2017 Mathematical Contest in Modeling (MCM) Summary Sheet

Summary

Urbanization has become an uprising phenomenon, where about 70% of World's population will be living in cities by 2050.To realize the sustainable development of city, it's essential to analyze the SG. In the paper, we construct the Indicator system of smart growth to measure the performance, whose indexes are collect by the mean of Fuzzy Aggregation ("Word frequency statistical analysis"). After the layers of summary and screening, we define a metric consisting of 16 indicators under the principles of SG. As urban development can not be drawn from a single indicator, we employ membership function to collect indicators along with the BPNN (short for Back Propagation Neural Network) to gain the weight of each index.

For the further, we have chosen two representative cities. After we summarize the government policies related to the SG targets in recent years , we draw up the tables to visualize our results. According to our definition, combined with the city's real data, we give each city's score by simple linear weighting. Simply put, Sonoma seems to be superior to sustainable urban development.

There is always a neglected part in urban development. According to the actual development of the city (reflected by the indicator) and the final city score, we give our urban development plan, taking full account of the weight of each indicator. Specific policies see the summary table. Our theory of policy design is system dynamics, because each of the urban development interacts with and restricts each other. The operating mechanism of system dynamics is in line with the real development of the city. In view of this, we have established three subsystems of environment, economy and society, each of which contains many variables. The interrelationship of these variables is characterized by the equation (the parameters are determined by the actual data).

In order to test the feasibility and advantages of our policy, first of all, we will quantify the policy, and then use software (vensim) to simulate the development of the city in the coming years. Again using the established Indicator system, given the city under the new policy, evaluate the success of our smart growth plans. In such an analysis, we can easily find out the development indicators for each city in a particular future year, that is, we can give a ranking table of the policy implications.

At the end of our paper, we also create a simulator to help us analyze the city development with regard with population based on system dynamics we generated. Through the three subsystems and the relationship between the system, we know the impact of population on urban sustainable development. Given the conditions of population growth, we can clearly see the pros and cons of our policy.

Contrast and design the model of sustainable cities based on SG

Abstract

In the paper, we construct the Indicator system of smart growth to measure the performance, whose indexes are collect by the mean of Fuzzy Aggregation ("Word frequency statistical analysis"). After the layers of summary and screening, we define a metric consisting of 16 indicators under the principles of SG. As urban development can not be drawn from a single indicator, we employ membership function to collect indicators along with the BPNN (short for Back Propagation Neural Network) to gain the weight of each index. For the further, we have chosen two representative cities (lishui and sonoma). According to our definition, combined with the city's real data, we give each city's score by simple linear weighting. Simply put, Sonoma seems to be superior to sustainable urban development. There is always a neglected part in urban development. According to the actual development of the city (reflected by the indicator) and the final city score, we give our urban development plan, taking full account of the weight of each indicator. Our theory of policy design is system dynamics, because each of the urban development interacts with and restricts each other. The operating mechanism of system dynamics is in line with the real development of the city. In view of this, we have established three subsystems of environment, economy and society, each of which contains many variables. The interrelationship of these variables is characterized by the equation (the parameters are determined by the actual data). In order to test the feasibility and advantages of our policy, first of all, we will quantify the policy, and then use software (vensim) to simulate the development of the city in the coming years. Again using the established Indicator system, given the city under the new policy, evaluate the success of our smart growth plans. In such an analysis, we can easily find out the development indicators for each city in a particular future year, that is, we can give a ranking table of the policy implications. At the end of our paper, we also create a simulator to help us analyze the city development with regard with population based on system dynamics we generated. Through the three subsystems and the relationship between the system, we know the impact of population on urban sustainable development. Given the conditions of population growth, we can clearly see the pros and cons of our policy.

contents

1 Introduction

Urbanization has become an uprising phenomenon here about 70% of World's population will be living in cities by 2050 [1]. This rapid urbanization caused a number of consequences; starting with the emergence of Urban Sprawl over the fringes of Metropolitan Areas, forming excessive pressure on environmental resources and infrastructure, in addition to negatively affecting the surrounding environment.

1.1 what is the Smart Growth ?

Smart Growth (SG) was first introduced by the American Planning Association in 2002, [2] .Although it was discussed earlier by academics and planners in 1990, but it became more formalized when the Smart Growth Network has issued the principles of SG and their application strategies. SG is an attempt to manage the growth of urban communities and limit its unplanned expansion. It promotes urban patterns that encourage compact zoning, high densities, mixed uses and walkable communities, in addition, it supports integration between policy making and social inclusion to achieve more sustainable development for existing communities [3].

1.2 Interpretation of the principle

Smart growth focuses on building cities that embrace the E's of sustainability—Economically prosperous, socially Equitable, and Environmentally Sustainable, which is also called three E's. The ten principles for smart growth are[4]:

1. **Mixed Land Use**: this principle is based on the relevance between achieving livability and heterogeneity and the integration between different aspects of life. The mixed zoning of spaces enhance social inclusion and interactions, as well as providing access to different services for different social classes.
2. **Compact Building Design**: The compact design has to follow appropriate desirable design standards which sustain privacy for families. It should also maintain an adequate ratio between buildings and street scales increasing the quality of life and sense of security.
3. **Housing Opportunities**: To ensure equity and fair distribution of resources, a diverse options for housing sector should be available for different types of families with different income levels. The diversity in social classes within the community should attract skillful workers, enhances local economy of the neighborhoods, promotes the environmental justice and decreases social segregation.
4. **Walkable Communities**: Walkability promotes a healthy society which interact repeatedly with local shops and services, as well as, open areas and public spaces. That decreases the excessive use of automobile, reducing the green gases emission and, enhances social relationships among citizens, as well as, local business.
5. **Foster Distinctive, Attractive Communities with a Strong Sense of Place:** The strong sense of place provokes civic pride and sense of identity among citizens which in return makes

them more interactively participate in enhancing the community quality of life, involved in development of decisions, as well as in the protection of the assets of the place.

6. **Preserve Open Space, Farmland, and Natural Beauty and Critical Environmental Areas**: Open spaces and Farmlands are considered to be a great asset environmentally, economically and socially. SG initiates programs and solutions to support investments in agriculture, and increase the awareness of communities with the importance of the ecological systems affecting social inclusion and enhancing their social interactions and quality of life.

7.**Direct Development toward Existing Communities**: Infill development and rehabilitation of existing buildings is much appreciated by existing communities than new communities' development. It directs the investments towards deteriorated zones and enhance the quality of life for local residents with different social classes.

