Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Robotic Systems Lab

Msc. - Written Exam

January 31th, 2018

Robot Dynamics - Exam

Marco Hutter, Roland Siegwart

Question	Points	Score
Multiple Choice	24	
Kinematics	12	
Dynamics	10	
Legged Robotics	8	
Rotary Wing	8	
Fixed-wing	8	
Total:	70	

Duration:	120min
Number of pages:	19
Allowed aids:	Calculator
	Two A4 sheets of personal notes, written on both sides
	Dictionary for foreign students

Write your name on every page in the box in the footer.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Cooperation is strictly forbidden.

Please draw your answer in the respective figure if required to do so in the respective questions.

Name: ______________________

Student number: ______________________

Signature: ______________________

A. Multiple Choice 24 pts

Decide whether the following statements are true or false. Cross the checkbox on the corresponding answer. You will be credited 1 point for a correct answer, while 1 pt will be subtracted from the total, if your answer is wrong.

(1) The quaternion $\mathbf{q} = \left(\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}\right)^T$ describes a 90° rotation around the x axis. The quaternion $\tilde{\mathbf{q}} = \left(\frac{-\sqrt{2}}{2}, 0, 0, \frac{-\sqrt{2}}{2}\right)^T$ therefore describes a 90° rotation around the negative x axis. ○ True ○ False [1 pt]

z

Solution: False

(2) The orientation of a free floating object in 3D space can only be described uniquely using 4 variables. ○ True ○ False [1 pt]

Solution: False

(3) A homogeneous transformation $\mathbf{T}_{\mathcal{AB}}$ from frame $\mathcal{B}$ to $\mathcal{A}$ applied to a vector only changes its representation but not the underlying vector. ○ True ○ False [1 pt]

Solution: False

(4) A rotation matrix $\mathbf{C}_{\mathcal{AB}}$ between from frame $\mathcal{B}$ to $\mathcal{A}$ applied to a vector only changes its representation but not the underlying vector. ○ True ○ False [1 pt]

Solution: True

(5) The inverse $\mathbf{T}_{\mathcal{AB}}^{-1}$ of a homogeneous transformation $\mathbf{T}_{\mathcal{AB}}$ is equal to its transpose $\mathbf{T}_{\mathcal{AB}}^{T}$. ○ True ○ False [1 pt]

Solution: False

(6) A planar two-link robot arm has a unique solution to the inverse kinematic problem. ○ True ○ False [1 pt]

Solution: False

(7) For a planar two-link robot arm, the differential inverse kinematic algorithm always converges to the same solution irrespective of initial configuration. ○ True ○ False [1 pt]

Solution: False

singular
redundant

(8) The contact constraint of a frictionless cube resting on flat ground is of dimension 3. ○ True ○ False [1 pt]

Solution: True

(9) For a floating base in 3-dimensional space there exists a choice of generalized coordinates such that the analytical and geometric orientation Jacobian are equal. ○ True ○ False [1 pt]

~~**Solution:** False~~

(10) In a static configuration $\dot{\mathbf{q}} = \ddot{\mathbf{q}} = \mathbf{0}$, the mass matrix of a dynamical system evaluates to zero. ○ True ○ False [1 pt]

Please fill in your name: ______________________________

Solution: False

(11) The total work done by the contact forces of a non-slipping, hard contact is zero. ◯ True ◯ False [1 pt]

Solution: True

(12) Given a perfect model, a robotic arm controlled by a PD controller with gravity compensation can achieve zero tracking error for any desired trajectory. ◯ True ◯ False [1 pt]

Solution: False

?4.7

(13) When choosing a unit quaternion as part of the generalized coordinates of a free-floating rigid body in 3D space, the mass matrix must have dimensions 7×7. ◯ True ◯ False [1 pt]

Solution: False

no mass matr?

?

(14) Joint space inverse dynamics control for a fixed-base manipulator with diagonal gain matrices decouples the dynamics of each generalized coordinate. ◯ True ◯ False [1 pt]

Solution: True

?

(15) For a bipedal system with two point feet on the ground, every torque command results in a unique acceleration. ◯ True ◯ False [1 pt]

Solution: True

(16) For a bipedal system with two point feet on the ground, every constraint consistent acceleration $\dot{u}^*_{consistent}$ is achieved by a unique torque command. ◯ True ◯ False [1 pt]

Solution: False

6.3

(17) A classic hexacopter (multi-rotor with 6 propellers) with all propellers spinning in the same plane is a fully actuated platform. ◯ True ◯ False [1 pt]

Solution: False

exclude the 6 base coordinates

(18) The generated thrust and drag torque from a spinning propeller are both proportional to the square of the propeller rotational speed. ◯ True ◯ False [1 pt]

Solution: True

(19) The attitude dynamics of a quadcopter can be stabilized by a proportional controller only. ◯ True ◯ False [1 pt]

P

Solution: False

(20) The yaw motion for a quadcopter is controlled by the drag moment of the propeller. ◯ True ◯ False [1 pt]

Solution: True

(21) The magnitude of the GPS velocity can be used directly as an airspeed measurement. ◯ True ◯ False [1 pt]

Please fill in your name: ______________________

A conventional fixed-wing aircraft flight control system consists of flight control surfaces, the respective cockpit controls, connecting linkages, and the necessary operating mechanisms to control an aircraft's direction in flight.

