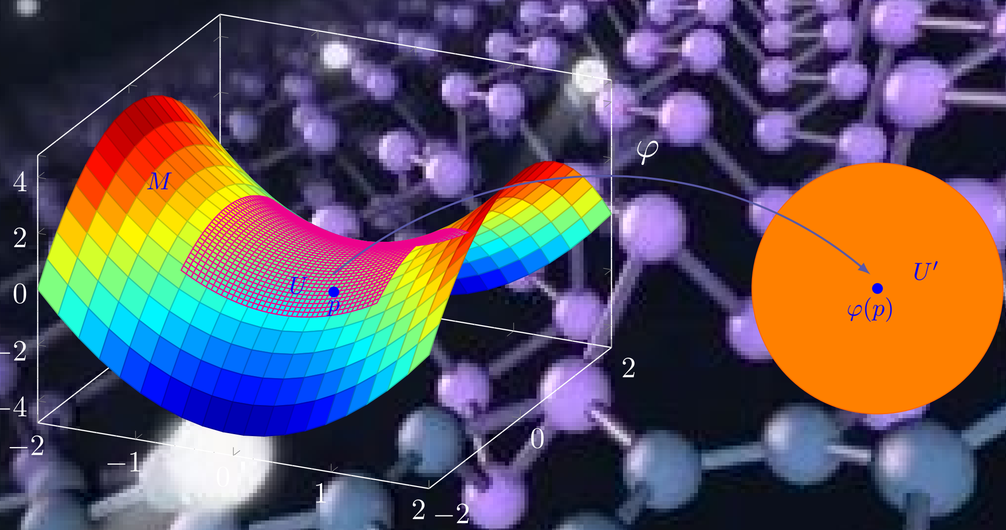
Awesome-drawing

By Using TikZ

Banach Spaces



$$\varphi : \mathbb{R}^3 \to \mathbb{R}^2$$

$$(x, y, z) \mapsto (u(x, y, z), v(x, y, z))$$

http://www.latexstudio.net

内容简介

Author: Yongxue Liu (E-mail: yongxue1487@foxmail.com.)
Author's GitHub: https://github.com/LiuYongxue-code/
The latest update can be found via: https://github.com/LiuYongxue-code/textbook

Update version:

Email: http://www.latexstudio.net.
Picture of title page comes from https://mp.weixin.qq.com/s/1pzt7iykiYmq33-NMkE95g.
The latest update can be found via: https://github.com/LiuYongxue-code/textbook the course.

本书原来模板源于原作者 **colin-young** colin-young@live.com，因计划用于教师教材用书，故在其中设置了诸多的 tcolorbox 环境，并整理于：https://marukunalufd0123.hatenablog.com/entry/2019/03/15/071717. 前模板：https://latexstudio.net/index/details/index/mid/1190.html.

模板使用说明

本书模板改编于原作者：**colin-young** colin-young@live.com 中 A-level 物理教材设计的模板，本模板是原模板的汉化版和升级版，原模板资源地址：https://github.com/yuhao-yang-cy/a2sphysics 模板的 tcolorbox 等环境支持中文，升级在于以下几点：

- 模板设置了可修改的封面设置。
- 模板在目录、章节设置，代码抄录等进行了单独的设置。
- 模板设置了大量的 tcolorbox 环境，适用于课前知识预习、知识点标题、段文关键词、练习，习题，解答、思考，知识回顾，延伸与探究等诸多环境。

Throughout the notes, key concepts are marked red, key definitions and important formulas are boxed. But to be honest, the main reason that I wrote up these notes was not to serve any of my students, but just to give myself a goal. Since physics is such a rich and interesting subject, I cannot help sharing a small part of topics beyond the syllabus that I personally find interesting.
Also very importantly, I am certain that there are tons of typos in the notes. If you spot any errors, please let me know.

