Assignment 5
Due on Wednesday, April 23

1. Find the 1st Euler-Lagrange system and the natural boundary conditions associated with maximizing or minimizing

$$J(y) = \int_0^1 [y_1'(x)^4 + y_2'(x)^2 - y_1'(x)y_2(x) + y_1(x)^3 - 2xy_2(x)]dx$$

on $\mathscr{Y} = \{y \in C^1([0,1];\mathbb{R}^2) : y_1(0) = y_2(0),\ y_1(1) + 2y_2(1) = 3\}$

2. Find all possible maximizers and minimizers for

$$J(y) = \int_0^{\frac{\pi}{2}} [y''(x)^2 - y'(x)^2 - 2y(x)]\, dx$$

on $\mathscr{Y} = \{y \in C^2[0,\frac{\pi}{2}] : y(0) = 0,\ y(\frac{\pi}{2}) = 0,\ y'(\frac{\pi}{2}) = 0\}$

3. Find all possible maximizers and minimizers for

$$J(y) = \int_1^2 [x^3 y''(x)^2 - 24xy(x)]dx$$

on $\mathscr{Y} = \{y \in C^2[1,2] : y(1) = 6,\ y'(1) = -1,\ y(2) = 8\ ,\ y'(2) = 4\}$

4. Let $\widehat{\mathscr{Y}} = \{y \in \widehat{C}^1[0,3] : y(0) = 0,\ y(3) = 2\}$ and define $J : \widehat{\mathscr{Y}} \to \mathbb{R}$ by

$$J(y) = \int_0^3 [y'(x)^4 - 8y'(x)^2]dx.$$

(a) Let $y \in \widehat{\mathscr{Y}}$ and $c \in S(y)$ be given and assume that y minimizes J on $\widehat{\mathscr{Y}}$. Put $\alpha = y'(c^-)$ and $\beta = y'(c^+)$. Use the Weierstrass-Erdmann corner conditions to determine the possible values of α and β?

(b) Find all possible minimizers for J on $\widehat{\mathscr{Y}}$ having exactly one corner point.