Supplementary File of "*ExTrEMO*: Transfer Evolutionary Multiobjective Optimization with Proof of Faster Convergence"

S-I. Benchmark Problems

Based on the widely known DTLZ and DTLZ^{-1} benchmark suite, eight base problems, i.e., DTLZ1b-$(\delta_1, \delta_2, \delta_3)$, DTLZ2b-$(\delta_1, \delta_2, \delta_3)$, DTLZ3b-$(\delta_1, \delta_2, \delta_3)$, DTLZ4b-$(\delta_1, \delta_2, \delta_3)$, DTLZ1b^{-1}-$(\delta_1, \delta_2, \delta_3)$, DTLZ2b^{-1}-$(\delta_1, \delta_2, \delta_3)$, DTLZ3b^{-1}-$(\delta_1, \delta_2, \delta_3)$, and DTLZ4b^{-1}-$(\delta_1, \delta_2, \delta_3)$ are constructed. These eight base problems are shown as follows:

- *DTLZ1b-$(\delta_1, \delta_2, \delta_3)$*:

$$\begin{aligned}
\min:\ & f_1(\mathbf{x}) = \frac{1}{2}\delta_1\delta_3 x_1 x_2(1+g(\mathbf{x})) + (1-\delta_3)(1+g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2) \\
& f_2(\mathbf{x}) = \frac{1}{2}\delta_1\delta_3 x_1(1-x_2)(1+g(\mathbf{x})) + (1-\delta_3)(1+g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\sin(\frac{\pi}{2}x_2) \\
& f_3(\mathbf{x}) = \frac{1}{2}\delta_1\delta_3(1-x_1)(1+g(\mathbf{x})) + (1-\delta_3)(1+g(\mathbf{x}))\sin(\frac{\pi}{2}x_1) \\
\text{s.t. }\ & 0 \le x_j \le 1, j = 1, \ldots, 10
\end{aligned} \tag{S-1}$$

where

$$\begin{aligned}
g(\mathbf{x}) = {} & \delta_3 \cdot 100 \cdot \left[8 + \sum_{j=3}^{10}((x_j - 0.5 - \delta_2)^2 - \cos(2\pi(x_j - 0.5 - \delta_2))\right] \\
& + (1-\delta_3)\cdot\left\{\left[-20\exp\left(-\frac{1}{5}\sqrt{\frac{1}{2}\sum_{j=3}^{10}(50\cdot(x_j - 0.5 - \delta_2))^2}\right)\right] - \exp\left[\frac{1}{8}\sum_{j=3}^{10}\cos(2\pi\cdot 50\cdot(x_j - 0.5 - \delta_2))\right] + 20 + e\right\}
\end{aligned} \tag{S-2}$$

- *DTLZ2b-$(\delta_1, \delta_2, \delta_3)$*:

$$\begin{aligned}
\min:\ & f_1(\mathbf{x}) = \delta_1\delta_3(1+g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2) + \frac{1}{2}(1-\delta_3)x_1x_2(1+g(\mathbf{x})) \\
& f_2(\mathbf{x}) = \delta_1\delta_3(1+g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\sin(\frac{\pi}{2}x_2) + \frac{1}{2}(1-\delta_3)x_1(1-x_2)(1+g(\mathbf{x})) \\
& f_3(\mathbf{x}) = \delta_1\delta_3(1+g(\mathbf{x}))\sin(\frac{\pi}{2}x_1) + \frac{1}{2}(1-\delta_3)(1-x_1)(1+g(\mathbf{x})) \\
\text{s.t. }\ & 0 \le x_j \le 1, j = 1, \ldots, 10
\end{aligned} \tag{S-3}$$

where

$$\begin{aligned}
g(\mathbf{x}) = {} & \delta_3 \cdot 100 \cdot \sum_{j=3}^{10}(x_j - 0.5 - \delta_2)^2 \\
& + (1-\delta_3)\cdot\left\{\left[-20\exp\left(-\frac{1}{5}\sqrt{\frac{1}{2}\sum_{j=3}^{10}(50\cdot(x_j - 0.5 - \delta_2))^2}\right)\right] - \exp\left[\frac{1}{8}\sum_{j=3}^{10}\cos(2\pi\cdot 50\cdot(x_j - 0.5 - \delta_2))\right] + 20 + e\right\}
\end{aligned} \tag{S-4}$$

- *DTLZ3b-$(\delta_1, \delta_2, \delta_3)$*:

$$\begin{aligned}
\min:\ & f_1(\mathbf{x}) = \delta_1\delta_3(1+g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2) + \frac{1}{2}(1-\delta_3)x_1x_2(1+g(\mathbf{x})) \\
& f_2(\mathbf{x}) = \delta_1\delta_3(1+g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\sin(\frac{\pi}{2}x_2) + \frac{1}{2}(1-\delta_3)x_1(1-x_2)(1+g(\mathbf{x})) \\
& f_3(\mathbf{x}) = \delta_1\delta_3(1+g(\mathbf{x}))\sin(\frac{\pi}{2}x_1) + \frac{1}{2}(1-\delta_3)(1-x_1)(1+g(\mathbf{x})) \\
\text{s.t. }\ & 0 \le x_j \le 1, j = 1, \ldots, 10
\end{aligned} \tag{S-5}$$

where

$$g(\mathbf{x}) = \delta_3 \cdot 100 \cdot \left[8 + \sum_{j=3}^{10}((x_j - 0.5 - \delta_2)^2 - \cos(2\pi(x_j - 0.5 - \delta_2))\right]$$
$$+ (1-\delta_3) \cdot \left\{ \left[-20\exp\left(-\frac{1}{5}\sqrt{\frac{1}{2}\sum_{j=3}^{10}(50\cdot(x_j - 0.5 - \delta_2))^2} \right) \right] - \exp\left[\frac{1}{8}\sum_{j=3}^{10}\cos(2\pi\cdot 50\cdot(x_j - 0.5 - \delta_2)) \right] + 20 + e \right\} \tag{S-6}$$