笔记内容

I borrow heavily from the following resources:

- Cambridge International AS and A Level Physics Coursebook, by *David Sang*, *Graham Jones*, *Richard Woodside* and *Gurinder Chadha*, Cambridge University Press
- International A Level Physics Revision Guide, by *Richard Woodside*, Hodder Education
- Longman Advanced Level Physics, by *Kwok Wai Loo*, Pearson Education South Asia
- Past Papers of Cambridge Internation A-Level Physics Examinations
- HyperPhysics Website: http://hyperphysics.phy-astr.gsu.edu/hbase/index.html
- Wikipedia Website: https://en.wikipedia.org

目　录 (CONTENTS)

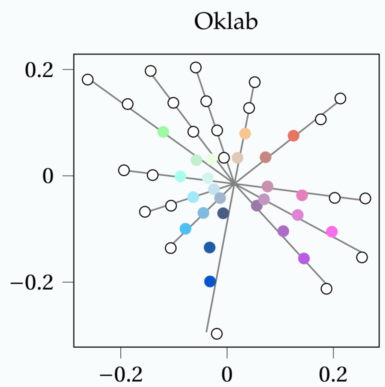
前 言

Angular quantities

movement or rotation of an object along a circular path is called **circular motion**

to describe a circular motion, we can use *angular quantities*, which turn out to be more useful than linear displacement , linear velocity , etc.

```
\begin{ascboxZ}{Angular␣quantities}
movement or rotation of an object along a
    circular path is called \keypoint{
    circular␣motion}
\end{ascboxZ}
```



angular displacement

angular displacement is angle swiped out by object moving along circular

注意几点

angular velocity is defined as angular displacement swiped out per unit time: $\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$

- unit of: $[\omega] =$ rad s^{-1}, also in radian measures
- angular velocity is a *vector* quantity

this vector points in a direction normal to the plane of circular motion

but in A-level course, we treat angular velocity as if it is a scalar

angular velocity and angular speed may be considered to be the same idea

知识点一

The vector points in the direction perpendicular to the circular motion plane, but in the A-level course, we treat the angular velocity as a scalar , That is, when we consider angular velocity, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most

- unit of: $[\omega] =$ rad s^{-1}.
- angular velocity is a *vector* quantity
- unit of: $[\omega] =$ rad s^{-1}.
- angular velocity is a *vector* quantity

重要公式列表

we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

- we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object. $W = \int_{\infty}^{r} F\mathrm{d}x = +\frac{GMm}{x}\Big|_{\infty}^{r} = \frac{GMm}{r}$.
- we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object. $W = \int_{\infty}^{r} F\mathrm{d}x = +\frac{GMm}{x}\Big|_{\infty}^{r} = \frac{GMm}{r}$

That is, when we consider angular velocity, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

重要笔记

we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

- we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object. $W = \int_{\infty}^{r} F\mathrm{d}x = +\frac{GMm}{x}\Big|_{\infty}^{r} = \frac{GMm}{r}$.
- we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object. $W = \int_{\infty}^{r} F\mathrm{d}x = +\frac{GMm}{x}\Big|_{\infty}^{r} = \frac{GMm}{r}$

That is, when we consider angular velocity, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

笔记

we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

- we regard it and velocity as the same.
- we regard it and linear velocity as the same physical quantity to describe the

The vector points in the direction perpendicular to the circular motion plane, but in the A-level course, we treat the angular velocity as a scalar , That is, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

! For the constant relationship between linear velocity and angular velocity, we can use linear velocity to describe the angular velocity, and conversely, we can use angular velocity to describe linear velocity.

The vector points in the direction perpendicular to the circular motion plane, but in the A-level course, we treat the angular velocity as a scalar ,

$$W = \int_{\infty}^{r} F \mathrm{d}x = +\left.\frac{GMm}{x}\right|_{\infty}^{r} = \frac{GMm}{r}$$

That is, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

练习 1 A simple harmonic oscillator is initially at rest. At $t = 0$, it is given an initial speed in the negative direction. Given that the frequency is 1.5 Hz and the amplitude is 5.0 cm, state an equation for its displacement-time relation.

解 ✍ angular frequency: $\omega = 2\pi f = 2\pi \times 1.5 = 3\pi$ rad s^{-1}

initial displacement $x(0) = 0$

for displacement-time relation, we use sine function

$$x(t) = -x_0 \cos\omega t \quad \Rightarrow \quad x = -5.0\sin(3\pi t)$$ □

Example 0.1 A simple harmonic oscillator is initially at rest. At $t = 0$, it is given an initial speed in the negative direction. Given that the frequency is 1.5 Hz and the amplitude is 5.0 cm, state an equation for its displacement-time relation.

