locations and that the samples are to be of equal size.)

8.84 Organic chemists often purify organic compounds by a method known as fractional crystallization. An experimenter wanted to prepare and purify 4.85 g of aniline. Ten 4.85-gram specimens of aniline were prepared and purified to produce acetanilide. The following dry yields were obtained:

3.85, 3.88, 3.90, 3.62, 3.72, 3.80, 3.85, 3.36, 4.01, 3.82

Construct a 95% confidence interval for the mean number of grams of acetanilide that can be recovered from 4.85 grams of aniline.

8.85 Two new drugs were given to patients with hypertension. The first drug lowered the blood pressure of 16 patients an average of 11 points, with a standard deviation of 6 points. The second drug lowered the blood pressure of 20 other patients an average of 12 points, with a standard deviation of 8 points. Determine a 95% confidence interval for the difference in the mean reductions in blood pressure, assuming that the measurements are normally distributed with equal variances.

8.86 Does the price paid for tuna depend on the packaging method? *Consumer Reports* gives the estimated average price for a 6-ounce can or a 7.06-ounce pouch of tuna based on prices paid nationally in supermarkets.[15] The prices are recorded for a variety of different tuna brands in the following table:

Light Tuna in Water		White Tuna in Oil	White Tuna in Water		Light Tuna in Oil	
0.99	0.53	1.27	1.49	1.29	2.56	0.62
1.92	1.41	1.22	1.29	1.00	1.92	0.66
1.23	1.12	1.19	1.27	1.27	1.30	0.62
0.85	0.63	1.22	1.35	1.28	1.79	0.65
0.65	0.67				1.23	0.60
0.69	0.60					0.67
0.60	0.66					

Assume that the tuna brands included in the survey represent a random sample of all tuna brands available in the United States. Find a 95% confidence interval

a for the average price of light tuna packed in water. Interpret the interval. Specifically, what does the "95%" refer to?

b for the average price of light tuna packed in oil. How does the width of this interval compare to the interval found in part (a)? Give three reasons that the length of the intervals differ.

8.87 Refer to Exercise 8.86.

a Construct a 90% confidence interval for the difference in the mean price for light tuna packed in water and light tuna packed in oil.

b Based on the interval obtained in part (a), do you think that the mean prices differ for light tuna packed in water and oil? Why?

8.88 The Environmental Protection Agency (EPA) has collected data on LC50 measurements (concentrations that kill 50% of test animals) for certain chemicals likely to be found in

15. *Source:* Case Study "Pricing of Tuna" Copyright 2001 by Consumers Union of U.S., Inc., Yonkers, N.Y. 1073–1057, a nonprofit organization. From the June 2001 issue of *Consumer Reports* © for educational purposes only. NO commercial use or reproduction permitted. www.ConsumerReports.org.

freshwater rivers and lakes. (See Exercise 7.13 for additional details.) For certain species of fish, the LC50 measurements (in parts per million) for DDT in 12 experiments were as follows:

$$16, \quad 5, \quad 21, \quad 19, \quad 10, \quad 5, \quad 8, \quad 2, \quad 7, \quad 2, \quad 4, \quad 9$$

Estimate the true mean LC50 for DDT with confidence coefficient .90. Assume that the LC50 measurements have an approximately normal distribution.

8.89 Refer to Exercise 8.88. Another common insecticide, diazinon, yielded LC50 measurements in three experiments of 7.8, 1.6, and 1.3.

a Estimate the mean LC50 for diazinon, with a 90% confidence interval.

b Estimate the difference between the mean LC50 for DDT and that for diazinon, with a 90% confidence interval. What assumptions are necessary for the method that you used to be valid?

8.90 Do SAT scores for high school students differ depending on the students' intended field of study? Fifteen students who intended to major in engineering were compared with 15 students who intended to major in language and literature. Given in the accompanying table are the means and standard deviations of the scores on the verbal and mathematics portion of the SAT for the two groups of students:[16]

	Verbal		Math	
Engineering	$\overline{y} = 446$	$s = 42$	$\overline{y} = 548$	$s = 57$
Language/literature	$\overline{y} = 534$	$s = 45$	$\overline{y} = 517$	$s = 52$

a Construct a 95% confidence interval for the difference in average verbal scores of students majoring in engineering and of those majoring in language/literature.

b Construct a 95% confidence interval for the difference in average math scores of students majoring in engineering and of those majoring in language/literature.

c Interpret the results obtained in parts (a) and (b).

d What assumptions are necessary for the methods used previously to be valid?

8.91 Seasonal ranges (in hectares) for alligators were monitored on a lake outside Gainesville, Florida, by biologists from the Florida Game and Fish Commission. Five alligators monitored in the spring showed ranges of 8.0, 12.1, 8.1, 18.2, and 31.7. Four different alligators monitored in the summer showed ranges of 102.0, 81.7, 54.7, and 50.7. Estimate the difference between mean spring and summer ranges, with a 95% confidence interval. What assumptions did you make?

