MATH 651: PROBLEM SET 6   SOLUTIONS ARE IN CLASS ON MON. NOV 19.

1. (10 points) A space Y is said to have the *universal extension property* if for every normal space X, every closed subset $A \subseteq X$ and every continuous mapping $f : A \to Y$ there exists a continuous extension of f to X.

   (i) Let I be a nonempty set. Show that $\mathbb{R}^I$ has the universal extension property.

   (ii) Show that $\{0, 1\}$ with discrete topology does not have the universal extension property.

2. (10 points) Let (X, τ) be a topological space and let $\{E_\alpha\}_{\alpha\in\Lambda}$ be a locally finite family of subsets of X. Prove that
$$\overline{\bigcup_{\alpha\in\Lambda} E_\alpha} = \bigcup_{\alpha\in\Lambda} \overline{E_\alpha}.$$
   In particular, the union of a locally finite family of closed sets is closed.

3. (10 points) Show that Sorgenfrey line is paracompact.

4. (10 points) Is every locally compact Hausdorff space paracompact? Prove or provide a counterexample.

5. (5 points) Provide an example of a 2nd countable Hausdorff space which is not metrizable (and justify your claims).

6. (5 points) Provide and example of a separable, 1st countable T4 space which is not metrizable (and justify your claims).