Solution

angular frequency: $\omega = 2\pi f = 2\pi \times 1.5 = 3\pi$ rad s^{-1}

initial displacement $x(0) = 0$

for displacement-time relation, we use sine function

$$x(t) = -x_0 \cos\omega t \quad \Rightarrow \quad x = -5.0\sin(3\pi t)$$ □

Example 0.2 A **geostationary satellite** moves in a circular orbit that appears motionless to ground observers. The satellite follows the Earth's rotation, so the satellite rotates from west to east above equator with an orbital period of 24 hours. Find the radius of this orbit.

✍
$$\frac{GMm}{r^2} = m\omega^2 r \quad \Rightarrow \quad \frac{GMm}{r^2} = m\left(\frac{2\pi}{T}\right)^2 r \quad \Rightarrow \quad r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left(\frac{6.67\times10^{-11}\times6.0\times10^{24}\times(24\times3600)^2}{4\pi^2}\right)^{1/3} \approx 4.23\times10^7 \text{ m}$$ □

Example 0.3 Assuming the planets in the solar system all move around the sun in circular orbits, show that the square of orbital period is proportional to the cube of orbital radius. [1]

✍
$$\frac{GMm}{r^2} = m\omega^2 r \quad \Rightarrow \quad \frac{GMm}{r^2} = m\left(\frac{2\pi}{T}\right)^2 r \quad \Rightarrow \quad T^2 = \frac{4\pi^2}{GM}\cdot r^3$$

G is gravitational constant, M is mass of the sun, so $\frac{4\pi^2}{GM}$ is a constant, so $T^2 \propto r^3$ □

Question 0.1 Given that it takes about 8.0 minutes for light to travel from the sun to the earth. (a) What is the mass of the sun? (b) At what speed does the earth move around the sun?

[1]This is known as *Kepler's 3rd law* for planetary motions. In the early 17th century, German astronomer Johannes Kepler discovered three scientific laws which describes how planets move around the sun. This $T^2 \propto r^3$ relation not only holds for circular orbits but are also correct for elliptical orbits.

Isaac Newton proved that Kepler's laws are consequences of his own law of universal gravitation, and therefore explained why the planets move in this way. (⋆)

CHAPTER 1 Circular Motion 圆周运动

1.1 gas molecules

we introduce **amount of substance** to measure the size of a collection of particles unit of amount of substance: $[n] = \text{mol}$. Speed of gas molecules depend on temperature molecules move faster at higher temperature[2]

1.1.1 motion of gas particles

重点难点梳理

- randomness results from *collisions* of fast-moving molecules in the gas.
- for an individual molecule, its velocity changes constantly as it collides with other molecules
- for the gas at any instant, there is a range of velocities for molecules

speed of gas molecules depend on temperature molecules move faster at higher temperature[3]

1.1.2 amount of molecules

there are a huge number of molecules in a gas

one **mole** is defined as the amount carbon-12 atoms in a sample of 12 grams

- 1 mole of substance contains 6.02×10^{23} particles

  this number is called **Avogadro constant**: $N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$ [4]

  conversion between number of molecules and amount of substance: $\boxed{N = nN_A}$
- it is useful to introduce the notion of molar mass M

【知识点衔接】

molar mass of a substance is defined as the mass of a given sample divided by the amount of substance: $M = \dfrac{m}{n}$. There are a huge number of molecules in a gas.

- amount of substance $= \dfrac{\text{mass of sample}}{\text{molar mass}}$, or $n = \dfrac{m}{M}$
- mass of single molecule $= \dfrac{\text{molar mass}}{\text{Avogadro constant}}$, or $m_0 = \dfrac{M}{N_A}$

Example 1.1 Find the number of molecules in 160 grams of argon-40 gas.

✍ amount of gas: $n = \dfrac{m}{M} = \dfrac{160\ \text{g}}{40\ \text{g mol}^{-1}} = 4.0\ \text{mol}$

number of gas molecules: $N = nN_A = 4.0\ \text{mol} \times 6.02 \times 10^{23}\ \text{mol}^{-1} \approx 2.41 \times 10^{24}$ □

Question 1.1 Find the mass of a sample of uranium-235 that contains 6.0×10^{20} atoms.

[2]We will prove this statement later in this chapter.

[3]We will prove this statement later in this chapter.

[4]In 2018, IUPAC suggested a new definition of the mole, which is defined to contain exactly 6.02×10^{23} particles. This new definition fixed numerical value of the Avogadro constant, and emphasized that the quantity ‘amount of substance’ is concerned with counting number of particles rather than measuring the mass of a sample.

