21-484 Notes

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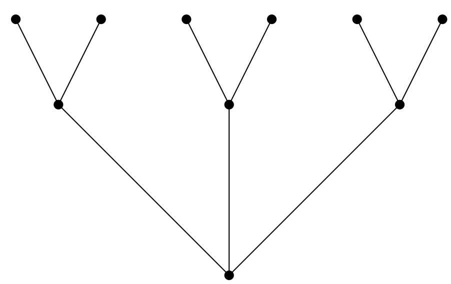
<u>Moore Bound:</u>

$$n(\delta, g) = \begin{cases} 1 + \delta \sum_{i=0}^{r-1} (\delta - 1)^i & g =: 2r+1 \text{ is odd} \\ 2 \sum_{i=0}^{r-1} (\delta - 1)^i & g =: 2r \text{ is even} \end{cases}$$

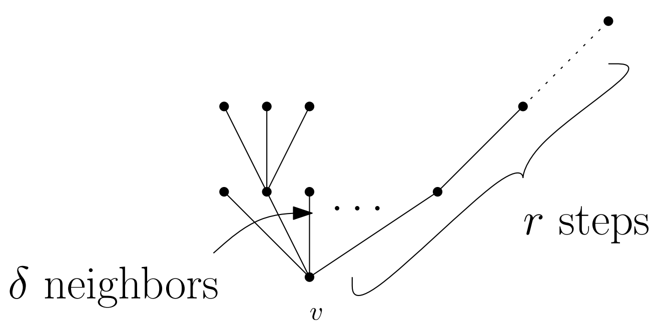
Every graph with minimal degree $\delta \geq 2$ and girth g has at least $n_0(\delta, g)$ vertices.

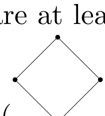
$\rightarrow \delta \geq 2 \Rightarrow g$ is finite

$\rightarrow$ <u>Main idea:</u> The ball of radius $\sim r$ around a vertex/edge is a tree.



$\rightarrow$ <u>Proof</u>: assume $g = 2r+1$ is odd. Pick a vertex v.



- $\rightarrow$ There are at least δ neighbors of v.
- $\rightarrow$ There are at least $\delta(\delta-1)$ neighbors of neighbors of v (assuming $g > 3$), otherwise we get a 4 cycle ().
- $\rightarrow$ There are $\delta(\delta-1)^i$ vertices in the i^{th} level (vertices of distance i from v) if $r \geq i$.

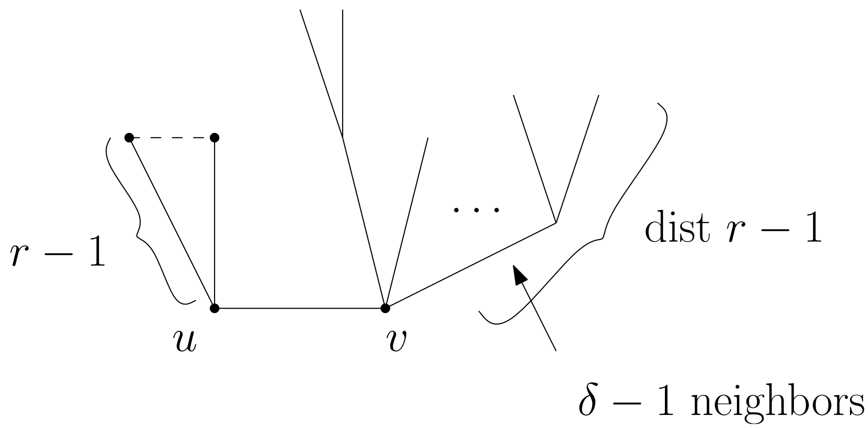
  Otherwise we get a cycle of length $2i \leq 2r < g$.

