Paper Summary

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1 Adapting to a Market Shock: Optimal Sequential Market-Making

The main result of this article is optimal bid-ask price under the strong assumption given by author.

1.1 Model

We use random variable V, obeying $p_V(v)$, to denote the underlying value of asset. At time t, the markert-maker raise its bid price b_t and ask price a_t. Then a trader comes in to find whether there exists some trading chance. Besides, the trader expect the value of asster is $w_t = V + \varepsilon$, where ε is random variable with CDF F_ε. For example, if $w_t < b_t$, then trader will sell asset; if $w_t > a_t$, then trader will buy the asset. Furthermore, we use x_t to denote the signal such that $x \in \{-1, 0, 1\}$ means trader buying, no action, selling. Hence, we can find signal distribution as,

$$p(x|v; a_t, b_t) = \begin{cases} F_\varepsilon(b_t - v) & x = 1 \\ F_\varepsilon(a_t - v) - F_\varepsilon(b_t - v) & x = 0 \\ 1 - F_\varepsilon(a_t - v) & x = -1 \end{cases} \tag{1.1}$$

If the bid price is acepted at time t, we can obtain a bid reward,

$$r_t^{\text{bid}} = \int dv(v - b_t)F_\varepsilon(b_t - v)p_t(v) = \int dvvF_\varepsilon(-v)p_t(v + b_t) \tag{1.2}$$

Similarly, we can obtain an ask reward,

$$r_t^{\text{ask}} = \int dv(a_t - v)(1 - F_\varepsilon(a_t - v))p_t(v) = \int dvvF_\varepsilon(-v)p_t(a_t - v), \tag{1.3}$$

where we have used the assumption that distribution of ε is symmetric, i.e. $F_\varepsilon(x) = 1 - F_\varepsilon(-x)$. Hence,

$$\begin{aligned} r_t &= r_t^{\text{bid}} + r_t^{\text{ask}} \qquad (1.4) \\ &= \int dvvF_\varepsilon(-v)(p_t(v + b_t) + p_t(a_t - v)). \qquad (1.5) \end{aligned}$$

For a zero-profit market, the sequential decision $\{a_t, b_t\}$ is very simple, given by equations $r_t^{\text{bid}} = r_t^{\text{ask}} = 0$, which means,

$$b_t = \frac{\int dvvF_\varepsilon(b_t - v)p_t(v)}{\int dvF_\varepsilon(b_t - v)p_t(v)}, \tag{1.6}$$

$$a_t = \frac{\int dvvF_\varepsilon(v - a_t)p_t(v)}{\int dvF_\varepsilon(v - a_t)p_t(v)}. \tag{1.7}$$

However, more interesting thing is that we want to be a monopolist in this market, which means we should maximize our reward.

$$\frac{\partial r_t^{\text{ask}}}{\partial a_t} = 0, \frac{\partial r_t^{\text{bid}}}{\partial b_t} = 0. \tag{1.8}$$

Analytically, it is,

$$b_t = \frac{\int dv p_t(v)(v f_\varepsilon(b_t - v) - F_\varepsilon(b_t - v))}{\int dv p_t(v) f_\varepsilon(b_t - v)}, \tag{1.9}$$

$$a_t = \frac{\int dv p_t(v)(v f_\varepsilon(a_t - v) + F_\varepsilon(v - a_t))}{\int dv p_t(v) f_\varepsilon(a_t - v)}, \tag{1.10}$$

where $f_\varepsilon(x) = F'_\varepsilon(x)$. Here is a problem stated in as: If you believer your current knowledge of random variable V, i.e. $p_t(v)$, you can choose the short time optimal decision at least numerically. However, the belief of $p_t(v)$ should not be very high, we also need to choose other decisions such that we can update our knowledge of V more quickly. This is 'explore-exploit' problem. Formally, we can restate our problem in more mathematical language instead of maximizing our local reward. Our aim to become a monopolist is to maximize the following value function,

$$V(p_t; \pi) = \mathbb{E}[r_t | p_t, a_t^\pi(p_t), b_t^\pi(p_t)] + \gamma V(p_{t+1}; \pi). \tag{1.11}$$