On The Energy-Efficiency Fairness of Reconfigurable Intelligent Surface-Aided Cell-Free Network

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***Abstract*—With the ability to overcome the inter-cell interferences, cell-free network is a promising network paradigm to achieve high spectrum efficiency for future 6G communications. However, the increasing number of distributed base stations in cell-free network introduces very high power consumption. To address this issue, inspired by the reconfigurable intelligent surface (RIS) technique with low power consumption, the concept of RIS-aided cell-free network has been recently proposed to improve the spectrum efficiency at a low cost of power consumption. In this paper, we take a further step to investigate the energy-efficiency fairness (EEF) of RIS-aided cell-free network. Specifically, we formulate the precoding design problem to maximize the energy efficiency of the worst user in a wideband RIS-aided cell-free network. To solve this problem, we then propose an iterative precoding algorithm by using Lagrangian transform and fractional programming (FP) to tackle the highly decoupled optimization objective. Through alternatingly solving subproblems of subcarrier assignment, power allocation, combining, and precoding, the objective will finally converge. Simulation results demonstrate the effectiveness of the proposed precoding algorithm, and the energy efficiency of users in cell-free network can be efficiently increased by the aid of RISs.**

***Index Terms*—energy efficiency, cell-free, reconfigurable intelligent surface (RIS), precoding**

I. Introduction

With the increase of network density in communication systems, inter-cell interferences are becoming the bottleneck to boost the network capacity [1]. To address this challenge, a new user-centric network paradigm called *cell-free network* has been recently proposed as a promising solution [2]. In the cell-free network, a large number of distributed base stations (BSs) connected to central processing units (CPUs) coherently serve all users, which makes the concept of cell boundary no longer exist. Benefiting from the cooperation among BSs, the serious inter-cell interferences can be almost removed, and thus the cell-free network breaks the theoretical capacity limit of the classical cellular network [3].

However, the deployment of massive distributed BSs in a cell-free network leads to very high power consumption. Fortunately, a novel technology named reconfigurable intelligent surface (RIS) developed from meta-materials is emerging as a cost-efficient and energy-efficient solution to improve the spectrum efficiency in communication systems [4]. With a large array with numerous low-cost and low-power elements, RIS is able to passively reflect the electromagnetic incident signals to any directions with high array gains by properly adjusting the phase shifts of its elements. Inspired by RIS, the concept of RIS-aided cell-free network was firstly proposed in [5], where the key performance metric of spectrum efficiency was extensively studied [5]–[7]. Apart from the spectrum efficiency, energy efficiency is another important performance metric for the wireless system design [8], which is still a blank for RIS-aided cell-free network.

To fill this gap, in this paper we investigate the energy-efficiency fairness (EEF) of RIS-aided cell-free network. Specifically, we formulate the EEF problem to maximize the energy efficiency of the worst user in a wideband RIS-aided cell-free network. To solve this problem, we further propose a precoding algorithm by using Lagrangian transform and fractional programming (FP) methods to tackle the highly decoupled objective. By alternatingly solving subproblems of subcarrier assignment, power allocation, combining, and precoding, the objective will finally converge. At last, simulation results demonstrate the effectiveness of the proposed precoding algorithm, and the energy efficiency of cell-free network can be efficiently increased by the aid of RISs.

The remainder of this paper is organized as follows. In Section II, the system model and the EEF problem formulation are introduced at first. Then, the proposed algorithm for solving the problem is described in Section III. Next, numerical simulation results are presented in Section IV to verify the effectiveness of the proposed precoding algorithm. Finally, conclusions are drawn in Section V.

Notations: $\mathbb{C}, \mathbb{R}$ and $\mathbb{R}_+$ denote the set of complex, real and positive real numbers, respectively; $[L]$ denotes the set of integers $\{1, 2, \cdots, L\}$; $\boldsymbol{A}^{-1}, \boldsymbol{A}^{*}, \boldsymbol{A}^{\top}$ and $\boldsymbol{A}^{\mathrm{H}}$ denote the inverse, conjugate, transpose and conjugate transpose of matrix $\boldsymbol{A}$, respectively; $\|\boldsymbol{x}\|$ denotes the ℓ_2-norm of vector $\boldsymbol{x}$; diag$(\cdot)$ denotes the diagonal operation; Re$\{\cdot\}$ denotes the real part of its argument; $\angle[\cdot]$ denotes the angle of its complex argument; $\boldsymbol{I}$ denotes the identity matrix; $\boldsymbol{e}_l$ denotes the vector with a one at the l-th position; $[\cdot]^+$ denotes the operation $\max\{0, \cdot\}$.

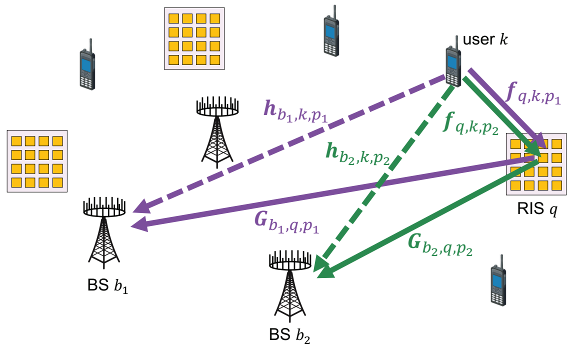

Fig. 1. An illustration of RIS-aided cell-free network where OFDMA is considered to realize uplink transmission.

II. System Model and Problem Formulation

In this section, we introduce the system model of wideband RIS-aided cell-free network first, and then we formulate the EEF optimization problem.

