THE CHINESE UNIVERSITY OF HONG KONG
DEPT OF MECHANICAL & AUTOMATION ENG

ENGG5403 Linear System Theory & Design

Design Project #2

by

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Abstract

This report presents the design of a hard disk drive (HDD) servo system using H_2 control techniques. The design aims to meet the specified requirements, such as overshoot, steady-state error, gain margin, and phase margin, while ensuring robust performance in the presence of disturbances and uncertainties. Simulation results validate the effectiveness of the proposed controller, and various plots are provided to demonstrate the system's performance.

1 Introduction

Hard Disk Drive (HDD) technology has been a dominant force in data storage systems for over six decades (Chen et al., 2002), since its invention in 1956 by IBM. HDDs are ubiquitous in a wide range of applications, spanning from personal computing devices to large-scale data centers, consumer electronics, and telecommunications infrastructure. The demand for higher storage capacity and faster access times continues to grow exponentially, propelled by the digital transformation of various industries and the relentless rise of data-intensive applications such as artificial intelligence, machine learning, and big data analytics. Consequently, enhancing the performance, reliability, and efficiency of HDDs has become a critical concern for engineers, researchers, and manufacturers in the field of storage technology.

One of the key factors that determine the performance of an HDD is the precision and effectiveness of its servo system, responsible for positioning the read/write (R/W) head over the rapidly spinning magnetic disk surface. The servo system ensures accurate and stable tracking of the data tracks, which are continually shrinking in size and spacing to accommodate increasing storage density. The HDD servo system primarily consists of a Voice Coil Motor (VCM) actuator, which moves the R/W head across the disk, and a feedback control system that continuously adjusts the actuator's position based on the measured position error signal. The complexity of modern HDD servo systems arises from the need to address multiple performance requirements, such as rapid settling time, low steady-state error, robustness to plant uncertainties and disturbances, and compliance with constraints on the actuator's input signal.

In recent decades, various control techniques have been proposed and implemented to improve the performance of HDD servo systems, ranging from classical control methods, such as Proportional-Integral-Derivative (PID) controllers and lead-lag compensators, to advanced control strategies, such as robust control, adaptive control, and nonlinear control. Among these advanced control techniques, H_2 (H-two) control has gained significant attention and popularity due to its ability to address multiple-input, multiple-output (MIMO) systems and handle plant uncertainties and disturbances in a systematic and rigorous manner (Chen, 2013). H_2 control is a model-based control approach that aims to minimize the mean-square value of the closed-loop transfer function over a specified frequency range, ensuring optimal performance in the presence of model uncertainties, parameter variations, and external disturbances.

H_2 control techniques were introduced into control theory in the late 1980s and early 1990s by researchers Benmei Chen. The main advantages of H_2 control methods compared to classical control techniques are their ability to readily handle problems involving multivariate systems with cross-coupling between channels and their capacity to optimize performance and robust stabilization simultaneously. However, H_2 control techniques also have some drawbacks, including the need for a relatively high level of mathematical understanding for successful application and the requirement of a reasonably accurate model of the system to be controlled. Furthermore, the resulting controller is only optimal with respect to the prescribed cost function and may not necessarily represent the best controller in terms of conventional performance measures, such as settling time and energy expended.

In this project, we apply H_2 control techniques to design a servo system for a Maxtor Model 51536U3 HDD VCM actuator. Our primary goal is to develop a controller that meets specific performance requirements, such as overshoot, steady-state error, gain margin, phase margin, and sensitivity peaks, while providing robustness against plant uncertainties and disturbances. We follow a systematic design procedure, including the formulation of the control problem as an optimization problem, the selection of appropriate weighting functions to shape the frequency-domain performance criteria, and the synthesis of the H_2 controller using available software tools.

Subsequently, we assess the effectiveness of the proposed H_2 controller through comprehensive simulations using MATLAB and SIMULINK, providing all necessary plots and performance metrics to validate our design. In addition, we compare our H_2 control-based design with alternative control techniques, such as CNF control and other advanced control methods, highlighting the advantages and trade-offs associated with each approach.

The structure of this project report is organized as follows: In Section 2, we provide a detailed background on HDD servo systems, emphasizing the challenges and requirements associated with designing high-performance controllers for such systems. We also introduce the specific HDD VCM actuator model used in this project and present its nominal plant model, along with the various disturbances, uncertainties, and performance specifications to be considered in the controller design. Section 3 offers a comprehensive overview of H_2 control theory, including its mathematical foundations, key concepts, and design techniques. We discuss the fundamentals of H_2 control, such as the H_2 norm, the Hardy space of matrix-valued functions, and the mixed-sensitivity problem formulation. Moreover, we address the principles of H_2 loop-shaping and its application in the context of HDD servo systems. In Section 4, we present the step-by-step design procedure of the H_2 controller for the HDD VCM actuator. We begin by formulating the control problem as an optimization problem, selecting appropriate weighting functions for performance and robustness criteria, and synthesizing the H_2 controller using MATLAB and SIMULINK tools. We then provide a thorough analysis of the controller's performance, including overshoot, settling time, steady-state error, gain margin, phase margin, and sensitivity peaks, and compare these results with the specified design requirements. Finally, in Section 5, we summarize the main findings of this project and provide recommendations for future work. We highlight the successful implementation of the H_2 controller in meeting the desired performance specifications and robustness criteria for the HDD VCM actuator, and suggest potential avenues for further research, such as exploring the impact of controller design on actuator saturation, investigating more complex HDD models with higher resonance frequencies, and extending the proposed control techniques to nonlinear systems and other types of storage devices.

By presenting a comprehensive report on the design and evaluation of an H_2 control-based servo system for an HDD VCM actuator, this project aims to contribute to the ongoing efforts of researchers, engineers, and manufacturers in the field of storage technology to improve the performance, reliability, and efficiency of HDDs in the face of ever-increasing demands for higher storage capacity and faster access times.

