Cost-Sharing Mechanism Design for Social Cost Minimization in Ridesharing Systems

Yangsu Liu, Chaoli Zhang, Zhenzhe Zheng(✉), and Guihai Chen

Department of Computer Science and Engineering, Shanghai Jiao Tong University, Shanghai, China
{liu_yangsu,chaoli_zhang,zhengzhenzhe}@sjtu.edu.cn,
gchen@cs.sjtu.edu.cn

Abstract. Ridesharing, as an emerging efficient solution for transportation congestion, has achieved great business success in recent years. A critical issue in the ridesharing system is to determine a group of passengers to share a ride and the corresponding payments to charge them to cover the cost of drivers. We present the desired properties of a cost-sharing mechanism in ridesharing systems, namely *economic efficiency*, *incentive compatibility*, *individual rationality* and *budget balance*. However, the existing classic mechanisms do not achieve these properties even in a simple case. In this work, we formulate a cost-sharing model for ridesharing systems, and design two VCG-based mechanisms for the simple case and the general case, respectively. The simple case can capture the ridesharing problem with a symmetric cost function, while the general case describes a more general ridesharing problem with a submodular cost function. We theoretically demonstrate that these two mechanisms are approximately economic efficient with other desirable properties guaranteed. Finally, we evaluate the proposed mechanisms on a real-world dataset. The evaluation results show that our mechanism could increase the user experience of passengers as well as the efficiency of the ridesharing system.

Keywords: Ridesharing · Smart transportation · Cost-sharing mechanism

1 Introduction

With the explosion of population and rapid development of urbanization in recent years, transportation services play a more and more significant role in urban management. In the past decade, ridesharing services, which group passengers with similar routes, were considered as an effective and efficient transportation solution to alleviate the problems of natural resource depletion, transportation congestion and air pollution [4]. The emerging ride-hailing platforms, such as Didi, Uber and Lyft, also provide various types of ridesharing services.

There are two compelling research problems in the context of ridesharing. One is how to efficiently match a group of passengers to a vehicle with variable objectives, such as maximizing the revenue of the platforms or minimizing the travel cost of the drivers. The other problem is determining the payments to passengers to cover

Z. Liu et al. (Eds.): WASA 2021, LNCS 12938, pp. 277–289, 2021.
https://doi.org/10.1007/978-3-030-86130-8_22

the travel cost. Even though these two problems are highly correlated, most works are only dedicated to the first optimization problem from the perspectives of demand prediction [12,24], dispatching [9,18,26] and online route selection [25,28], paying less attention to the second pricing problem. On the other hand, existing pricing mechanisms, which provide the same fare rate for all passengers [21,23], ignore different travel costs and willingness-to-pay of passengers. In this work, we jointly consider the passengers assignment and cost allocation, and design a cost-sharing mechanism to minimize the social cost in ridesharing systems.

Designing an efficient and effective cost-sharing mechanism needs to consider the following three challenges. The first challenge comes from the heterogeneity of passengers in terms of willingness-to-pay. For example, some impatient passengers would like to pay more to prioritize their routes. In this case, it is necessary to take the utilities of passengers into consideration during the process of passengers assignment. We resort to the auction mechanism, in which passengers submit a bid to reveal their maximum willingness-to-pay. Considering the strategic behaviors of passengers, passengers would manipulate the bids to take advantages of the passengers assignment. It is non-trivial to design a mechanism to guarantee incentive compatibility and individual rationality[1].

The second challenge is due to the complexity of travel cost in ridesharing systems. A desirable cost-sharing mechanism should satisfy the property of budget balance, *i.e.*, the total payments of passengers should at least cover the travel cost of the driver. However, the incurred travel cost could be influenced by diverse factors like detour, congestion or travel time. As we will show in Sect. 3, classic incentive compatible mechanisms cannot guarantee budget balance even in a simple case with a symmetric cost function.

The third challenge is to maximize the social welfare of the ridesharing system, which is defined as the difference between the passengers' valuations and the incurred cost of the system. Unfortunately, it has been proved that there exists no such a mechanism [13]. Furthermore, there is no efficient mechanism with guaranteed approximation ratio [22]. Thus, we need to find another metric to measure the efficiency loss of the ridesharing system.

