Final Project

Quantitative Investment Analysis(Fall 2021)

Group:

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Read Data

In [1]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
import statsmodels.api as sm
from statsmodels.api import OLS
import math
#TS
from statsmodels.tsa import stattools
from arch import arch_model
from statsmodels.tsa import arima_model
from statsmodels.graphics.tsaplots import *
from arch.unitroot import ADF
#datetime
from matplotlib.dates import DateFormatter,WeekdayLocator,DateLocator,MONDAY,date2num
from datetime import datetime
#k线
%matplotlib inline
import mplfinance as mpf
# This allows multiple outputs from a single jupyter notebook cell:
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
#cvx
import cvxopt as cvx
from cvxopt import matrix,solvers
```

In [2]:

```
project = pd.read_csv('/Users/liuke/Downloads/HW2021Fall/Quant/FinalProj/finalproj/finalproj.csv',
                      index_col='DATE')
```

In [3]:

```
project.index = pd.to_datetime(project.index, format='%Y%m%d')
```

原数据

In [4]:

```
project.head(3)
```

Out[4]:

	PERMNO	exret	mkt_beta	logme	logbeme	r_2_12	gp	invest_asset
DATE								
1963-07-31	10006	-0.0479	1.049360	4.947052	0.182105	0.698340	0.182	0.459617
1963-07-31	10014	-0.0027	0.706807	2.408498	0.080984	0.035714	0.257	0.424276

	PERMNO	exret	mkt_beta	logme	logbeme	r_2_12	gp	invest_asset
DATE								
1963-07-31	10030	-0.0663	0.569758	4.493456	NaN	0.246521	NaN	NaN

检查空值

In [5]:

```
project.isnull().sum()
```

Out[5]:

```
PERMNO               0
exret            12841
mkt_beta             0
logme             1766
logbeme         171927
r_2_12              61
gp              118989
invest_asset    123295
dtype: int64
```

创建一个包括所有月份的list，原数据共有619个月

In [6]:

```
dateID = []
for i in project.index.unique():
    dateID.append(str(i).split()[0])
len(dateID)
```

Out[6]:

```
619
```

`1.1 Optimal portfolio via a BARRA model`

Step 1

①Reset outliers of 5 firm−level characteristics

用循环计算每个月的1%与99%分位数，并令大于或小于的等于对应分位数

In [7]:

```
for d in dateID:
    Q01 = project.loc[d].iloc[:,-5:].quantile(0.01)
    Q99 = project.loc[d].iloc[:,-5:].quantile(0.99)
    for j in range(3,8):
        project.loc[d].iloc[:,j][project.loc[d].iloc[:,j]>Q99[j-3]] = Q99[j-3]
        project.loc[d].iloc[:,j][project.loc[d].iloc[:,j]<Q01[j-3]] = Q01[j-3]
```

②Standardize its value

用循环计算每个月，并对该月内的五列五个不同特征值进行当月该列特征值的标准化

In [8]:

```
project_std = project
for d in dateID:
    for j in range(3,8):
        mean = project.loc[d].iloc[:,j].mean()
        std = project.loc[d].iloc[:,j].std()
        project_std.loc[d].iloc[:,j]=project.loc[d].iloc[:,j].apply(lambda x : (x-mean)/std)
```

```
/Applications/anaconda3/lib/python3.9/site-packages/pandas/core/indexing.py:1773: SettingWithCopyW
arning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/inde
```

