

香港中文大學

The Chinese University of Hong Kong

**THE CHINESE UNIVERSITY OF HONG KONG**

**DEPARTMENT OF MECHANICAL & AUTOMATION ENGINEERING**

# **MAEG5070 Nonlinear Control Systems**

## **Assignment #9**

by

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## Problem 1

Consider the following system

$$\begin{aligned}\dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= x_3 + \cos(x_1)u \\ \dot{x}_3 &= x_1 + x_2^2 + \lambda x_3 \\ y &= x_1\end{aligned}\tag{1}$$

- (a) For what values of  $\lambda$  is the system minimum phase? nonminimum phase?
- (b) Assume a state feedback control law  $u = k(x)$  is such that  $\ddot{y}(t) + 4\dot{y}(t) + 2y(t) = 0$ . Is the equilibrium point at the origin of the closed-loop system (locally) asymptotically stable for all  $\lambda \in \mathbb{R}$ ? Why or Why not?

**Solution:**

- (a) The Jacobian linearization of the system at the origin is given by

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}\tag{2}$$

with

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & \lambda \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\tag{3}$$

The transfer function of the above linear system is then given by

$$\begin{aligned}H(s) &= C(sI - A)^{-1}B \\ &= \frac{1}{s(s-1)(s-\lambda)+1} C \begin{bmatrix} s(s-\lambda) & s-\lambda & 1 \\ 1 & (s-1)(s-\lambda) & s-1 \\ s & 1 & s(s-1) \end{bmatrix} B \\ &= \frac{s-\lambda}{s(s-1)(s-\lambda)+1}\end{aligned}\tag{4}$$

The system has a zero at  $s = \lambda$ . Thus, it is minimum phase for all  $\lambda < 0$  and it is nonminimum phase for all  $\lambda \geq 0$ .

- (b) No. Since when  $\lambda \geq 0$ , the system is nonminimum phase at the origin, the equilibrium cannot be made locally asymptotically stable.

## Problem 2

The motion equation of a single-link robot rigid-joint manipulator is given by

$$\ddot{y} + a\dot{y} \sin(y) = \beta(x) u \quad (5)$$

- (a) Give the state space equation of Equation (5) with  $x_1 = y$  and  $x_2 = \dot{y}$ .
- (b) Assume  $1 \leq a \leq 2$  and  $0.5 < \beta(x) < 2.5$ , using  $\hat{a} = 1.5$  to design a sliding mode control law  $u$  such that

$$\frac{ds^2}{dt} \leq -|s| \quad (6)$$

where  $s = \dot{e} + 2e$  with  $e = y - y_d$ .

- (c) Assume  $y_d$  is a unit step function, simulate your design on the closed-loop system consisting of the plant Equation (5) with  $a = 2$  and the sliding mode control law with  $\hat{a} = 1.5$ . Illustrate the performance of your control law by plotting  $y(t)$  and  $y_d(t)$  in the same figure for  $0 \leq t \leq 20$ . Also, plot  $u(t)$  for  $0 \leq t \leq 20$ .
- (d) In the control law designed in part (b), replace  $\text{sgn}(s)$  by  $\text{sat}(s/0.2)$ , and repeat part (c).

Hint: In simulation, you can let  $F(x) = |\alpha(x) - \hat{\alpha}(x)|$  and  $\beta(x) = 1$ .

### Solution:

- (a) Let  $x_1 = y$  and  $x_2 = \dot{y}$ , the state-space equation is

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -ax_2 \sin x_1 + \beta(x) u \\ y = x_1 \end{cases} \quad (7)$$

- (b) Define the estimate  $\hat{\beta}$  of  $\beta(x)$  by  $\hat{\beta} = (b_{\min} b_{\max})^{1/2} = \frac{\sqrt{5}}{2}$  and let  $b = (b_{\max}/b_{\min})^{1/2} = \sqrt{5}$ .

Using  $\hat{a} = 1.5$  yields that

$$\hat{\alpha} = -1.5x_2 \sin x_1 \quad (8)$$

$$\Delta\alpha = \alpha(x) - \hat{\alpha} \implies |\Delta\alpha| = |\alpha(x) - \hat{\alpha}| \leq 0.5 |x_2 \sin x_1| \implies F(x) = 0.5 |x_2 \sin x_1| \quad (9)$$