In this paper, we conduct a study on the problem of cost-sharing mechanism design for social cost minimization in ridesharing systems. First, we explore the desirable properties that a desirable cost-sharing mechanism is supposed to satisfy, including economic efficiency, incentive compatibility, individual rationality and budget balance. Second, we demonstrate disadvantages of the classical mechanisms, and give an impossibility result of designing a mechanism satisfying the above four desirable properties simultaneously, even for a simple case. Then, we relax the constraint of economic efficiency and turn to a new metric, social cost, to measure the approximate efficiency of the ridesharing system. Based on this metric, we design a VCG-based cost-sharing mechanism for the simple case, and extend it to the general case with the general submodular cost function. We theoretically prove that these two mechanisms are $O(n)$ and $\mathcal{H}_n$ approximately economic efficient with other the desirable properties guaranteed,

[1] Please refer to Sect. 2 for the detailed definitions.

respectively. The n is the number of candidate passengers and $\mathcal{H}_n = \sum_{l=1}^{n} \frac{1}{l}$ denotes the sum of the first nth Harmonic number.

In summary, our main contributions are shown as follows:

- We formulate a cost-sharing model for ridesharing systems and show that existing mechanisms are not qualified for desirable properties in ridesharing systems.
- We introduce the social cost metric to measure efficiency of ridesharing systems and propose two incentive compatible, budget balanced, individually rational and guaranteed approximately efficient mechanisms for the simple and general scenarios, respectively.
- We evaluate the performance of the proposed mechanisms on a real-world dataset, and evaluation results shows that our mechanism could increase user experience and efficiency of ridesharing system.

This paper is structured as follows: In Sect. 2, we present a cost sharing model of ridesharing and introduce basic notions of the cost-sharing mechanism. We discuss some existing truthful mechanisms in the context of cost-sharing problem in Sect. 3. In Sect. 4, we design two VCG-based mechanisms for the simple case and general case, respectively. We show our evaluation results in Sect. 5, and briefly review the related works in Sect. 6. Finally, we conclude this paper in Sect. 7.

2 Preliminaries

In this section we briefly introduce the notions used in this paper and give an overview of the cost-sharing problem.

We focus on an under-supply scenario during rush hour, where drivers are much more than drivers. In this case, maximizing the efficiency of the system is approach to maximizing the efficiency of each driver. Thus, we reduce the problem with multiple drivers to the mechanism design problem with single driver. Assuming that a set $U = \{1, 2, \ldots, n\}$ of passengers are requiring to be lifted. The travel cost of the driver is associated with the selected passenger set $W \subseteq U$ with a set function $C : 2^U \to R$. The cost function is defined as a general set function which may depend on travel time, travel distance or other factors of passengers. We assume that $C(\emptyset) = 0$ and that C is monotonically non-decreasing, implying that $\forall T \subseteq S$, $C(T) \leq C(S)$. Furthermore, we note that the cost of a group $S \cup T$ would be less than the sum of the separate cost of them, *i.e.*, $C(S \cup T) \leq C(S) + C(T)$. Otherwise, passengers in S and T would not be willing to share the ride as it takes less to ride separately.

Each passenger $i \in U$ has a fixed private valuation v_i for getting the travel service, *i.e.*, she is willing to pay at most v_i to get lifted. Since passengers only care about whether arriving at their destinations or not, the fixed valuation v_i of the passenger i would not be affected by other passengers. Without loss of generality, we assume that passengers are sorted in descending order of their valuations, *i.e.*, $v_i \geq v_j$ for any $i < j$.

Now we demonstrate the cost-sharing problem in the ridesharing system. As illustrated in the Fig. 1, the ridesharing platform first collects a non-negative valuation v_i from each passenger $i \in U$ for one ride. Then, the platform selects a set $W \subseteq U$ of passengers to share the ride, and finally charges every selected passenger $i \in W$ with a

