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Quantum Computation (UIT2205) Second Essay

Is the Universe a Quantum Computer?

Section 1: Introduction

For thousands of years of human civilization, humankind has been engaged in the endeavour to fathom the workings of the world around us. Mythologies, religions, observations, theories, and even machines have been used in our numerous attempts to explain how the universe operates. Around 205 BC, ancient Greeks created the Antikythera mechanism to model and predict movements of celestial bodies decades in advance (Carman & Evans, 2014). Boasting surprisingly intricate design and complexity, it is the earliest known man-made computing device aiming to simulate a part of the universe. Since then, advancements in technology and humans' understanding of nature have driven the development of more and more complex machines to model nature. In 125 AD, Chinese astronomer Zhang Heng from the Han dynasty invented the world's first water-powered armillary sphere to assist the observation of the stars for calendrical computations (Needham et al., 1986).

The advent of science in the West provided us with rigorous theoretical frameworks to better understand nature. In 1687, Isaac Newton published *Philosophiæ Naturalis Principia Mathematica*, formulating the laws of classical mechanics. Its implications were monumental, as every object in the universe, from gigantic stars in the sky to tiny particles constituting matter, could have its movement perfectly described by a simple set of mathematical equations under Newtonian mechanics. This discovery subsequently led to the conception of a Clockwork Universe, referring to the "Notion of the World's being a great Machine, going on without the Interposition of God, as a Clock continues to go without the Assistance of a Clockmaker" as stated by English philosopher Samuel Clarke (Davis, 1991).

With the emergence of quantum physics in the last century, the notion of a Newtonian clockwork universe lost its momentum. Nevertheless, as humankind entered the age of modern computers in the second half of the 20th century and the subsequent theorisation of quantum computation based on the postulates of quantum mechanics, it has appeared that we may be a step closer to modelling the universe as a giant machine obeying certain pre-defined laws. In view of these developments, this essay will discuss the following question: is the universe a quantum computer? Here, the universe is defined as all physical matter and energy in the entire space and time. Answering this question may give us invaluable insights and deepen our understanding of how the universe works. I will first discuss what a quantum computer is, before delving into discussions on whether the universe fits the quantum computation model with regards to its states, its processes and its measurement. Finally, I will discuss the complexity of the universe and its relevance to our topic, as well as the implications of the universe being a quantum computer, before concluding the essay.

Section 2: What is a quantum computer?

In order to answer the question whether the universe is a quantum computer, we need to first clarify what a quantum computer is. A quantum computer (Fig. 1) is a machine that takes some qubits with certain initial states as inputs, processes them by transforming their states according to the laws of quantum physics, and eventually has the final states of the qubits measured in order to read out classical results. The states, processes and measurements of a quantum computer follow the following four postulates of quantum computing (Nielsen & Chuang, 2019). Postulate 1 states that our knowledge about a system (a qubit) is

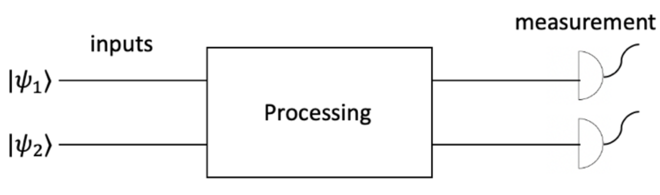

Figure 1: Diagram of Quantum Computer

described by a normalised ket vector in a complex vector space equipped with an inner product. Postulate 2 states that a composite system is described by a tensor product of the component physical systems. Postulate 3 states that a gate acting on a quantum system (one or many qubits) is described by a unitary operator. Last but not least, Postulate 4 states that a measurement performed on a quantum system is described by a set of operators $\{A_j\}, j = 1 \ldots M$, where index j refers to the measurement outcomes that may occur and such that $\sum_{j=1}^{M} A_j^\dagger A_j = \mathbb{I}$. The probability that the outcome j occurs given a system in state $|\psi\rangle$ is given by $p(j|\psi) = \langle\psi|A_j^\dagger A_j|\psi\rangle$. The updated state of knowledge of the system after measurement is given by $|\psi_j'\rangle = \frac{A_j|\psi\rangle}{\sqrt{p(j|\psi)}}$.

