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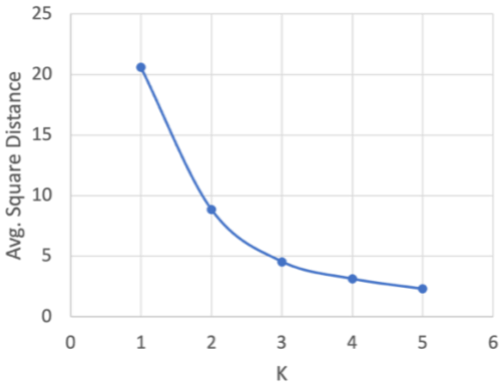
Theoretical:

Q1: Give a set of data points shown in figure 1, plot how the **average square distance** of a data sample with respect to its corresponding cluster center change with different k values. The initialization for each k is in table. Kindly use `sklearn.cluster.KMeans` to help for calculation.

Answers:

k	Init centers	Avg. square distance
1	(4,3)	20.58
2	(4,3), (8,10)	8.86
3	(4,3), (8,10), (9,7)	4.55
4	(4,3), (8,10), (9,7), (3,9)	3.13
5	(4,3), (8,10), (9,7), (3,9),(4,8)	2.3

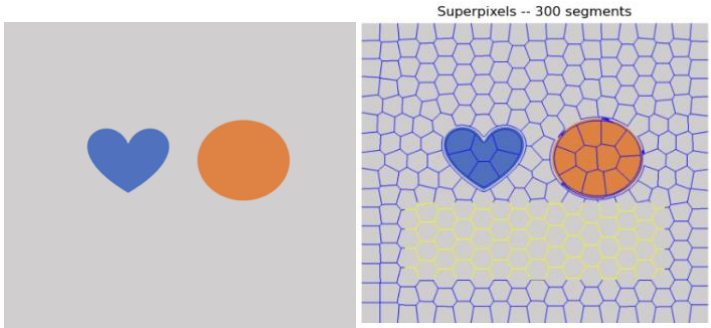


Step-by-step Explanation for K=2:
Data points: [1,5], [2,2], [3,9], [4,8], [6,6], [4,3], [8,10], [9,7], [11,5], [13,4]

iteration	centers	labels
0	(4,3), (8,10)	[0, 0, 1, 1, 0, 0, 1, 1, 1, 1]
1	(3.25, 4.0), (8.0, 7.17)	[0, 0, 0, 0, 1, 0, 1, 1, 1, 1]
2	(2.8, 5.4), (9.4, 6.4)	[0, 0, 0, 0, 0, 0, 1, 1, 1, 1]
3	(3.33, 5.5), (10.25, 6.5)	[0, 0, 0, 0, 0, 0, 1, 1, 1, 1]

Final centers: (3.33, 5.5), (10.25, 6.5)

Q2: Apply SLIC algorithm to Fig 2, which consists of large flatten regions. The results is shown in Fig 3 when number of segments is set to 200. Why the resulting super-pixels in central uniform region tend to be hexagonal and not square or rectangular like the border?

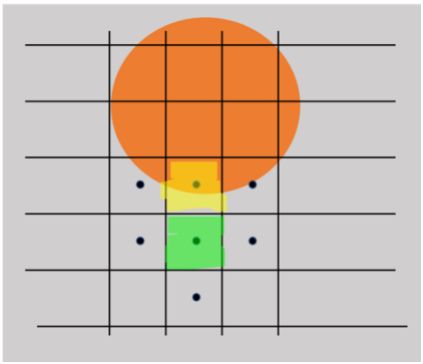


(a) (b)

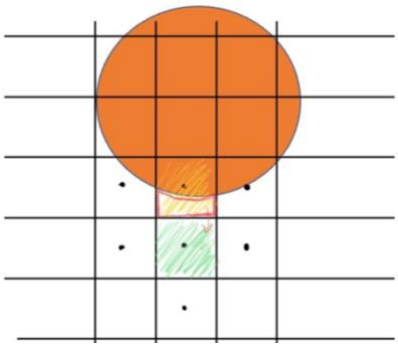
Fig 2

Answers:

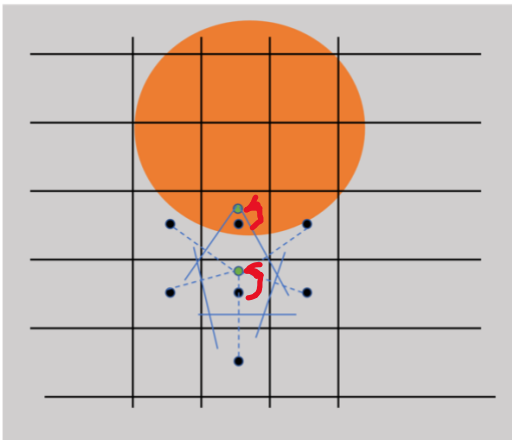
To summary, the reason is the shift of centers around the edges. For example,



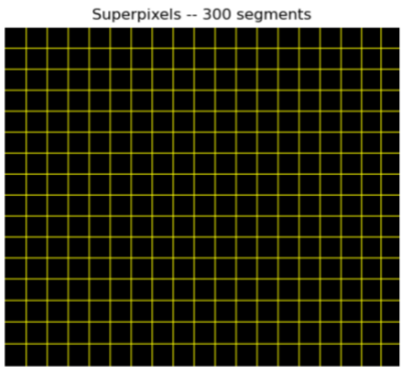
We initialize the clustering with rigid grid and sample the center of grids as initial cluster centers (represented as black dots). Here we show the change of yellow and green region after the first update.



Within the neighbourhood, the color is dominant fact of feature distance, thus white region in the yellow grid will be assigned to the center of green grid.



Then the positions of centers of these two grids shift (to the green dots). We focus on the reassign of pixels around the green dot below. As around region is of the same color, position determines the feature distance. Pixels are assigned to the spatially closest new center (illustrated by the perpendicular bisector of the line between two centers), which changes the “shape” of superpixels to be hexagonal.



This will not happen if no centers shift. For example, SLIC superpixels of a purely black image will remain rectangular.

Q3: For Mean-Shift clustering, what happens to the number of clusters as the window size or bandwidth is increased? Explain your answer.

Answers:
Reference: [Understand MeanShift]
The number of clusters should decrease. Bandwidth is the radius of kernels. Qualitatively, large bandwidth tend to consider more data points as a cluster when computing mean vector and clusters are more likely to be similar. Therefore, the number of cluster decrease.