8. **Provision of Variety of Transportation Options**: It is essential to study carefully the regular trips performed daily by citizens, to design and connect routes and different modes of transportation adapting with the community needs, as well as providing guiding instructions for users. The objective of this is to provide citizens with equal access to these transportation nodes, as well as services, jobs and housing opportunities, and finally enhancing social equity.

9. **Predictable, Fair and Cost Efficient Development Decisions**: The production of feasibility studies for development projects, enhances the chances of their success and decreases their negative impact on the exiting culture / social interactions. The main concept included is to develop self-sustained communities that encourage investments and acquire appropriate standard of living.

10. **Community and Stakeholder Collaboration in Development Decisions:** Community members have accurate information about their neighborhoods, and they can introduce innovative solutions for complex problems that respond to their values and needs. Enhancing social inclusion by community participation through capacity building programs, is an effective strategy to meet the expectations of people.

1.3 assumptions

- During the process of forecast and design of city plan, large economic fluctuations are not taken into account.
- The development of our city is continuous. The intending values of these different indexes and indicators are predictive on the basis of values of previous data and derivation.
- Ignore the influence from other cities. And we also ignore any impact from outer space.
- Ignore the the political and economical interaction between other cities or counties.
- Ignore any catastrophe made by nature or people just like earthquake, wars, nuclear weapon employment and so on.

2 construct the Indicator system of smart growth

In this paper, we define the indicator system of smart growth, which function as the metric to measure the degree of the goal to adhere to the three E's. Though the connotation of "smart growth" has been defined clearly before, researchers and experts still cannot get any agreement on the approaches of measuring this conception. In this perspective, all indicator selects based on Fuzzy Aggregation, which is also call "Word frequency statistical analysis".

2.1 synthesis of indicator system

Synthesis of index system refers to the current influential domestic and international smart growth evaluation index system, merged into an overall indicator library. We arduously collect articles (18 articles) and summarize the current evaluation indicators at home and abroad to build the database on the rules of three E's of sustainability and/or the 10 principles. We meet 34 indicators varying from residential density, population density, greenhouse gases emission to cultural facilities. We consider the breadth of the source of the indicator.

2.2 selection of indicator

At present, foreign scholars mainly adopt the empirical choice in the process of selecting the index hoofs. Therefore, the method used to conduct indicator is subjective.
Some who pays high attention to statistical analysis, in order to improve the rationality of the choice of indicators, introduces mainly methods as follows: the guiding ideology based on relevance analysis method, Analytic Hierarchy Process(AHP), representative selection index, expert advice screening method - expert law, regression equation

In this paper, we do not preclude the use of the above-mentioned kinds of indexing method. Membership function is constructed. Membership function (Membership function) is also known as membership function or fuzzy membership function is a fuzzy set function will be used, is generally set in the function indicating a generalized [1]. Function may indicate a set of instructions belongs to a particular subset of elements. Value indicates the function of an element may be 0 or 1, and a membership function value of the element would be between 0 and 1, represents an element belonging to a fuzzy set "true degree" (degree of truth).According to our condition ,we get：

$$\mathcal{F}(\mathrm{x}) = {}^{x}/_{18} \tag{1}$$

X denotes the frequency of appearance of the indicator,18 is the number of articles collected. The indicators with high membership degree are reserved and the indicators with low membership degree are excluded (there, the degree is 0.3).

Selecting principles

- The first round of election, after retaining one of synonymous substitution, the synonymous expression, the near-synonymous expression or antonym, we delete redundant indicators. (It has been done in the synthesis of indicator system).
- the second round of election, considering some fail to be quantified or unobservable

indicators, like “community participation in decision”, we delete unobserved indicators.

- the third based on the membership of the fuzzy set, the indicator with too low membership degree is deleted (the preamble has been completed).

In the end, we construct the Indicator system of smart growth which has 16 indicator Specific indicators. The first level indicator is three E’s and the 10 principles.

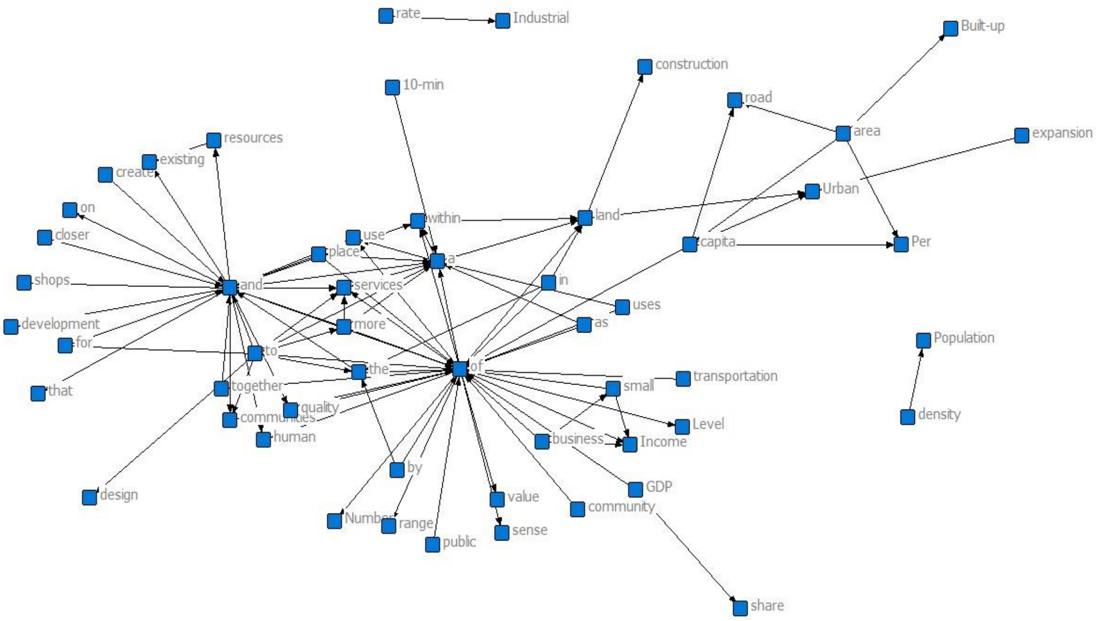

Figure 1: results of Word frequency statistical analysis

Figure only retains high-frequency indicators. Each center is the most frequently occurring word, surrounded by its related vocabulary.

