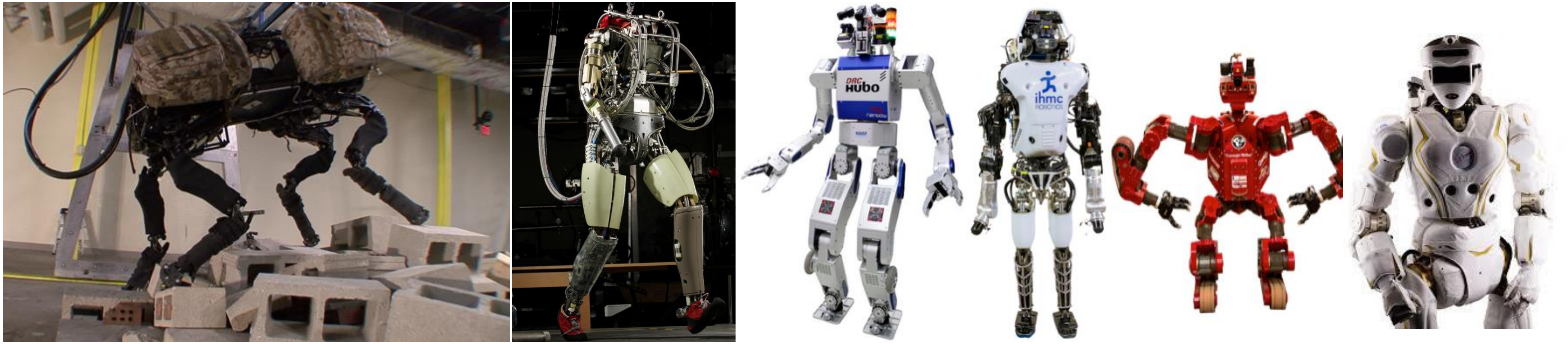
ETH zürich



Lecture «Robot Dynamics»: Legged Robots

151-0851-00 V

lecture:	HG F3	Tuesday 10:15 – 12:00, every week
exercise:	HG F3, D7.1	Wednesday 8:15 – 10:00, according to schedule (about every 2nd week)

Marco Hutter

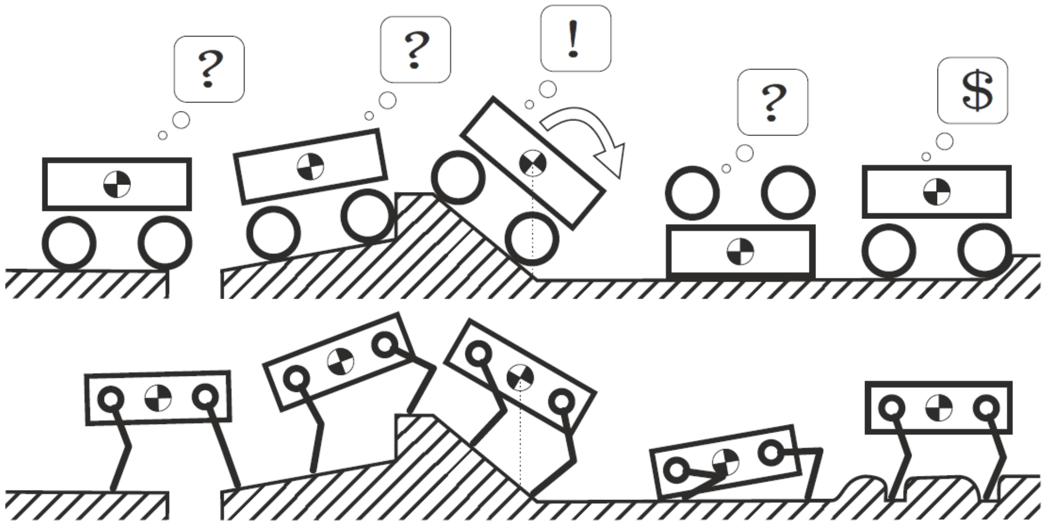
17.09.2019	Intro and Outline	Course Introduction; Recapitulation Position, Linear Velocity			
24.09.2019	Kinematics 1	Rotation and Angular Velocity; Rigid Body Formulation, Transformation	25.09.2019	Exercise 1a	Kinematics Modeling the ABB arm
01.10.2019	Kinematics 2	Kinematics of Systems of Bodies; Jacobians	02.10.2019	Exercise 1a	Differential Kinematics of the ABB arm
08.10.2019	Kinematics 3	Kinematic Control Methods: Inverse Differential Kinematics, Inverse Kinematics; Rotation Error; Multi-task Control	09.10.2019	Exercise 1b	Kinematic Control of the ABB Arm
15.10.2019	Dynamics L1	Multi-body Dynamics	16.10.2019	Midterm 1	Programming kinematics with matlab
22.10.2019	Dynamics L2	Floating Base Dynamics	23.10.2019	Exercise 2a	Dynamic Modeling of the ABB Arm
29.10.2019	Dynamics L3	Dynamic Model Based Control Methods	30.10.2019	Exercise 2b	Dynamic Control Methods Applied to the ABB arm
05.11.2019	Legged Robot	Dynamic Modeling of Legged Robots & Control	06.11.2019	Midterm 2	Programming dynamics with matlab
12.11.2019	Case Studies 1	Legged Robotics Case Study	13.11.2019	Exercise 3	Legged robot
19.11.2019	Rotorcraft	Dynamic Modeling of Rotorcraft & Control	20.11.2019		
26.11.2019	Case Studies 2	Rotor Craft Case Study	27.11.2019	Exercise 4	Modeling and Control of Multicopter
03.12.2019	Fixed-wing	Dynamic Modeling of Fixed-wing & Control	04.12.2019		
10.12.2019	Case Studies 3	Fixed-wing Case Study (Solar-powered UAVs - AtlantikSolar, Vertical Take-off and Landing UAVs – Wingtra)	11.12.2019	Exercise 5	Fixed-wing Control and Simulation
17.12.2019	Summery and Outlook	Summery; Wrap-up; Exam			

Midterm exam tomorrow

- WED 8.15-10
  - Like the exercises, the exam will be done with matlab. Bring your own laptop! We will distribute the exam files at the beginning of the exam through piazza.
  - Additionally, you will get a printout version of the exam to write down your name and to answer a couple of questions.
  - We will ask you to write down numeric results that are generated with matlab on your paper
  - You have to hand in the exam paper and upload the matlab files in a single compressed folder (.zip, .gz, .tar, .rar).
  - We check the matlab files if they produce the correct results.
  - The exam will be open book, which means you can use the script, slides, exercises, etc.
  - You are not allowed to communicate or share results.
  - The use of internet (beside for licenses) is forbidden.
- Due to limited space, the group will be split based on your surname:
  - A-K in HG F3
  - L-Z in HG D7.1

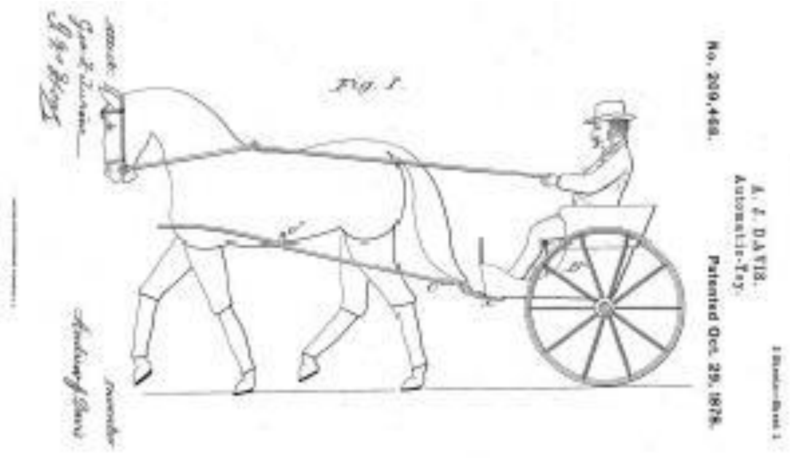
Why legged robots?

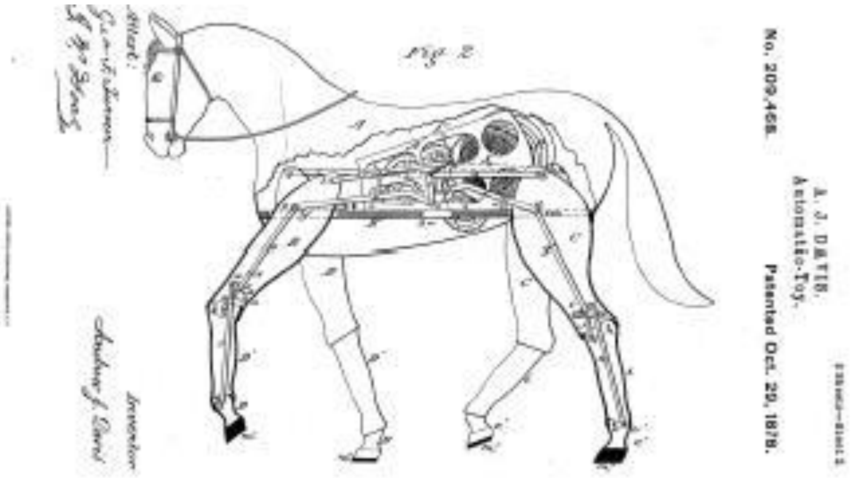
- Legged systems can overcome many obstacles



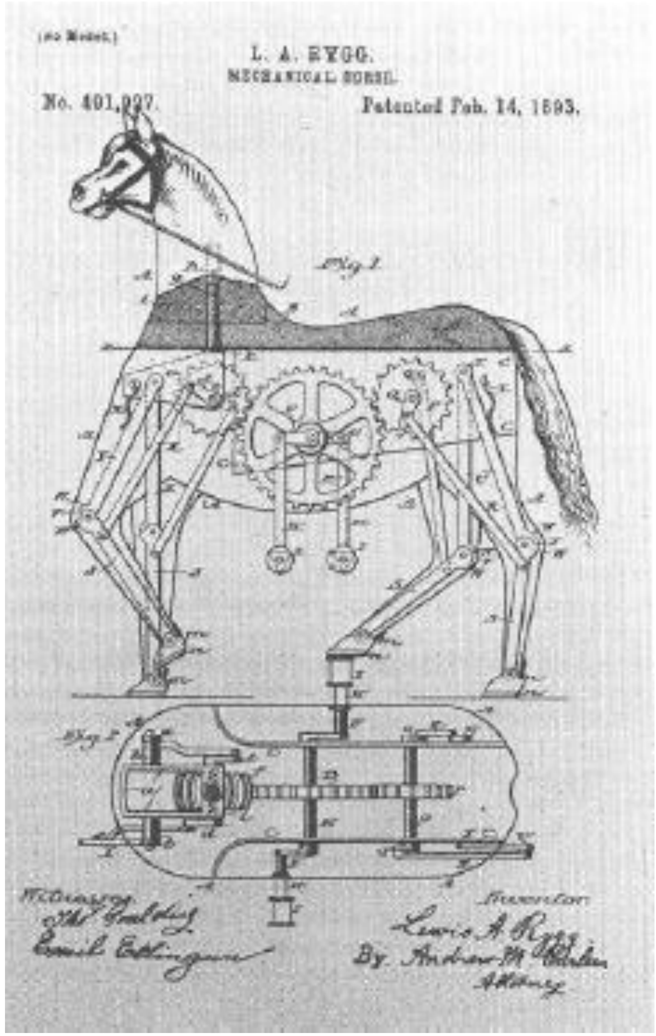
- But it is quite hard to achieve this since
  - many DOFs must be controlled in a coordinated way
  - the robot must interact with (uncertain) terrain

History of Legged Robotics
Walking Mechanism – First patents



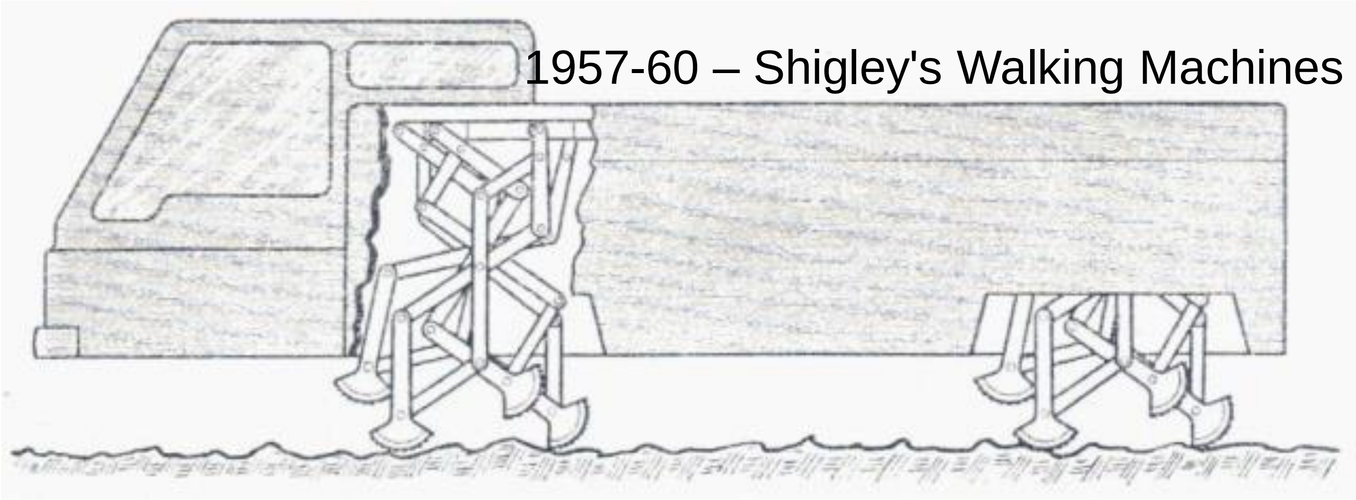


- Davis, 1878



Rygg, 1893

Walking Mechanisms



1957-60 – Shigley's Walking Machines

1984 - OSU ASV
(Adaptive Suspension Vehicle)

