36 - 226 Introduction to Statistical Inference

Homework assignment 8

Due: Wednesday, March 20, 2013

- Write your full name, the course number, and the homework number at the top of each page.
- **STAPLE** your entire assignment together with a staple.
- Write clearly. Electronic submission of homework assignments is not accepted.

1. Question 1 on Exam 1

2. Question 4 on Exam 1

3. Wackerly 9.71.

4. Wackerly 9.74.

5. Wackerly 9.77.

6. Wackerly 9.78.

7. Wackerly 9.96.

8. Wackerly 9.97.

9. Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a probability density function $f(y \mid \theta)$, where θ is an unknown parameter. Let $\hat{\theta}$ be an unbiased estimator for θ. Define the *Fisher Information* or *expected information* to be

$$I_Y(\theta) = -nE\left[\frac{\partial^2 \log f(y \mid \theta)}{\partial \theta^2}\right].$$

Then under very general conditions: $var(\hat{\theta}) \geq [I_Y(\theta)]^{-1}$. This result is known as the Cramer-Rao inequality. If equality is obtained, i.e. $var(\hat{\theta}) = [I_Y(\theta)]^{-1}$, then the estimator $\hat{\theta}$ is called *efficient*. The inequality holds for discrete probability mass functions $p(y)$ as well.

Please use the Cramer-Rao inequality to answer the following:

(a) Suppose that $p(y \mid \lambda)$ is Poisson with mean λ. Show that $\bar{Y}$ is an efficient estimator of λ.

(b) Suppose $f(y \mid \mu, \sigma^2)$ is the normal probability density with mean μ and variance σ^2. show that $\bar{Y}$ is an efficient estimator of μ.

10. Wackerly 10.3.