21-621 Introduction to Lebesgue Integration Test 2 December 7, 2017
due December 14, 2012

Read all the problems prior to beginning to work. Check with the text, where there may be additional hints. You are, of course, challenged to complete this exercise on your own.

1. page 93 Problem 19: Suppose that $f \in L^1(R^d)$. For $\alpha > 0$, let $E_\alpha = \{x : |f(x)| > \alpha\}$. Show that

$$\int_{R^d} |f(x)|dx = \int_0^\infty m(E_\alpha)d\alpha$$

Note Corollary 3.8, page 85. This is a useful fact.

2. page 94 Problem 22. Prove that if $f \in L^1(R^d)$ and

$$\hat{f}(\xi) = \int_{R^d} f(x)e^{-2\pi i x\xi}dx$$

then

$$\hat{f}(\xi) \to 0, \text{ as } |\xi| \to \infty.$$

(Note that there is a hint in the book.)

3. page 260 Problem 6. It is not true that $f \in L^1$ implies that $f^* \in L^1$, where f^* is the maximal function of f. However $f \in L^2 \;\Rightarrow\; f^* \in L^2$ and there is a constant C such that

$$\|f^*\|_{L^2} \leqq C\|f\|_{L^2}$$

(There is a sketch for the proof with steps in the text.)