Homework 9; Due Wednesday, 11/16
(Real Analysis, 1 – Schwab)

Answer these questions and prove these lemmas. All problems require proof/justification, as noted by the style of proof used in the examples in class.

Question 0.1. *Chp 5 #3*

Question 0.2. *Chp 5 #5*

Question 0.3. *Chp 5 #7*

Question 0.4. *Chp 5 #9*

Lemma 0.5 (best affine approximation)**.** *Let f be a real valued function defined on a neighborhood of $x_0 \in \mathbb{R}$.*

(a) *f is differentiable at x_0 if and only if there exists a real number, c, such that*

$$\lim_{h \to 0} \left| \frac{f(x_0 + h) - f(x_0) - c \cdot h}{h} \right| = 0,$$

in which case the value of $f'(x_0)$ is $f'(x_0) = c$.

(b) *a corollary of (a) is that if f is differntiable at x_0, then there exists a function, $err(x - x_0)$ such that for all x sufficiently close to x_0*

$$f(x) = f(x_0) + f'(x_0) \cdot (x - x_0) + err(x - x_0),$$

and most importantly err satisfies

$$\lim_{h \to 0} \frac{err(h)}{h} = 0.$$

(commentary: thanks to the uniquess of $f'(x_0)$, if it exists, part (a) says that the function $l(x) = f(x_0) + f'(x_0) \cdot (x - x_0)$ is the best affine approximation to f in a neighborhood of x_0.)

Question 0.6. *Chp 5 #11 (hint: it is just my personal preference, but I suggest trying to incorporate Lemma 0.5 a few times here.)*

Question 0.7. *Chp 5 #12*

Question 0.8. *Chp 5 #22*

Question 0.9. *Chp 5 #24*

Question 0.10. *Give an explanation using Thm 5.12 as to why the function $f(x) = |x|$ is not differentiable on all of the domain, $[-1, 1]$.*