A. System architecture

We consider a wideband RIS-aided cell-free network [5] shown in Fig. 1, where multiple BSs and multiple RISs are deployed to cooperatively serve multiple users. Specifically, the considered network consists of B BSs, Q RISs, and K single-antenna users. The number of antennas at the b-th BS is M_b, and the number of elements at the q-th RIS is N_q. For simplicity but without loss of generality, we assume M_b and N_q are equal to M and N. In addition, orthogonal frequency division multiple access (OFDMA) is considered as the standard way to realize uplink transmission, where there are P subcarriers, and each subcarrier can be assigned to at most one user.

B. Channel model

Let $\boldsymbol{h}_{b,k,p} \in \mathbb{C}^{M\times 1}$, $\boldsymbol{f}_{q,k,p} \in \mathbb{C}^{N\times 1}$, and $\boldsymbol{G}_{b,q,p} \in \mathbb{C}^{N\times M}$ denote the frequency-domain channel on subcarrier p from user k to BS b, from user k to RIS q, and from RIS q to BS b, for all $k \in [K], b \in [B], p \in [P]$ and $q \in [Q]$, respectively. For simplicity, we assume that all channels experience quasi-static block fading. In addition, all channel state information (CSI) is assumed to be known at the BSs, which can be obtained by channel estimation methods [9]. Then, we denote

$$\boldsymbol{\Theta}_q := \mathrm{diag}(\boldsymbol{\theta}_q) = \mathrm{diag}\left(\left[\theta_{q,1}, ..., \theta_{q,N}\right]^{\top}\right) \tag{1}$$

for all $q \in [Q]$ as the phase shift matrix at RIS q. Besides, two assumptions of the feasible set of reflection coefficient (RC) $\theta_{q,n}$ is considered:

- *Continuous Phase Shift*: Only the phase of $\theta_{q,n}$ can be controlled continuously, and the amplitude of $\theta_{q,n}$ is fixed as 1, which corresponds to the set
$$\mathcal{F}_1 = \left\{\theta \middle| \theta = e^{j\varphi}, \varphi \in [-\pi, \pi]\right\}. \tag{2}$$
- *Discrete Phase Shift*: Compared to the continuous phase shift case, only D equally spaced discrete values can be chosen for the phase of $\theta_{q,n}$. It's easier for hardware implementation of metamaterials. The feasible set can be written as
$$\mathcal{F}_2 = \left\{\theta \middle| \theta = e^{j\frac{2\pi d}{D}}, d \in [D]\right\}. \tag{3}$$

C. Transmission and consumption at user k

Denote the transmitted data symbol by user k on subcarrier p as $\hat{s}_{k,p}$ for all k and p, which are assumed to be independent identically distributed with zero mean and unit variance. Let $\boldsymbol{y}_{b,k,p} \in \mathbb{C}^{M\times 1}$ denote the signals from the user k arriving at BS b on subcarrier p, which can be written as

$$\begin{aligned}\boldsymbol{y}_{b,k,p} &= \left(\boldsymbol{h}_{b,k,p} + \sum_{q=1}^{Q} \boldsymbol{G}_{b,q,p}^{\mathrm{H}} \boldsymbol{\Theta}_q \boldsymbol{f}_{q,k,p}\right)\sqrt{W_{k,p}}\hat{s}_{k,p} + \boldsymbol{z}_{b,p} \\ &:= \boldsymbol{H}_{b,k,p}\sqrt{W_{k,p}}\hat{s}_{k,p} + \boldsymbol{z}_{b,p}\end{aligned} \tag{4}$$

where $W_{k,p}$ is the transmit power of user k on subcarrier p, $\boldsymbol{z}_{b,p} \in \mathbb{C}^{M\times 1}$ denotes the additive white Gaussian noise (AWGN) introduced at BS b on subcarrier p, and

$$\boldsymbol{H}_{b,k,p} = \boldsymbol{h}_{b,k,p} + \sum_{q=1}^{Q} \boldsymbol{G}_{b,q,p}^{\mathrm{H}} \boldsymbol{\Theta}_q \boldsymbol{f}_{q,k,p} \tag{5}$$

denotes the equivalent channel from user k on subcarrier p to BS b, containing one direct link and Q reflective links. Then, after combining at BS, which can be represented by a combining vector $\boldsymbol{w}_{b,p} \in \mathbb{C}^{M\times 1}$, the decoding SNR of $\hat{s}_{k,p}$ is

$$\mathrm{SNR}_{k,p} = \frac{W_{k,p}\left|\sum_{b=1}^{B} \boldsymbol{w}_{b,p}^{\mathrm{H}} \boldsymbol{H}_{b,k,p}\right|^2}{\sum_{b=1}^{B}\left\|\boldsymbol{w}_{b,p}^{\mathrm{H}}\right\|^2 \sigma_0^2}. \tag{6}$$

Define

$$\boldsymbol{w} = \begin{bmatrix} \boldsymbol{w}_{1,1} & \cdots & \boldsymbol{w}_{1,P} \\ \vdots & \ddots & \vdots \\ \boldsymbol{w}_{B,1} & \cdots & \boldsymbol{w}_{B,P} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\theta}_1 \\ \vdots \\ \boldsymbol{\theta}_Q \end{bmatrix}, \tag{7}$$

and then the sum-rate R_k can be given by

$$R_k(\boldsymbol{\rho}, \boldsymbol{P}, \boldsymbol{w}, \boldsymbol{\theta}) = \sum_{p=1}^{P} \rho_{k,p} \log_2\left(1 + \mathrm{SNR}_{k,p}\right), \tag{8}$$

where $\rho_{k,p}$ is an indicator of whether the subcarrier p is allocated to user k.