Therefore,

$$\hat{u} = -\hat{\alpha}(x) + \ddot{y}_d - \alpha_1 \dot{e} \quad (10)$$

Because  $s = \dot{e} + 2e$ ,  $\alpha_1 = 2$ . Therefore,

$$\hat{u} = 1.5x_2 \sin x_1 + \ddot{y}_d - 2\dot{e} \quad (11)$$

Because the sliding mode control law  $u$  should satisfy

$$\frac{ds^2}{dt} \leq -|s| \quad (12)$$

we can know that  $\eta = \frac{1}{2}$ . Therefore, we can let  $\phi(x)$  be

$$\begin{aligned}
 \phi(x) &= b(F(x) + \eta) + (b-1)|\hat{u}| \\
 &= \sqrt{5} \left( 0.5|x_2 \sin x_1| + \frac{1}{2} \right) + (\sqrt{5}-1)|1.5x_2 \sin x_1 + \ddot{y}_d - 2\dot{e}| \\
 &= \sqrt{5} \left( 0.5|x_2 \sin x_1| + \frac{1}{2} \right) + (\sqrt{5}-1)|1.5x_2 \sin x_1 + \ddot{y}_d - 2(\dot{y} - \dot{y}_d)| \\
 &= \sqrt{5} \left( 0.5|x_2 \sin x_1| + \frac{1}{2} \right) + (\sqrt{5}-1)|1.5x_2 \sin x_1 + \ddot{y}_d - 2(x_2 - \dot{y}_d)|
 \end{aligned} \tag{13}$$

Hence, we can design a sliding mode control law  $u$  as follows:

$$\begin{aligned}
 u &= \hat{\beta}^{-1} [\hat{u} - \phi(x) \operatorname{sgn}(s)] \\
 &= \frac{2}{\sqrt{5}} [1.5x_2 \sin x_1 + \ddot{y}_d - 2\dot{e} - \phi(x) \operatorname{sgn}(s)] \\
 &= \frac{2}{\sqrt{5}} [1.5x_2 \sin x_1 + \ddot{y}_d - 2(x_2 - \dot{y}_d) - \phi(x) \operatorname{sgn}(s)]
 \end{aligned} \tag{14}$$

where

$$\phi(x) = \sqrt{5} \left( 0.5|x_2 \sin x_1| + \frac{1}{2} \right) + (\sqrt{5}-1)|1.5x_2 \sin x_1 + \ddot{y}_d - 2(x_2 - \dot{y}_d)| \tag{15}$$

$$s = \dot{e} + 2e = (\dot{y} - \dot{y}_d) + 2(y - y_d) = (x_2 - \dot{y}_d) + 2(x_1 - y_d) \tag{16}$$

The MATLAB shown below is used to simulate the performance of the designed controller.

```

1  clc; clf; clear all;
2  %% sgn part
3  [t,x] = ode45('Q9_2_Systemsgn',[0,20],[0 0]);
4  phi = 0.5+0.5*abs(x(:,2)).*sin(x(:,1));
5  y_d = 1;
6  y_ddot = 0;
7  y_dddots = 0;
8  y = x(:,1);
9  ydot = x(:,2);
10 s = ydot-y_ddot+2*(y-y_d);
11 u = -phi.*sgn(s)+1.5*x(:,2).*sin(x(:,1))+y_dddots-2*(ydot-y_ddot);
12 figure(1);
13 hold on;
14 plot(t, y,'color',[0.667 0.667 1],'LineWidth',2.5);
15 plot(t, y_d+t*0,'color',[1 0.5 0],'LineWidth',2.5);
16 hold off;
17 grid on;
18 legend(' $y\left(t\right)$ ',' $y_d\left(t\right)$ ','interpreter','latex');
19 xlabel(' $t$, \ (\mathrm{s})$ ','interpreter','latex');
20 ylabel(' $ \theta$, \ (\mathrm{rad})$ ','interpreter','latex');
21 a = get(gca,'XTickLabel');
22 set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);

```

```

23 set(gca,'position',[0.15 0.20 0.6 0.6]);
24 set(gcf,'position',[100 100 800 600]);
25 set(gcf,'renderer','painters');
26 filename = "Q9-2-yyd-sgn"+"pdf";
27 saveas(gcf,filename);
28 figure(2);
29 plot(t, u,'color',[0.667 0.667 1],'LineWidth',2.5);
30 grid on;
31 xlabel('$t, \ (\mathrm{s})$', 'interpreter','latex');
32 ylabel('$u, \ (\mathrm{N\dot{m}})$', 'interpreter','latex');
33 a = get(gca,'XTickLabel');
34 set(gca,'XTickLabel',a,'FontName','Times','fontSize',12);
35 set(gca,'position',[0.15 0.20 0.6 0.6]);
36 set(gcf,'position',[100 100 800 600]);
37 set(gcf,'renderer','painters');
38 filename = "Q9-2-u-sgn"+"pdf";
39 saveas(gcf,filename);
40 %% sat part
41 [t,x] = ode45('Q9_2_Systemsat',[0,20],[0 0]);
42 phi = 0.5+0.5*abs(x(:,2)).*sin(x(:,1)));
43 y_d = 1;
44 y_ddot = 0;
45 y_ddd = 0;
46 y = x(:,1);
47 ydot = x(:,2);
48 s = ydot-y_ddot+2*(y-y_d);
49 u = -phi.*sgn(s)+1.5*x(:,2).*sin(x(:,1))+y_ddd-2*(ydot-y_ddot);
50 figure(3);
51 hold on;
52 plot(t, y,'color',[0.667 0.667 1],'LineWidth',2.5);
53 plot(t, y_d+t*0,'color',[1 0.5 0],'LineWidth',2.5);
54 hold off;
55 grid on;
56 legend('$y\left(t\right)','$y_d\left(t\right)', 'interpreter','latex');
57 xlabel('$t, \ (\mathrm{s})$', 'interpreter','latex');
58 ylabel('$\theta, \ (\mathrm{rad})$', 'interpreter','latex');
59 a = get(gca,'XTickLabel');
60 set(gca,'XTickLabel',a,'FontName','Times','fontSize',12);
61 set(gca,'position',[0.15 0.20 0.6 0.6]);
62 set(gcf,'position',[100 100 800 600]);
63 set(gcf,'renderer','painters');
64 filename = "Q9-2-yyd-sat"+"pdf";
65 saveas(gcf,filename);
66 figure(4);
67 plot(t, u,'color',[0.667 0.667 1],'LineWidth',2.5);
68 grid on;