Any machine that has its states and interactions governed by these four postulates can be considered a quantum computer. Thus, the question "*is the universe a quantum computer?*" can be rephrased as: "*does the universe follow the four postulates of quantum computing?*". Or more precisely, do the postulates of quantum computing provide a complete description of all physical matter and interactions in our universe? This is because, if the postulates do offer a complete description of the universe, then every element of the universe has a corresponding representation in the abstract model of quantum computing, and consequently the universe is a quantum computer. We should note here that the question whether the universe *is* a quantum computer is different from whether the universe *can be modelled by* a quantum computer. A model only provides a partial description of some properties of the object being modelled. For example, if a linear regression models the population growth of a country, it simply means that the model captures some aspects of the population growth, perhaps the trend as well as the estimates of actual numbers of people at different points in time. But this model does not encapsulate every possible piece of knowledge about the growth, for example which exact new member is added to the population. A model thus captures useful but incomplete knowledge of a system. After having established what a quantum computer is, we shall now explore whether the universe *is* indeed a quantum computer.

Section 3: States

This section will discuss the parallelism between the state of the universe and the state of a quantum computer. There may be two ways to interpret what a "state" is. The first way is that the state of a system is a complete description of all physical properties of the system which exist objectively independent of external observation. However, the famous "EPR" paper (Einstein et al., 1935) shows that the assumption of realism, that a physical property can only be considered "real" if its value can be predicted with certainty without observation, may not hold. Thus, the first definition is rendered less useful for our discussion, since this "state" defined this way would not capture all physical properties of the system. We can then turn to a second definition, which defines a "state" as what an observer knows about all the physical properties of a system. In quantum computing, a qubit is the most fundamental unit that carries information, and it is defined in Postulate 1 as an object described by a complex ket vector describing our (user's or programmer's) *state of knowledge* of the single-qubit system. This ket vector is then the "state" of the system that it describes. Since the postulates of quantum computing are directly based on the postulates of quantum mechanics which reflect our most updated understanding of the universe at the most fundamental level, the universe follows Postulate 1 as well. For example, our state of knowledge of an electron, which is one of the fundamental particles of matter, can be described by such a complex vector.

We should note here that it is possible to talk about the "observer's" state of knowledge of a particle in the universe as long as the observer is not the particle itself. However, is it still possible to talk about an "observer" if we talk about the system encompassing the entire universe? This brings us to Postulate 2 of quantum computing, which deals with a composite system with multiple qubits. If we view the universe as a composite system comprising all the fundamental particles and energy fields that exist in the physical universe, it seems that we can just take the tensor product of all these qubits and then it will give us the state of the universe. However, if an observer's knowledge is needed in order to define "state", then this observer has to exist to describe the state of the universe. By definition of the universe, the observer is part of the universe. Thus, it is either not possible to describe the state of the universe, or the observer is able to know its own state. Either way, I would argue that it is still possible to speak of the universe as a quantum computer in this regard.

If it is not possible to describe the state, or "state of knowledge", of the universe, it is still possible for the universe to function as a quantum computer. There are three scenarios where we may need to deal with states in a quantum computer: (1) to specify the initial states of the inputs, (2) to analyse the system during its execution, and (3) to take a measurement. When we conceive the universe as a quantum computer, whether there is a being analysing the entire system of the universe does not affect the functioning of the universe, so (2) does not matter in our discussion. (3) happens only at the end of the execution of a quantum programme, so it does not affect the functioning of a quantum computer before the measurement. Even if no measurement is taken, the computer can still compute according to the current state and a predetermined set of rules (Processes and measurement will be discussed in more detail in Sections 4 and 5 respectively). Thus, we shall focus on the use of states in (1). When we conceive the universe as a quantum computer, its inputs were specified at the time of the Big Bang, after which the states of all the qubits in the universe evolved according to the programme of this computer, or the laws of nature (more detail in Section 4). So, when we talk about the state of knowledge of the universe, it is the initial state of the universe that matters. We do not know what specified the initial states of all the qubits and whose "state of knowledge" is the initial state of the universe at the time of the Big Bang. Nevertheless, whether or not there was an "observer" observing the initial state of the universe has no effect on what happened after the Big Bang as all the subsequent states of the universe were a result of the initial states and the processes occurring within the universe, we can still think of the universe as a quantum computer as if there was an "observer" specifying the initial states.

