**CS5229 ADVANCED COMPUTER NETWORKS**

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1. You roll two fair dice and sum up the values shown on the top of the two dice. What is the expected value of this sum?

Answer:

Let Xi be the random variable which shows on the top of one dice. So, we have events X1, X2, X3, X4, X5 & X6 representing all possible scenario, and the possibility of each scenario of a fair dice is 1/6.

So the E[Xi] for rolled one dice is (X1+X2+X3+X4+X5+X6)/6 which is (1+2+3+4+5+6)/6 = 3.5

Rolling the 2 dice are independent, E(X+Y) = E(X) + E(Y) if X, Y are independent.

Sum = 3.5 +3.5 = 7

The expected value of the sum is 7.

1. Customers arrive at a fast food restaurant at a rate of five per minute (assume Poisson arrivals) and wait to receive their order for an average of 5 minutes.

Arrive rate is 5ppl/min => λ= 5

Waiting time is 5 min => T = 5

a. (10pt) How many customers are there in the restaurant on the average?

Answer:

λ= 5, T = 5

N = λ \* T = 5 \* 5 = 25

Average number of customer in the restaurant is 25.

b. If customers eat in the restaurant with probability 0.5 and carry their order without eating with probability 0.5. A meal requires an average of 20min. What is the average number of customers in the restaurant? Show your working.

Half customers eat in and half eat out, so let λ1 reflect the arrival rate of those eating outside, and λ2 be those eating in the restaurant.

λ1= 5 \*0.5 = 2.5

λ2= 5 \*0.5 = 2.5

T1 = 5 min (only wait then eat outside)

T2 = 5+20 = 25min (wait and eat in the restaurant)

N = λ \* T = λ1\*T1 + λ2\*T2 = 2.5\*5+2.5\*(5+25) = 75

Average number of customer in the restaurant is 75

1. Customers arrive at a service counter at a rate of 0.5 per minute (assume Poisson arrivals) and service time is exponentially distributed with mean 90 seconds. a. (5pt)

Arrive rate is 0.5ppl/min =>λ = 0.5  
Service time is exponentially distributed with mean 90sec => = 1/1.5min = 2/3

1. What is the probability that the service counter is idle?

λ = 0.5 = 1/1.5min = 2/3

P0 respects there is no customer in the system:

P0 = (1- λ/) = 1 – 0.75 = 0.25

The probability that the service counter is idle is 25%

1. (10pt) What is the average number of customers in the system

= λ/ = ¾

N = /(1-) = 0.75/(1-0.75) = 3

The average number of customers in the system is 3.

1. What is the probability there are more than 3 customers in the system?

P(N) = (λ/ )^N \* P0

P(N>3) = 1 – P(N<=3) = 1 – P(N=0) – P(N=1) – P(N=2) - P(N=3)

= 1 – ¼ - (¾)^1\*¼ -(¾)^2\*¼ - (¾)^3\*¼ = 0.31640625

The probability more than 3 customers in the system is 31.640625%

1. Using the Erlang B table (available on www.pitt.edu/~dtipper/2110/erlang-table.pdf), answer the following questions:
2. There are 60 servers and the target blocking probability is 1%. What is the largest load that can be supported?

Row 60 Colum 5(1%) in Erlang B table => 46.95

The largest load that can be support is 46.95

1. Load is 30 Erlang and the target blocking probability is to be 10. How many servers are needed?

Row 32 Colum 8(10%) in Erlang B table => 30.24

The servers number needed is 32

1. There are 40 servers and the load is 25 Erlang. What is the expected blocking probability?

Row 40 Colum 3(0.1%) in Erlang B table => 24.44

Row 40 Colum 4(0.5%) %) in Erlang B table => 27.38

The expected blocking probability is between 0.1% and 0.5%

From the Erlang table on Wiki: P = 0.2% N=40, largest load supported = 25.59

So for 40 servers with load 25, the blocking probability should be more than 0.2%

1. Equation 3.4.2 in the Data Networks textbook (2-Queueing\_Data\_Nets.pdf in IVLE workbin) is for a M/G/1 system. Assume that arrival is a Poisson process with mean 0.1. For each of the following cases, calculate the average waiting time in the system for:

Equation 3.4.2 for a M/G/1 system:

λ = 0.1

= E(X) = 1/

T = + λ\*/2\*(1-)

1. (10pt) service time is exponentially distributed with mean 5;

For exponentially distributed:

= E(X) = 1/ = 5 = 1/5

Var(X) = E(X^2) − (E(X))^2 = 1/ ^2.

E(X^2) = 2/ ^2 = 25\*2 = 50

=λ/ = 0.5

T = + λ\*/2\*(1-) = 5+0.1\*50/2\*(1-0.5) = 10

The average waiting time is 10

1. (10pt) service time is always 5;

= E(X) = 1/ = 5 = 1/5

=λ/ = 0.5

If Xi is always 5, X^2 is always 25

= E(X^2) = 25

T = + λ\*/2\*(1-) = 5+0.1\*25/2\*(1-0.5) = 7.5

The average waiting time is 7.5

1. (10pt) service times can be either 1 or 9 with equal probability.

1 and 9 with same probability P = 0.5

= 1\*0.5 + 9\*0.5 = 5 = 1/5

=λ/ = 0.5

E(X^2) = 1^2\*0.5+9^2\*0.5 = 41

T = + λ\*/2\*(1-) = 5+0.1\*41/2\*(1-0.5) = 9.1

The average waiting time is 9.1