Table1 :Summary of indicators

	First-level indicators	second-level indicators
	Economically prosperous	Per capita GDP
		The tertiary industry Proportion
		Per capita infrastructure financial expenditure
		Urban residents per capita disposable income
		Cultivated land proportion
	Socially Equitable	Built-up area of green area
SG		PM2.5
		employment rate
	Environmentally Sustainable	Per capita number of buses
		The number of tourist attractions
		The number of tourist attractions
		High school graduates proportion
		Population density
		Per capita road area
		Per capita number of beds
		Per capita car ownership

2.3 calculation the weight of indicators based on artificial neural network

2.3.1 Introduction of Artificial Neural Network

BPNN(short for Back Propagation Neural Network) is a variety of ANN, whose attribute is forward transmission of signal and reversed shift of error. In the process of forward transmission, input signals are operated through a few middle layers and exported from output layer ultimately. The state of every nerve cell could only influence the state of the next layer's nerve cell.

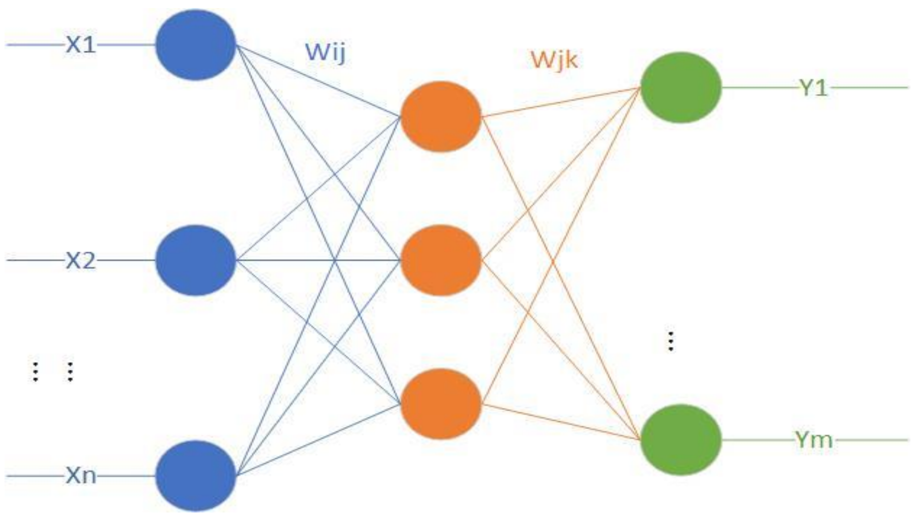

Figure2 BPNN Topological Structure Diagram

The training of BPNN includes following steps.

- Step 1 Initialization of Network

According to the system's input-output sequence(X, Y), decide the number of input layer's nodes, middle layer's nodes and output layer's nodes. Initialize weights among input layer, middle layer and output layer. Initialize thresholds from hidden layer and output layer. And preset other constants just like learning rate and nerve cells' driving function.

- Step 2 Calculate the Output of Hidden Layer

Calculate the output H of hidden layer's nodes on the basis of input variables, connection weights and thresholds from hidden layer. And we can have

$$\mathrm{H} = \mathrm{f}\left(\sum_{i=1}^{n} \omega_{ij} x_i - a_j\right), \mathrm{j} = 1,2, \dots, \mathrm{l} \quad (2)$$

Where H represents the output of hidden layer, l denotes the number of hidden layer's nodes and f is the driving function of hidden layer. There are many different kinds of expression for f, and we select

$$f(x) = \frac{1}{1+\mathrm{e}^{-\mathrm{x}}} \quad (3)$$

as its expression.

- Step 3 Calculate the Output of Output Layer

Refer to values of the output of hidden layer H, connection weights ω_{ij} and thresholds b, and then calculate the output of output layer. The basic formula is shown as:

$$O_k = \sum_{j=1}^{l} H_j \omega_{jk} - b_k, \mathrm{k} = 1,2, \dots, \mathrm{m} \quad (4)$$

Where O_k represents the output of output layer.

- Step 4 Calculate Errors

Calculate errors of the predictive values of the network. And here is the formula:

$$e_k = Y_k - O_k, \mathrm{k} = 1,2, \dots, \mathrm{m} \tag{5}$$

- Step 5 Update Weights

We should use errors calculated from the previous step to update weights of network connections. We can have

$$\omega_{ij} = \omega_{ij} + \eta H_j \left(1 - H_j\right) x(i) \sum_{k=1}^{m} \omega_{jk} e_k \,, i = 1,2, \dots, n; j = 1,2, \dots, l \tag{6}$$

$$\omega_{jk} = \omega_{jk} + \eta H_j e_k, j = 1,2, \dots, l; k = 1,2, \dots, m \tag{7}$$

Where η is learning rate.

- Step 6 Update Thresholds

$$a_j = a_j + \eta H_j \left(1 - H_j\right) \sum_{k=1}^{m} \omega_{jk} e_k \,, j = 1,2, \dots, l \tag{8}$$

$$b_k = b_k + e_k, k = 1,2, \dots, m \tag{9}$$

- Step 7 Estimate whether the iteration of algorithm is finished. If it is not finished, go back to the step 2.

2.3.2 Implementation to Weight Calculation on ANNs

- **Initialize parameters**

We choose 16 indicators involving serval different aspects including economy, environment, education, social equality and so on. So we regard these 16 indicators as input nodes of ANN(short for Artificial Neural Network), which can be marked as $X_1, X_2, X_3, \dots, X_{16}$. Of course, we should normalize the value of every indicator.

Then we are supposed to look for an index, which can express the comprehensive meaning of smart growth, to be regarded as the only one output node for the ANN model. After the assessment of serval indexes, we select HDI (short for Human Development Index) as this critical index. The Human Development Index (HDI) is a composite statistic (composite index) of life expectancy, education, and per capita income indicators, which are used to rank countries into four tiers of human development.

- **Select the Number of Middle Nodes**

According to the reference, we can get the inequality for the number of middle layer's nodes:

$$\mathrm{l} < \sqrt{(m + n)} + a \tag{10}$$

Where n is the number of input layer's nodes, l denotes the number of hidden(middle) layer's nodes and m represents the number of output layer's nodes.

So we preset the number of middle layer's nodes range from 4 to 15. And we build a neural network with variable numbers of middle layer's nodes. At the same time, we use this trained network to get output results with test input data. And we can determine the best number of hidden layer's nodes through errors of these new results.