Section 4: Processes

This section will discuss whether the processing of information in a quantum computer occurs in the universe as well. This point is related to Postulate 3 of quantum computing, which stipulates how the states of qubits evolve with time by unitary transformations. The quantum gates in a quantum computer can be generalised to be the "laws" of the universe. There are two possible ways to understand what "the laws of the universe" mean. First, when we talk about the laws of nature, we often mean those theories, laws and equations written in physics textbooks, such as Newton's laws of motion, Maxwell's electromagnetic equations, Einstein's general relativity, Schrödinger's equation and so on. These are the laws of physics formulated by human beings in an attempt to model (seeming) regularities in nature and make predictions about the future. These "laws" of the universe are humans' perception of regularities in nature, and they are not an inherent feature of the universe itself. By virtue of the falsifiability principle of science, we are never sure whether a theory is definitely correct, but we only know whether a theory is wrong when it is falsified by experimental results. These "laws", as conceived by human beings, thus evolve over time when we have better understanding of the universe.

However, if the universe is a quantum computer and its laws are the programme running on this computer, how can these laws be dependent on humans' ever-changing understanding of the universe? Therefore, it may be more appropriate to instead define the laws of the universe as the inherent regularities in nature independent of human discovery. Here, we assume that there is indeed inherent regularity in nature and it does obey a set of pre-defined laws consistently. What does regularity mean? The notion of regularity is closely related to predictability. I would define that there is predictability in nature if and only if there exists a complete set of rules such that given the state of any element in the universe at any point of time, by using these rules, we can calculate its state at a future time with outcomes following a known probability distribution. Since it is impossible to know whether the universe is inherently regular and predictable, there are two possibilities: the universe is predictable and the laws of the universe exist, or the universe is unpredictable and there are no laws of the universe.

On one hand, if there are indeed laws of the universe, then it is possible for the universe to be a computing machine, as all these laws can be conceived as programmes or logic gates on this computing machine. However, even if the universe is predictable, can we say with certainty that the universe is a *quantum* computer? We know that quantum mechanics by far gives us the most accurate prediction of the motion and interaction of matter and energy at both the microscopic and macroscopic levels. At the macroscopic level, interactions of matter are still consistent with quantum mechanics, which is simply reduced to classical mechanics for classical objects. It may appear that the laws of the universe follow a quantum computer, since the universe seems to operate according to the laws of quantum mechanics. However, this may not be true for two reasons. Firstly, quantum theory currently does not provide a complete description of all interactions in the universe. There are four known fundamental forces, namely, electromagnetic, strong, weak and gravitational forces. However, quantum mechanics only provides theoretical explanation for the electromagnetic, strong and weak forces. It has proven extremely challenging to incorporate gravity into the quantum theory. Although gravity and quantum theory do not necessarily contradict each other, "there is as yet no logically consistent and complete relativistic quantum field theory" (Landau & Lifšic, 1971). Thus, the universe may not be a quantum computer whose processes are completely governed by quantum mechanics. Secondly, even if we manage to incorporate gravity into quantum theory and come up with a theory that appears to explain everything in the universe, there may be other phenomena that this theory fails to explain but we are not aware of. We can never be sure whether our theory provides a complete explanation of the universe, and thus we cannot be sure whether the universe is indeed a computer built upon such a theory.