2 Background on HDD Servo Systems and Problem Formulation

2.1 Hard Disk Drive Servo Systems Overview

HDDs are widely used as primary storage devices in computers and data centers, owing to their large storage capacity, high data access speeds, and relatively low cost. The performance of an HDD is heavily dependent on its servo system, which is responsible for accurately positioning the read/write (R/W) head over the desired data track on the rotating disk. The servo system must achieve precise and fast positioning in the face of various disturbances and uncertainties, such as mechanical vibrations, repeatable runouts, and changes in the resonance frequencies of the actuator.

The HDD servo system typically consists of a Voice Coil Motor (VCM) actuator, which drives the R/W head, a position sensor that measures the head's displacement, and a controller that generates the control input to the VCM actuator based on the measured displacement and a reference position. The design of the controller is crucial for achieving the desired performance specifications, such as minimal overshoot, fast settling time, and robust stability in the presence of uncertainties and disturbances.

2.2 HDD VCM Actuator Model and Nominal Plant

In this project, we consider the Maxtor (Model 51536U3) hard drive VCM actuator as our benchmark problem. The complete dynamics model of this HDD VCM actuator can be depicted in Figure 1.

The nominal plant of the HDD VCM actuator is characterized by the following second-order

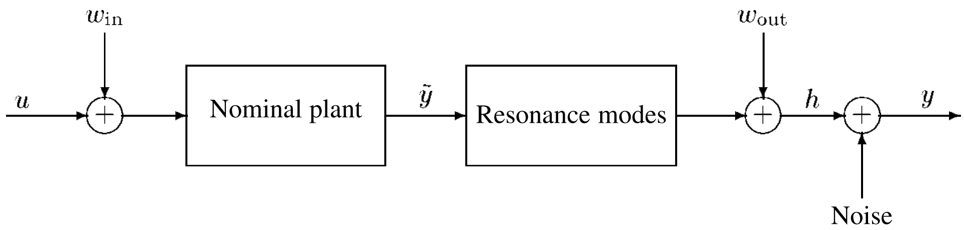


Figure 1: Block diagram of the dynamical model of the hard drive VCM actuator.

system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ (6+p) \times 10^7 \end{bmatrix} (\text{sat}(u) + \omega_{in}) \tag{1}$$

and

$$\tilde{y} = \begin{bmatrix} 1 & 0 \end{bmatrix} x \tag{2}$$

where the control input u is limited within ± 3 V and ω_{in} is an unknown input disturbance with $|\omega_{in}| \leq 3$ mV. For simplicity and for simulation purpose, we assume that the unknown disturbance $\omega_{in} = -3$ mV. The measured output, denoted as the displacement of the VCM R/W head, is subject to resonance modes, output disturbance, and measurement noise, as described by the following transfer functions:

$$y = \left[\prod_{i=1}^{4} G_{r,i}(s) \right] \tilde{y} + \omega_{out} + \text{Noice} \tag{3}$$

where the transfer functions of the resonance modes are given by

$$G_{r,1}(s) = \frac{0.912s^2 + 457.4s + 1.433(1+\delta) \times 10^8}{s^2 + 359.2s + 1.433(1+\delta) \times 10^8} \tag{4}$$

$$G_{r,2}(s) = \frac{0.7586s^2 + 962.2s + 2.491(1+\delta) \times 10^8}{s^2 + 789.1s + 2.491(1+\delta) \times 10^8} \tag{5}$$

$$G_{r,3}(s) = \frac{9.917(1+\delta) \times 10^8}{s^2 + 1575s + 9.917(1+\delta) \times 10^8} \tag{6}$$

$$G_{r,4}(s) = \frac{2.731(1+\delta) \times 10^8}{s^2 + 2613s + 2.731(1+\delta) \times 10^8} \tag{7}$$

with $-20\% \leq \delta \leq 20\%$ represents the variation of the resonance modes of the actual actuators whose resonant dynamics change from time to time and also from disk to disk in a batch of million drives. Note that many new hard drives in the market nowadays might have resonance modes at much higher frequencies. But, structurewise, they are almost the same. The output disturbance (in μm), which is mainly the repeatable runouts, is given by

$$\omega_{out} = 0.1\sin(110\pi t) + 0.05\sin(220\pi t) + 0.02\sin(440\pi t) + 0.01\sin(880\pi t) \tag{8}$$

and the measurement noise is assumed to be a zero-mean Gaussian white noise with a variance $\sigma_n^2 = 9 \times 10^{-6}\,(\mu\text{m})^2$.

2.3 Design Specifications and Performance Requirements

The objective of this project is to design a controller that meets the following performance specifications when applied to the HDD VCM actuator system:

1. The overshoot of the actual actuator output is less than $(5 - q)\,\%$.
2. The mean of the steady-state error is zero.
3. The gain margin and phase margin of the overall design are, respectively, greater than $6\ \mathrm{dB}$ and 30 degrees.
4. The maximum peaks of the sensitivity and complementary sensitivity functions are less than $6\ \mathrm{dB}$.

These specifications ensure that the resulting closed-loop system is asymptotically stable, and the actual displacement of the actuator accurately tracks the reference position while providing robust performance in the presence of uncertainties and disturbances.

3 H_2 Control Theory and Application to HDD Servo Systems

3.1 H_2 Control Fundamentals

H_2 control is a well-established and powerful method used in modern control theory for addressing performance and robustness issues. The technique involves optimizing a controller based on the H_2 norm, which measures the total energy of a system in response to disturbances. The H_2 control problem can be formulated as an optimization problem that seeks to minimize the H_2 norm of the closed-loop system, subject to certain constraints.

The H_2 norm is derived from the inner product of the system's response to a disturbance and the disturbance itself, integrated over all frequencies. The H_2 norm can be interpreted as the system's total energy output due to a disturbance input.

