

Homework 7

36-705

Due: Thursday October 30 by 3:00

1. Chapter 10, problem 9.
2. Chapter 10, problem 15.
3. Chapter 10, problem 16.
4. In hypothesis testing, we often study the power of tests at “local alternatives.” For example, let $Y_1, \dots, Y_n \sim N(\theta, 1)$. Suppose want to test

$$H_0 : \theta = 0 \quad \text{versus} \quad \theta > 0.$$

Suppose we reject H_0 when $\bar{Y}_n > c_n$.

- (a) Find c_n so that the test has level α .
- (b) Find the power of the test at the local alternative $\theta_n = a/\sqrt{n}$ where $a > 0$ is a fixed constant.

The reasons we study the power at local alternatives are: (i) The power at a fixed alternative goes to 1 quickly as $n \rightarrow \infty$ which is not very informative. (ii) In practice, the difficult cases are when the alternative is close to the null. The local alternative captures this. (iii) If we want to compare two different tests, there will be very little difference at any fixed alternative. But the difference is magnified by looking at local alternatives. So we get a more informative comparison by looking at local alternatives.