Example 1.2 A **geostationary satellite** moves in a circular orbit that appears motionless to ground observers. The vector points in the direction perpendicular to the circular motion plane, but in the A-level course, we treat the angular velocity as a scalar ,

习题训练解答

n interval Δt, distance moved along arc

$$\Delta s = v\Delta t = r\Delta\theta \quad\Rightarrow\quad \omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r} \quad\Rightarrow\quad \boxed{v = \omega r}$$

this relation between linear speed and angular speed holds at any instant

Example 1.3 A **circular orbit** moves in a circular orbit that appears motionless to ground observers. The satellite follows the Earth's rotation, so the satellite rotates from west to east above equator with an orbital period of 24 hours. Find the radius of this orbit.

✍

$$\frac{GMm}{r^2} = m\omega^2 r \quad\Rightarrow\quad \frac{GMm}{r^2} = m\left(\frac{2\pi}{T}\right)^2 r \quad\Rightarrow\quad r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left(\frac{6.67\times10^{-11}\times6.0\times10^{24}\times(24\times3600)^2}{4\pi^2}\right)^{1/3} \approx 4.23\times10^7\ \text{m}$$ □

Example 1.4 Assuming the planets in the solar system all move around the sun in circular orbits, show that the square of orbital period is proportional to the cube of orbital radius. [5]

✍

$$\frac{GMm}{r^2} = m\omega^2 r \quad\Rightarrow\quad \frac{GMm}{r^2} = m\left(\frac{2\pi}{T}\right)^2 r \quad\Rightarrow\quad T^2 = \frac{4\pi^2}{GM}\cdot r^3$$

命题 1.1: Theorem

G is gravitational constant, M is mass of the sun, so $\frac{4\pi^2}{GM}$ is a constant, so $T^2 \propto r^3$ □

Question 1.2 Given that it takes about 8.0 minutes for light to travel from the sun to the earth. (a) What is the mass of the sun? (b) At what speed does the earth move around the sun?

The vector points in the direction perpendicular to the circular motion plane, but in the A-level course, we treat the angular velocity as a scalar , That is, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

! For the constant relationship between linear velocity and angular velocity, we can use linear velocity to describe the angular velocity, and conversely, we can use angular velocity to describe linear velocity.

F_c is not a new force by nature, it can have a variety of origins

F_c acts at right angle to direction of velocity

or equivalently, if $F_{\text{net}} \perp v$ and F_{net} is of constant magnitude

then this net force provides centripetal force for circular motion

$$W = \int_\infty^r F\mathrm{d}x = +\frac{GMm}{x}\bigg|_\infty^r = \frac{GMm}{r}$$

That is, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

练习 2 A simple harmonic oscillator is initially at rest. At $t = 0$, it is given an initial speed in the negative direction. Given that the frequency is 1.5 Hz and the amplitude is 5.0 cm, state an equation for its displacement-time relation.

[5] This is known as *Kepler's 3rd law* for planetary motions. In the early 17th century, German astronomer Johannes Kepler discovered three scientific laws which describes how planets move around the sun. This $T^2 \propto r^3$ relation not only holds for circular orbits but are also correct for elliptical orbits.

Isaac Newton proved that Kepler's laws are consequences of his own law of universal gravitation, and therefore explained why the planets move in this way. (⋆)

知识点总结

Angular quantities

movement or rotation of an object along a circular path is called **circular motion**

to describe a circular motion, we can use *angular quantities*, which turn out to be more useful than linear displacement , linear velocity , etc.

angular displacement

angular displacement is angle swiped out by object moving along circular

注意几点

angular velocity is defined as angular displacement swiped out per unit time: $\boxed{\omega = \dfrac{\Delta\theta}{\Delta t}}$

- unit of: $[\omega] = \text{rad s}^{-1}$, also in radian measures
- angular velocity is a *vector* quantity

this vector points in a direction normal to the plane of circular motion

but in A-level course, we treat angular velocity as if it is a scalar

angular velocity and angular speed may be considered to be the same idea

知识点一

The vector points in the direction perpendicular to the circular motion plane, but in the A-level course, we treat the angular velocity as a scalar , That is, when we consider angular velocity, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

注意几点列表

- unit of: $[\omega] = \text{rad s}^{-1}$.
- angular velocity is a *vector* quantity
- unit of: $[\omega] = \text{rad s}^{-1}$.
- angular velocity is a *vector* quantity