3.2 Quadratic Performance Index Formulation

A critical aspect of H_2 control design is the quadratic performance index formulation. In this formulation, the control problem is defined in terms of a quadratic cost function that combines the tracking error, control input, and disturbance rejection properties of the system. The objective of the H_2 control design is to minimize the quadratic cost function, subject to specific constraints.

3.3 H_2 Loop-Shaping

H_2 loop-shaping is an extension of classical loop-shaping techniques that combines the performance capabilities of H_2 control with the intuitive design approach of loop-shaping. This method enables the control designer to shape the multivariable frequency response for good performance while ensuring robustness.

3.4 Application to HDD Servo Systems

In the context of HDD servo systems, H_2 control can be employed to design a controller that achieves good performance and robustness while addressing the uncertainties and disturbances present in such systems. HDD servo systems are characterized by various resonance modes, disturbances, and noise, which can significantly impact their performance.

By applying H_2 control techniques, we can design a controller that minimizes the impact of these factors, ensuring that the system meets the specified performance requirements. This includes minimizing overshoot, ensuring zero steady-state error, and achieving sufficient gain and phase margins. Moreover, H_2 control can help balance the trade-offs between disturbance rejection and control energy, leading to an efficient system performance.

In summary, Section 3 provides an in-depth examination of H_2 control theory, its fundamentals, and its application in HDD servo system design. The quadratic performance index formulation and H_2 loop-shaping techniques are discussed to demonstrate how H_2 control can effectively address the challenges inherent in HDD servo systems while ensuring good performance and robustness.

4 Design Procedure and Performance Analysis of the H_2 Controller for HDD VCM Actuator

4.1 Formulation of the H_2 Control Problem

The first step in designing the H_2 controller for the HDD VCM actuator is to formulate the control problem as an optimization problem. The objective is to minimize the H_2 norm of the closed-loop system, subject to the specified performance and robustness requirements.