```

---

```

69 xlabel('$t, \ (\mathrm{s})$', 'interpreter', 'latex');
70 ylabel('$u, \ (\mathrm{N\cdot m})$', 'interpreter', 'latex');
71 a = get(gca, 'XTickLabel');
72 set(gca, 'XTickLabel', a, 'FontName', 'Times', 'fontsize', 12);
73 set(gca, 'position', [0.15 0.20 0.6 0.6]);
74 set(gcf, 'position', [100 100 800 600]);
75 set(gcf, 'renderer', 'painters');
76 filename = "Q9-2-u-sat"+"pdf";
77 saveas(gcf, filename);

```

---

where the codes for the system representation are shown below:

---

```

1 function xd = Q9_2_Systemsgn(t,x)
2     xd(1) = x(2);
3     phi = 0.5+0.5*abs(x(2)*sin(x(1)));
4     y_d = 1;
5     y_ddot = 0;
6     y_dddots = 0;
7     y = x(1);
8     ydot = x(2);
9     s = ydot-y_ddot+2*(y-y_d);
10    u = -phi*sgn(s)+1.5*x(2)*sin(x(1))+y_dddots-2*(ydot-y_ddot);
11    xd(2) = -2*x(2)*sin(x(1))+u;
12    xd = xd';
13 end

```

---

and the sgn function is designed as follows:

---

```

1 function y = sgn(s)
2     if s == 0
3         y = 0;
4     else
5         y = s./abs(s);
6     end
7 end

```

---

The simulation results are shown in Figure 1.

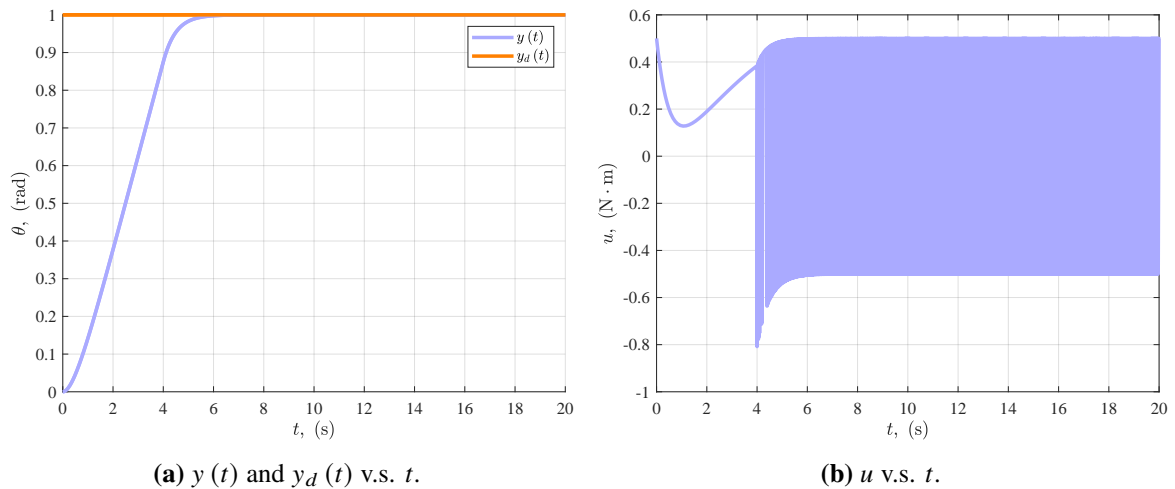
(c) The codes for the controller are changed for sat function as shown below:

---

```

1 function xd = Q9_2_Systemsat(t,x)
2     xd(1) = x(2);
3     phi = 0.5+0.5*abs(x(2)*sin(x(1)));
4     y_d = 0*t+1;
5     y_ddot = 0;
6     y_dddots = 0;
7     y = x(1);
8     ydot = x(2);

```



**Figure 1:** Simulation results for the controller with sgn.

```

9      s = ydot-y_ddot+2*(y-y_d);
10     u = -phi*sat(s)+1.5*x(2)*sin(x(1))+y_ddd-2*(ydot-y_ddot);
11     xd(2) = -2*x(2)*sin(x(1))+u;
12     xd = xd';
13 end