  <u>Summing up:</u> $\underset{v\uparrow}{1} + \underset{N(v)\uparrow}{\delta} + \underset{N(N(v))\uparrow}{\delta(\delta-1)} \dots + \underset{\underbrace{N(N(\cdots(N(v))\cdots))\uparrow}_{r \text{ times}}}{\delta(\delta-1)^{r-1}}$

$\longrightarrow$ If g is even, we do the same around an edge:

$\rightarrow$ Pick an edge $e = uv$. The tree of depth $r-1$ around each endpoint has $\sum_{i=0}^{r-1}(\delta-1)^i$ vertices, as before (otherwise we get a cycle of length $< r-1+r-1 = 2r-2 < g$)

$\rightarrow$ The two trees are disjoint since otherwise we get a cycle of length $r-1+1+r-1 = 2r-1 < g$.



<u>Q2:</u> diam $G = \max_{u,v} \text{dist}(u,v)$

$\text{ecc}_G(u) = \max_v \text{dist}(u,v)$

$\text{radius}(G) = \min_u \text{ecc}_G(u)$.

$$\text{rad} \le \text{diam} \le 2\text{rad}$$

$\text{rad} = \min_u \max_v \text{dist}(u,v) \le \max_u \max_v \text{dist}(u,v) = \text{diam}$

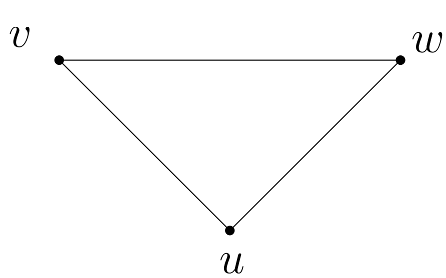
For the left inequality, let u be a vertex such that $\text{ecc}_G(u) = \text{rad}(G)$ (u is called a center of G). Let v and w be two vertices such that

$$\text{dist}(u,w) = \text{diam}(G)$$

Notice that by the triangle inequality we have

$$\text{diam} = \text{diam}(v,w) \le \text{dist}(v,u) + \text{dist}(u,w) \le 2\text{rad}(G)$$

To see that the triangle inequality holds, consider the greatest v–u path followed by the geometric u–w path.



$\text{dist}(v,w) \le \text{dist}(v,u) + \text{dist}(u,w)$

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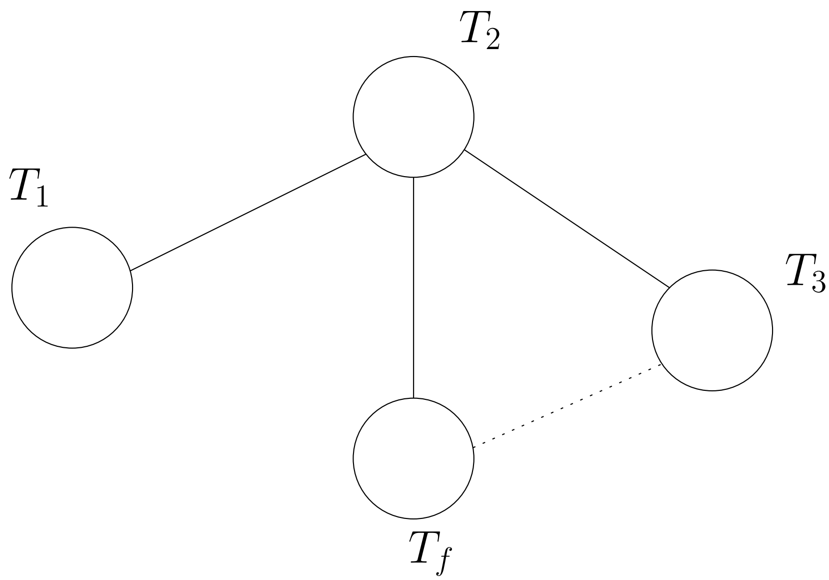
Take care of disconnected graphs.

$F = T_1 \cup \cdots \cup T_f$, $V(F) = [n]$.

$\rightarrow$ Main idea: Think of T_i as vertices.



$f^{f-2} t_1 t_2$