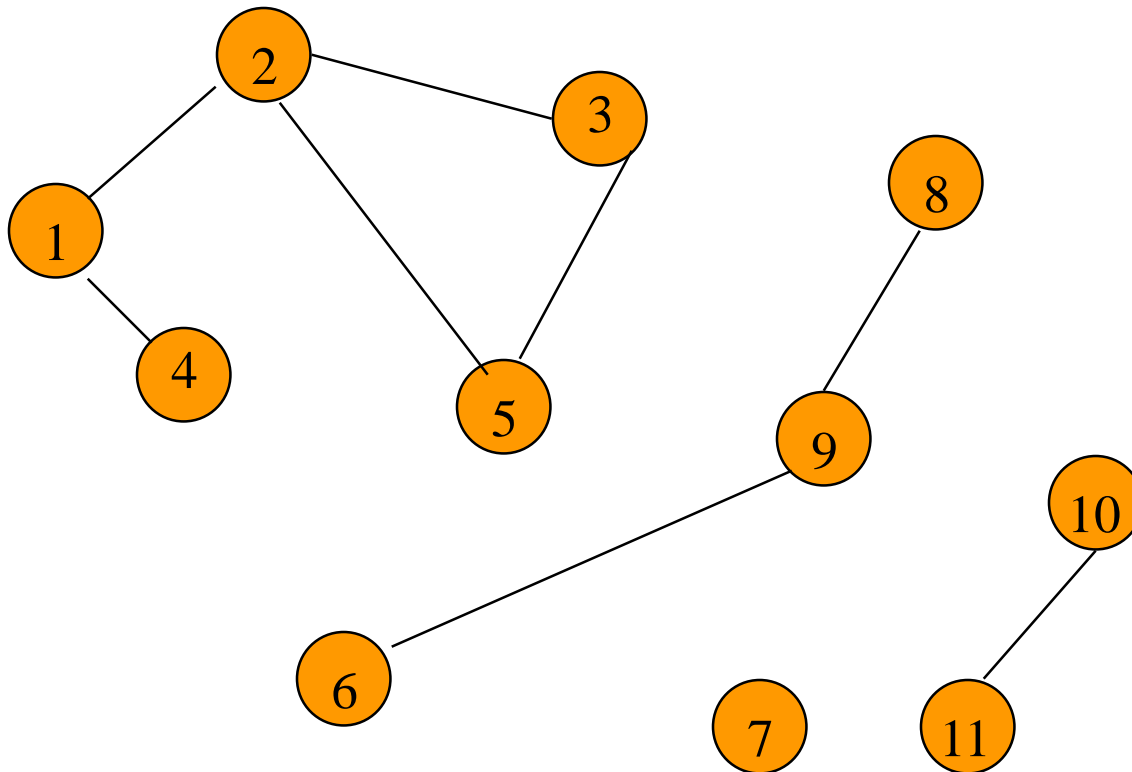


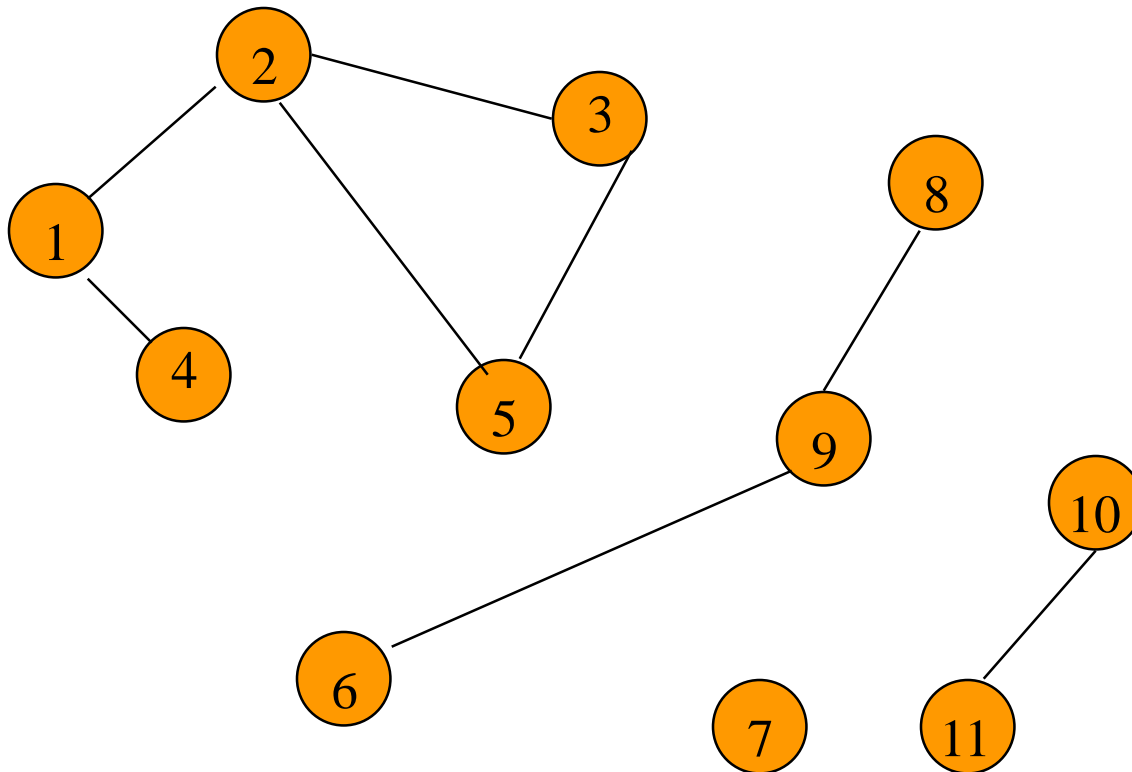
Graph Search Methods

- A vertex **u** is **reachable** from vertex **v** iff there is a path from **v** to **u**.



Graph Search Methods

- A search method starts at a given vertex **v** and visits/labels/marks every vertex that is reachable from **v**.



Graph Search Methods