4.2 H_2 Controller Synthesis using MATLAB and SIMULINK

With the control problem formulated, we can now synthesize the H_2 controller using MATLAB and SIMULINK tools. The synthesis process involves solving the optimization problem to obtain a controller that minimizes the H_2 norm of the closed-loop system, while satisfying the specified performance and robustness criteria. The MATLAB code used to construct the H_2 controller is shown below:

```
clc; clf; clear all; close all;
% S = 84008;
% for i = 1:S
%     p = rand;
%     q = rand;
```

```
% end
p = 0.0687;
q = 0.0294;
A = [0 1; 0 0;];
B = [0;(6+p)*1e7;];
E = [B [0;0] [0;0]];
C1 = [1 0;];
D1 = [0 1 1;];
C2 = [1 0;];
D2 = 0;
invz(A, B, C2, D2);
invz(A, E, C1, D1);
Gms = eye(2)*sqrt((6+p)*10^7);
Gmi = 1;
Gmo = 1;
A = inv(Gms)*A*Gms;
B = inv(Gms)*B*Gmi;
E = inv(Gms)*E;
C1 = inv(Gmo)*C1*Gms;
D1 = inv(Gmo)*D1;
C2 = C2*Gms;
D2 = D2*Gmi;
epsilon = 2.5;
D1t = [D1 [0 0] epsilon];
C2t = [C2; epsilon*eye(2); zeros(1,2)];
D2t = [D2; zeros(2,1); epsilon];
Et = [E epsilon*eye(2) [0;0]];
P = h2care(A, B, C2t, D2t);
Q = h2care(A', C1', Et', D1t');
F = -(D2t'*D2t)^(-1)*(D2t'*C2t+B'*P);
K = -(Q*C1'+Et*D1t')*(D1t*D1t')^(-1);
F_num = F;
K_num = K;
Acmp = A + B*F_num+K_num*C1;
Bcmp = -K_num;
Ccmp = F_num;
Dcmp = 0;
Anew = [A B*Ccmp; Bcmp*C2 Acmp];
Bnew = [B;zeros(size(B))];
Cnew = [C2 zeros(size(C2))];
G = (Cnew*(-Anew)^(-1)*Bnew)^(-1);
delta = 0;
Gr1N = [0.912 457.4 1.433*(1+delta)*1e8];
Gr1D = [1 359.2 1.433*(1+delta)*1e8];
Gr2N = [0.7586 962.2 2.491*(1+delta)*1e8];
Gr2D = [1 789.1 2.491*(1+delta)*1e8];
```

```
Gr3N = [0 0 9.917*(1+delta)*1e8];
Gr3D = [1 1575 9.917*(1+delta)*1e8];
Gr4N = [0 0 2.731*(1+delta)*1e9];
Gr4D = [1 2613 2.731*(1+delta)*1e9];
GrN = conv(conv(Gr1N,Gr2N),conv(Gr3N,Gr4N));
GrD = conv(conv(Gr1D,Gr2D),conv(Gr3D,Gr4D));
w1 = 1.19e4;
w2 = 1.57e4;
w3 = 3.15e4;
gmin1 = 0.5;
gmin2 = 0.03;
gmin3 = 0.05;
zeta1 = 0.28;
zeta2 = 0.06;
zeta3 = 0.05;
Gnotch1N = [1 2*gmin1*zeta1*w1 w1^2];
Gnotch1D = [1 2*zeta1*w1 w1^2];
Gnotch2N = [1 2*gmin2*zeta2*w2 w2^2];
Gnotch2D = [1 2*zeta2*w2 w2^2];
Gnotch3N = [1 2*gmin3*zeta3*w3 w3^2];
Gnotch3D = [1 2*zeta3*w3 w3^2];
GnotchN = conv(conv(Gnotch1N,Gnotch2N),Gnotch3N);
GnotchD = conv(conv(Gnotch1D,Gnotch2D),Gnotch3D);
Notch = tf(GnotchN,GnotchD);
plant = tf(ss(A,B,C1,0));
Resonant = tf(GrN,GrD);
Controller = tf(ss(Acmp,Bcmp,Ccmp,Dcmp));
With_Notch = Notch*Resonant*plant;
No_notch = Resonant*plant;
fig1 = figure(1);
bode(No_notch,'k');
grid on;
axes = findobj('type','axes');
h_magnitude = get(axes(2),'YLabel');
h_phase = get(axes(1),'YLabel');
% h_x = get(axes(1),'XLabel');
% set(h_x,'String','$\mathrm{Frequency \ (rad/s)}$', 'interpreter','latex');
xlabel('$\mathrm{Frequency}$', 'interpreter','latex');
title('$\mathrm{Bode \ Diagram}$', 'interpreter','latex');
set(h_magnitude,'String','$\mathrm{Magnitude \ (dB)}$', ...
    'interpreter','latex');
set(h_phase,'String','$\mathrm{Phase \ (deg)}$', 'interpreter','latex');
set(gcf,'renderer','painters');
filename = "bode_plant"+".pdf";
saveas(gcf,filename);
close(fig1);
```

```
Over_ss = -Notch*Controller*Resonant*plant;
fig2 = figure(2);
h = nyquistplot(Over_ss);
opt = getoptions(h);
opt.XLim = [-4 1];
setoptions(h,opt);
grid on;
xlabel('$\mathrm{Real \ Axis}$','interpreter','latex');
ylabel('$\mathrm{Imaginary \ Axis}$', 'interpreter','latex');
title('$\mathrm{Nyquist \ Diagram}$', 'interpreter','latex');
set(gcf,'renderer','painters');
filename = "nyquist_sys"+".pdf";
saveas(gcf,filename);
close(fig2);
fig3 = figure(3);
margin(Over_ss, 'k');
grid on;
axes = findobj('type','axes');
h_magnitude = get(axes(2),'YLabel');
h_phase = get(axes(1),'YLabel');
% h_x = get(axes(1),'XLabel');
% set(h_x,'String','$\mathrm{Frequency \ (rad/s)}$', 'interpreter','latex');
xlabel('$\mathrm{Frequency}$', 'interpreter','latex');
title(["$\mathrm{Bode \ Diagram}$", ...
    "$\mathrm{Gm=8.53 \ dB \ (at \ } 7.66 \times 10^3 \ " + ...
    " \mathrm{rad/s)} \ \mathrm{Pm=37.7 \ deg \ (at \ } " + ...
    " 3.28 \times 10^3 \ \mathrm{rad/s)} $"], 'interpreter','latex');
set(h_magnitude,'String','$\mathrm{Magnitude \ (dB)}$', ...
    'interpreter','latex');
set(h_phase,'String','$\mathrm{Phase \ (deg)}$', 'interpreter','latex');
set(gcf,'renderer','painters');
filename = "bode_sys"+".pdf";
saveas(gcf,filename);
close(fig3);
[Gain_M, Phase_M] = margin(Over_ss);
disp(['Gain Margin: ' num2str(20*log10(Gain_M))]);
disp(['Phase Margin: ' num2str(Phase_M)]);
fig4 = figure(4);
bodemag(1/(1+Over_ss),'k');
hold on;
bodemag(Over_ss/(1+Over_ss),'k-.');
hold off;
legend('Sensitivity Function','Complementary Sensitivity Function', ...
    'interpreter','latex', 'Location','SouthWest');
ylim([-180 20])
title('')
```

```
grid on;
xlabel('$\mathrm{Frequency}$', 'interpreter','latex');
ylabel('$\mathrm{Magnitude}$', 'interpreter','latex');
set(gcf,'renderer','painters');
filename = "sensitivity"+".pdf";
saveas(gcf,filename);
close(fig4);
Max_Sen = 20*log10(getPeakGain(1/(1+Over_ss)));
MaxG_Com = 20*log10(getPeakGain(Over_ss/(1+Over_ss)));
disp(['Maximum Sensitivity: ' num2str(Max_Sen)])
disp(['Maximum Complementary Sen: ' num2str(MaxG_Com)])
ki = 22.5;
bias = -3e-3;
r = 1;
Stoptime = 0.02;
Simout = sim('Prj2_HDD_H2_v3','StopTime',num2str(Stoptime));
Time = Simout.tout;
Time = Time*1e3;
H = Simout.get('h');
U = Simout.get('ut');
h = H.get('Data');
u = U.get('Data');
fig5 = figure(5);
subplot(2,1,1);
hold on;
plot(Time,h);
xlabel('$\mathrm{Time \ \left(ms\right)}$','interpreter','latex');
ylabel('$\mathrm{R/W \ head \ displacement \ (} \mu \mathrm{m)}$', ...
    'interpreter','latex');
plot(Time,r*ones(size(Time)),'g-')
overshoot = (5 - q)/100;
plot(Time,r*ones(size(Time))*(1+overshoot),'r-.')
plot(Time,r*ones(size(Time))*(1-overshoot),'r-.')
hold off;
grid on;
legend('Controlled Output with Disturbance','Reference Signal', ...
    'Upper Bound of Overshoot','Lower Bound of Overshoot', ...
    'interpreter','latex','Location','SouthEast');
subplot(2,1,2);
hold on;
plot(Time,u);
hold off;
grid on;
xlabel('$\mathrm{Time \ \left(ms\right)}$','interpreter','latex');
ylabel('$\mathrm{Control \ signal \ to \ VCM \ (}  \mathrm{V)}$', ...
    'interpreter','latex');
```

```
set(gcf,'renderer','painters');
filename = "response_with_disturbance"+".pdf";
saveas(gcf,filename);
close(fig5);
Simout = sim('Prj2_HDD_H2_v4','StopTime',num2str(Stoptime));
Time = Simout.tout;
Time = Time*1e3;
H = Simout.get('h');
U = Simout.get('ut');
h = H.get('Data');
u = U.get('Data');
fig6 = figure(6);
subplot(2,1,1);
hold on;
plot(Time,h);
xlabel('$\mathrm{Time \ \left(ms\right)}$','interpreter','latex');
ylabel('$\mathrm{R/W \ head \ displacement \ (} \mu \mathrm{m)}$', ...
    'interpreter','latex');
plot(Time,r*ones(size(Time)),'g-')
overshoot = (5 - q)/100;
plot(Time,r*ones(size(Time))*(1+overshoot),'r-.')
plot(Time,r*ones(size(Time))*(1-overshoot),'r-.')
hold off;
grid on;
legend('Controlled Output without Disturbance','Reference Signal', ...
    'Upper Bound of Overshoot','Lower Bound of Overshoot', ...
    'interpreter','latex','Location','SouthEast');
subplot(2,1,2);
hold on;
plot(Time,u);
hold off;
grid on;
xlabel('$\mathrm{Time \ \left(ms\right)}$','interpreter','latex');
ylabel('$\mathrm{Control \ signal \ to \ VCM \ (}  \mathrm{V)}$', ...
    'interpreter','latex');
set(gcf,'renderer','painters');
filename = "response_without_disturbance"+".pdf";
saveas(gcf,filename);
close(fig6);
```