- *DTLZ4b-($\delta_1, \delta_2, \delta_3$)*:

$$\begin{aligned}
\min:\ & f_1(\mathbf{x}) = \delta_1\delta_3(1+g(\mathbf{x}))\cos(\frac{\pi}{2}x_1^5)\cos(\frac{\pi}{2}x_2^5) + \frac{1}{2}(1-\delta_3)x_1x_2(1+g(\mathbf{x}))\\
& f_2(\mathbf{x}) = \delta_1\delta_3(1+g(\mathbf{x}))\cos(\frac{\pi}{2}x_1^5)\sin(\frac{\pi}{2}x_2^5) + \frac{1}{2}(1-\delta_3)x_1(1-x_2)(1+g(\mathbf{x}))\\
& f_3(\mathbf{x}) = \delta_1\delta_3(1+g(\mathbf{x}))\sin(\frac{\pi}{2}x_1^5) + \frac{1}{2}(1-\delta_3)(1-x_1)(1+g(\mathbf{x}))\\
& \text{s.t. } 0 \le x_j \le 1, j = 1,\ldots,10
\end{aligned} \tag{S-7}$$

where

$$g(\mathbf{x}) = \delta_3 \cdot 100 \cdot \sum_{j=3}^{10}(x_j - 0.5 - \delta_2)^2$$
$$+ (1-\delta_3) \cdot \left\{ \left[-20\exp\left(-\frac{1}{5}\sqrt{\frac{1}{2}\sum_{j=3}^{10}(50\cdot(x_j - 0.5 - \delta_2))^2} \right) \right] - \exp\left[\frac{1}{8}\sum_{j=3}^{10}\cos(2\pi\cdot 50\cdot(x_j - 0.5 - \delta_2)) \right] + 20 + e \right\} \tag{S-8}$$

- *DTLZ1b^{-1}-($\delta_1, \delta_2, \delta_3$)*:

$$\begin{aligned}
\min:\ & f_1(\mathbf{x}) = -\frac{1}{2}\delta_1\delta_3x_1x_2(1-g(\mathbf{x})) - (1-\delta_3)(1-g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2)\\
& f_2(\mathbf{x}) = -\frac{1}{2}\delta_1\delta_3x_1(1-x_2)(1-g(\mathbf{x})) - (1-\delta_3)(1-g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\sin(\frac{\pi}{2}x_2)\\
& f_3(\mathbf{x}) = -\frac{1}{2}\delta_1\delta_3(1-x_1)(1-g(\mathbf{x})) - (1-\delta_3)(1-g(\mathbf{x}))\sin(\frac{\pi}{2}x_1)\\
& \text{s.t. } 0 \le x_j \le 1, j = 1,\ldots,10
\end{aligned} \tag{S-9}$$

where

$$g(\mathbf{x}) = \delta_3 \cdot 100 \cdot \left[8 + \sum_{j=3}^{10}((x_j - 0.5 - \delta_2)^2 - \cos(2\pi(x_j - 0.5 - \delta_2))\right]$$
$$+ (1-\delta_3) \cdot \left\{ \left[-20\exp\left(-\frac{1}{5}\sqrt{\frac{1}{2}\sum_{j=3}^{10}(50\cdot(x_j - 0.5 - \delta_2))^2} \right) \right] - \exp\left[\frac{1}{8}\sum_{j=3}^{10}\cos(2\pi\cdot 50\cdot(x_j - 0.5 - \delta_2)) \right] + 20 + e \right\} \tag{S-10}$$

- *DTLZ2b^{-1}-($\delta_1, \delta_2, \delta_3$)*:

$$\begin{aligned}
\min:\ & f_1(\mathbf{x}) = -\delta_1\delta_3(1-g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2) - \frac{1}{2}(1-\delta_3)x_1x_2(1-g(\mathbf{x}))\\
& f_2(\mathbf{x}) = -\delta_1\delta_3(1-g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\sin(\frac{\pi}{2}x_2) - \frac{1}{2}(1-\delta_3)x_1(1-x_2)(1-g(\mathbf{x}))\\
& f_3(\mathbf{x}) = -\delta_1\delta_3(1-g(\mathbf{x}))\sin(\frac{\pi}{2}x_1) - \frac{1}{2}(1-\delta_3)(1-x_1)(1-g(\mathbf{x}))\\
& \text{s.t. } 0 \le x_j \le 1, j = 1,\ldots,10
\end{aligned} \tag{S-11}$$

where

$$
\begin{aligned}
g(\mathbf{x}) = {} & \delta_3 \cdot 100 \cdot \sum_{j=3}^{10} (x_j - 0.5 - \delta_2)^2 \\
& + (1-\delta_3) \cdot \left\{ \left[-20 \exp\left(-\frac{1}{5} \sqrt{\frac{1}{2} \sum_{j=3}^{10} (50 \cdot (x_j - 0.5 - \delta_2))^2} \right) \right] - \exp\left[\frac{1}{8} \sum_{j=3}^{10} \cos(2\pi \cdot 50 \cdot (x_j - 0.5 - \delta_2)) \right] + 20 + e \right\}
\end{aligned} \tag{S-12}
$$

