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Mechanical Design 1

10 Assignment

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Mechanical Design 1

Class Section 01

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Problem 1

A solid square rod is cantilevered at one end. The rod is 0.6 m long and supports a completely reversing transverse load at the other end of ±2 kN. The material is AISI 1080 hot-rolled steel (S_{ut} = 770 MPa, S_y = 420 MPa). If the rod must support this load for 10^4 cycles with a factor of safety of 1.5, what dimension should the square cross section have? Neglect any stress concentrations at the support end.

Solution:

For this question, we are asked to determine what dimension should the square cross section have.

$$M_{max} = (2000\ \ \mathrm{N}) \times (0.6\ \ \mathrm{m}) = 1200\ \ \mathrm{N \cdot m}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{M\left(\frac{b}{2}\right)}{\frac{bb^4}{12}} = \frac{6M}{b^3} = \frac{7200}{b^3}\ \ \mathrm{Pa}$$

Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
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		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
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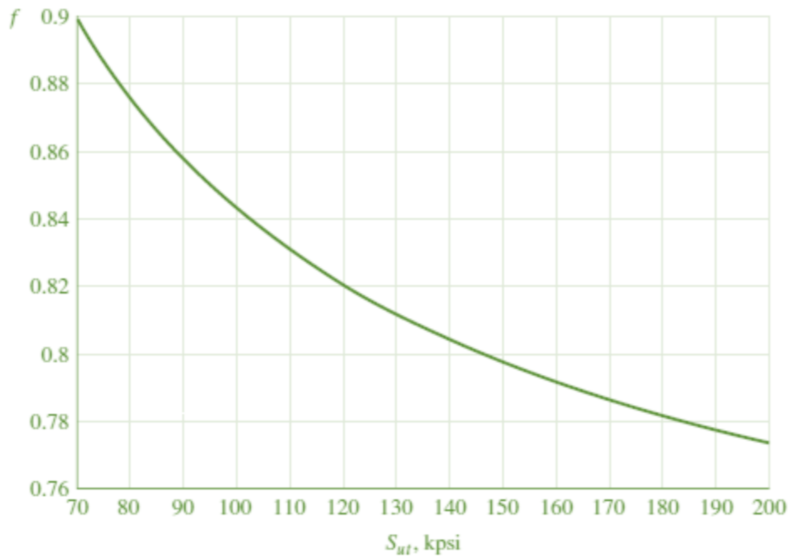
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

$$S_{ut} = 770\ \text{MPa} \Rightarrow \begin{cases} S'_e = 0.5S_{ut} = 0.5 \times (770\ \text{MPa}) = 385\ \text{MPa} \\ \text{Figure } 6.18\text{:}\ f = 0.83 \end{cases}$$

$$S_y = 420\ \text{MPa}$$

Figure 6–18

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S'_e = 0.5S_{ut}$ at 10^6 cycles.



Assume:

$$k_b = 0.85$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis*, vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook*, McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

And from the question and Table 6-2, I can know that

$$k_a = aS_{ut}^b = 57.7 \times (770\ \ \text{MPa})^{-0.718} = 0.488$$

Therefore,

$$S_e = k_a k_b S'_e = (0.488) \times (0.85) \times (385\ \ \text{MPa}) = 159.79\ \ \text{MPa}$$

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{[(0.83) \times (770\ \ \text{MPa})]^2}{(159.79\ \ \text{MPa})} = 2256\ \ \text{MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{(0.83) \times (770\ \ \text{MPa})}{(159.79\ \ \text{MPa})}\right] = -0.2007$$

$$S_f = aN^b = (2256\ \ \text{MPa}) \times (10^4)^{-0.2007} = 402.6\ \ \text{MPa}$$

$$FS = \frac{S_f}{\sigma_{max}} = \frac{(402.6\ \ \text{MPa})}{\left(\frac{7200}{b^3}\ \ \text{Pa}\right)} = 1.5$$

$$\Rightarrow b = 0.0299\ \ \text{m}$$

$$\Rightarrow b = 30\ \ \text{mm}$$

Verify:

$$d_e = 0.808(hb)^{\frac{1}{2}} = 0.808b = 24.24\ \ \text{mm}$$

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{24.24\ \ \text{mm}}{7.62}\right)^{-0.107} = 0.88$$