Walking Mechanism

Strandbest, by Theo Jansen

History of Legged Robotics

GE Walking Truck – human controlled 4-ped

Large Scale Legged Locomotion and Manipulation

Youtube:
menzi muck extreme



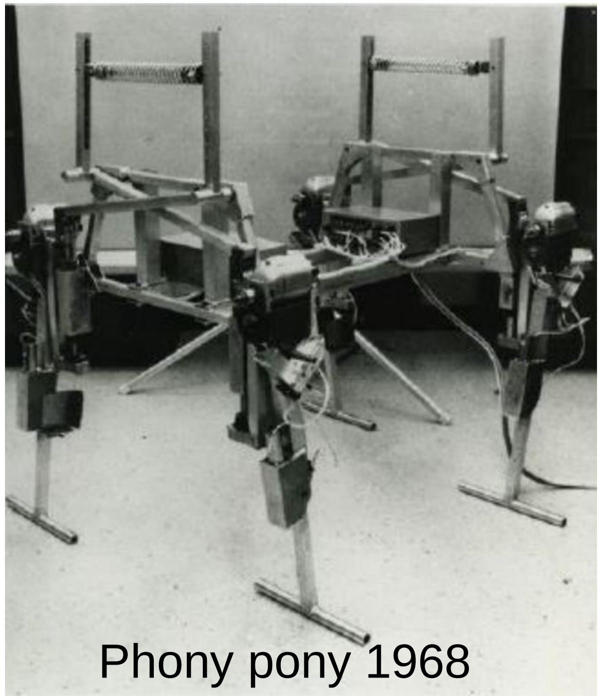




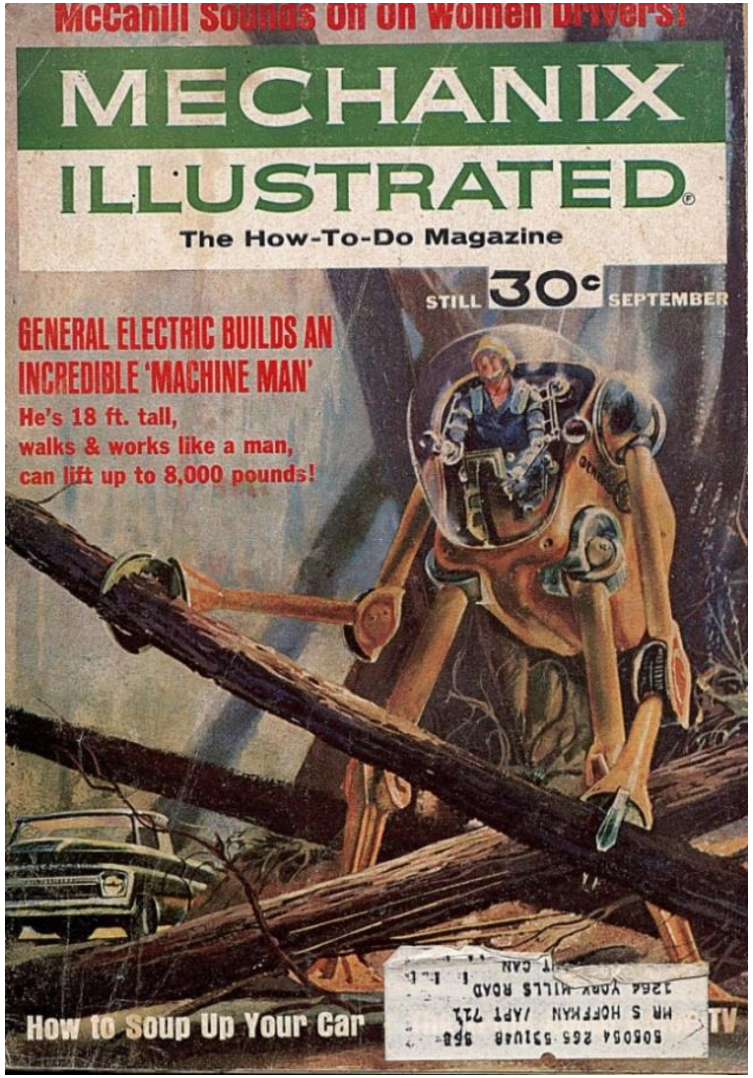
History of Legged Robotics

Phony Pony, GE Hardiman and many more…

- More on http://cyberneticzoo.com/
  - steam-actuated humans
  - mechanical elephants
  - …


Phony pony 1968


Hardiman1965



History of Legged Robotics
Humanoid robots after 2000

- Honda Asimo

- Toyota Humanoid

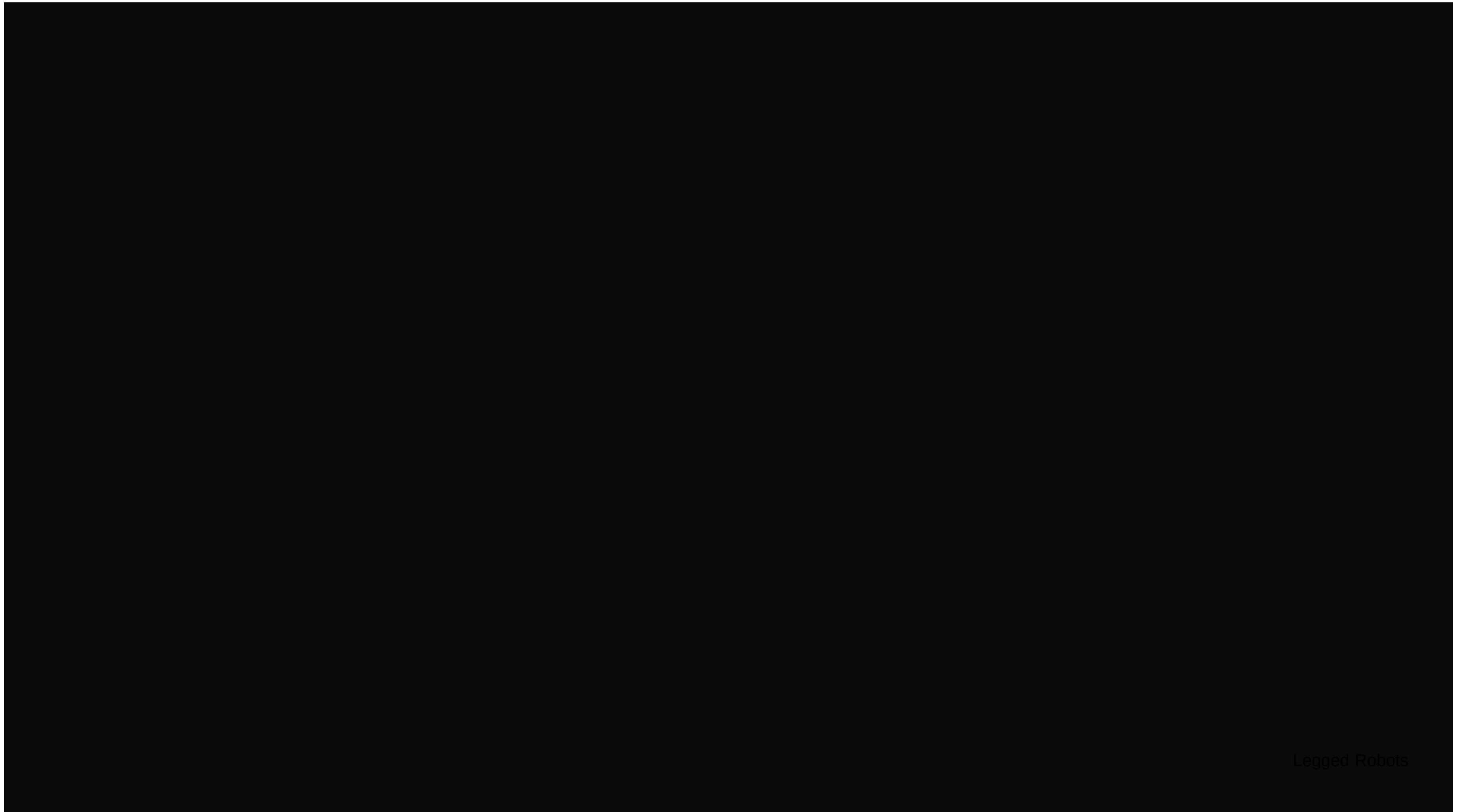
History of Legged Robotics
Humanoid robots after 2000

- Fukushima 2011

- DARPA Robotics Challenge 2012

Legged Robotics

Where are we really and what are the challenges?



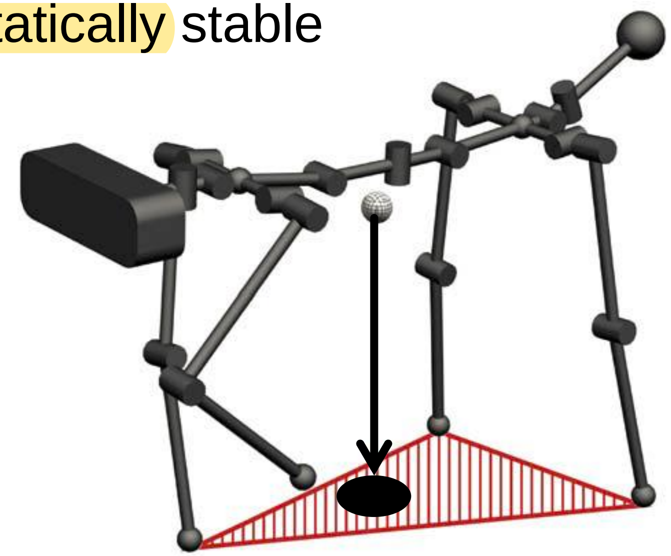
DARPA Robotics Challenge

... and a thing we learned after the DRC Finals

- Walking is still difficult

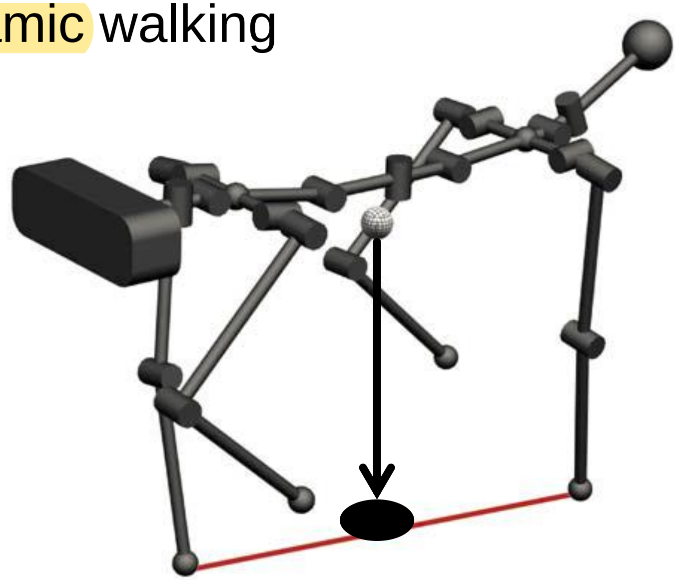
Static vs. Dynamic Stability

- Statically stable



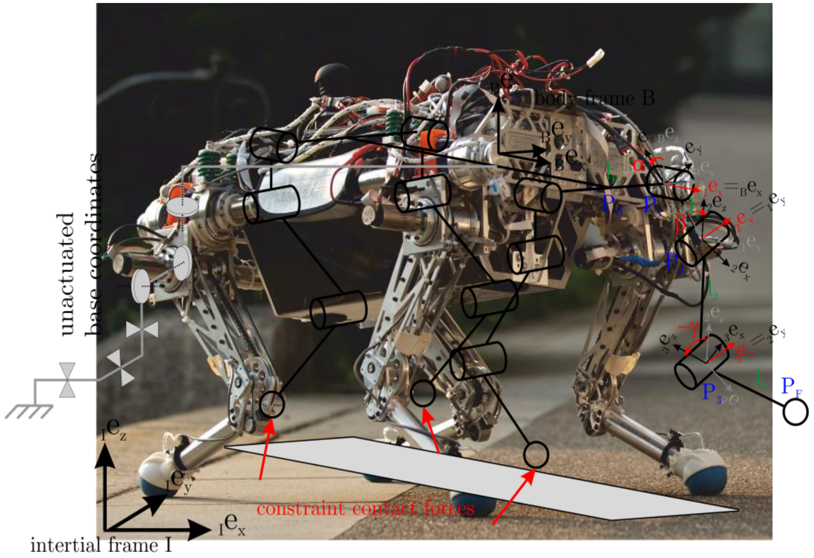
- Bodyweight supported by at least three legs
- Even if all joints 'freeze' instantaneously, the robot will not fall
- Safe, slow and inefficient

- Dynamic walking



- The robot will fall if not continuously moving
- Less than three legs can be in ground contact
- fast, efficient and demanding for actuation and control

