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T1________________	**22395**	F1________________
T2________________		F2________________
T3________________	Problem Chosen	F3________________
T4________________	**C**	F4________________

2013 Mathematical Contest in Modeling (MCM) Summary Sheet

Summary

With economy and population boom, humans have also changed ecosystems more rapidly and extensively than in any comparable period of time in human history. In our paper, we construct a global ecosystem health network to predict the earths health state varying with time, human activity and governments policy. We focus our research on the environmental pollution-related health assessment, for that the environmental pollution has the most directly harmful effects on human and other creature living.

In this perspective, we define the earths health as the biosphere health state. Since the biosphere is global sum of ecosystems, we regard each regional ecosystem in our global as the node in our network. Ecosystem health assessment is then applied to count the value of each node, which we call it, the Health Index. We employ the prevalently used vigor-organization-resilience indicator system along with the AHP model to gain the weight of each index. The interaction between two nodes can be quantified as the weight of the link. According to our definition on health, the link can be simplified to represent the transboundary pollutant diffusion between two nodes. Two models, the Atmospheric Dispersion Model and the Water Diffusion Model, are applied here to quantify the diffusion procedure in the air pathway and water pathway respectively. Introduction of the concept of Per-Unit-System simplify our calculation and make us understand the Health Index in a more visual way, with value 1 standing for the health evaluation standard.

We construct two typical networks, basing on the concept of nodes and links we defined. The basic network ignores the human behavior, policy effect as well as the local feedback, only depicting a spontaneous natural pollutant diffusion procedure. However, the operation of the basic network can help us to understand the complex interactions in the network. To make our network approach the real world global network, the PSR (Pressure-State-Response) model then be utilized in our modified network. Instead of only using the PSR as an indicator system, we depict the pressure and the response fitting to the two functions vary with time, based on the competing-species model.

At the end of our paper, we create a simulator to help us analyze the global health state with regard to the time, human activity and policy influence based on pseudo-global we generate. Some research-focus problems have been analyzed by operating our simulator, such as the human behavior and policy effect to the global health, the emergency case. We conclude from our critical node analysis that the most polluted urban ecosystem will influence global health index most. Other simulation results have also been shown in our paper.

Construct and Analyze the Global Ecosystem Health Network

Abstract

In our paper,we focus our research on the environmental pollution-related health assessment. Considering the big influence, we define the earths health as the biosphere health state. Since the biosphere is global sum of ecosystems, we regard each regional ecosystem in our global as the node in our network. Ecosystem health assessment is then applied to count the value of each node, which we call it, the Health Index. We employ the prevailing used vigor-organization-resilience indicator system along with the AHP to gain the weight of each index. Two models, the Atmospheric Dispersion Model and Water Diffusion Model, are applied here to quantify the diffusion procedure in the air pathway and water pathway respectively. The link function then can be written combining the air and water diffusion to calculate links weight with regard to the distance and the nodes Health Index. We construct two typical networks, basing on the concept of nodes and links we defined. The basic network ignores the human behavior, policy effect as well as the local feedback, only depicting a spontaneous natural pollutant diffusion procedure. However, the operation of the basic network can help us to understand the complex interactions in the network. To make our network approach the real world global network, the PSR (Pressure-State-Policy) model then be utilized in our modified network. Instead of only using the PSR as an indicator system, we depict the pressure and the response fitting to the two functions, based on the competing-species model. At the end of our paper, we create a simulator to help us analysis the global health state with regard to the time, human activity and policy influence based on pseudo-global, we generate. Some research-focus problems have been analyzed by operating our simulator.

Contents

1 Introduction

Over the past 50 years, humans have changed ecosystems more rapidly and extensively than in any comparable period of time in human history, largely to meet rapidly growing demands for food, fresh water, timber, fiber and fuel [1]. This has resulted in a substantial and largely irreversible loss in the diversity of life on Earth.

In our paper, we construct a global ecosystem health network to predict the earth's health state varying with time, human activity and government's policy. Although some scholars may take the economic or the cultural factor into account when define the earth's health, we focus our research on the environmental pollution-related health assessment. We define the earth's health as the biosphere health state. The biosphere is the global sum of all ecosystems [2]. It can also be called the zone of life on Earth, a closed (apart from solar and cosmic radiation), and self-regulating system. We assess and predict the biosphere health state by evaluating the ecosystems' health it embraces.

1.1 Outline of Our Approach

Our paper begins with presenting the definition of our network's nodes and links. The basic network then be constructed limiting to the perspective in the environmental pollutant diffuse interaction. Latter the policy decision and human behavior have been taken into account and added to the modified network. Simulation has been adopted to test and validate our model, as well as analyzing the hotspot issues concerning the environment and human behaviors. The outline of our paper can be shown as following:

- Utility the ecosystem health concept to define the value of our network's node.
- Modify the Gaussion Plume Model and Water Diffusion Model to calculate the link value between nodes
- Build the basic model, a only pollutant-diffusion considered network
- Add the human influence function and policy impact function to compose the Modified Network.
- Our simulation results and analysis.

1.2 What is the Ecosystem-Health?

As we mentioned before we define the earth's health as the biosphere health state. The health value of the biosphere can be appropriately evaluated by the ecosystem-health assessment by decomposing the biosphere into small ecosystems. In this part, we explain the basic concept of the ecosystem health.

Ecosystem health assessment is a kind of systemic diagnostic method to evaluate the ecosystem characteristics associated with human health under the framework of ecology. Till now, researchers and experts still cannot get any agreement on the precise definition of this conception. The definition of ecosystem health can be divided into two types, biological-ecological definition and ecological-economic definition [3]. The former was put forward in the early 1990s by Costanza [4], which emphasizes the natural ecological aspect of the ecosystem while ignores the aspects of social economics and human health. The latter definition, on the contrary, regards human beings as one of the components of the ecosystem and considers both the health of ecosystem itself and the ecosystem service functions, which can express the extent of the natural ecosystem in meeting the human demands and requirements. It was brought forward in the late 1990s by Rapport et al [5].