重要公式列表

we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

- we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object. $W = \int_\infty^r F\mathrm{d}x = +\dfrac{GMm}{x}\Big|_\infty^r = \dfrac{GMm}{r}$.
- we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object. $W = \int_\infty^r F\mathrm{d}x = +\dfrac{GMm}{x}\Big|_\infty^r = \dfrac{GMm}{r}$

That is, when we consider angular velocity, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

重要笔记

we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

- we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object. $W = \int_\infty^r F\mathrm{d}x = +\dfrac{GMm}{x}\Big|_\infty^r = \dfrac{GMm}{r}$.
- we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object. $W = \int_\infty^r F\mathrm{d}x = +\dfrac{GMm}{x}\Big|_\infty^r = \dfrac{GMm}{r}$

That is, when we consider angular velocity, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

笔记

we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

- we regard it and velocity as the same.
- we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

we regard it and linear velocity as the same physical quantity to describe the most.

The vector points in the direction perpendicular to the circular motion plane, but in the A-level course, we treat the angular velocity as a scalar , That is, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

! For the constant relationship between linear velocity and angular velocity, we can use linear velocity to describe the angular velocity, and conversely, we can use angular velocity to describe linear velocity.

The vector points in the direction perpendicular to the circular motion plane, but in the A-level course, we treat the angular velocity as a scalar ,

$$W = \int_{\infty}^{r} F \mathrm{d}x = + \left.\frac{GMm}{x}\right|_{\infty}^{r} = \frac{GMm}{r}$$

That is, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

练习 3 A simple harmonic oscillator is initially at rest. At $t = 0$, it is given an initial speed in the negative direction. Given that the frequency is 1.5 Hz and the amplitude is 5.0 cm, state an equation for its displacement-time relation.

解 angular frequency: $\omega = 2\pi f = 2\pi \times 1.5 = 3\pi$ rad s^{-1}

initial displacement $x(0) = 0$

for displacement-time relation, we use sine function

$$x(t) = -x_0 \cos \omega t \quad \Rightarrow \quad x = -5.0 \sin(3\pi t)$$ □

Example 1.5 A simple harmonic oscillator is initially at rest. At $t = 0$, it is given an initial speed in the negative direction. Given that the frequency is 1.5 Hz and the amplitude is 5.0 cm, state an equation for its displacement-time relation.

Solution

angular frequency: $\omega = 2\pi f = 2\pi \times 1.5 = 3\pi$ rad s^{-1}

initial displacement $x(0) = 0$

for displacement-time relation, we use sine function

$$x(t) = -x_0 \cos \omega t \quad \Rightarrow \quad x = -5.0 \sin(3\pi t)$$ □

1.1.3 Uniform circular motion

定义与概念

when studying linear motion, we started from motion with constant velocity v

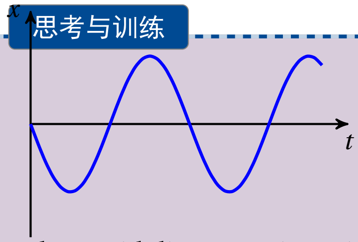
consider the simplest possible circular motion $\longrightarrow$ circular motion with constant ω

change in velocity: $\Delta v = 2v \sin \frac{\Delta\theta}{2} \approx v\Delta\theta \quad (\text{as } \Delta\theta \to 0, \sin\Delta\theta \approx \Delta\theta)$

acceleration: $a = \frac{\Delta v}{\Delta t} \approx v\frac{\Delta\theta}{\Delta t} = v\omega \quad (\text{as } \omega = \frac{\Delta\theta}{\Delta t})$

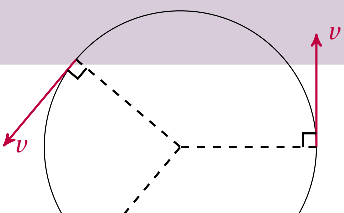
consider motion along a circular path from A to B with constant speed v

思考与训练



analogy with linear motion with constant v

uniform linear motion: $s = vt$



displacement $s \leftrightarrow \theta$, velocity $v \leftrightarrow \omega$

for uniform circular motion, one has: $\boxed{\theta = \omega t}$

➢ time taken for one complete revolution is called **period** T

in one T, angle swiped is 2π, so $\boxed{\omega = \frac{2\pi}{T}}$

➢ uniform circular motion is still *accelerated* motion

speed is unchanged, but *velocity* is changing

direction of velocity always *tangential* to its path, so direction of velocity keeps changing

in general, any object moving along circular path is accelerating.