Kinematics of Floating Base / Mobile Systems

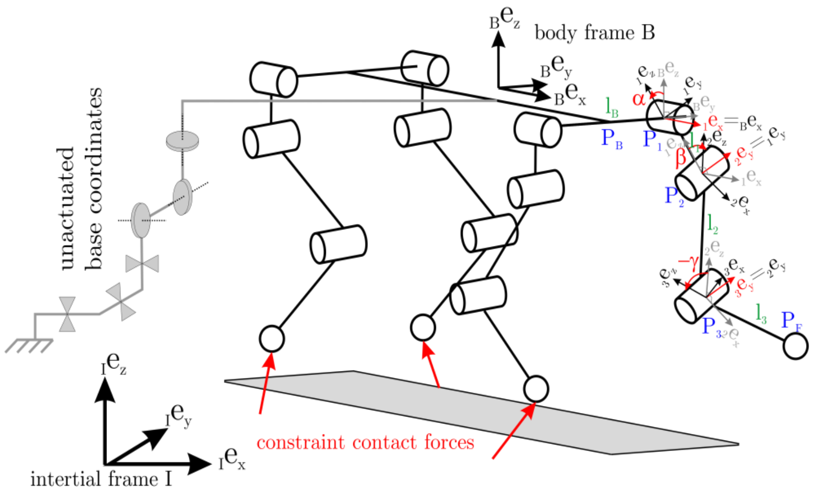


- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
- 1 in swing [NR 4]

5

Quiz 2min

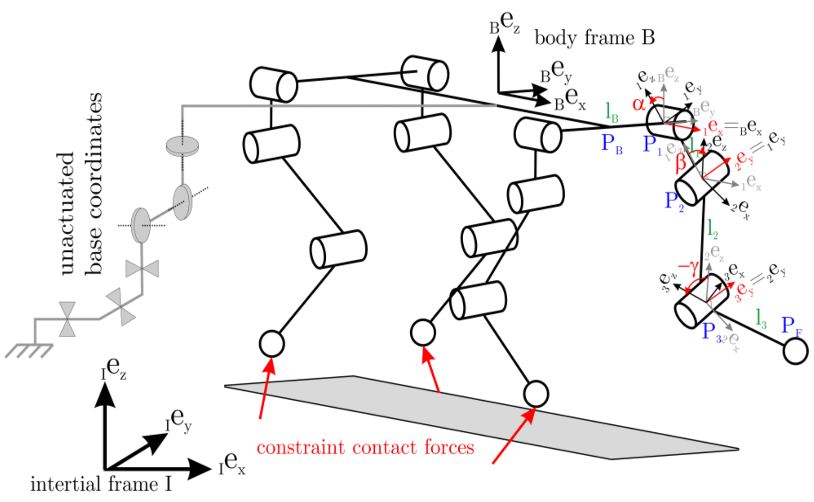
Kinematics of Floating Base / Mobile Systems



- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
- 1 in swing [NR 4]

1. How many generalized coordinates?
2. How many base coordinates?
3. How many actuated joint coordinates?
4. How many contact constraints?

Kinematics of Floating Base / Mobile Systems



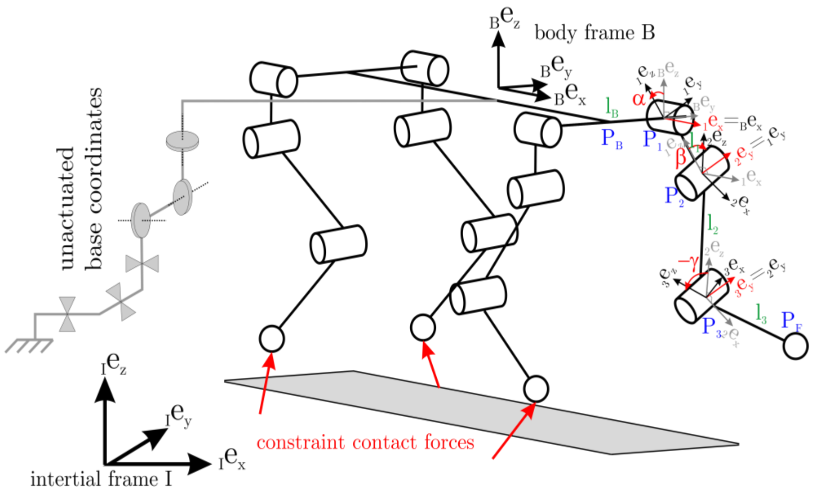
- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
- 1 in swing [NR 4]

1. How many generalized coordinates? 12+6
2. How many base coordinates? 6
3. How many actuated joint coordinates? 12
4. How many contact constraints? 3*3=9

5

Quiz
2min

Differential Kinematics



- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
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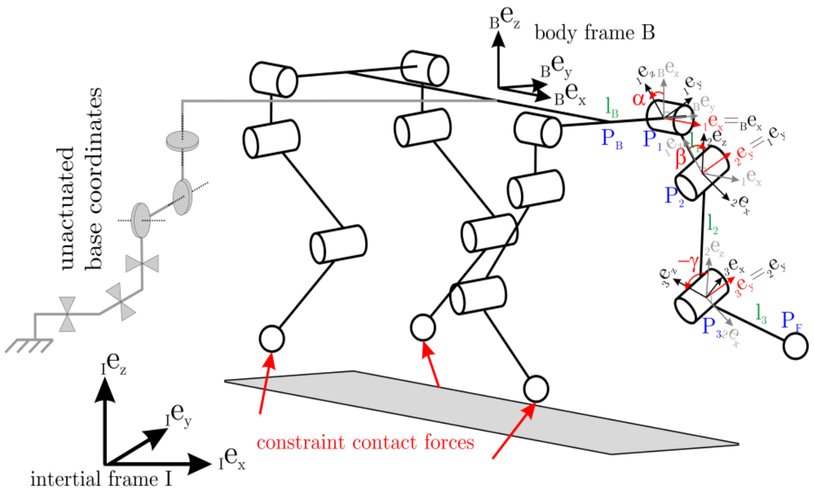
5. Write down the contact constraint
6. How many DoFs remain adjustable?
7. Which DoFs remain adjustable?

Differential Kinematics



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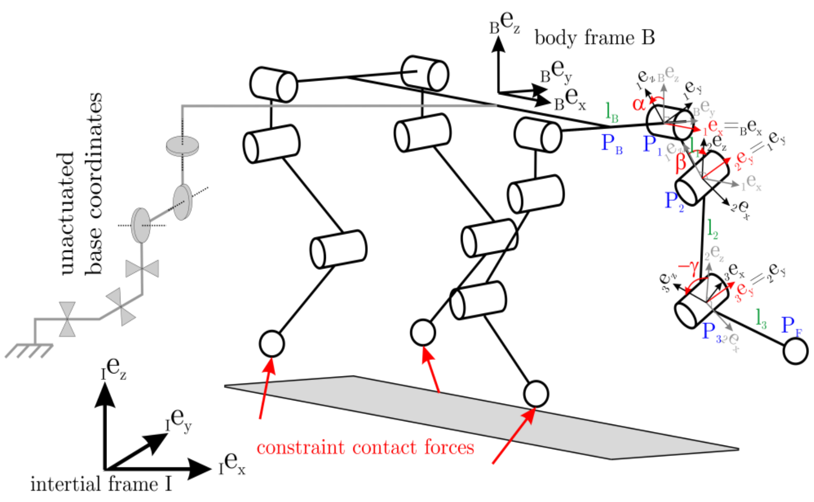
$$
\begin{aligned}
{}_I\dot{\mathbf{r}}_{OP1} &= {}_I\mathbf{J}_{OP1}\dot{\mathbf{q}} = \mathbf{0} \\
{}_I\dot{\mathbf{r}}_{OP2} &= {}_I\mathbf{J}_{OP2}\dot{\mathbf{q}} = \mathbf{0} \\
{}_I\dot{\mathbf{r}}_{OP3} &= {}_I\mathbf{J}_{OP3}\dot{\mathbf{q}} = \mathbf{0}
\end{aligned}
\qquad
\begin{aligned}
{}_I\ddot{\mathbf{r}}_{OP1} &= {}_I\mathbf{J}_{OP1}\ddot{\mathbf{q}}_I + \dot{\mathbf{J}}_{OP1}\dot{\mathbf{q}} = \mathbf{0} \\
{}_I\ddot{\mathbf{r}}_{OP2} &= {}_I\mathbf{J}_{OP2}\ddot{\mathbf{q}} + {}_I\dot{\mathbf{J}}_{OP2}\dot{\mathbf{q}} = \mathbf{0} \\
{}_I\ddot{\mathbf{r}}_{OP3} &= {}_I\mathbf{J}_{OP3}\ddot{\mathbf{q}} + {}_I\dot{\mathbf{J}}_{OP3}\dot{\mathbf{q}} = \mathbf{0}
\end{aligned}
$$

9 independent constraints
6 independent base constraints
3 DoF Swing leg
3 internal force directions

5

Quiz 2min

Inverse Differential Kinematics



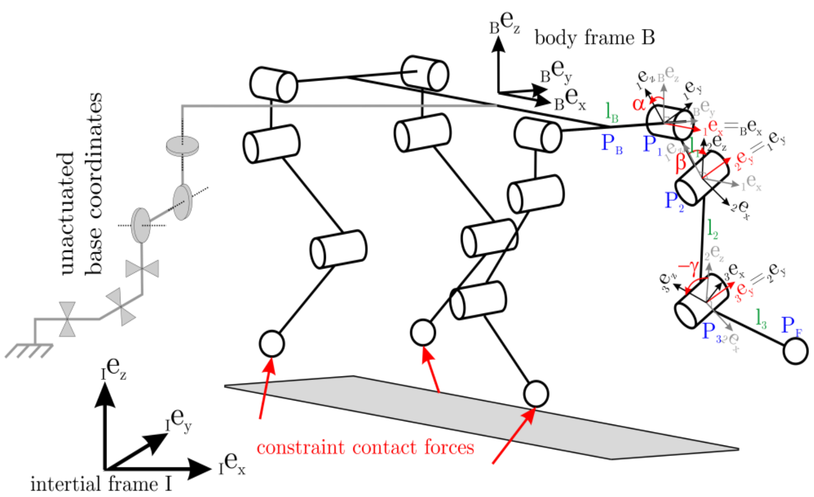
- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
- 1 in swing [NR 4]

8. Given a desired swing velocity $_I\dot{\mathbf{r}}_{OP_{F4}}^{des}$ what is the generalized velocity?

$$\dot{\mathbf{q}} = f\left(\mathbf{q}, {}_I\dot{\mathbf{r}}_{OP4}^{des}\right)$$

9. Is it unique?
10. Is it possible to follow the desired swing trajectory without moving the joints of leg 4? How?

Inverse Differential Kinematics



- Quadrupedal robot
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- 3 legs in stance [NR 1,2,3]
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$\dot{q} : 18 \times 1$

8. Given a desired swing velocity $_I\dot{\mathbf{r}}_{OP_{F4}}^{des}$ what is the generalized velocity?

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$$\begin{aligned} {}_I\dot{\mathbf{r}}_{OP1} &= {}_I\mathbf{J}_{OP1}\dot{\mathbf{q}} = \mathbf{0} \\ {}_I\dot{\mathbf{r}}_{OP2} &= {}_I\mathbf{J}_{OP2}\dot{\mathbf{q}} = \mathbf{0} \\ {}_I\dot{\mathbf{r}}_{OP3} &= {}_I\mathbf{J}_{OP3}\dot{\mathbf{q}} = \mathbf{0} \\ {}_I\dot{\mathbf{r}}_{OP4} &= {}_I\mathbf{J}_{OP4}\dot{\mathbf{q}} = {}_I\dot{\mathbf{r}}_{OP4}^{*} \end{aligned}$$

$$\dot{\mathbf{q}}_{leg4} = \begin{bmatrix}\mathbf{0} & \mathbf{I}_{3x3}\end{bmatrix}\dot{\mathbf{q}} = \mathbf{0}$$

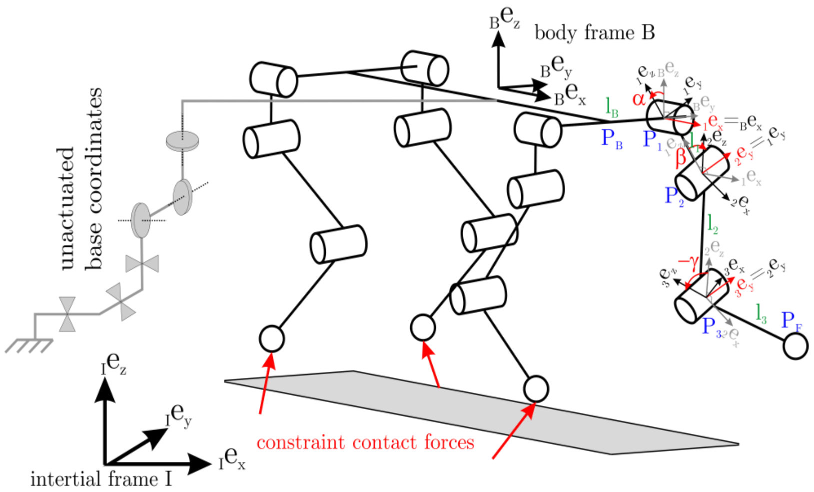
$$\mathbf{J}_1 = \begin{bmatrix} {}_I\mathbf{J}_{OP1} \\ {}_I\mathbf{J}_{OP2} \\ {}_I\mathbf{J}_{OP3} \\ {}_I\mathbf{J}_{OP4} \\ \begin{bmatrix}\mathbf{0} & \mathbf{I}_{3x3}\end{bmatrix} \end{bmatrix}, \mathbf{w}_1 = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ {}_I\dot{\mathbf{r}}_{OP4}^{des} \\ \mathbf{0} \end{pmatrix} \qquad \dot{\mathbf{q}} = \mathbf{J}_1^{+}\mathbf{w}_1$$

There exist multiple solutions that fulfill task 8

5

Quiz 2min

Kinematic Singularity



- There exist different formulations for “moving the foot (task1) while keeping the base position and orientation (task2)”. Write down the solution for A and B!
- What is the difference?
- What happens in singular configs?