- *DTLZ3b*$^{-1}$*-(*$\delta_1, \delta_2, \delta_3$*)*:

$$
\begin{aligned}
\min: \; & f_1(\mathbf{x}) = -\delta_1 \delta_3 (1-g(\mathbf{x})) \cos(\frac{\pi}{2} x_1) \cos(\frac{\pi}{2} x_2) - \frac{1}{2}(1-\delta_3) x_1 x_2 (1-g(\mathbf{x})) \\
& f_2(\mathbf{x}) = -\delta_1 \delta_3 (1-g(\mathbf{x})) \cos(\frac{\pi}{2} x_1) \sin(\frac{\pi}{2} x_2) - \frac{1}{2}(1-\delta_3) x_1 (1-x_2)(1-g(\mathbf{x})) \\
& f_3(\mathbf{x}) = -\delta_1 \delta_3 (1-g(\mathbf{x})) \sin(\frac{\pi}{2} x_1) - \frac{1}{2}(1-\delta_3)(1-x_1)(1-g(\mathbf{x})) \\
\text{s.t. } & 0 \le x_j \le 1, j = 1, \ldots, 10
\end{aligned} \tag{S-13}
$$

where

$$
\begin{aligned}
g(\mathbf{x}) = {} & \delta_3 \cdot 100 \cdot \left[8 + \sum_{j=3}^{10} ((x_j - 0.5 - \delta_2)^2 - \cos(2\pi(x_j - 0.5 - \delta_2)) \right] \\
& + (1-\delta_3) \cdot \left\{ \left[-20 \exp\left(-\frac{1}{5} \sqrt{\frac{1}{2} \sum_{j=3}^{10} (50 \cdot (x_j - 0.5 - \delta_2))^2} \right) \right] - \exp\left[\frac{1}{8} \sum_{j=3}^{10} \cos(2\pi \cdot 50 \cdot (x_j - 0.5 - \delta_2)) \right] + 20 + e \right\}
\end{aligned} \tag{S-14}
$$

- *DTLZ4b*$^{-1}$*-(*$\delta_1, \delta_2, \delta_3$*)*:

$$
\begin{aligned}
\min: \; & f_1(\mathbf{x}) = -\delta_1 \delta_3 (1-g(\mathbf{x})) \cos(\frac{\pi}{2} x_1^5) \cos(\frac{\pi}{2} x_2^5) - \frac{1}{2}(1-\delta_3) x_1 x_2 (1-g(\mathbf{x})) \\
& f_2(\mathbf{x}) = -\delta_1 \delta_3 (1-g(\mathbf{x})) \cos(\frac{\pi}{2} x_1^5) \sin(\frac{\pi}{2} x_2^5) - \frac{1}{2}(1-\delta_3) x_1 (1-x_2)(1-g(\mathbf{x})) \\
& f_3(\mathbf{x}) = -\delta_1 \delta_3 (1-g(\mathbf{x})) \sin(\frac{\pi}{2} x_1^5) - \frac{1}{2}(1-\delta_3)(1-x_1)(1-g(\mathbf{x})) \\
\text{s.t. } & 0 \le x_j \le 1, j = 1, \ldots, 10
\end{aligned} \tag{S-15}
$$

where

$$
\begin{aligned}
g(\mathbf{x}) = {} & \delta_3 \cdot 100 \cdot \sum_{j=3}^{10} (x_j - 0.5 - \delta_2)^2 \\
& + (1-\delta_3) \cdot \left\{ \left[-20 \exp\left(-\frac{1}{5} \sqrt{\frac{1}{2} \sum_{j=3}^{10} (50 \cdot (x_j - 0.5 - \delta_2))^2} \right) \right] - \exp\left[\frac{1}{8} \sum_{j=3}^{10} \cos(2\pi \cdot 50 \cdot (x_j - 0.5 - \delta_2)) \right] + 20 + e \right\}
\end{aligned} \tag{S-16}
$$

Based on the eight base problems, five kinds of source tasks, i.e., high-correlation source tasks, medium-correlation source tasks, low-correlation source tasks, medium-negative-correlation source tasks, and high-negative-correlation source tasks, are generated based on the settings defined in Table S-I. Moreover, the averaged Pearson correlation between the target tasks and the various categories of source tasks, determined by Monte Carlo sampling, is given in Table S-II.

S-II. Additional Discussions

A. Performance of ExTrEMO with Medium- & Low-Correlation Source Data

In Section VI-D, we employed a set of five diverse source datasets, encompassing a HS dataset, a MS dataset, a LS dataset, a MNS dataset, and a HNS dataset, to aid in optimizing ExTrEMO. However, there is a possibility that ExTrEMO's outstanding performance is primarily attributed to the HS/HNS datasets, owing to their exceptionally high positive/negative correlations. It might be of interest to evaluate how ExTrEMO performs when using only MS, LS, and MNS datasets. In this subsection, we use five source datasets (a MS dataset, a MNS dataset, and three LS datasets), each containing 100 samples, to assist ExTrEMO. The results are recorded as ExTrEMO-M/L and summarized in Table S-IV. For the sake of comparison, we also present the results of SASSEA, MOEA/D-EGO, CSEA, K-RVEA, and KTA2 in Table S-IV. The outcomes indicate that even when exclusively utilizing MS, LS, and MNS datasets, ExTrEMO exhibits significantly faster convergence than its competitors. This suggests that ExTrEMO is capable of effectively harnessing the information within the source data, even when correlations are of medium or low magnitude, to expedite convergence.