✍ $$\omega = \frac{2\pi}{T} = \frac{2\pi}{40} \approx 0.157 \text{ rad s}^{-1} \qquad v = \omega r = 0.157 \times 2.5 \approx 0.39 \text{ m s}^{-1}$$ □

练习 4 What is the angular velocity of the minute hand of a clock?

练习 5 A spacecraft moves around the earth in a circular orbit. The spacecraft has a speed of 7200 m s^{-1} at a height of 1300 km above the surface of the earth. Given that the radius of the earth is 6400 km. (a) What is the angular speed of this spacecraft? (b) What is its period?

1.1.4 centripetal acceleration

知识归纳与探究

centripetal acceleration is the acceleration due to the change in direction of velocity vector, it points toward the centre of circular path

consider motion along a circular path from A to B with constant speed v

under small (infinitesimal) duration of time Δt[a]

[a] A more rigorous derivation can be given by using differentiation techniques

change in velocity: $\Delta v = 2v \sin\frac{\Delta\theta}{2} \approx v\Delta\theta \quad (\text{as } \Delta\theta \to 0, \sin\Delta\theta \approx \Delta\theta)$

acceleration: $a = \frac{\Delta v}{\Delta t} \approx v\frac{\Delta\theta}{\Delta t} = v\omega \quad (\text{as } \omega = \frac{\Delta\theta}{\Delta t})$

consider motion along a circular path from A to B with constant speed v

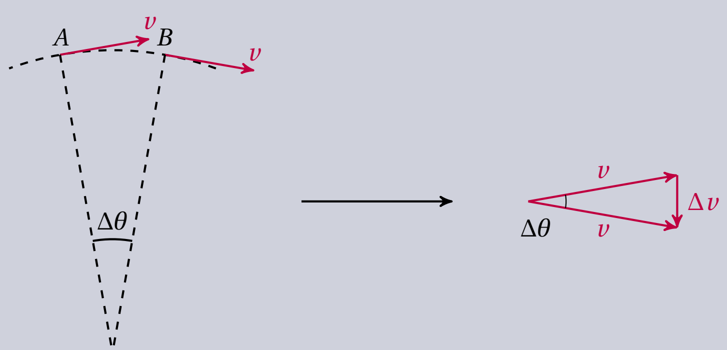
思考与解答

recall relation $v = \omega r$, we find centripetal acceleration: $\boxed{a_c = \frac{v^2}{r} = \omega^2 r}$

➢ direction of centripetal acceleration: always towards centre of circular path

➢ centripetal acceleration is only responsible for the change in *direction* of velocity

change in *magnitude* of velocity will give rise to *tangential acceleration*



this is related to *angular acceleration*[6] , which is beyond the syllabus

【练习与思考】

A racing car makes a 180° turn in 2.0 s. Assume the path is a semi-circle with a radius of 30 m and the car maintains a constant speed during the turn. (a) What is the angular velocity of the car? (b) What is the centripetal acceleration?

1.2 centripetal force

circular motion must involve change in velocity, so object is not in equilibrium

there must be a *net force* on an object performing circular motion

课前预习与思考

centripetal force (F_c) is the resultant force acting on an object

- moving along a circular path, and it is always directed towards centre of the circle
- centripetal force causes centripetal acceleration

using Newton's 2nd law: $\boxed{F_c = m\frac{v^2}{r} = m\omega^2 r}$

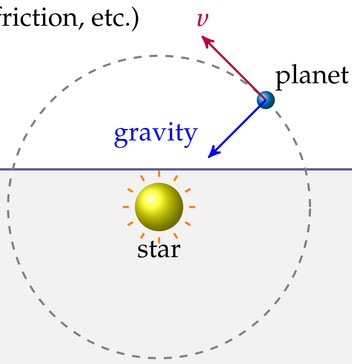
F_c is not a new force by nature, it can have a variety of origins

F_c is a resultant of forces you learned before (weight, tension, contact force, friction, etc.)