```

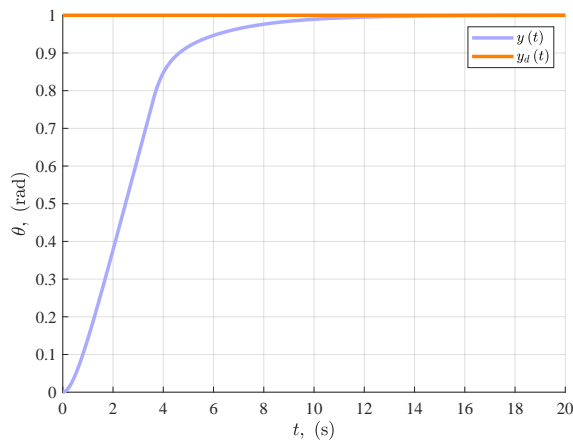
and the sat function is designed as follows:

```

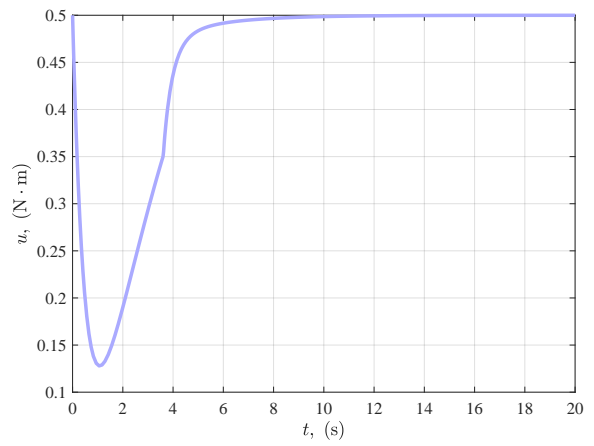
1 function y = sat(s)
2     if abs(s) < 0.2
3         y = s;
4     elseif s < 0
5         y = -1;
6     else
7         y = 1;
8     end
9 end

```

The simulation results are shown in Figure 2.



(a)  $y(t)$  and  $y_d(t)$  v.s.  $t$ .



(b)  $u$  v.s.  $t$ .

**Figure 2:** Simulation results for the controller with sat.



### Problem 3

Consider the motion equation of a single-link robot rigid-joint manipulator given in Equation (5) where  $\beta(x) = 1$  for all  $x$ .

- Assume  $a = 1.5$ ,  $y_d = 2 \sin t$  and  $s = \dot{e} + 3e$ . Design a control law of the form (8.7) of the lecture note with  $k = 2$  and simulate the performance of your control law by plotting  $y(t)$  and  $y_d(t)$  in the same figure for  $0 \leq t \leq 20$ . Also, plot  $u(t)$  for  $0 \leq t \leq 20$ .
- Assume the actual value of  $a = 2$ . Use the same control law as the one in Part (i) to simulate the performance of your control law by plotting  $y(t)$  and  $y_d(t)$  in the same figure for  $0 \leq t \leq 20$ . Also, plot  $u(t)$  for  $0 \leq t \leq 20$ .
- Assume  $a$  is unknown, put the system in the form (8.3) of the lecture note and identify  $a_0, a_1, a_2$  and  $f_1, f_2$ .
- Design an adaptive control law of the form (8.10) and (8.12) of the lecture note with  $\gamma_i = 3$ . Assume the actual value of  $a = 2.5$ , respectively. Simulate the performance of your control law by plotting  $y(t)$  and  $y_d(t)$  in the same figure for  $0 \leq t \leq 20$ . Also, plot  $u(t)$  and  $\hat{a}_i(t)$  for  $0 \leq t \leq 20$ .

Hint: Note Part (iv) of Remark 8.1.

#### Solution:

- Because  $\beta(x) = 1$ ,  $a_0 = 1$ . Consider the control law,

$$u = a_0 f_0(x, t) - ks + \sum_{i=1}^m a_i f_i(x, t) \quad (17)$$

where  $k = 2$ , and

$$f_0(x, t) = \ddot{y}_d - \alpha_1 \dot{e} \quad (18)$$

Because  $s = \dot{e} + 3e$ ,  $\alpha_1 = 3$ . Therefore, the designed control law for the system is as follows:

$$\begin{aligned} u &= \ddot{y}_d - 3\dot{e} - 2s + 1.5\dot{y} \sin(y) \\ &= \ddot{y}_d - 3(x_2 - \dot{y}_d) - 2((x_2 - \dot{y}_d) + 3(x_1 - y_d)) + 1.5x_2 \sin x_1 \end{aligned} \quad (19)$$

