with 1 numerator degree of freedom and ν denominator degrees of freedom. You can easily verify that the square of $t_{.025} = 2.228$ (used for the two-tailed test with $\alpha = .05$ and $\nu = 10$ df) is equal to $F_{.05} = 4.96$. Had the t test been used for Example 13.1, we would have obtained $t = -1.6967$, which satisfies the relationship $t^2 = (-1.6967)^2 = 2.88 = F$. ■

Exercises

13.1 The reaction times for two different stimuli in a psychological word-association experiment were compared by using each stimulus on independent random samples of size 8. Thus, a total of 16 people were used in the experiment. Do the following data present sufficient evidence to indicate that there is a difference in the mean reaction times for the two stimuli?

Stimulus 1	1	3	2	1	2	1	3	2
Stimulus 2	4	2	3	3	1	2	3	3

a Use the ANOVA approach to test the appropriate hypotheses. Test at the $\alpha = .05$ level of significance.

b **Applet Exercise** Use the applet *F-Ratio Probabilities and Quantiles* to determine the exact p-value for the test in part (a).

c Test the appropriate hypotheses by using the two-sample t test for comparing population means, which we developed in Section 10.8. Compare the value of the t statistic to the value of the F statistic calculated in part (a).

d What assumptions are necessary for the tests implemented in the preceding parts?

13.2 Refer to Exercises 8.90 and 10.77.

a Use an F test to determine whether there is sufficient evidence to claim a difference in the mean verbal SAT scores for high school students who intend to major in engineering and language/literature. Give bounds for the associated p-value. What would you conclude at the $\alpha = .05$ level of significance?

b **Applet Exercise** Use the applet *F-Ratio Probabilities and Quantiles* to determine the exact p-value for the test in part (a).

c How does the value of the F statistic obtained in part (a) compare to the value of the t statistic that you obtained in Exercise 10.77?

d What assumptions are necessary for the analyses performed in part (a)?

13.3 Comparison of More Than Two Means: Analysis of Variance for a One-Way Layout

An ANOVA to compare more than two population means is a simple generalization of the ANOVA presented in Section 13.2. The random selection of independent samples from k populations is known as a *one-way layout*. As indicated in Section 12.4, the data in a one-way layout may correspond to data obtained from a completely randomized

Table 13.3 ANOVA table for a one-way layout

Source	df	SS	MS	F
Treatments	$k-1$	SST	$\text{MST} = \dfrac{\text{SST}}{k-1}$	$\dfrac{\text{MST}}{\text{MSE}}$
Error	$n-k$	SSE	$\text{MSE} = \dfrac{\text{SSE}}{n-k}$	
Total	$n-1$	$\sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij}-\overline{y})^2$		

Table 13.4 ANOVA table for Example 13.2

Source	df	SS	MS	F
Treatments	3	712.6	237.5	3.77
Error	19	1196.6	63.0	
Total	22	1909.2		

The ANOVA table for Example 13.2, shown in Table 13.4, gives a compact presentation of the appropriate computed quantities for the analysis of variance.

Exercises

13.3 State the assumptions underlying the ANOVA of a completely randomized design.

13.4 Refer to Example 13.2. Calculate the value of SSE by pooling the sums of squares of deviations within each of the four samples and compare the answer with the value obtained by subtraction. This is an extension of the pooling procedure used in the two-sample case discussed in Section 13.2.

***13.5** In Exercise 6.59, we showed that if Y_1 and Y_2 are independent χ^2-distributed random variables with ν_1 and ν_2 df, respectively, then $Y_1 + Y_2$ has a χ^2 distribution with $\nu_1 + \nu_2$ df. Now suppose that $W = U + V$, where U and V are *independent* random variables, and that W and V have χ^2 distributions with r and s df, respectively, where $r > s$. Use the method of moment-generating functions to prove that U must have a χ^2 distribution with $r - s$ df.[1]

13.6 Suppose that independent samples of sizes $n_1, n_2, \ldots, n_k$ are taken from each of k normally distributed populations with means $\mu_1, \mu_2, \ldots, \mu_k$ and common variances, all equal to σ^2. Let Y_{ij} denote the jth observation from population i, for $j = 1, 2, \ldots, n_i$ and $i = 1, 2, \ldots, k$, and let $n = n_1 + n_2 + \cdots + n_k$.

a Recall that

$$\text{SSE} = \sum_{i=1}^{k}(n_i - 1)S_i^2 \qquad \text{where } S_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}(Y_{ij} - \overline{Y}_{i\bullet})^2.$$

Argue that SSE/σ^2 has a χ^2 distribution with $(n_1 - 1) + (n_2 - 1) + \cdots + (n_k - 1) = n - k$ df.