```
xing.html#returning-a-view-versus-a-copy
  self._setitem_single_column(ilocs[0], value, pi)
```

标准化完毕

In [9]:
```
project_std
```

Out[9]:

	PERMNO	exret	mkt_beta	logme	logbeme	r_2_12	gp	invest_asset
DATE								
1963-07-31	10006	-0.0479	1.049360	0.286460	0.774220	1.434258	-0.655549	3.186066
1963-07-31	10014	-0.0027	0.706807	-1.390264	0.618245	-1.050796	-0.325663	2.896638
1963-07-31	10030	-0.0663	0.569758	-0.013142	NaN	-0.260204	NaN	NaN
1963-07-31	10057	-0.0967	1.135995	-0.680445	NaN	1.124172	NaN	NaN
1963-07-31	10102	-0.0190	0.963979	0.715342	0.021303	-0.516088	-0.202505	0.587275
...	...	...	...	...	...	...	...	...
2015-01-30	93428	-0.0734	1.224872	0.030170	-0.661853	-0.121359	0.386886	0.391627
2015-01-30	93429	0.0166	0.581365	0.895934	-2.339622	0.441129	1.640668	0.779888
2015-01-30	93433	0.5939	2.559842	-2.110162	-3.145233	-2.096343	-0.861179	-1.841682
2015-01-30	93434	0.2850	-0.036769	-1.274439	0.383626	-1.532435	-0.809712	5.509782
2015-01-30	93436	-0.0846	0.778791	1.671654	-3.064301	1.846522	-0.128138	3.964574

1971903 rows × 8 columns

进行训练集为前120个月的演示，定义training数据

In [10]:
```
trainingdate = dateID[:120]
training_start = '1963-7-31'
training_end = '1973-6-30'
training = project_std[training_start:training_end]
training.tail(3)
training.head(3)
```

Out[10]:

	PERMNO	exret	mkt_beta	logme	logbeme	r_2_12	gp	invest_asset
DATE								
1973-06-29	65825	0.0300	1.275393	-1.266623	0.754495	0.323338	0.204975	-0.228146
1973-06-29	68195	-0.0265	1.179756	0.567518	-1.548156	-1.440666	0.412852	0.044301
1973-06-29	68523	-0.0264	1.450590	-0.080775	NaN	-1.045893	NaN	NaN

Out[10]:

	PERMNO	exret	mkt_beta	logme	logbeme	r_2_12	gp	invest_asset
DATE								
1963-07-31	10006	-0.0479	1.049360	0.286460	0.774220	1.434258	-0.655549	3.186066
1963-07-31	10014	-0.0027	0.706807	-1.390264	0.618245	-1.050796	-0.325663	2.896638
1963-07-31	10030	-0.0663	0.569758	-0.013142	NaN	-0.260204	NaN	NaN

③Cross-sectional regression

Collect the resulting coefficient estimates

收集θ的120个月的时间序列数据，先建立用来存放的表格

In [11]:
```
θ = pd.DataFrame(data=np.zeros([120,6]),index=training.index.unique(),
                 columns=['mkt_beta','logme','logbeme','r_2_12','gp','invest_asset'])
```

对每个月进行6个特征值的回归，将系数记录在上面的表格

In [12]:
```python
for t in trainingdate:
    θ.loc[t] = OLS(training.loc[t].iloc[:,1],training.loc[t].iloc[:,2:],missing='drop').fit().para
```

得到如下的θ的120个月的数据

In [13]:
```python
θ
```

Out[13]:

DATE	mkt_beta	logme	logbeme	r_2_12	gp	invest_asset
1963-07-31	-0.009225	-0.001787	-0.006311	0.007078	0.000406	0.001267
1963-08-30	0.047990	0.007174	0.001092	0.009503	0.002422	-0.002563
1963-09-30	-0.022506	0.001859	0.005765	-0.003435	0.004645	-0.003438
1963-10-31	0.020559	-0.001818	0.001067	0.016890	0.004495	-0.000174
1963-11-29	-0.006861	0.000740	-0.000229	-0.000181	-0.006763	-0.001882
...	...	...	...	...	...	...
1973-02-28	-0.052802	0.006952	0.006234	-0.005770	-0.006178	-0.003055
1973-03-30	-0.017573	-0.001362	0.009965	0.009215	-0.005966	0.002347
1973-04-30	-0.051029	0.001509	0.013480	0.009644	-0.006367	-0.008786
1973-05-31	-0.057780	0.017399	0.000217	0.004706	-0.001829	-0.001384
1973-06-29	-0.025241	-0.005282	-0.001805	0.008627	-0.002079	-0.002957

120 rows × 6 columns

In [14]:
```python
#θ.to_csv('/Users/liuke/Downloads/HW2021Fall/Quant/FinalProj/θ.csv')
```

Collect the residuals of cross-sectional regressions

建立收集η的表格

In [15]:
```python
η=pd.DataFrame(data=np.zeros([120,2510]),index=training.index.unique(),
              columns=training.PERMNO.unique())
```

由于不是每一个公司都参与回归，有的公司数据残缺，所以这一步需要将残差与公司对号入座

In [16]:
```python
for t in trainingdate:
    testtheta =  training.loc[t]
    testtheta.index = training.loc[t].PERMNO
    dicttest = dict(OLS(testtheta.iloc[:,1],testtheta.iloc[:,2:],missing='drop').fit().resid)
    η.loc[t]=dicttest
```

In [17]:
```python
#η.to_csv('/Users/liuke/Downloads/HW2021Fall/Quant/FinalProj/η.csv')
```

得到如下的120个月的η数据，可以发现，越早上市的公司（100XX），数据越完整，越晚上市的公司（483XX），数据就比较少

In [18]:
```python
η
```

Out[18]:

DATE	10006	10014	10030	10057	10102	10137	10145	10153	10161	10188	..
1963-07-31	-0.046743	0.009138	NaN	NaN	-0.005704	0.004607	0.028751	-0.063615	-0.090622	-0.038800	..

DATE	10006	10014	10030	10057	10102	10137	10145	10153	10161	10188	...
1963-08-30	0.064502	0.032025	NaN	NaN	0.030122	0.088553	-0.052194	0.005873	-0.005439	0.010307	...
1963-09-30	-0.043745	-0.016979	NaN	NaN	-0.007585	-0.082890	0.031948	-0.051777	-0.038148	0.041821	...
1963-10-31	0.015083	-0.109031	NaN	NaN	0.020517	0.029229	0.048320	-0.046128	-0.014519	-0.044726	...
1963-11-29	0.253260	0.006279	NaN	NaN	0.000964	-0.047379	0.019463	-0.008019	0.055317	-0.054098	...
...	...	...	...	...	...	...	...	...	...	...	...
1973-02-28	-0.033271	-0.144079	NaN	-0.062460	0.020427	0.006745	0.103987	-0.041335	0.102895	0.025197	...
1973-03-30	0.118891	0.063162	NaN	-0.073148	0.005686	-0.091395	0.071867	-0.065489	0.019426	0.019332	...
1973-04-30	0.035180	-0.131197	NaN	0.027674	-0.032529	0.061821	0.003653	0.053667	-0.012713	0.019283	...
1973-05-31	0.020027	-0.034882	NaN	-0.017352	0.016574	0.053122	0.013590	0.004050	-0.036701	0.063209	...
1973-06-29	0.048558	0.083457	NaN	-0.037822	-0.026682	-0.008492	0.040335	0.043194	-0.016248	-0.004694	...

120 rows × 2510 columns

Step 2 Compute the sample average of factors θt over past 120 months

求简单均值

In [19]:

```
θ_1_120 = θ.mean()
θ_1_120
```

Out[19]:

```
mkt_beta        0.003850
logme          -0.001872
logbeme         0.001044
r_2_12          0.004283
gp              0.000598
invest_asset   -0.001727
dtype: float64
```

Step 3 Compute the sample covariance matrix of factors θt over past 120 months

求θ的协方差，注意到分母是训练长度120 – 自由度6，意味着之后训练长度变长，分母也会跟随着变

In [20]:

```
Var_120 = 0
for t in trainingdate:
    temp = np.array(θ.loc[t] - θ_1_120).reshape([6,1]).dot(np.array(θ.loc[t] - θ_1_120).reshape([1
    Var_120 += temp
Var_120_θ = Var_120/114
Var_120_θ
```