Solution: False

(22) The heading of a conventional aircraft is controlled primarily by the rudder. ◯ True ◯ False [1 pt]

Solution: False

(23) Wind disturbances are typically modeled/mitigated within the guidance-level loops of a fixed-wing autopilot. ◯ True ◯ False [1 pt]

Solution: ~~True~~

(24) Minimum airspeed demand during a coordinated turn increases as the turning radius decreases, assuming constant angle of attack. ◯ True ◯ False [1 pt]

Solution: True

B. Kinematics 12 pts

The two dimensional legged robot depicted in Fig. 1 has two rotation degrees of freedom (DOF) per leg and an arm with three DOF. The robot's base itself has three DOF in the 2D plane.

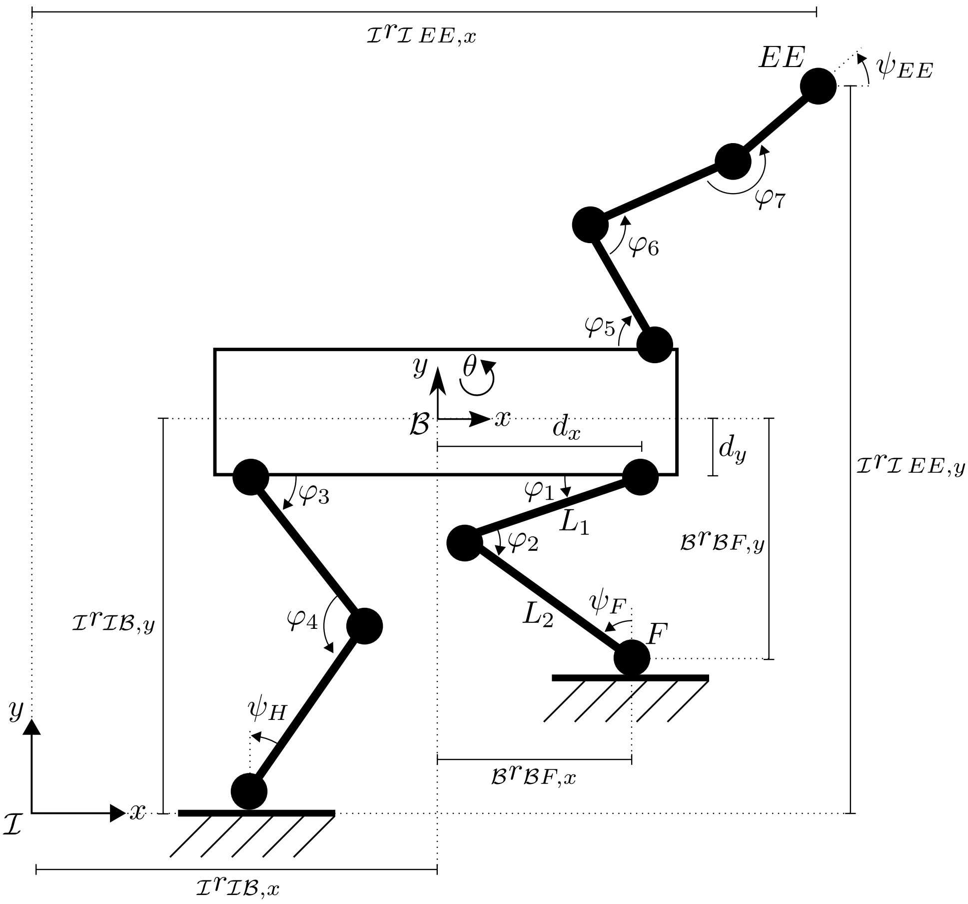

Figure 1: Two dimensional legged robot with arm.

(1) What are the generalized coordinates for the robot in Fig. 1? [1 pt]

$\mathbf{q} =$

Solution:

$$\mathbf{q} = \begin{pmatrix} _{\mathcal{I}}r_{\mathcal{I}\mathcal{B},x} & _{\mathcal{I}}r_{\mathcal{I}\mathcal{B},y} & \theta & \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 & \varphi_5 & \varphi_6 & \varphi_7 \end{pmatrix}^T \quad \text{(1Pt)}$$

Please fill in your name: ______________________________

(2) We first focus on the kinematics of the front leg only. Thus, we ignore the base (e.g. assume the robot's base is fixed). The front leg has local generalized coordinates $(\varphi_1, \varphi_2)^T$. Provide the expression for foot position ${}_\mathcal{B}\mathbf{r}_{\mathcal{B}F} = ({}_\mathcal{B}\mathbf{r}_{\mathcal{B}F,x}, {}_\mathcal{B}\mathbf{r}_{\mathcal{B}F,y})^T$ expressed in the base frame, and the lower leg orientation ψ_F [2 pts]

$${}_\mathcal{B}\mathbf{r}_{\mathcal{B}F} = \begin{pmatrix} \quad \end{pmatrix}$$

$$\psi_F =$$

Solution:

$${}_\mathcal{B}\mathbf{r}_{\mathcal{B}F} = \begin{pmatrix} d_x - L_1 \cos(\varphi_1) + L_2 \cos(\varphi_2 - \varphi_1) \\ -d_y - L_1 \sin(\varphi_1) - L_2 \sin(\varphi_2 - \varphi_1) \end{pmatrix} \quad \text{(1 Pts)}$$

