Homework 4

15-423 Digital Signal Processing for CS
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1. (a) By Linearity and the Time-Shifting property of the Z-transform,
$$Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) = \boxed{(b_0 + b_1 z^{-1} + b_2 z^{-2}) X(z).}$$
   (b) By Linearity and the Differentiation property of the Z-transform,
$$Y(z) = \boxed{-z\frac{dX(z)}{dz} - 5X(z).}$$

2. The snap of a whip should closely approximate a delta function. Thus, assuming effect of the concert hall on the music was time-invariant (pretty reasonable) and linear (probably still reasonable), given a recording of the violinist, we can perform the same transformation on the sound as did the concert hall, by writing the signal as a linear combination of delta functions and then replacing the delta functions with the recording of the whip.

3. Since convolutions in the time domain correspond to products in the frequency domain, we can simply convolve the signal with system corresponding to $H_1(Z)H_2(Z)$, rather than convolving it separately with the signal corresponding to $H_1(Z)$ and the signal corresponding to $H_2(Z)$.

4. (a) Decomposing into partial fractions, we note
$$\frac{z(2z-a-b)}{(z-a)(z-b)} = \frac{z}{z-a} + \frac{z}{z-b}. = \frac{1}{1-az^{-1}} + \frac{1}{1-bz^{-1}}.$$
   Then, by linearity of the inverse Z-transform,
$$x[n] = \mathcal{Z}^{-1}\left\{\frac{z}{z-a}\right\} + \mathcal{Z}\left\{\frac{z}{z-b}\right\} = \boxed{a^n u[n] + b^n u[n].}$$
   (b) Since $\mathcal{Z}^{-1}\{1\} = \delta[n]$, by linearity and the time-shifting property of Z-transform,
$$x[n] = \boxed{\delta[n] + 2\delta[n-1] + 5\delta[n-2] + 7\delta[n-3] + \delta[n-5].}$$

5. By Parseval's Theorem, the sum of squared values of the original signal is equal to the integral of the square of the Z-transform (over a contour dependent on the region of convergence). Thus, we could just approximate this integral and compare it to the threshold.

6. Since convolutions in the time domain correspond to products in the frequency domain, we can invert the effects of the systems in Problem 3 by convolving with the signal corresponding to $\frac{1}{H_1(Z)H_2(Z)}$.

7. (a) The original time-domain signal must have bandwidth no greater than $\frac{1}{2}\min\{Y, X\}$.
   (b) The signal must have no frequencies greater than Y.
   (c) The signal should first be low-pass filtered to remove any frequencies greater than Y, and then all but 1 in α of the samples should be removed from the signal.