Task 1:

- contact constraints
- foot motion

$$\mathbf{J}_1 = \begin{bmatrix} {}_I\mathbf{J}_{OP1} \\ {}_I\mathbf{J}_{OP2} \\ {}_I\mathbf{J}_{OP3} \\ {}_I\mathbf{J}_{OP4} \end{bmatrix}, \mathbf{w}_1 = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ {}_I\dot{\mathbf{r}}_{OP4}^{des} \end{pmatrix}$$

Task 2:

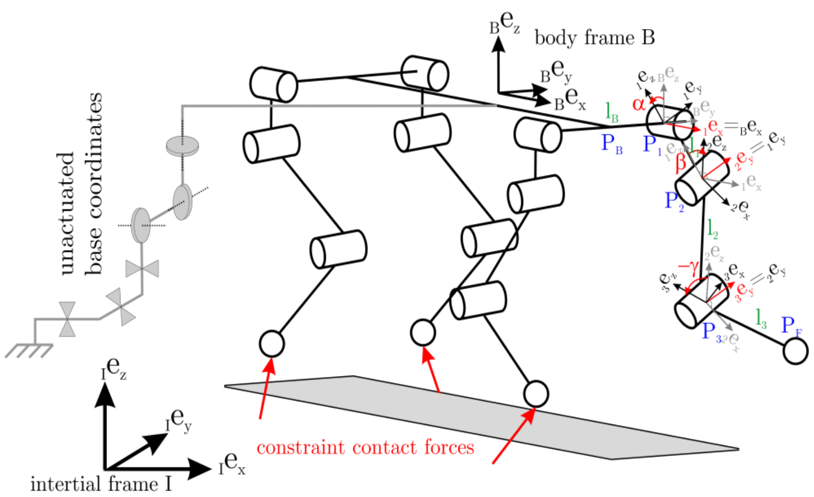
- base position
- base orientation

$$\mathbf{J}_2 = \begin{bmatrix} {}_I\mathbf{J}_{OB} \\ {}_I\mathbf{J}_{BRot} \end{bmatrix}, \mathbf{w}_2 = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

A) Single Task

B) Multiple Tasks

Kinematic Singularity



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Task 1:

- contact constraints
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$$\mathbf{J}_1 = \begin{bmatrix} {}_I\mathbf{J}_{OP1} \\ {}_I\mathbf{J}_{OP2} \\ {}_I\mathbf{J}_{OP3} \\ {}_I\mathbf{J}_{OP4} \end{bmatrix}, \mathbf{w}_1 = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ {}_I\dot{\mathbf{r}}_{OP4}^{des} \end{pmatrix}$$

Task 2:

- base position
- base orientation

$$\mathbf{J}_2 = \begin{bmatrix} {}_I\mathbf{J}_{OB} \\ {}_I\mathbf{J}_{BRot} \end{bmatrix}, \mathbf{w}_2 = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

A) Single Task

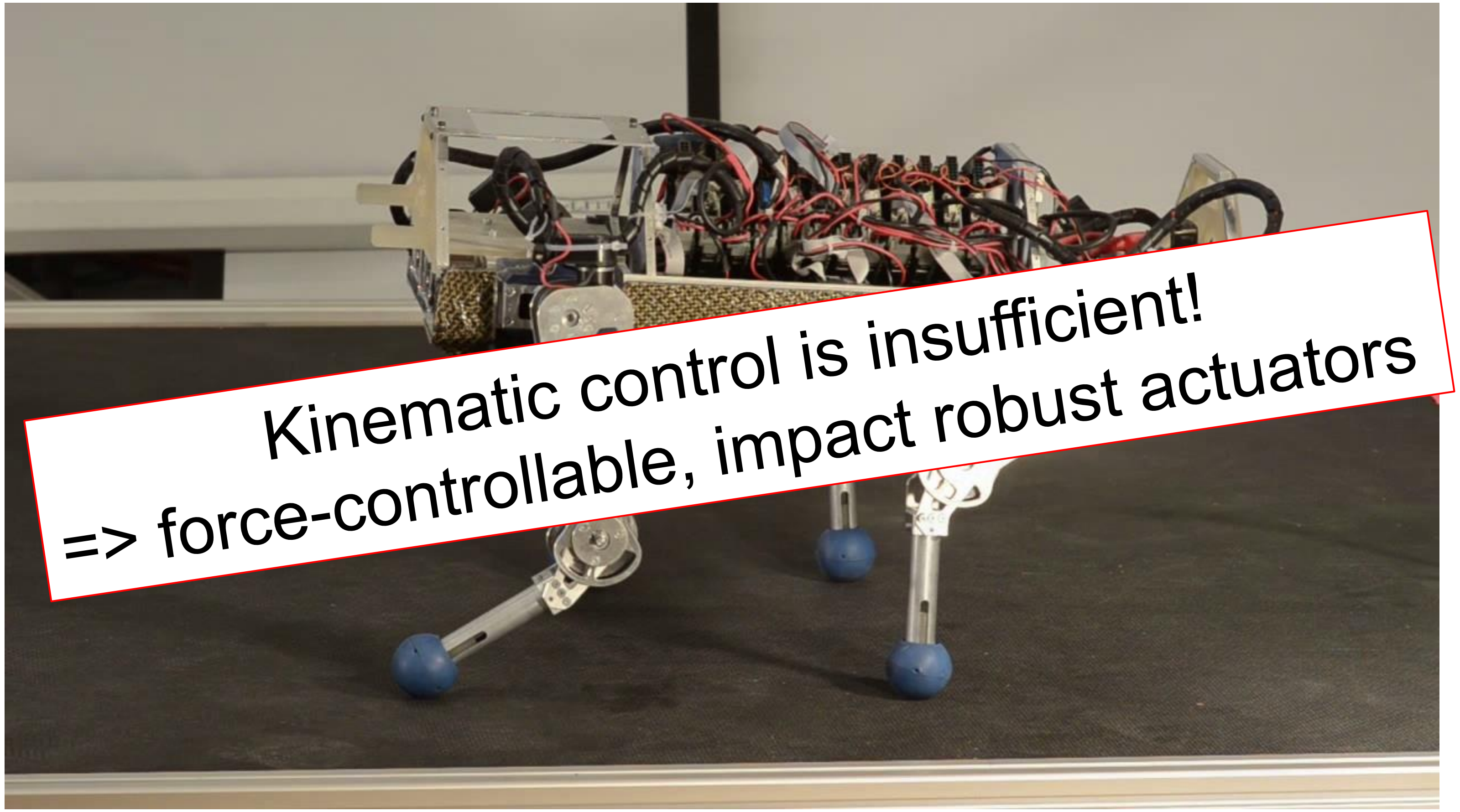
$$\dot{\mathbf{q}} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \end{bmatrix}^+ \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{pmatrix} \qquad \min \left\| \mathbf{J}\dot{\mathbf{q}} - \mathbf{w} \right\|^2$$

B) Multiple Tasks

$$\dot{\mathbf{q}} = \mathbf{J}_1^+ \mathbf{w}_1 + \mathbf{N}_1 \dot{\mathbf{q}}_0 \qquad \min \left\| \dot{\mathbf{q}} \right\|^2$$

$$\mathbf{J}_2 \dot{\mathbf{q}} = \mathbf{J}_2 \left(\mathbf{J}_1^+ \mathbf{w}_1 + \mathbf{N}_1 \dot{\mathbf{q}}_0 \right) = \mathbf{w}_2 \qquad \min \left\| \mathbf{J}_2 \dot{\mathbf{q}} - \mathbf{w}_2 \right\|^2$$

$$\dot{\mathbf{q}}_0 = \left(\mathbf{J}_2 \mathbf{N}_1 \right)^+ \left(\mathbf{w}_2 \cancel{+} \mathbf{J}_2 \mathbf{J}_1^+ \mathbf{w}_1 \right) \qquad \text{s.t.} \quad \mathbf{J}_1 \dot{\mathbf{q}} - \mathbf{w}_1 = \mathbf{0}$$


Kinematic control is insufficient!
=> force-controllable, impact robust actuators

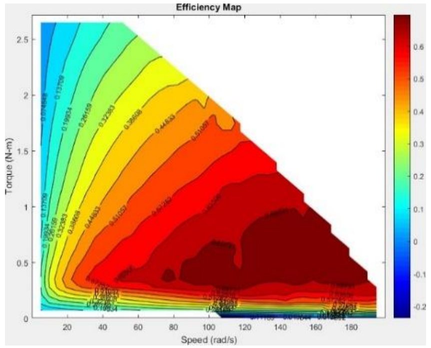
Actuation principles in legged robots

- The ideal actuator for versatile dynamic legged robots
  - Ideal Torque source (high bandwidth and versatility)
  - Energy Efficiency
  - High maximum joint torque
  - High maximum joint velocity
  - Small size and weight
  - Robustness (to impacts,failure)
  - Large range of motion
  - Low price
  - User friendliness

Actuation principles in legged robots

Geared motor

- High-geared motor with torque sensor
  - \+ Very compact
  - \+ Motor can be operated at high speed
  - – High reflected inertia
  - – Low gearbox efficiency
  - – Impact loads can destroy the gear



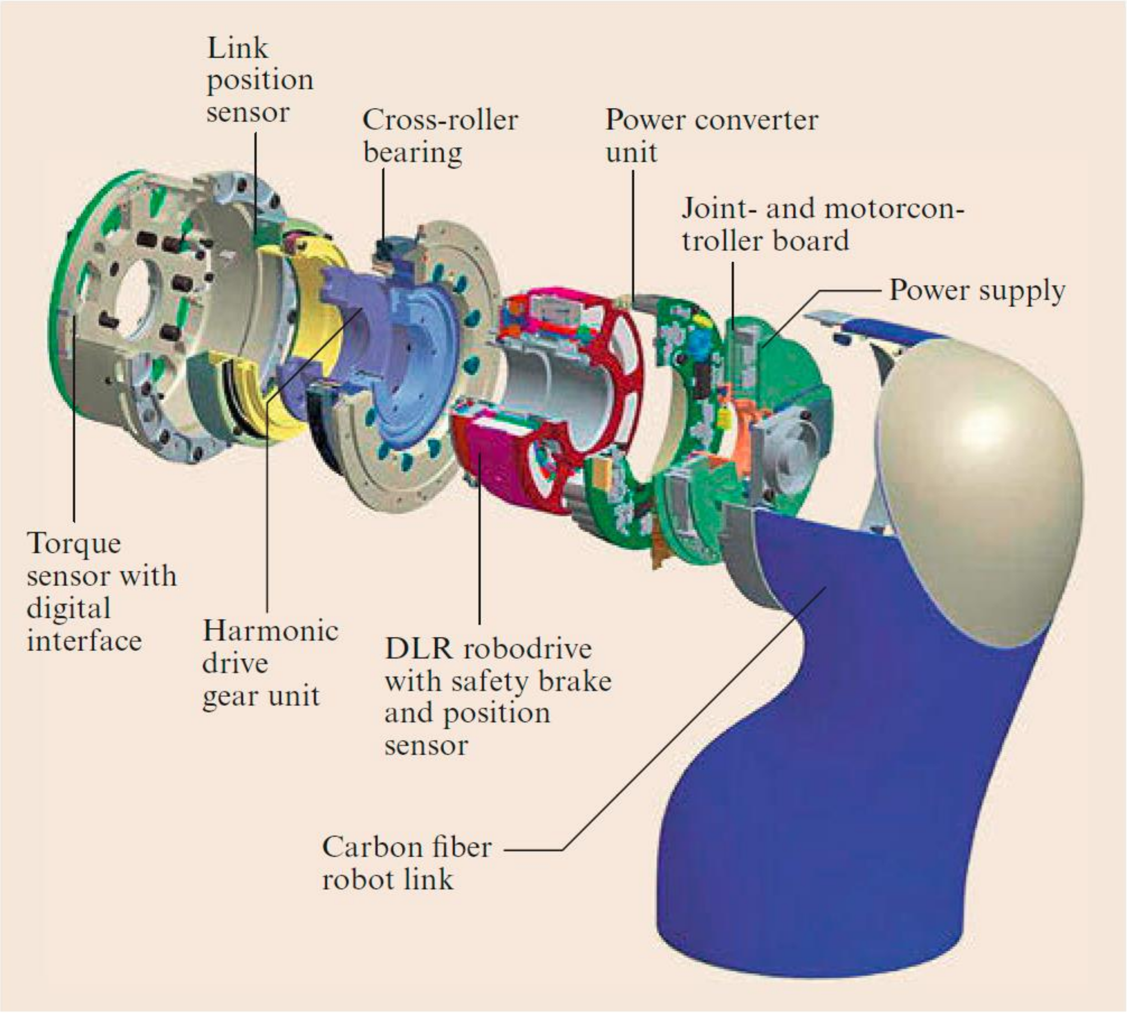


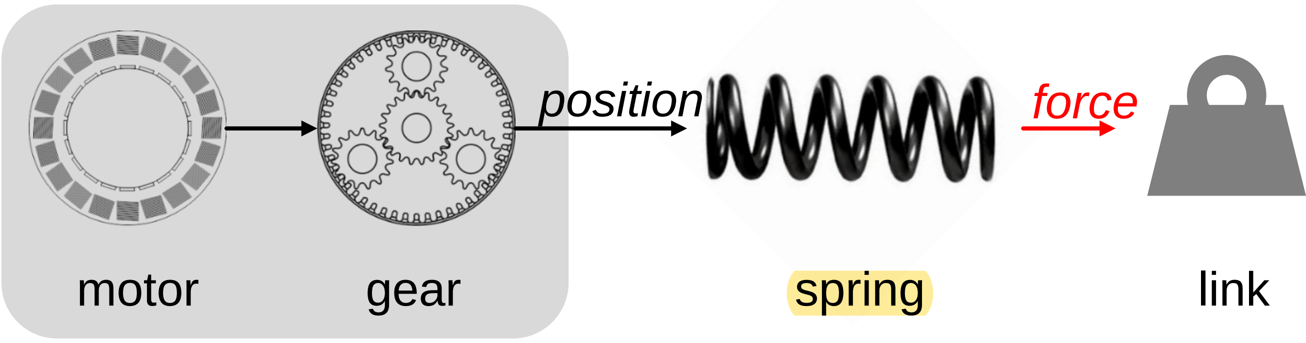
Fig. 11.8 Exploded view of a joint of the *DLR LWR-III* lightweight manipulator and its sensor suite