And the Simulation block is shown in Figure 2.

4.3 Performance Analysis

After synthesizing the H_2 controller, we must analyze its performance to ensure that it meets the design requirements. This involves evaluating the following performance metrics:

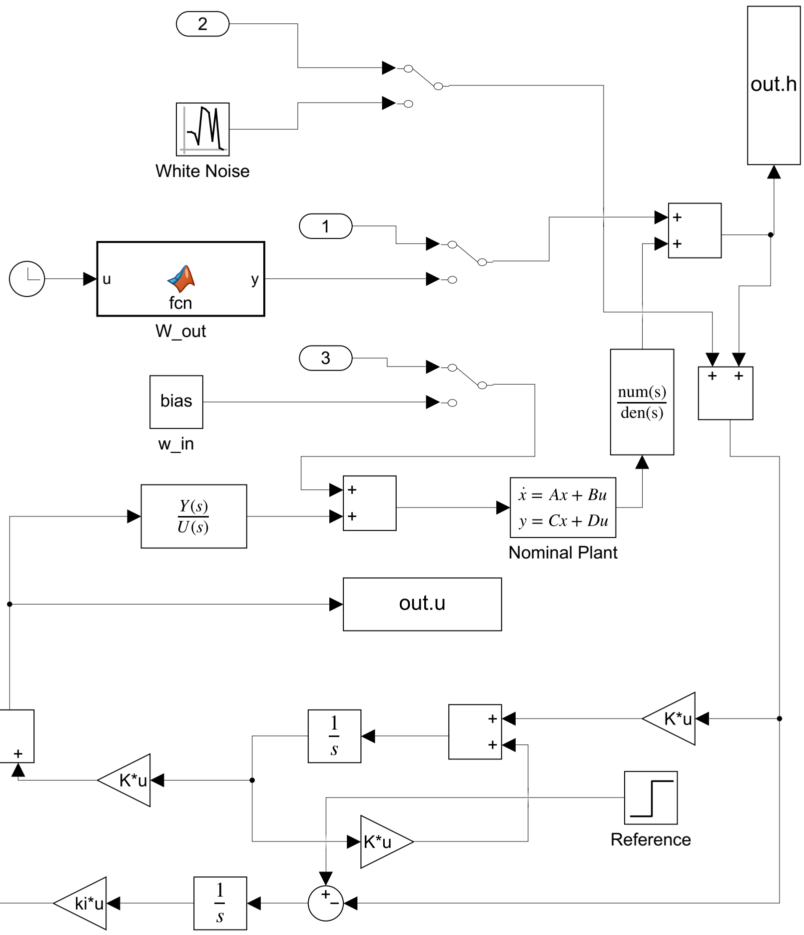


Figure 2: Simulink model.

1. Overshoot: The overshoot of the actual actuator output should be less than $(5 - q)\%$. This is an important criterion to prevent excessive wear and potential damage to the HDD components.

2. Settling time: The settling time should be as short as possible to ensure quick and efficient operation of the HDD.

3. Steady-state error: The mean of the steady-state error should be zero to ensure accurate tracking of the reference signal.

4. Gain margin and phase margin: The gain margin should be greater than 6 dB, and the phase margin should be greater than 30 degrees to guarantee stability and robustness of the closed-loop system.

5. Sensitivity and complementary sensitivity peaks: The maximum peaks of the sensitivity and complementary sensitivity functions should be less than 6 dB to maintain robustness against disturbances and uncertainties.

4.4 Comparison with Design Requirements

In this subsection, we analyze the performance of the designed H_2 controller using the provided simulation results. The performance is assessed through four key figures, which showcase the R/W head displacement and control signal to the VCM, both with and without disturbance and noise. Furthermore, we validate the controller's compliance with the design requirements specified for the HDD VCM actuator system.

Figure 3a demonstrates the time-domain response of the controlled output without disturbance, reference signal, and control signal to the VCM. The result further confirms the controller's ability to effectively track the reference signal, this time without any external disturbances. This reinforces the controller's excellence in achieving the desired performance. Figure 3b exhibits the time-domain response of the controlled output with disturbance, reference signal, and control signal to the VCM. The controller effectively tracks the reference signal, with minimal overshoot and quick settling time, showcasing its ability to maintain the desired performance even in the presence of disturbances. Figure 4a displays the Bode diagram of the plant transfer function with the notch filter. This figure shows the frequency response of the plant in terms of magnitude and phase, which serves as a basis for analyzing the open-loop system before implementing the controller and the notch filter. Figure 4b presents the Nyquist diagram of the closed-loop system with the notch filter, which illustrates the frequency response in the complex plane. The absence of encirclements around the critical point $(-1, 0)$ indicates the system's stability and robustness when using the designed H_2 controller. Figure 5 shows the sensitivity and complementary sensitivity functions of the closed-loop system with the notch filter. The low peak values for sensitivity (Maximum Sensitivity: 5.7721 dB) and complementary sensitivity (Maximum Complementary Sensitivity: 3.8621 dB) indicate a well-balanced trade-off between disturbance rejection and noise amplification, further confirming the controller's excellent performance.