By the biological-ecological definition, ecosystem health can be assessed using measures of resilience, vigor and Organization [6].

Vigor is measured in terms of activity, metabolism or primary productivity.

Organization can be assessed as the diversity and number of interactions between system components.

Resilience is measured in terms of a system's capacity to maintain structure and function in the presence of stress. When resilience is exceeded, the system can 'flip' to an alternate state.

The ecological-economic definition, however, add an extra factor to assessment **the ecosystem service functions**, which can present the effect of spatial adjacent relationships of different ecosystems.

Beyond the prevailing used the "vigor-organization-resilience" indicator, there are a variety of indicators proposed owing to various understandings to ecosystem health, its evaluation as well as the difference between detailed ecosystem types and regional ecological environment characteristics. The application of them is flexible and various, we'll explain more when applying these indicators in the latter part of our paper.

1.3 Assumptions

- *We regard each regional ecosystem as a node in the network.* The major regional ecosystems existing can be classified into the forest ecosystem, ocean ecosystem, farm, prairie, wetland, river, urban and desert ecosystem, each of which covers a certain amount of area in the global. To simplify our model and interaction procedure, we take each of these regional ecosystems as a node, whose location is defined in the central of the ecosystem covering area. That is necessary, for we calculate the ecosystem health by taking the ecosystem as a whole, regardless of the geometry shape and distributed factor in the area.

- *We ignore the political and economical interaction between two urban or farm ecosystems.* Political interaction between two cities have too many uncertain factors which are widely affected by the social regime, social culture etc. These factors are hard to estimate accurately. So we just consider the political interactions in the local region. The interactions between two ecosystems are operated just by the transboundary pollution.

- *The influence between two indexes is beyond our consideration.* We assume that each index in our assessment system evaluate the health independently. The increasing of the SO_2 will not affect the amount of CO_2 in certain region.

- *The pollutant residence time is assumed as 30 days.* Each ecosystem has its own self-cleaning effect to the pollutant. Once pollutant has been emitted into the atmosphere or water area, their ultimate destination depends in part on their chemical and physical characteristics, and in part on the prevailing meteorological and fluid dynamic conditions. The pollutant residence time varies with many factors, such as temperature, humidity, fluid velocity and cloud cover. We assume the pollutant resident time when calculating the pollutant diffuse as 30 days. That is for we throw away the indicators such as large dust particles, which remain in the atmosphere for only a matter of minutes and only exert a local influence due to its heavy weight. And the indicators we choose to measure the pollutant diffusion have a average residence time about 30 days [7].

- *During the time span we predict the earth's health there are no disasters from the outer space.* We image the earth is an isolated, closed, and self-regulating entity in the outer space. Once destructive disasters happen to the earth, our prediction results to the earth's health and evolvement in the future will surely be influenced.

- *We ignore the natural disaster and human destroying activities such as wars, nuclear weapon* *employment*. The natural disasters such as earthquake, tsunami will greatly change the existing ecosystem structure. So as the wars and nuclear weapon. However, the above situations relatively rarely happen in modern society. So we ignore these factors in our major model. More detailed consideration about these "emergency case" can be found in the simulation analysis part of our paper.

2 Construct the Basic Global Ecosystem Health Network

In this part, we would introduce our basic global ecosystem health network, which only depicts a spontaneous natural pollutant diffusion procedure. We temporarily ignore the policy and the human behavior influence. Each node is regarded as being political independent.

2.1 Regional Ecosystem Defined as the Node

Since we define the earth's health as the biosphere health state and biosphere is the global sum of all ecosystems, we come up to use the regional ecosystems as the nodes in our global ecosystem health network. Ecosystem is defined as a community of living organisms in conjunction with the nonliving components of their environment interacting as a system [8]. As we can see in this definition, ecosystem can be set in any size. The continent or country scale ecosystem is not an exercisable spatial scale to discuss ecosystem health due to the significant ecological and environment differences in the continent or the country. The regional scale ecosystem is the important foundation in the global scale ecosystem research. It can link the health issues at macro and micro scales and also relate ecosystem health and its influences to social economics. Hence, the regional scale is the key scale for ecosystem health researches [3].

In our paper, **we regard each regional ecosystem in the globe as one of our network's node**, whose location is defined at the centre of the geometry shape of region area. The regional ecosystems can be categorized into eight major partitions: the forest ecosystem, ocean ecosystem, farm ecosystem, prairie ecosystem, wetland ecosystem, river ecosystem, urban ecosystem and desert ecosystem. Each regional ecosystem node represents the geometry location and ecosystem health condition, the latter of which can be evaluated using ecosystem health assessment method mentioned in the following part. The partition and mapping of each regional ecosystem would be a huge work, the GIS technology can be applied to collect and partition the boundary. The detailed steps can be found in [9].

2.2 The Health Value of Node Entity

The global ecosystem health network we construct should reflect the environmental interaction between each two regions in the globe. To satisfy this function, the value of our node entity should be some magnitude which can manifest the health condition of the region the node entity represents. The assessment of the ecosystem health can evaluate this magnitude. **We define the value of the node the Health Index**. The indicator system we adopt here is the most commonly used "vigor-organization-resilience" indicator. We do not take the ecosystem service function into account. For in our present basic network, we ignore the human and policy factors, which make the consideration of the service functions redundant. The index we adopted can be seen in the Table 1, classified into 3 indicator layers. We applied the Analytic Hierarchy Process (AHP) to gain the weight of each index, the results also shows in the Table 1.