8.92 Solid copper produced by sintering (heating without melting) a powder under specified environmental conditions is then measured for porosity (the volume fraction due to voids) in a laboratory. A sample of $n_1 = 4$ independent porosity measurements have mean $\overline{y}_1 = .22$ and variance $s_1^2 = .0010$. A second laboratory repeats the same process on solid copper formed from an identical powder and gets $n_2 = 5$ independent porosity measurements with $\overline{y}_2 = .17$ and $s_2^2 = .0020$. Estimate the true difference between the population means $(\mu_1 - \mu_2)$ for these two laboratories, with confidence coefficient .95.

***8.93** A factory operates with two machines of type A and one machine of type B. The weekly repair costs X for type A machines are normally distributed with mean μ_1 and variance σ^2. The weekly repair costs Y for machines of type B are also normally distributed but with mean μ_2

16. *Source:* "SAT Scores by Intended Field of Study," *Riverside* (Calif.) *Press Enterprise*, April 8, 1993.

times at random intervals. The six readings, in parts per million, were 9.54, 9.61, 9.32, 9.48, 9.70, and 9.26. Estimate the population variance σ^2 for readings on this standard, using a 90% confidence interval.

8.102 The ages of a random sample of five university professors are 39, 54, 61, 72, and 59. Using this information, find a 99% confidence interval for the population standard deviation of the ages of all professors at the university, assuming that the ages of university professors are normally distributed.

8.103 A precision instrument is guaranteed to read accurately to within 2 units. A sample of four instrument readings on the same object yielded the measurements 353, 351, 351, and 355. Find a 90% confidence interval for the population variance. What assumptions are necessary? Does the guarantee seem reasonable?

8.10 Summary

The objective of many statistical investigations is to make inferences about population parameters based on sample data. Often these inferences take the form of estimates—either point estimates or interval estimates. We prefer unbiased estimators with small variance. The goodness of an unbiased estimator $\hat{\theta}$ can be measured by $\sigma_{\hat{\theta}}$ because the error of estimation is generally smaller than $2\sigma_{\hat{\theta}}$ with high probability. The mean square error of an estimator, $\text{MSE}(\hat{\theta}) = V(\hat{\theta}) + [B(\hat{\theta})]^2$, is small only if the estimator has small variance and small bias.

Interval estimates of many parameters, such as μ and p, can be derived from the normal distribution for large sample sizes because of the central limit theorem. If sample sizes are small, the normality of the population must be assumed, and the t distribution is used in deriving confidence intervals. However, the interval for a single mean is quite robust in relation to moderate departures from normality. That is, the actual confidence coefficient associated with intervals that have a nominal confidence coefficient of $100(1-\alpha)\%$ is very close to the nominal level even if the population distribution differs moderately from normality. The confidence interval for a difference in two means is also robust in relation to moderate departures from normality and to the assumption of equal population variances if $n_1 \approx n_2$. As n_1 and n_2 become more dissimilar, the assumption of equal population variances becomes more crucial.

If sample measurements have been selected from a normal distribution, a confidence interval for σ^2 can be developed through use of the χ^2 distribution. These intervals are very sensitive to the assumption that the underlying population is normally distributed. Consequently, the actual confidence coefficient associated with the interval estimation procedure can differ markedly from the nominal value if the underlying population is not normally distributed.

References and Further Readings

Casella, G., and R. L. Berger. 2002. *Statistical Inference*, 2d ed. Pacific Grove, Calif.: Duxbury.

Hoel, P. G. 1984. *Introduction to Mathematical Statistics*, 5th ed. New York: Wiley.

Notice that $L(\theta)$ is a function only of θ and $\overline{y}$ and that if

$$g(\overline{y}, \theta) = \frac{e^{-n\overline{y}/\theta}}{\theta^n} \quad \text{and} \quad h(y_1, y_2, \ldots, y_n) = 1,$$

then

$$L(y_1, y_2, \ldots, y_n \mid \theta) = g(\overline{y}, \theta) \times h(y_1, y_2, \ldots, y_n).$$

Hence, Theorem 9.4 implies that $\overline{Y}$ is a sufficient statistic for the parameter θ. ■

Theorem 9.4 can be used to show that there are many possible sufficient statistics for any one population parameter. First of all, according to Definition 9.3 or the factorization criterion (Theorem 9.4), the random sample itself is a sufficient statistic. Second, if $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a distribution with a density function with parameter θ, then the set of order statistics $Y_{(1)} \leq Y_{(2)} \leq \cdots \leq Y_{(n)}$, which is a function of $Y_1, Y_2, \ldots, Y_n$, is sufficient for θ. In Example 9.5, we decided that $\overline{Y}$ is a sufficient statistic for the estimation of θ. Theorem 9.4 could also have been used to show that $\sum_{i=1}^{n} Y_i$ is another sufficient statistic. Indeed, for the exponential distribution described in Example 9.5, any statistic that is a one–to–one function of $\overline{Y}$ is a sufficient statistic.