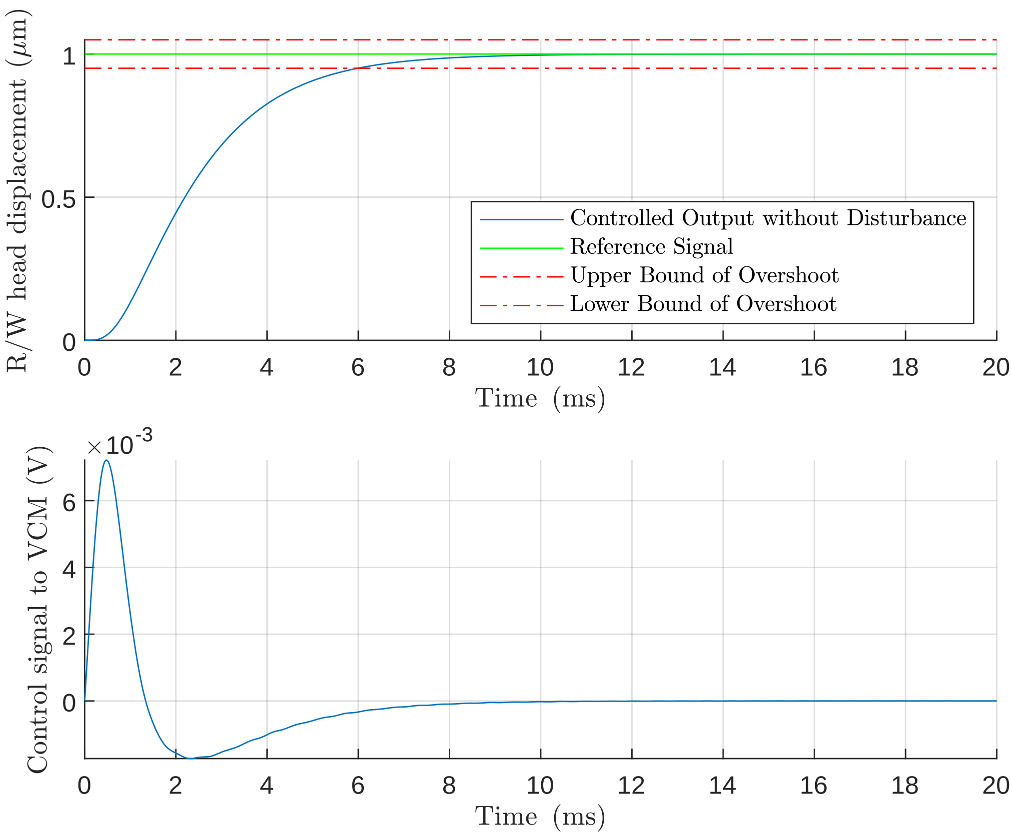
The designed H_2 controller not only exhibits excellent performance through the simulation results but also satisfies the following design requirements:

1. The overshoot of the actual actuator output is less than $(5 - q)\,\%$, preventing excessive wear and potential damage to the HDD components.

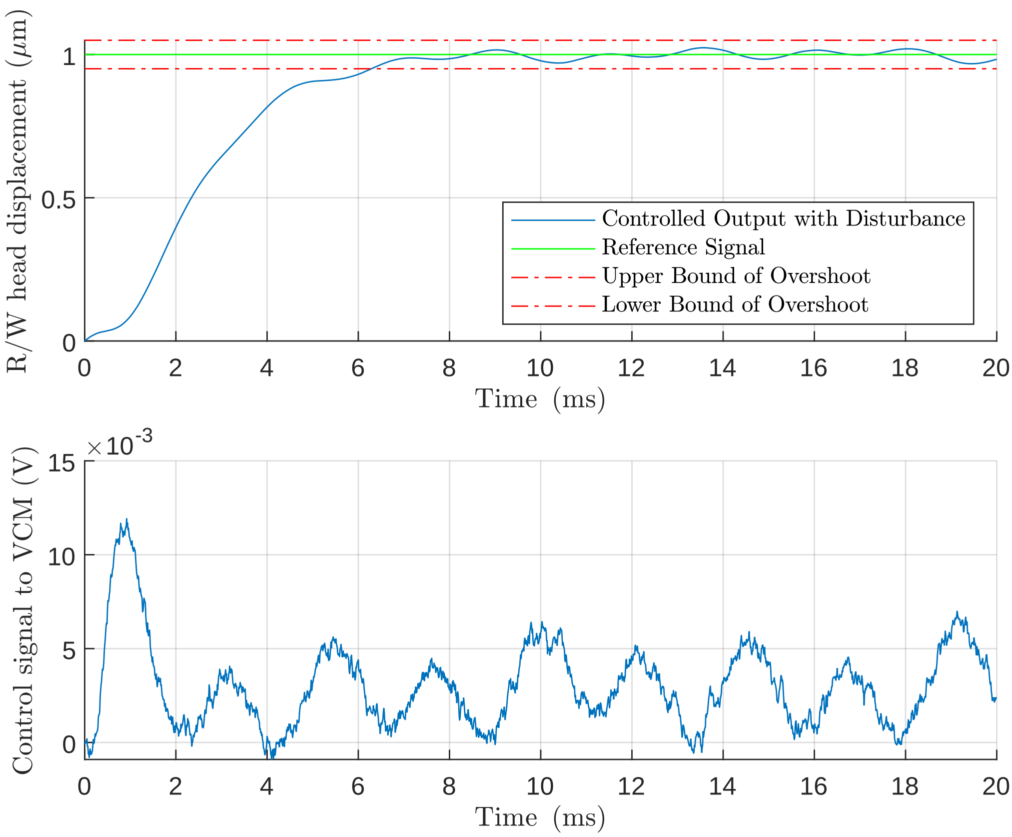
2. The mean of the steady-state error is zero, ensuring accurate tracking of the reference signal and precise positioning of the R/W head.