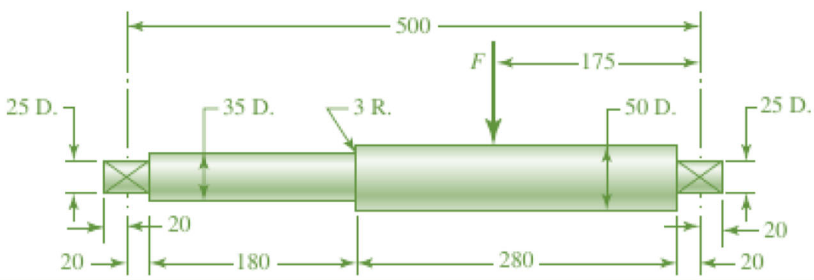
$$\sigma_{max} = \frac{7200}{(30\ \ \text{mm})^3} = 267\ \ \text{MPa}$$

$$n_y = \frac{S_y}{\sigma_{max}} = \frac{420\ \ \text{MPa}}{267\ \ \text{MPa}} = 1.57$$

This is suitable.

Problem 2

The rotating shaft shown in the figure (all dimensions in mm) is machined from AISI 1020 CD steel. It is subjected to a force of F = 6 kN. Find the minimum factor of safety for fatigue based on infinite life. If the life is not infinite, estimate the number of cycles. Be sure to check for yielding



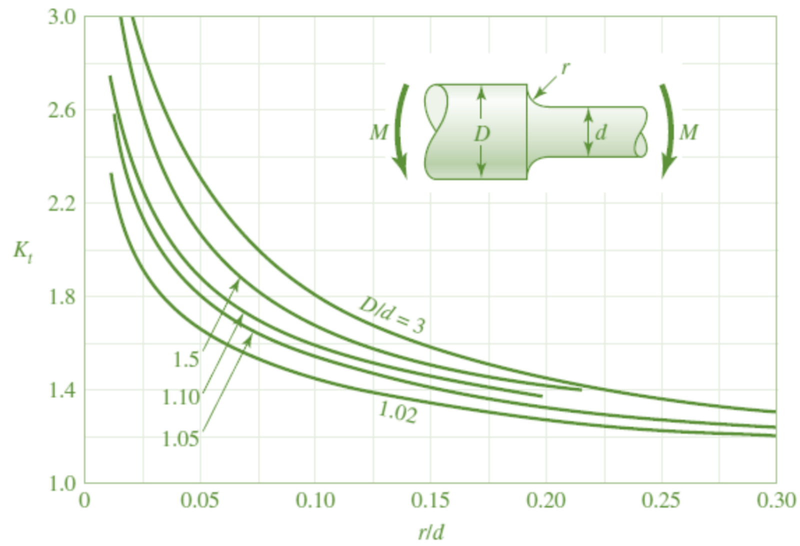
Solution:

For this question, we are asked to find the minimum factor of safety for fatigue based on infinite life. If the life is not infinite, estimate the number of cycles. Be sure to check for yielding.

$$\sigma_{max} = \frac{Mc}{I} = \frac{(2.1\ \ \text{kN}) \times (200\ \ \text{mm}) \times \left[\frac{(35\ \ \text{mm})}{2}\right]}{\frac{\pi \times (35\ \ \text{mm})^4}{64}} = 99.78\ \ \text{MPa}$$

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



$$\begin{cases} \dfrac{r}{d} = \dfrac{3\ \text{mm}}{35\ \text{mm}} = 0.76 \\ \dfrac{D}{d} = \dfrac{50\ \text{mm}}{35\ \text{mm}} = 1.43 \end{cases} \Rightarrow K_t = 1.7$$

Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
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		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
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		CD	630 (91)	530 (77)	12	35	179
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G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

$$S_{ut} = 470 \ \text{MPa} = 68 \ \text{kpsi}$$

$$S_y = 390 \ \text{MPa}$$

$$\sqrt{a} = 0.246 - 3.08\times10^{-3}\times S_{ut} + 1.51\times10^{-5}\times S_{ut}^2 - 2.67\times10^{-8}\times S_{ut}^3$$
$$= 0.246 - 3.08\times10^{-3}\times(68 \ \text{kpsi}) + 1.51\times10^{-5}\times(68 \ \text{kpsi})^2 - 2.67\times10^{-8}\times(68 \ \text{kpsi})^3 = 0.09771$$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.09771}{\sqrt{0.118}}} = 0.778$$

$$K_f = 1 + q(K_t - 1) = 1 + (0.778)\times(1.7-1) = 1.54$$

$$S_e' = 0.5S_{ut} = 0.5\times(470 \ \text{MPa}) = 235 \ \text{MPa}$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

