15-451 Algorithms, Spring 2012

Homework 1 (50 pts) **Due: Wed. Jan. 25.**

You should type up your solutions in TeX and submit electronically by 23:59 on Jan. 25.

If you struggle with TeX/LaTeX, you may use any other editor. However, in this case you will have to print your work and hand it in recitation on Jan. 25. Our submission system can only accept TeX files.

Question	Points	Score
1	6	
2	16	
3	12	
4	10	
5	6	
Total:	50	

(6) 1. **Review.** Compute the following series. Show your work.

(a) $\sum_{k=0}^{\infty} \frac{k+1}{2^k}$

(b) $\sum_{k=0}^{n} 2^{k+1}$

(16) 2. **Asymptotic Notations.**

(a) Describe the asymptotic behavior of each of the following expressions in the simplest form.

i. $2n^3 + 25n^2 + \log n = \Theta($

ii. $\log_4(n^3) = \Theta($

iii. $4^{\log_8 n} = \Theta($

iv. $\log_{2n}(n^{3n}) = \Theta($

(b) Give a tightest expression using o, O, Θ, or Ω, relating the two functions.

i. $f(n) = n^2$ and $g(n) = 4n^2 + 3n \log n$

i. ____________________

ii. $f(n) = n^{15}$ and $g(n) = 3^n$

ii. ____________________

iii. $f(n) = \log(\sqrt{n})$ and $g(n) = \log(n^{12})$

iii. ____________________

iv. $f(n) = 2^{\log_3 n}$ and $g(n) = 3^{\log_5 n}$

iv. ____________________

(12) 3. **Solving Recurrence Equations.** Give a tight asymptotic bound for the following recurrences. Use the master theorem. Show your work.

(a) $T(n) = 3T(\frac{n}{3}) + 1, \quad T(1) = 1$

(b) $T(n) = 3T(\frac{n}{2}) + n, \quad T(1) = 1$

(c) Give an asymptotic solution to this recurrence. Use the tree method. Show your work.

$$T(n) = T(\tfrac{n}{2}) + T(\tfrac{n}{3}) + n, \quad T(1) = T(2) = T(3) = 1$$

4. **Strassen's Algorithm.**

(5) (a) Consider Strassen's algorithm for matrix multiplication. Derive and then solve a recurrence relation for the number of additions and subtractions. Assume that the matrix size is a power of 2.

(5) (b) For the lower order matrices Strassen's algorithm is slower compare to a regular divide and conquer. Find that threshold value for the matrix size n when you want to switch from Strassen's algorithm to the regular one.

5. **Karatsuba's Algorithm.**

(6) (a) Compute the number of addition and subtractions in Karatsuba's algorithm by deriving a recurrence equation and then solving it. Assume that the number length is a power of 2.