

2 Cvatál Erdős  $\alpha(G) \leq \kappa(G)$

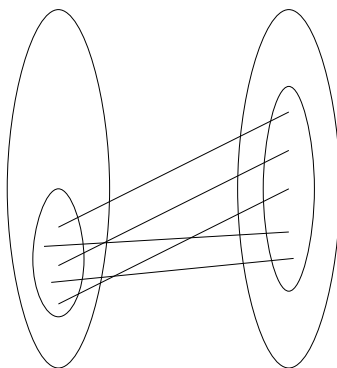
4 Take a maximum cycle

3 Find one

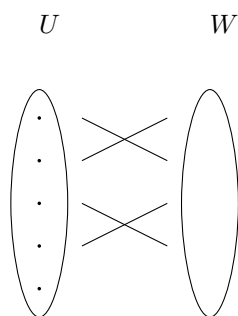
1 Ore:  $\forall u, v$  non-adjacent  $\deg(u) + \deg(v) \geq n$

$r = |U| \leq |W|$ .  $G$  has a matching of cardinality  $r$  iff

$$\forall S \subseteq U, |N(S)| \geq |S|$$

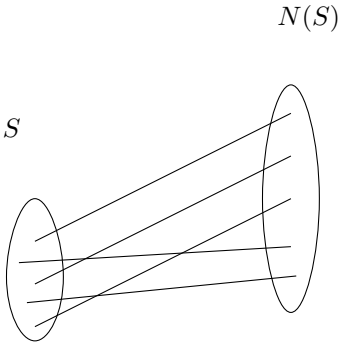


$G$  is  $d$ -regular



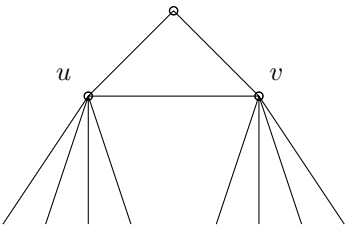
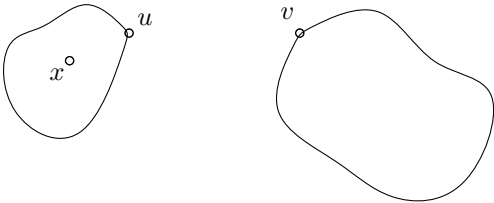
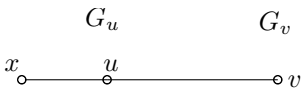
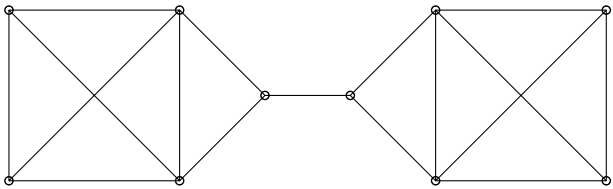
$$\begin{aligned} \sum_{u \in U} \deg(u) &= \sum_{v \in W} \deg(v) \\ \sum_{u \in U} d &= \sum_{v \in W} d \\ |U|d &= |W|d \\ |U| &= |W| \end{aligned}$$

Let  $S \subseteq U$



$$\begin{aligned} \sum_{u \in S} \deg(u) &= \sum_{v \in N(S)} \deg(v) \\ \sum_{u \in S} d &\leq \sum_{v \in N(S)} d \\ |S| \cdot d &\leq |N(S)| \cdot d \\ |N(S)| &\geq |S| \end{aligned}$$

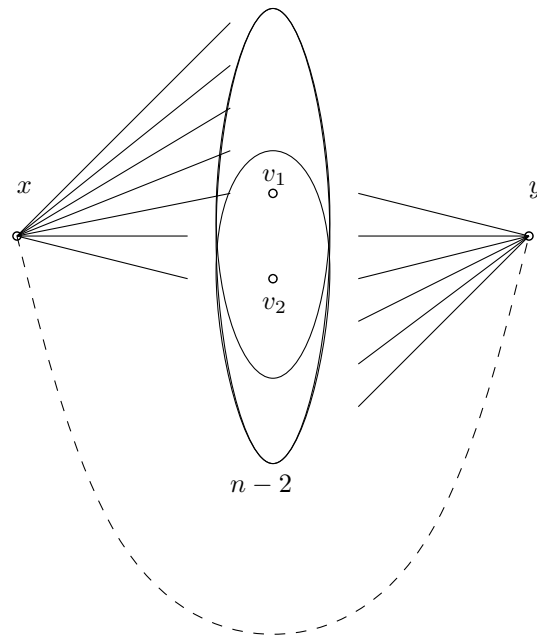
$$\delta(G) \geq \kappa(G)$$



$G$   $n \geq 3$

$\deg(v) \geq n/2$

nonseparable



2 internally disjoint paths

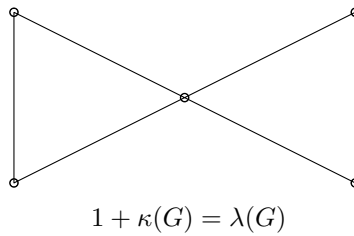
Menger's  $\Rightarrow$  nonseparable.

$G$   $n \geq 3$

$(n-1)$ -Connected

$G$  is  $K_n$

Whitney's Theorem:  $\kappa(G) \leq \lambda(G) \leq \delta(G)$



$$1 + \kappa(G) = \lambda(G)$$

Menger's Theorem

Fan Lemma