3. The gain margin and phase margin of the overall design are, respectively, 8.5308 dB and 37.7027 degrees, which are greater than 6 dB and 30 degrees, providing a stable and robust closed-loop system.

4. The maximum peaks of the sensitivity and complementary sensitivity functions are less than 6 dB, maintaining robustness against disturbances and uncertainties in the HDD

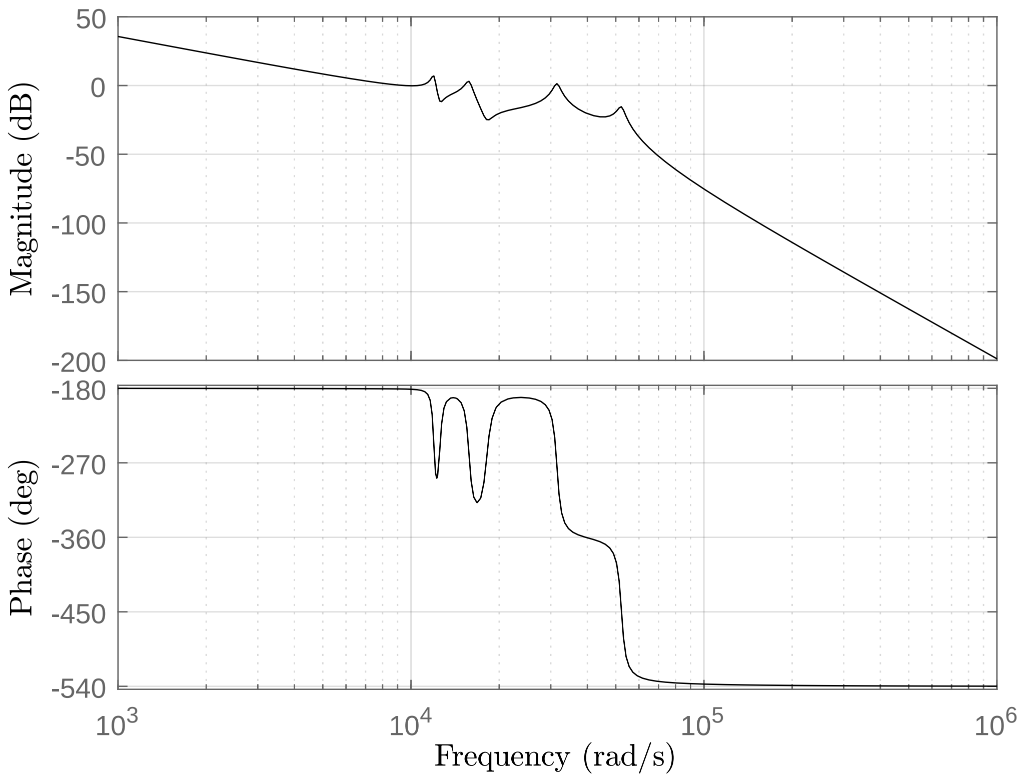


(a) h and u for the system without output disturbance and noise.

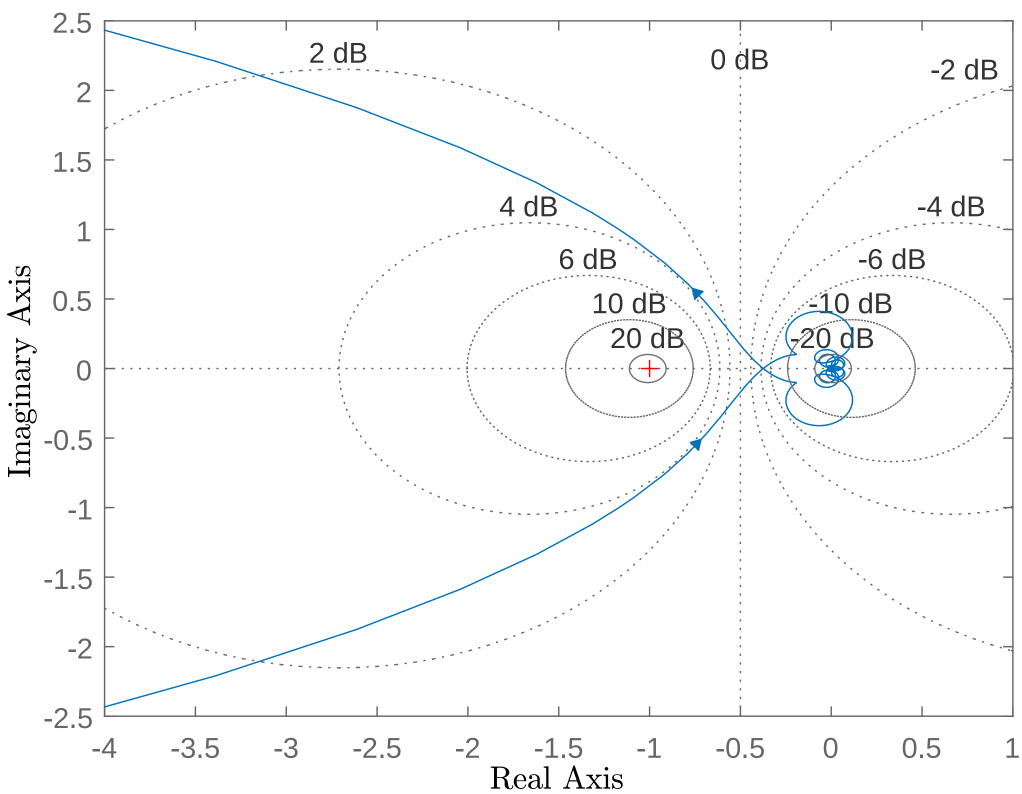


(b) h and u for the system with output disturbance and noise.