The MATLAB shown below is used to simulate the performance of the designed controller.

```
1  clc; clf; clear all;
2  %% Q9-3-a
3  [t,x] = ode45('Q9_3_a_System',[0,20],[0 0]);
4  y_d = 2*sin(t);
5  y_ddot = 2*cos(t);
6  y_dddott = -2*sin(t);
```

```

7  y = x(:,1);
8  ydot = x(:,2);
9  s = ydot-y_ddot+3*(y-y_d);
10 a0 = 1;
11 f0 = y_ddd-3*(ydot-y_ddot);
12 u = a0*f0-2*s+1.5*x(:,2).*sin(x(:,1));
13 figure(1);
14 hold on;
15 plot(t, y,'color',[0.667 0.667 1],'LineWidth',2.5);
16 plot(t, y_d+t*0,'color',[1 0.5 0],'LineWidth',2.5);
17 hold off;
18 grid on;
19 legend('$y\left(t\right)$','$y_d\left(t\right)$','interpreter','latex');
20 xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
21 ylabel('$\theta, \ (\mathrm{rad})$','interpreter','latex');
22 a = get(gca,'XTickLabel');
23 set(gca,'XTickLabel',a,'FontName','Times','fontSize',12);
24 set(gca,'position',[0.15 0.20 0.6 0.6]);
25 set(gcf,'position',[100 100 800 600]);
26 set(gcf,'renderer','painters');
27 filename = "Q9-3-a-yyd"+"pdf";
28 saveas(gcf,filename);
29 figure(2);
30 plot(t, u,'color',[0.667 0.667 1],'LineWidth',2.5);
31 grid on;
32 xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
33 ylabel('$u, \ (\mathrm{N\cdotp m})$','interpreter','latex');
34 a = get(gca,'XTickLabel');
35 set(gca,'XTickLabel',a,'FontName','Times','fontSize',12);
36 set(gca,'position',[0.15 0.20 0.6 0.6]);
37 set(gcf,'position',[100 100 800 600]);
38 set(gcf,'renderer','painters');
39 filename = "Q9-3-a-u"+"pdf";
40 saveas(gcf,filename);
41 %% Q9-3-b
42 [t,x] = ode45('Q9_3_b_System',[0,20],[0 0]);
43 y_d = 2*sin(t);
44 y_ddot = 2*cos(t);
45 y_ddd = -2*sin(t);
46 y = x(:,1);
47 ydot = x(:,2);
48 s = ydot-y_ddot+3*(y-y_d);
49 a0 = 1;
50 f0 = y_ddd-3*(ydot-y_ddot);
51 u = a0*f0-2*s+1.5*x(:,2).*sin(x(:,1));
52 figure(3);

```

---

```

53 hold on;
54 plot(t, y, 'color', [0.667 0.667 1], 'LineWidth', 2.5);
55 plot(t, y_d+t*0, 'color', [1 0.5 0], 'LineWidth', 2.5);
56 hold off;
57 grid on;
58 legend('$y\left(t\right)$', '$y_d\left(t\right)$', 'interpreter', 'latex');
59 xlabel('$t, \ (\mathrm{s})$', 'interpreter', 'latex');
60 ylabel('$\theta, \ (\mathrm{rad})$', 'interpreter', 'latex');
61 a = get(gca, 'XTickLabel');
62 set(gca, 'XTickLabel', a, 'FontName', 'Times', 'fontsize', 12);
63 set(gca, 'position', [0.15 0.20 0.6 0.6]);
64 set(gcf, 'position', [100 100 800 600]);
65 set(gcf, 'renderer', 'painters');
66 filename = "Q9-3-b-yyd"+" .pdf";
67 saveas(gcf, filename);
68 figure(4);
69 plot(t, u, 'color', [0.667 0.667 1], 'LineWidth', 2.5);
70 grid on;
71 xlabel('$t, \ (\mathrm{s})$', 'interpreter', 'latex');
72 ylabel('$u, \ (\mathrm{N\cdot m})$', 'interpreter', 'latex');
73 a = get(gca, 'XTickLabel');
74 set(gca, 'XTickLabel', a, 'FontName', 'Times', 'fontsize', 12);
75 set(gca, 'position', [0.15 0.20 0.6 0.6]);
76 set(gcf, 'position', [100 100 800 600]);
77 set(gcf, 'renderer', 'painters');
78 filename = "Q9-3-b-u"+" .pdf";
79 saveas(gcf, filename);

```

---

where the codes for the system representation are shown below:

---

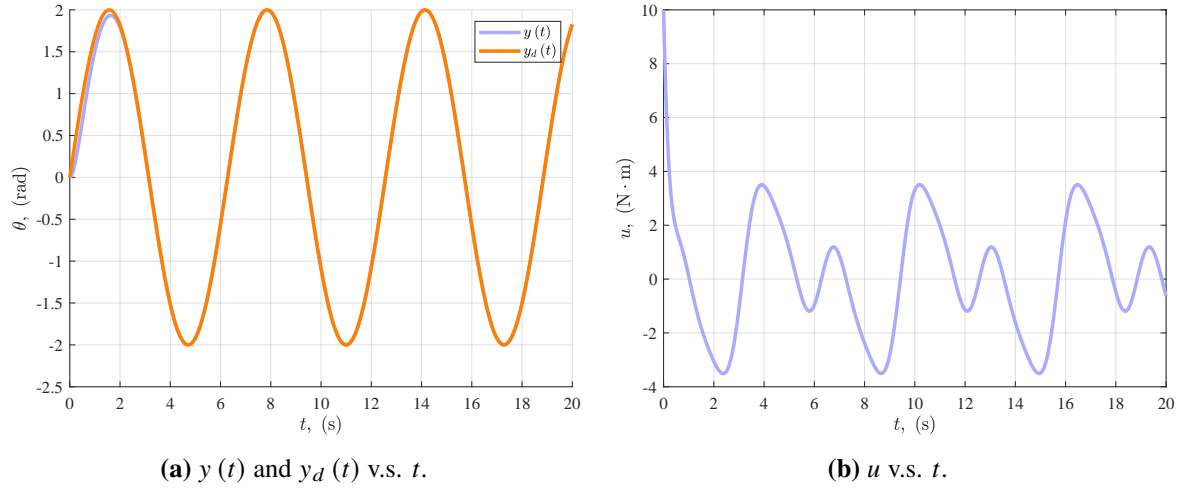
```

1 function xd = Q9_3_a_System(t,x)
2     xd(1) = x(2);
3     % phi = 0.5+0.5*abs(x(2)*sin(x(1)));
4     y_d = 2*sin(t);
5     y_ddot = 2*cos(t);
6     y_ddd = -2*sin(t);
7     y = x(1);
8     ydot = x(2);
9     s = ydot-y_ddot+3*(y-y_d);
10    a0 = 1;
11    f0 = y_ddd-3*(ydot-y_ddot);
12    u = a0*f0-2*s+1.5*x(2)*sin(x(1));
13    xd(2) = -1.5*x(2)*sin(x(1))+u;
14    xd = xd';
15 end

```

---

The simulation results are shown in Figure 3.



**Figure 3:** Simulation results for the controller with for the adaptive control with  $k = 2$ .

(b) The codes for the system representation are changed as shown below:

---

```

1 function xd = Q9_3_b_System(t,x)
2     xd(1) = x(2);
3     % phi = 0.5+0.5*abs(x(2)*sin(x(1)));
4     y_d = 2*sin(t);
5     y_ddot = 2*cos(t);
6     y_ddd = -2*sin(t);
7     y = x(1);
8     ydot = x(2);
9     s = ydot-y_ddot+3*(y-y_d);
10    a0 = 1;
11    f0 = y_ddd-3*(ydot-y_ddot);
12    u = a0*f0-2*s+1.5*x(2)*sin(x(1));
13    xd(2) = -2*x(2)*sin(x(1))+u;
14    xd = xd';
15 end

```

---

The simulation results are shown in Figure 4.

(c)

$$a_0 y^{(n)} + \sum_{i=1}^m a_i f_i(x, t) = u \quad (20)$$

Here,  $a_0 = 1$ ,  $a_1 = a$ ,  $a_2 = 0$ ,  $f_1(x) = x_2 \sin x_1$ ,  $f_2(x) = 0$

(d)

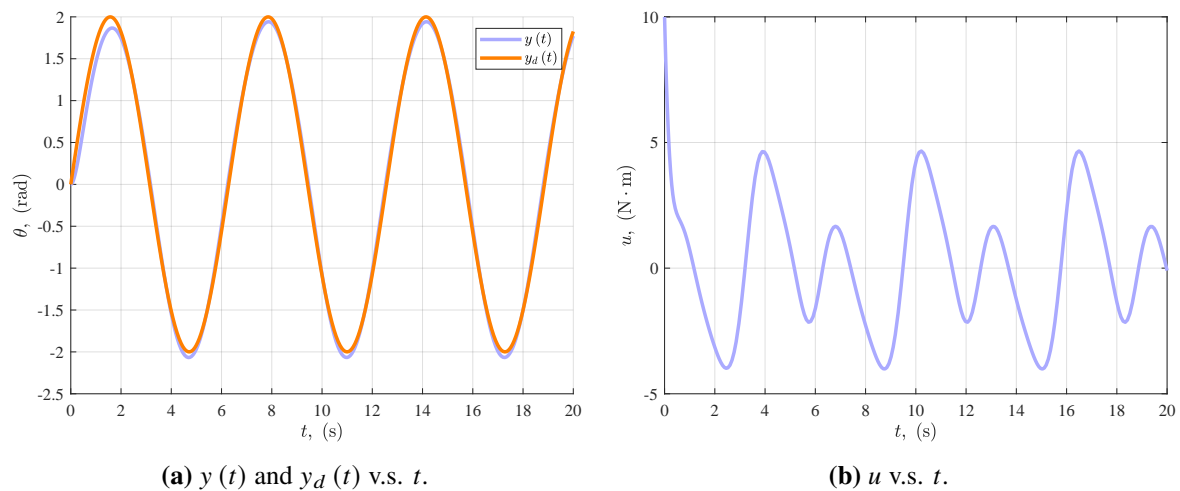
$$u = a_0 f_0(x, t) - ks + \sum_{i=1}^m \hat{a}_i f_i(x, t) \quad (21)$$

$$\dot{\hat{a}}_i = -\gamma_i \text{sgn}(a_0) s f_i, i = 1, \dots, m$$

Because  $\gamma_i = 3$  and  $a_0 = 1$ , Equation (21) can be simplified into

$$u = (\ddot{y}_d - 3(x_2 - \dot{y}_d)) - 2((x_2 - \dot{y}_d) + 3(x_1 - y_d)) + \hat{a}_1 x_2 \sin x_1 \quad (22)$$

$$\dot{\hat{a}}_1 = -3((x_2 - \dot{y}_d) + 3(x_1 - y_d)) x_2 \sin x_1$$



**Figure 4:** Simulation results for the controller with for the adaptive control with  $k = 2$  and the actual value of  $a = 2$ .