```
array([[ 1.66198661e-03, -2.56172075e-04,  2.52606496e-05,
```

```
Out[20]:          2.69375116e-05, -3.04900035e-06,  3.23134228e-06],
                [-2.56172075e-04,  2.60906246e-04,  3.35365257e-05,
                 -8.01462900e-06,  1.90901247e-05,  7.43214303e-06],
                [ 2.52606496e-05,  3.35365257e-05,  4.66774031e-05,
                 -1.73587973e-05,  6.05526330e-06, -5.57541035e-06],
                [ 2.69375116e-05, -8.01462900e-06, -1.73587973e-05,
                  1.56667992e-04,  1.49194906e-05,  9.51424575e-06],
                [-3.04900035e-06,  1.90901247e-05,  6.05526330e-06,
                  1.49194906e-05,  3.51728290e-05,  9.38034383e-06],
                [ 3.23134228e-06,  7.43214303e-06, -5.57541035e-06,
                  9.51424575e-06,  9.38034383e-06,  2.23785584e-05]])
```

Step 4 Compute the sample covariance matrix of residuals ηt

筛选数据主要有三个问题

- 第一是选择在训练最后一期仍在交易的股票
- 第二是选择未来一期仍有交易的股票
- 第三是该股票可用数据大于100的

这里注意到窗口长度为120时，可用数据筛选的下限是100，之后窗口增大，按理来说应该相应增大可用数据的下限，比如训练200个月，选择可用数据为160以上的股票，但是在任务中没有明确说明，之后我仍以可用数据数量为100来计算

In [21]:
```
ffdf = training.loc['1973-6-29']
ffdf.index = ffdf.PERMNO
ff1 = ffdf.exret.dropna().index
ff2 = list(η[ff1].columns[η[ff1].count()>100])

ffdf2 = project_std.loc['1973-7-31'].dropna()
ffdf2.index = ffdf2.PERMNO
ffdf2 = ffdf2.exret.dropna()
fff = list(set(ff2).intersection(set(ffdf2.index)))
```

股票数据筛选完毕，选出501个股票进行计算，这里对号入座，选则这501个股票的η

In [22]:
```
η_select = η[fff]
η_select
```

Out[22]:

DATE	10241	20482	22533	22541	18446	24600	12319	26657	16424	26665
1963-07-31	-0.011251	0.148298	-0.022518	0.024621	-0.063550	-0.055881	0.023817	0.016498	0.003755	-0.132050
1963-08-30	0.017603	0.044109	0.042719	-0.006570	0.000386	-0.002275	0.040321	-0.041226	0.074791	0.005764
1963-09-30	-0.024069	0.005664	-0.079853	0.012928	0.014544	-0.014532	-0.073607	-0.046684	-0.026716	-0.133519
1963-10-31	-0.051019	-0.117208	0.064395	-0.032517	0.003532	0.030844	-0.003088	-0.122484	-0.139527	0.223472
1963-11-29	-0.025544	0.057984	0.017964	0.010278	-0.008238	-0.126051	-0.048952	-0.003100	0.023677	-0.022102
...	...	...	...	...	...	...	...	...	...	...
1973-02-28	-0.024358	0.034486	-0.022188	0.017539	0.037831	0.084977	0.067960	0.020546	-0.021639	-0.100893
1973-03-30	0.028547	0.068959	-0.115656	-0.010005	0.029383	0.116961	0.129184	-0.137262	0.024042	0.057223

DATE	10241	20482	22533	22541	18446	24600	12319	26657	16424	26665
1973-04-30	0.015737	-0.032109	-0.053337	0.012179	-0.017282	0.249953	0.132061	0.003020	-0.002956	-0.034201
1973-05-31	0.031575	-0.060948	0.072191	-0.017717	0.019475	-0.132928	0.118459	-0.069412	0.021419	-0.040187
1973-06-29	-0.015166	0.003780	0.040293	0.007963	0.047552	0.019463	-0.032955	0.115305	-0.030013	0.058070

120 rows × 501 columns

In [23]:

```
η_select.mean()
len_of_select = len(η_select.T)
Var_120_η = np.eye(len_of_select);
```

为了储存残差的对角阵，建立储存的表格

In [24]:

```
Var_120_η = pd.DataFrame(data=Var_120_η,index=[np.arange(len_of_select)+1],columns=[np.arange(len_
Var_120_η
```

Out[24]:

	1	2	3	4	5	6	7	8	9	10	...	492	493	494	495	496	497	498	499	500	501
1	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
497	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
498	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
501	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0

501 rows × 501 columns

计算并输入对角阵，这里注意到分母是随着公司可用数据的多少决定的，所以不是常数，最后得到该对角阵

In [25]:

```
for i in range(len_of_select):
    Var_120_η.iloc[i,i] = (η_select.iloc[:,i].var()*(η_select.iloc[:,i].count()-1))/(η_select.iloc
```

In [26]:

```
Var_120_η
```

Out[26]:

	1	2	3	4	5	6	7	8	9	10	...	492	493	494	495	496	
1	0.002088	0.000000	0.000000	0.000000	0.000000	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.000
2	0.000000	0.003625	0.000000	0.000000	0.000000	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.000
3	0.000000	0.000000	0.005042	0.000000	0.000000	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.000
4	0.000000	0.000000	0.000000	0.001483	0.000000	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.000
5	0.000000	0.000000	0.000000	0.000000	0.004387	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.000
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	

	1	2	3	4	5	6	7	8	9	10	...	492	493	494	495	496	
497	0.000000	0.000000	0.000000	0.000000	0.000000	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.001
498	0.000000	0.000000	0.000000	0.000000	0.000000	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.000
499	0.000000	0.000000	0.000000	0.000000	0.000000	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.000
500	0.000000	0.000000	0.000000	0.000000	0.000000	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.000
501	0.000000	0.000000	0.000000	0.000000	0.000000	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.000

501 rows × 501 columns

Step 5 Compute time–t (June 1973) forecast

In [27]:

```
θ_1_120
```

Out[27]:

```
mkt_beta        0.003850
logme          -0.001872
logbeme         0.001044
r_2_12          0.004283
gp              0.000598
invest_asset   -0.001727
dtype: float64
```

由于滞后的关系，找到下一期的被选中的公司的特征数据，提取相关的数据

In [28]:

```
test_e121 = project_std.loc['1973-7-31']
test_e121.index = test_e121.PERMNO
test_e121.loc[fff]
```

Out[28]:

	PERMNO	exret	mkt_beta	logme	logbeme	r_2_12	gp	invest_asset
PERMNO								
10241	10241	-0.0321	0.557337	1.302578	0.706663	1.452011	-0.297104	-0.633175
20482	20482	0.0834	0.600614	1.538218	-1.210529	0.440673	0.622965	0.373311
22533	22533	0.1499	0.831745	1.212344	-0.679286	-0.258592	-0.162983	0.551184
22541	22541	0.0139	0.847198	1.513147	0.488195	0.988619	-1.203315	0.148674
18446	18446	0.0404	1.450027	1.306052	-0.663061	0.135971	-0.356118	-0.121538
...	...	...	...	...	...	...	...	...
26614	26614	-0.0348	0.276632	-0.074015	-0.040445	0.951292	-1.042370	0.010116
10233	10233	0.1584	1.587835	1.093097	-0.787116	-0.143723	0.343099	-0.115342
20474	20474	0.2209	1.951782	0.389532	1.076514	-0.095613	0.760216	-0.955284
24571	24571	-0.0265	0.551069	1.705449	0.672304	2.217551	-0.664149	-0.416640
26622	26622	0.2529	1.056934	1.240255	-0.926005	0.311634	1.047234	-0.004718

501 rows × 8 columns

In [29]:

```
X_120 = test_e121.loc[fff].iloc[:,2:]
X_120
```

Out[29]:

	mkt_beta	logme	logbeme	r_2_12	gp	invest_asset
PERMNO						
10241	0.557337	1.302578	0.706663	1.452011	-0.297104	-0.633175
20482	0.600614	1.538218	-1.210529	0.440673	0.622965	0.373311
22533	0.831745	1.212344	-0.679286	-0.258592	-0.162983	0.551184
22541	0.847198	1.513147	0.488195	0.988619	-1.203315	0.148674
18446	1.450027	1.306052	-0.663061	0.135971	-0.356118	-0.121538

PERMNO	mkt_beta	logme	logbeme	r_2_12	gp	invest_asset
...	...	...	...	...	...	...
26614	0.276632	-0.074015	-0.040445	0.951292	-1.042370	0.010116
10233	1.587835	1.093097	-0.787116	-0.143723	0.343099	-0.115342
20474	1.951782	0.389532	1.076514	-0.095613	0.760216	-0.955284
24571	0.551069	1.705449	0.672304	2.217551	-0.664149	-0.416640
26622	1.056934	1.240255	-0.926005	0.311634	1.047234	-0.004718

501 rows × 6 columns

求得这些被选中公司的下一期的条件期望超额收益

In [30]:
```
E_121 = (X_120*θ_1_120).sum(axis=1)
```

Step 6 Compute time-t (June 1973) conditional variance of month-t+1 return (July 1973)

计算方差

In [31]:
```
X_120_v = np.array(X_120)
```

In [32]:
```
Var_s6 = X_120_v.dot(Var_120_θ).dot(X_120_v.T)+Var_120_η
```

In [33]:
```
Var_s6
```

Out[33]:

	1	2	3	4	5	6	7	8	9	10	...	
1	0.003047	0.000728	0.000678	0.001051	0.001171	0.000822	0.000570	0.000949	0.001193	0.001227	...	0.00
2	0.000728	0.004401	0.000733	0.000876	0.001121	0.000521	0.000304	0.000881	0.001195	0.001133	...	0.00
3	0.000678	0.000733	0.006011	0.001007	0.001602	0.001240	-0.000198	0.001044	0.001026	0.002052	...	0.00
4	0.001051	0.000876	0.001007	0.002804	0.001727	0.001352	0.000291	0.001164	0.001327	0.002039	...	0.00
5	0.001171	0.001121	0.001602	0.001727	0.007271	0.002639	-0.000344	0.001767	0.001694	0.003968	...	0.0
...	...	...	...	...	...	...	...	...	...	...	...	...
497	0.000327	0.000193	0.000258	0.000410	0.000621	0.000731	0.000195	0.000208	0.000436	0.000900	...	0.00
498	0.001175	0.001162	0.001738	0.001793	0.003164	0.002993	-0.000554	0.001968	0.001775	0.004491	...	0.0
499	0.001416	0.001224	0.002102	0.002192	0.004037	0.004317	-0.000947	0.002592	0.001921	0.006083	...	0.00
500	0.001182	0.000883	0.000694	0.001218	0.001175	0.000658	0.000968	0.000981	0.001498	0.001020	...	0.0
501	0.000969	0.000972	0.001194	0.001310	0.002084	0.001700	-0.000052	0.001461	0.001539	0.002730	...	0.00

501 rows × 501 columns

Step 7a GMV

按照最小方差公式计算权重

In [34]:
```
e = np.ones([len_of_select,1])
```

In [35]:
```
w_gmv_120 = np.linalg.inv(Var_s6).dot(e)/(e.T.dot(np.linalg.inv(Var_s6)).dot(e))
```

In [36]:
```
R_e_next = test_e121.loc[fff].exret
```

得到权重之后，用下一期收益率来获得这个用120个月数据选择的最小方差组合的下一个月的表现

In [37]:
```
Excess_ret_GMV = w_gmv_120.T.dot(np.array(R_e_next).reshape(len(R_e_next),1))
float(Excess_ret_GMV)
```

Out[37]:
```
-0.03047561418154594
```

Step 7b

使用CVXOPT包求解二次规划问题

不等式左边系数矩阵g的形成

In [38]:
```
g1 = pd.DataFrame(data=np.eye(len_of_select))
g2 = pd.DataFrame(data=-1*np.eye(len_of_select))
g = g1.append(g2)
```

不等式右边矩阵h

In [39]:
```
h = pd.DataFrame(data=0.01*np.ones(2*len_of_select).reshape([len_of_select*2,1]))
```

等式约束的等式左边系数矩阵A

In [40]:
```
A1 = X_120[['mkt_beta','logme']]
A1['e'] = 1
A = A1.T;
```

```
/var/folders/t0/8lchz22d7wjbvmdj2j9vt7jh0000gn/T/ipykernel_1104/3107406243.py:2: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  A1['e'] = 1
```

等式约束等式右边矩阵b

In [41]:
```
b1 = pd.DataFrame(data=np.array([0,0,0])).astype(float)
```

二次规划的标准化矩阵

In [42]:
```
P = matrix(np.matrix(3*Var_s6))
q = matrix(np.array(-1*E_121).reshape([len_of_select,1]))
G = matrix(np.array(g))
h = matrix(np.array(h))
A = matrix(np.array(A))
b = matrix(np.array(b1))
```

求解得到多空组合的权重

In [43]:
```
result = solvers.qp(P,q,G,h,A,b)
w_optLS = np.array(result['x'])
```