B. Longer Term Performance of ExTrEMO

In our previous experiments, our focus was primarily on assessing the performance of ExTrEMO under stringent evaluation budgets, typically ranging from 50 to 100 evaluations. However, it is of interest to gauge whether ExTrEMO can maintain its competitive edge over more extended evaluation budgets. To address this, we conducted an investigation of ExTrEMO's performance under an extended evaluation budget of 300. For this evaluation, we employed five source datasets (a HS dataset, a MS dataset, a LS dataset, a MNS dataset, and a HNS dataset), each containing 100 samples, to facilitate the optimization process. As a point of comparison, we also executed ExTrEMO-ZeroT with a 300 evaluation budget. The results of these two algorithms when applied to solve DTLZ1b-$(1, 0, 1)$ and DTLZ2b-$(1, 0, 1)$ are presented in Table S-V. The findings indicate that, in comparison to ExTrEMO-ZeroT, ExTrEMO exhibits the ability to sustain its superior performance over the course of 300 evaluations. This observation underscores ExTrEMO's remarkable long-term convergence capabilities.

C. Investigation of CPU Time

In Section III-C, we conducted an analysis of the computational complexity of ExTrEMO, demonstrating that its computational complexity increases linearly with the number of sources. To validate this conclusion, we ran ExTrEMO with varying numbers of sources, including zero, one, three, and five, to solve DTLZ1b-$(1, 0, 1)$ and DTLZ2b-$(1, 0, 1)$ problems 20 times each. We recorded the average CPU time, as shown in Fig. S-1, where it is denoted as ExTrEMO-ZeroT, ExTrEMO-1S, ExTrEMO-3S, and ExTrEMO-5S. The figure illustrates that for both DTLZ1b-$(1, 0, 1)$ and DTLZ2b-$(1, 0, 1)$, the average CPU time increases almost linearly with the number of sources. This empirical observation aligns with the theoretical analysis of the computational complexity as discussed in Section III-C.

D. Pareto Front Approximations

Figs. S-2 and S-3 show the Pareto front approximation provided by MOEA/D-EGO, CSEA, KTA2 and ExTrEMO on DTLZ1b-$(1, 0, 1)$ and DTLZ2b-$(1, 0, 1)$ with two objectives after 100 evaluations. It is observed that, compared with MOEA/D-EGO, CSEA, and KTA2, ExTrEMO obtains solutions that offer better objective function values, indicating improved convergence.

S-III. Results

TABLE S-I
DETAILS OF THE GENERATION OF SOURCE TASKS. IN THE TABLE, *rand* IS A RANDOMLY GENERATED NUMBER BETWEEN 0 AND 1.

Base Problems	Sources Tasks	Parameter Settings		
		δ_1	δ_2	δ_3
DTLZ1b-$(\delta_1,\delta_2,\delta_3)$	High-Correlation Source Tasks	1	$0.1\times rand-0.05$	1
	Medium-Correlation Source Tasks	1	$0.1\times rand+0.2$	1
	Low-Correlation Source Tasks	1	$0.1\times rand+0.4$	0
	Medium-Negative-Correlation Source Tasks	-1	$0.1\times rand+0.2$	1
	High-Negative-Correlation Source Tasks	-1	$0.1\times rand-0.05$	1
DTLZ2b-$(\delta_1,\delta_2,\delta_3)$	High-Correlation Source Tasks	1	$0.1\times rand-0.05$	1
	Medium-Correlation Source Tasks	1	$0.1\times rand+0.3$	1
	Low-Correlation Source Tasks	1	$0.1\times rand+0.4$	0
	Medium-Negative-Correlation Source Tasks	-1	$0.1\times rand+0.3$	1
	High-Negative-Correlation Source Tasks	-1	$0.1\times rand-0.05$	1
DTLZ3b-$(\delta_1,\delta_2,\delta_3)$	High-Correlation Source Tasks	1	$0.1\times rand-0.05$	1
	Medium-Correlation Source Tasks	1	$0.1\times rand+0.2$	1
	Low-Correlation Source Tasks	1	$0.1\times rand+0.4$	0
	Medium-Negative-Correlation Source Tasks	-1	$0.1\times rand+0.2$	1
	High-Negative-Correlation Source Tasks	-1	$0.1\times rand-0.05$	1
DTLZ4b-$(\delta_1,\delta_2,\delta_3)$	High-Correlation Source Tasks	1	$0.1\times rand-0.05$	1
	Medium-Correlation Source Tasks	1	$0.1\times rand+0.3$	1
	Low-Correlation Source Tasks	1	$0.1\times rand+0.4$	0
	Medium-Negative-Correlation Source Tasks	-1	$0.1\times rand+0.3$	1
	High-Negative-Correlation Source Tasks	-1	$0.1\times rand-0.05$	1
DTLZ1b^{-1}-$(\delta_1,\delta_2,\delta_3)$	High-Correlation Source Tasks	1	$0.1\times rand-0.05$	1
	Medium-Correlation Source Tasks	1	$0.1\times rand+0.2$	1
	Low-Correlation Source Tasks	1	$0.1\times rand+0.4$	0
	Medium-Negative-Correlation Source Tasks	-1	$0.1\times rand+0.2$	1
	High-Negative-Correlation Source Tasks	-1	$0.1\times rand-0.05$	1
DTLZ2b^{-1}-$(\delta_1,\delta_2,\delta_3)$	High-Correlation Source Tasks	1	$0.1\times rand-0.05$	1
	Medium-Correlation Source Tasks	1	$0.1\times rand+0.3$	1
	Low-Correlation Source Tasks	1	$0.1\times rand+0.4$	0
	Medium-Negative-Correlation Source Tasks	-1	$0.1\times rand+0.3$	1
	High-Negative-Correlation Source Tasks	-1	$0.1\times rand-0.05$	1
DTLZ3b^{-1}-$(\delta_1,\delta_2,\delta_3)$	High-Correlation Source Tasks	1	$0.1\times rand-0.05$	1
	Medium-Correlation Source Tasks	1	$0.1\times rand+0.2$	1
	Low-Correlation Source Tasks	1	$0.1\times rand+0.4$	0
	Medium-Negative-Correlation Source Tasks	-1	$0.1\times rand+0.2$	1
	High-Negative-Correlation Source Tasks	-1	$0.1\times rand-0.05$	1
DTLZ4b^{-1}-$(\delta_1,\delta_2,\delta_3)$	High-Correlation Source Tasks	1	$0.1\times rand-0.05$	1
	Medium-Correlation Source Tasks	1	$0.1\times rand+0.3$	1
	Low-Correlation Source Tasks	1	$0.1\times rand+0.4$	0
	Medium-Negative-Correlation Source Tasks	-1	$0.1\times rand+0.3$	1
	High-Negative-Correlation Source Tasks	-1	$0.1\times rand-0.05$	1