Table 1: The indicator and index weight evalating the ecosystem health [1]

	Indicator	Index		Weight
State	Vigor	metabolic capacity		0.1095
	Organization	Population density		0.1545
		Greenbelt coverage		0.1545
	Resilience	Atmosphere pollution	SO_2	0.122
			NO	0.0544
			O_3	0.0208
			CO	0.0208
		Water pollution	PH	0.0545
			BOD_5	0.0545
			COD	0.0545
			SS	0.0545
		Soil pollution		0.0727
		Greenhouse gas emittingCO_2		0.0727

[1].The unit of the atmosphere pollution index is ug/m^3, the unit of water pollution index(except PH) is mg/l, the unit of greenhouse gas emitting is ton.The BOD_5 index presents the amount of dissolved oxygen consumed in five days by biological processes breaking down organic matter. The COD index stands for the chemical oxygen demand. The SS index stands for the suspended substance in the water.

Weight represents the relative importance of all indicators or indexes. The method to gain the weight varies. AHP model which we adopted here is the most widely used method in current ecosystem health evaluation [3], though its objectivity is pretty poor. There are also some methods to generate weight more objectively, such as entropy model, factors analysis model, standard deviation model, based on the statistics of original data. Nevertheless, the weight defined by objective method cannot match its importance in practice completely. As to the subjective method, it can reflect difference of relative importance among all evaluation indicators in spite of the subject process to define indicators' weight. Considering that ecosystem health evaluation itself is a human subjective judgment, weight defining through the subjective method possesses more scientific rationality. So we utilize the more subjective AHP method rather than other more objective model to gain the weight.

Another limitation to the model to apply is the large amount of data to operate the assessment. We are usually not accessible to all these data. We then employ the per unit value concept to define our per unit health index. The per-unit concept has been widely used in the power transmission field of electrical engineering and can be expressed as system quantities as fractions of a defined base unit quantity:

$$S_q{}^* = \frac{S_q}{S_b} \tag{1}$$

Where, $S_q{}^*$ represents the per unit value of S_q , S_q denotes the actual quantity and S_b denotes the defined base unit quantity, such as rated power or current etc.Calculations are simplified because quantities expressed as per-unit are the same regardless of the unit level. Impedances, voltage drops and losses can be the same when expressed as a per-unit fraction of the equipment rating, even if the unit size varies widely. Similarly, we can also apply this strategy here as:

$$HI_{ij}^* = \frac{I_{r_{ij}}}{I_{b_{ij}}} \tag{2}$$

Where, *i* denotes the indicator layer, *j* represents the index level, HI^* denotes the per unit health value to each index, $I_{r_{ij}}$ denotes the real index number, and $I_{b_{ij}}$ denotes the based or rated index number. We define HI^* here a "rated" index that represents the existing health standard index proposed by the government or the environmental institute. So when the per unit value equals 1, it means the corresponding ecosystem-related factor exactly meets the health standard requirement. The lower the per unit health index means the healthier the relevant index and vice versa.

The total ecosystem health index of the node in our defined per unit system can be expressed as:

$$HI^* = \sum_{i=1}^{3}\sum_{j=1}^{n} w_{ij} \cdot HI_{ij}^* \tag{3}$$

Where, i, j, HI_{ij}^* have the similar meaning as above, w_{ij} denotes the index in indicator layer's weight. The HI^* then is our node value in our network.

The introduction of the per unit system concept simplify our assessment calculation greatly, allowing us apply the model regardless of the unit transform. The per-unit processing can show significant effect in the latter simulation part.

2.3 Atmospheric Dispersion Model and the Water Diffusion Model

The interaction between two ecosystems is just relevant to the pollutant diffuse between the two nodes without human and policy impact in our pollution-related research perspective. The health index of the node can also only be affected by the neighbors' health indexes. This kind of pollutant diffuse affecting to the node is defined as the transboundary pollution. Transboundary pollution is the pollution that originates in one region but is able to cause damage in another region's environment, by crossing borders through pathways like water or air [10]. The main travel pathways for the transboundary pollution include wind, river, and ocean. We then model the pollutant diffuse procedures through air and water respectively, applying the Atmospheric Dispersion Model and the Water Diffusion Model.

2.3.1 Atmospheric Dispersion Model

Atmospheric dispersion modeling refers to the mathematical description of contaminant transport in the atmosphere. Analytical and approximate solutions for the atmospheric dispersion problem have been derived under a wide range of simplifying assumptions, as well as various boundary conditions and parameter dependencies, while the simplest of these exact solutions is called the Gaussian plume Model.

We restrict our attention at the outset to the transport of a single contaminant whose mass concentration (or density) at location $\vec{x}= (x, y, z)$. The Gaussian plume function can be described as:

$$c(x, y, z) = \frac{Q}{4\pi ur} exp(-\frac{y^2}{4r}) \left[exp(-\frac{(z-H)^2}{4r}) + exp(-\frac{(z+H)^2}{4r}) \right] \tag{4}$$

Where, *c* denotes the pollutant concentration in the atmospheric,*r*is a constant multiple of the variance of the concentration distribution. The contaminant is emitted at a constant rate *Q[kg/s]* from a single point source $\vec{x}$ = (0, 0, H) located at height H above the ground surface. The wind velocity (u) is constant.

It is worthwhile noticing that in the limit of vanishing velocity

$$\lim_{u \to 0^+} C(x,y,z) = \frac{Q}{2\pi Kx} \tag{5}$$

2.3.2 Water Diffusion Model

The Water Diffusion Model is proposed to describe the pollutant diffusion through the water pathway. The basic model can be expressed as:

$$c(x) = c_0 e^{-\frac{ux}{2Dx}(\sqrt{1+\frac{4kD_x}{u^2}}-1)} \tag{6}$$

When the velocity of the fluid is large enough, regardless of dispersion effect, we can get:

$$c(x) = c_0 e^{-\frac{kx}{u}} \tag{7}$$

Where,

$$\begin{cases} c_0 = \frac{Qc_Q + qc_w}{Q+q} \\ qc_w = M \end{cases} \tag{8}$$

In which,$c_0(mg/l)$ and $c(mg/l)$ are the initial concentrations of the solution and the concentration at level of the diffusion cell, respectively. denotes the diffusion coefficient,$Q(m^3/s)$ is the flow of the upstream, $q(m^3/s)$ represents the flow of the waste water into the river, $c_0(mg/l)$ denotes the existing pollutant concentration in the river, $c_w(mg/l)$ denotes the pollutant concentration in the waste water ,$M(g/s)$ denotes the emission rate of the pollutant.