The Simulink as shown in Figure 5 is used to simulate the performance of the designed controller.

And we use the following code to plot the results:

```
1 clear all; clc;
2 figg1 = openfig('Q9-3-d-ahat.fig','reuse');
3 grid on;
4 xlabel('$t, \mathrm{\left(s\right)}$', 'interpreter','latex');
5 ylabel('$\hat{a}$', 'interpreter','latex');
6 % legend('$y_d$', '$y$', 'interpreter','latex','Location','southeast');
7 title('');
8 a = get(gca,'XTickLabel');
9 set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
10 set(gca,'position',[0.15 0.20 0.6 0.6]);
11 set(gcf,'position',[100 100 800 600]);
12 set(gcf,'renderer','painters');
13 filename = "Q9-3-d-ahat"+"pdf";
14 saveas(gcf,filename);
15 close(figg1);
16 figg2 = openfig('Q9-3-d-u.fig','reuse');
17 grid on;
18 xlim([0.07 20]);
19 ylim([-5 6]);
20 xlabel('$t, \mathrm{s})$', 'interpreter','latex');
21 ylabel('$u, \mathrm{N\cdot m})$', 'interpreter','latex');
22 % legend('$y_d$', '$y$', 'interpreter','latex','Location','southeast');
23 title('');
24 a = get(gca,'XTickLabel');
25 set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
26 set(gca,'position',[0.15 0.20 0.6 0.6]);
```