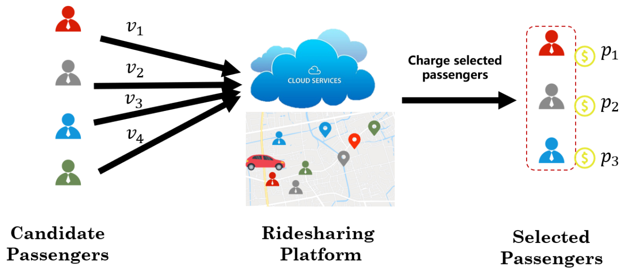

Fig. 1. An example of ridesharing system

non-negative payment p_i. The goal of every passenger i is to maximize her quasi-linear utility $u_i = (v_i - p_i)x_i$, where $x_i = 1$ if the passenger i gets served and $x_i = 0$ otherwise. For the sustainable development and economic efficiency of the system, the goal of the platform is to select the passenger set which maximizes the social welfare of the system and distribute incurred cost among passengers. Formally, the *Social Welfare* of the ridesharing system is defined as $SW(W) = \sum_{i \in W} v_i - C(W)$, where W is the set of passengers being served. With private valuations of passengers, truthfulness of the mechanism is necessary for the economic efficiency, because only when passengers report true valuation we can get real social welfare. Both budget balance and individual rationality are basic properties in ridesharing systems as the platform aims at gaining profit and the passenger would not accept a negative utility. Totally, we have identified four desirable properties that a cost-sharing mechanism for ridesharing systems should have: *incentive compatibility*, *budget balance*, *individual rationality* and *economic efficiency*, which are defined as follows:

Definition 1 (Truthfulness or Incentive Compatibility). *A mechanism satisfies incentive compatibility if truthfully bidding is the dominant strategy for every passenger. Formally, for any passenger $i \in U$, with others' valuation unchangeable, $v_i - p_i \geq v_i - p'_i$ where p_i and p'_i are the payments of i when truthfully bidding v_i and misreporting v'_i, respectively.*

Definition 2 (Individual Rationality). *A mechanism satisfies individual rationality if passengers never pay more than their willingness-to-pay,* i.e., $(v_i - p_i)x_i \geq 0, \forall i \in U$.

Definition 3 (Budget Balance). *A mechanism satisfies budget balance if the whole platform having no deficit. That means that the sum of payments from passengers should cover the travel cost,* i.e., $\sum_{i \in W} p_i \geq C(W)$ *where W is the set of passengers served.*

Definition 4 (Economic Efficiency). *A mechanism satisfies economic efficiency if the mechanism selects the subset of passengers that maximizes the social welfare.*

As we will show in the Sect. 3, designing a cost-sharing mechanism satisfying these four properties simultaneously is not trivial even in a very simple case and we will relax the economic efficiency property in Sect. 4.

Algorithm 1. Maximal Social Welfare Algorithm for The Simplified Case

Input: Sorted passengers' valuations: $\{v_1, v_2, \cdots, v_n\}$, where $v_i \geq v_j$ if $i < j$.
Output: Maximal social welfare SW and corresponding passengers Set W

Initialize $SW \leftarrow 0$, $W \leftarrow \emptyset$, $S \leftarrow \emptyset$, $i \leftarrow 1$.
while $i \leq n$ **do**
  $S \leftarrow S \cup \{i\}$
  if $SW \leq \sum_{j \in S} v_i - C(i)$ **then**
    $SW \leftarrow \sum_{j \in S} v_i - C(i)$
    $W \leftarrow S$
  end if
  $i \leftarrow i + 1$
end while
return SW, W

3 Discussion of Existing Mechanisms

It is non-trivial to maximize social welfare with all the critical properties guaranteed. In this section, we will discuss some widely used truthful mechanisms. We will show that all of them are not feasible for the ridesharing system even in a simplified case.

3.1 Simplified Case

We first consider a simple case where the cost function is symmetric and monotone, *i.e.*, $C(S) = C(T)$ whenever $|S| = |T|$ and $C(S) < C(T)$ where $|S| < |T|$. By abuse of notation, we write $C(|S|) = C(S)$ in the simple case. This scenario could be described as a special case in a prioritized ridesharing problem [27]. With such an assumption, as long as the number of passengers in the group does not change, neither will the incurred cost. When considering social welfare maximization, it is simplified in the sense that we can mainly focus on valuations of passengers, assuming that we have got the real valuations from passengers. To choose the passenger group with maximal social welfare, we greedily pick the passengers with maximal social welfare as winners (as shown in Algorithm 1). It makes the problem much easier and clearer. Even in such a setting, we will show that existing widely used mechanisms fail to satisfy budget balance and incentive compatibility simultaneously.

3.2 Critical Payment Method

First, we show that the critical payment method, which plays a significant role in truthful mechanisms, fails to guarantee budget balance. By critical payment method, the platform charges every winner with the highest valuation of passengers who do not win. We give a simple example to illustrate the mechanism.

Example 1. There are four passengers $\{1, 2, 3, 4\}$ with true valuations $v_1 = 10, v_2 = 6, v_3 = 3, v_4 = 1$ and $C(0) = 0, C(1) = 1, C(2) = 2, C(3) = 3, C(4) = 5$.

