Homework 3

21-470 Calculus of Variations

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Due: Friday, February 14, 2014

Problem 1

Figure 1 and Table 1 show the relationship between β and B.

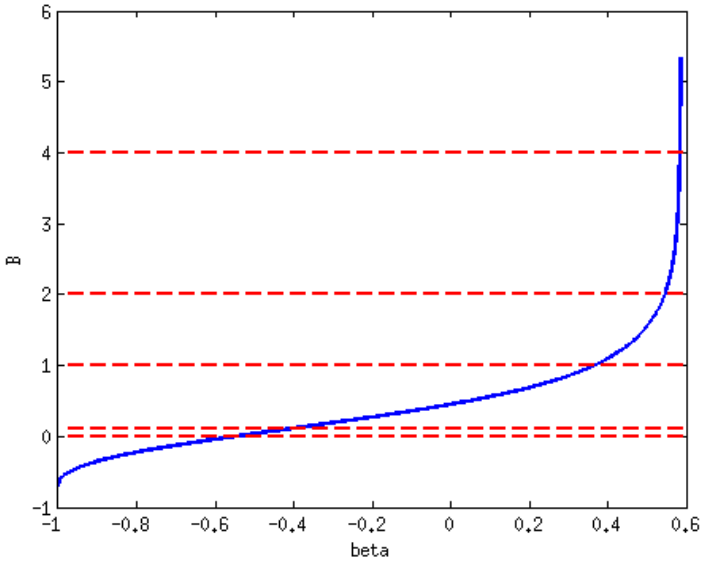

Figure 1: Plot of B over $\beta \in (-1, 0.6)$. B values selected in Table 1 are marked by dotted lines.

B	0	0.1	1	2	4
β	-0.5471	-0.4178	0.3649	0.5454	0.5841

Table 1: Values of β for selected values of B

Observing that B increases monotonically with β, the β corresponding to each B was computed numerically using the following iterative procedure:

```
beta_H <- 0.586; beta_L <- -0.8
while beta_H - beta_L > 0.0001
  beta_A <- (beta_H + beta_L)/2
  if B(beta_A) > B
    beta_H <- beta_A
  else
    beta_L <- beta_A
return beta_L
```

where $B(\beta_A)$ was computed by numerically integrating Equation (6).

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Problem 2

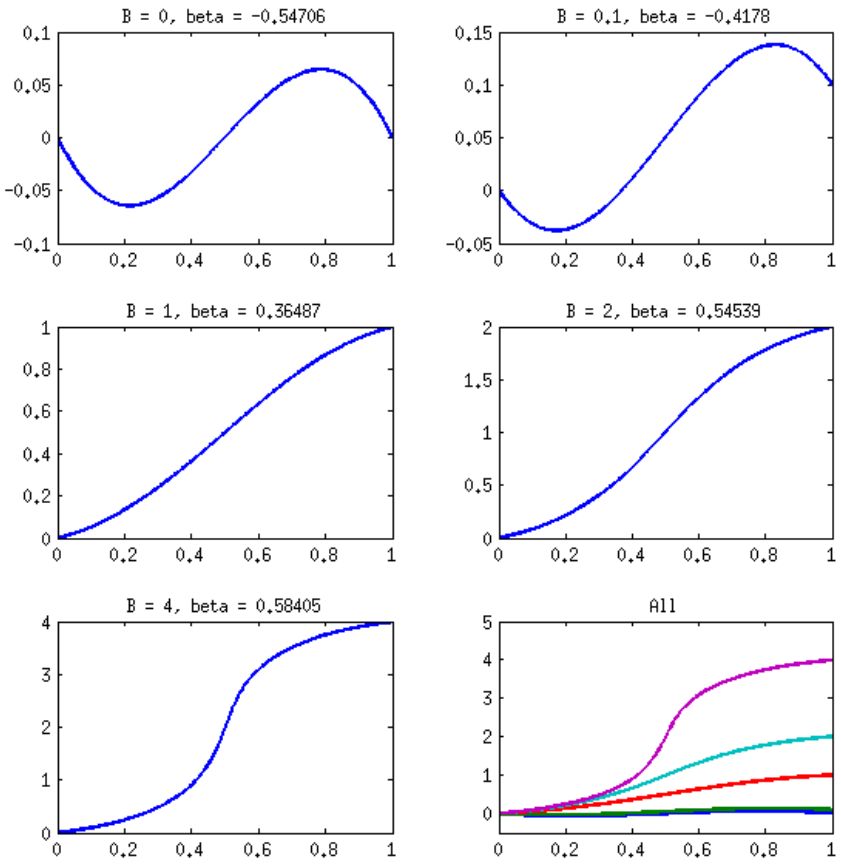

Figure 2: Graphs of y for $B = 0, 0.1, 1, 2, 4$, computed from equation (6) and β values in Table 1. The last plot shows all 5 graphs, for comparison.

Problem 3

Figure 1 suggests that β is roughly a sigmoid function of B. A least squares approximation to a sample of β points gives

$$\beta(B) \approx \frac{0.56 + 1.3}{1 + 10^{1.1(0.12-B)}} - 1.3,$$

as illustrated in Figure 3. The case $\beta \geq 0$ results in a path (y) that is monotonically increasing

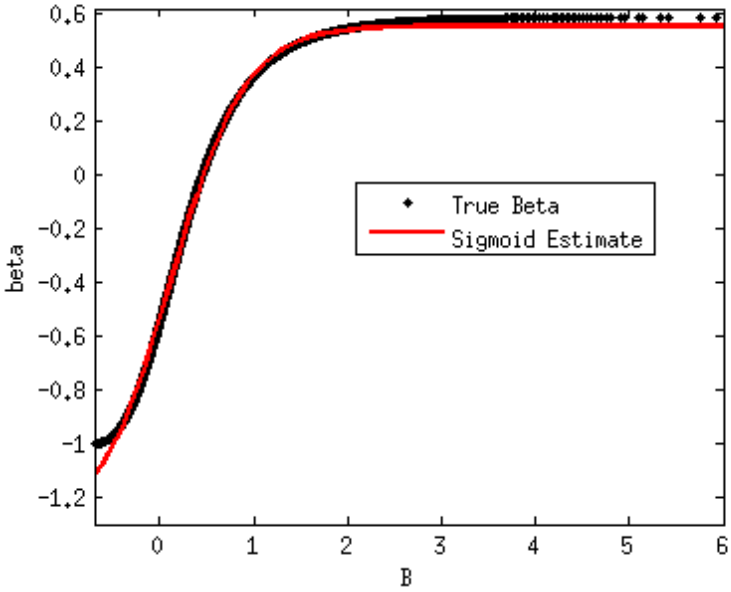

Figure 3: Plot of β over B and a fitted sigmoid function.

in time and x, whereas, when $\beta < 0$, the path is more sinusoidal, and, y is not monotonically increasing. This corresponds roughly to $B = 0.45$.