On the other hand, it is possible that there are no inherent laws of the universe at all. The universe might be unpredictable, and everything about nature that the human endeavour has unravelled might just be our futile attempt at fitting apparent regularity on a nature that is inherently irregular and unpredictable. The view that the universe may not follow a finite set of rules is not uncommon among the scientific community. For example, Stephen Hawking (2002), after considering Gödel's Incompleteness Theorem, stated publicly that a theory of everything is not possible in his lecture "Gödel and the End of Physics". He claimed that it is impossible to find an ultimate theory, that can be formulated as a finite number of principles, which is both consistent and complete. If this view is true, then the universe cannot be a computing machine which follows a pre-written finite sequence of instructions to compute, since such finite sequence of instructions may not exist in the first place.

However, even if the universe does not follow a fixed set of laws to operate and it is thus impossible to predict the results of computation based on known rules, is there still value in modelling the universe as a computing machine? The universe may not be following any laws inherently, but it is still computing results in the sense that the state of every fundamental particle in the universe is calculated at every instant of time (Whether an "instant" of time exists will be further discussed in Section 7). Since there are no "laws" or "algorithms" that the universe follows, the universe may just be running on its own and we have to wait and see the results. Does this imply that the universe goes beyond just computation?

To answer the above question, we shall consider whether there are computational tasks that the universe can solve but a quantum computer cannot. If the universe is a quantum computer, then the set S_u of computational tasks that the universe can theoretically solve should be equal to the set of computational tasks S_q that a quantum computer can solve. In other words, the universe and a quantum computer should be able to simulate each other. We already know that S_q is a subset of S_u, since a quantum computer, such as the IMB Q System One computer, is part of the universe. We need to decide whether S_u is also a subset of S_q in order to prove that S_u is equal to S_q. If we can find a task in S_u that is not in S_q , we can determine that S_u is *not* equal to S_q. In order to find a possible example of such a task, let us consider a classical problem in computability theory. The halting problem is the problem of determining, from a description of an arbitrary computer programme and an input, whether the program will finish running or continue to run forever. Alan Turing (1936) proved that a general algorithm to solve the halting problem for all possible program-input pairs cannot exist. Such an algorithm as defined by Turing is a Turing machine. Since a classical computer (universal Turing machine) can compute everything that a quantum computer could, by contraposition, a quantum computer cannot compute a problem that a classical computer cannot. Since the halting problem is undecidable over classical computers, we can say that a quantum computer also cannot compute the task of the halting problem. The question, then, is whether the universe can determine the result of the halting problem. If we consider the conclusion from the previous paragraph that the universe can compute results by just running itself and waiting for the results, then we can consider, given an arbitrary programme and an input, whether the universe can decide whether the programme halts by letting it run. The moment that the programme finishes running, it is known for sure that this programme halts. However, if the programme does not finish by a given time t, then at time t, one cannot say for sure whether the programme will or will not halt, since this programme may or may not halt at a future time t'. Thus, for a programme that does not halt, at any moment in time, the universe cannot tell whether this programme will halt eventually or it will run forever. We are not sure whether S_u is equal to S_q until we find a task that the universe can solve but a quantum computer cannot do. Therefore, it is still possible that the universe is a quantum computer.

Section 5: Measurement

This section will discuss Postulate 4 of quantum computing in relation to the conception of the universe as a quantum computer. Postulate 4 is related to measurement in quantum computing. According to the postulate, measurement outcomes are classical information in nature. First of all, we need to establish whether it is even possible to take a measurement of the state of the universe in the first place. There may be two things necessary for a measurement to take place: an observer and a measuring device.

We shall first explore whether it is possible to have an observer observing the universe. If there is a physical being, the "observer", that attempts to take a measurement of the universe, then this physical being must be part of the universe by definition of the universe. Then, the observer has to measure itself in order to obtain a complete state of knowledge of the universe. This then brings in the question of whether an observer is able to observe itself. However, I argue that it is not necessary for an observer to exist. In a quantum computer, when we place a measuring device to measure a qubit without reading the outcome of the measurement, the process of measurement still takes place in the sense that the measuring device interacts with the qubit so that the measuring device has some information regarding the state of the qubit. Therefore, measurement does not require an external observer, and can be conceived as the measuring device interacting with the system being measured.