Assuming that all passengers report truthfully, we will select $W = \{1, 2, 3\}$ as the winner set. By the critical payment technique, the payments are $p_1 = v_4 = 1, p_2 = v_4 = 1, p_3 = v_4 = 1$, while the social welfare is still maximized. The utilities are $u_1 = v_1 - p_1 = 9$, $u_2 = v_2 - p_2 = 5$, $u_3 = v_3 - p_3 = 2$.

However, the critical payment based mechanism cannot guarantee budget balance. For example, we keep the same setting in Example 1 except that $v_4 = 0.9$. In this case, $\{1, 2, 3\}$ are picked as winners with $p_1 = p_2 = p_3 = 0.9$. Thus, $p_1+p_2+p_3-C(3) < 0$, indicating the mechanism does not satisfy budget balance.

3.3 VCG Mechanism

We consider another general incentive compatible mechanism, the VCG mechanism, which also guarantees the optimal social welfare. The payment of the passenger $i \in W$ is given as

$$p_i = \sum_{j \in W'_i} v_j - h(W'_i) - (\sum_{j \in W \setminus \{i\}} v_j - h(W)), \tag{1}$$

where $h : 2^U \rightarrow R$ is any set function that is independent of v_i and W'_i denotes the winner set generated by the Algorithm 1 by setting $v_i = 0$ and other valuations identical. When the function h is the cost function C, the passenger i pays the "damage" she causes, *i.e.*, the difference between the optimal social welfare when i does not participate and social welfare of other passengers in the outcome decision. In such case, the mechanism satisfies both economic efficiency and incentive compatibility [20]. Unfortunately, we will show that budget balance is not guaranteed by the following example:

Example 2. Keep the main setting of Example 1 except that $v_4 = 0.9$ and $C(0) = 0, C(1) = 2, C(2) = 3, C(3) = 4, C(4) = 6$.

By the VCG mechanism, $\{1, 2, 3\}$ will be picked as winners and $p_1 = (v_2+v_3-C(2)) - (v_1 + v_2 + v_3 - C(3) - v_1) = 1, p_2 = 1, p_3 = 1, p_4 = 0$. Thus, $p_1 + p_2 + p_3 - C(3) = -1 < 0$. Thus, the budget balance is not guaranteed.

4 Modified VCG Mechanism

In this section, we first try to modify the VCG mechanism to satisfy budget balance. Unfortunately, we show an impossibility result for maximizing social welfare while keeping budget balance, even in the simple case. Thus we resort to another metric, social cost, to measure the loss of efficiency in the ridesharing system and design two modified VCG mechanisms satisfying budget balance, incentive compatibility, individual rationality and approximate economic efficiency for the simple case and the general case, respectively.

4.1 Beneficial Attempt on Budget Balance

First, we modify the VCG mechanism to satisfy budget balance. Note that W_i' is the winner set without participation of the passenger i. We can get that $\sum_{j\in W_i'} v_j - h(W_i') \geq \sum_{j\in W\backslash\{i\}} v_j - h(W\backslash\{i\})$. Combining it with the Eq. (1), we have $p_i \geq h(W) - h(W\backslash\{i\})$. Therefore, if the function h satisfies that

$$\sum_{i\in W} h(W) - h(W\backslash\{i\}) \geq C(S), \tag{2}$$

the mechanism could guarantee budget balance. Therefore, we can define a simple function $h(W) = \mathcal{H}(W) = \sum_{l=1}^{|W|} \frac{C(l)}{l}$. Thus, the payment p_i is

$$p_i = \sum_{j\in W_i'} v_j - \mathcal{H}(W_i') - (\sum_{j\in W\backslash\{i\}} v_j - \mathcal{H}(W)). \tag{3}$$

We demonstrate the mechanism by an example shown below (Example 3).

Example 3. There are five passengers $\{1,2,3,4,5\}$ with valuations $v_1 = 20, v_2 = 18, v_3 = 15, v_4 = 10, v_5 = 5$. The cost is only decided by the number of passengers where $C(0) = 0, C(1) = 10, C(2) = 19, C(3) = 27, C(4) = 34, C(5) = 40$.

With the definition of $\mathcal{H}$, we get $\mathcal{H}(0) = 0$, $\mathcal{H}(1) = 10$, $\mathcal{H}(2) = 19.5$, $\mathcal{H}(3) = 28.5$, $\mathcal{H}(4) = 37$, $\mathcal{H}(5) = 45$. To maximize social welfare, we still pick passengers $\{1,2,3,4\}$ as the winner set with $p_1 = (v_2 + v_3 + v_4 - \mathcal{H}(3)) - (v_2 + v_3 + v_4 - \mathcal{H}(4)) = 8.5$, $p_2 = 8.5$, $p_3 = 8.5$ and $p_4 = 8.5$. Here, $p_1 + p_2 + p_3 + p_4 - C(4) = 0$.