$$\psi_F = \frac{\pi}{2} + \varphi_1 - \varphi_2 \quad \text{(1 Pt)}$$

(3) Still focusing on the front leg kinematics, what are the position Jacobian ${}_\mathcal{B}\mathbf{J}_{\mathcal{B}F,P}$ and rotational Jacobian ${}_\mathcal{B}\mathbf{J}_{\mathcal{B}F,R}$ of the foot w.r.t the local generalized coordinates $(\varphi_1, \varphi_2)^T$? [2 pts]

assume geometric jac. and
geometric jac = analytic jac here

$${}_\mathcal{B}\mathbf{J}_{\mathcal{B}F,P} = \begin{bmatrix} \quad \end{bmatrix}$$

$${}_\mathcal{B}\mathbf{J}_{\mathcal{B}F,R} = \begin{bmatrix} \quad \end{bmatrix}$$

Solution:

$${}_\mathcal{B}\mathbf{r}_{\mathcal{B}F} = \begin{pmatrix} d_x - L_1 \cos(\varphi_1) + L_2 \cos(\varphi_2 - \varphi_1) \\ -d_y - L_1 \sin(\varphi_1) - L_2 \sin(\varphi_2 - \varphi_1) \end{pmatrix}$$

$${}_\mathcal{B}\dot{\mathbf{r}}_{\mathcal{B}F} = \begin{pmatrix} L_1 \dot{\varphi}_1 \sin(\varphi_1) - L_2 (\dot{\varphi}_2 - \dot{\varphi}_1) \sin(\varphi_2 - \varphi_1) \\ -L_1 \dot{\varphi}_1 \cos(\varphi_1) - L_2 (\dot{\varphi}_2 - \dot{\varphi}_1) \cos(\varphi_2 - \varphi_1) \end{pmatrix}$$

$${}_\mathcal{B}\mathbf{J}_{\mathcal{B}F,P} = \begin{bmatrix} L_1 \sin(\varphi_1) + L_2 \sin(\varphi_2 - \varphi_1), & -L_2 \sin(\varphi_2 - \varphi_1) \\ -L_1 \cos(\varphi_1) + L_2 \cos(\varphi_2 - \varphi_1), & -L_2 \cos(\varphi_2 - \varphi_1) \end{bmatrix} \quad \text{(1Pt)}$$

Please fill in your name: ____________________

$$\psi_F = \frac{\pi}{2} + \varphi_1 - \varphi_2$$
$$\dot{\psi}_F = \dot{\varphi}_1 - \dot{\varphi}_2$$
$$_{\mathcal{B}}\mathbf{J}_{\mathcal{B}F,R} = \begin{bmatrix} 1 & -1 \end{bmatrix} \quad \text{(1Pt)}$$

NOTE: A correct Jacobian of a wrong answer in the previous question provides full points.

(4) We now take into account the motion of the base, hence we consider the full robot. Assume you are given the end-effector rotational Jacobian w.r.t the arm's local joint angles, i.e. $_{\mathcal{I}}\tilde{\mathbf{J}}_{\mathcal{B}EE,R} = \begin{bmatrix} \frac{\partial \varphi_{EE}}{\partial \varphi_5} & \frac{\partial \varphi_{EE}}{\partial \varphi_6} & \frac{\partial \varphi_{EE}}{\partial \varphi_7} \end{bmatrix}$. What is the rotational Jacobian, $_{\mathcal{I}}\mathbf{J}_{\mathcal{I}EE,R} = \frac{\partial \varphi_{EE}}{\partial \mathbf{q}}$, w.r.t. the full generalized coordinates? [1 pt]

$$_{\mathcal{I}}\mathbf{J}_{\mathcal{I}EE,R} = \begin{bmatrix} \quad \end{bmatrix}$$

Solution:

phiEE = 7 - PI + 6 - 5 + theta

$$_{\mathcal{I}}\mathbf{J}_{\mathcal{I}EE,R} = \frac{\partial \varphi_{EE}}{\partial \mathbf{q}}$$
$$= \begin{bmatrix} \frac{\partial \varphi_{EE}}{\partial _{\mathcal{I}}r_{\mathcal{I}\mathcal{B},x}} & \frac{\partial \varphi_{EE}}{\partial _{\mathcal{I}}r_{\mathcal{I}\mathcal{B},y}} & \frac{\partial \varphi_{EE}}{\partial \theta} & \frac{\partial \varphi_{EE}}{\partial \varphi_1} & \frac{\partial \varphi_{EE}}{\partial \varphi_2} & \frac{\partial \varphi_{EE}}{\partial \varphi_3} & \frac{\partial \varphi_{EE}}{\partial \varphi_4} & \frac{\partial \varphi_{EE}}{\partial \varphi_5} & \frac{\partial \varphi_{EE}}{\partial \varphi_6} & \frac{\partial \varphi_{EE}}{\partial \varphi_7} \end{bmatrix},$$

where $\frac{\partial \varphi_{EE}}{\partial \theta} = 1$, $\begin{bmatrix} \frac{\partial \varphi_{EE}}{\partial \varphi_5} & \frac{\partial \varphi_{EE}}{\partial \varphi_6} & \frac{\partial \varphi_{EE}}{\partial \varphi_7} \end{bmatrix} = {_{\mathcal{I}}\tilde{\mathbf{J}}_{\mathcal{I}EE,R}}$ is given, and the others are 0.

$$_{\mathcal{I}}\mathbf{J}_{\mathcal{I}EE,R} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & {_{\mathcal{I}}\tilde{\mathbf{J}}_{\mathcal{B}EE,R}} \end{bmatrix} \quad \text{(1Pt)}$$

NOTE: The entries can appear in different order based on the order of generalized coordinates.