If a measuring device is to measure the universe, then it has to do two things: (1) measure other parts of the universe other than itself, and (2) measure itself. (1) is possible in principle, since we just need the measuring device to interact with every other qubit in the universe. But for (2), a natural question to ask is: can a measuring device measure itself? To answer this question, we should look at what is a measuring device, and its role in a quantum computer. A measuring device is a qubit that interacts with another qubit (usually the qubit to be measured) so as to get information from the other qubit. The measuring device can thus be seen as a qubit that is part of the quantum computer, interacting with other qubits in the quantum computer. To illustrate this with an example, let us consider a qubit $|x\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and a measuring device with its own state described by $|M\rangle$. Initially, when the qubit and the measuring device have not interacted, the state of the entire system consisting of this qubit and the measuring device is $|x\rangle \otimes |M\rangle$. After the measuring device interacts and measures the state of $|x\rangle$, the measuring device now takes on the state of $|x\rangle$ such that the qubit and the measuring device are now in an entangled state of $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, since when the measurement outcome of the qubit is "0", the measuring device will register a state of $|0\rangle$ and vice versa. This shows that (2) is indeed possible since a measuring device registers its own state. Therefore, just like a quantum computer, it is possible to take measurement of the universe without an external observer.

However, what is the universe measuring in the process of measurement? I would argue that the universe is measuring the results of its own computation. Computation refers to the continual process of information processing happening in the entire universe. We can view the universe as being made of bits of information at the most fundamental level. This is possible by adopting a particulate view of nature where everything, including matter, energy and forces, can be modelled as particles. For instance, the Standard Model of elementary particles offers such a particulate theory of the universe (Oerter, 2006). Each particle has some properties, such as its position, momentum, charge and spin, encoded by several bits of information. Since these properties of fundamental particles are currently known to be governed by the laws of quantum physics, the information they carry is thus qubits. As the universe evolves with time, the universe computes its next state based on the information carried by all the qubits in the universe and following the laws of nature. The results of computation that the universe "measures" are thus the states of all the qubits in the universe, where each qubit is a measuring device that keeps track of its own state after each interaction. Thus, in this regard, the universe can be viewed as a quantum computer processing information on a gigantic scale.

Section 6: The Universe and its Complexity

Whenever we look around us, it is difficult not to marvel at the complexity of even the most ordinary things that exist in our world, from the bird that just flew by the window to the laptop on which this essay is being typed. These wonders of the universe have long been attributed to God's creativity. However, modern scientific knowledge points to the result of *chance* over a grand scale of space and time. If the universe is a quantum computer, how could this computer generate the level of complexity we see in our world today? First, we need to operationalise complexity. The human brain, for example, is an epitome of complexity in our universe. Microscopically, the human brain is made up of around 10^{26} atoms of hydrogen, oxygen, carbon and so on, which are perhaps not much different from the constituent atoms of a bowl of banana milkshake. However, it is obvious that the human brain is much more complex than the latter. So, what characterises this complexity? The fundamental particles that constitute the brain are arranged ingeniously in such a way that it can serve as the command centre of a human body and generate intelligent thoughts and emotions. It is the *structure* of the arrangement of its particles that makes the brain complex. This structure is complex on many levels, from a protein molecule to the interconnecting nerve cells, from the cerebellum to the whole brain structure. If any tiny part of this structure is altered, the brain may lose its original functionalities. Compare this to a banana milkshake, which is made up of around the same number of atoms but is *far less* complex than the human brain (but certainly still *much more* complex than just a random collection of atoms). When the structure of the milkshake changes slightly from its original structure, it will most likely taste the same. Then, how do we characterise the complexity of this structure? I would suggest that a complex object is one that exhibits a low degree of disorder in its structure. Since entropy is a measure of disorder in a system, we can quantify the complexity of a system as the absolute difference between the entropy of the current state of the system and the maximum attainable entropy of the same system (i.e. when the original structure is broken and all the constituent atoms are randomly arranged in a disordered manner).

