Graph Theory — Exercise 2

Due Wednesday 22^{nd}, 2012

1. A tree T has n vertices all of degree 1 or 3. Find the number of leaves in T.
2. Find all graphs with at least four vertices such that the subgraph induced by every three vertices is a tree (or show that no such graph exists).
3. Prove that an edge of a connected graph is a bridge if and only if it belongs to every spanning tree of G.
4. Let T and T' be two spanning trees of a connected graph G with n vertices. Show that there exists a sequence $T = T_0, T_1, \ldots, T_k = T'$ of spanning trees of G such that

$$|E(T_i) \cap E(T_{i+1})| \geq n - 2 \text{ for all } \; 0 \leq i \leq k - 1.$$

5. Prove Cayley's formula using Kirchoff's Theorem.
6. Let G be a graph, let E_G be its oriented incidence matrix and let L_G be its Laplacian. Prove that $L_G = E_G E_G^T$.
7. Let G be a graph with vertex set $[n]$ and adjacency matrix $A = (A_{i,j})$. Show that the (i, j) entry of the matrix A^k is the number of i–j walks of length k in G.
8. Let G be the graph obtained by removing a regular spanning subgraph of degree 1 from the complete graph K_8. Find the number of spanning trees in G. You may use (any) matrix manipulation software (submit your code as part of your solution).