2.3.3 The Definition of Link in the Network

The link weight in a network should reflect the interaction between the nodes it connects. In the two kinds of spread pathways, air and water, the link weight has different forms. When pollutant spreads through the air pathway, for example the gas, the pollutant diffusion procedure is bidirectional, which means each node's air pollutant can spread to the other node, while in the water pathway pollutant can only diffuse from upstream to the downstream. The bidirectional pollutant dispersion procedure is shown in the Figure 1.

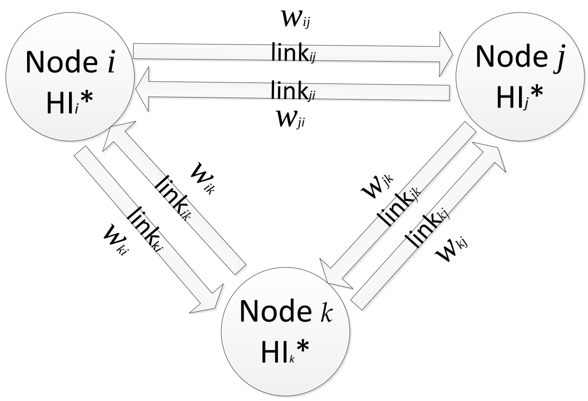


Figure 1: Bidirectional pollutant dispersion procedure

We define our network link between two nodes the pollution diffusion procedure, and the weight on the link can reflect the amount of pollutant it spreads to the neighbor node. The link function to calculate links' weight can be written combining the air and water diffusion with regard to the distance and the nodes' Health Index:

$$\begin{cases} W^*_{ij} = c^*_{air_{ij}} + c^*_{w_{ij}} \cdot \delta_{ij} \\ W^*_{ji} = c^*_{air_{ji}} + c^*_{w_{ij}} \cdot \delta_{ji} \end{cases} \quad (i,j = 1,2,...,n) \tag{9}$$

Where,*i*,*j* denotes the node *i* and node *j* in the network respectively, ${W_{ij}}^*$ represents the link weight from node to the node *j*. ${c_{air_{ij}}}^*$ denotes the per-unit air pollutant concentration value from the *i*th to *j*th nodes with the standard health index as the base value, ${c_{w_{ij}}}^*$ has the similar meaning but the ${c_{w_{ij}}}^*$ and ${c_{wji}}^*$ have equal value, δ_{ij} can only be 0 or 1. When there there are river connections between *i* node and *j* node and the node presents the upstream towards the *j* node, δ_{ij} equals 1. Otherwise δ_{ij} equals 0.

Noticing we apply the per-unit value in our formula. The link weight spreads to the influenced node and the influenced node then modifies its health value due to the pollution impact from the other nodes:

$$
\begin{aligned}
{HI_i}^{*'} = {HI_i}^* + w_{31} \cdot \sum_j c^*_{air_{ji}} - w_{31} \cdot \sum_j c^*_{air_{ij}} \\
+ w_{32} \cdot \sum_j c^*_{wji} \cdot \delta_{ji} - w_{32} \cdot \sum_j c^*_{wji} \cdot \delta_{ij}
\end{aligned} \tag{10}
$$

Where the w_{31} and w_{32} indicate the assessment weight concerning the air pollution and water pollution. The other variables have the similar meaning as the above mentioned.

As we can see in the formula (10) the proposal of the per-unit value concept avoid the unit transform and therefore is significant in simplifying the calculation.

We get our basic network model $Net\{HI^*, W\}$, $HI^* = ({HI_1}^*, {HI_2}^* ... {HI_i}^* + ... {HI_n}^*)$

2.3.4 The Operation of Our Global Ecosystem Health Network

Our basic global ecosystem health network only depicts the interaction between two ecosystems, (or as we define here two nodes) in the level of the transboundary pollution. Imagine there are an original node and its neighbor nodes. The health value of the original node can reflect the amount of pollutant in the corresponding ecosystem, thus the more pollutant the node stands for, the more pollution diffusion to the neighbor. The transboundary pollution can be quantificationally expressed using the Atmospheric Dispersion Model and the Water Diffusion Model. When the neighbors are influenced by the diffusion, their health indexes thus are modified. However our pollution diffusion is not unidirectional. So the modified health value of the neighbor node will modify their link weights to the original node as well, and therefore the health index of the original node will also be modified. How the node health value be modified is expresses in the formula (10).The real global ecosystem health network is more complex than this. The modification of the health value and the link weight between each two nodes will be processing all the time until the modification is approaching to zero. We use the iteration method to realize the whole procedure in our computer and we ultimately gain the stable global ecosystem health network.

We ignore the human element and obscure the time element in our basic network. Although our basic global ecosystem health network is far from clearly depicting all the complex ecosystem operations in real world, it provides a basic method and model to express the interaction between each two ecosystems in the global scale.

3 Add the Human ,Policy Effect and Time Element into Our Network

We do not take the effects of human behaviors and government policy into account in our basic network model. In this part we are to improve our network by quantifying the effects of human behaviors and government policy.

3.1 The PSR Model

The pressure-state-response (PSR) model is a conceptual framework consisting of a feedback system of pressures, states, and responses. It is widely used as a tool to model human related environmental systems. The PSR framework has proven to be a logical, comprehensive tool to picture environmental issues from an anthropocentric perspective [11]. In fact, the PSR model suggests linear relationships between man and the environment without employing implicit feedback loops into the system which implies a loss of complex information. Figure2 shows a framework of this model.