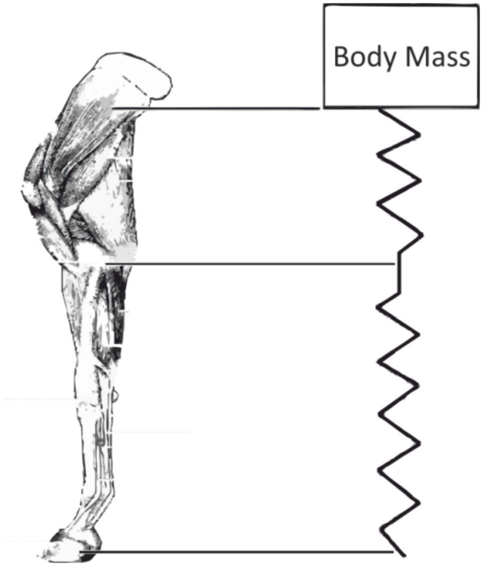
SCHAFT

Actuation principles in legged robots

Series Elastic Actuator

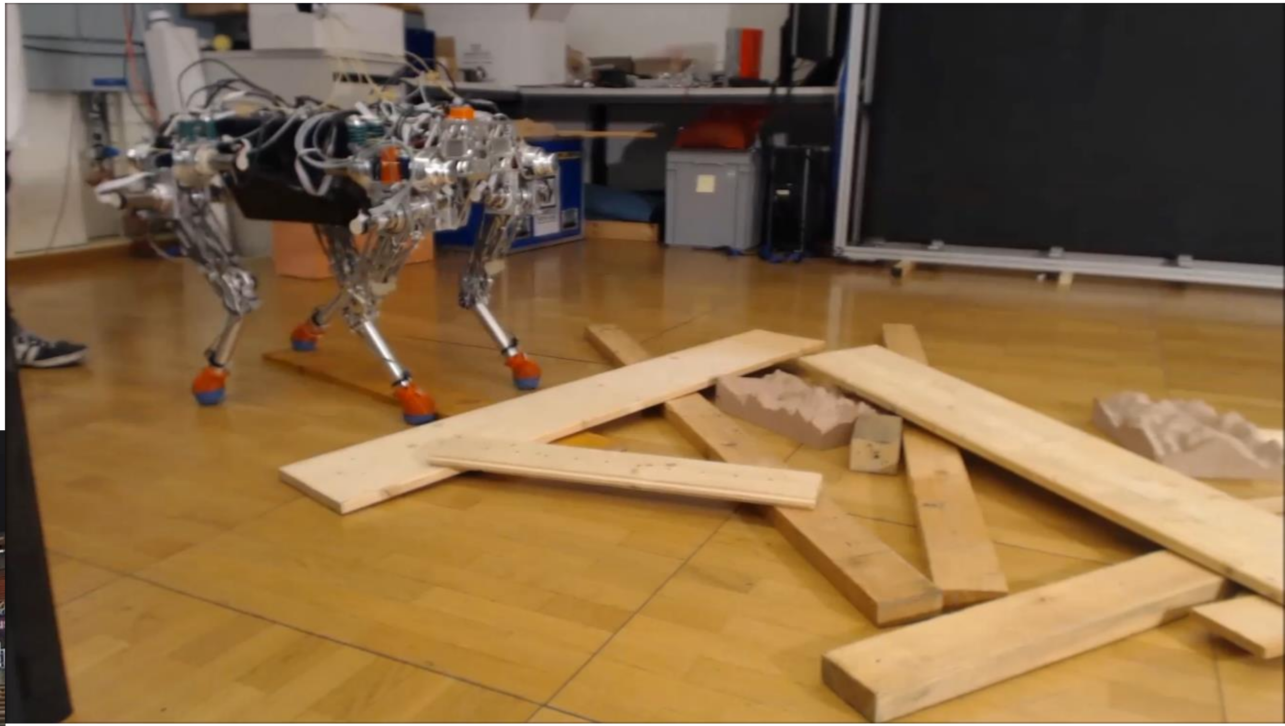
- High-geared motor with serial spring
  - \+ Very compact
  - \+ Precise torque regulation
  - \+ Decoupled actuator and link inertia
    - \+ robustness
  - Additional spring dynamics
    - Temporary energy storage
    - Power/speed amplification
  - – Low control bandwidth





Actuation principles in legged robots

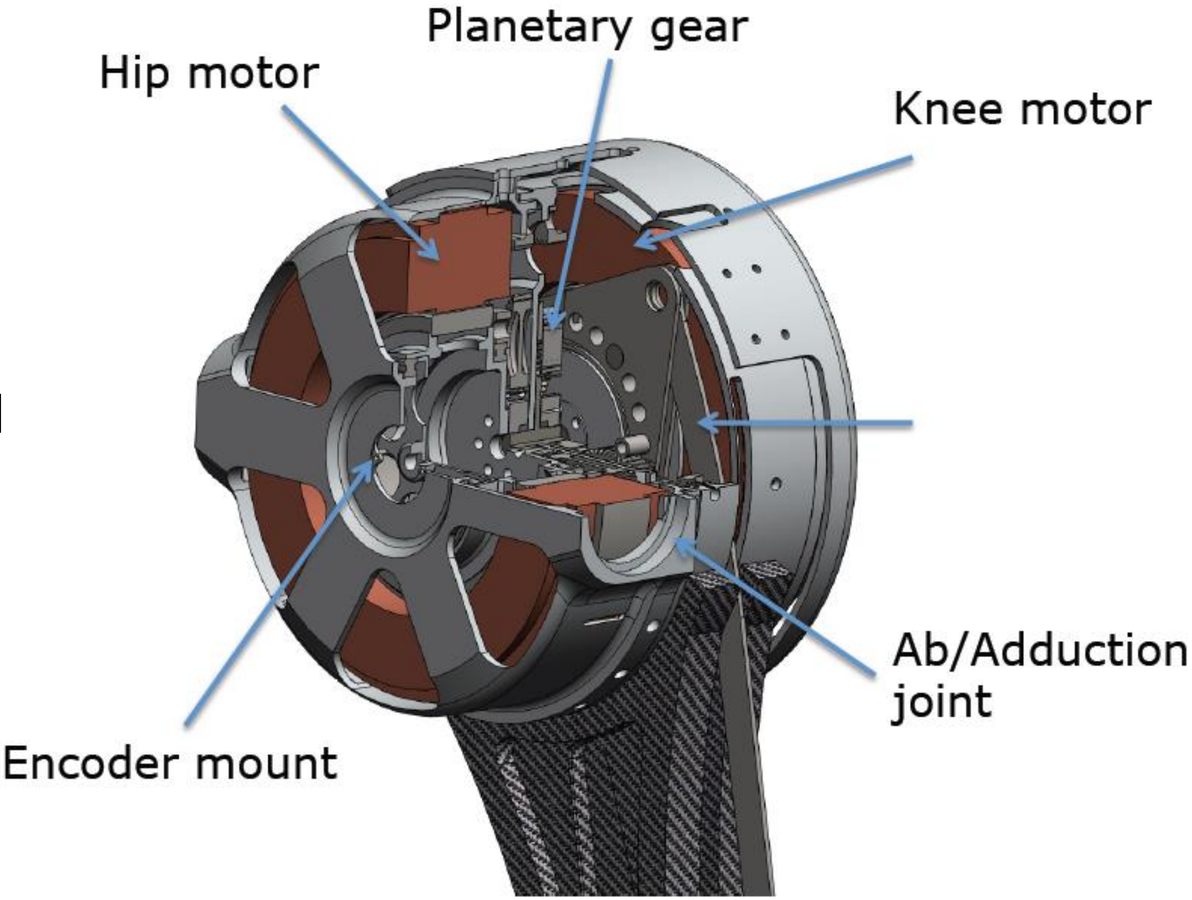
Series Elastic Actuator



Actuation principles in legged robots

Pseudo direct drive

- Low-geared high-torque motor
  - Low reflected inertia due to low gear ratio
    - Impact robust
    - High speed and power
  - High-bandwidth current control ⇔ force control
  - Relatively big, hard to integrate



Actuation principles in legged robots
Pseudo direct drive

- Low-geared high-torque motor
  - Low reflected inertia due to low gear ratio
    - Impact robust
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  - High-bandwidth current control ⇔ force control
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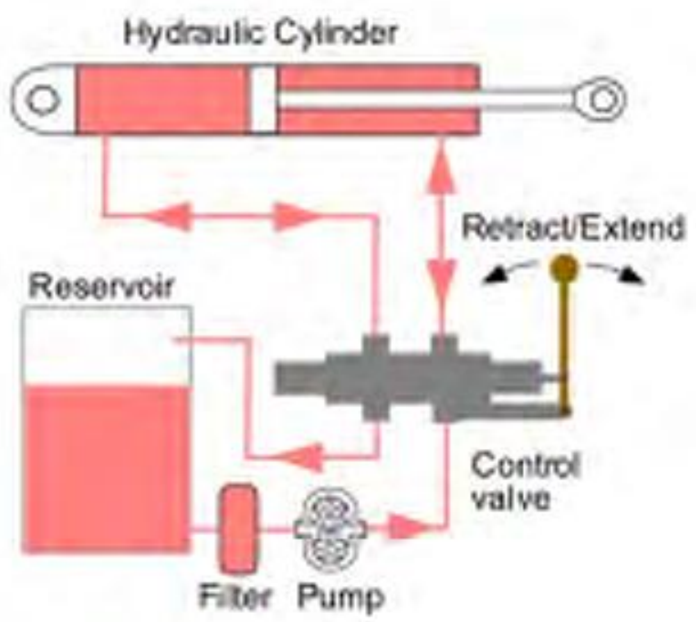
Actuation principles in legged robots

Hydraulic Actuation

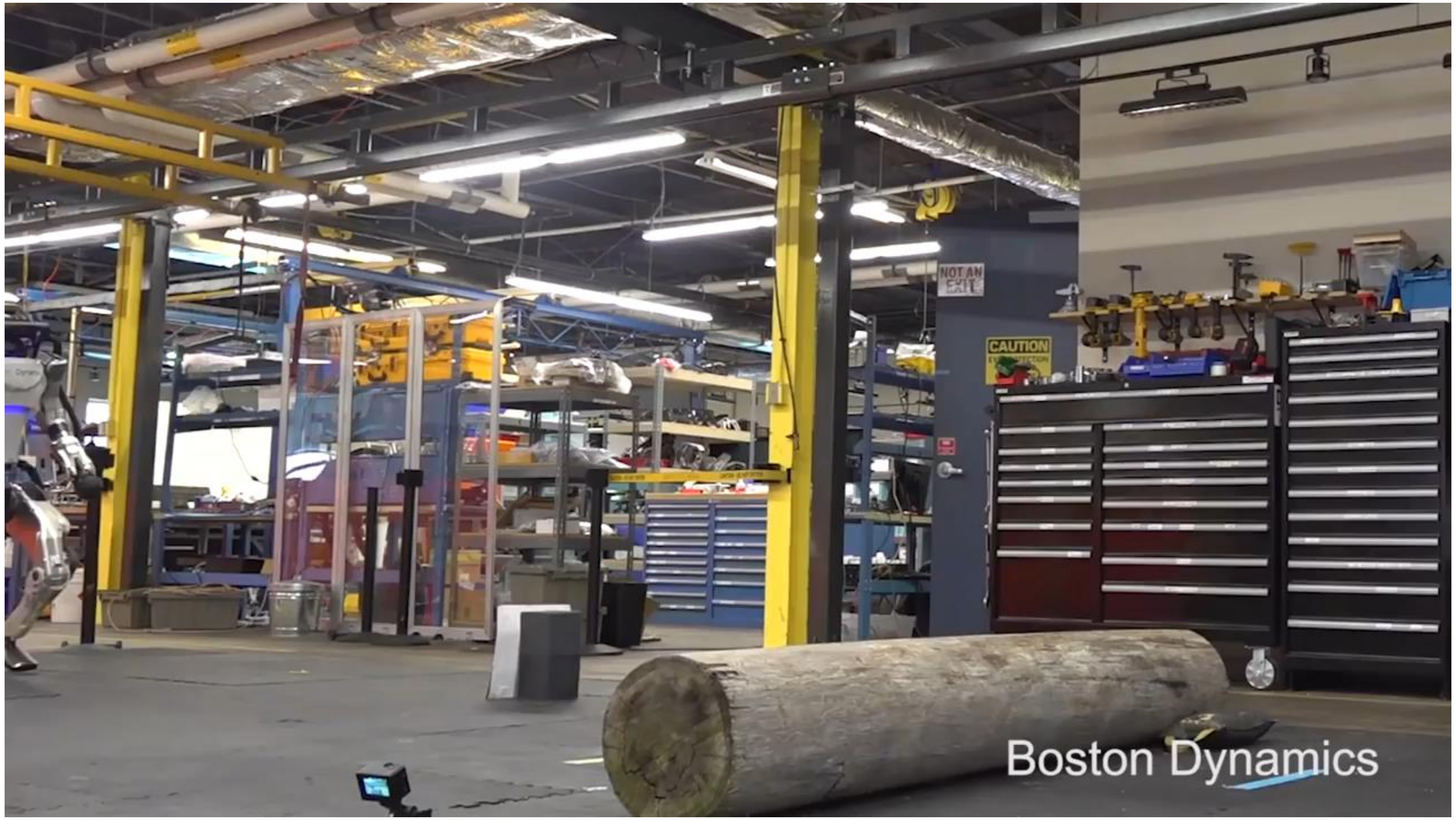
- Hydraulic actuation
  - + High force at small size/weight
  - + Very rugged
  - + Pressure sensor provides direct force feedback
  - – Onboard pump required
  - – Hard to downscale
  - – Energetically inefficient
  - – Can leak



P_max = 200bar = 20N/mm
r=1cm
A=pi*r^2 = 3.14cm2=314mm2
=> Fmax = 6kN!!

