36 - 226 Introduction to Statistical Inference

Homework assignment 7

Due: Wednesday, March 6, 2013

- Write your full name, the course number, and the homework number at the top of each page.
- **STAPLE** your entire assignment together with a staple.
- Write clearly. Electronic submission of homework assignments is not accepted.

1. During the lecture on Wednesday, 2/20/13, we considered a special case of the Pareto distribution:

$$Y_i \text{ iid sample from } f_Y(y) = \frac{\alpha\beta^\alpha}{y^{\alpha+1}} = \alpha\beta^\alpha y^{-(\alpha+1)} \text{ where } \beta < y < \infty$$

   Interested in joint sufficient statistics for α and β:

   (a) Show that $\left(\prod Y_i, Y_{(1)}\right)$ and $\left(\sum logY_i, Y_{(1)}\right)$ are two possible sets of joint sufficient statistics for α, β

   (b) Show that, for α known (i.e. constant) and β unknown, the MVUE for β is $\frac{\alpha n-1}{\alpha n} Y_{(1)}$.

   (c) During the MVUE part, when finding the distribution of the minimum, check that you see it is also a Pareto distribution but with parameters $(\alpha^* = \alpha n, \beta^* = \beta)$. Inidcate this in your solutions.

2. Wackerly 9.16. *Hint: See HW4 problem 1 for the variance of* $\hat{\sigma}_1^2$.

3. Wackerly 9.19.

4. Wackerly 9.21.

5. Wackerly 9.80.

6. Wackerly 9.82.

7. Wackerly 9.89.

8. Suppose you work for an automotive engineering group that is responsible for modeling the length of life of brake components. While consumers know that brake pads have to be replaced every so often, repeated replacements in a short length of time will drive away customers. Your boss has asked you to help calculate the probability of replacing brake pads six times in fewer than three years (for example).

   The life of a component X is often assumed to be $Exp(\beta)$. We're interested in the life of six components, replaced one after the other.

   Your group doesn't have an established β value for this component type, so you'll need to estimate it. Let $Y = \sum_{i=1}^{6} X_i$.

   (a) Find the maximum likelihood estimator for β. *Hint: first you will need to find the distribution of* Y*, and then the corresponding likelihood.*

(b) You're also asked for a possible confidence interval for β. Find a 95% confidence interval for β using the pivotal quantity $\frac{Y}{\beta}$.

(c) Suppose you gather a sample of size $n = 20$, and find that $\sum_{i=1}^{20} y_i = 85.6$. Estimate β.

(d) Use collected customer data to estimate the probability of six replacements in fewer than three years. *Hint: Y represents the customer's total component life length, and we are interested in $P(Y \leq 3)$. In R, it may be helpful to use the function 'pgamma'.*