1:

$n > 1$ lightbulbs are arranged in a circle, and each is initially on. A mischievous gnome runs around the circle clockwise and whenever he sees a light that is on, flips the switch for the next light (changing it from on to off or from off to on). Formally, we might number the lightbulbs $1, \ldots, n$, and say that the gnome starts at 1, and when he is at lighbulb i changes the setting of lightbulb $i + 1 \pmod n$ if i is on, and then proceeds to $i + 1 \pmod n$ to repeat.

Prove that the gnome's mischief is in vain, and eventually all of the lights will again be turned on!

2:

Prove that for any Eulerian plane graph there exists an Eulerian tour that never crosses itself in the plane. That is, it can "touch" briefly at a vertex (perhaps entering from the East, leaving South, and then later entering from the West and leaving North), but it could not travel from West to East through the vertex then from North to South.

3:

Prove that if H is a triangulation, then H^* is 1-factorable (that is, the edges of H^* can be partitioned into perfect matchings).

4:

An *outerplanar* graph is a graph that can be drawn on the plane with all vertices on the boundary of the exterior region. Prove that every outerplanar graph is 3-colorable.

5:

Given an arbitrary Hamiltonian plane graph G, prove that G^* is 4-colorable. [You may not use the Four Color Theorem.]