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And from the question and Table 6-2, I can know that

$$k_a = aS_{ut}^b = 4.51 \times (470\ \ \mathrm{MPa})^{-0.265} = 0.883$$

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{35\ \ \mathrm{mm}}{7.62}\right)^{-0.107} = 0.849$$

$$k_c = 1$$

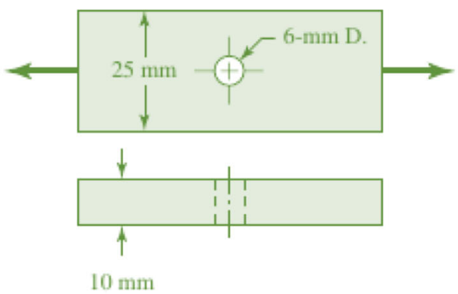
Therefore,

$$S_e = k_a k_b k_c S_e' = (0.883) \times (0.849) \times (1) \times (235\ \ \mathrm{MPa}) = 176.32\ \ \mathrm{MPa}$$

$$n_f = \frac{S_e}{K_f \sigma_{max}} = \frac{(176.32\ \ \mathrm{MPa})}{(1.54) \times (99.78\ \ \mathrm{MPa})} = 1.14$$

Problem 3

The cold-drawn AISI 1040 steel bar shown in the figure is subjected to a completely reversed axial load fluctuating between 28 kN in compression to 28 kN in tension. Estimate the fatigue factor of safety based on achieving infinite life, and the yielding factor of safety. If infinite life is not predicted, estimate the number of cycles to failure.



Solution:

For this question, we are asked to estimate the fatigue factor of safety based on achieving infinite life, and the yielding factor of safety. If infinite life is not predicted, estimate the number of cycles to failure.

Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

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G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

$$S_{ut} = 590\ \text{MPa} = 85\ \text{kpsi}$$

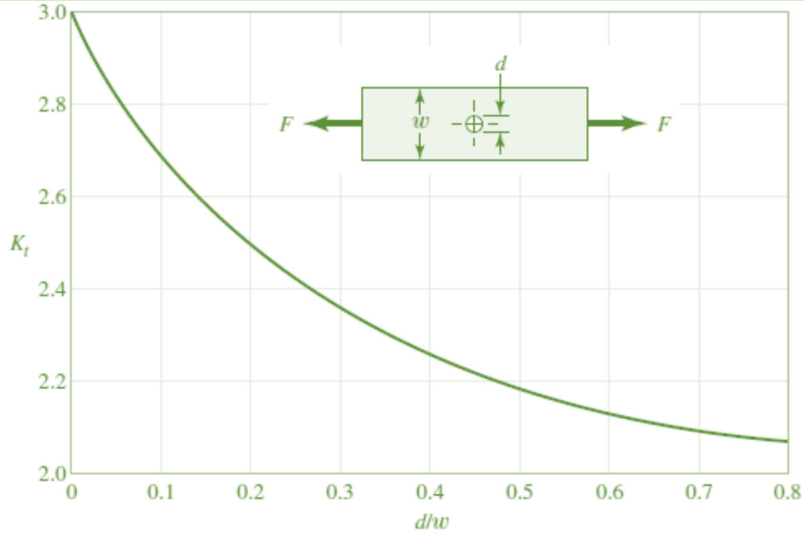
$$S_y = 490\ \text{MPa}$$

$$\sigma_{max} = \frac{F_{max}}{A} = \frac{(28\ \text{kN})}{(10\ \text{mm}) \times [(25\ \text{mm}) - (6\ \text{mm})]} = 147.37\ \text{MPa}$$

$$n_y = \frac{S_y}{\sigma_{max}} = 3.32$$

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.



$$\frac{d}{w} = \frac{6\ \text{mm}}{25\ \text{mm}} = 0.24$$

$$\Rightarrow K_t = 2.44$$

$$S_e' = 0.5S_{ut} = 0.5 \times (590\ \text{MPa}) = 295\ \text{MPa}$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
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And from the question and Table 6-2, I can know that

$$k_a = aS_{ut}^b = 4.51 \times (590\ \text{MPa})^{-0.265} = 0.832$$

$$k_b = 1$$

$$k_c = 0.85$$

Therefore,

$$S_e = k_a k_b k_c S_e' = (0.832) \times (1) \times (0.85) \times (295\ \text{MPa}) = 208.5\ \text{MPa}$$

Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*

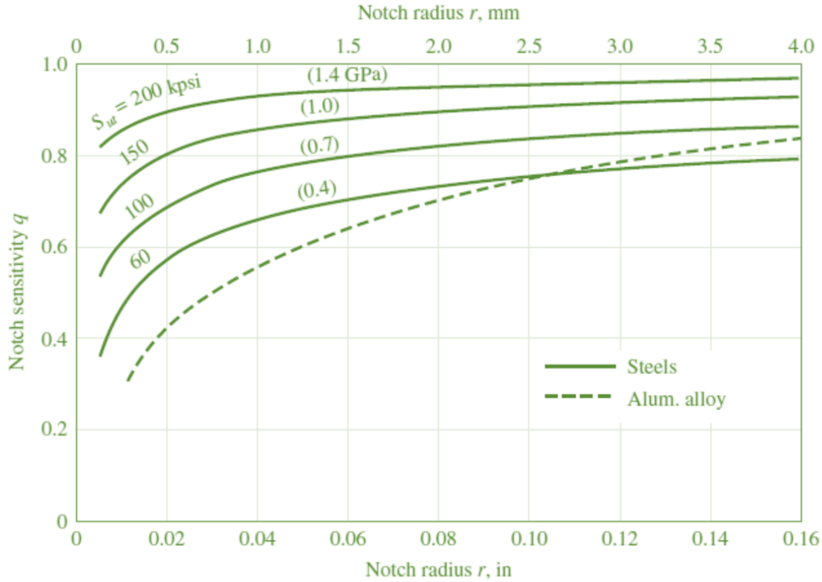


Figure 6-20:

$$q = 0.83$$

$$K_f = 1 + q(K_t - 1) = 1 + (0.83) \times (2.44 - 1) = 2.1952$$

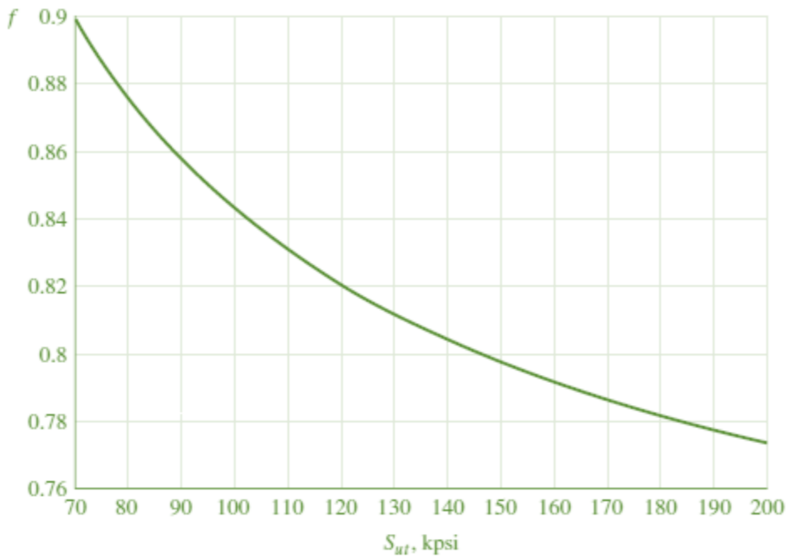
$$\sigma_a = K_f \left| \frac{F_{max} - F_{min}}{2A} \right| = (2.1952) \times \left| \frac{(28\ \ \text{kN}) - (-28\ \ \text{kN})}{2 \times (10\ \ \text{mm}) \times [(25\ \ \text{mm}) - (6\ \ \text{mm})]} \right|$$
$$= 323.5\ \ \text{MPa}$$

$$\sigma_m = K_f \left| \frac{F_{max} + F_{min}}{2A} \right| = (2.1952) \times \left| \frac{(28\ \ \text{kN}) + (-28\ \ \text{kN})}{2 \times (10\ \ \text{mm}) \times [(25\ \ \text{mm}) - (6\ \ \text{mm})]} \right| = 0\ \ \text{MPa}$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \Rightarrow n_f = \frac{S_e}{\sigma_a} = 0.64$$

Figure 6–18

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S'_e = 0.5S_{ut}$ at 10^6 cycles.



$$f = 0.87$$

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{[(0.87)\times(590\ \ \mathrm{MPa})]^2}{(208.5\ \ \mathrm{MPa})} = 1263.6\ \ \mathrm{MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{(0.87)\times(590\ \ \mathrm{MPa})}{(208.5\ \ \mathrm{MPa})}\right] = -0.1304$$

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{\frac{1}{b}} = \left[\frac{(323.5\ \ \mathrm{MPa})}{(1263.6\ \ \mathrm{MPa})}\right]^{\frac{1}{-0.1304}} = 34000$$

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