In the above example, we achieve optimal social welfare and budget balance at the same time if passengers report truthfully. Unfortunately, the mechanism cannot guarantee the individual rationality and incentive compatibility although the payment is independent of the passenger's reporting valuation. For example, if we set $v_5 = 7$ in Example 3, we have $p_5 = (\sum_{i=1}^{4} v_i - H(4)) - (\sum_{i=1}^{5} v_i - H(5) - v_5) = 8$. Thus, the utility of the passenger 5 is negative, so that the truth-telling is not the dominant strategy of the passenger 5. This is due to that the allocation algorithm cannot guarantee maximization of social welfare and $\sum_{i\in W} v_i - \mathcal{H}(W)$ simultaneously.

Actually, as pointed out by Green [13], there is no cost-sharing mechanism satisfying the truthfulness, budget balance, individual rationality and economic efficiency simultaneously. This impossibility result indicates that we should relax at least one of our desirable properties. As a consequence, we turn to looking for a mechanism approximately maximizing social welfare. Unfortunately, another impossibility result given by Feigenbaum [11] implies that there is no such approximation guarantee of social welfare. The hardness of approximating efficiency is mainly due to mixed-sign property of social welfare. Hence, we need measure the efficiency loss of the mechanism in another way. For this purpose, we refer to the social cost metric proposed by Roughgarden [22]. Formally, the social cost $SC(W)$ of selected passenger set W is defined as

$$SC(W) = C(W) + \sum_{i\in U\backslash W} v_i \tag{4}$$

Algorithm 2. VCG-Based Mechanism for Simplified Case

Input: Sorted passengers' valuation profiles:$\{v_1, v_2, \cdots, v_n\}$, where $v_i \geq v_j$ if $i < j$.
Output: Passenger set W and corresponding Payment vector $\mathcal{X} = \{p_i | i \in W\}$

Compute $\mathcal{H}(|I|) \leftarrow \sum_{l=1}^{|I|} \frac{C(l)}{l}$ as: $\{\mathcal{H}(0), \mathcal{H}(1), \mathcal{H}(2), ..., \mathcal{H}(n)\}$.
Initialize $i \leftarrow 1$, $SW \leftarrow 0$, $W \leftarrow \emptyset$, $S \leftarrow \emptyset$, $\mathcal{X} \leftarrow \emptyset$.
while $i \leq n$ **do**
    $S \leftarrow S \cup \{i\}$
    if $SW \leq \sum_{i \in S} v_i - \mathcal{H}(i)$ **then**
        $SW \leftarrow \sum_{i \in S} v_i - \mathcal{H}(i)$
        $W \leftarrow S$
    end if
    $i \leftarrow i + 1$
end while
for $i \in W$ **do**
    Recompute W_i' by setting $v_i = 0$ for each passenger $i \in W$
    $p_i \leftarrow \sum_{j \in W_i'} v_j - \mathcal{H}(|W_i'|) - (\sum_{j \in W \setminus \{i\}} v_j - \mathcal{H}(|W|))$
    $\mathcal{X} \leftarrow \mathcal{X} \cup \{p_i\}$
end for
return $W, \mathcal{X}$

Social cost is the sum of incurred cost and valuations fail to receive the service (*i.e.*, opportunity cost). As the sum of social welfare and social cost is a constant, minimizing social cost is equivalent to maximizing social welfare. Definitely, the two objectives are not equivalent from an approximation perspective. Thus, we give another definition of approximate economic efficiency with the social cost, which is commonly adopted by other cost-sharing mechanisms [10].

Definition 5 (α-approximate economic efficiency). *The α-approximately efficient allocation satisfies that, $SC(W) \leq (\alpha + 1)SC(W^*)$, where W^* is the set minimizing the social cost and W denotes the set generated by the allocation algorithm.*

4.2 Mechanism Design for Simplified Case

For the simple case where the cost function is symmetric, we design a VCG-based mechanism similar to the method in Sect. 4.1. As shown in Algorithm 2, we first select the passenger set W with

$$W = \arg\max_{W \subseteq U} \sum_{j \in W} v_j - \mathcal{H}(W), \tag{5}$$

and charge the passenger $i \in W$ with the Eq. (3), where W_i' is the outcome of allocation by setting $v_i = 0$. Then, we briefly demonstrate the desirable properties guaranteed by Algorithm 2.