```
     pcost       dcost       gap    pres   dres
```

```
 0: -2.4398e-03 -1.0071e+01  1e+01  9e-17  3e-15
 1: -2.4828e-03 -1.2773e-01  1e-01  6e-17  4e-15
 2: -4.9536e-03 -1.7305e-02  1e-02  7e-17  4e-16
 3: -9.5832e-03 -1.1659e-02  2e-03  1e-16  1e-16
 4: -1.0634e-02 -1.1024e-02  4e-04  1e-16  1e-16
 5: -1.0849e-02 -1.0885e-02  4e-05  9e-17  1e-16
 6: -1.0869e-02 -1.0871e-02  1e-06  2e-16  1e-16
 7: -1.0870e-02 -1.0870e-02  4e-08  2e-16  1e-16
Optimal solution found.
```

检查所得多空组合权重的和是否为0

In [44]:
```
round(w_optLS.sum())
```

Out[44]:
```
0
```

检查所得多空组合的每个元素绝对值是否都小于0.01

In [45]:
```
(abs(w_optLS) <0.01).sum() == len_of_select
```

Out[45]:
```
True
```

检查是否所得多空组合β和为0，log(ME)和为0

In [46]:
```
w_optLS.T.dot(A.T).astype(int)
```

Out[46]:
```
array([[0, 0, 0]])
```

所得多空组合的表现

In [47]:
```
float(w_optLS.T.dot(np.array(R_e_next).reshape(len(R_e_next),1)))
```

Out[47]:
```
-0.045698140434420194
```

Step 8

建立一个储存两个组合收益表现的表格

In [50]:
```
ExcessRet_of_2 = pd.DataFrame(data=np.zeros([619,2]),index = project.index.unique(),
                             columns=['GMV','long-short']).iloc[120:]
```

In [51]:
```
ExcessRet_of_2.iloc[:,:] = np.nan
ExcessRet_of_2
```

Out[51]:

	GMV	long-short
DATE		
1973-07-31	NaN	NaN
1973-08-31	NaN	NaN
1973-09-28	NaN	NaN
1973-10-31	NaN	NaN
1973-11-30	NaN	NaN
...	...	...
2014-09-30	NaN	NaN
2014-10-31	NaN	NaN
2014-11-28	NaN	NaN
2014-12-31	NaN	NaN

	GMV	long-short
DATE		
2015-01-30	NaN	NaN

499 rows × 2 columns

将以上步骤进行整合，再循环求得两个组合表现的时间序列数据

In [ ]:

```
for month in range(119,618):
    #设置训练集，每次循环都会使得训练月长度越来越大，逐渐变大的窗口，但是开始的时间固定
    trainingdate = dateID[:month+1]
    training = project_std['1963-7-31':dateID[month]]

    #Step1 标准化早已完成，现在收集本次循环的θ和η
    #创建收集表，形状是当次循环包含训练集的长度*6
    θ_s8 = pd.DataFrame(data=np.zeros([month+1,6]),index=training.index.unique(),
                    columns=['mkt_beta','logme','logbeme','r_2_12','gp','invest_asset'])
    #进行回归，month-by-month的回归并写入数据表中，收集θ
    for t1 in trainingdate:
        θ_s8.loc[t1] = OLS(training.loc[t1].iloc[:,1],training.loc[t1].iloc[:,2:],missing='drop').
    #建立η表，收集每一个公司的每一个月的η，表格形状为月份数量*公司数量，
    η_s8 = pd.DataFrame(data=np.zeros([month+1,len(training.PERMNO.unique())]),index=training.inde
              columns=training.PERMNO.unique())
    #回归并提取η，利用dict将每个η和它所属于的公司对号入座，确保公司对应自己的当月的η
    for t2 in trainingdate:
        testtheta =  training.loc[t2]
        testtheta.index = training.loc[t2].PERMNO
        dicttest = dict(OLS(testtheta.iloc[:,1],testtheta.iloc[:,2:],missing='drop').fit().resid)
        η_s8.loc[t2]=dicttest

    #Step2 θ样本均值
    θ_1_120_s8 = θ_s8.mean()

    #Step3 θ样本协方差
    Var_120_s8 = 0
    for t3 in trainingdate:
        temp = np.array(θ_s8.loc[t3] - θ_1_120_s8).reshape([6,1]).dot(np.array(θ_s8.loc[t3] - θ_1_
        Var_120_s8 += temp
    Var_120_θ_s8 = Var_120_s8/(month-5) #month + 1 - 6,取120个月时，month=119，自由度：119+1-6 = 114

    #Step4 η协方差
    ffdf = training.loc[dateID[month]]
    ffdf.index = ffdf.PERMNO
    ff1 = ffdf.exret.dropna().index
    ff2 = list(η_s8[ff1].columns[η_s8[ff1].count()>100])

    ffdf2 = project_std.loc[dateID[month+1]].dropna()
    ffdf2.index = ffdf2.PERMNO
    ffdf2 = ffdf2.exret.dropna()
    fff = list(set(ff2).intersection(set(ffdf2.index)))
    #数据选择完毕，进行计算
    η_select_s8 = η_s8[fff]
    len_of_select_s8 = len(η_select_s8.T)
    #建立收集表
    Var_120_η_s8 = np.eye(len_of_select_s8);
    Var_120_η_s8 = pd.DataFrame(data=Var_120_η_s8,index=[np.arange(len_of_select_s8)+1],columns=[n
    #求对角线上每一个数字，分母Bi是变化的，视每个公司情况而定
    for t4 in range(len_of_select_s8):
        Var_120_η_s8.iloc[t4,t4] = (η_select_s8.iloc[:,t4].var()*(η_select.iloc[:,i].count()-1))/(

    #Step 5 下一期期望收益，由于滞后一期的关系，系数用下一期的
    e_s5 = project_std.loc[dateID[month+1]];
    e_s5.index = e_s5.PERMNO;
    X_120_s8 = e_s5.loc[fff].iloc[:,2:];
    E_121_s8 = (X_120_s8*θ_1_120_s8).sum(axis=1);

    #Step 6 下一期条件方差
    X_120_s8_v = np.array(X_120_s8)
    Var_s6_s8 = X_120_s8_v.dot(Var_120_θ_s8).dot(X_120_s8_v.T)+Var_120_η_s8;

    #Step 7a GMV
    R_e_next =e_s5.loc[fff].exret
```

```
    e = np.ones([len_of_select_s8,1])
    w_gmv_120_s8 = np.linalg.inv(Var_s6_s8).dot(e)/(e.T.dot(np.linalg.inv(Var_s6_s8)).dot(e))
    Excess_ret_GMV_s8 = float(w_gmv_120_s8.T.dot(np.array(R_e_next).reshape(len(R_e_next),1)))

