Assignment 2
Due on Friday, February 7

In Problems 1-6 find all possible maximizers and minimizers for J on $\mathscr{Y}$. For each "candidate" that you find, try to determine if this candidate is a minimizer, maximizer, or neither.

1. $J(y) = \displaystyle\int_0^1 [xy(x)^4 + 2x^2 y(x)^3 y'(x)]dx$

   $\mathscr{Y} = \{y \in C^2[0,1] : y(0) = 0,\ y(1) = 1\}$

2. $J(y) = \displaystyle\int_1^2 [x^2 y'(x)^2 + 2y(x)^2]dx$

   $\mathscr{Y} = \{y \in C^2[1,2] : y(1) = -1,\ y(2) = 5\}$

3. $J(y) = \displaystyle\int_0^\pi [y(x)^2 - y'(x)^2]dx$

   $\mathscr{Y} = \{y \in C^2[0,\pi] : y(0) = y(\pi) = 0\}$

4. $J(y) = \displaystyle\int_0^\pi [y(x)^2 - y'(x)^2]dx$

   $\mathscr{Y} = \{y \in C^2[0,\pi] : y(\pi) = 0\}$

5. $J(y) = \displaystyle\int_0^1 [(y'(x) - x)^2 + 2xy(x)]dx$

   $\mathscr{Y} = \{y \in C^2[0,1] : y(0) = 1\}$

6. $J(y) = \displaystyle\int_1^8 xy'(x)^4 dx$

   $\mathscr{Y} = \{y \in C^2[1,8] : y(1) = 4,\ y(8) = 7\}$

Let $a, b, A, B, \alpha \in \mathbb{R}$ with $a < b$ and $f \in C^2([a,b] \times \mathbb{R} \times \mathbb{R})$ be given.

7. Let $g \in C[a,b]$ be given and put $\overline{\mathscr{V}} = \left\{ v \in C^2[a,b] : v(a) = v(b) = 0,\ \displaystyle\int_a^b v(x)dx = 0 \right\}$.

   Assume that

   $$\int_a^b g(x)v(x)dx = 0 \quad \text{for all } v \in \overline{\mathscr{V}}.$$

   What can you conclude about g?

8. Let $\overline{\mathscr{Y}} = \left\{ y \in C^2[a,b] : y(a) = A,\ y(b) = B,\ \int_a^b y(x)dx = \alpha \right\}$ and put

$$J(y) = \int_a^b f(x, y(x), y'(x))dx \quad \text{for all } y \in \overline{Y}.$$

Use the result of problem 7 to discuss minimizing (or maximizing) J on $\overline{\mathscr{Y}}$

9. Let $y \in C^2[a,b]$ be given and assume that y satisfies $(E-L)_1$. Show that there exists $c \in \mathbb{R}$ such that y satisfies $(E-L)_2$.