TABLE S-II
THE AVERAGED PEARSON CORRELATION COEFFICIENTS BETWEEN THE TARGET TASKS AND THE VARIOUS CATEGORIES OF SOURCE TASKS, NAMELY, HIGH-CORRELATION, MEDIUM-CORRELATION, LOW-CORRELATION, MEDIUM-NEGATIVE-CORRELATION, HIGH-NEGATIVE-CORRELATION.

	Averaged Pearson Correlation Coefficients			
Target Tasks	DTLZ1b -$(1,0,1)$	DTLZ2b -$(1,0,1)$	DTLZ3b -$(1,0,1)$	DTLZ4b -$(1,0,1)$
Source Tasks	DTLZ1b -$(\delta_1,\delta_2,\delta_3)$	DTLZ2b -$(\delta_1,\delta_2,\delta_3)$	DTLZ3b -$(\delta_1,\delta_2,\delta_3)$	DTLZ4b -$(\delta_1,\delta_2,\delta_3)$
High-Correlation (HS)	0.9927	0.9862	0.9925	0.9851
Medium-Correlation (MS)	0.6516	0.6255	0.6015	0.6268
Low-Correlation (LS)	-0.2132	-0.0231	-0.1714	0.0437
Medium-Negative-Correlation (MNS)	-0.5616	-0.6240	-0.6081	-0.6183
High-Negative-Correlation (HNS)	-0.7214	-0.8197	-0.8052	-0.7313
Target Tasks	DTLZ1b^{-1} -$(1,0,1)$	DTLZ2b^{-1} -$(1,0,1)$	DTLZ3b^{-1} -$(1,0,1)$	DTLZ4b^{-1} -$(1,0,1)$
Source Tasks	DTLZ1b^{-1} -$(\delta_1,\delta_2,\delta_3)$	DTLZ2b^{-1} -$(\delta_1,\delta_2,\delta_3)$	DTLZ3b^{-1} -$(\delta_1,\delta_2,\delta_3)$	DTLZ4b^{-1} -$(\delta_1,\delta_2,\delta_3)$
High-Correlation (HS)	0.9926	0.9859	0.9822	0.9868
Medium-Correlation (MS)	0.6501	0.6158	0.6135	0.6193
Low-Correlation (LS)	-0.2228	-0.0445	-0.1681	0.0033
Medium-Negative-Correlation (MNS)	-0.5567	-0.6087	-0.6049	-0.6133
High-Negative-Correlation (HNS)	-0.7223	-0.8207	-0.8071	-0.8319

TABLE S-III
RESULTS OF SASSEA, MOEA/D-EGO, CSEA, K-RVEA, KTA2, AND EXTREMO OVER 20 INDEPENDENT RUNS. THE KRUSKAL-WALLIS TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED TO COMPARE EXTREMO WITH THE OTHER ALGORITHMS. RESULTS AFTER 50 EVALUATIONS ARE HIGHLIGHTED BY GRAY CELLS TO MARK THE PERFORMANCE GAINS OF EXTREMO UNDER TIGHT EVALUATION BUDGETS.