Finally we use MATLAB to find the best results in Figure 2:

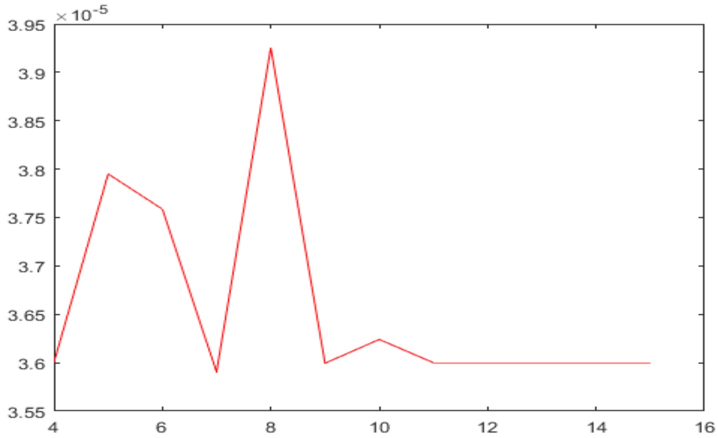

Figure3： Relationship Between Errors and Numbers of Hidden Layer's Nodes

From the above figure, we can learn errors could be the smallest when the number of hidden layer's nodes is 7.

- **Get Weights**

Finally, we can get the weights of different indexes with the best number of hidden layer's nodes.

2.3.3 Results Analysis

We have used mathematics modeling of ANNs and programmed with MATLAB to analyze weights of different indexes. And here are the Results.

Table2 :Calculating Weights by Different Indexes

Index	Weight
PM2.5	-0.031975742
GDP per capita	0.161897337
proportion of the third industry	0.059201123
housing per capita	0.057884302
green area per capita	0.057884302
proportion of high school education	0.073173823
population density	0.053315083
road area per capita	0.057616019
number of road per capita	0.052075813
number of bus per capita	0.056897074
disposable income per capita	0.059353508
rate of employment	0.064163208
investment of infrastructure per capita	0.073565065
number of cars per capita	0.073565065
proportion of cultivated land area	0.064275641
number of scenic spots	0.067108379

We calculate the success of smart growth, using:

$$s = \sum_{1}^{16} w_{ij} * x_{ij} \tag{12}$$

where, i denotes city and j denotes indicators. x_{ij} represents i city's j indicator and w_{ij}

represents this indicator's weight. The value of s is in the range of 0 to 1. Obviously, the bigger the value of s is the better the city performs.

3 Empirical test of the Indicator system of smart growth

3.1 the selected cities

We select Lishui District, Nanjing, china(Asia), and Sonoma California, United States(America).（Give full consideration to city strength, population, area, climate and data availability, etc.）

- Located in central and southern Nanjing City, Lishui District, an important agricultural science and technology base, is the East China's major transport hub and logistics center. Lishui District is one of the top 100 comprehensive strength areas in the china. The developed transportation makes it the most investment-value area in the Yangtze River Delta.
- Sonoma is a city in Sonoma Valley, Sonoma County, California, United States, surrounding its historic town plaza, a remnant of the town's Mexican colonial past. Today, Sonoma is a center of the state's wine industry for the Sonoma Valley AVA Appellation, as well as the home of the Sonoma International Film Festival.

Table3: Basic geographic information

	Population (2016)	Areas (m^2)	latitude	longitude	continent
Sonoma	497776	1076.6	38° 17′ N,	122° 27′ W	America
lishui	429553	1063.6	31 ° 23 'N	118 ° 51 W	Asia

3.2 review of the current growth plan

Sonoma:2020 general plan, Sonoma's current growth plan is the fifth adopted by Each of these, adopted n roughly ten-year intervals, since its first general plan in 1964.The plan has had a different scope and focus, but one quality has remained consistent: these plans have represented the collective vision of the community and expressed its desire to preserve and improve upon the essential characteristics that define. [5]

City Council Vision Statements [5]:

•Innovative, creative and sustainably-designed development respects the availability of natural resources and enhances the scale, character, and natural setting of the community.

• The community's history and its role as a cultural center are enhanced through public art, special events, and careful preservation of historic features.

• A vibrant, entrepreneurial economy is fueled largely by retention and incubation of locally-owned businesses that complement the small-town atmosphere and provide high paying jobs.

• Housing is available and affordable to the residents and the local workforce to support an economically diverse population.

• Creeks, trees, other natural features are valued and preserved, and open space and agricultural lands are protected—both in and around the city.

• Residents have access to a variety of high-quality recreational opportunities.

• Walking and bicycling are safe and the use of clean-fuel transit is popular. Traffic congestion is mitigated.

• Residents enjoy peace, quiet, and security, as well as efficient, high-quality public services.

• The City enjoys productive relationships with neighboring communities to effectively address regional issues, including planning, service provision and capital improvements.

Amazingly, the government's 9 goals coincide with the 10 principles. Therefore, in the article,

Table4:government's 9 goals of Sonoma

Sonoma General Plan Element	Required Elements	Examples ofTopics Covered
Community Development	Land Use	Development patterns, neighborhoods, visual character,
Circulation	Circulation	Traffic, street network, parking, transit services, bike routes
Public Safety	Safety	Development in hazardous areas, hazardous materials management,
Noise	Noise	Noise sources and mitigation
Housing	Housing	Demographics, housing needs, affordability, constraints on production
Environmental Resources	Conservation, Open Space	Open space, hillsides, riparian areas, sensitive plants and animals
Local Economy	Optional	Commercial and industrial land uses, economic diversification, job opportunities, tourism

Lishui : Lishui has developed a five-year urban planning, which calls "Lishui District, Nanjing City, the recent construction plan (2015-2020)" In recent years, Lishui's urban development has been steadily marching forward. Lishui Light Rail S7 completed, disposable income per capita is up to twice than before and urban and rural basic pension and basic medical insurance achieve 100% coverage. In the city planning for 2015-2020, the government proposes six major construction priorities. Now, we summarize and analyze its compliance with the SG's principles:

Table5: government's goals of lishui

goals	actions
"konggang " City	Focus on building "one heart and three areas" One for the integrated service center, three for the economic development center
Innovation City	The introduction of research and development, culture and other industrial projects Improve the public facilities system, efforts to enhance urban service soft power
Health Island	From north to south: build retirement health industry chain In-out: Develop eco-tourism energetically to boost population concentration
Culture and education park	To point with the surface: improve the quality of the environment landscape; To shaft pieces: cultural and leisure belt.