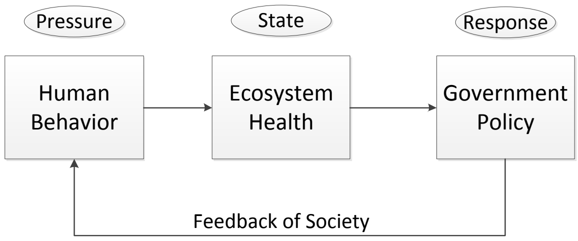


Figure 2: PSR model

However, the existing PSR model is just widely applied to define the indicators in the ecosystem health assessment. In our paper, we employ the PSR model's concept and framework to quantify the respond and the pressure indicator.

3.2 Application

3.2.1 Human Pressure on Ecosystem

As we can imagine, when human live in an ecosystem without any government control, they will destroy the ecosystem around them for getting adequate food, enough living space and some other basic demands and requirements for making a living. There will be two effects of these behaviors towards the ecosystem, one of which is **destroying the local ecosystem** where they settle, while the other is **expanding their activity area**. The latter effect will be considered in our simulation part. In this part, we only consider the effect towards the local ecosystem. There is a maximum limitation of pollutant caused by human behavior in any existing ecosystem due to the source limitation and capacity of the ecosystem. We define α as the function to depict influence of human behaviors to the ecosystem. The human behaviors functions will vary with time.

The function can be depicted as shown in Figure 3. We let $\alpha(t)>1$ in the function for the human's spontaneous behaviors for meeting the demands of their living will make the environment worse. With the time goes by, the pressure to the environment caused by human behaviors will augment. But it approaches to a maximum capacity of the human pressure.

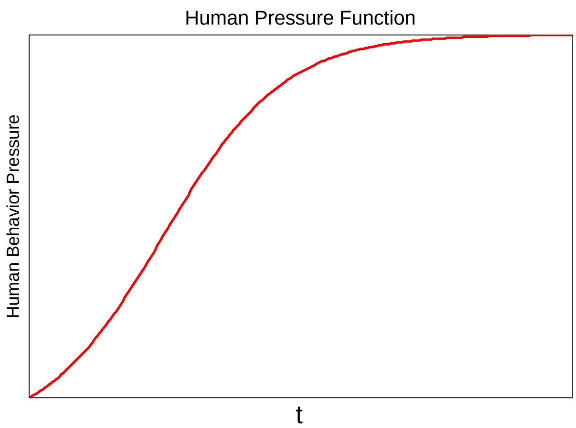


Figure 3: The Human pressure function

Hence, we get the description equation as follows:

$$\alpha(t) = \frac{x_m}{1 + ({x_m}/{x_0} - 1) \cdot e^{-r \cdot t}} + 1 \tag{11}$$

In this function, we use x_m to describe the maximum multiplying power of pollution in a specific existing ecosystem and x_0 represents the initial multiplying power of pollution in a specific existing ecosystem. In this case, we get the modified health index of a node

$$HI_i^{*\prime\prime} = \alpha(\mathrm{t}) \cdot HI_i^{*} \tag{12}$$

3.2.2 Government Policy Respond to the Ecosystem

Generally speaking, a policy can either be environment friendly or environment unfriendly. As we can learn from the PSR model, the probability of a government making the environment-friendly or the environment-unfriendly policy is relevant to the ecosystem health state. We define a probability function with regard to the health value of the ecosystem (also the ecosystem state):

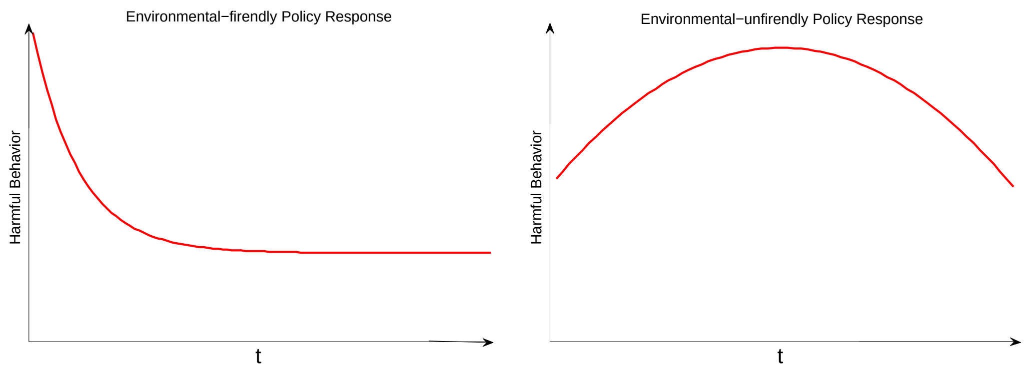
$$P = F(HI_i^{*}) \tag{13}$$

The environment-friendly and the environment-unfriendly policies will also have distinct effect to the human behaviors; also we can say the human pressure to the environment.

The environment-friendly policies' respond to Human Behaviors

Once a policy is proposed by the local government, its effect to the human behavior can then be quantified. Here we define the effect as r/ (1+r). At the same time, we define β as the effect of the policy. Generally speaking the environment-friendly policies influence will enhance with the time increasing. So we choose the following function to depict the effect, what the function looks like shows in the Figure 4:

$$\beta(t) = \frac{e^{-t} + r}{1 + r} \tag{14}$$



(a) Friendly (b) Unfriendly

Figure 4: Policy Respond to the Human Behavior

Noticing that here $\beta(t)$ represents the harmful behavior inducted by human. As we can see in the Figure 4, when the policy just come out, it has little effect to the harmful behavior, when the time increases, the harmful behavior will then decrease. However the effect of the policy also has its limitation, so the curve flows to straight line ultimately.

The environment-unfriendly policies

Similarly, we can get:

$$\beta(\mathrm{t}) = \frac{1-M}{t_0{}^2}(\mathrm{t}-\mathrm{t}_0)^2 + M \tag{15}$$

Where, we define M as the biggest damage of the policy to environment. In the other hand, t_0 represents the time when the damage becomes the maximum. What the function looks like can also be seen in the Figure 4. The environment-harmful behavior will enhance due to the policy, but with time increased the harmful behavior will reduce due to the respond to the deteriorating environmental state.