Target Tasks	Evaluations	SASSEA		MOEA/D-EGO		CSEA		K-RVEA		KTA2		ExTrEMO
		Average HV $\pm$ Std		Average HV $\pm$ Std		Average HV $\pm$ Std		Average HV $\pm$ Std		Average HV $\pm$ Std		Average HV$\pm$ Std
DTLZ1b-(1,0,1)	25	1.2742$\pm$0.0028	+	1.2876$\pm$0.0081	+	1.2780$\pm$0.0054	+	1.2718$\pm$0.0040	+	1.2717$\pm$0.0028	+	**1.3111$\pm$0.0126**
	50	1.3008$\pm$0.0077		1.3154$\pm$0.0047		1.3039$\pm$0.0071		1.2930$\pm$0.0086		1.3044$\pm$0.0071		**1.3297$\pm$0.0007**
	75	1.3148$\pm$0.0045		1.3220$\pm$0.0027		1.3154$\pm$0.0068		1.3035$\pm$0.0069		1.3204$\pm$0.0047		**1.3305$\pm$0.0003**
	100	1.3254$\pm$0.0036		1.3236$\pm$0.0022		1.3225$\pm$0.0059		1.3070$\pm$0.0064		1.3263$\pm$0.0034		**1.3308$\pm$0.0001**
DTLZ2b-(1,0,1)	25	1.1956$\pm$0.0011	+	1.1986$\pm$0.0032	+	1.2081$\pm$0.0216	+	1.2065$\pm$0.0129	+	1.2004$\pm$0.0059	+	**1.2974$\pm$0.0186**
	50	1.2097$\pm$0.0103		1.2749$\pm$0.0158		1.2610$\pm$0.0230		1.2385$\pm$0.0219		1.2926$\pm$0.0168		**1.3281$\pm$0.0018**
	75	1.2417$\pm$0.0327		1.2856$\pm$0.0166		1.3005$\pm$0.0118		1.2631$\pm$0.0227		1.3186$\pm$0.0073		**1.3302$\pm$0.0004**
	100	1.2766$\pm$0.0329		1.2985$\pm$0.0142		1.3146$\pm$0.0066		1.2669$\pm$0.0230		1.3264$\pm$0.0028		**1.3307$\pm$0.0002**
DTLZ3b-(1,0,1)	25	1.0159$\pm$0.0000	+	1.0477$\pm$0.0430	+	1.0447$\pm$0.0232	+	1.0281$\pm$0.0150	+	1.0418$\pm$0.0155	+	**1.2127$\pm$0.0626**
	50	1.0834$\pm$0.0511		1.1511$\pm$0.0330		1.1777$\pm$0.0368		1.0815$\pm$0.0403		1.2563$\pm$0.0299		**1.3242$\pm$0.0039**
	75	1.1266$\pm$0.0732		1.2073$\pm$0.0392		1.2582$\pm$0.0294		1.1126$\pm$0.0533		1.3006$\pm$0.0234		**1.3293$\pm$0.0011**
	100	1.1952$\pm$0.0782		1.2250$\pm$0.0354		1.2944$\pm$0.0160		1.1289$\pm$0.0586		1.3240$\pm$0.0056		**1.3305$\pm$0.0003**
DTLZ4b-(1,0,1)	25	1.1833$\pm$0.0000	+	1.1985$\pm$0.0162	+	1.1992$\pm$0.0188	+	1.1875$\pm$0.0084	+	1.1892$\pm$0.0050	+	**1.2566$\pm$0.0352**
	50	1.2113$\pm$0.0293		1.2849$\pm$0.0166		1.2734$\pm$0.0269		1.2354$\pm$0.0447		1.3013$\pm$0.0195		**1.3260$\pm$0.0093**
	75	1.2499$\pm$0.0379		1.3064$\pm$0.0081		1.3071$\pm$0.0199		1.2758$\pm$0.0334		1.3185$\pm$0.0122		**1.3305$\pm$0.0005**
	100	1.2999$\pm$0.0240		1.3085$\pm$0.0066		1.3174$\pm$0.0154		1.2818$\pm$0.0246		1.3281$\pm$0.0025		**1.3308$\pm$0.0003**
DTLZ1b^{-1}-(1,0,1)	25	1.2808$\pm$0.0106	+	1.2849$\pm$0.0093	+	1.2823$\pm$0.0083	+	1.2782$\pm$0.0068	+	1.2805$\pm$0.0176	+	**1.3179$\pm$0.0087**
	50	1.3017$\pm$0.0095		1.3109$\pm$0.0060		1.3015$\pm$0.0070		1.2971$\pm$0.0099		1.3097$\pm$0.0137		**1.3257$\pm$0.0017**
	75	1.3120$\pm$0.0085		1.3164$\pm$0.0033		1.3135$\pm$0.0076		1.3053$\pm$0.0095		1.3208$\pm$0.0081		**1.3268$\pm$0.0010**
	100	1.3200$\pm$0.0055		1.3191$\pm$0.0021		1.3193$\pm$0.0052		1.3096$\pm$0.0096		1.3245$\pm$0.0061		**1.3272$\pm$0.0001**
DTLZ2b^{-1}-(1,0,1)	25	1.1284$\pm$0.0478	+	1.1537$\pm$0.0448	+	1.1247$\pm$0.0249	+	1.1308$\pm$0.0487	+	1.1651$\pm$0.0433	+	**1.2601$\pm$0.0250**
	50	1.1672$\pm$0.0523		1.2218$\pm$0.0431		1.2196$\pm$0.0276		1.1773$\pm$0.0404		1.2675$\pm$0.0201		**1.2931$\pm$0.0015**
	75	1.2149$\pm$0.0539		1.2534$\pm$0.0221		1.2587$\pm$0.0200		1.2069$\pm$0.0344		1.2900$\pm$0.0062		**1.2957$\pm$0.0026**
	100	1.2667$\pm$0.0272		1.2668$\pm$0.0122		1.2716$\pm$0.0167		1.2254$\pm$0.0337		1.2944$\pm$0.0040		**1.2977$\pm$0.0032**
DTLZ3b^{-1}-(1,0,1)	25	0.9884$\pm$0.0687	+	0.9947$\pm$0.0685	+	1.0020$\pm$0.0769	+	0.9882$\pm$0.0673	+	1.0166$\pm$0.0785	+	**1.2407$\pm$0.0352**
	50	1.0492$\pm$0.0722		1.1052$\pm$0.0672		1.1408$\pm$0.0557		1.0651$\pm$0.0586		1.2172$\pm$0.0499		**1.3149$\pm$0.0075**
	75	1.1354$\pm$0.0910		1.1637$\pm$0.0405		1.2304$\pm$0.0456		1.1025$\pm$0.0463		1.2911$\pm$0.0315		**1.3235$\pm$0.0037**
	100	1.2064$\pm$0.0936		1.1897$\pm$0.0420		1.2741$\pm$0.0268		1.1286$\pm$0.0572		1.3192$\pm$0.0088		**1.3267$\pm$0.0005**
DTLZ4b^{-1}-(1,0,1)	25	1.1287$\pm$0.0546	+	1.1803$\pm$0.0380	+	1.1821$\pm$0.0271	+	1.1639$\pm$0.0419	+	1.1737$\pm$0.0423	+	**1.2107$\pm$0.0556**
	50	1.1690$\pm$0.0532		1.2426$\pm$0.0156		1.2373$\pm$0.0218		1.2030$\pm$0.0329		1.2539$\pm$0.0350		**1.2915$\pm$0.0043**
	75	1.2243$\pm$0.0456		1.2591$\pm$0.0151		1.2689$\pm$0.0129		1.2191$\pm$0.0313		1.2864$\pm$0.0078		**1.2945$\pm$0.0007**
	100	1.2570$\pm$0.0361		1.2672$\pm$0.0119		1.2803$\pm$0.0097		1.2301$\pm$0.0283		1.2933$\pm$0.0036		**1.2957$\pm$0.0025**
$+/-/=$		8/0/0		8/0/0		8/0/0		8/0/0		8/0/0		