Agricultural Valley	Promote the construction of national agricultural demonstration garden Driven agricultural tourism, leisure and tourism development.
Leisure Ridge	Construction of special tourism projects, Organization interesting travel traffic, good tourist service facilities

3.3 scores of selected cities

Since we conclude the equation and pertaining indicators under the principles of SG theory, and calculate the weight based on ANNs, we get the scores of selected cities aiming to measure the current plans:

Table6: score of each city

city	Score ([0 ,1])
Lishui	0.314
Sonoma	0.4798

As we have talked before, the high score means the better performance. Due to data limitations, we are unable to collect a large amount of city data and hard to rate the scores. The different stages and characteristics of urban development are also a headache so that all factors make the score a relative quantity. But in a narrow sense, Sonoma's urban development (based on SG) seems more attractive.

4. design of dynamics-based smart growth planning

In the study of smart growth, it is regarded as a large complex system, which can be divided into economic, social, environment and other sub-systems. The measure of smart growth is affected by many factors, including but not limited to the resource environment and social livelihood. Due to the above characteristics of urban development, the idea of system dynamics equation is introduced to construct the urban development plan.

4.1 development of the growth plan

Principles based on system dynamics model

The system model must be constructed before the impact of various factors in the system are organized and analyzed detailly. First, economic prosperity is a major factor affecting the development, including the proportion of the tertiary industry, infrastructure, financial expenditure, per capita disposable income. Secondly, the factors of social stability are divided into government factors and non-government factors. In this paper, the main research is the government factors such as the number of buses per capita, the road area per capita and the number of beds per capita. Finally, the impact of environmental sustainability depends on the urban greening, the urban pollution index, etc. In summary, the system model should follow the following principles:

- **the principle of Causal relationship.** Starting from the development of the city, a causal cycle diagram is drawn based on the feedback relationship among the variables in the system dynamics. Therefore, a complete system dynamics model (SD model) can end up integrating various subsystem into an interaction of the whole.

- **Coordination principle.** In view of mutual promotion, mutual restraint relationship in the city's social, economic and environmental development, to make the city more intelligent development requires the development of the entire system, so when building the system model, we need to comprehensively consider all variables that can affect the coordinated development of the system.
- **Dynamic principle.** In system dynamics, both system and variables are dynamic and change according to the time scale. As the object of system research, the score of smart growth is also dynamic, and each of the indicators will be show different values and states in the different time periods. Therefore, dynamic thinking needs to be used to study the dynamic changes of system model flowcharts and the dynamic behaviors of system variables, so as to make the whole study more comprehensive and comprehensive.
- **The principle of scientific objectivity.** By combining the theory with practice and unifying the methods of qualitative and quantitative analysis, we can calculate the internal indexes of each subsystem by referring to the relevant statistical data and regression interpolation, and then make a comprehensive analysis of the calculated results so as to make the research more scientific and efficient objectivity.

Design of the growth plan

Considering the weights of metrical algorithm and the structural network of different indicators and indexes, we have reached serval valuable conclusions. Here are the results:

1. The proportion of infrastructure investment is critical node that has made a significant impact on the other nodes. So we think we should think it over repeatedly.
2. And as an important reason for infrastructure investment, GDP is definitely supposed to be taken into our plan's consideration.
3. In the long term, it is necessary to develop education and high technology industries to improve the potential of our city.

And in view of the three E's standard of smart growth, we can establish serval tips of city development plan:

Table 7: Plan for Lishui

Prosperity	**plan**
Economic	Take economic construction as the center of daily development goals.
	Attract foreign investment and build splendid commercial environments.
	Reduce taxes and improve the efficiency of government officials.
Social Equality	Increase the proportion of the investment of education
	and promote the development of balanced education.
	Attract talents positively and give favor policies to high-tech companies.
Environmental	Plan the layout of our city's built-up area reasonably
Sustainability	and build some natural parks or green road for amusement and rest.

Table8: Plan for Sonoma:

Prosperity	plan
Economic	Develop characteristic agriculture and enlarge the area of grape planting Find peripheral earning industry just like tourism and agricultural by-products.
Social Equality	Make a balance for the earning of the rich and the poor. Increase the quantity of the investment of infrastructure construction. Improve the level of medicine and the quality of education.
Environmental Sustainability	Stabilize existing environmental policies enhance their capacity for sustainable development.

4.2 Support of theory and system dynamics model

4.2.1Build system dynamics equations

In the model, all the precise relationships between variables are connected and determined by structural equations and then the internal structure of the model is expressed. The system model constructed in this paper based on the data from 2013 of two cities. The system equations were constructed according to the different development plans of the two cities and the data statistics of previous years, combining the interrelationships among the indicators and local actual conditions and statistical laws.

In order to be able to accurately describe the complex relationship between the various factors of the system and to re-predict its future development trend through the known initial state, we introduce three kinds of system dynamics equations, namely the equation of state (L), rate equation (R), and auxiliary equation (A)

- Equation of state (L)

The equation of state (L) is the basic equation of the system dynamics and describes the changes of the stock (state variables) in the system dynamics model. The formula is

$$L(t) = L(0) + \int_0^t (\sum R_{in}(t) - \sum R_{out}(t))dt \qquad (13)$$

Where , the initial value of L is L (0). R_{in} denotes the input flow of the state variable. R_{out} denotes the output flow of the state variable .So we get the net flow of the state variable -- $(\sum R_{in}(t) - \sum R_{out}(t))$.Then $L(t)$ represents the value of the state variable Lat time t.From that ,we can see that the value of the state variable(at time t) equals the change of the initial value of the state variable plus the net flow accumulated at [0, t] . In system dynamics, it is sometimes described in the form of difference equations:

$$L.K = L.J + (\sum R.JK_{in} - \sum R.JK_{out}) \times DT \qquad (14)$$

Where, the value of state variable$L$ at time$J$ is$L.J$. $R.JK_{in}$ represents the inflow rate variable R_{in}'s the value in the interval JK, and $R.JK_{out}$ represents the outflow rate variable R_{out} 's the value in the interval JK

- Rate equation (R)

The rate equation is an equation that defines the flow formation within a unit time. The inherent nature of rate equation includes the natural law of flow variation and the rule of decision of mankind to adjust the stock. The addition of auxiliary variables makes the formulation of the rate equation more accurate and explicit in the decision-making process. The rate equation can be expressed as a function of constants or state variables, often with some auxiliary variables:

$$R = f(L, Constant) \tag{15}$$

- Auxiliary equation(A)

By introducing auxiliary variables, the rate equation is decomposed into several simple auxiliary equations, and the operation rules between variables are determined according to the actual meaning in order to simplify the complicated decision-making process.