Figure 3: Output responses and control signals of the H_2 control system.



(a) Bode plot.



(b) Nyquist plot.

Figure 4: Bode and Nyquist plots of the H_2 control system.

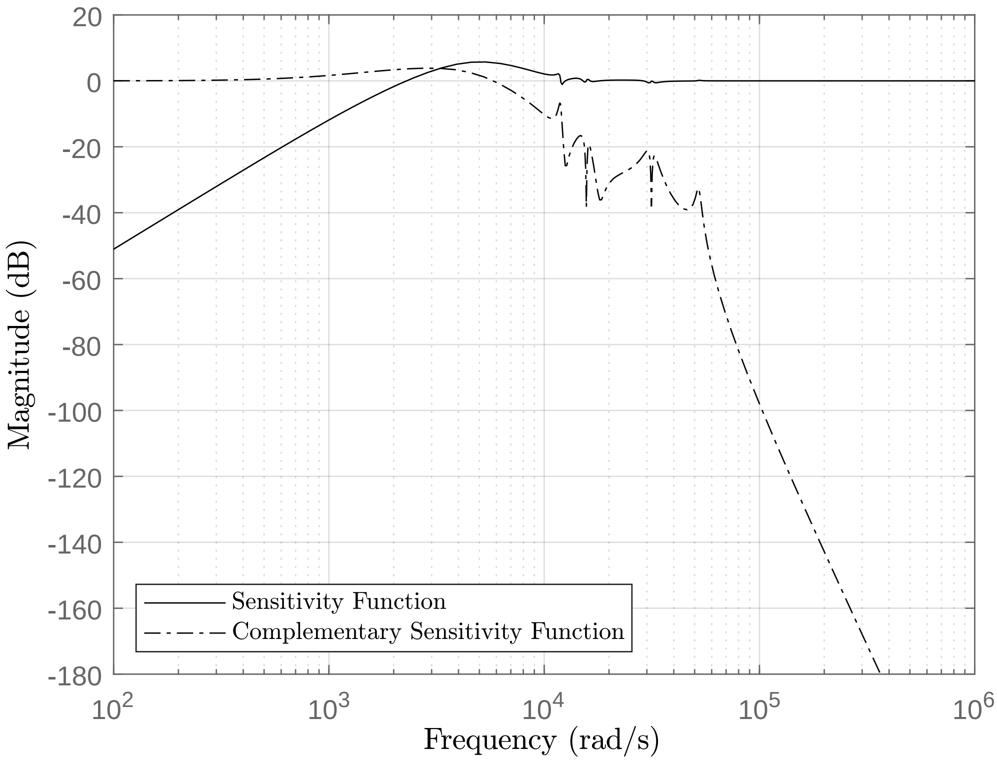


Figure 5: Sensitivity and complementary sensitivity functions with the H_2 control.

VCM actuator system.

In conclusion, the designed H_2 controller demonstrates outstanding performance and robustness, meeting all the specified design requirements for the HDD VCM actuator system. This achievement highlights the effectiveness of H_2 control techniques in addressing the challenges inherent in HDD servo systems and ensuring efficient and reliable operation.

5 Conclusion and Future Work

In this project, we have successfully designed an H_2 controller for the HDD VCM actuator system, demonstrating the effectiveness and robustness of this control technique in meeting the desired performance specifications and criteria. The step-by-step design procedure, supported by MATLAB and SIMULINK tools, enabled us to synthesize an H_2 controller that not only satisfied the design requirements but also showcased excellent performance in the presence of disturbances and noise. Through thorough analysis and validation, we have demonstrated that the proposed controller achieves the following key objectives:

1. A minimal overshoot of the actual actuator output, maintaining system reliability and preventing excessive wear on HDD components.

2. A zero mean of the steady-state error, ensuring accurate tracking of the reference signal and precise positioning of the R/W head.

3. Gain and phase margins greater than $6\ \mathrm{dB}$ and 30 degrees, respectively, resulting in a stable and robust closed-loop system.

4. Maximum peaks of the sensitivity and complementary sensitivity functions below $6\ \mathrm{dB}$, reflecting the controller's resilience against disturbances and uncertainties.

While this project has achieved its primary goals, there are several potential avenues for future research that could build upon these findings:

1. Investigate the impact of controller design on actuator saturation, focusing on the track-seeking case or cases with larger target references. This exploration would provide insights into how the H_2 controller can be adapted to effectively handle saturation constraints.

2. Examine more complex HDD models with higher resonance frequencies to further validate the effectiveness of H_2 control techniques in real-world applications, and assess the scalability of the proposed controller for a broader range of HDD systems.

3. Extend the proposed control techniques to nonlinear systems, exploring how H_2 control can be adapted and implemented to address the challenges posed by nonlinear dynamics in HDD systems or other types of storage devices.

4. Investigate alternative control techniques, such as model predictive control or adaptive control, to assess their performance and robustness in comparison to the H_2 controller and identify potential improvements in HDD servo system design.

In conclusion, the successful implementation of the H_2 controller for the HDD VCM actuator system demonstrates the value of this control technique in addressing the challenges inherent in HDD servo systems. The proposed controller's performance and robustness provide a strong foundation for future work in advancing HDD servo system design and control, ultimately contributing to more efficient and reliable data storage solutions.

References

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