Actuation principles in legged robots

Hydraulic Actuation


NOT AN EXIT
CAUTION
Boston Dynamics

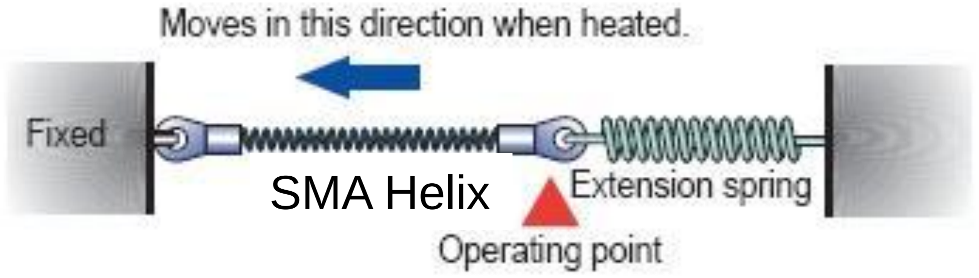
Actuation principles in legged robots

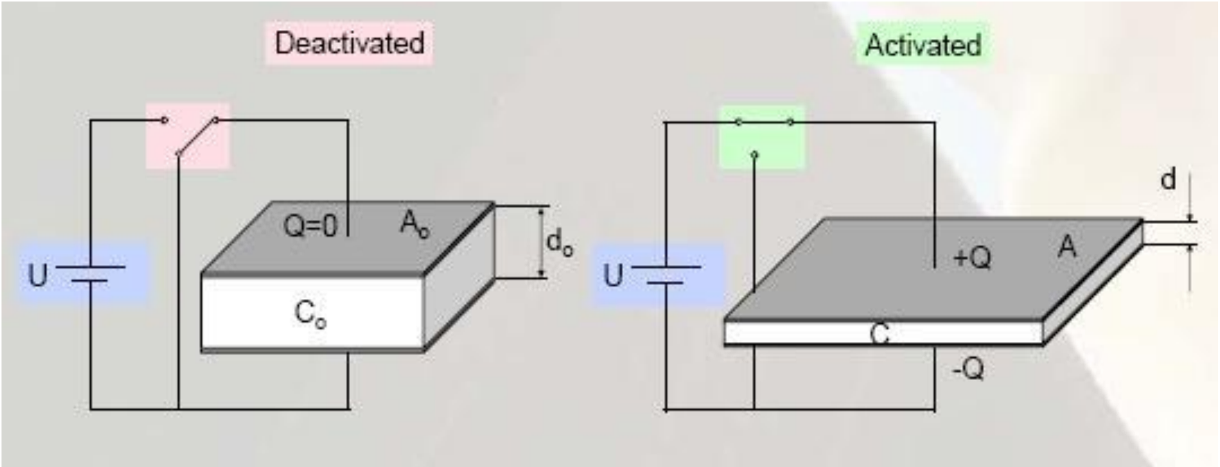
Pneumatics

- Pneumatic Muscle Actuators
  - \+ light weight
  - \+ high maximum contraction force
  - – often with off-board pump
  - – works only in contraction
  - – highly non-linear contraction-force-pressure characteristics
  - – difficult to control
  - – can be quite loud

Actuation principles in legged robots
Other types of actuators

- New, unconventional actuators types
  - Shape Memory Alloy (SMA)
  - Electro-Active Polymer (EAP)
  - Piezo-electric
- Open Issues
  - Low output force levels
  - Low displacement (strain)
  - Need kV power supplies
  - Low control bandwidth



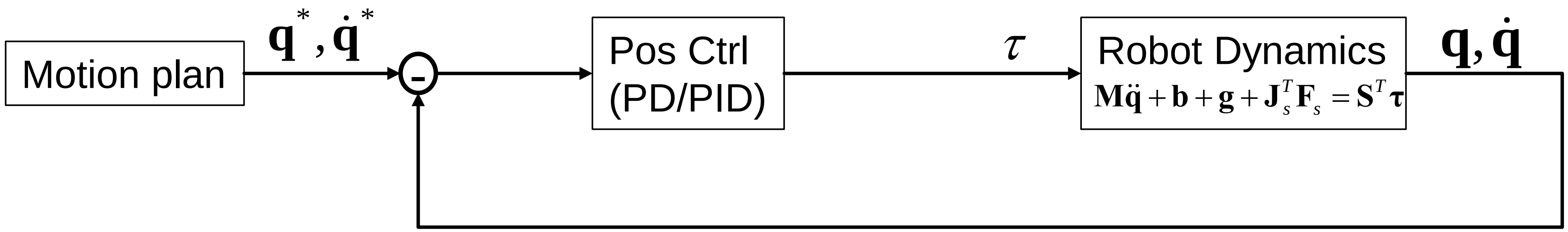


Dielectric EAP actuator in deactivated ***(left)*** *and activated state* ***(right)****.*

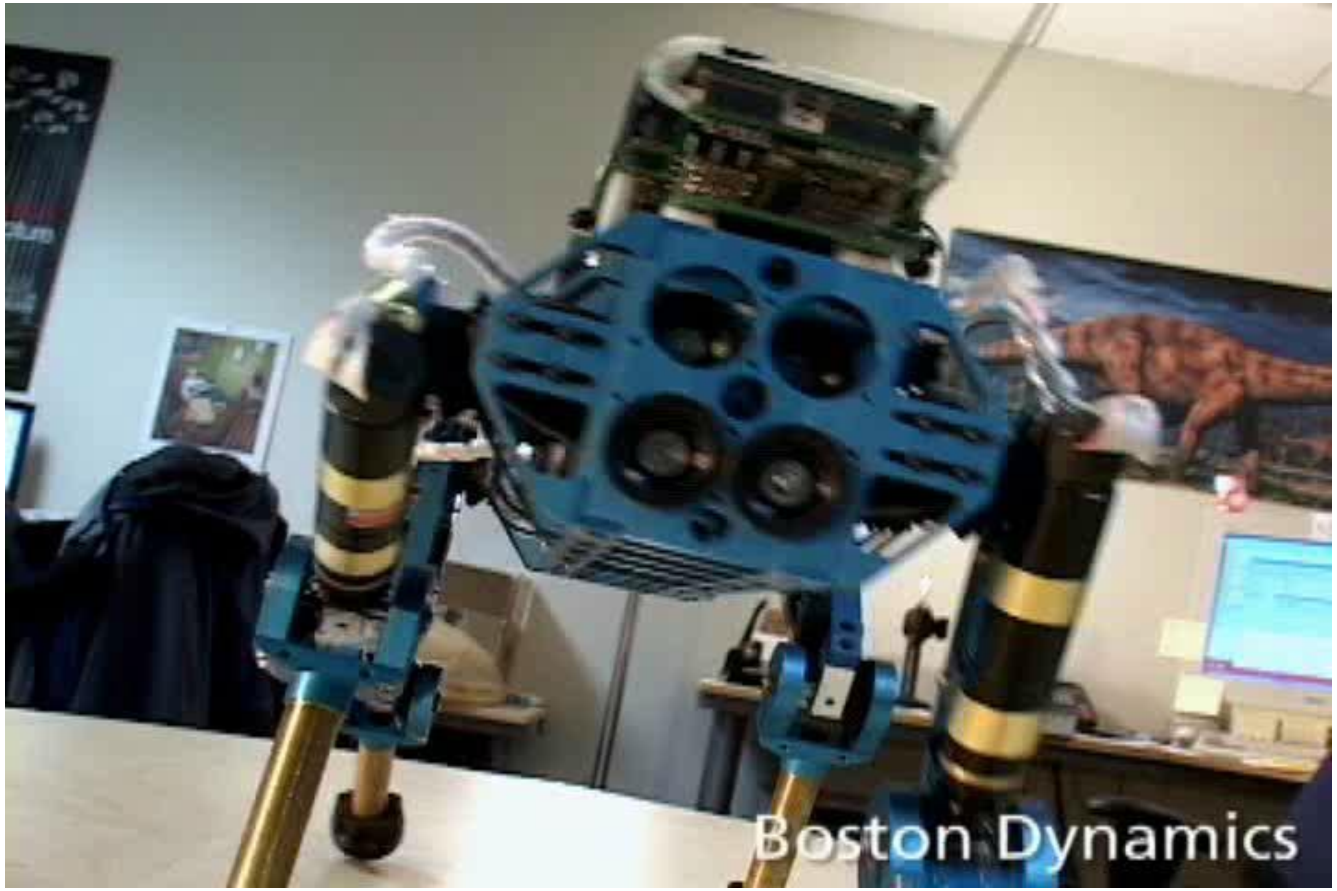
Control Concepts
an overview

- Kinematic control:
  - High-gain joint position trajectory tracking
- Impedance control with joint space inverse dynamics
  - Low-gain joint control with model compensation
- Support-consistent inverse dynamics control
  - Projection of dynamics and desired acceleration into the null-space of contact constraint
- Task-space inverse dynamics control
  - Directly regulating in «task space» as sequential QP

Motion planning and high-gain kinematic trajectory following



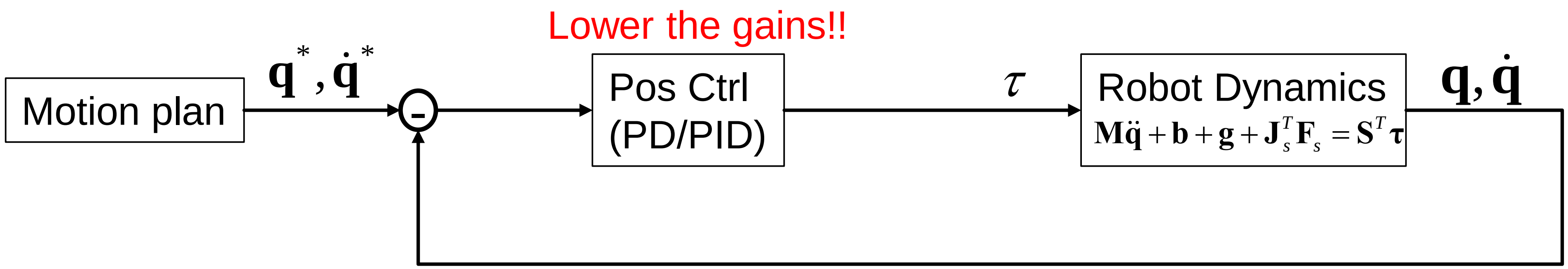
Motion planning and high-gain kinematic trajectory following



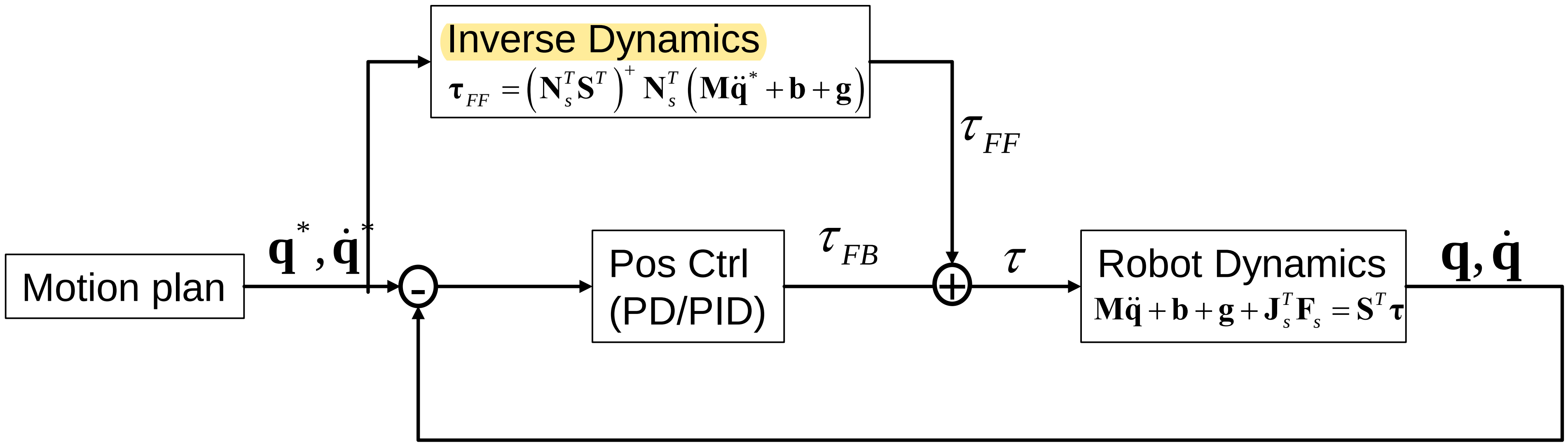
Motion planning and high-gain kinematic trajectory following