1. Exercises preceded by an asterisk are optional.

b Argue that under the null hypothesis, $H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$ all the Y_{ij}'s are independent, normally distributed random variables with the *same mean and variance.* Use Theorem 7.3 to argue further that, under the null hypothesis,

$$\text{Total SS} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (Y_{ij} - \overline{Y})^2$$

is such that $(\text{Total SS})/\sigma^2$ has a χ^2 distribution with $n - 1$ df.

c In Section 13.3, we argued that SST is a function of only the sample means and that SSE is a function of only the sample variances. Hence, SST and SSE are independent. Recall that Total SS = SST + SSE. Use the results of Exercise 13.5 and parts (a) and (b) to show that, under the hypothesis $H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$, SST/σ^2 has a χ^2 distribution with $k-1$ df.

d Use the results of parts (a)–(c) to argue that, under the hypothesis $H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$, $F = \text{MST}/\text{MSE}$ has an F distribution with $k - 1$ and $n - k$ numerator and denominator degrees of freedom, respectively.

13.7 Four chemical plants, producing the same products and owned by the same company, discharge effluents into streams in the vicinity of their locations. To monitor the extent of pollution created by the effluents and to determine whether this differs from plant to plant, the company collected random samples of liquid waste, five specimens from each plant. The data are given in the accompanying table.

Plant	Polluting Effluents (lb/gal of waste)				
A	1.65	1.72	1.50	1.37	1.60
B	1.70	1.85	1.46	2.05	1.80
C	1.40	1.75	1.38	1.65	1.55
D	2.10	1.95	1.65	1.88	2.00

a Do the data provide sufficient evidence to indicate a difference in the mean weight of effluents per gallon in the effluents discharged from the four plants? Test using $\alpha = .05$.

b **Applet Exercise** Find the p-value associated with the test in part (a) using the applet *F-Ratio Probabilities and Quantiles.*

13.8 In a study of starting salaries for assistant professors, five male assistant professors at each of three types of doctoral-granting institutions were randomly polled and their starting salaries were recorded under the condition of anonymity. The results of the survey (measured in \$1000) are given in the following table.[2]

Public Universities	Private-Independent	Church-Affiliated
49.3	81.8	66.9
49.9	71.2	57.3
48.5	62.9	57.7
68.5	69.0	46.2
54.0	69.0	52.2

2. *Source:* Adapted from "Average Salary for Men and Women Faculty, by Category, Affiliation, and Academic Rank 2002–2003," *Academe: Bulletin of the American Association of University Professors,* March–April 2003, 37.

a What type of experimental design was utilized when the data were collected?

b Is there sufficient evidence to indicate a difference in the average starting salaries of assistant professors at the three types of doctoral-granting institutions? Use the table in the text to bound the p-value.

c **Applet Exercise** Determine the exact p-value by using the applet *F-Ratio Probabilities and Quantiles*.

13.9 In a comparison of the strengths of concrete produced by four experimental mixes, three specimens were prepared from each type of mix. Each of the 12 specimens was subjected to increasingly compressive loads until breakdown. The accompanying table gives the compressive loads, in tons per square inch, attained at breakdown. Specimen numbers 1–12 are indicated in parentheses for identification purposes.

Mix A	Mix B	Mix C	Mix D
(1) 2.30	(2) 2.20	(3) 2.15	(4) 2.25
(5) 2.20	(6) 2.10	(7) 2.15	(8) 2.15
(9) 2.25	(10) 2.20	(11) 2.20	(12) 2.25

a Assuming that the requirements for a one-way layout are met, analyze the data. State whether there is statistical support at the $\alpha = .05$ level of significance for the conclusion that at least one of the concretes differs in average strength from the others.

b **Applet Exercise** Use the applet *F-Ratio Probabilities and Quantiles* to find the p-value associated with the test in part (a).

