21-484 Notes

JD Nir
jnir@andrew.cmu.edu
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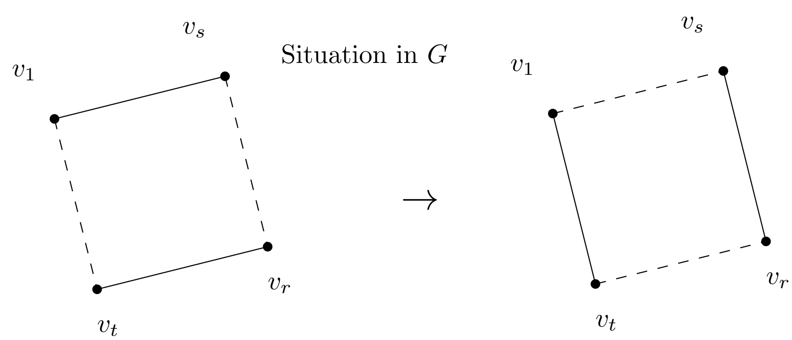
<u>Recall:</u> A graphical sequence.

- Want to prove: $S = d_1, d_2, \dots, d_n$, $d_1 \geq 1, n \geq 2$, s monotonically non increasing is graphical iff $s_1 = d_2 - 1, d_3 - 1, \dots, d_{d_1+1} - 1, d_{d_1+2}, \dots, d_n$ is graphical.
- need to show that if s is graphical then there is a graph G with vertex set $\{v_1, \dots, v_n\}$ such that
    - the degree sequence of G is S
    - $\deg_G(v_1) = d_1$
    - the degrees of the neighbors of v_1 are $d_2, \dots, d_{d_1+1}$
- Assume that there is no such graph. Let G be the graph with $V(G) = \{v_1, \dots, v_n\}$ such that
    1. the degree sequence of G is s
    2. $\deg_G(v_1) = d_1$
    3. $\sum\limits_{v \in N_G(v_1)} \deg_G(v)$ is maximal (over all verticies of degree d_1 in G and over all graphs satisfying 1 and 2)

$\rightarrow$ There is a neighbor of v_1, v_2, and a nonneighbor of v_1, v_t, such that $\deg(v_t) > \deg(v_s)$

$\rightarrow$ $\exists v_r$. $v_r v_t \in E(G)$ and $v_r v_s \notin E(G)$.



Situation in G

$\rightarrow$ Define G' by removing $v_1 v_s$ and $v_t v_r$ and adding $v_s v_r, v_1 v_t$

- <u>Notice:</u>
    1. $V(G') = \{v_1, \dots, v_n\}$
    2. $d'_G(v_1) = d_1$
    3. The degree sequence of G' is s
    4. $\sum\limits_{v \in N_{G'}(v_1)} \deg_{G'}(v) > \sum\limits_{v \in N_G(v_1)} \deg_G(v)$ $\lightning$

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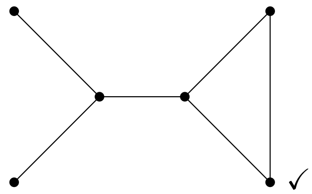
Example (like 2.12 but shorter)

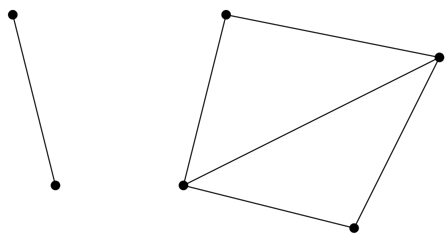
Is 3,3,2,2,1,1 graphical?



2,1,1,1,1

0,0,1,1 $\rightarrow$ 1,1,0,0

0,0,0



Graphical sequences:

Do not define a graph

Do not define connectivity