F_c acts at right angle to direction of velocity

or equivalently, if $F_{net} \perp v$ and F_{net} is of constant magnitude

then this net force provides centripetal force for circular motion



练习 6 ➢ effect of F_c: change *direction* of motion, or maintain circular orbits

to change *magnitude* of velocity, there requires a *tangential* component for the net force

again the idea of tangential force is beyond the syllabus

planet orbiting around a star

解 gravity by the star provides centripetal force for the planet

A rock is able to orbit around the earth near the earth's surface. Let's ignore air resistance for this question, so the rock is acted by weight only. Given that radius of the earth $R = 6400$ km.

(a) What is the orbital speed of the rock? (b) What is the orbital period?

✍ weight of object provides centripetal force: $mg = \frac{mv^2}{R}$

orbital speed: $v = \sqrt{gR} = \sqrt{9.81 \times 6.4 \times 10^6} \approx 7.9 \times 10^3$ m s^{-1}

period: $T = \frac{2\pi R}{v} = \frac{2\pi \times 6.4 \times 10^6}{7.9 \times 10^3} \approx 5.1 \times 10^3$ s ≈ 85 min □

解题思路分析： A turntable can rotate freely about a vertical axis through its centre. A small object is placed on the turntable at distance $d = 40$ cm from the centre. The turntable is then set to rotate, and the angular speed of rotation is slowly increased. The coefficient of friction between the object and the turntable is $\mu = 0.30$. If

[6]Angular acceleration is analogous to linear acceleration α, defined as rate of change of angular velocity: $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ (⋆). Similar to $v = \omega r = \frac{ds}{dt}$, the relation $a = \alpha r = \frac{dv}{dt}$ also holds.

the object does not slide off the turntable, find the maximum number of revolutions per minute.
✍ if object stays on turntable, friction provides the centripetal force required: $f = m\omega^2 d$
increasing ω requires greater friction to provide centripetal force

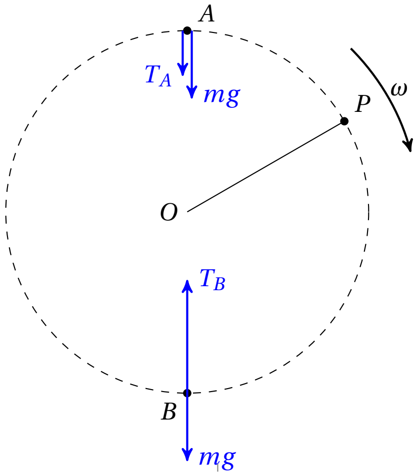
but maximum limiting friction possible is: $f_{\text{lim}} = \mu N = \mu mg$, therefore

$$f \le f_{\text{lim}} \quad \Rightarrow \quad m\omega^2 d \le \mu mg \quad \Rightarrow \quad \omega^2 \le \frac{\mu g}{d} \quad \Rightarrow \quad \omega_{\max} = \sqrt{\frac{0.30 \times 9.81}{0.40}} \approx 2.71 \text{ rad s}^{-1}$$

period of revolution: $T_{\min} = \dfrac{2\pi}{\omega_{\max}} = \dfrac{2\pi}{2.71} \approx 2.32$ s

umber of revolutions in one minute: $n_{\max} = \dfrac{t}{T_{\min}} = \dfrac{60}{2.32} \approx 25.9$

【解答与反思】



Particle P of mass $m = 0.40$ kg is attached to one end of a light inextensible string of length $r = 0.80$ m. The particle is whirled at a constant angular speed ω in a vertical plane. (a) Given that the string never becomes slack, find the minimum value of ω. (b) Given instead that the string will break if the tension is greater than 20 N, find the maximum value of ω.

✍ at top of circle (point A): $F_c = T_A + mg = m\omega^2 r \quad \Rightarrow \quad T_A = m\omega^2 r - mg$
at bottom of circle (point B): $F_c = T_B - mg = m\omega^2 r \quad \Rightarrow \quad T_B = m\omega^2 r + mg$
tension is minimum at A, but string being taut requires $T \ge 0$ at any point, so $T_A \ge 0$