13.10 A clinical psychologist wished to compare three methods for reducing hostility levels in university students. A psychological test (HLT) was used to measure the degree of hostility. High scores on this test indicate great hostility. Eleven students obtaining high and nearly equal scores were used in the experiment. Five were selected at random from among the 11 problem cases and treated by method A. Three were taken at random from the remaining 6 students and treated by method B. The other 3 students were treated by method C. All treatments continued throughout a semester. Each student was given the HLT test again at the end of the semester, with the results shown in the accompanying table.

Method A	Method B	Method C
73	54	79
83	74	95
76	71	87
68		
80		

a Do the data provide sufficient evidence to indicate that at least one of the methods of treatment produces a mean student response different from the other methods? Give bounds for the attained significance level.

b **Applet Exercise** Find the exact p-value by using the applet *F-Ratio Probabilities and Quantiles*.

c What would you conclude at the $\alpha = .05$ level of significance?

13.11 It is believed that women in the postmenopausal phase of life suffer from calcium deficiency. This phenomenon is associated with the relatively high proportion of bone fractures were selected, and their blood phenotypes were recorded. The observed numbers with each phenotype are given in the following table.

A	B	AB	O
89	18	12	81

a Is there sufficient evidence, at the .05 level of significance, to claim that current proportions differ from the historic values?

b **Applet Exercise** Use the applet *Chi-Square Probability and Quantiles* to find the p-value associated with the test in part (a).

14.2 Previous enrollment records at a large university indicate that of the total number of persons who apply for admission, 60% are admitted unconditionally, 5% are conditionally admitted, and the remainder are refused admission. Of 500 applicants to date for next year, 329 were admitted unconditionally, 43 were conditionally admitted, and the remainder were not admitted. Do the data indicate a departure from previous admission rates?

a Test using $\alpha = .05$.

b **Applet Exercise** Use the applet *Chi-Square Probability and Quantiles* to find the p-value associated with the test in part (a).

14.3 A city expressway with four lanes in each direction was studied to see whether drivers preferred to drive on the inside lanes. A total of 1000 automobiles were observed during the heavy early-morning traffic, and their respective lanes were recorded. The results are shown in the accompanying table. Do the data present sufficient evidence to indicate that some lanes are preferred over others? (Test the hypothesis that $p_1 = p_2 = p_3 = p_4 = 1/4$, using $\alpha = .05$.) Give bounds for the associated p-value.

Lane	1	2	3	4
Count	294	276	238	192

14.4 Do you hate Mondays? Researchers in Germany have provided another reason for you: They concluded that the risk of heart attack on a Monday for a working person may be as much as 50% greater than on any other day.[1] The researchers kept track of heart attacks and coronary arrests over a period of 5 years among 330,000 people who lived near Augsberg, Germany. In an attempt to verify the researcher's claim, 200 working people who had recently had heart attacks were surveyed. The day on which their heart attacks occurred appear in the following table.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
24	36	27	26	32	26	29

Do these data present sufficient evidence to indicate that there is a difference in the percentages of heart attacks that occur on different days of the week? Test using $\alpha = .05$.

14.5 After inspecting the data in Exercise 14.4, you might wish to test the hypothesis that the probability that a heart attack victim suffered a heart attack on Monday is $1/7$ against the alternative that this probability is greater than $1/7$.

1. *Source:* Daniel Q. Haney, "Mondays May Be Hazardous," *Press-Enterprise* (Riverside, Calif.), 17 November 1992, p. A16.

Americans are not so sure about this conclusion. Do you think that we know all of the relevant facts associated with Kennedy's assassination, or do you think that some information has been withheld? The following table contains the results of a nationwide poll of 900 registered voters.[4]

	We Know All Relevant Facts	Some Relevant Facts Withheld	Not Sure
Democrat	42	309	31
Republican	64	246	46
Other	20	115	27

a Do the data provide sufficient evidence to indicate a dependence between party affiliation and opinion about a possible cover-up? Test using $\alpha = .05$.

b Give bounds for the associated p-value and interpret the result.

c **Applet Exercise** Use the χ^2 applet to obtain the approximate p-value.

d Why is the value you obtained in part (c) "approximate"?

14.14 A study was conducted by Joseph Jacobson and Diane Wille to determine the effect of early child care on infant–mother attachment patterns.[5] In the study, 93 infants were classified as either "secure" or "anxious" using the Ainsworth strange-situation paradigm. In addition, the infants were classified according to the average number of hours per week that they spent in child care. The data appear in the accompanying table.