To implement EEF design, we define the transmit power W_k and the overall power consumption $\widetilde{W}_k$ of user k as

$$W_k(\boldsymbol{\rho}, \boldsymbol{P}) = \sum_{p=1}^{P} \rho_{k,p} W_{k,p}, \quad \widetilde{W}_k(\boldsymbol{\rho}, \boldsymbol{P}) = A_k W_k(\boldsymbol{\rho}, \boldsymbol{P}) + \overline{W}_k, \tag{9}$$

where $A_k, \overline{W}_k$ are the factor characterizing energy conversion efficiency and the static power consumption of user k. Thus, the energy efficiency of user k can be derived as

$$\eta_k(\boldsymbol{\rho}, \boldsymbol{P}, \boldsymbol{w}, \boldsymbol{\theta}) = \frac{R_k(\boldsymbol{\rho}, \boldsymbol{P}, \boldsymbol{w}, \boldsymbol{\theta})}{\widetilde{W}_k(\boldsymbol{\rho}, \boldsymbol{P})}, \tag{10}$$

which is our optimization objective.

D. Problem formulation

Based on the system model above, the EEF problem can be formulated as

$$\begin{aligned}\max_{\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta}}\ \min_{k}\quad & \eta_k(\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta}) \\ \text{s.t.}\quad & \text{C1:}\ \ R_k(\boldsymbol{\rho},\boldsymbol{P},\boldsymbol{w},\boldsymbol{\theta}) \geq R_k^{\text{req}}, \forall k \\ & \text{C2:}\ \ W_k(\boldsymbol{\rho},\boldsymbol{P}) \leq W_k^{\max}, \forall k \\ & \text{C3:}\ \ \theta_{q,n} \in \mathcal{F}_1 \text{ or } \mathcal{F}_2, \forall q,n \\ & \text{C4:}\ \ \sum_{k=1}^{K} \rho_{k,p} \leq 1, \forall p \\ & \text{C5:}\ \ \rho_{k,p} \in \{0,1\}, \forall k,p\end{aligned} \tag{11}$$

Our design goal is to maximize the worst user's energy efficiency, and the subjectives are: C1 guarantees the rate requirements of users where R_k^{req} is the required transmit rate. C2 is a transmit power constraint where $W_k^{\max}$ is the maximum transmit power. C3 is the constraint of RIS phase shift. C4 and C5 are the OFDMA requirements that each subcarrier can only be assigned to at most one user.

Since the optimization (11) is a mixed-integer nonconvex problem with fractional objective, obtaining the optimal utilizations is very challenging. Fortunately, inspired by the FP methods introduced in [10], we propose a joint precoding algorithm to find a sub-optimal solution, which will be presented in the following Section III.

III. Energy-Efficiency Fairness Design

To address the non-convexity of the problem (11), in this section, we use FP methods with Lagrangian dual decomposition and quadratic transform to decouple the problem.

A. Preprocessing

According to Dinkelbach's algorithm, the optimal energy efficiency $\eta^\star$ of problem (11) satisfies

$$\begin{aligned}&\max_{\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta}}\ \min_{k}\left[R_k(\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta}) - \eta^\star \widetilde{W}_k(\boldsymbol{\rho},\boldsymbol{W})\right] \\ &\quad= \min_{k}\left[R_k(\boldsymbol{\rho}^\star,\boldsymbol{W}^\star,\boldsymbol{w}^\star,\boldsymbol{\theta}^\star) - \eta^\star \widetilde{W}_k(\boldsymbol{\rho}^\star,\boldsymbol{W}^\star)\right] = 0.\end{aligned} \tag{12}$$

This implies that the optimal $\eta^\star$ can be achieved by updating η via alternatingly optimizing

$$\begin{aligned}\max_{\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta}}\ \min_{k}\ & \left[R_k(\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta}) - \eta \widetilde{W}_k(\boldsymbol{\rho},\boldsymbol{W})\right] \\ \text{s.t.}\ & \text{C1, C2, C3, C4, C5.}\end{aligned} \tag{13}$$

To solve the mixed-integer problem (13), we first relax $\rho_{k,p}$ from $\{0,1\}$ to $[0,1]$. By introducing a new variable φ to deal with the problem's nonsmoothness, the jointly convex problem can be reformulated as

$$\begin{aligned}\max_{\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta},\varphi}\quad & \varphi \\ \text{s.t.}\quad & \text{C1, C2, C3, C4} \\ & \text{C5:}\ \ 0 \leq \rho_{k,p} \leq 1, \forall k,p \\ & \text{C6:}\ \ R_k - \eta \widetilde{W}_k \geq \varphi, \forall k\end{aligned} \tag{14}$$

Algorithm 1 Resource allocation optimization of (13)