Having operationalised complexity, we can now discuss how the universe generates this complexity and whether it is possible with a quantum computer. We shall first look at the previous example of the human brain. How did the universe generate such a complex object as our brain? There are two characteristics of the universe that have led to this creation, namely, randomness and memory. The creation of increasingly complex structures in the universe is a long process of trial and error. The inherent randomness that governs the interaction of matter and energy generates many possibilities of the next states of these particles. According to chaos theory, a slight difference in the initial state can result in huge differences in a later state (Weisstein, 2020). This creates a large number of vastly different possible states in a system. The vast majority of these possibilities do not generate low-entropy complex structures that sustain. But over time, even a miniscule probability over many times of trial and error can result in a relatively more complex structure (arrangement of particles) to emerge, sustain and be "remembered" by the memory of the universe. The memory of the universe is the individual fundamental particles themselves. They sustain this complex structure and the universe computes its states as time evolves. This possibility has a structure that distinguishes it from other possibilities in terms of complexity, and from which increasingly more complex structures can be built. For example, by pure randomness, a group of atoms might have congregated together to form a structure that had allowed a star to form and sustain via nuclear fusion in the enormous void of the universe. At some point, the star died in a supernova, and its remnants formed a solar system with the Earth as a planet—again through randomness and trial and error. Afterwards, the formation of life, the process of evolution through natural selection, and eventually the creation of the human brain all resulted in increasingly complex structures built upon the previous ones through randomness and memory. This process is how the universe generates the complex world we witness today.

Now, we shall discuss whether a quantum computer is capable of generating such levels of complexity. A quantum computer has memory as realised by its constituent qubits. Moreover, quantum computation allows for the superposition of different states of the qubits. Superposition is essentially a linear combination of states, where the coefficients of the linear combination indicate the probability amplitude of each state upon measurement. This linearity encompasses the notion of multiple possibilities and inherent randomness in the interaction of qubits in a quantum computer. This characteristic aligns with how the universe generates complexity. Thus, a quantum computer can theoretically generate the level of complexity we see in our world today, given sufficient number of operations performed on the qubits and sufficient memory (number of qubits). Consequently, it is possible that the universe is a quantum computer.

Section 7: Implications of the Universe as a Quantum Computer

From the discussions in the previous sections, we can see that it is possible that the universe is a quantum computer. The universe can be seen to consist of a vast number of fundamental particles and each particle can be represented by several qubits that encapsulate its properties. The interactions between these qubits are then governed by the "programme/logic gates" of this computer, or by the so-called laws of the universe if they exist in the first place, as discussed in Section 4. Suppose the universe is indeed a quantum computer, what implications does it have on our understanding of the universe?

Firstly, time may be discrete. In a quantum computer, the evolution of time is discretised by the individual unitary operations performed on the qubits. In the circuit model of quantum computation, we can consider time to be the left-to-right progression in the circuit (Fig. 2). At time t_0 and time t_1, there is no difference between the states of the qubits at these two time steps, so t_0 and t_1 are indistinguishable from each other. We can say that $t_0 = t_1$. At time t_2, since the qubits have undergone some transformation, they have evolved with time, and we can say that t_2 happens strictly after t_0 or t_1. From this, we can see that, instead of following a continuum, time can be discretised into individual time steps. If the universe is a quantum computer, then the progression of time is the same as the progression of computation steps, which are discrete. However, since our current understanding of the physics of the universe does not show with certainty whether time is discrete or continuous, we cannot be sure of this parallelism between the universe and quantum computers.

