

Homework 4

36-705

Due: Thursday October 2 by 3:00

1. Let  $X_1, \dots, X_n \sim \text{Uniform}(\theta, \theta + 1)$ .
  - (a) Find a minimal sufficient statistic.
  - (b) Show that  $X_3$  is not a sufficient statistic.
2. Chapter 6, problem 1.
3. Chapter 9, Problem 2(a), 2(b), 2(c).
4. Let  $X_1, \dots, X_n \sim N(\mu, \sigma^2)$  where  $\sigma^2$  is known. Let  $\mu \sim N(a, b^2)$ .
  - (a) Find the Bayes estimator for  $\mu$  under squared error loss.
  - (b) Find the risk of the estimator.
  - (c) Find the maximum risk.
  - (d) Find the Bayes risk.
5. Let  $X_1, \dots, X_n \sim \text{Bernoulli}(p)$ . Let  $L(p, \hat{p}) = (p - \hat{p})^2$ .
  - (a) Let  $\hat{p}$  be the Bayes estimator using a  $\text{Beta}(\alpha, \beta)$  prior. Find the Bayes estimator.
  - (b) Compute the risk function.
  - (c) Compute the Bayes risk.
  - (d) Find  $\alpha$  and  $\beta$  to make the risk constant and hence find the minimax estimator.