21-484 Notes

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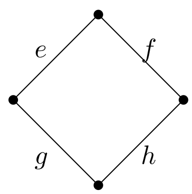
wmacrae@andrew.cmu.edu Def: (p. 184): A set of edges in a grpah G is independent or is a matching if every two edges are disjoint.

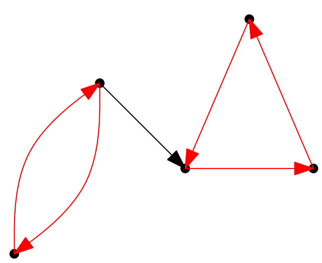
Example:

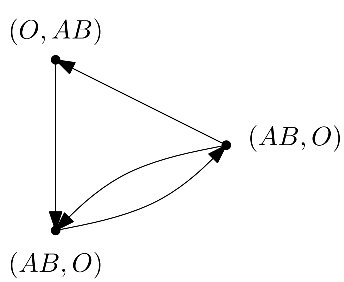


$$\varnothing, \{e\}, \{f\}, \{g\}, \{h\}, \quad \underbrace{\{e,g\}, \{f,g\}}_{\text{Perfect matchings (p.194)}}$$

Application:

(Patient, Doner)





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<u>Def:</u> (p. 185): Let G be a bipartite graph, $G = (V = U \cup W, E)$. For a set $X \subseteq U$ we define the neighborhood of X, $N(X)$, to be

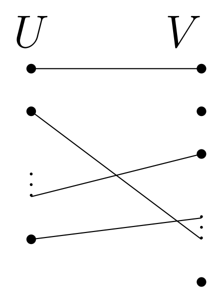
$$N(X) = \{w \in W | \exists u \in X. uw \in E\}$$

Theorem (Hall, Theorem 8.3): $G = (V = U \cup W, E)$ be a bipartite graph. Then there is a matching of size $|U|$ if and only if for every $X \subset U$,

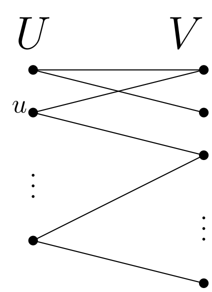
$$|N(X)| \geq |X| \; (*).$$

<u>Proof:</u> If G has a matching M of size $|U|$, then every vertex of U lies in a unique edge. For every $X \subseteq U$

$$|X| = |\{w \in W | \exists u \in X. uw \in M\}| \leq |N(X)|$$



Assume the condition $(*)$, assume for the sake of contradiction that there is no matching of size $|U|$. Pick a maximum matching and let $u \in U$ be an unmatched vertex.



An alternating path is a path in which the edges alternate between matching edges and nonmatching edges. Let S be the set of all vertices s such that there is a u–s alternating path of maximal length.

$\rightarrow$ $S \cap W = \varnothing$. Otherwise, there is a maximal alternating path of odd length. Such a path starts and ends with a nonmatching edge. Define $M' = M \setminus$ the matching edges in the path $\cup$ the nonmatching edges in the path $|M'| > |M|$. ↯