21-484 Notes

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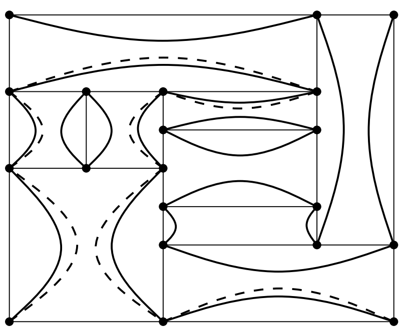
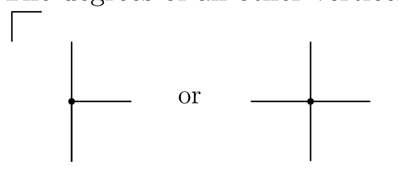


→ The degree of a vertex is the number of small rectangles containing it
→ The degrees of the corners of the big rectangle are 1
→ The degrees of all other vertices are either 2 or 4



→ So (1), start a trail from the lower left corner (of the big rectangle) and continue as long as you can
→ notice, you will not stop on a non-corner vertex.
⇒ there is a trail between two corners.
→ the trail is made entirely of integer edges.
→ there is an integer length side in the big rectangle.

→ So (2), the connected component containing the lower left corner should contain another corner (the sum of the degrees in the connected component is even)

→ ∃ path from the lower left corner to another corner.

Remark: Also works if instead of integer we have algebraic.

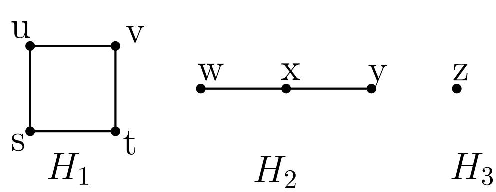
Def:(p.13-14)

- If a graph contains a path from u to v, then u and v are connected in the graph.
- If every two verticies in a graph G are connected, then G is connected.
- If G is not connected, it is disconnected.

Remark: The path $p = v_0$ shows that v_0 is connected to itself (by a path of length 0). So, the trivial graph—the simple graph with one vertex—is connected.

Def: (p.14) A connected subgraph of a graph G that is not a proper subgraph of any other connected subgraph, is called a “component” of G or a connected component.

Example: $H = H_1 \cup H_2 \cup H_3$:



Fact: (Theorem 1.7): The connectivity relation is an equivalence relation.

That is: if uRv iff there is a path from u to v, then R is an equivalence relation.

Proof:

1. R is reflexive ✓
2. R is symmetric since if $v_0, \ldots, v_\ell$ is a u-v path, then $v_\ell, \ldots, v_0$ is a v-u path.
3. R is transitive; if $v_0, \ldots, v_\ell$ is a u-v path and $v_0', \ldots, v_k'$ is a v-w path, then $v_0, \ldots, v_\ell, v_01, \ldots, v_k'$ is a u-w walk. A u-w walk contains a u-w path.

claim (Thrm 2.4): If for any two vertices x, y in a greaph G with n verticies we have

$$\deg\, x + \deg\, y \geq n - 1$$

then G is connected.

Proof: If $x = y$ then x is an x-y path (len. 0)

If $xy \in E(G)$ then x, y is an x-y path (len. 1)

If $xy \notin E(G)$, then $y \notin N(x)$ and $y \notin N(y)$ and $x \notin N(y)$ and $x \notin N(x)$.

$\rightarrow N(X) \cup N(y) \subseteq |V(G) \setminus \{x, y\}\,| = n - 2$

$|N(x)| + |N(y)| \geq n - 1$

$\Rightarrow N(x) \cap N(y) \neq \varnothing$

$\Rightarrow \exists w$ such that $wx, wy \in E(G)$; x, w, y is x-y path in G.