TABLE S-IV
RESULTS OF SASSEA, MOEA/D-EGO, CSEA, K-RVEA, KTA2, AND EXTREMO-M/L ON DTLZ2B-(1,0,1) AND DTLZ3B-(1,0,1) OVER 20 INDEPENDENT RUNS. THE KRUSKAL-WALLIS TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED TO COMPARE EXTREMO-M/L WITH THE OTHER ALGORITHMS.

Target Tasks	Evaluations	SASSEA		MOEA/D-EGO		CSEA		K-RVEA		KTA2		ExTrEMO-M/L
		Average HV ± Std		Average HV ± Std		Average HV ± Std		Average HV ± Std		Average HV ± Std		Average HV± Std
DTLZ2b-(1,0,1)	25	1.1284±0.0478	+	1.1537±0.0448	+	1.1247±0.0249	+	1.1308±0.0487	+	1.1651±0.0433	+	**1.2601±0.0250**
	50	1.1672±0.0523		1.2218±0.0431		1.2196±0.0276		1.1773±0.0404		1.2675±0.0201		**1.2931±0.0015**
	75	1.2149±0.0539		1.2534±0.0221		1.2587±0.0200		1.2069±0.0344		1.2900±0.0062		**1.2957±0.0026**
	100	1.2667±0.0272		1.2668±0.0122		1.2716±0.0167		1.2254±0.0337		1.2944±0.0040		**1.2977±0.0032**
DTLZ3b-(1,0,1)	25	1.0159±0.0021	+	1.0477±0.0430	+	1.0447±0.0232	+	1.0281±0.0150	+	1.0418±0.0155	+	**1.2760±0.0453**
	50	1.0834±0.0511		1.1511±0.0330		1.1777±0.0368		1.0815±0.0403		1.2563±0.0299		**1.3271±0.0017**
	75	1.1266±0.0732		1.2073±0.0392		1.2582±0.0294		1.1126±0.0533		1.3006±0.0234		**1.3298±0.0004**
	100	1.1952±0.0782		1.2250±0.0354		1.2944±0.0160		1.1289±0.0586		1.3240±0.0056		**1.3306±0.0002**
$+/-/=$		2/0/0		2/0/0		2/0/0		2/0/0		2/0/0		

TABLE S-V
RESULTS OF EXTREMO-ZEROT AND EXTREMO ON DTLZ1B-(1,0,1) AND DTLZ2B-(1,0,1) WITH 300 EVALUATIONS OVER 20 INDEPENDENT RUNS.

Target Tasks	Evaluations	ExTrEMO-ZeroT	ExTrEMO	Target Tasks	Evaluations	ExTrEMO-ZeroT	ExTrEMO
		Average HV ± Std	Average HV ± Std			Average HV ± Std	Average HV ± Std
DTLZ1b-$(1,0,1)$	25	1.3084±0.0057	**1.3240±0.0044**	DTLZ2b-$(1,0,1)$	25	1.2427±0.0184	**1.3210±0.0073**
	50	1.3283±0.0019	**1.3306±0.0004**		50	1.3156±0.0103	**1.3301±0.0015**
	75	1.3301±0.0010	**1.3309±0.0001**		75	1.3282±0.0022	**1.3309±0.0001**
	100	1.3308±0.0002	**1.3309±0.0001**		100	1.3301±0.0005	**1.3309±0.0001**
	125	1.3309±0.0001	**1.3310±0.0001**		125	1.3306±0.0002	**1.3310±0.0001**
	150	1.3309±0.0000	**1.3310±0.0000**		150	1.3307±0.0002	**1.3310±0.0000**
	175	1.3309±0.0000	**1.3310±0.0000**		175	1.3308±0.0001	**1.3310±0.0000**
	200	**1.3310±0.0000**	**1.3310±0.0000**		200	1.3309±0.0001	**1.3310±0.0000**
	225	**1.3310±0.0000**	**1.3310±0.0000**		225	**1.3310±0.0000**	**1.3310±0.0000**
	250	**1.3310±0.0000**	**1.3310±0.0000**		250	**1.3310±0.0000**	**1.3310±0.0000**
	275	**1.3310±0.0000**	**1.3310±0.0000**		275	**1.3310±0.0000**	**1.3310±0.0000**
	300	**1.3310±0.0000**	**1.3310±0.0000**		300	**1.3310±0.0000**	**1.3310±0.0000**