Compound effects on the ecosystem

In real world, there are a lot of factors including human behavior and government policy affecting a specific ecosystem's environment. To make the problem can be solved and the research meaningful, we apply the PSR concept and just take human behavior (Pressure) and government policy(Respond) into consideration, in which case, we can get the complex effects on the Ecosystem State expressed as the following function:

$$
\begin{aligned}
{HI_i}^{*'''} &= \alpha \cdot \beta \cdot {HI_i}^* \\
&= \begin{cases} (\frac{x_m}{1+({x_m}/{x_0}-1)\cdot e^{-r\cdot t}}+1)\cdot\frac{e^{-t}+r}{1+r} & P > c \\ (\frac{x_m}{1+({x_m}/{x_0}-1)\cdot e^{-r\cdot t}}+1)\cdot(\frac{1-M}{{t_0}^2}(t-t_0)^2+M) & P < c \end{cases}
\end{aligned} \tag{16}
$$

The ${HI_i}^{*'''}$ is the health value of the node in per unit form considering the compound effect of the "Human pressure—ecosystem state" and the "Policy Respond—Human pressure–ecosystem". Is some constant to measure the ability to respond and make accurate decision of the government.

Thus we get the modified network $Net\{HI^{*'''}, W\}$, with

$$HI^{*'''} = \{{HI_1}^{*'''}, {HI_2}^{*'''}, ...{HI_i}^{*'''}, ...{HI_n}^{*'''}\}$$

4 Simulator

We develop a simulator to simulate the operation of our constructed network and the complex interaction between the two ecosystems. The goal of our simulator is that, given the large amount certain ecosystems' geometry locations and categories, our simulator can tell the operator the health state of each put-in ecosystem, and thus the health degree of the earth, during the given time steps. Due to time limit, we are hard to access to all the global geometry information, we then create a pseudo global, in which we create some ecosystem circles. The generation of our pseudo work as follows:

- We create a sphere the same size as our earth.
- We about 100 ecosystem circles, categorized into 8 classification as the urban, farm, wetland, forest, river, ocean, desert, prairie ecosystems. Each ecosystem circle has different radius. And category of ecosystem circle has different range of radius, according to its feature in the real world. That is, for example the ocean ecosystem circles have generally larger radius than those of the urban ecosystem circles.
- The ecosystem health—measure indexes were produced as the same index in table1. The initial state of these indexes in the each ecosystem is produced randomly but within some range that also depends on the category of the ecosystems. That is easy to understand that CO_2 emitting number would surely be higher in the urban ecosystem than the forest ecosystem.

Now we get a Pseudo world embracing 100 ecosystem circles all with its health indexes respectively, distributed nonuniformly and real—globe based.

As we define in our constructed network, the node then is the center of the each ecosystem circles. And as we mentioned in the assumption part, we assume that the indexes we choose reside in the air or water about 30 days. So we take average diffusion distance of the pollutant about 100 miles, and the link weight only defined between the nodes whose distance is above this value.

Our simulator operates as follows, obeying some rules to fulfill our requirements:

- Calculating the node value (ecosystem health value) of each ecosystem circles with the indexes in the table 1, and the formula (10). The per-unit strategy has been employed to avoid transforming the unit. With every index values being united, the health value represents the health standard, the smaller value, means the healthier state. When considering the human behavior and the policy response. The impact factor $alpha$(t),$beta$(t) should be multiplied to the health index as (16), considering the governments have the different ability to decision-making and response, the constant c varies to different ecosystem circles. However, we should notice that, the human behavior and policy responses are only to be taken into account in the urban and farm ecosystem. That is for, in the other, ecosystem, human directly impacts are relatively tiny.

- Another human activity effect is simulated here, that is human enlarge their settlement to fulfill their boosting population living requirements. We realize this by decreasing the number of forest, wetland and prairie ecosystem circles, and replace it by the urban or farm ecosystems.

- Calculating the link weight between each two efficient nodes, applying the formula (9). The inner river or water connection is generated randomly.

- To each time step moves, we modify the health value of every node once, and thus the link weight between each two efficient nodes also modified once. We call that interaction procedure.

- We repeat the interaction procedure until the maximum modification of the health value and weight value is less than the 10^{-3}.

What needs to be noticed is that the time step here is about 30 days, nearly to the time the pollutant can spread, and the time in the $\alpha(t)$, $\beta(t)$ is recognized about 12 times about this time, thus one year. For the policy effect is always measured yearly.

We then operate our simulator to analysis some research—focus problems connecting to the ecosystem, human behaviors and policy decision.

4.1 The Human Behavior Influence

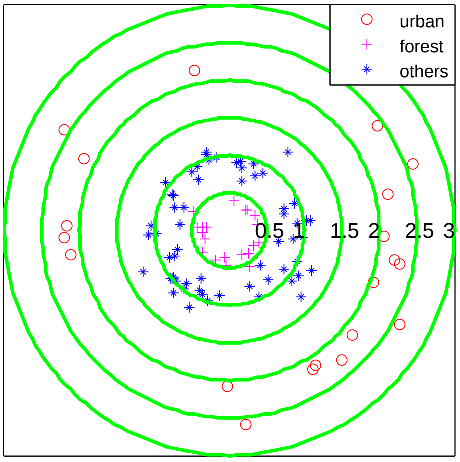
We operate our simulator with three setting situations:
(1) Applying our basic model regardless of human behavior and the policy effect.
(2) Taking the local effect of human behavior into account, thus multiply the human pressure factor α.
(3) Taking the enlargement of human behavior into account.

The initial health state of each ecosystem and the results of the above three situations are be shown in the Figure 5.