Theorem 1. *The VCG-based mechanism described in Algorithm 2 is truthful, individually rational, budget balanced and provides an approximation ratio of $O(n)$ in social cost where the cost function is symmetric.*

The proof is included in our technical report [17] but is omitted in this work due to space limitations.

4.3 Mechanism Design for General Case

Now we extend the mechanism to the general case where cost function is extremely complex due to the complexity of travel routing. Unfortunately, not every cost function can lead to a well-performed mechanism. It has been shown that if the cost function C is supermodular, no mechanism can achieve a guaranteed social cost approximation ratio [10]. Except for supermodular cost function, another class of general cost function satisfies submodularity, *i.e.*, for any $S \subseteq T \subseteq U$ and $i \in U \backslash T$, $C(S \cup \{i\}) - C(S) \geq C(T \cup \{i\}) - C(T)$. It is consist with the practice that the marginal cost is always monotonically decreasing in the ridesharing system.

For a general submodular cost function, we use Hart and Mas-Colell's potential function [14] to define the function $h(W)$ in the Eq. (3), which is defined as:

$$h(W) = \sum_{S \subseteq W} \frac{(|S|-1)!(|W|-|S|)!}{|W|!} C(S),$$

where W is the allocation outcome. Similar to the mechanism for the simple case, the allocation outcome is

$$W = \arg\max_{W} \sum_{i \in W} v_i - h(W). \tag{6}$$

Due to the generality of cost function, here we skip the discussion on the complexity of calculating the optimal result and focus on designing the cost-sharing mechanism.

Theorem 2. *The VCG-based mechanism for general case satisfies the truthfulness, budget balance, individual rationality and provides an approximation ratio of $\mathcal{H}_n$ in the social cost.*

Due to the limitation of space, we leave the detailed proof in our technical report [17].

5 Evaluations

In this section, we conduct evaluations to demonstrate the performance of the VCG-based mechanisms for the general case in different levels of supply and demand. We first describe our simulation setup and then present simulation results.

5.1 Evaluation Setup

We evaluate the cost-sharing mechanism on a real taxi dataset of New York, which contains information of individual taxi rides in New York city [1]. Each entry in the dataset records information including start location, destination location, passenger number, travel time, travel distance. For studying the situation during rush hour, we sample the records in (10:00:00–10:02:00) on Jan 1st 2019 and Jan 2nd 2019 as our experiment data. We regard each entry as an individual request of a passenger. The individual valuation v_i of passenger i is sampled from a Gaussian distribution whose mean value is μ_i and variance $\delta_i = 10$. We let μ_i be linear with the travel distance of her request.

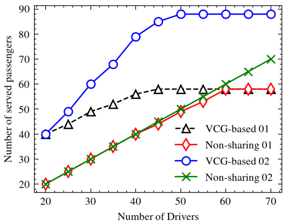

Fig. 2. The effect of supply on user experience

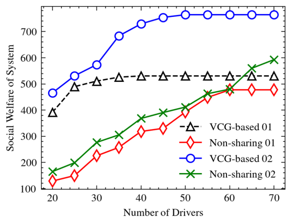

Fig. 3. The effect of supply on social welfare

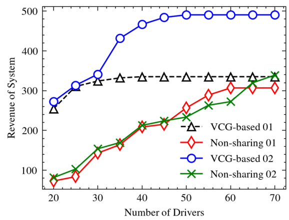

Fig. 4. The effect of supply on revenue

To investigate the influence of the vehicle supply on the performance of our mechanism, we set different numbers of drivers for the rush hour setting in random location which is in range [20] with step 5. We define the cost function for each driver as $C(S) = \sum_{i \in S} dis(s_i, d_i)$, where S is the group of passengers share the same ride and the $dis(s_i, d_i)$ denotes the distance between the start point of the first passenger and the destination of the last passenger in a ride. We adopt the bipartite graph matching algorithm for vehicle sharing to allocate candidate passengers to each driver in advance.

