Homework 1
15-423 Digital Signal Processing for CS
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1 Composing signals

1. The following signal is a pulse train of period $N_1 + N_2$ and duty cycle $\dfrac{N_1}{N_1 + N_2}$:

$$s_1[n] := \sum_{k \in \mathbb{Z}} p[n + k(N_1 + N_2)], \text{ where } p[n] := u[n] - u[n - N_1] \text{ is a pulse of length } N_1.$$

2. The following signal is an exponential signal beginning at $n = n_0$ and tapering as α^{n-n_0}:

$$s_2[n] := u[n - n_0]e_\alpha[n - n_0].$$

3. The following signal is a sinusoid of zero phase and amplitude decaying as α^n:

$$s_3[n] := \sin[n]e_\alpha[-n].$$

4. The following signal is a complex exponential of frequency ω and zero phase, ending at $n = n_0$:

$$s_4[n] := e_{\omega,0}[n]u[-n + n_0].$$

5. The following signal is an impulse train of phase -1 and period 4:

$$s_5[n] := \sum_{k \in \mathbb{Z}} \delta[n - (4k + 1)].$$

s_5 can also be written in terms of s_1 (see problem 1), with $N_1 = 1$, $N_2 = 3$:

$$s_5[n] = s_1[n - 1].$$

2 System properties

1. The system is causal ($y[n]$ depends only on $x[n-4]$), is not memoryless ($y[n]$ must remember $x[n-4]$), is linear (y is a linear combination of terms of x), and is shift-invariant (n appears only as an argument to x).

   The system is invertible if and only if α is non-zero (the mapping $x[n] \mapsto \frac{y[n+4]}{\alpha}$ is an inverse).

2. The system is not causal ($y[n]$ anticipates $x[n+1]$), is memoryless ($y[n]$ depends only on $x[n]$ and $x[n+1]$), is linear ($y[n]$ is a linear combination of terms of x), and is shift-invariant (n appears only as an argument to x).

3. It can be shown by induction on n that, $\forall n \in \mathbb{N},\ y[n] = \sum_{i=0}^{\infty} \beta^i(\alpha_0 x[n-i] + \alpha_1 x[n-i-1])$. Thus, the system is causal ($y[n]$ depends only on $x[n], x[n-1]$, and $y[n-1]$), is not memoryless ($y[n]$ depends on $x[n-1]$), is linear ($y[n]$ is a linear combination of terms of x), and is shift-invariant (n appears only as an argument to x).

4. The system is causal ($y[n]$ depends only on previous terms), is not memoryless (if $K > 0$, $y[n]$ depends on $x[n-1]$), is linear ($y[n]$ is a linear combination of terms of x), and is not shift-invariant (e.g., if x is the DC signal $s[n]$, and $K = 0$, then $y[n] = n$, which is clearly not shift-invariant).

5. The system is causal ($y[n]$ depends only on previous terms), is not memoryless (if $K > 0$, $y[n]$ depends on $x[n-1]$), is linear ($y[n]$ is a linear combination of terms of x), and is shift-invariant (n appears only as an argument to x).

6. The system is causal and memoryless ($y[n]$ depends only on $x[n]$), is non-linear (if x_1 and x_2 are the DC signal $s[n]$, then the sum of their quartics, the constant signal of amplitude 2, is not the quartic of their sum, constant signal of amplitude 16), and is shift-invariant (n appears only as an argument to x).