(5) Assume you want to move the end-effector (point EE) along a horizontal trajectory with the velocity $_{\mathcal{I}}\mathbf{v}^* = (v_x, 0)^T$, while at the same priority ensuring that both the front and hind legs stay on the ground. What generalized velocity produces such motion? (In addition to all previously defined variables, assume you have access to positional Jacobians $_{\mathcal{I}}\mathbf{J}_{\mathcal{I}EE,P}$, $_{\mathcal{I}}\mathbf{J}_{\mathcal{I}F,P}$, and $_{\mathcal{I}}\mathbf{J}_{\mathcal{I}H,P}$ for the arm, front, and hind leg respectively w.r.t. the full generalized coordinates.) [2 pts]

$\dot{\mathbf{q}} =$

Please fill in your name: ______________________

Solution:

$$\mathbf{J}_{task} = \begin{bmatrix} {}_{\mathcal{I}}\mathbf{J}_{\mathcal{I}EE,P} \\ {}_{\mathcal{I}}\mathbf{J}_{\mathcal{I}F,P} \\ {}_{\mathcal{I}}\mathbf{J}_{\mathcal{I}H,P} \end{bmatrix}, \quad \mathbf{w}_{task} = \begin{bmatrix} {}_{\mathcal{I}}\mathbf{v}^* \\ \mathbf{0}_{2\times 1} \\ \mathbf{0}_{2\times 1} \end{bmatrix} \qquad \text{(1Pt)}$$

$$\dot{\mathbf{q}} = \mathbf{J}_{task}^{\dagger}\mathbf{w}_{task} \qquad \text{(1Pt)}$$

(6) After fulfilling the task in Question 5, how many degrees of freedom are available to fulfill additional tasks? [1 pt]

4

Solution: The tasks is 6 dimensional, there are 10 DoF. Therefore there are **4** DoF left to fulfill additional tasks.

(7) Provide a generalized velocity that moves the base to a desired position ${}_{\mathcal{I}}\mathbf{r}_{\mathcal{IB}}^* = ({}_{\mathcal{I}}r_{\mathcal{IB},x}^*, {}_{\mathcal{I}}r_{\mathcal{IB},y}^*)^T$ while performing the task specified in Question 5. You are allowed to use the symbols $\mathbf{J}_{Q5}$, and $\mathbf{w}_{Q5}$ to refer to the task Jacobian and task generalized velocity of Question 5. The task in Question 5 should have a higher priority. [3 pts]

$\dot{\mathbf{q}} =$

Solution:

$$\mathbf{w}_0 = k_p({}_{\mathcal{I}}\mathbf{r}_{\mathcal{IB}}^* - {}_{\mathcal{I}}\mathbf{r}_{\mathcal{IB}}) \qquad \text{(1Pt)}$$

$$\mathbf{q}_0 = \begin{bmatrix} \mathbf{w}_0 \\ \mathbf{0}_{8\times 1} \end{bmatrix} \qquad \text{(1Pt)}$$

$$\dot{\mathbf{q}} = \mathbf{J}_{Q5}^{\dagger}\mathbf{w}_{Q5} + \left(\mathbb{I}_{10\times 10} - \mathbf{J}_{Q5}^{\dagger}\mathbf{J}_{Q5}\right)\mathbf{q}_0 \qquad \text{(1Pt)}$$

?

C. Dynamics 10 pts

In this section, we consider the ABB IRB 120 robot arm that was introduced in the exercises. An annotated image of this system is displayed in Figure 2.

The equations of motion for this system are given by

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} \ . \tag{1}$$

The position and rotation Jacobians of each frame i are denoted by $\mathbf{J}_{iP}(\mathbf{q})$ and $\mathbf{J}_{iR}(\mathbf{q})$, respectively.