	Hours in Child Care		
Attachment Pattern	Low (0–3 hours)	Moderate (4–19 hours)	High (20–54 hours)
Secure	24	35	5
Anxious	11	10	8

a Do the data indicate a dependence between attachment patterns and the number of hours spent in child care? Test using $\alpha = .05$.

b Give bounds for the attained significance level.

14.15 Suppose that the entries in a contingency table that appear in row i and column j are denoted n_{ij}, for $i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, c$; that the row and column totals are denoted r_i, for $i = 1, 2, \ldots, r$, and c_j, for $j = 1, 2, \ldots, c$; and that the total sample size is n.

a Show that

$$X^2 = \sum_{j=1}^{c}\sum_{i=1}^{r}\frac{[n_{ij} - \widehat{E(n_{ij})}]^2}{\widehat{E(n_{ij})}} = n\left(\sum_{j=1}^{c}\sum_{i=1}^{r}\frac{n_{ij}^2}{r_i c_j} - 1\right).$$

Notice that this formula provides a computationally more efficient way to compute the value of X^2.

b Using the preceding formula, what happens to the value of X^2 if every entry in the contingency table is multiplied by the same integer constant $k > 0$?

14.16 A survey to explore the relationship between voters' church-attendance patterns and their choice of presidential candidate was reported in the *Riverside Press-Enterprise* prior to the

4. *Source:* Adapted from Dana Blanton, "Poll: Most Believe 'Cover-Up' of JFK Assassination Facts," http://www.foxnews.com/story/0,2933,102511,00.html, 10 February 2004.

5. *Source:* Linda Schmittroth (ed.), *Statistical Record of Women Worldwide* (Detroit and London: Gale Research, 1991), pp. 8, 9, 335.

The critical value of χ^2 for $\alpha = .05$ and $(r-1)(c-1) = (1)(3) = 3$ df is 7.815. Because X^2 exceeds this critical value, we reject the null hypothesis and conclude that the fraction of voters favoring candidate A is not the same for all four wards. The associated p-value is given by $P(\chi^2 > 10.72)$ when χ^2 has 3 df. Thus, $.01 \leq p\text{-value} \leq .025$. The χ^2 applet gives $P(\chi^2 > 10.72) = .01334$. ■

This example was worked out in Exercise 10.106 by the likelihood ratio method. Notice that the conclusions are the same.

The test implemented in Example 14.4 is a test of the equality of four binomial proportions based on independent samples from each of the corresponding populations. Such a test is often referred to as a *test of homogeneity* of the binomial populations. If there are more than two row categories and the column totals are fixed, the χ^2 test is a test of the equivalence of the proportions in c multinomial populations.

Exercises

14.22 A study to determine the effectiveness of a drug (serum) for the treatment of arthritis resulted in the comparison of two groups each consisting of 200 arthritic patients. One group was inoculated with the serum whereas the other received a placebo (an inoculation that appears to contain serum but actually is not active). After a period of time, each person in the study was asked whether his or her arthritic condition had improved. The results in the accompanying table were observed. Do these data present sufficient evidence to indicate that the proportions of arthritic individuals who said their condition had improved differed depending on whether they received the serum?

Condition	Treated	Untreated
Improved	117	74
Not improved	83	126

a Test by using the X^2 statistic. Use $\alpha = .05$.

b Test by using the Z test of Section 10.3 and $\alpha = .05$. Compare your result with that in part (a).

c Give bounds for the attained significance level associated with the test in part (a).

14.23 The χ^2 test used in Exercise 14.22 is equivalent to the two-tailed Z test of Section 10.3, provided α is the same for the two tests. Show algebraically that the χ^2 test statistic X^2 is the square of the test statistic Z for the equivalent test.

14.24 How do Americans in the "sandwich generation" balance the demands of caring for older and younger relatives? The following table contains the results of a telephone poll of Americans aged 45 to 55 years conducted by the *New York Times*.[9] From each of four subpopulations, 200 individuals were polled and asked whether they were providing financial support for their parents.

9. *Source:* Adapted from Tamar Lewin, "Report Looks at a Generation, and Caring for Young and Old," *New York Times* online, 11 July 2001.