Assignment 3
Due on Friday, February 14

Minimum Transit Time of a Boat: Let $b = 1$ and $\omega = \sqrt{2}$ so that the functional for the crossing time becomes

$$T(y) = \frac{1}{\sqrt{2}} \int_0^1 \frac{\sqrt{1 - e(x)^2 + y'(x)^2} - e(x)y'(x)}{1 - e(x)^2} dx. \tag{1}$$

We want to minimize $T(y)$ over all smooth functions y satisfying

$$y(0) = 0, \quad y(1) = B, \tag{2}$$

where B is a given real number.
Recall that $e(x)$ is the normalized current and is given by

$$e(x) = \frac{w(x)}{\omega}, \tag{3}$$

where $w(x)$ is the actual current velocity (in the positive y-direction). For this assignment you may use either

$$w(x) = \sin \pi x, \quad 0 \le x \le 1 \tag{4}$$

or

$$w(x) = 4x(1 - x), \quad 0 \le x \le 1. \tag{5}$$

Recall also that the solutions to the 1$^{\text{st}}$ Euler-Lagrange equation satisfying the boundary conditions are given by

$$y(x) = \int_0^x G(t, \beta) dt, \quad 0 \le x \le 1, \tag{6}$$

where

$$G(x, \beta) = \frac{\gamma(x, \beta)\sqrt{1 - e(x)^2}}{\sqrt{1 - \gamma(x, \beta)^2}}, \text{ and} \tag{7}$$

$$\gamma(x, \beta) = e(x) + \beta(1 - e(x)^2) \tag{8}$$

and β is a constant to be chosen so that $y(1) = B$.

a. Try to determine β numerically for each of the cases $B = 0$, $B = .1$, $B = 1$, $B = 2$, and $B = 4$.

b. Plot the graph of the minimizer y for each of the β-values found in part (a).

c. Try some other values of B, and/or other current profiles. Make some interesting observations and/or conjectures.