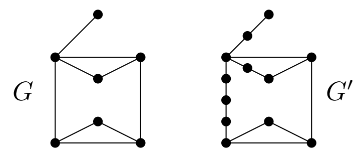
21-484 Notes

JD Nir
jnir@andrew.cmu.edu
April 18, 2012

Def (p. 235): A graph G' is a subdivision of a graph G, if G' can be obtained from G by replacing edges by paths.

Example:



G' is a subdivision of G.

Thm: (9.7, Kuratowski's theorem): G is planar if and only if it does not contain a subdivision of K_5 as a subgraph or a subdivision of $K_{3,3}$ as a subgraph.

$\rightarrow$ If G is planar, then G does not contain a K_5 subgraph or a $K_{3,3}$ subgraph because these graphs are not planar.

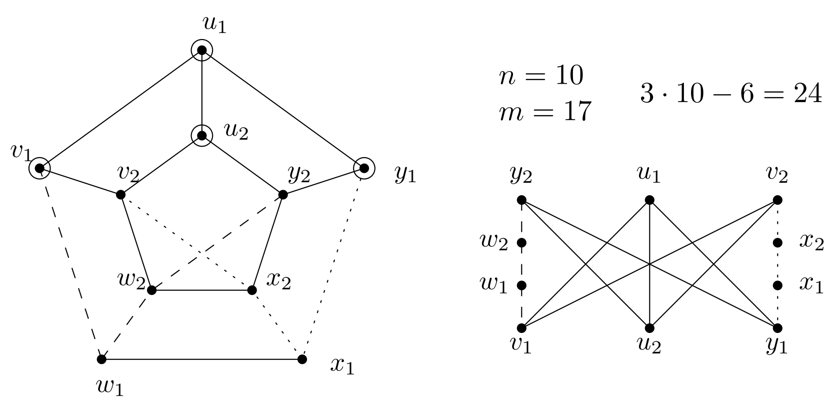
$\rightarrow$ K_5 is not planar since it has 5 vertices and 10 edges, $10 > 3 \cdot 5 - 6$

$\rightarrow$ a subdivision operation is replacing one edge uv by a path uwv where w is a new vertex adjacent only to u and v.

$\rightarrow$ If we do k subdivision operations of K_5 we end with $5 + k$ vertices, $10 + k$ edges, so...

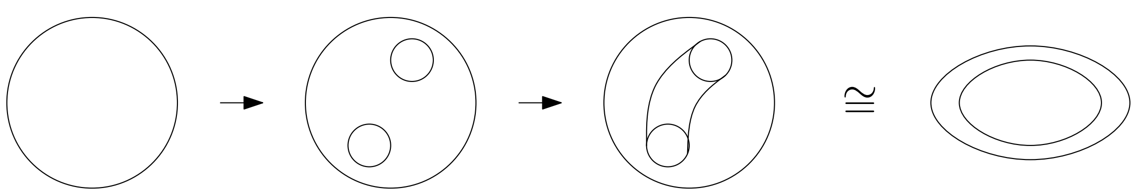
$\rightarrow$ Should prove similarly to the proof that $K_{3,3}$ is not planar.

Example (9.8):



placeholder

"Adding a handle to a surface"



→ If you add k handles to the sphere, you get S_k, which is a surface of genus k.

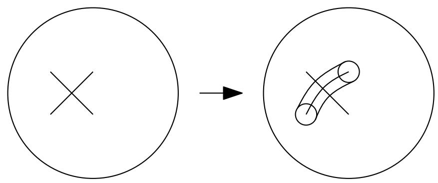
→ Def (p. 244): - A graph G is embeddable in S_k if it can be drawn on S_k such that two edges do not intersect.

- A Graph G has genus if it can be embedded in $S_{\gamma(G)}$ but can not be embedded in $S_{\gamma G-1}$.

Claim: $\delta(G)$ is finite for all graphs G.

Proof: Draw G on the sphere such that every intersection point which is not a vertex is an intersection point of at most 2 edge.

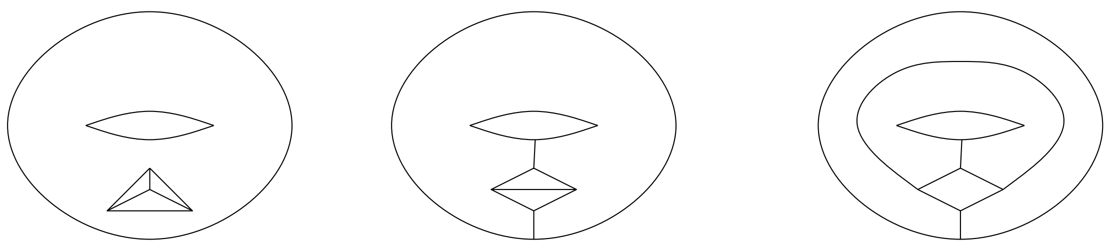
→ Add a handle for each intersection point



Def: A region of a surface is called 2-cell if any closed curve can be continuously contracted in the region to a single point.

- A 2-cell embedding of a graph is an embedding such that every region is a 2-cell region.

Example: (9.25 + 9.27)



$r = 4$ $r = 3$ $r = 2$