    #Step 7b long-short
    g1 = pd.DataFrame(data=np.eye(len_of_select_s8))
    g2 = pd.DataFrame(data=-1*np.eye(len_of_select_s8))
    g = g1.append(g2)
    h = pd.DataFrame(data=0.01*np.ones(2*len_of_select_s8).reshape([len_of_select_s8*2,1]))
    A1 = X_120_s8[['mkt_beta','logme']]
    A1['e'] = 1
    A = A1.T;
    b1 = pd.DataFrame(data=np.array([0,0,0])).astype(float)
    P = matrix(np.matrix(3*Var_s6_s8)) #!
    q = matrix(np.array(-1*E_121_s8).reshape([len_of_select_s8,1])) #!
    G = matrix(np.array(g))#!
    h = matrix(np.array(h))#
    A = matrix(np.array(A));
    b = matrix(np.array(b1));
    result = solvers.qp(P,q,G,h,A,b);
    w_optLS_s8 = np.array(result['x']);
    Excess_ret_ls_s8 = float(w_optLS_s8.T.dot(np.array(R_e_next).reshape(len(R_e_next),1)));

    #Record
    ExcessRet_of_2.loc[dateID[month+1]]['GMV'] = Excess_ret_GMV_s8
    ExcessRet_of_2.loc[dateID[month+1]]['long-short'] = Excess_ret_ls_s8
```

In [48]:

```
#ExcessRet_of_2.to_csv('/Users/liuke/Downloads/HW2021Fall/Quant/FinalProj/record.csv')
```

将两组合表现数据存档

In [49]:

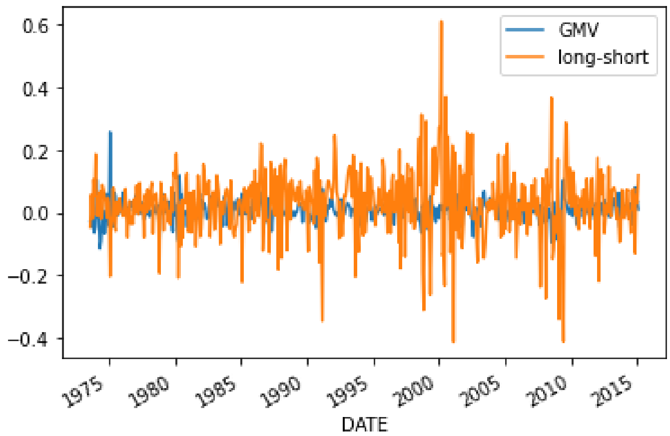
```
Perfomence = pd.read_csv('/Users/liuke/Downloads/HW2021Fall/Quant/FinalProj/ExcessRet_TS.csv',inde
Perfomence.index = pd.to_datetime(Perfomence.index)
```

画出收益率的折线图，大体在0附近上下波动

In [50]:

```
Perfomence.plot()
```

Out[50]: <AxesSubplot:xlabel='DATE'>



具体数据如下

In [51]:

```
Perfomence
```

Out[51]:

DATE	GMV	long-short
1973-07-31	-0.030070	-0.045562
1973-08-31	-0.044310	0.057972
1973-09-28	0.033216	-0.020284

DATE	GMV	long-short
1973-10-31	-0.051882	0.105971
1973-11-30	-0.064184	0.055607
...	...	...
2014-09-30	-0.005530	0.077052
2014-10-31	0.081516	-0.130269
2014-11-28	0.019264	0.053737
2014-12-31	0.038766	0.032163
2015-01-30	0.009577	0.120301

499 rows × 2 columns

1.2 Performance Evaluation

Preparation

In [52]:
```
riskfactor = pd.read_csv('/Users/liuke/Downloads/HW2021Fall/Quant/FinalProj/finalproj/riskfactor.c
riskfactor.index = pd.to_datetime(riskfactor.index,format='%Y%m%d')
```

In [53]:
```
Perfomence['Enhanced_index'] = riskfactor.Mkt_RF[-499:] + Perfomence['long-short']
```

两个组合在499个月中，收益大于0的天数为

In [54]:
```
(Perfomence>0).sum()
```

Out[54]:
```
GMV               331
long-short        326
Enhanced_index    337
dtype: int64
```

无风险收益率与市场超额收益大于0的天数，发现无风险收益率在绝大多数情况是大于零的，市场超额收益大于0的天数，与组合的超额完成收益大于0的天数相仿

In [55]:
```
(riskfactor>0).sum()
```

Out[55]:
```
Mkt_RF    362
RF        573
dtype: int64
```

记录最小方差组合与指数增强组合，接下来进行业绩评价

In [56]:
```
Min_var = Perfomence['GMV']
Enhanced_index = Perfomence['Enhanced_index']
```

1.Annualized average excess return:

Minimal variance portfolio

In [57]:
```
Min_var.mean()*12
```

Out[57]:
```
0.10247648406700628
```

Enhanced index portfolio

In [58]:
```
Enhanced_index.mean()*12
```
Out[58]:
```
0.419551841705897
```

2.Annualized standard deviation of excess return:

Minimal variance portfolio

In [59]:
```
Min_var.std()*(12**0.5)
```
Out[59]:
```
0.11560336815664732
```

Enhanced index portfolio

In [60]:
```
Enhanced_index.std()*(12**0.5)
```
Out[60]:
```
0.389980992988152
```

3. Annualized Sharpe ratio:

Minimal variance portfolio

In [61]:
```
(Min_var.mean()*12) / (Min_var.std()*(12**0.5))
```
Out[61]:
```
0.8864489478208493
```

Enhanced index portfolio

In [62]:
```
(Enhanced_index.mean()*12) / (Enhanced_index.std()*(12**0.5))
```
Out[62]:
```
1.0758263844890599
```

4. Annualized CAPM alpha and t-stats:

Minimal variance portfolio

In [63]:
```
model_minvar = OLS(Min_var,sm.add_constant(riskfactor.Mkt_RF[-499:])).fit()
print(model_minvar.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    GMV   R-squared:                       0.327
Model:                            OLS   Adj. R-squared:                  0.326
Method:                 Least Squares   F-statistic:                     241.7
Date:                Sat, 22 Jan 2022   Prob (F-statistic):           1.06e-44
Time:                        21:55:01   Log-Likelihood:                 1087.9
No. Observations:                 499   AIC:                            -2172.
Df Residuals:                     497   BIC:                            -2163.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0062      0.001      5.027      0.000       0.004       0.009
Mkt_RF         0.4132      0.027     15.547      0.000       0.361       0.465
==============================================================================
Omnibus:                       98.968   Durbin-Watson:                   1.943
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              722.210
Skew:                           0.635   Prob(JB):                    1.49e-157
Kurtosis:                       8.755   Cond. No.                         21.7
==============================================================================
```

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Minimal variance portfolio αp

In [64]:
```
model_minvar.params[0]*12
```

Out[64]:
```
0.07454201250238879
```

Minimal variance portfolio t(αp)

In [65]:
```
model_minvar.tvalues[0]
```

Out[65]:
```
5.026864104356599
```

Enhanced index portfolio

In [66]:
```
model_Enhanced = OLS(Enhanced_index,sm.add_constant(riskfactor.Mkt_RF[-499:])).fit()
print(model_Enhanced.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:         Enhanced_index   R-squared:                       0.112
Model:                            OLS   Adj. R-squared:                  0.110
Method:                 Least Squares   F-statistic:                     62.78
Date:                Sat, 22 Jan 2022   Prob (F-statistic):           1.53e-14
Time:                        21:55:01   Log-Likelihood:                 412.00
No. Observations:                 499   AIC:                            -820.0
Df Residuals:                     497   BIC:                            -811.6
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0304      0.005      6.341      0.000       0.021       0.040
Mkt_RF         0.8161      0.103      7.923      0.000       0.614       1.018
==============================================================================
Omnibus:                       46.686   Durbin-Watson:                   1.881
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              245.214
Skew:                          -0.112   Prob(JB):                     5.66e-54
Kurtosis:                       6.427   Cond. No.                         21.7
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Enhanced index portfolio: αp