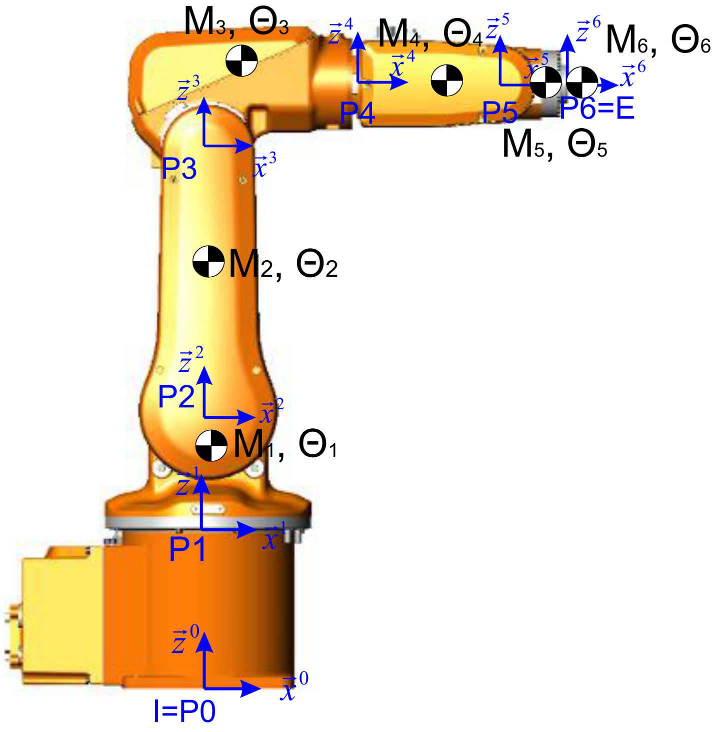

Figure 2: ABB IRB 120 with coordinate systems and joints

(1) The robot arm is supposed to pick up an object with its end-effector. What torque needs to be applied to statically hold a load of mass m_{load} (assume point mass) at the end-effector? Please write a symbolic expression for the desired joint torque vector $\boldsymbol{\tau}^*$ using the terms introduced above. [2 pts]

$\boldsymbol{\tau}^* =$

Solution:

$$\boldsymbol{\tau}^* = \mathbf{g}(\mathbf{q}) - {}_I\mathbf{J}_{EP}(\mathbf{q})^\top \begin{bmatrix} 0 \\ 0 \\ -m_{\text{load}}\, g \end{bmatrix}$$

Please fill in your name: ______________________

(2) The robot now starts to move in order to move the object to a new location. At a given generalized configuration $\mathbf{q}$ and velocity $\dot{\mathbf{q}}$, what is the total kinetic energy of the system including the payload? [2 pts]

$E_{\text{kin}} =$

Solution:

$$E_{\text{kin}} = \frac{1}{2}\dot{\mathbf{q}}^\top \mathbf{M}(\mathbf{q})\dot{\mathbf{q}} + \frac{1}{2}(\mathbf{J}_{EP}\dot{\mathbf{q}})^\top m_{\text{load}}(\mathbf{J}_{EP}\dot{\mathbf{q}})$$
$$= \frac{1}{2}\dot{\mathbf{q}}^\top(\mathbf{M}(\mathbf{q}) + \mathbf{J}_{EP}^\top m_{\text{load}}\mathbf{J}_{EP})\dot{\mathbf{q}}$$

(3) The object was successfully moved to its destination and dropped there. We now want to command the end-effector with task space references. Write a symbolic expression for the joint torque vector $\boldsymbol{\tau}^*$ that achieves an end-effector linear acceleration ${}_I\mathbf{a}_E^* = \begin{bmatrix} a_x^* & a_y^* & 0 \end{bmatrix}^\top$ while exerting a contact force ${}_I\mathbf{f}_E^* = \begin{bmatrix} 0 & 0 & f_z^* \end{bmatrix}^\top$ on the environment with the end-effector. [2 pts]
Is your answer unique?

$\boldsymbol{\tau}^* =$

Answer unique (yes / no)?

Solution:

$$\ddot{\mathbf{q}} = ({}_I\mathbf{J}_{PE})^\dagger({}_I\mathbf{a}_E^* - {}_I\dot{\mathbf{J}}_{PE}\dot{\mathbf{q}}) \quad (1\text{ Pt})$$
$$\boldsymbol{\tau}^* = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + {}_I\mathbf{J}_{PE}(\mathbf{q})^\top {}_I\mathbf{f}_E^* \,. \quad (1\text{ Pt})$$

(4) If instead you want to move the end-effector to a desired position ${}_I\mathbf{r}_{IE}^*$, how would you calculate the acceleration ${}_I\mathbf{a}_E^*$ that was given in the previous question? You may use the current position ${}_I\mathbf{r}_{IE}$ and velocity ${}_I\mathbf{v}_E$ in your solution. [1 pt]

${}_I\mathbf{a}_E^* =$

note the PD control in both generalized coord. and end-eff. coord.

Solution:

$${}_I\mathbf{a}_E^* = \mathbf{K}_p({}_I\mathbf{r}_{IE}^* - {}_I\mathbf{r}_{IE}) - \mathbf{K}_d({}_I\mathbf{v}_E)$$

$\ddot{q} = K(q^* - q) + K_d(\dot{q}^* - \dot{q}) +$

Question (3) can also be solved by a hierarchical control approach by successively minimizing objectives (optimization problems) of the form

$$\min_{\mathbf{x}} ||\mathbf{A}\mathbf{x} - \mathbf{c}||_2 \ . \tag{2}$$

(5) What would be a suitable decision variable? [1 pt]

$\mathbf{x} = \left[\quad \right]$

Solution: $\mathbf{x} = \left[\begin{array}{ccc} \ddot{\mathbf{q}}^\top & \boldsymbol{\tau}^\top & {}_I\mathbf{f}_E^\top \end{array} \right]^\top$

(6) Formulate the objective for physical feasibility (obeying the equations of motion). [1 pt]

$\mathbf{A} = \left[\quad \right]$

$\mathbf{c} =$

Solution:

$$\mathbf{A} = \left[\begin{array}{ccc} \mathbf{M}(\mathbf{q}) & -\mathbb{I} & -{}_I\mathbf{J}_{PE}^\top(\mathbf{q}) \end{array} \right]$$
$$\mathbf{c} = -\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{g}(\mathbf{q})$$

