21-484 Notes
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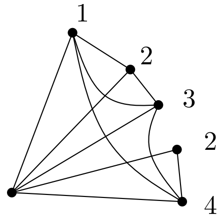
→ Kempe:

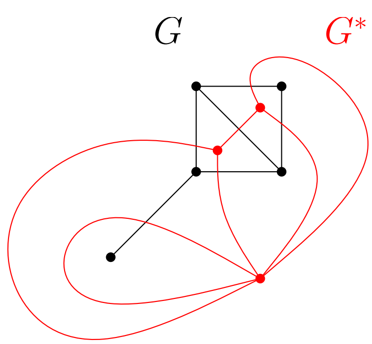


Heawood

Def: (p. 267)

Let G be a plane graph. The dual of G, denoted G^*, is a plane multigraph. From each region of G we pick one inner point to be a vertex of G^*. For every edge of G we add a curve connecting the vertices of G^* corresponding to the regions incident with e, such that this curve intersects e once and does not intersect anything else (including itself).

Example:



→ The name "dual" is justified.

→ We also talk about the dual of a planar graph, which is not unique.

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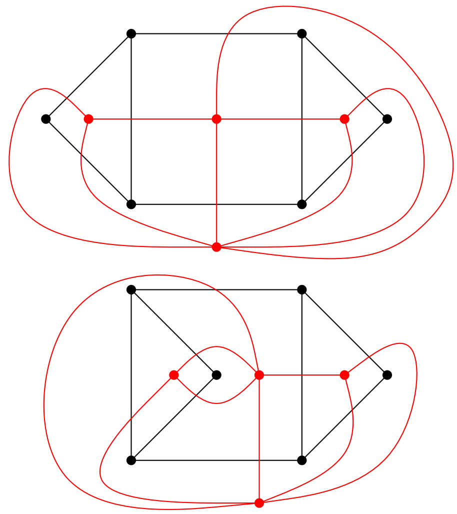
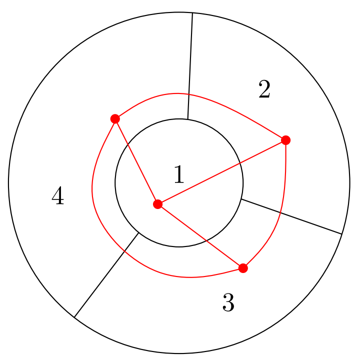


$\Rightarrow$ The dual of a planar graph depends on the embedding.

<u>Thm:</u> "Every map can be colored in 4 colors."



$\rightarrow$ Taking the dual of a map gives a planar graph.

$\rightarrow$ Every planar graph is 4-colorable

<u>About the proof:</u>

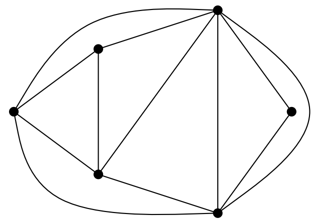
$\rightarrow$ reducible configurations.

$\rightarrow$ unavoidable set of reducible configurations.

<u>5-color thm</u>

$$\left.\begin{array}{c}\text{1 vertex of deg 0}\\ \text{1 vertex of deg 1}\\ \vdots\\ \text{1 vertex of deg 5}\end{array}\right\}\text{unavoidable}$$

$\rightarrow$ Triangulation:



$$2e = 3r$$

Ramsey Theory

Def: (p. 299)

The ramsey number $r(F_1, F_2)$ is the minimal number r such that in every red-blue coloring of the edges of K_r there is either a red copy of F_1 or a blur copy of F_2.

$\rightarrow$ $r(n, m)$ is $r(K_n, K_m)$

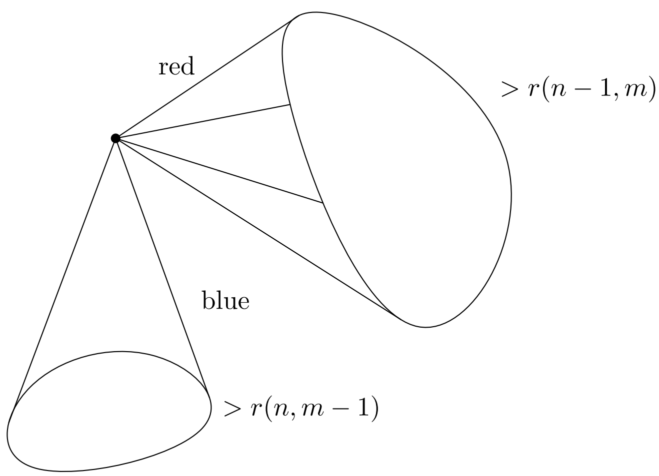
$\rightarrow$ $r(n) = r(n, n)$

Thm (Ramsey): $r(n, m)$ is finite.

Proof: by double induction.

$r(1, n) = r(m, 1) = 1$

$\rightarrow$ Assume that $r(n', m')$ is finite for all pairs $n', m' < n, m$.



$\rightarrow$ Let $r = r(n-1, m) + r(n, m-1)$

$\rightarrow$ Fix a vertex v.

$\rightarrow$ Let N_{red} be the set of vertices adjacent to v via a red edge.

$\rightarrow$ N_{blue}.

$\rightarrow$ If both $|N_{\text{red}} < r(n-1, m)$ and $|N_{\text{blue}}| < r(n, m-1)$ ↯

$$r = |N_{\text{red}}| + |N_{\text{blue}}| + 1 < r(n-1, m) + r(n, m-1) + 1$$

$\rightarrow$ One of them is large enough and we can finish