In [67]:
```
model_Enhanced.params[0]*12
```

Out[67]:
```
0.3643800562456526
```

Enhanced index portfolio: t(αp)

In [68]:
```
model_Enhanced.tvalues[0]
```

Out[68]:
```
6.340938117781441
```

5. CAPM beta βp and annualized systematic volatility

Minimal variance portfolio

In [69]:
```
model_minvar.params[1]
```

Out[69]:
```
0.4132073287428889
```

In [70]:
```
model_minvar.params[1] * (Min_var.std()*(12**0.5))
```
Out[70]: 0.047768158949688985

Enhanced index portfolio βp and annualized systematic volatility

In [71]:
```
model_Enhanced.params[1]
```
Out[71]: 0.8161022856390496

In [72]:
```
model_Enhanced.params[1] * (Enhanced_index.std()*(12**0.5))
```
Out[72]: 0.31826437973341704

6. Annualized idiosyncratic volatility (or tracking errors):

Minimal variance portfolio

In [73]:
```
model_minvar.resid.std()*(12**0.5)
```
Out[73]: 0.09482333805923192

Enhanced index portfolio

In [74]:
```
model_Enhanced.resid.std()*(12**0.5)
```
Out[74]: 0.36746193220935164

7. R2 of the preceding time series regression, p=1,3

Minimal variance portfolio

In [75]:
```
model_minvar.rsquared
```
Out[75]: 0.32719454482498966

Enhanced index portfolio

In [76]:
```
model_Enhanced.rsquared
```
Out[76]: 0.11215362234422033

8. Information ratio (relative to the CAPM): αp σ(et) , p=1,3

Minimal variance portfolio

In [77]:
```
model_minvar.params[0] / model_minvar.resid.std()*(12**0.5)
```
Out[77]: 0.7861146214429374

Enhanced index portfolio

In [78]:
```
model_Enhanced.params[0] / model_Enhanced.resid.std()*(12**0.5)
```
Out[78]: 0.9916130741892925

9. Maximal Drawdown (not annualized) of the cumulative portfolio return, p=1,3

def MaxDrawdown

In [79]:

```
def MaxDrawdown(return_list):
    '''最大回撤率'''
    i = np.argmax((np.maximum.accumulate(return_list) - return_list) / np.maximum.accumulate(retur
    if i == 0:
        return 0
    j = np.argmax(return_list[:i])  #最高点
    a = (return_list[j] - return_list[i]) / (return_list[j])
    b = (return_list.index[i])
    c = (return_list.index[j])
    return (a,c,b)
def MaxDrawdownPlot(Fund):
    date1 = MaxDrawdown((Fund+1).cumprod())[1]
    date2 = MaxDrawdown((Fund+1).cumprod())[2]
    plt.plot((Fund+1).cumprod())
    x = [date1,date2]
    y = [(Fund+1).cumprod()[date1],(Fund+1).cumprod()[date2]]
    plt.scatter(x, y, color='r')
```

Cumulative wealth

In [80]:

```
CW_Min_var = (Min_var + riskfactor.RF[-499:]+1).cumprod()
CW_Enhanced_index = (Enhanced_index + riskfactor.RF[-499:]+1).cumprod()
```

Minimal variance portfolio

In [81]:

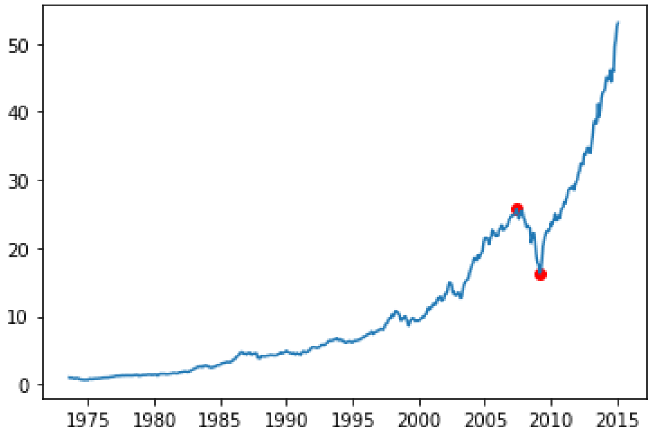
```
MaxDrawdown(CW_Min_var)[0]
```

Out[81]:

```
0.34718172346861464
```

In [97]:

```
MaxDrawdownPlot((Min_var+riskfactor.RF[-499:]))
```



Enhanced index portfolio

In [83]:

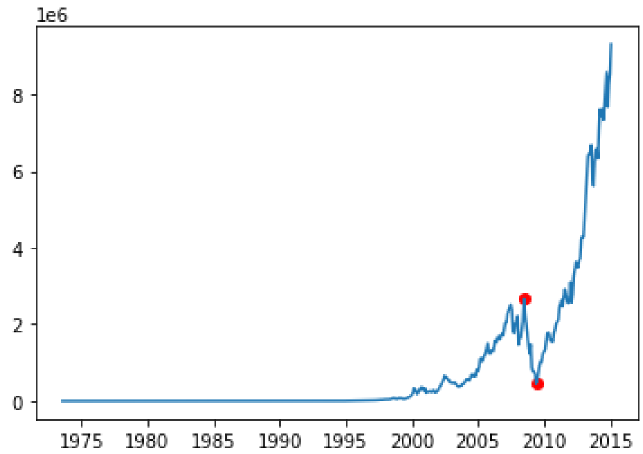
```
MaxDrawdown(CW_Enhanced_index)[0]
```

Out[83]:

```
0.8282986861801551
```

In [100…

```
MaxDrawdownPlot((Enhanced_index+riskfactor.RF[-499:]))
```



10. Maximal Recovery Period (not annualized) of the cumulative portfolio wealth, p=1,3

def MaximalRecoveryPeriod

In [85]:
```
def MaximalRecoveryPeriod(test2):
    test3=0
    test4=[]
    for i in range(len(test2)):
        a = (test2.iloc[i] - test3)
        if float(a) < 0:
            pass
        else:
            test3 = test2.iloc[i]
            test4.append(i)
    test5 = []
    for o in range(len(test4)-1):
        test5.append( test4[o+1] - test4[o] )
        b =  max(test5)
    return b
```

Minimal variance portfolio

In [86]:
```
print('Maximal Recovery Period for the {} is: '.format(i),MaximalRecoveryPeriod(CW_Min_var),'month
```

```
Maximal Recovery Period for the 500 is:  30 months
```

Enhanced index portfolio

In [87]:
```
print('Maximal Recovery Period for the {} is: '.format(i),MaximalRecoveryPeriod(CW_Enhanced_index)
```

```
Maximal Recovery Period for the 500 is:  36 months
```

`1.3Discussions`

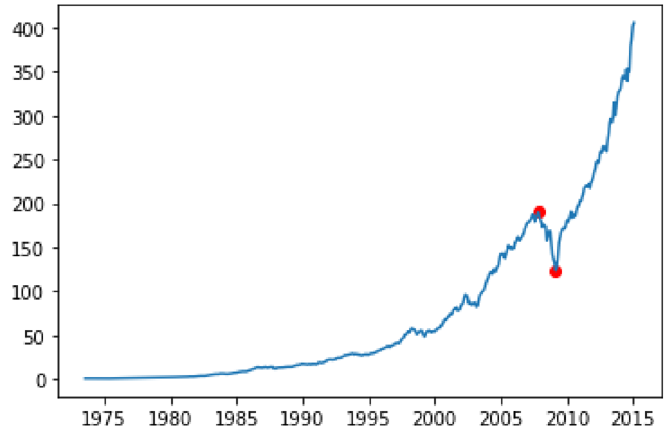
1.Summarize the way in which you construct the two trading strategies

A:For the minimum variance portfolio, it is the framework of the mean variance model, in which the calculation of stock selection and expected return uses the BARRA model. The long-short portfolio uses the BARRA model to calculate the required parameters, and applies these parameters to the quadratic programming with constraints to obtain the weight of the long-short portfolio. The procedure above is explained in more detail.

2.Draw the cumulative wealth series of the two strategies.

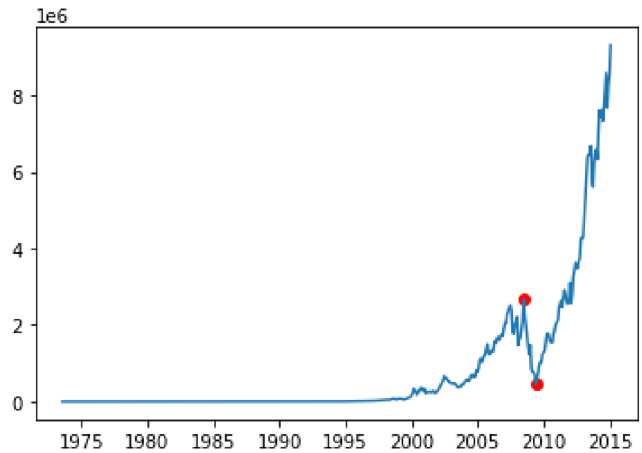
Minimal variance portfolio

In [88]:
```
MaxDrawdownPlot((Min_var+riskfactor.RF[-499:]))
```



Enhanced index portfolio

In [89]:
```
MaxDrawdownPlot((Enhanced_index+riskfactor.RF[-499:]))
```



3.Compute the first three performance measures of the market excess return

Annualized average excess return: MinVar `<` Enhanced_index;
Annualized standard deviation of excess return: MinVar `<` Enhanced_index;
Annualized Sharpe ratio:MinVar `<` Enhanced_index

4.For the rest of performance measures, you compare their values

Annualized CAPM alpha and t-stats:MinVar `<` Enhanced_index
CAPM beta βp and annualized systematic volatility:MinVar `<` Enhanced_index
Annualized idiosyncratic volatility:MinVar `<` Enhanced_index

R2:MinVar `>` Enhanced_index
Information ratio:MinVar `<` Enhanced_index
Maximal Drawdown:MinVar `<<` Enhanced_index
Maximal Recovery Period:MinVar `<` Enhanced_index

5.Describe the performance of the two strategies

Average Returns:MinVar `<` Enhanced_index
Alpha: MinVar `<` Enhanced_index
Volatility: MinVar `<` Enhanced_index
Market beta: MinVar `<` Enhanced_index
Maximal drawdown: MinVar `<` Enhanced_index
Sharpe ratio: MinVar `<` Enhanced_index
Information ratio:MinVar `<` Enhanced_index

6.Suppose you plan to invest your total wealth in one of the two strategies

A:Choose the one with a high Sharpe ratio, so choose enhanced index portfolio, because the Sharpe ratio is a measure of total risk

7.Finally suggest at least one way to improve the two strategies

A:Both strategies have no stop loss, and a stop loss condition should be added to the strategy.

`END`

`Quantitative Investment Analysis(Fall 2021)`

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