Input: Channels $\boldsymbol{h}_{b,k,p}, \boldsymbol{f}_{q,k,p}, \boldsymbol{G}_{b,q,p}$ and parameters.
Output: Optimized energy efficiency η.

```
Initialize θ, w_p and W;
while no convergence of η do
    Update W by (20);
    Update ρ by (23);
    Update μ by (28);
    Update ν by (31);
    Update w by (32);
    Update θ by solving (36);
    Update φ by (44);
end while
return η = R_k(ρ,P,w,θ) / W̃_k(ρ,P).
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Define $\boldsymbol{H}_{k,p} = \left[\boldsymbol{H}_{1,k,p}^\top, ..., \boldsymbol{H}_{B,k,p}^\top\right]^\top \in \mathbb{C}^{MB\times 1}$, and then the user SNR can be rewritten as

$$\text{SNR}_{k,p} = \frac{W_{k,p}\left|\boldsymbol{w}_p^{\text{H}}\boldsymbol{H}_{k,p}\right|^2}{\left\|\boldsymbol{w}_p^{\text{H}}\right\|\sigma_0^2}. \tag{15}$$

Thus, the Lagrangian of (14) can be derived as

$$\begin{aligned}&\mathcal{L}(\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta},\varphi,\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\kappa}) \\ &= \varphi + \sum_{k=1}^{K}\alpha_k\left[\sum_{p=1}^{P}\rho_{k,p}\log_2\left(1+\text{SNR}_{k,p}\right) - R_k^{\text{req}}\right] \\ &\quad + \sum_{k=1}^{K}\beta_k\left(W_k^{\max} - \sum_{p=1}^{P}\rho_{k,p}W_{k,p}\right) \\ &\quad + \sum_{k=1}^{K}\gamma_k\Big\{\sum_{p=1}^{P}\left[\rho_{k,p}\log_2\left(1+\text{SNR}_{k,p}\right) - \eta A_k\rho_{k,p}W_{k,p}\right] \\ &\qquad - \eta\overline{W}_k - \varphi\Big\} + \sum_{p=1}^{P}\kappa_p\left(1 - \sum_{k=1}^{K}\rho_{k,p}\right)\end{aligned} \tag{16}$$

where $\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma} \in \mathbb{R}_+^K$ and $\boldsymbol{\kappa} \in \mathbb{R}_+^P$ are the Lagrange multipliers corresponding to C1, C2, C6 and C4 in (14). Problem (14) can be then transformed to

$$\max_{\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta},\varphi}\quad \mathcal{L}(\boldsymbol{\rho},\boldsymbol{W},\boldsymbol{w},\boldsymbol{\theta},\varphi) \tag{17}$$

where the Lagrange multipliers can be updated by traditional subgradient projection methods.

In the next subsection, we discuss the optimization for subcarrier assignment $\boldsymbol{\rho}$, power allocation $\boldsymbol{W}$, combining vectors $\boldsymbol{w}$, RIS precoding vectors $\boldsymbol{\theta}$, and the auxiliary variable φ in (17), respectively.

B. Resource allocation optimization

The proposed resource allocation algorithm is summarized in **Algorithm 1**, where the sub-optimal solution is achieved by alternately optimizing seven variables. The optimal solutions of these variables are introduced respectively as follows.

1) Subcarrier assignment and power allocation optimization: Denote $s_{k,p} = \rho_{k,p} W_{k,p}$, and substitute all $W_{k,p}$ with $\frac{s_{k,p}}{\rho_{k,p}}$. The subproblem corresponding to subcarrier assignment $\boldsymbol{\rho}$ and power allocation $\boldsymbol{W}$ can be reformulated as

$$\begin{aligned} \max_{\boldsymbol{\rho},\boldsymbol{s}} \quad & \mathcal{L}_1(\boldsymbol{\rho},\boldsymbol{s}) \\ \text{s.t.} \quad & 0 \le \rho_{k,p} \le 1, \forall k,p \end{aligned} \tag{18}$$

where

$$\begin{aligned} &\mathcal{L}_1(\boldsymbol{\rho},\boldsymbol{s}) \\ &= \sum_{k=1}^{K} (\alpha_k+\gamma_k) \left[\sum_{p=1}^{P} \rho_{k,p} \log_2 \left(1 + \frac{s_{k,p} \left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2}{\rho_{k,p} \left\| \boldsymbol{w}_p^{\mathrm{H}} \right\|^2 \sigma_0^2} \right) \right] \\ &\quad - \sum_{k=1}^{K} (\beta_k + \gamma_k \eta A_k) \left(\sum_{p=1}^{P} s_{k,p} \right) - \sum_{p=1}^{P} \kappa_p \left(\sum_{k=1}^{K} \rho_{k,p} \right). \end{aligned} \tag{19}$$

By using the Karush-Kuhn-Tucker (KKT) condition w.r.t $s_{k,p}$, we obtain

$$W_{k,p}^{\star} = \left[\frac{\alpha_k+\gamma_k}{(\beta_k+\gamma_k \eta A_k)\ln 2} - \frac{\left\| \boldsymbol{w}_p^{\mathrm{H}} \right\|^2 \sigma_0^2}{\left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2} \right]^+. \tag{20}$$