Determine the main parameters in the system

We refer to the relevant literature and standards, taking into account the stability of the system. The parameters are divided into three categories: 1) some parameters stable better. So, after referring to the existing model, we can directly calculate using the methods like the experience, trends Push method and arithmetic average. For example: population growth rate (0.012), employment rate (0.97). 2) there is a simple functional relationship between variables. For example: urban area = built-up area + cultivated area and per capita car = total car / total population .3) For data that is not perfect or not obvious between variables, you can use table functions or logistic functions to describe the nonlinear relationship between variables.

4.2.2 Analysis of each subsystem model

- **Economic subsystem analysis**

The economic subsystem consists of 12 variables, of which 3 are speed variables, 3 are state variables, 2 are constants, 4 are auxiliary variables. The proportion of infrastructure expenditure reflect the degree of economic input, and changes according to GDP. The greater the annual growth of GDP performs, the bigger the proportion of infrastructure shows. Meanwhile, the tertiary industry share has a high relationship with GDP. In this paper, the GDP, the output of the tertiary industry and the fiscal expenditures of the infrastructure are all determined by their annual growth and initial value. All variables in this paper are based on the data collected in 2013. Considering the reality, the annual growth of infrastructure expenditure and GDP are expressed as a linear fit function. The remaining variables are calculated using simple mathematical methods.

- **social development subsystem**

The social development subsystem consists of 19 variables, of which 2 are speed variables, 2 are horizontal variables, 3 are constants and 12 are auxiliary variables. We think that the employment rate of a city should be a constant in the development of a city, in which case, there is a linear relationship between employment and population. In the process of urban development, the characteristics of education, medical care, transportation, welfare and community should be closely linked with the government expenditures on infrastructure. That is to say, a series of variables such as road area, number of buses per capita, number of tourist attractions and so on in the indicator system are calculated using the table functions related to infrastructure finance to predict the future data. As population density and housing per capita are indicators which show the pressure of population, we consider the total population and total housing as a horizontal variable, the value of which is determined by the annual growth and the initial value(the 2013 data are also used as initial values).

- **Ecological environment subsystem**

The subsystem model consists of 10 variables, of which two constants, a rate variable, a state variable, six auxiliary variables. Starting from the ecological environment of urban construction, we think the greening area of the built-up area can be expressed by the table function of the government built-up area and infrastructure fiscal expenditure. The model limits its growth. Avoiding that the proportion of green space exceeds 50%, its annual growth should be controlled at 100 hectares, so as to avoid any unreasonable forecasting during the urban growth simulation. Similarly, PM2.5 is related to the green area of the built-up area and the change of car ownership per capita. However, the value of a city PM2.5 is within a fixed range, so we need to consider all the above factors while building the model.

Although the whole model is simply divided into three subsystems, the subsystems are not closed or isolated. The subsystems are interrelated and influence each other. The variables of one subsystem are also used for the variables of other subsystems.

4.2.3 Verification and revision of the model

When building the system dynamics model, the model represents only the approximation or abstraction of the actual system, so we need to test the validity of the model. This is the basis for simulation and prediction of the model

Theoretical test: First of all, we analyze the reasonableness, authenticity and scientific aspects of the parameter values, the relationship between variables, the consistency of units, the boundary of the model and so on. Then test the reason of the statements, causality and variables set

- First, we should rule out the errors on the mechanism. We need to test the types of variables, flowcharts, parameters, mathematical structural equations among variables, mathematical relations, and the rationality of causal relationships.
- Second, test the consistency of the dimension of each mathematical structural relation: After the simulation of Lishui District in Nanjing City, the simulation shows that the structural equation of the system can accurately describe the relationship between the variables, and can prove the causal relationship of the system . Using Units check tool for dimensional inspection, prove dimension unity of the mathematical structural equation.
- Third, run the model with Vensim. Rely on the software's own error detection and tracking functions to test the correctness of the system model expression, and use the software's equation test and dimensional inspection function to detect the consistency of model dimension. In addition, analyze the results of the operation of the model to determine the reasonableness of the rationale and make amendments in due course.

Historical test

The historical test of the model is the test of the overall prediction accuracy of the system model. Thanks to the model, we can analyze the historical data of each indicator, analyze the source of the error and its impact on the accuracy of the model so as to reduce or eliminate the impact of unfavorable factors. In the end, it can improve the accuracy of the model. Historical test means the simulation started from the historical statistics .Then, the simulation results will be compared and error will be checked.

For models with large deviations of operating results and historical data, it needs to be amended so that the system can accurately reflect the objective system. Similarly, we can get :

$$D_t = \frac{X'_t - X_t}{X_t} \times 100\%$$

Where X'_t denotes the predicted value at time t. X_t denotes the actual historical value at time t, and $D_t$denotes the degree of deviation.

The correctness of the model structure is very important in the system dynamics method. When testing the model, it is usually based on the theory test, which focuses on the consistency, correctness and adaptability of the model structure. After verification and amendment of the model, the general requirements of the deviation (the actual results and historical results) should not exceed 10%. We applied the system dynamics model to design the city plan, the model can similarly show in the table as follows

4.3.The results of policy design

The policies we design are those for specific city planning, which are too general to calculate. By quantifying the policy, we conclude with specific indicator values (16, three categories, and 10 principles) for each future period as follows:

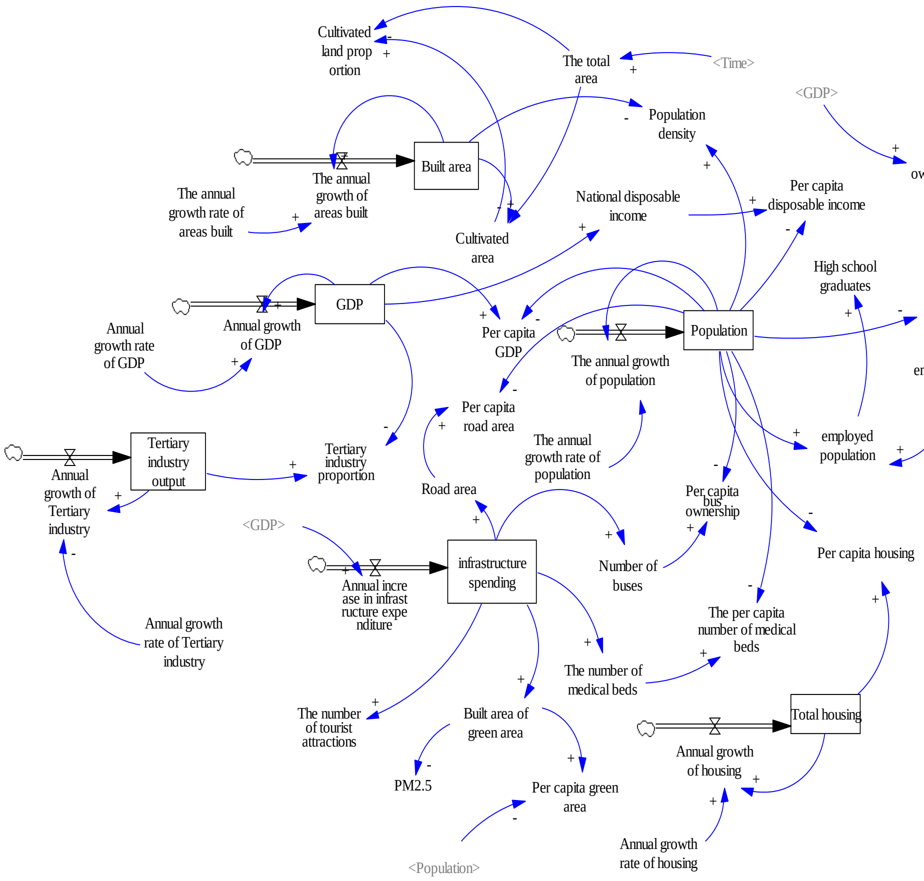

Figure4: system dynamics of the city

5 Further Study and Analysis of System Dynamics Simulation

5.1 compare and contrast the initiatives

The final ranking of its policies in both cities is:

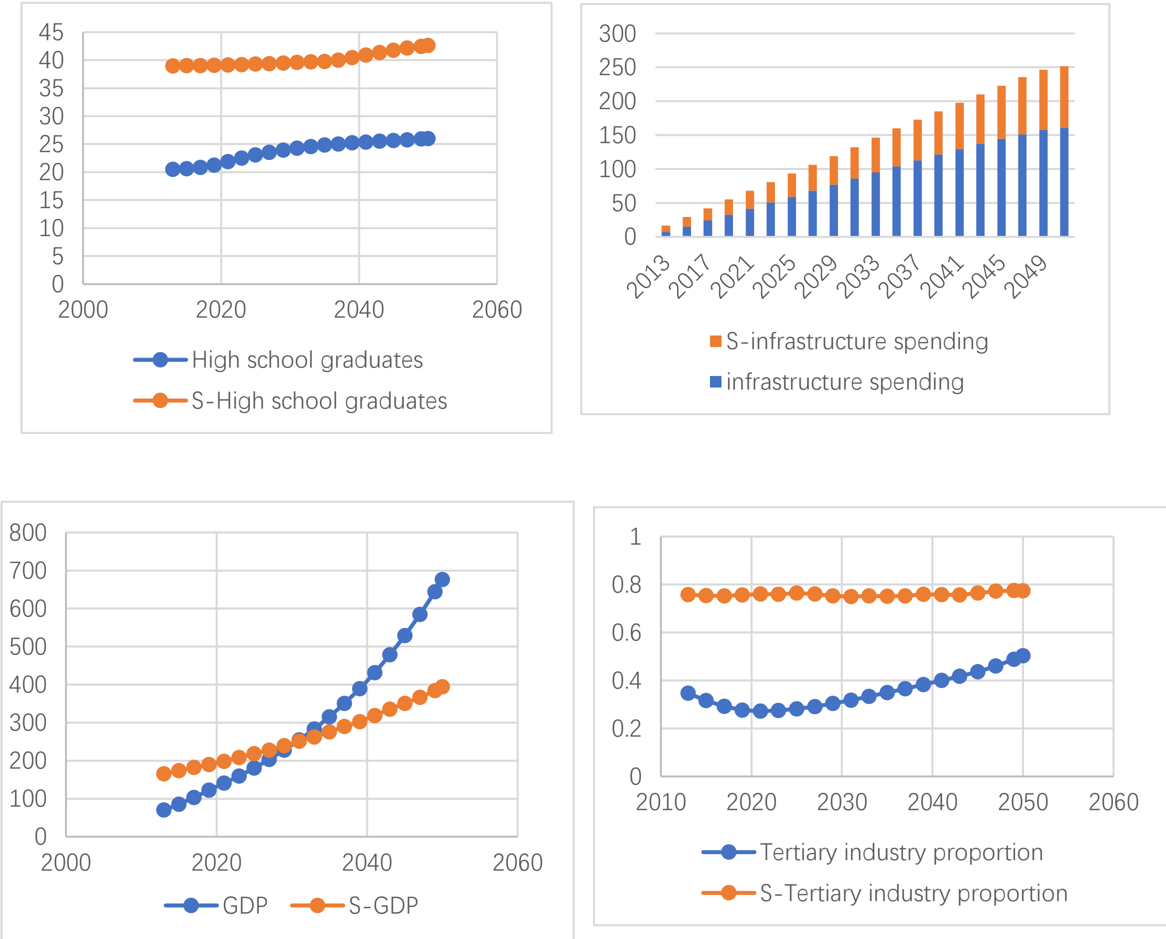

Figure5:Two cities index forecast（part）

16 indicators are so more that we can't be enumerated. list four important: GDP, tertiary industry output, high school graduates, infrastructure financial investment. It's found GDP and the output of the tertiary industry in lishui increases more rapidly. Although the number of high school graduates of Sonoma does not increase insignificantly, it was still at a high level. Lishui and Sonoma are both increasing their investments in infrastructure finance. In absolute terms, investment of lishui seems more.

Table9:compare and contrast the initiatives

First-level indicators	Score(lishui)	rank	Score(Sonoma)	rank
Economically prosperous	0.41	1	0.25	3
Socially Equitable	0.30	2	0.39	1
Environmentally Sustainable	0.29	3	0.36	2

5.2 effect of population growth

By consulting the literature, it is found that the U.S. population growth rate is 0.75% (per annum). In our dynamic model, we also found that the predicted population is much lower than the question asked--the additional 50%. If there was a population increase of 50% by 2050 in Sonoma (United States), there must have been a population surge. For Lishui (China), the situation is relatively complicated because of this year's second child policy and the improvement of population education.