5.2 Evaluation Results

We compare the performance of the VCG-based mechanism with the non-sharing mechanism which adopts a fixed rate of charge for the single passenger. The Fig. 2 shows the number of served passengers with different riding resource supplies, where the *VCG-based 01* and *Non-sharing 01* represent results conducted by the VCG-based mechanism and non-sharing mechanism using dataset on Jan 1st 2019, respectively. Similarly, the *VCG-based 02* and *Non-sharing 02* represent results using dataset on Jan 2nd 2019. Although the demand levels are different on these two days, it leads to the similar overall trend. The number of served passengers climbs and stay stable with the number of drivers rises except that the number of served passenger keep rising with the increasing number of drivers in Non-sharing 02. This is due to that 70 drivers cannot serve all passengers over the Non-sharing mechanism on the second day while all passengers get served with 50 drivers over VCG-based mechanism. Moreover, more passengers get serviced in VCG-based mechanism in comparison to the non-sharing mechanism, indicating that the experience of passengers are better by adopting VCG-based mechanism.

In our second simulation, we investigate the influence of riding resource on the social welfare and revenue of the system. As the Fig. 3 and Fig. 4 illustrate, both social welfare and revenue of the system first increases and then stay stable with drivers increasing. And the system could get extremely more social welfare and revenue by adopting VCG-based mechanism comparing to Non-sharing mechanism.

6 Related Works

We briefly review related works in this section. The most related topic is pricing and dispatching in ridesharing systems [3,5,6], which focused on optimizing efficiency of ridesharing systems. However, they all provide fixed fare rate and dismiss the heterogeneity of passengers' willingness-to-pay. There are a line of works based on the auction to recognize the heterogeneity of passengers. Kleiner *et al.* [15] proposed an incentive compatible mechanism based on parallel auctions considering the individual preference of passengers. Based on the Vickery auction, Asghari *et al.* [2] designed a truthful and individually rational mechanism for online ridesharing. Chen and Wang [7] proposed a pricing scheme aiming at optimizing social welfare of ridesharing system. But neither of them considers the travel cost of drivers.

Cheng *et al.* [8] proposed a fare splitting mechanism for last-mile ride-sharing which satisfies budget balance, individual rationality and incentive compatibility. Zhao *et al.* [29] showed that the VCG mechanism in ridesharing system results in a high deficit and proposed a mechanism with deficit control. However, these works ignore the efficiency of the ridesharing system.

There are several works designing the cost-sharing mechanism from a perspective of fairness [16,19]. They turned to cooperative game theory solution concepts, the Shapley value and the nucleolus, to allocate incurred cost fairly. However, the Shapley value and nucleolus cannot guarantee neither individual rationality nor incentive compatibility.

7 Conclusion

In this work, we have proposed a cost-sharing framework based on VCG mechanism to recognize the heterogeneity of passengers' willingness-to-pay and approximately minimize social cost of systems. First, we explored desirable properties for ridesharing and demonstrated why existing mechanisms fail. We have proposed two VCG-based mechanisms for the simple case and general case to guarantee the incentive compatibility, budget balance, individual rationality and provide a guaranteed approximation ratio for the economic efficiency. Finally, we evaluated the performance of our mechanism over a real-world data set. The evaluation results showed that the cost-sharing mechanism could increase the efficiency of the system as well as the user experience of passengers.

Acknowledgements. This work was supported in part by National Key R&D Program of China No. 2019YFB2102200, in part by China NSF grant No. 62025204, 62072303, 61972252, and 61972254, in part by Alibaba Group through Alibaba Innovation Research Program, and in part by Tencent Rhino Bird Key Research Project. The opinions, findings, conclusions, and recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the funding agencies or the government.