For the optimization of subcarrier assignment $\rho_{k,p}$ in (18), using KKT condition w.r.t $\rho_{k,p}$, we have

$$\rho_{k,p}^{\star} = \begin{cases} 1, & \kappa_p < L_{k,p} \\ 0, & \kappa_p > L_{k,p} \end{cases} \tag{21}$$

where

$$\begin{aligned} &L_{k,p} \\ &= (\alpha_k+\gamma_k) \Bigg\{ \left[\log_2 \left(\frac{\alpha_k+\gamma_k}{(\beta_k+\gamma_k\eta A_k)\ln 2} \cdot \frac{\left\| \boldsymbol{w}_p^{\mathrm{H}} \right\|^2 \sigma_0^2}{\left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2} \right) \right]^+ \\ &\quad - \frac{1}{\ln 2} \left[1 - \frac{1}{\frac{\alpha_k+\gamma_k}{(\beta_k+\gamma_k\eta A_k)\ln 2} \cdot \frac{\left\| \boldsymbol{w}_p^{\mathrm{H}} \right\|^2 \sigma_0^2}{\left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2}} \right]^+ \Bigg\}. \end{aligned} \tag{22}$$

To satisfy the constraint C4 in (14), for a specific subcarrier p, there should be at most one k among all $k \in [K]$ satisfying $\rho_{k,p} = 1$. Thus, the optimal subcarrier assignment is given by

$$\rho_{k,p}^{\star} = \begin{cases} 1, & k = \arg\max_k L_{k,p} \\ 0, & \text{otherwise} \end{cases}. \tag{23}$$

2) Combining vector optimization and RIS precoding: The subproblem corresponding to the optimization of combining vectors and RIS precoding can be written as

$$\begin{aligned} \max_{\boldsymbol{w},\boldsymbol{\theta}} \quad & \mathcal{L}_2(\boldsymbol{w},\boldsymbol{\theta}) \\ \text{s.t.} \quad & \theta_{q,n} \in \mathcal{F}_1 \text{ or } \mathcal{F}_2, \forall q,n \end{aligned} \tag{24}$$

where

$$\begin{aligned} &\mathcal{L}_2(\boldsymbol{w},\boldsymbol{\theta}) \\ &= \sum_{k=1}^{K} (\alpha_k+\gamma_k) \left[\sum_{p=1}^{P} \rho_{k,p} \log_2 \left(1 + \frac{W_{k,p} \left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2}{\left\| \boldsymbol{w}_p^{\mathrm{H}} \right\|^2 \sigma_0^2} \right) \right]. \end{aligned} \tag{25}$$

Since the function $\mathcal{L}_2$ is not convex, here we use Lagrangian dual transform [10] to reformulate the problem. Specifically, we introduce an auxiliary variable $\boldsymbol{\mu} \in \mathbb{C}^{K \times P}$ to assist the reformulation, i.e.,

$$\begin{aligned} \max_{\boldsymbol{w},\boldsymbol{\theta},\boldsymbol{\mu}} \quad & \mathcal{L}_3(\boldsymbol{w},\boldsymbol{\theta},\boldsymbol{\mu}) \\ \text{s.t.} \quad & \theta_{q,n} \in \mathcal{F}_1 \text{ or } \mathcal{F}_2, \forall q,n \end{aligned} \tag{26}$$

where

$$\begin{aligned} \mathcal{L}_3(\boldsymbol{w},\boldsymbol{\theta},\boldsymbol{\mu}) &= \sum_{k=1}^{K} (\alpha_k+\gamma_k) \Bigg\{ \sum_{p=1}^{P} \rho_{k,p} \\ &\left[\ln(1+\mu_{k,p}) - \mu_{k,p} + \frac{(1+\mu_{k,p}) W_{k,p} \left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2}{W_{k,p} \left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2 + \left\| \boldsymbol{w}_p^{\mathrm{H}} \right\|^2 \sigma_0^2} \right] \Bigg\}. \end{aligned} \tag{27}$$

By setting $\partial \mathcal{L}_3 / \partial \mu_{k,p}$ to zero, the optimal $\mu_{k,p}$ can be obtained by

$$\mu_{k,p}^{\star} = \frac{W_{k,p} \left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2}{\left\| \boldsymbol{w}_p^{\mathrm{H}} \right\|^2 \sigma_0^2}. \tag{28}$$

After fixing $\boldsymbol{\mu}$, we further apply quadratic transform [10] to address the non-convexity, which transfers the optimization problem into

$$\begin{aligned} \max_{\boldsymbol{w},\boldsymbol{\theta},\boldsymbol{\nu}} \quad & \mathcal{L}_4(\boldsymbol{w},\boldsymbol{\theta},\boldsymbol{\nu}) \\ \text{s.t.} \quad & \theta_{q,n} \in \mathcal{F}_1 \text{ or } \mathcal{F}_2, \forall q,n \end{aligned} \tag{29}$$

where

$$\begin{aligned} \mathcal{L}_4(\boldsymbol{w},\boldsymbol{\theta},\boldsymbol{\nu}) &= \sum_{k=1}^{K} (\alpha_k+\gamma_k) \Bigg\{ \sum_{p=1}^{P} \rho_{k,p} \\ &\Bigg[\ln(1+\mu_{k,p}) - \mu_{k,p} + 2\sqrt{(1+\mu_{k,p})\, W_{k,p}} \mathrm{Re}\left\{ \nu_{k,p}^{*} \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right\} \\ &- |\nu_{k,p}|^2 \left(W_{k,p} \left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2 + \left\| \boldsymbol{w}_p^{\mathrm{H}} \right\|^2 \sigma_0^2 \right) \Bigg] \Bigg\} \end{aligned} \tag{30}$$

