Math 21-236, Mathematical Studies Analysis II, Spring 2012
Assignment 8

The due date for this assignment is Wednesday May 2.

1. Let $E \subseteq \mathbb{R}^N$ and let $f : E \to \mathbb{R}$. Assume that f^+ and f^- are Riemann integrable in the improper sense with

$$\int_E f^+(\mathbf{x})\, d\mathbf{x} = \int_E f^-(\mathbf{x})\, d\mathbf{x} = \infty.$$

   Prove that f cannot be Riemann integrable in the improper sense.

2. Prove that (ii) implies (i) in Proposition 219.

3. Let $V := \left\{(u,v) \in \mathbb{R}^2 : 0 < v < 1,\, 0 < u < \sqrt{v}\right\}$, consider the function $\varphi : V \to \mathbb{R}^3$ defined by

$$\varphi(u,v) := \left(u+v, 1+v^2, u\right),$$

   and let $M := \varphi(V)$.

   (a) Prove that M is a 2-dimensional surface of class C^∞.

   (b) Calculate the integral

$$\int_M z\, d\mathcal{H}^2.$$

4. Prove that the set

$$M := \left\{(x,y,z) \in \mathbb{R}^3 : z = \sqrt{x^2+y^2},\, (x,y) \in \mathbb{R}^2\right\}$$

   is not a 2-dimensional surface of class C^1.