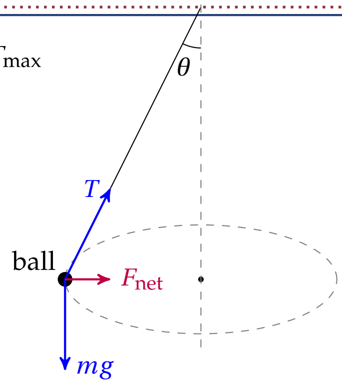
$$m\omega^2 r - mg \ge 0 \quad \Rightarrow \quad \omega^2 \ge \frac{g}{r}$$

$$\omega_{\min} = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.81}{0.80}} \approx 3.5 \text{ rad s}^{-1}$$

tension is maximum at B, but string does not break requires $T \le T_{\max}$, so $T_B \le T_{\max}$

$$m\omega^2 r + mg \le T_{\max} \quad \Rightarrow \quad \omega^2 \le \frac{T_{\max}}{m} - \frac{g}{r}$$

$$\omega_{\max} = \sqrt{\frac{T_{\max}}{m} - \frac{g}{r}} = \sqrt{\frac{20}{0.40} - \frac{9.81}{0.80}} \approx 6.1 \text{ rad s}^{-1} \qquad \square$$



✍ vertical component of tension T_y equals weight

$$T_y = mg \quad \Rightarrow \quad T\cos\theta = mg$$

$$T = \frac{mg}{\cos\theta} = \frac{0.12 \times 9.81}{\cos 25°} \approx 1.3 \text{ N}$$

net force equals horizontal component of tension T_x
so component T_x provides centripetal force

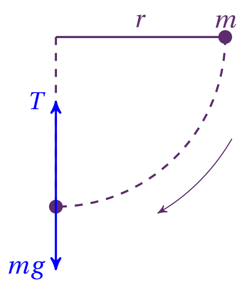
$$F_c = T_x \quad \Rightarrow \quad T\sin\theta = \frac{mv^2}{r}$$

by eliminating T and m, one can find

$$v^2 = \frac{r\tan\theta}{g} = \frac{0.10 \times \tan 25°}{9.81} \quad \Rightarrow \quad v \approx 0.069 \text{ m s}^{-1} \qquad \square$$

When the ball reaches lowest point, find its speed and the tension in the string in terms of m and l.

思考与练习



✍ energy conservation: G.P.E. loss = K.E. gain

$$mgr = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gr}$$

at lowest point: $F_c = T - mg = m\dfrac{v^2}{r}$

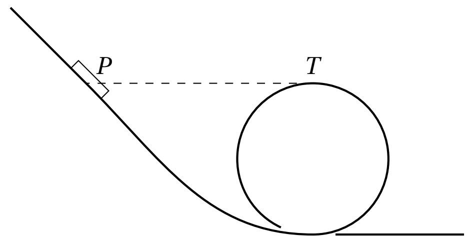
$$T = mg + m\frac{v^2}{r} = mg + m\frac{2gr}{r} = 3mg \qquad \square$$

Question 1.3 Suggest what provides centripetal force in the following cases. (a) An athlete running on a curved track. (b) An aeroplane banking at a constant altitude. (c) A satellite moving around the earth.

Question 1.4 A turntable that can rotate freely in a horizontal plane is covered by dry mud. When the angular speed of rotation is gradually increased, state and explain whether the mud near edge of the plate or near the mud will first leave the plate?

Question 1.5 A bucket of water is swung at a constant speed and the motion describes a circle of radius $r = 1.0$m in the vertical plane. If the water does not pour down from the bucket even when it is at the highest position, how fast do you need to swing the bucket?

This question is about the design of a roller-coaster. We consider a slider that starts from rest from a point P and slides along a frictionless circular track as sketched below. P is at the same height as the top of the track T. (a) Show that the slider cannot get to T. (b) As a designer for a roller-coaster, you have to make sure the slider can reach point T and continue to slide along the track, what is the minimum height for the point of release?



Question 1.6 A turntable that can rotate freely in a horizontal plane is covered by dry mud. When the angular speed of rotation is gradually increased, state and explain whether the mud near edge of the plate or near the mud will first leave the plate?

Question 1.7 A bucket of water is swung at a constant speed and the motion describes a circle of radius $r = 1.0$m in the vertical plane. If the water does not pour down from the bucket even when it is at the highest position, how fast do you need to swing the bucket?

【知识点衔接】

Newton's law of gravitation states that gravitational force between two *point* masses is proportional to the product of their masses and inversely proportional to the square of their distance $\left(F_{\text{grav}} \propto \dfrac{Mm}{r^2}\right)$

this law was formulated in *Issac Newton*'s work 'The Principia', or 'Mathematical Principles of Natural Philosophy', first published in 1687