where $\boldsymbol{\nu} \in \mathbb{C}^{K \times P}$ is another auxiliary variable. By setting $\partial \mathcal{L}_4 / \partial \nu_{k,p}$ to zero, the optimal $\nu_{k,p}$ can be obtained by

$$\nu_{k,p}^{\star} = \frac{\sqrt{(1+\mu_{k,p})\, W_{k,p}} \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p}}{W_{k,p} \left| \boldsymbol{w}_p^{\mathrm{H}} \boldsymbol{H}_{k,p} \right|^2 + \left\| \boldsymbol{w}_p^{\mathrm{H}} \right\|^2 \sigma_0^2}. \tag{31}$$

Note that, $\mathcal{L}_5(\boldsymbol{w},\boldsymbol{\theta}) := \mathcal{L}_4(\boldsymbol{w},\boldsymbol{\theta},\boldsymbol{\nu}^{\star})$ is convex w.r.t $\boldsymbol{w}$. Thus, we can obtain $\boldsymbol{w}^{\star}$ by setting $\partial \mathcal{L}_5 / \partial \boldsymbol{w}_p$ to zero, which yields

$$\begin{aligned} \boldsymbol{w}_p &= \left[\sum_{k=1}^{K} (\alpha_k+\gamma_k) \rho_{k,p} |\nu_{k,p}|^2 \left(W_{k,p} \boldsymbol{H}_{k,p} \boldsymbol{H}_{k,p}^{\mathrm{H}} + \sigma_0^2 \boldsymbol{I} \right) \right]^{-1} \\ &\quad \sum_{k=1}^{K} (\alpha_k+\gamma_k) \rho_{k,p} \sqrt{(1+\mu_{k,p})\, W_{k,p}} \nu_{k,p}^{*} \boldsymbol{H}_{k,p}. \end{aligned} \tag{32}$$

For RIS precoding optimization, define

$$a_{k,p} = \sum_{b=1}^{B} \boldsymbol{w}_{b,p}^{\mathrm{H}} \boldsymbol{h}_{b,k,p}, \quad \boldsymbol{b}_{k,p}^{\mathrm{H}} = \sum_{b=1}^{B} \boldsymbol{w}_{b,p}^{\mathrm{H}} \boldsymbol{G}_{b,p}^{\mathrm{H}} \mathrm{diag}(\boldsymbol{f}_{k,p}), \tag{33}$$

where

$$\boldsymbol{G}_{b,p}=\left[\boldsymbol{G}_{b,1,p}^{\top},...,\boldsymbol{G}_{b,Q,p}^{\top}\right]^{\top},\boldsymbol{f}_{k,p}=\left[\boldsymbol{f}_{1,k,p}^{\top},...,\boldsymbol{f}_{Q,k,p}^{\top}\right]^{\top}. \tag{34}$$

Then $\boldsymbol{w}_p^{\mathrm{H}}\boldsymbol{H}_{k,p}$ can be rewritten as

$$\begin{aligned}\boldsymbol{w}_p^{\mathrm{H}}\boldsymbol{H}_{k,p}&=\sum_{b=1}^{B}\boldsymbol{w}_{b,p}^{\mathrm{H}}\boldsymbol{h}_{b,k,p}+\sum_{b=1}^{B}\boldsymbol{w}_{b,p}^{\mathrm{H}}\boldsymbol{G}_{b,p}^{\mathrm{H}}\mathrm{diag}(\boldsymbol{f}_{k,p})\boldsymbol{\theta}\\&=a_{k,p}+\boldsymbol{b}_{k,p}^{\mathrm{H}}\boldsymbol{\theta}.\end{aligned} \tag{35}$$

Using (35), the subproblem corresponding to RIS precoding can be written as

$$\begin{aligned}\max_{\boldsymbol{\theta}}\quad&\mathcal{L}_6(\boldsymbol{\theta})\\ \text{s.t.}\quad&\theta_{q,n}\in\mathcal{F}_1\text{ or }\mathcal{F}_2,\forall q,n\end{aligned} \tag{36}$$

where

$$\begin{aligned}&\mathcal{L}_6(\boldsymbol{\theta})=\sum_{k=1}^{K}(\alpha_k+\gamma_k)\\&\left\{\sum_{p=1}^{P}\rho_{k,p}\left[2\sqrt{(1+\mu_{k,p})\,W_{k,p}}\mathrm{Re}\left\{\nu_{k,p}^{*}\left(a_{k,p}+\boldsymbol{b}_{k,p}^{\mathrm{H}}\boldsymbol{\theta}\right)\right\}\right.\right.\\&\left.\left.-\left|\nu_{k,p}\right|^2\left(W_{k,p}\left|a_{k,p}+\boldsymbol{b}_{k,p}^{\mathrm{H}}\boldsymbol{\theta}\right|^2+\left\|\boldsymbol{w}_p^{\mathrm{H}}\right\|^2\sigma_0^2\right)\right]\right\}.\end{aligned} \tag{37}$$

Moreover, $\left|a_{k,p}+\boldsymbol{b}_{k,p}^{\mathrm{H}}\boldsymbol{\theta}\right|^2$ can be further expanded as

$$\left|a_{k,p}+\boldsymbol{b}_{k,p}^{\mathrm{H}}\boldsymbol{\theta}\right|^2=\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{b}_{k,p}\boldsymbol{b}_{k,p}^{\mathrm{H}}\boldsymbol{\theta}+2\mathrm{Re}\{a_{k,p}\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{b}_{k,p}\}+|a_{k,p}|^2, \tag{38}$$

so (37) can be simplified as

$$\mathcal{L}_6(\boldsymbol{\theta},\boldsymbol{\lambda})=-\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{U}\boldsymbol{\theta}+2\mathrm{Re}\{\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{v}\}+C \tag{39}$$

where

$$\boldsymbol{U}=\sum_{k=1}^{K}(\alpha_k+\gamma_k)\left[\sum_{p=1}^{P}\rho_{k,p}|\nu_{k,p}|^2W_{k,p}\boldsymbol{b}_{k,p}\boldsymbol{b}_{k,p}^{\mathrm{H}}\right], \tag{40a}$$

