36 - 226 Introduction to Statistical Inference

Homework assignment 1

Due: Wednesday, January 23, 2013

Format reminders:

- Write your full name, the course number, and the homework number at the top of each page.
- **STAPLE** your entire assignment together with a staple.
- Write clearly. Electronic submission of homework assignments is not accepted.

1. **Let the cumulative distribution function $F(y)$ be given by**

$$F(y) = \begin{cases} 0, & y < 0 \\ y^3, & 0 \le y \le 1 \\ 1, & y > 1 \end{cases}$$

   (a) **Write down the probability density fucntion.**

   (b) **Compute $V(Y)$.**

   (c) **What is the probability that an observed random variable Y takes on a value less than $1/2$, given that it is greater than $1/4$.**

2. **A student prepares for an exam by studying a list of nine questions. She can solve five of them. For the exam, the instructor selects four problems at random from the list of nine. What is the probability that the student can solve all four problems on the exam? *Hint: consider the hypergeometric distribution.***

3. **Assume $Y_1 \sim N(2,2)$, $Y_2 \sim Gamma(2,2)$, and $Y_3 \sim Beta(2,2)$ are independent random variables. Let $Y = Y_1 + 2Y_2 + 3Y_3$. Find $E(Y)$ and $V(Y)$.**

4. **The moment-generating function for the Laplace distribution is**

$$m(t) = \frac{e^{at}}{1 - b^2t^2}.$$

   Derive $E(Y)$ for this distribution.

5. **Wackerly, 3.147. If Y has a geometric distribution with probability of success p, show that the moment-generating function for Y is**

$$m(t) = \frac{pe^t}{1 - qe^t}, \textbf{ where } q = 1 - p.$$

6. **Wackerly, 4.12. The length of time to failure (in hundreds of hours) for a transistor is a random variable Y with distribution function given by**

$$F(y) = \begin{cases} 0, & \textbf{if } y < 0 \\ 1 - e^{-y^2}, & \textbf{if } y \ge 0. \end{cases}$$

(a) **Show that $F(y)$ has the properties of a distribution function.**

(b) **Find the 0.30 -quantile, $\phi_{0.30}$, of Y. =**

(c) **Find $f(y)$.**

(d) **Find the probability that the transistor operates for at least 200 hours.**

(e) **Find $P(Y > 100 \mid Y \leq 200)$.**

7. **Wackerly, 5.8. Let Y_1 and Y_2 have the joint probability density function given by**

$$f(y_1, y_2) = \begin{cases} ky_1y_2, & 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1 \\ 0, & \textbf{elsewhere.} \end{cases}$$

(a) **Find the value of k that makes this a probability density function.**

(b) **Find the joint distribution function for Y_1 and Y_2.**

(c) **Find $P(Y_1 \leq 1/2, Y_2 \leq 3/4)$.**

8. **Wackerly, 5.31. Suppose that the random variables Y_1 and Y_2 have joint probability density function, $f(y_1, y_2)$ given by**

$$f(y_1, y_2) = \begin{cases} 30y_1y_2^2, & y_1 - 1 \leq y_2 \leq 1 - y_1, 0 \leq y_1 \leq 1 \\ 0, & \textbf{elsewhere.} \end{cases}$$

(a) **Show that the marginal density of Y_1 is a beta density with $\alpha = 2$ and $\beta = 4$.**

(b) **Derive the marginal desnity of Y_2.**

(c) **Derive the conditional density of Y_2 given $Y_1 = y_1$.**

(d) **Find $P(Y_2 > 0 \mid Y_1 = 0.75)$.**