(7) Formulate the objective for desired task space acceleration ${}_I\mathbf{a}_E$. [1 pt]

$\mathbf{A} = \left[\quad \right]$

$\mathbf{c} =$

Solution:

$$\mathbf{A} = \left[\begin{array}{ccc} {}_I\mathbf{J}_{PE} & \mathbf{0} & \mathbf{0} \end{array} \right]$$
$$\mathbf{c} = {}_I\mathbf{a}_E - {}_I\dot{\mathbf{J}}_{PE}\dot{\mathbf{q}}$$

Please fill in your name: ____________________

D. Legged Robotics 8 pts

In this exercise, consider the case of a planar legged robot with two legs and 4 joints overall. The robot consists of a rectangular base and legs comprised of links of same length, as is shown in Fig. 3. The configuration of the robot is parametrized by a vector of generalized coordinates:

$$\mathbf{q} =: \left({}_{\mathcal{I}}x_{\mathcal{I}B}, {}_{\mathcal{I}}z_{\mathcal{I}B}, \theta_B, \varphi_1, \varphi_2, \varphi_3, \varphi_4 \right)^\top$$

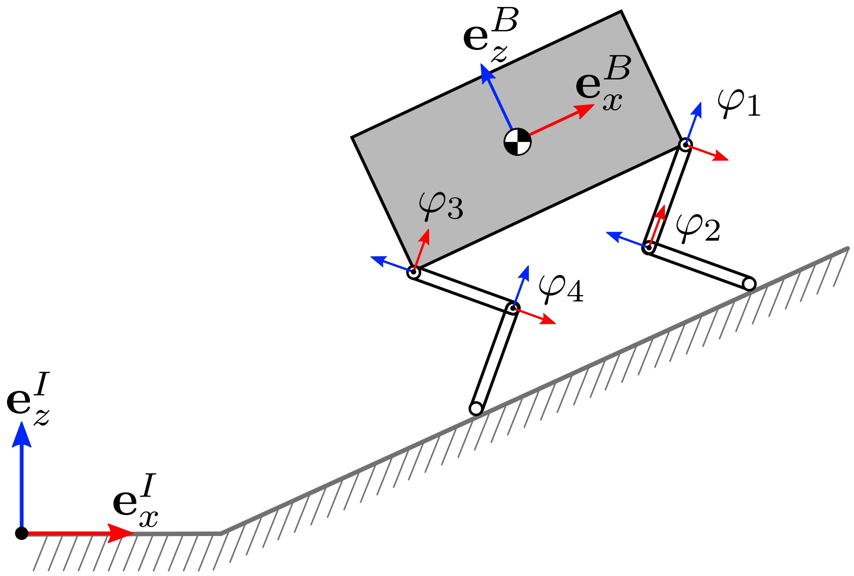

Figure 3: Planar legged robot standing on an incline.

(1) The robot in Fig. 3 can have different contact configurations (without slipping). Analyze the number of independent contact constraints and the number of uncontrollable degrees-of-freedom in each case. Fill in the table below. [1 pt]

Contact Configuration	Number of contact constraints	Number of uncontrollable DoFs
No legs in contact	0	3
One leg in contact	2	1
Both legs in contact	4	0

Solution:

Contact Configuration	Number of contact constraints	Number of uncontrollable DoFs
No legs in contact	0	3
One leg in contact	2	1
Both legs in contact	4	0

Please fill in your name: ______________________

(2) Consider now that the contact surface is frictionless (e.g. ice). Analyze the contact configurations in this new situation and fill in the table below. [1 pt]

Contact Configuration	Number of contact constraints	Number of uncontrollable DoFs
No legs in contact	0	3
One leg in contact	1	2
Both legs in contact	2	1

Solution:

Contact Configuration	Number of contact constraints	Number of uncontrollable DoFs
No legs in contact	0	3
One leg in contact	1	2
Both legs in contact	2	1

(3) The equation of motion for the system in Fig. 3 can be written as $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} + {}_I\mathbf{J}_c^\top \, {}_I\mathbf{F}_c = \mathbf{S}^\top \boldsymbol{\tau}$. Assume that the base has mass m_B and that mass of the legs is negligible. Write the expressions for: [1 pt]

1. The generalized gravity force vector $\mathbf{g}$.
2. The floating-base selection matrix $\mathbf{S}$.

$$\mathbf{g} = \Big(\qquad \Big)$$

$$\mathbf{S} = \Big[\qquad \Big]$$

Solution:

$$\mathbf{g} = (0, -m_B\, g, 0, 0, 0, 0, 0)^\top, (0.5pt)$$

$$\mathbf{S} = \begin{bmatrix} 0_{4\times 3} & \mathbb{I}_{4\times 4} \end{bmatrix}, (0.5pt)$$

