Signal Detection Theory Problem Set
42-699B/86-595 Neural Data Analysis
Due 11/15/12 at noon.

1. (27 pts.) Suppose you have a neuron that emits spike counts that are well described by a Poisson process. With no signal present, the rate of the process is λ_N=20; with a signal present, the rate increases to λ_S=30.

   a. (9 pts.) Suppose you don't know anything about how often the signal is presented. If you want to maximize your odds of guessing whether or not a signal was present, how many spikes should you observe before reporting that you saw it? With this criterion, what are P(TP) and P(FA)?

   b. (9 pts.) Suppose now you find out that signals are quite rare, i.e., the priors for noise and signal are p(n) = 0.8 and p(s)=0.2. Now how many spikes should you observe before reporting that you saw the signal? With this new criterion, what are P(TP) and P(FA)?

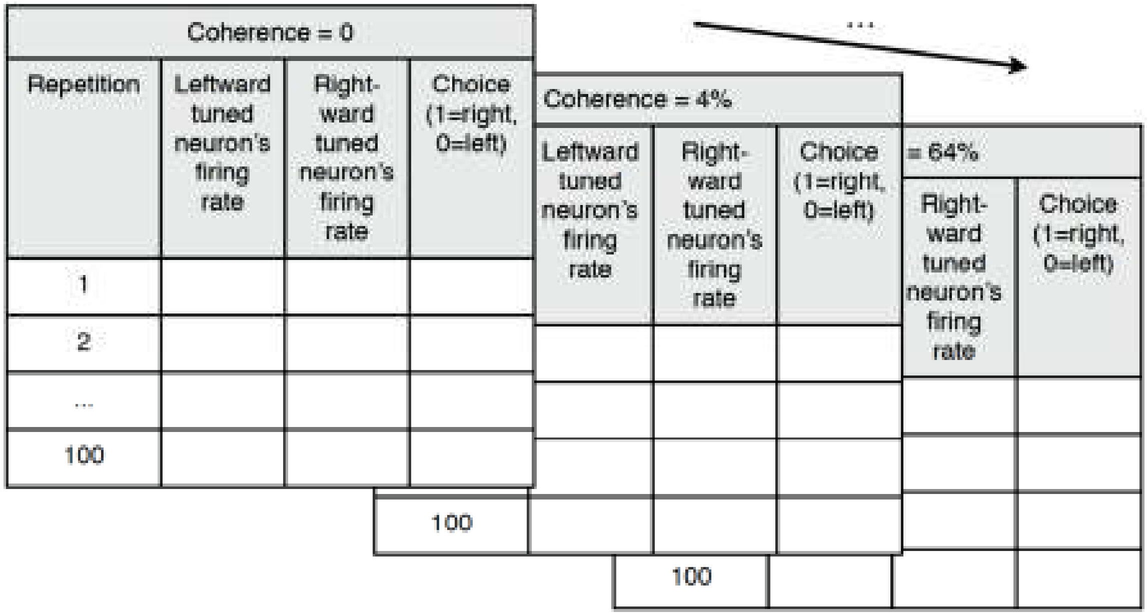
   c. (9 pts.) Suppose I were to pay you \$1 every time you correctly surmise that a signal is present, but I deduct \$5 every time you report a signal that isn't there. (You get nothing for false negatives and true negatives.) To maximize your payoff, how many spikes should you now observe before reporting that a signal is there? Assume the same priors as in part (b). What are P(TP) and P(FA) with this new criterion?

2. **(68 pts.) Matlab analysis: neurometric curves and choice probability.**

   General suggestion: the more you turn in, the more you boost your chances for partial credit. Feel free to turn in all code you use to solve these problems.

   In this problem set, you will recreate the seminal analyses performed by Britten, Newsome and colleagues in their 1992 and 1996 papers that we discussed in class. You will compare the activity of the neurons to the behavior of the monkey, and observe how fluctuations in the neurons' responses influence the monkey's judgments. The dataset MTdata.mat contains data from an MT recording experiment. Two neurons were isolated, that (remarkably) were tuned for perfectly opposite directions. They were recorded while the monkey viewed a moving dot pattern. Simultaneously, the animal made perceptual judgments about the direction of dot motion, and reported them with a saccadic eye movement. The monkey performed 200 repetitions of each task condition while the neurons were recorded. Six different dot coherences were used: 0, 4%, 8%, 16%, 32%, and 64%. At 0% coherence, the monkey just had to guess (and he was rewarded randomly), and at 64% coherent, the task was easy.

The three-dimensional matrix MTdata is structured like this:



Data for 100 repetitions per coherence are given to you. To make things easier, I've only given you the data collected when the dot motion was actually to the right. (Note that for the 0% coherent stimulus, a randomly-selected 100 trials is provided.)

Thanks to Aaron Batista for providing us with this data set.

a. (17 pts.) Construct a psychometric curve for the monkey's behavior. Fit it with the following function:

$$p = 1 - \frac{1}{2}\exp\left(-\left(\frac{c}{\alpha}\right)^{\beta}\right)$$

One way to do this is to use the Matlab function nlinfit.m. Another option is to use the curve fitting toolbox, cftool. (I have provided this function in the code psychometric_curve.m, take a look at it.) Plot the choice data and your curve fit, and put on the plot your estimates of α (the animal's psychophysical threshold) and β (the animal's sensitivity to coherence).

b. (17 pts.) Use the neurons to implement an ROC analysis. For each coherence, slide a criterion from 0 to 100, and calculate what percentage of the null and preferred distributions are greater than that criterion (an easy way to do this is to use the commands sort, find, and sum). Plot those values against each other, and when you connect those points, it yields a curve. On one plot, graph the ROC curve for each coherence level. (Make sure you label your curves!) For each value of coherence, report the accuracy of the most accurate classifier you could have, and indicate with a colored circle the point on the ROC curve that would yield the most accurate classifier.

c. (17 pts.) Construct a neurometric curve from the pair of neurons. To do this, calculate the area under each of the ROC curves you generated above. You don't have to cut out the shapes onto a piece of cardboard – trapz.m is a good way to do numerical integration in Matlab. As we discussed in class, these area-under-curve measurements (normalized to be a fraction of the whole box) for each of the coherence values yields a neurometric curve. Fit it with the same equation you fit the psychometric curve, and determine α and β for the best fit curve. How do the neurometric and psychometric curves compare?

d. (17 pts.) Calculate the choice probability for this neuron. To what degree can you predict the monkey's choice on a trial-by-trial basis based on the fluctuations in the firing rates of the rightward-tuned and leftward-tuned neurons? To do this, first generate histograms of the firing rates conditioned on leftward or rightward movement for each cell and each coherence value. (Do not turn in plots of all of these histograms. Instead, turn in one plot with the two histograms for left and right movements at 0 coherence for the leftward preferring cell, both histograms overlaid, and one plot of the two histograms at 0 coherence for the rightward preferring cell, again with both histograms overlaid.)

   Next, for each of these pairs of histograms, compute the area under the ROC curve. This is the choice probability for that neuron at that coherence. You should get 12 numbers in total (6 coherences * 2 neurons). Turn in a plot of the histogram of the 12 choice probabilities. What is the mean choice probability for this monkey?

3. (5 pts.) About how much time did you spend on each question of this problem set? Which (sub)problem taught you the most, and which taught you the least?