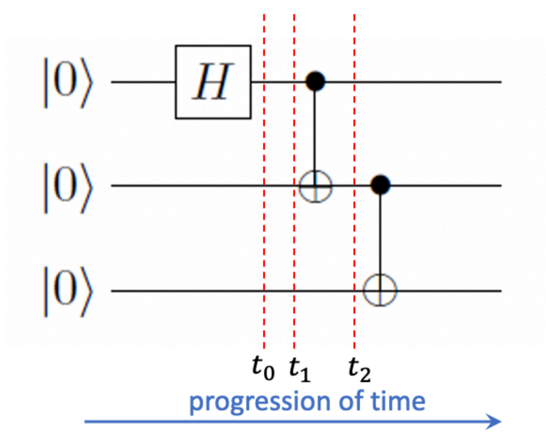

Figure 2: Diagram of Quantum Circuit

Secondly, space and energy may be discrete, too. If each fundamental particle in the universe can be represented using a finite number of qubits to express its properties such as energy and position in space, then these properties should be discrete. This is because, if only a finite number of qubits are available, then only a finite level of precision can be specified for each of these physical properties. Hence, their values cannot be continuous (i.e. any real numbers of arbitrary precision). In addition, if the position of a particle is discrete, then space must also be discrete, since an arbitrary particle cannot occupy any arbitrarily defined space coordinates, but only certain allowable discrete values. Again, since our current understanding of the physics of the universe does not show with certainty whether energy and space are discrete or continuous, we cannot be sure of this parallelism between the universe and quantum computers.

Section 8: Conclusion

From the above discussions, I shall conclude that it is possible for the universe to be a quantum computer, based on the four postulates of quantum computing. Our current understanding of the universe is not sufficient for us to conclude decisively whether the universe *is* or *cannot be* a quantum computer, as our theory of the universe keeps evolving over time. Nevertheless, even if the quantum computer, as defined by the four postulates, is not an adequate representation of our universe, the universe may still be a machine that computes. By theorising the universe using information-processing principles, there are significant implications for our understanding of nature, such as the discreteness of space and time. Our aim is not to prove whether the universe is indeed some computer, but to gain novel insights into the inner workings of the universe, and hopefully advancing the boundaries of natural science in this process.

References

Carman, C. C., & Evans, J. (2014). On the epoch of the Antikythera mechanism and its eclipse predictor. *Archive for History of Exact Sciences*, *68*(6), 693–774. https://doi.org/10.1007/s00407-014-0145-5

Davis, E. B. (1991). Newton's rejection of the "Newtonian world view": the role of divine will in Newton's natural philosophy. *Science and Christian Belief 3*, no. 2: 103-117.

Einstein, A., Podolsky, B., & Rosen, N. (1935). Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review*, *47*(10), 777–780. https://doi.org/10.1103/physrev.47.777

Hawking, S. W. (2002, July). *Gödel and the end of physics*. Lecture. Retrieved 20 April 2020 from http://www.hawking.org.uk/godel-and-the-end-of-physics.html

Landau Lev Davidovič., & Lifšic Evgenij Michajlovič. (1971). *Course of theoretical physics* (2nd ed., Vol. 4). Pergamon-Press.

Needham, J., Tsien, T.-hsuin, Kuhn, D., Bray, F., Daniels, C., Menzies, N. K., … Huang, H. T. (1986). *Science and Civilisation in China* (Vol. 4). Cambridge Univ. Press.

Nielsen, M. A., & Chuang, I. L. (2019). *Quantum Computation and Quantum Information*. Cambridge University Press.

Oerter, R. (2006). *The theory of almost everything the standard model, the unsung triumph of modern physics*. Penguin Group.

Turing, A. M. (1936). On Computable Numbers, with an Application to the Entscheidungsproblem. *Proceedings of the London Mathematical Society*, *s2-42*(1), 230–265. https://doi.org/10.1112/plms/s2-42.1.230

Weisstein, E. W. (2020). *Chaos*. MathWorld -- A Wolfram Web Resource. https://mathworld.wolfram.com/Chaos.html