Unperceived/unplanned obstacles

Low-gain joint control with model compensation



Low-gain joint control with model compensation



Low-gain kinematic trajectory following + Inv. Dynamics
Unperceived/unplanned obstacles

Support consistent inverse dynamics control

Joint acceleration from multiple objectives

- Find the desired acceleration that:
  - track the swing leg $\ddot{\mathbf{r}}_{OF} = \mathbf{J}_F\ddot{\mathbf{q}} + \dot{\mathbf{J}}_F\dot{\mathbf{q}} = \ddot{\mathbf{r}}_{OF,des}(t) = k_p\left(\mathbf{r}^* - \mathbf{r}\right) + k_d\left(\dot{\mathbf{r}}^* - \dot{\mathbf{r}}\right) + \ddot{\mathbf{r}}^*$
  - to move the base $\dot{\mathbf{w}}_B = \mathbf{J}_B\ddot{\mathbf{q}} + \dot{\mathbf{J}}_B\dot{\mathbf{q}} = \dot{\mathbf{w}}_{B,des}(t) = k_p\left(\begin{pmatrix}\mathbf{r}\\ \boldsymbol{\varphi}\end{pmatrix}^* - \begin{pmatrix}\mathbf{r}\\ \boldsymbol{\varphi}\end{pmatrix}\right) + k_d\left(\mathbf{w}^* - \mathbf{w}\right) + \dot{\mathbf{w}}^*$
  - ensure contact constraint $\ddot{\mathbf{r}}_c = \mathbf{J}_c\ddot{\mathbf{q}} + \dot{\mathbf{J}}_c\dot{\mathbf{q}} = \mathbf{0}$

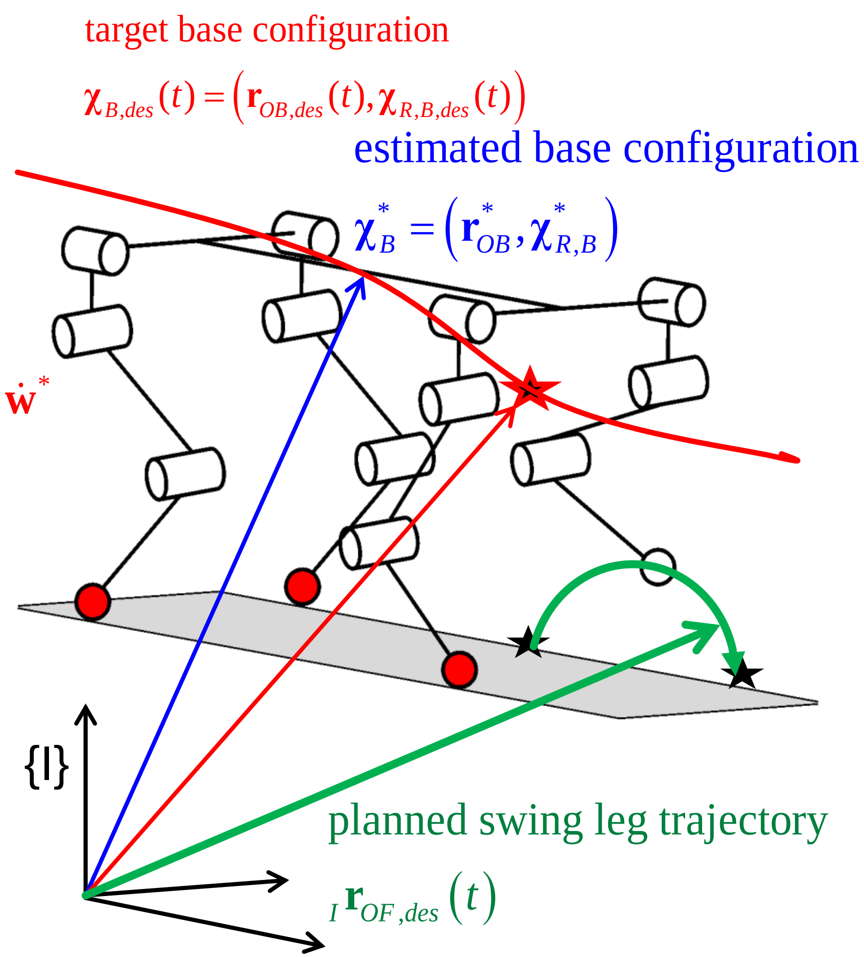
  $\Rightarrow$ fully defines the motion of the system

  12 actuated + 6 unactuated = 18 DOF
  6 motion + 9 constraints + 3 swing leg = 18

If more tasks, use pseudo-inverse

$$\ddot{\mathbf{q}}_{des} = \begin{bmatrix}\mathbf{J}_F\\ \mathbf{J}_B\\ \mathbf{J}_c\end{bmatrix}^{-1}\left(\begin{pmatrix}\ddot{\mathbf{r}}_{OF,des}(t)\\ \dot{\mathbf{w}}_{B,des}(t)\\ 0\end{pmatrix} - \begin{bmatrix}\dot{\mathbf{J}}_F\\ \dot{\mathbf{J}}_B\\ \dot{\mathbf{J}}_c\end{bmatrix}\dot{\mathbf{q}}\right)$$

- Inverse dynamics control $\boldsymbol{\tau} = \left(\hat{\mathbf{N}}_c^T\mathbf{S}^T\right)^+ \hat{\mathbf{N}}_c^T\left(\hat{\mathbf{M}}\ddot{\mathbf{q}}_{des} + \hat{\mathbf{b}} + \hat{\mathbf{g}}\right)$



Operational Space Control as Quadratic Program

A general problem

$$\boxed{\min_{\mathbf{x}} \quad \|\mathbf{A}_i\mathbf{x} - \mathbf{b}_i\|_2 \qquad \mathbf{x} = \begin{pmatrix} \dot{\mathbf{u}} \\ \mathbf{F}_c \\ \boldsymbol{\tau} \end{pmatrix}}$$

- We search for a solution that fulfills the equation of motion

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{u}} + \mathbf{b}(\mathbf{q},\mathbf{u}) + \mathbf{g}(\mathbf{q}) + \mathbf{J}_c^T\mathbf{F}_c = \mathbf{S}^T\boldsymbol{\tau} \quad \Rightarrow \mathbf{A} = \begin{bmatrix} \hat{\mathbf{M}} & \hat{\mathbf{J}}_c^T & -\mathbf{S}^T \end{bmatrix} \qquad \mathbf{b} = -\hat{\mathbf{b}} - \hat{\mathbf{g}}$$

- Motion tasks: $\mathbf{J}\dot{\mathbf{u}} + \dot{\mathbf{J}}\mathbf{u} = \dot{\mathbf{w}}^* \quad \Rightarrow \mathbf{A} = \begin{bmatrix} \hat{\mathbf{J}}_i & \mathbf{0} & \mathbf{0} \end{bmatrix} \qquad \mathbf{b} = \dot{\mathbf{w}}^* - \hat{\dot{\mathbf{J}}}_i\mathbf{u}$
- Force tasks: $\mathbf{F}_i = \mathbf{F}_i^* \quad \Rightarrow \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbb{I} & \mathbf{0} \end{bmatrix} \qquad \mathbf{b} = \mathbf{F}_i^*$
- Torque min: $\min\|\boldsymbol{\tau}\|_2 \quad \Rightarrow \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbb{I} \end{bmatrix} \qquad \mathbf{b} = \mathbf{0}$

Locomotion as Multi-task Optimization Problem

- Write inverse dynamics as constraint (prioritized) optimization
  - Step 1: move base

$$\min_{\ddot{\mathbf{q}}} \left\| \dot{\mathbf{w}}_{B,des}(t) - \mathbf{J}_B \ddot{\mathbf{q}} - \dot{\mathbf{J}}_B \dot{\mathbf{q}} \right\|$$

s.t.

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} + \mathbf{J}_c^T \mathbf{F}_c = \mathbf{S}^T \boldsymbol{\tau}$$ <= equation of motion holds

$$\mathbf{J}_B \ddot{\mathbf{q}} + \dot{\mathbf{J}}_B \dot{\mathbf{q}} = \mathbf{0}$$ <= contact constraint holds

$$\mathbf{F}_{c,n_i} > F_{n,\min}$$ <= minimal normal contact force

$$\mu \mathbf{F}_{c,n_i} > \left\| \mathbf{F}_{c,t_i} \right\|_2$$ <= contact force in friction cone

  - Step 2: move swing leg

$$\min_{\ddot{\mathbf{q}}} \left\| \ddot{\mathbf{r}}_{OF,des}(t) - \mathbf{J}_F \ddot{\mathbf{q}} - \dot{\mathbf{J}}_F \dot{\mathbf{q}} \right\|$$

s.t.

$$c_1 = \ddot{\mathbf{x}}_{B,des}(t) - \mathbf{J}_B \ddot{\mathbf{q}} - \dot{\mathbf{J}}_B \dot{\mathbf{q}}$$ <= higher priority task is not influenced

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} + \mathbf{J}_c^T \mathbf{F}_c = \mathbf{S}^T \boldsymbol{\tau}$$ <= equation of motion holds

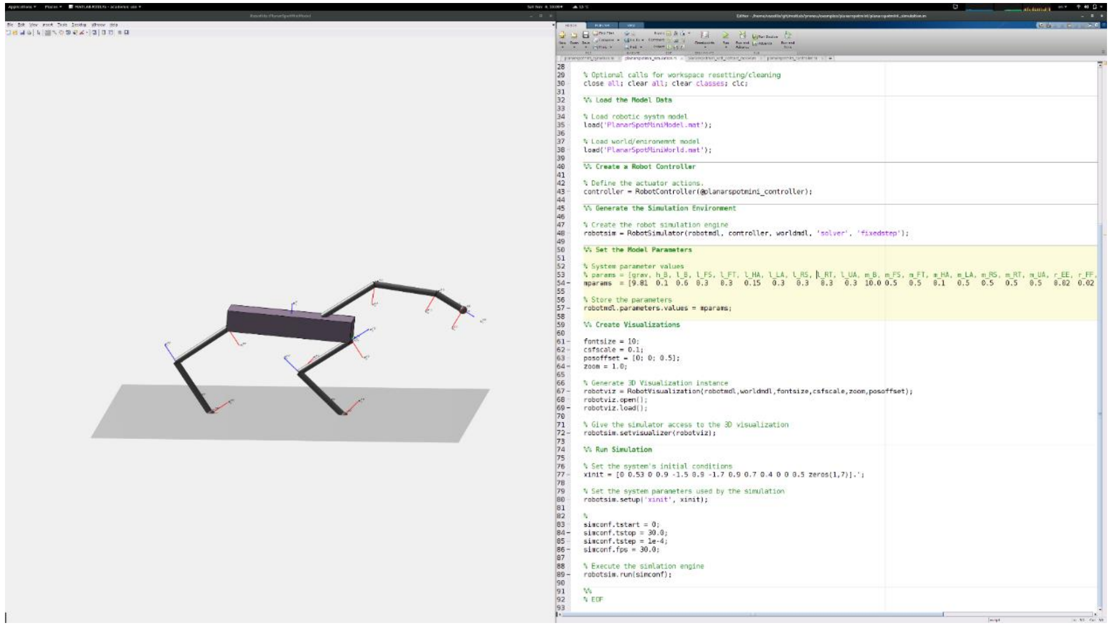
$$\mathbf{F}_{c,n_i} > F_{n,\min}$$ <= minimal normal contact force

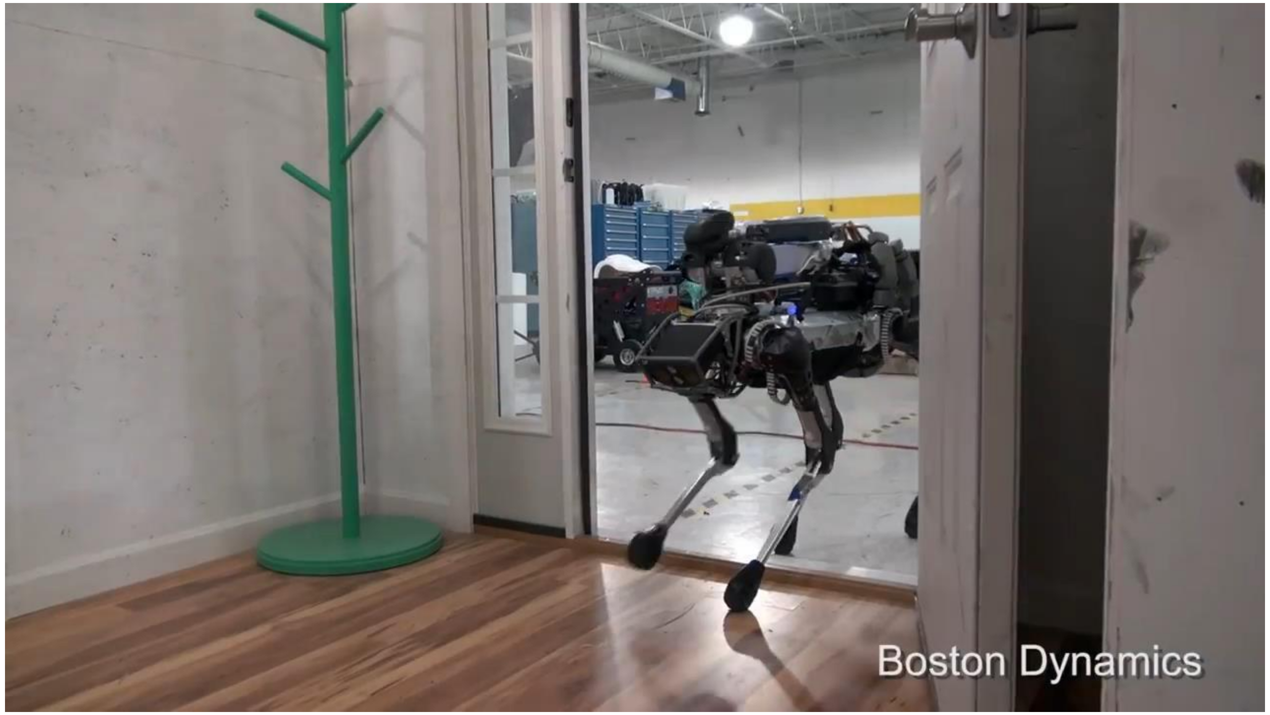
$$\mu \mathbf{F}_{c,n_i} > \left\| \mathbf{F}_{c,t_i} \right\|_2$$ <= contact force in friction cone