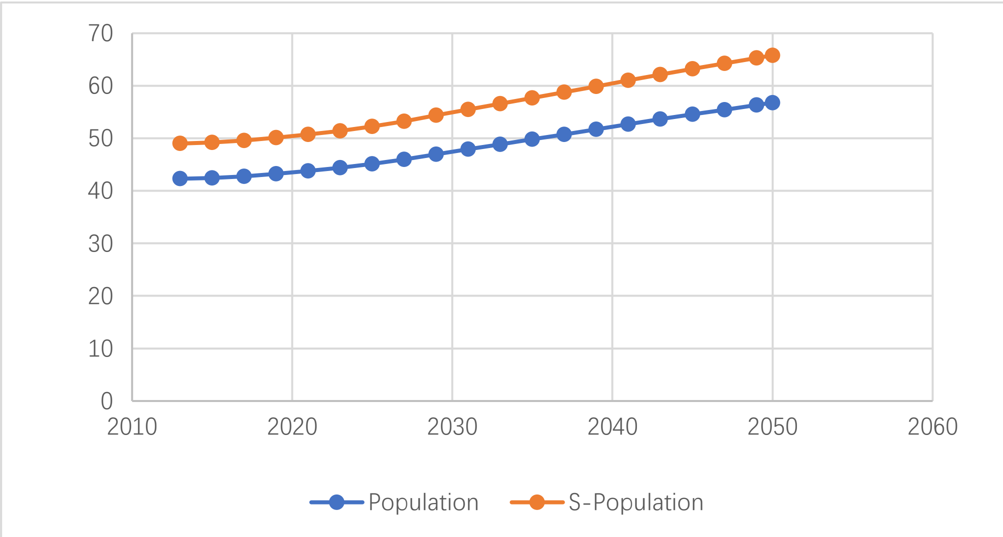

Figure6:Population growth forecast

In the subsystem, in addition to the indicator of population itself, the important indicators are GDP, afforestation area, and basic fiscal expenditure, all of which have a deep relationship with population. Population growth and urban expansion are mutually causal. It is clear that population growth has a crucial impact on the future of the city. In summary, we use Vensim to modify the population function, and finally get:

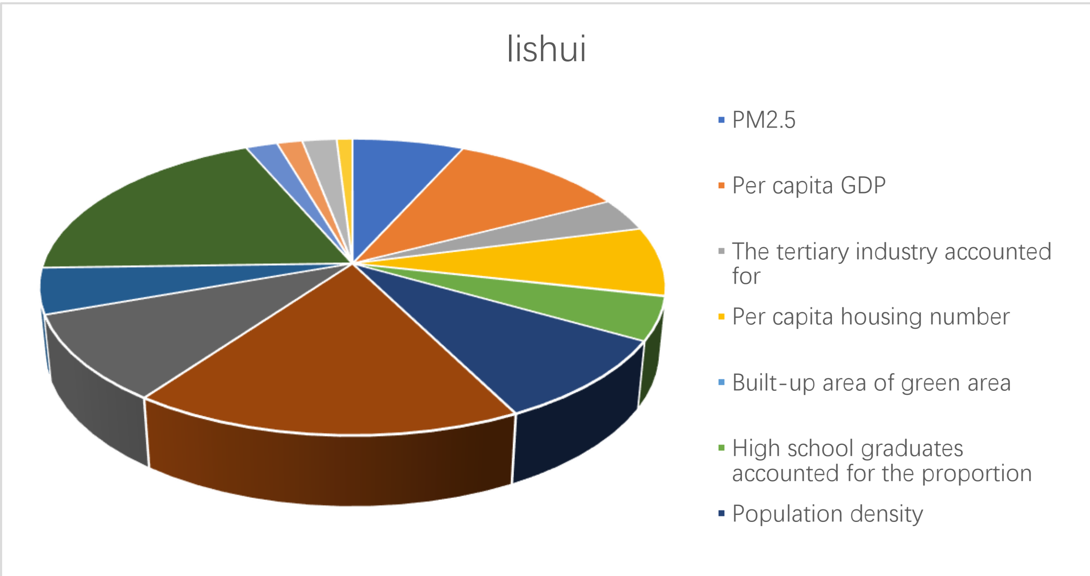

Figure7: lishui's score of each index

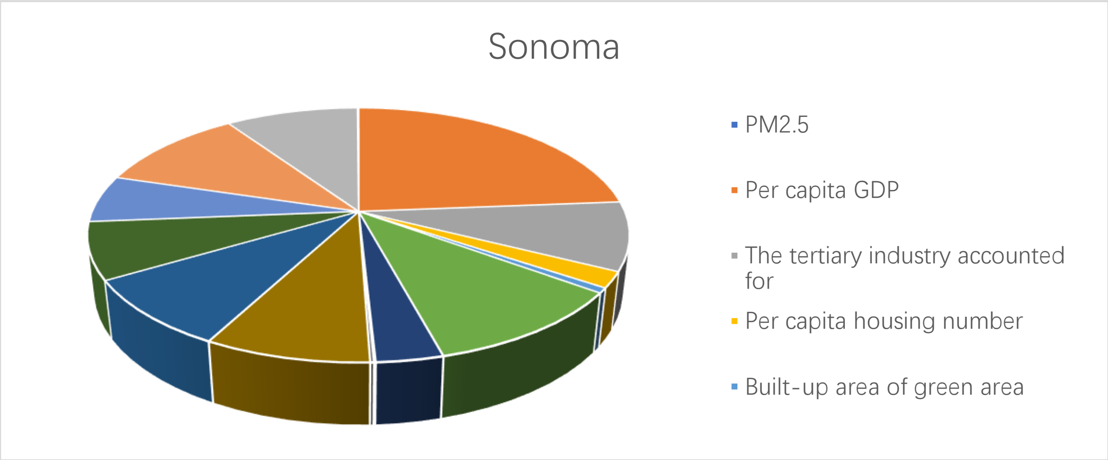

Figure8: sonoma's score of each index

6 Strengths and Weaknesses

Strengths

- Adopt objective weighting method (entropy weight method) and the subjective (group decision making method). We get gain the weight of each index based on BPNN . More effective weights are obtained.
- Employing combined prediction model combines the advantages of two prediction models, eliminates shortcomings in a certain extent and improves prediction accuracy.
- The operating mechanism of system dynamics is in line with the real development of the city.

Weaknesses

- Although we have try our best. Time is finite, and some data are missed. As a result, the missing data can still bring the errors in evaluation.
- Some of the parameters are based on common sense because few data are available

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