$$\begin{aligned}\boldsymbol{v}&=\sum_{k=1}^{K}(\alpha_k+\gamma_k)\left[\sum_{p=1}^{P}\rho_{k,p}\right.\\&\left.\left(\nu_{k,p}\sqrt{(1+\mu_{k,p})\,W_{k,p}}-|\nu_{k,p}|^2W_{k,p}a_{k,p}\right)\boldsymbol{b}_{k,p}\right],\end{aligned} \tag{40b}$$

$$\begin{aligned}C&=\sum_{k=1}^{K}(\alpha_k+\gamma_k)\left\{\sum_{p=1}^{P}\rho_{k,p}\left[2\sqrt{(1+\mu_{k,p})\,W_{k,p}}\right.\right.\\&\left.\left.\mathrm{Re}\{\nu_{k,p}a_{k,p}^{*}\}-|\nu_{k,p}|^2\left(W_{k,p}|a_{k,p}|^2+\left\|\boldsymbol{w}_p^{\mathrm{H}}\right\|^2\sigma_0^2\right)\right]\right\}.\end{aligned} \tag{40c}$$

For the feasible set $\mathcal{F}_1$ which represents the continuous phase shift RIS, i.e. $|\theta_{q,n}|=1$, relax the constraint as

$$\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{e}_j\boldsymbol{e}_j^{\mathrm{H}}\boldsymbol{\theta}\leq 1,\quad\forall j\in[QN]. \tag{41}$$

Then, the problem (36) has convex constraints and convex objective function since $\boldsymbol{U}$ is a positive semidefinite matrix. Thereby, $\boldsymbol{\theta}^{\star}$ can be obtained by standard optimization tools.

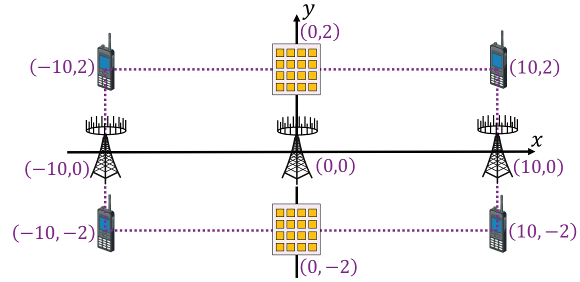

Fig. 2. Scenario where 3 BSs assisted by 2 RISs serve 4 users.

For the feasible set $\mathcal{F}_2$ which represents the discrete phase shift RIS, the simplified subproblem (36) becomes non-convex. Based on the optimal solution $\theta_{q,n}^{\star}$ in case $\mathcal{F}_1$, we apply projection method to obtain a suboptimal solution $\theta_{q,n}^{\mathrm{sub}}$ with

$$\angle\theta_{q,n}^{\mathrm{sub}}=\arg\min_{\phi\in\mathcal{F}_2}\left|\angle\theta_{q,n}^{\star}-\angle\phi\right|,\quad\forall q,n. \tag{42}$$

3) Adaptive φ selection: Finally, we need to update φ which is involved in the following subproblem.

$$\begin{aligned}\max_{\varphi}\quad&\left(1-\sum_{k=1}^{K}\gamma_k\right)\varphi\\ \text{s.t.}\quad&0\leq\varphi\leq\sum_{p=1}^{P}\rho_{k,p}\log_2\left(1+\mathrm{SNR}_{k,p}\right)\\&\qquad-\eta\left(A_k\sum_{p=1}^{P}\rho_{k,p}W_{k,p}+\overline{W}_k\right),\forall k\end{aligned} \tag{43}$$

It's obvious to conclude that $\varphi^{\star}=0$ when $\sum_{k=1}^{K}\gamma_k>1$, and otherwise

$$\begin{aligned}\varphi^{\star}=\min_{k}&\left\{\sum_{p=1}^{P}\rho_{k,p}\log_2\left(1+\mathrm{SNR}_{k,p}\right)\right.\\&\left.-\eta\left(A_k\sum_{p=1}^{P}\rho_{k,p}W_{k,p}+\overline{W}_k\right)\right\}.\end{aligned} \tag{44}$$

IV. Simulation Results

In this section, we provide extensive simulation results to further verify the superior performance of the proposed EEF design.

A. Simulation setup

We consider the simulation setups as shown in Fig. 2, where 3 BSs serve 4 single-antenna users simultaneously, and 2 RISs are deployed to construct extra reflection links. We assume the 3 BSs are respectively located at $(-10\mathrm{m},0\mathrm{m}),(0\mathrm{m},0\mathrm{m})$ and $(10\mathrm{m},0\mathrm{m})$, and 2 RISs are respectively located at $(0\mathrm{m},2\mathrm{m})$ and $(0,-2\mathrm{m})$. The number of antennas at each BS is set as $M=8$. The number of RIS elements is set as $N=32$. The number of subcarriers is set as $P=16$, and the noise power is set as $\sigma^2=1\times10^{-5}$. For the initialization step in Algorithm 1, $\boldsymbol{\theta}$ is initialized by random values in its feasible set, and $\boldsymbol{w}$ and $\boldsymbol{W}$ are set to ones.

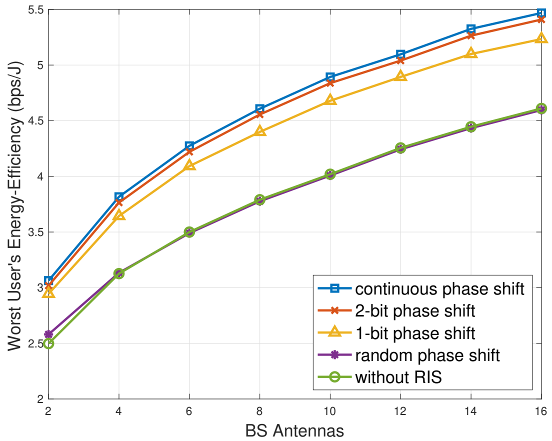

Fig. 3. Performance of the worst user's energy efficiency against the number of BS antennas.

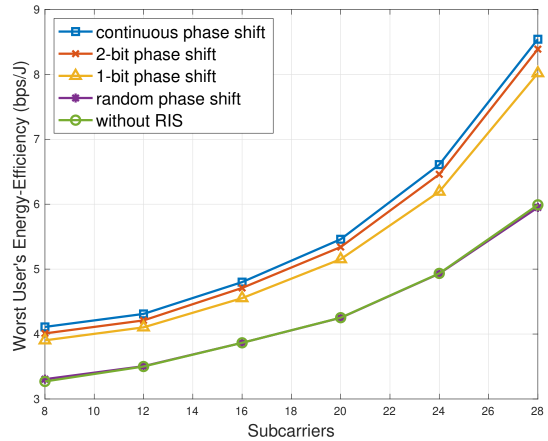

Fig. 4. Performance of the worst user's energy efficiency against the number of subcarriers.

For the channel model, at a reference distance 1m, the signal attenuation is set as 30 dB for the user-BS link and 40 dB for the user-RIS-BS link, respectively. The path loss exponent of the RIS-BS link, the user-RIS link, and the user-BS link are set as 2, 2, and 3, respectively [6]. We also assume that the user-RIS link and the user-BS link satisfy Rayleigh fading, while the RIS-BS link satisfies LoS fading.

B. Performance of the EEF design