Please fill in your name: ______________________

0

(4) Assume that $\mathbf{M}(\mathbf{q})$, $\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{g}(\mathbf{q})$ and $\mathbf{S}$ are given, and that we also know the contact Jacobians ${}_I\mathbf{J}_{IC,1}$ and ${}_I\mathbf{J}_{IC,2}$ for the front and rear legs (no slippage). We wish to find a matrix $\mathbf{P}_c$ which removes the contact forces ${}_I\mathbf{F}_{c,1}$ and ${}_I\mathbf{F}_{c,2}$ from the equations-of-motion: [3 pts]

$$\boldsymbol{\tau}^* = \mathbf{P}_c \left(\mathbf{M}\,\ddot{\mathbf{q}}^* + \mathbf{b} + \mathbf{g}\right) + \mathbf{N}_{\mathbf{P}_c}\,\boldsymbol{\tau}_0^* \tag{3}$$

4 × 7

1. What are the dimensions of $\mathbf{P}_c$?
2. Show how to compute $\mathbf{P}_c$ using the given dynamic and kinematic properties.
3. Explain in a single sentence what $\boldsymbol{\tau}_0^*$ can be used for.

to compute multiple solutions of tau

Solution:

$$\mathbf{P}_c \in \mathbb{R}^{4\times 7} \qquad \text{(1 Pt)}$$

Using the contact-consistent null-space projection:

$${}_I\mathbf{J}_c = \begin{bmatrix} {}_I\mathbf{J}_{IC,1} \\ {}_I\mathbf{J}_{IC,2} \end{bmatrix}$$

$$\mathbf{N}_c = \mathbb{I} - \mathbf{M}^{-1}\,{}_I\mathbf{J}_c\left({}_I\mathbf{J}_c\,\mathbf{M}^{-1}\,{}_I\mathbf{J}_c^{\top}\right)^{-1}\,{}_I\mathbf{J}_c$$

$$\mathbf{P}_c = \left(\mathbf{N}_c\,\mathbf{S}^{\top}\right)^{+}\mathbf{N}_c \qquad \text{(1 Pt)}$$

The null space torque can be used to control the interal forces / contact forces. (1 Pt)

(5) We want to achieve a desired task-space acceleration vector $\dot{\mathbf{w}}_T^* = \begin{bmatrix}\ddot{x}_B^* & \ddot{z}_B^* & \ddot{\theta}_B^*\end{bmatrix}^{\top}$ with the center of the base. Specify the corresponding task-space Jacobian matrix $\mathbf{J}_T$. [1 pt]

we could write jacobian by our own when generating tasks

$$\mathbf{J}_T = \begin{bmatrix} \quad \end{bmatrix}$$

Please fill in your name: ______________________

Solution:

$$\mathbf{J}_T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(6) Given all previously introduced kinematic and dynamic quantities, write an expression for the desired torque $\boldsymbol{\tau}^*$ which achieves the desired task space acceleration $\dot{\mathbf{w}}_T^*$. [1 pt]

$\boldsymbol{\tau}^* =$

Solution:

$$\ddot{\mathbf{q}}^* = \begin{bmatrix} \mathbf{J}_T \\ {}_I\mathbf{J}_{IC,1} \\ {}_I\mathbf{J}_{IC,2} \end{bmatrix}^+ \left(\dot{\mathbf{w}}_T^* - \begin{bmatrix} \dot{\mathbf{J}}_T \\ {}_I\dot{\mathbf{J}}_{IC,1} \\ {}_I\dot{\mathbf{J}}_{IC,2} \end{bmatrix} \dot{\mathbf{q}} \right)$$

$$\boldsymbol{\tau}^* = \mathbf{P}_c \left(\mathbf{M}\, \ddot{\mathbf{q}}^* + \mathbf{b} + \mathbf{g} \right)$$

E. Rotary Wing 8 pts

You bought a quadrotor in cross configuration with four propellers for your project. The properties of the quadrotor and the environment can be seen in Table 1 and Figure 4. You want to experimentally verify the efficiency of the currently mounted propellers.

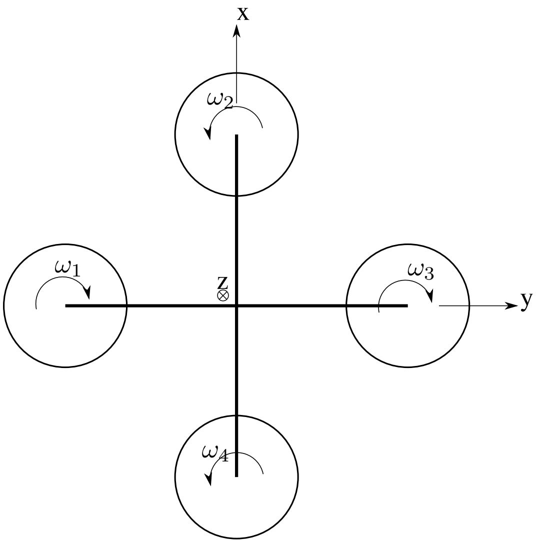

Figure 4: Schematic of the quadrotor.

Property	Value
Mass	$0.5\,\mathrm{kg}$
Thrust constant	$b = 8.5 \times 10^{-6}\,\mathrm{N\,rad^{-2}\,s^2}$
Drag constant	$d = 1.36 \times 10^{-7}\,\mathrm{N\,m\,rad^{-2}\,s^2}$
Gravitational acceleration	$9.81\,\mathrm{m\,s^{-2}}$

Table 1: Properties of the UAS and the environment.