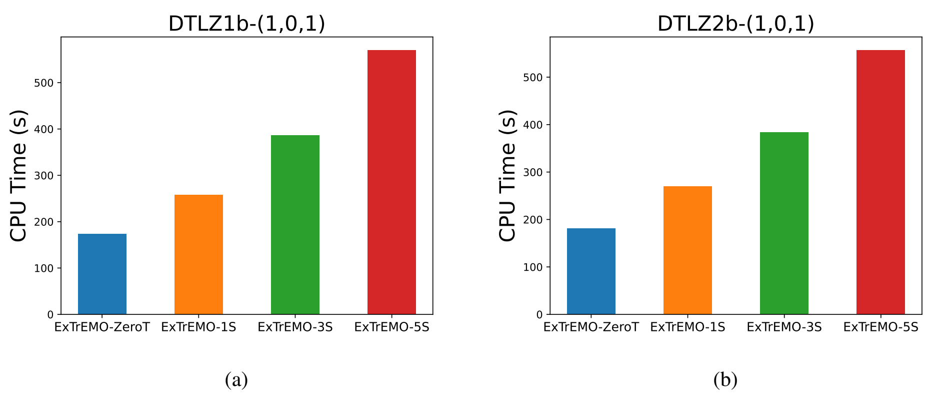
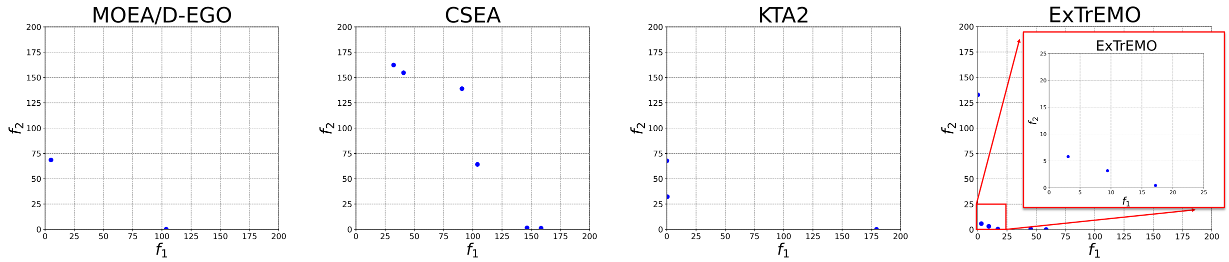
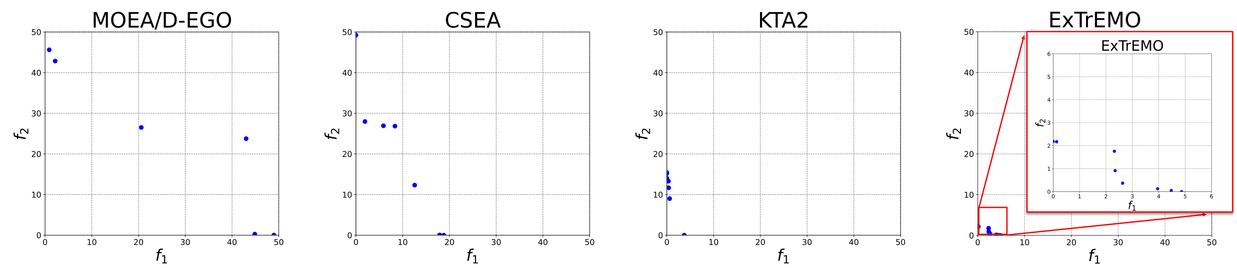


Fig. S-1. Average CPU time of ExTrEMO-ZeroT, ExTrEMO-1S, ExTrEMO-3S, and ExTrEMO-5S. (a) DTLZ1b-$(1,0,1)$. (b) DTLZ2b-$(1,0,1)$.



(a) (b) (c) (d)

Fig. S-2. Pareto front approximation by MOEA/D-EGO, CSEA, KTA2 and ExTrEMO on DTLZ2b-$(1, 0, 1)$ over 100 evaluations. Note that, only the solutions with the objective function values in the region $[0, 200] \times [0, 200]$ are shown in the figure. (a) MOEA/D-EGO. (b) CSEA. (c) KTA2. (d) ExTrEMO



(a) (b) (c) (d)

Fig. S-3. Pareto front approximation by MOEA/D-EGO, CSEA, KTA2 and ExTrEMO on DTLZ2b-$(1, 0, 1)$ over 100 evaluations. Note that, only the solutions with the objective function values in the region $[0, 50] \times [0, 50]$ are shown in the figure. (a) MOEA/D-EGO. (b) CSEA. (c) KTA2. (d) ExTrEMO