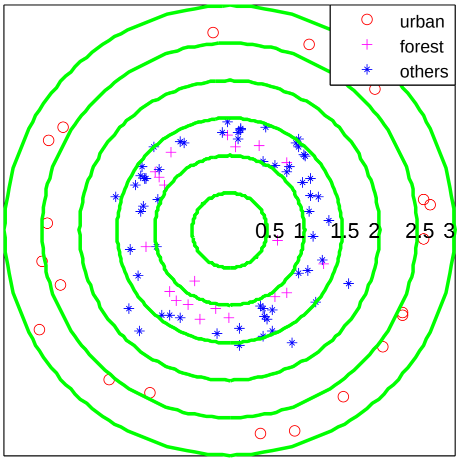
The number 0.5-3 represent the health degree of the ecosystem nodes. Since our per unit system regard the standard health value as the based value, the 0.5-3 have the following meanings:

Table 2: Health value meaning

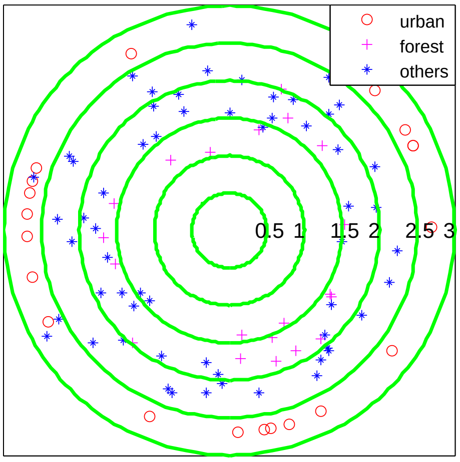
Health Value	health degree
0.5	very healthy
1	healthy
1.5	sub-healthy
2	slight-polluted
2.5	polluted
3	serious polluted



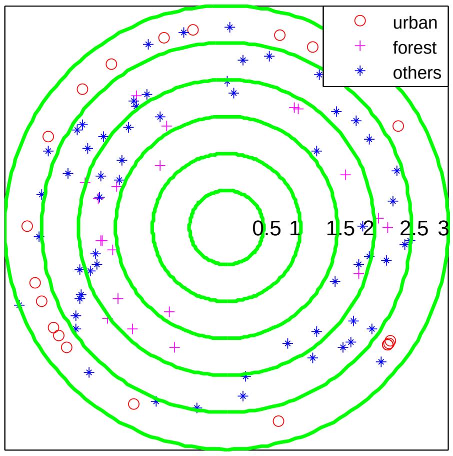
(a) Initial state



(b) Regardless of human effect



(c) The First human effect



(d) The Second human effect

Figure 5: The Human Behavior Influence

Table 3: The paremeter

x_m	x_0	r
1	0.1	0.03

Table 3 presents the key parameter in the formula(11)

The six circles in the Figure 5 stands for the health value 0.5, 1, 1.5, 2, 2.5, 3 respectively. And the nodes which represent the ecosystem distributed in the circle according to its health value.

As we are informed from the figure, the initial state of ecosystem we generate mainly within the 1 in (a), and in the first situation which do not consider the human behavior, the final health state do not vary much as shown in (b). However when the human activity is considered, we can notice that most of the ecosystems' health state is located between the 1.5-2 in(c). That means most of the ecosystems have already been polluted due to the human activity. Things get worse when the second human activity happen, thus human begin replace the forest, wetland, and prairie ecosystems for the urban and farm. Most ecosystems become serious polluted in this situation as we can see in (d).

4.2 The Policy Decision Effect

The government may make environment friendly policy or environment unfriendly policy due to the various political intentions. The probability function can imitate our deciders' decision making procedure. Here we let the decider make the environment friendly policy and the environment unfriendly policy respectively to see how it works to the whole global health value. The result can be seen in the Figure 6.

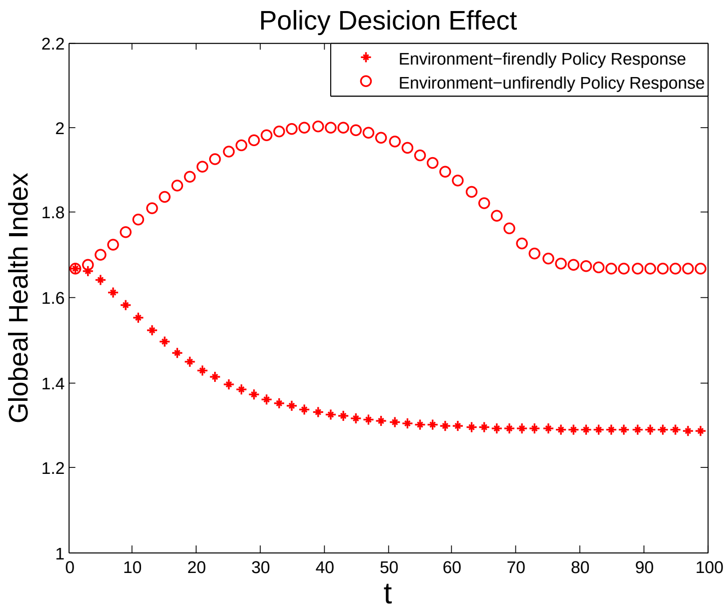


Figure 6: Policy Decision Effect

The global health value can be calculated by computing the average value of all the nodes' health value. The initial state here is polluted globe of which health index is about 1.68 rather than the state we first generate. The Figure 4 can tell us the environment friendly policy can decrease the global health value, which means the ecosystem pollutant decreases. The environment harmful policy, however, will produce more pollutant. Its worthwhile noticing that the reaction of one policy needs some time, the global health value will have a significant decrease in about 20 time steps, thus nearly 2 years time needs for the policy to ease the current ecosystem stress.

4.3 The Emergency Situation

Some emergency hazard towards ecosystem always happens such as earthquake, epidemic, nuclear explosion.etc. We simulate one of those cases in our simulator. We imagine the nuclear explosion happens in certain urban ecosystem, we trace its health index in our simulator. The result can be viewed in Figure 7.