(1) Calculate each rotor speed in hovering condition. [4 pts]

Please fill in your name: ______________________

Solution:

$$T_{prop} = \frac{mg}{4} = 1.23\,\mathrm{N} \qquad (1.5pt)$$
$$T_{prop} = b\omega_{prop}^2 \qquad (1.5pt)$$
$$\omega_{prop} = \sqrt{\frac{T_{prop}}{b}} = 380.4\,\mathrm{rad\,s^{-1}} \qquad (1pt)$$

(2) You want to achieve linear acceleration of $3\,\mathrm{m\,s^{-2}}$ along x axis while maintaining the same altitude. Neglecting the drag forces, calculate the pitching angle θ to achieve the desired acceleration. [4 pts]

Solution:

$$F_x = m \cdot a_x = 0.5 \cdot 3 = 1.5\,\mathrm{N} \qquad (1pt)$$
$$F_z = -m \cdot g = -0.5 \cdot 9.81 = -4.91\,\mathrm{N} \qquad (1pt)$$
$$\theta = \arctan\left(\frac{F_x}{F_z}\right) = -17^\circ = -0.297\,\mathrm{rad} \qquad (2pt)$$

抬头为正 低头为负

F. Fixed-wing 8 pts

Use the parameters from the table in Figure 5 with the additional environmental and platform specific information, below, to answer the following questions. **Do not interpolate** when using the table, simply take the closest value. Assume the aircraft is in steady conditions, and that its *lift* and *drag* forces are only a function of angle of attack, $c_L = f(\alpha)$, $c_D = f(\alpha)$.

Enivronment: density $\rho = 1.225 kg/m^3$, gravity $g = 9.81 m/s^2$ (assume sea-level values)

Aircraft specs: wing area $S = 0.39 m^2$, mass $m = 2.65 kg$

α [deg]	c_L	c_D	c_L/c_D	c_L^3/c_D^2
-5	-0.227	0.0268	-8.486	-16.393
-4	-0.0544	0.0313	-1.733	-0.163
-3	0.106	0.0367	2.908	0.904
-2	0.256	0.0429	5.969	9.136
-1	0.394	0.0499	7.885	24.503
0	0.519	0.0578	8.992	42.041
1	0.633	0.0664	9.536	57.641
2	0.735	0.0759	9.691	69.120
3	0.826	0.0862	9.581	75.847
4	0.904	0.0973	9.293	78.126
5	0.971	0.109	8.888	76.716
6	1.025	0.121	8.408	72.517
7	1.068	0.135	7.882	66.404
8	1.099	0.149	7.333	59.137
9	1.118	0.165	6.774	51.343
10	1.125	0.181	6.215	43.504
11	1.121	0.197	5.664	35.979
12	1.105	0.215	5.124	29.016
13	1.0767	0.234	4.599	22.777
14	1.036	0.253	4.090	17.348
15	0.984	0.273	3.600	12.761

stall

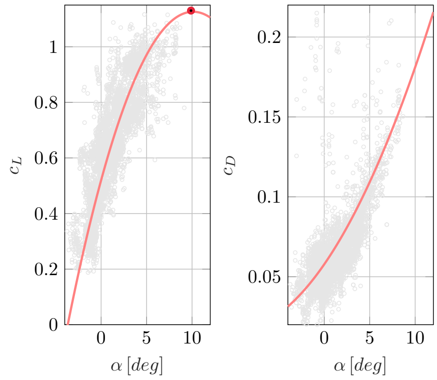

Figure 5: Aerodynamic Data

(1) The Techpod is flying at its *endurance*-optimal angle of attack, $\alpha = 4°$, in level flight. What is its airspeed? [1 pt]

$mg = F_L =$

Solution:
$V = \sqrt{\frac{2mg}{\rho S c_L(\alpha=4°)}} = 10.97 m/s$ 1pt

Please fill in your name: ____________________

(2) A tight loiter is commanded with radius $R = 20m$. What roll angle ϕ can achieve this loiter while keeping the above calculated airspeed? Assume **coordinated** turning. [2 pts]

Solution:
$L \sin\phi = \frac{mV^2}{R}$
$L \cos\phi = mg$
$\rightarrow \tan\phi = \frac{V^2}{gR}$
$\rightarrow \phi = \tan^{-1}\left(\frac{V^2}{gR}\right) = 31.53°$ 2pt

(3) What new angle of attack α' is necessary to maintain the coordinated turn from question (2) with the airspeed calculated in question (1)? Is this a safe flight configuration? Why or why not? [3 pts]

Solution:
$c_L(\alpha') = \frac{2mg}{\rho S V^2 \cos\phi} = 1.06 \approx 1.068$
$\rightarrow \alpha' = 7°$ 2pt
$\alpha' = 7°$ is getting very close to *stall*, where any small perturbation could potentially push the plane over the boundary. This is not very safe. 1pt

(4) What *percentage* increase in *thrust* force ΔT over the original level flight condition is needed to hold this loiter pattern at a constant altitude? [2 pts]

Solution:

$D = \frac{1}{2}\rho V^2 S c_D(\alpha)$

$D' = \frac{1}{2}\rho V^2 S c_D(\alpha')$

$T = D, \quad T' = D'$

$\Delta T = \frac{T'-T}{T} = \frac{D'-D}{D} = 38.75\%$ 2pt