

Name	Distribution	$E[X]$	$\text{Var}[X]$	z-transform
Bernoulli( $p$ ), $p \in [0, 1]$	$p_X(k) = \begin{cases} p & k=1 \\ 1-p & k=0 \end{cases}$	$p$	$p(1-p)$	$1-p+zp$
Binomial( $n, p$ )	$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$	$np$	$np(1-p)$	$(zp + (1-p))^n$
Geometric( $p$ )	$p_X(k) = (1-p)^{k-1} p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{zp}{1-z(1-p)}$
Poisson( $\lambda$ )	$p_X(k) = \frac{e^{-\lambda} \lambda^k}{k!}$	$\lambda$	$\lambda$	$e^{(z-1)\lambda}$

Name	p.d.f. ( $f_X$ )	c.d.f. ( $F_X$ )	$E[X]$	$\text{Var}[X]$
Uniform( $a, b$ ), $a < b$	$\frac{1}{b-a} \quad a \leq x \leq b,$ $0 \quad \text{otherwise}$	$\begin{matrix} 0 & x \leq a, \\ \frac{x-a}{b-a} & a \leq x \leq b, \\ 1 & x \geq b \end{matrix}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential( $\lambda$ ), $0 < \lambda$	$\lambda e^{-\lambda x} \quad 0 \leq x,$ $0 \quad x < 0$	$\begin{matrix} 1 - e^{-\lambda x} & 0 \leq x, \\ 0 & x < 0 \end{matrix}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Pareto( $\alpha$ ), $0 < \alpha < 2$	$\alpha x^{-\alpha-1} \quad 1 \leq x,$ $0 \quad x < 1$	$\begin{matrix} 1 - x^{-\alpha} & 1 \leq x, \\ 0 & x < 1 \end{matrix}$	$\begin{matrix} \infty & \alpha \leq 1, \\ \frac{\alpha}{\alpha-1} & 1 < \alpha \end{matrix}$	$\infty$
Normal( $\mu, \sigma^2$ ),	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	GROSS	$\mu$	$\sigma^2$

3 axioms ( $E, F$  mutually exclusive): 1)  $P(E) \geq 0$ , 2)  $P(E \cup F) = P(E) + P(F)$ , 3)  $P(\Omega) = 1$

Bayes' Law (discrete):  $P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{P(E|F) \cdot P(F)}{P(E)}$  (cont.):  $f_X(x|Y=y) = \frac{P(Y=y|X=x)f_X(x)}{P(Y=y)}$

Two events are independent if  $P(E|F) = P(E)$  or  $P(E \cap F) = P(E)P(F)$ , they are conditionally independent if  $P(E \cap F|G) = P(E|G)P(F|G)$

If  $X$  and  $Y$  are independent, then  $E[XY] = E[X]E[Y]$ .

Memorilessness:  $P(X > s + t | X > s) = P(X > t)$ .

$X$  and  $Y$  are independent if  $f_{X,Y}(x, y) = f_X(x) \cdot f_Y(y)$ .

Z-transform of  $X$  is:  $\hat{X}(z) = E[z^X] = \sum_{i=0}^{\infty} p_X(i) z^i$ .  $\hat{X}'(1) = E[X]$ ,  $\hat{X}''(1) = E[X(X-1)]$ .

If  $X, Y$  are independent, and  $Z = X + Y$ , then  $\hat{Z}(z) = \hat{X}(z) \cdot \hat{Y}(z)$ .

Also  $\hat{X}(z) = p \cdot \hat{A}(z) + (1-p) \cdot \hat{B}(z)$  if  $X = A$  with probability  $p$  and  $X = B$  with probability  $1-p$ .

$\text{Var}(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2$

If  $X, Y$  independent, then  $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ .

The  $k^{\text{th}}$  moment is  $E[X^k] = \sum_i p_X(i) i^k$ , and the  $k^{\text{th}}$  central moment is defined as  $E[(X - E[X])^k]$ .

If  $\{X\}_n$  are independent and identically distributed random variables, where  $N$  is the random variable indicating how many there are and  $S = \sum_{i=1}^N X_i$ , then  $E[S] = E[N]E[X]$ ,  $E[S^2] = E[N]\text{Var}(X) + E[N^2](E[X])^2$  and  $\text{Var}(S) = E[N]\text{Var}(X) + \text{Var}(N)(E[X])^2$

**Yao's:** Let  $\mathcal{A}$  be a class of deterministic algorithms, and let  $\mathcal{I}$  be a class of all inputs. Then,  $\min_{A \in \mathcal{A}} E[T_A(I_\tau)] \leq \max_{I \in \mathcal{I}} E[T_{A_\sigma}(I)]$  where  $\tau$  and  $\sigma$  are distributions on  $\mathcal{I}$  and  $\mathcal{A}$ .

**Chebyshev:** If  $X$  has finite expected value  $\mu$  and variance  $\sigma^2 \neq 0$ , then,  $\forall k > 0$ ,  $P(|X - \mu| \geq \frac{1}{k^2})$ .

**Markov:** If  $a > 0$ , the  $P(|X| \geq a) \leq \frac{E[|X|]}{a}$ .

If  $X \sim \mathcal{N}(\mu, \sigma^2)$  and  $Y = aX + b$ , then  $Y \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$ .