References

1. Taxi and limousine commission: Tlc trip record data. https://www1.nyc.gov/site/tlc/about/tlc-trip-record-data.page

2. Asghari, M., Shahabi, C.: An on-line truthful and individually rational pricing mechanism for ride-sharing. In: Proceedings of SIGSPATIAL, pp. 1–10 (2017)
3. Banerjee, S., Johari, R., Riquelme, C.: Pricing in ride-sharing platforms: a queueing-theoretic approach. In: Economics and Computation, p. 639 (2015)
4. Caulfield, B.: Estimating the environmental benefits of ride-sharing: a case study of Dublin. Transp. Res. Part D: Transp. Environ. **14**(7), 527–531 (2009)
5. Chen, L., Zhong, Q., Xiao, X., Gao, Y., Jin, P., Jensen, C.S.: Price-and-time-aware dynamic ridesharing. In: Proceedings of ICDE, pp. 1061–1072 (2018)
6. Chen, M., Shen, W., Tang, P., Zuo, S.: Dispatching through pricing: modeling ride-sharing and designing dynamic prices. In: Proceedings of IJCAI, pp. 165–171 (2019)
7. Chen, Y., Wang, H.: Pricing for a last-mile transportation system. Transp. Res. Part B: Methodol. **107**, 57–69 (2018)
8. Cheng, S., Nguyen, D.T., Lau, H.C.: Mechanisms for arranging ride sharing and fare splitting for last-mile travel demands. In: Proceedings of AAMAS, pp. 1505–1506 (2014)
9. Dickerson, J.P., Sankararaman, K.A., Srinivasan, A., Xu, P.: Allocation problems in ride-sharing platforms: online matching with offline reusable resources. In: Proceedings of AAAI (2018)
10. Dobzinski, S., Ovadia, S.: Combinatorial cost sharing. In: Proceedings of EC, pp. 387–404 (2017)
11. Feigenbaum, J., Papadimitriou, C.H., Shenker, S.: Sharing the cost of multicast transmissions. J. Comput. Syst. Sci. **63**(1), 21–41 (2001)
12. Geng, X., et al.: Spatiotemporal multi-graph convolution network for ride-hailing demand forecasting. In: Proceedings of AAAI (2019)
13. Green, J., Kohlberg, E., Laffont, J.J.: Partial equilibrium approach to the free-rider problem. J. Public Econ. **6**(4), 375–394 (1976)
14. Hart, S., Mas-Colell, A.: Potential, value, and consistency. Econometrica: J. Econom. Soc. **57**, 589–614 (1989)
15. Kleiner, A., Nebel, B., Ziparo, V.A.: A mechanism for dynamic ride sharing based on parallel auctions. In: Proceedings of IJCAI, Spain, July 16–22, 2011, pp. 266–272 (2011)
16. Levinger, C., Hazon, N., Azaria, A.: Fair sharing: the Shapley value for ride-sharing and routing games. arXiv preprint arXiv:1909.04713 (2019)
17. Liu, Y., Zhang, C., Zheng, Z., Chen, G.: Cost-sharing mechanism design for ridesharing systems. In: Technical report (2021). https://www.dropbox.com/s/rjb5x3fzb48q6pe/WASA_2021_full
18. Liu, Y., Skinner, W., Xiang, C.: Globally-optimized realtime supply-demand matching in on-demand ridesharing. In: Proceedings of WWW, pp. 3034–3040 (2019)
19. Lu, W., Quadrifoglio, L.: Fair cost allocation for ridesharing services - modeling, mathematical programming and an algorithm to find the nucleolus. Transp. Res. Part B-Methodol. **121**, 41–55 (2019)
20. Suzuki, M., Vetta, A.: How many freemasons are there? The consensus voting mechanism in metric spaces. In: Harks, T., Klimm, M. (eds.) SAGT 2020. LNCS, vol. 12283, pp. 322–336. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-57980-7_21
21. Pandit, V.N., Mandar, D., Hanawal, M.K., Moharir, S.: Pricing in ride sharing platforms: static vs dynamic strategies. In: Proceedings of COMSNETS, pp. 208–215 (2019)
22. Roughgarden, T., Sundararajan, M.: Quantifying inefficiency in cost-sharing mechanisms. J. ACM (JACM) **56**(4), 1–33 (2009)
23. Sun, L., Teunter, R.H., Babai, M.Z., Hua, G.: Optimal pricing for ride-sourcing platforms. Eur. J. Oper. Res. **278**(3), 783–795 (2019)
24. Tong, Y., et al.: The simpler the better: a unified approach to predicting original taxi demands based on large-scale online platforms. In: Proceedings of SIGKDD, pp. 1653–1662. ACM (2017)

25. Xu, Y., Tong, Y., Shi, Y., Tao, Q., Xu, K., Li, W.: An efficient insertion operator in dynamic ridesharing services. In: Proceedings of ICDE, pp. 1022–1033 (2019)
26. Xu, Z., et al.: Large-scale order dispatch in on-demand ride-hailing platforms: a learning and planning approach. In: Proceedings of SIGKDD, pp. 905–913 (2018)
27. Yengin, D.: Characterizing the Shapley value in fixed-route traveling salesman problems with appointments. Int. J. Game Theory **41**(2), 271–299 (2012)
28. Yuen, C.F., Singh, A.P., Goyal, S., Ranu, S., Bagchi, A.: Beyond shortest paths: route recommendations for ride-sharing. In: Proceedings of WWW, pp. 2258–2269 (2019)
29. Zhao, D., Zhang, D., Gerding, E.H., Sakurai, Y., Yokoo, M.: Incentives in ridesharing with deficit control. In: Proceedings of AAMAS, pp. 1021–1028 (2014)