In our initial example of this section, involving the number of successes in n trials, $Y = \sum_{i=1}^{n} X_i$ reduces the data $X_1, X_2, \ldots, X_n$ to a single value that remains sufficient for p. Generally, we would like to find a sufficient statistic that reduces the data in the sample as much as possible. Although many statistics are sufficient for the parameter θ associated with a specific distribution, application of the factorization criterion typically leads to a statistic that provides the "best" summary of the information in the data. In Example 9.5, this statistic is $\overline{Y}$ (or some one-to-one function of it). In the next section, we show how these sufficient statistics can be used to develop unbiased estimators with minimum variance.

Exercises

9.37 Let $X_1, X_2, \ldots, X_n$ denote n independent and identically distributed *Bernoulli* random variables such that

$$P(X_i = 1) = p \quad \text{and} \quad P(X_i = 0) = 1 - p,$$

for each $i = 1, 2, \ldots, n$. Show that $\sum_{i=1}^{n} X_i$ is sufficient for p by using the factorization criterion given in Theorem 9.4.

9.38 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a normal distribution with mean μ and variance σ^2.

a If μ is unknown and σ^2 is known, show that $\overline{Y}$ is sufficient for μ.

b If μ is known and σ^2 is unknown, show that $\sum_{i=1}^{n}(Y_i - \mu)^2$ is sufficient for σ^2.

c If μ and σ^2 are both unknown, show that $\sum_{i=1}^{n} Y_i$ and $\sum_{i=1}^{n} Y_i^2$ are jointly sufficient for μ and σ^2. [Thus, it follows that $\overline{Y}$ and $\sum_{i=1}^{n}(Y_i - \overline{Y})^2$ or $\overline{Y}$ and S^2 are also jointly sufficient for μ and σ^2.]

9.39 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Poisson distribution with parameter λ. Show by conditioning that $\sum_{i=1}^{n} Y_i$ is sufficient for λ.

9.40 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Rayleigh distribution with parameter θ. (Refer to Exercise 9.34.) Show that $\sum_{i=1}^{n} Y_i^2$ is sufficient for θ.

9.41 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Weibull distribution with known m and unknown α. (Refer to Exercise 6.26.) Show that $\sum_{i=1}^{n} Y_i^m$ is sufficient for α.

9.42 If $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a geometric distribution with parameter p, show that $\overline{Y}$ is sufficient for p.

9.43 Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed random variables from a power family distribution with parameters α and θ. Then, by the result in Exercise 6.17, if $\alpha, \theta > 0$,

$$f(y \mid \alpha, \theta) = \begin{cases} \alpha y^{\alpha-1}/\theta^{\alpha}, & 0 \le y \le \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

If θ is known, show that $\prod_{i=1}^{n} Y_i$ is sufficient for α.

9.44 Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed random variables from a Pareto distribution with parameters α and β. Then, by the result in Exercise 6.18, if $\alpha, \beta > 0$,

$$f(y \mid \alpha, \beta) = \begin{cases} \alpha \beta^{\alpha} y^{-(\alpha+1)}, & y \ge \beta, \\ 0, & \text{elsewhere.} \end{cases}$$

If β is known, show that $\prod_{i=1}^{n} Y_i$ is sufficient for α.

9.45 Suppose that $Y_1, Y_2, \ldots, Y_n$ is a random sample from a probability density function in the (one-parameter) exponential family so that

$$f(y \mid \theta) = \begin{cases} a(\theta) b(y) e^{-[c(\theta) d(y)]}, & a \le y \le b, \\ 0, & \text{elsewhere,} \end{cases}$$

where a and b do not depend on θ. Show that $\sum_{i=1}^{n} d(Y_i)$ is sufficient for θ.

9.46 If $Y_1, Y_2, \ldots, Y_n$ denote a random sample from an exponential distribution with mean β, show that $f(y \mid \beta)$ is in the exponential family and that $\overline{Y}$ is sufficient for β.

9.47 Refer to Exercise 9.43. If θ is known, show that the power family of distributions is in the exponential family. What is a sufficient statistic for α? Does this contradict your answer to Exercise 9.43?

9.48 Refer to Exercise 9.44. If β is known, show that the Pareto distribution is in the exponential family. What is a sufficient statistic for α? Argue that there is no contradiction between your answer to this exercise and the answer you found in Exercise 9.44.

***9.49** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the uniform distribution over the interval $(0, \theta)$. Show that $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$ is sufficient for θ.

***9.50** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the uniform distribution over the interval (θ_1, θ_2). Show that $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$ and $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$ are jointly sufficient for θ_1 and θ_2.

***9.51** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y \mid \theta) = \begin{cases} e^{-(y-\theta)}, & y \ge \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$ is sufficient for θ.