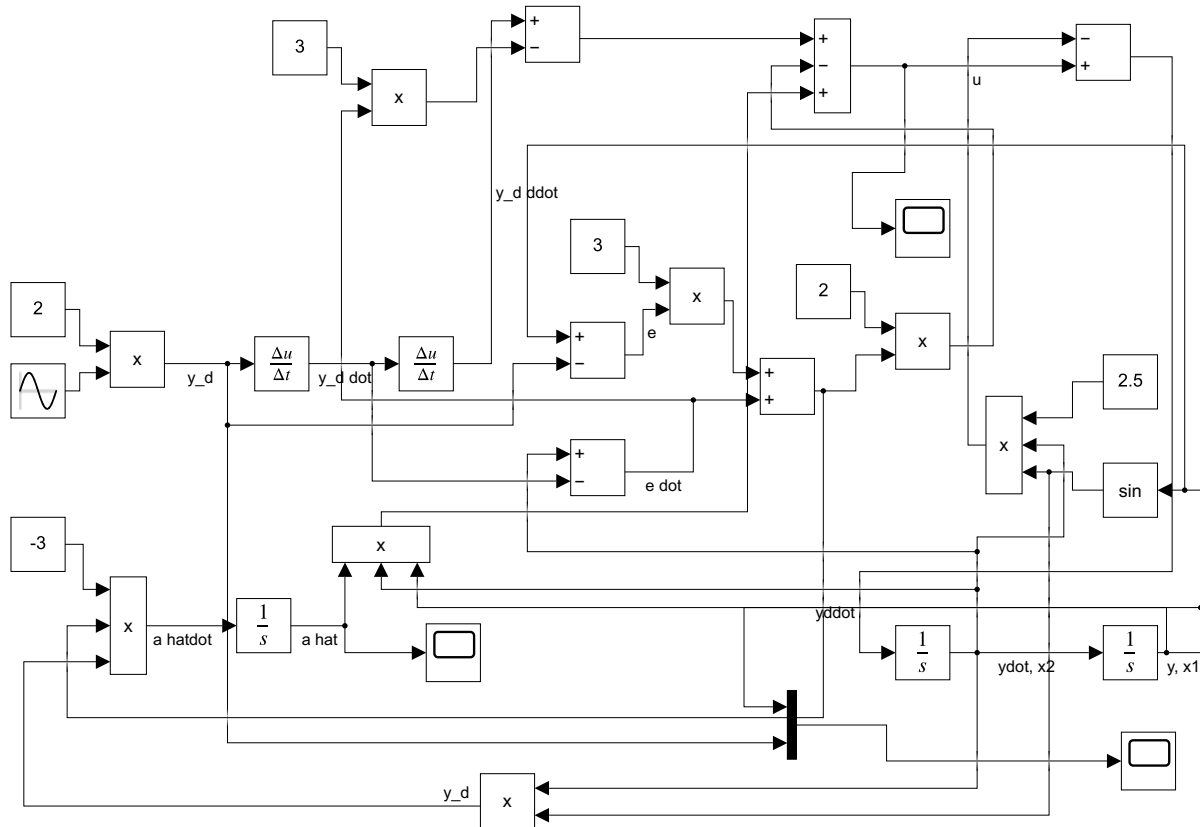


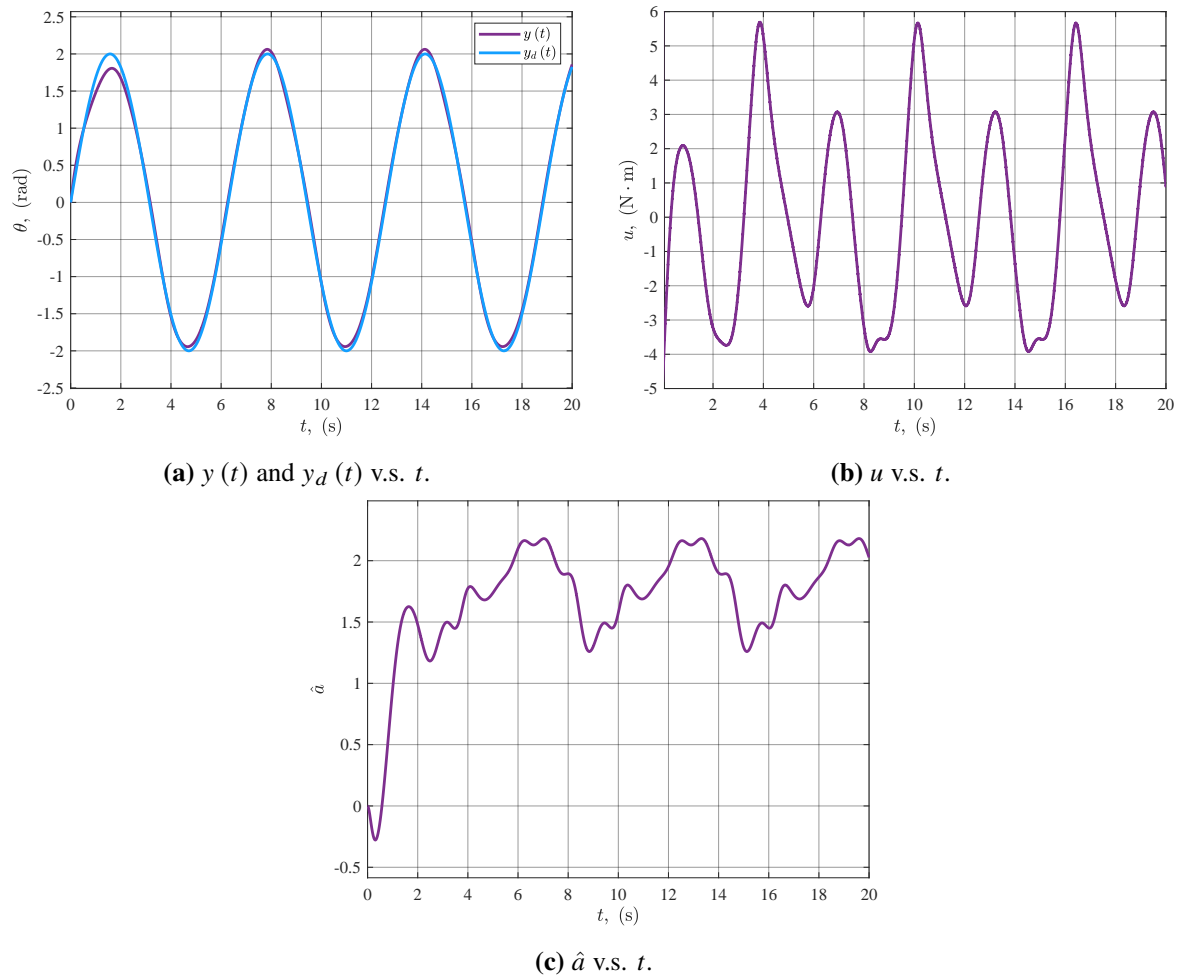
Figure 5: Block diagram for the system.

```

27 set(gcf,'position',[100 100 800 600]);
28 set(gcf,'renderer','painters');
29 filename = "Q9-3-d-u"+"pdf";
30 saveas(gcf,filename);
31 close(fig2);
32 figg3 = openfig('Q9-3-d-yyd.fig','reuse');
33 grid on;
34 legend('$y\left(t\right)$','$y_d\left(t\right)$','interpreter','latex');
35 xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
36 ylabel('$\theta, \ (\mathrm{rad})$','interpreter','latex');
37 title('');
38 a = get(gca,'XTickLabel');
39 set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
40 set(gca,'position',[0.15 0.20 0.6 0.6]);
41 set(gcf,'position',[100 100 800 600]);
42 set(gcf,'renderer','painters');
43 filename = "Q9-3-d-yyd"+"pdf";
44 saveas(gcf,filename);
45 close(figg3);

```

The simulation results are shown in Figure 6.



**Figure 6:** Simulation results for the controller with for the adaptive control law of the form (8.10) and (8.12) of the lecture note.