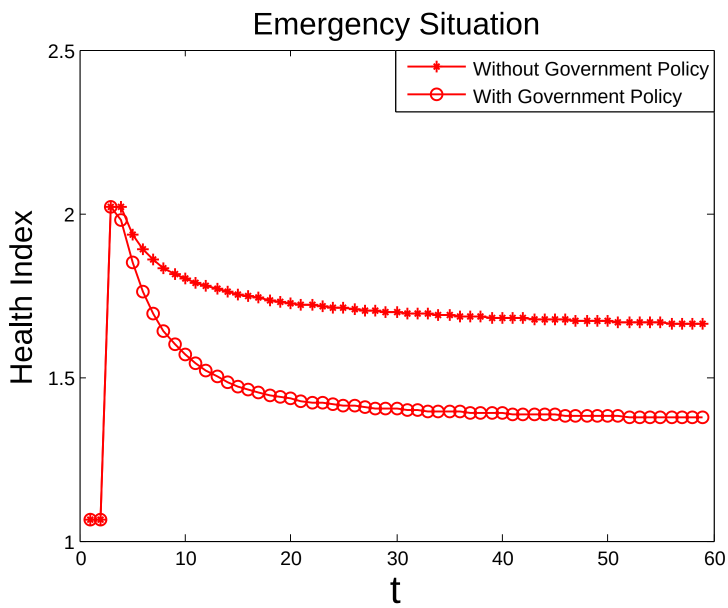


Figure 7: Emergency Situation

When the emergency hazard happens in the third time steps, the health index of the urban node augment rapidly. The Health Index will then decrease slowly in both government control and anarchy situation. However, the depth of the decline with government policy will be obviously lager than the one without government policy.

4.4 The Critical Nodes Analysis

We utilize the leave-one-out strategy to find out the critical nodes in our network. We leave each of the 100 ecosystems out of the network to analyze the effect to the global health index. Figure 8 shows the results. The red straight line represents the global health index when no node is thrown out. Each mark in the Figure 8 denotes the global health index when the corresponding node being thrown away. From the figure we can notice easily that urban ecosystems are relatively more critical in the network.

We then just picture the urban ecosystem's leave one out results, with the initial state of each node distinguished by the color.

As we can conclude from the Figure8, the urban ecosystem, thus cities, with higher health index will have more critical effect on the global ecosystem health state. We can draw conclusion from this analysis that to improve our whole global health state, it is important to improve the health state of some highly-polluted cities.

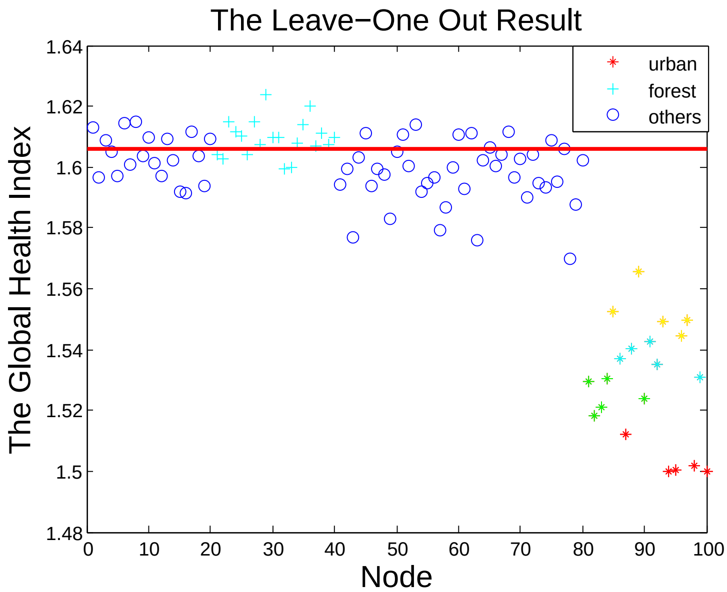


Figure 8: The leave- one -out result

5 Conclusion

In our paper, we construct a global ecosystem health network to predict the earths health state varying with time, human activity and government's policy.

Verification

To verify our model, one would have to obtain global data related to all biological ecosystems existing in the earth and compare the actual results to the predicted calculated by our model. However, we cannot verify our model because complete data for verifying our model is not that easily accessible and available. But we can verify our model through historical data and additional research. We will talk about this in the future work.

Strengths

- **Extendable:** In our initial model, we use 8 kinds of ecosystems to represent the earth, which is significant for further research and the model would work well even if the standard of selecting the ecosystem changes. The ecological and environment feature in each ecosystem are similar, so it is easy to model them accurately. Whats more, when we add some important factors such as human behavior and government policy into our model, it works as well.

- **Flexible:** The equations and approaches analyzing biological problems used in our model could be applied to other relevant problem.

- **Simplicity:** In our model, first we study every node of the whole ecosystems respectively without considering their inter-effect and then we study the relationship between nodes, which is easy to study while given the relation of nodes.

- **Quantified Research:**we quantify the pressure and response indicator in the PSR model, making it depict the heath state of our earth more fit.

- **Application:**Our model is applied to our simulator to analyze the focus problem relevant to the environment. Conclusion and results can draw from our simulation.

Weaknesses

- **Lack of Data Support:** This model requires a large amount of data, some of which is hard to obtain. Although our created pseudo-global can simulate our global geometry quite well, it can hardly inform us the corresponding node in the real world. So we cannot know what the exactly city is that need to act environment-friendly policy or what time should we must implement a policy.

- **Simplified Assumption:** Simplifying assumptions had to be made in order to create a solvable model. We only consider the pollution-related health evaluation and the political influence between two ecosystems have been omitted.

- **Estimated Parameters:** Due to lack of data, some values used in the calculations had to be estimated.

Future Work

Due to lack of the accessible data, our model can hardly be based on the real world. The necessary data we need can be accessed by applying the GIS technology. The reference [9] has detailed explanation of how it works to collect the geometry information. Unfortunately, limited to the time, we cannot apply it to our model. Future work may process in this field. Since we regard ecosystem as our network's node, we can build different models to depict each ecosystems health, so some specific features in ecosystem can be considered. There are large amounts of existing models to each kind of regional ecosystem now.

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