Fig. 3 and Fig. 4 show the worst user's energy efficiency against the number of BS antennas and the number of subcarriers under various design, respectively. Both of the two figures imply the same tendency that the worst user's energy efficiency can be improved by consuming more hardware or software resources. Particularly, the deployment of RISs can bring more than 20% performance gain to the system. Comparing different schemes, one can see that even when the phase shift of RIS is restricted to finite bits due to implementational limitation, the RIS-aided cell-free network still performs much better than the conventional cell-free network without RIS. Also, when applying random phase shifts to RISs, the worst user's energy efficiency performs nearly the same as the system without RIS. This indicates that without passive beamforming at RIS, the signals reaching RIS cannot be accurately directed to the users, which demonstrates the necessity of passive precoding at RISs.

V. Conclusions

In this paper, we have taken a further step to investigate the EEF of RIS-aided cell-free network. Specifically, we have formulated the precoding design problem to maximize the energy efficiency of the worst user in a wideband RIS-aided cell-free network. To solve this problem, we have then proposed an iterative precoding algorithm by using Lagrangian transform and FP methods to tackle the highly decoupled optimization objective that alternatingly solves the subproblems of subcarrier assignment, power allocation, combining, and precoding. Simulation results have demonstrated the effectiveness of the proposed precoding algorithm, and the energy efficiency of users in cell-free network can be efficiently increased by the aid of RISs.

Acknowledgment

This work was supported in part by the National Key Research and Development Program of China (Grant No. 2020YFB1805005), in part by the National Natural Science Foundation of China (Grant No. 62031019), and in part by the European Commission through the H2020-MSCA-ITN META WIRELESS Research Project under Grant 956256.

References

[1] A. Lozano, R. W. Heath, and J. G. Andrews, "Fundamental limits of cooperation," *IEEE Trans. Inf. Theory*, vol. 59, no. 9, pp. 5213–5226, Sep. 2013.

[2] E. Nayebi, A. Ashikhmin, T. L. Marzetta, and H. Yang, "Cell-free massive MIMO systems," in *Proc. 2015 49th Asilomar Conf. Signals Sys. Comput. (ACSSC'15)*, Nov. 2015, pp. 695–699.

[3] H. Q. Ngo, A. Ashikhmin, H. Yang, E. G. Larsson, and T. L. Marzetta, "Cell-free massive MIMO versus small cells," *IEEE Trans. Wireless Commun.*, vol. 16, no. 3, pp. 1834–1850, Mar. 2017.

[4] Q. Wu and R. Zhang, "Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming," *IEEE Trans. Wireless Commun.*, vol. 18, no. 11, pp. 5394–5409, Nov. 2019.

[5] Z. Zhang and L. Dai, "A joint precoding framework for wideband reconfigurable intelligent surface-aided cell-free network," *arXiv preprint arXiv:2002.03744*, Feb. 2020.

[6] Z. Zhang and L. Dai, "Capacity improvement in wideband reconfigurable intelligent surface-aided cell-free network," in *Proc. 2020 IEEE 21st International Workshop on Signal Processing Advances in Wireless Communications (SPAWC'20)*, 2020, pp. 1–5.

[7] S. Huang, Y. Ye, M. Xiao, H. V. Poor, and M. Skoglund, "Decentralized beamforming design for intelligent reflecting surface-enhanced cell-free networks," *IEEE Wireless Commun. Lett.*, Dec. 2020.

[8] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, "Reconfigurable intelligent surfaces for energy efficiency in wireless communication," *IEEE Trans. Wireless Commun.*, vol. 18, no. 8, pp. 4157–4170, Aug. 2019.

[9] X. Wei, D. Shen, and L. Dai, "Channel estimation for RIS assisted wireless communications: Part I–Fundamentals, solutions, and future opportunities," *IEEE Commun. Lett.*, Jan. 2021.

[10] K. Shen and W. Yu, "Fractional programming for communication systems—Part I: Power control and beamforming," *IEEE Trans. Signal Process.*, vol. 66, no. 10, pp. 2616–2630, May 2018.