**MATH 759: PROBLEM SET 4** **DUE ON MON. NOVEMBER 4TH**

**Exercises 5, 7, 8, 9, 11, and 12** at the end of Chapter 3 in the book by do Carmo.

1. Let $\mathcal{M}$ be an $n$ dimensional manifold, and $(\overline{M}, \overline{g})$ an $n+k$ dimensional Riemannian manifold. Consider an immersion $f : \mathcal{M} \to \overline{\mathcal{M}}$. Then $g = f^*\overline{g}$ is a metric on $\mathcal{M}$. Let $p \in \mathcal{M}$ and $U$ an open neighborhood such that $f(U)$ is a submanifold of $\overline{M}$. Furthermore let $X$ and $Y$ be vector fields on $f(U)$. Let $\overline{X}$ and $\overline{Y}$ be their extensions to an open set in $\overline{M}$. Define $\nabla_X Y|_p = (df)^{-1}(\Pi_T \overline{\nabla}_{\overline{X}}\overline{Y}|_p)$ where $\overline{\nabla}$ is the Riemann connection on $\overline{M}$, $\Pi_q$ is, for $q \in f(U)$, the orthogonal projection of $T_q\mathcal{M}$ to $df(T_{f^{-1}(q)}\mathcal{M})$, that is the projection to the tangent space of the submanifold. Show that $\nabla$ defined above is the Riemannian connection on $\mathcal{M}$.