21-373, Algebraic Structures, Department of Mathematical Sciences, Carnegie Mellon University
Fall 2011: (Math Studies Section) Monday, Wednesday, Friday, 10:30 am, Porter Hall 226B.
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Assignment 10 - Wednesday November 30, 2011. Due Monday December 5

Exercise 64: Show that the polynomial $P = -1 + (x-1)\,(x-2)\cdots(x-n)$ is irreducible in $\mathbb{Z}[x]$ for all $n \geq 1$.

Exercise 65: For $n \geq 2$, show that $P = 1 + x + \ldots + x^{n-1}$ is irreducible in $\mathbb{Z}[x]$ if and only if n is prime.

Exercise 66: Determine the splitting field extensions $F \subset \mathbb{C}$ for P_j over $\mathbb{Q}$ and compute $[F\!:\!\mathbb{Q}]$ for
i) $P_1 = x^4 - 2$,
ii) $P_2 = x^4 + 2$,
iii) $P_3 = x^4 + x^2 + 1$,
iv) $P_4 = x^6 - 4$.

Exercise 67: Show that the product of the non-zero elements of any finite field E is -1.

Exercise 68: Find the number of monic irreducible polynomials of degree 4 in $\mathbb{Z}_3[x]$.

Exercise 69: Find the number of monic irreducible polynomials of degree d in $\mathbb{Z}_p[x]$, when both d and p are prime.

Exercise 70: (Putnam 2001-A3) For each integer m, consider the polynomial

$$P_m(x) = x^4 - (2m+4)x^2 + (m-2)^2.$$

For what values of m is $P_m(x)$ the product of two non-constant polynomials with integer coefficients?