  - Last step: minimize e.g torque $\min \|\boldsymbol{\tau}\|$ or tangential contact forces $\min \left\| \mathbf{F}_{s,t_i} \right\|$

    s.t. all other task are still fulfilled

Exercise next week



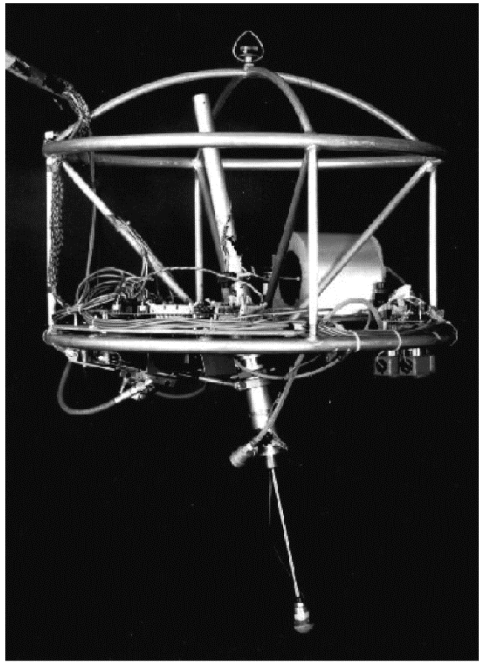


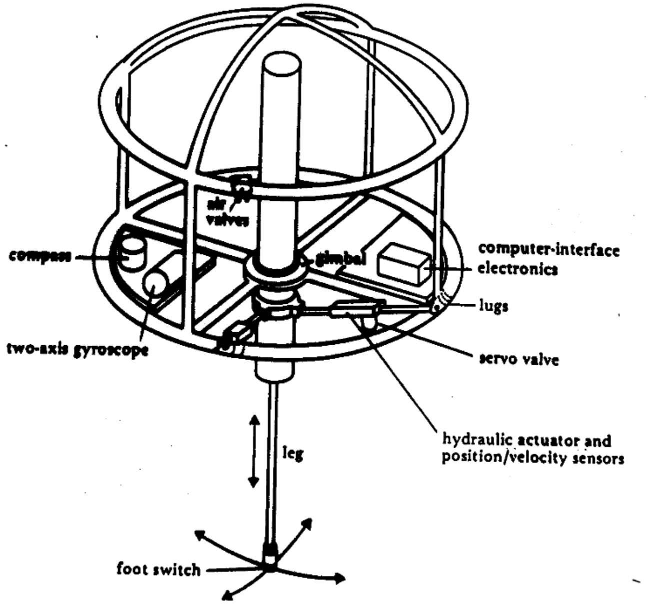
Stability

- How is stability defined?
  - Difference of static vs dynamic stability
  - Limit cycle analysis: look at running on a step-to-step basis

Dynamic Locomotion
SLIP principles in robotics

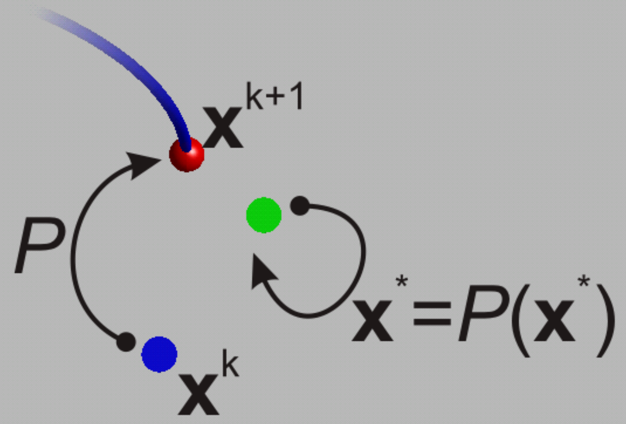
- Early Raibert hoppers (MIT leg lab) [1983]
  - Pneumatic piston
  - Hydraulic leg “angle” orientation

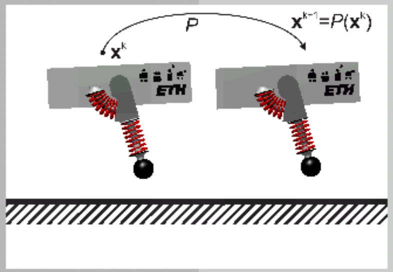




Analyzing Stability through Limit Cycles

- Poincaré Map $\mathbf{x}_{k+1} = P\left(\mathbf{x}_k\right)$
- Fix-Point $\mathbf{x}^* = P\left(\mathbf{x}^*\right)$
- Linearization of mapping $\Delta\mathbf{x}_{k+1} = \dfrac{\partial P}{\partial \mathbf{x}} \Delta\mathbf{x}_k = \mathbf{\Phi}\Delta\mathbf{x}_k$
- The system is stable iff: $\lambda_i\left(\mathbf{\Phi}\right) < 1$

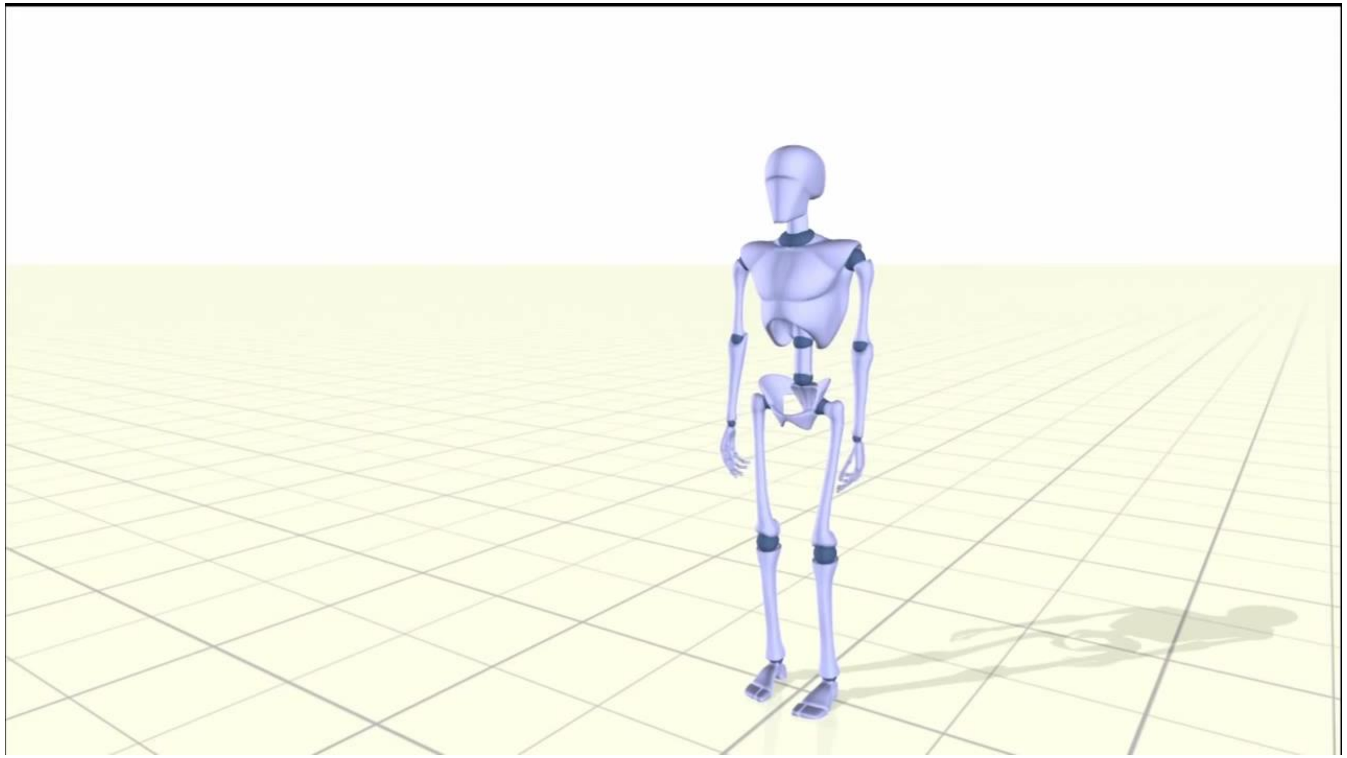




[C. David Remy, 2011]

Stability and Control of Locomotion
Example of a biped

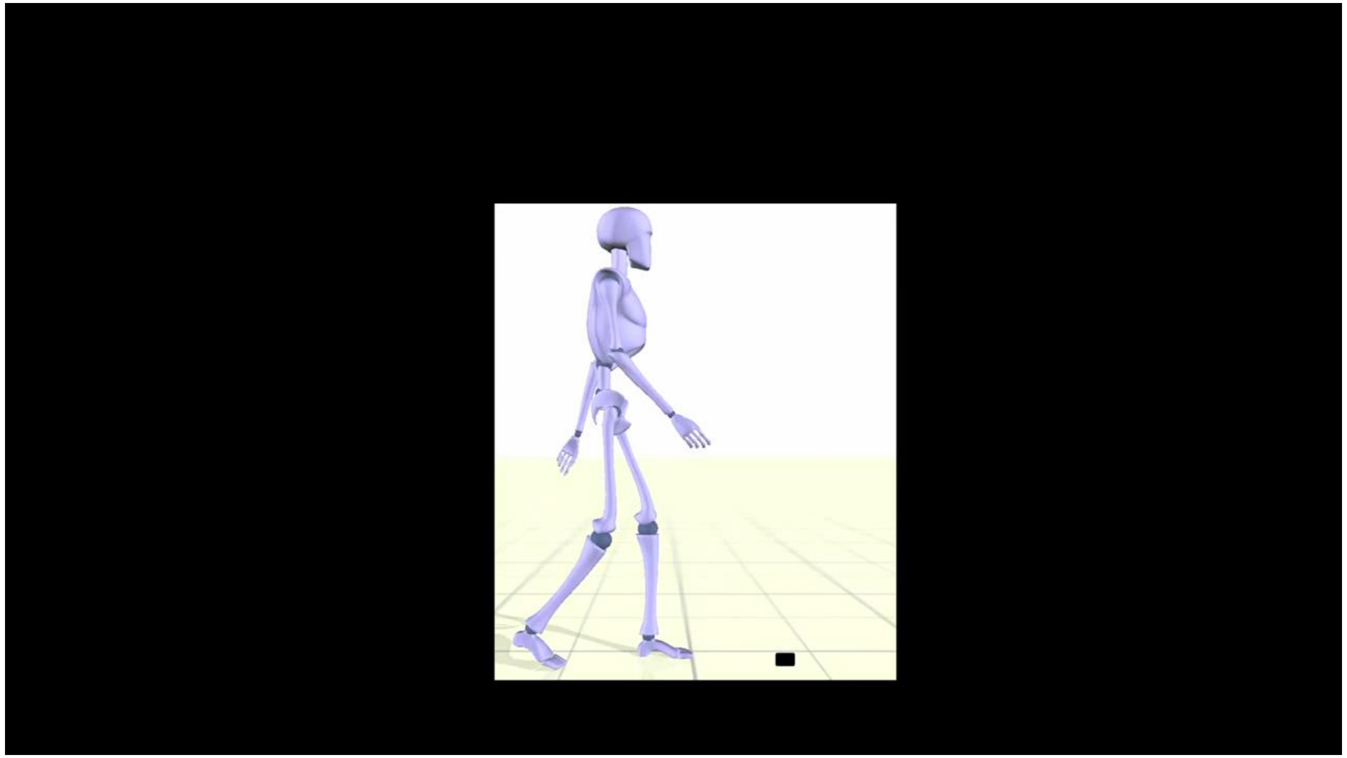
- External disturbances lead to instability



[Coros 2012]

Stability and Control of Locomotion
Example of a biped

- External disturbances lead to instability
- Foot step control for fall recovery



[Coros 2012]

Stability and Control of Locomotion

Example of a quadruped

- Dynamic gaits
  - Inverted pendulum

[Raibert 1986]

$$\mathbf{r}_F = \frac{1}{2}\dot{\mathbf{r}}_{HC,des}T_{st} + k_R^{FB}\left(\dot{\mathbf{r}}_{HC,des} - \dot{\mathbf{r}}_{HC}\right)\sqrt{h_{HC}}$$