- Many graph problems solved using a search method.
 - Path from one vertex to another.
 - Is the graph connected?
 - Find a spanning tree.
 - Etc.
- Commonly used search methods:
 - Depth-first search.
 - Breadth-first search.

Depth-First Search

DFS(**v**)

{

Label vertex **v** as reached.

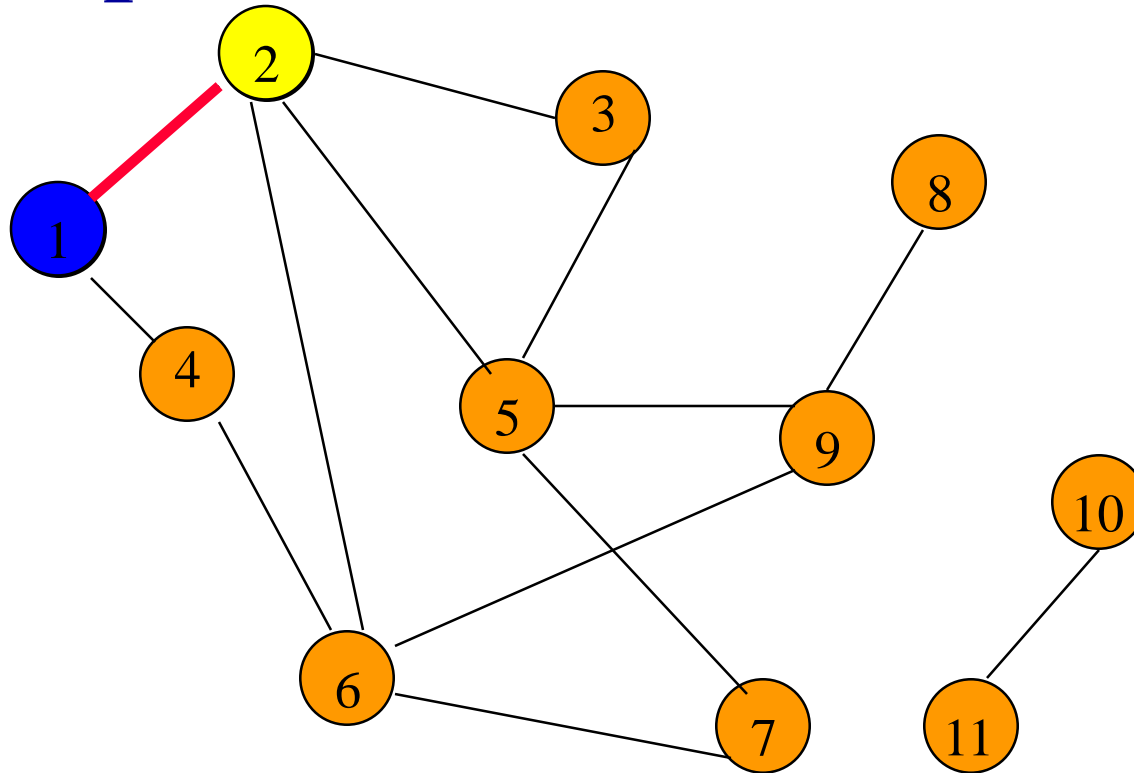
for (each unreached vertex **u**

adjacent from **v**)

DFS(**u**);

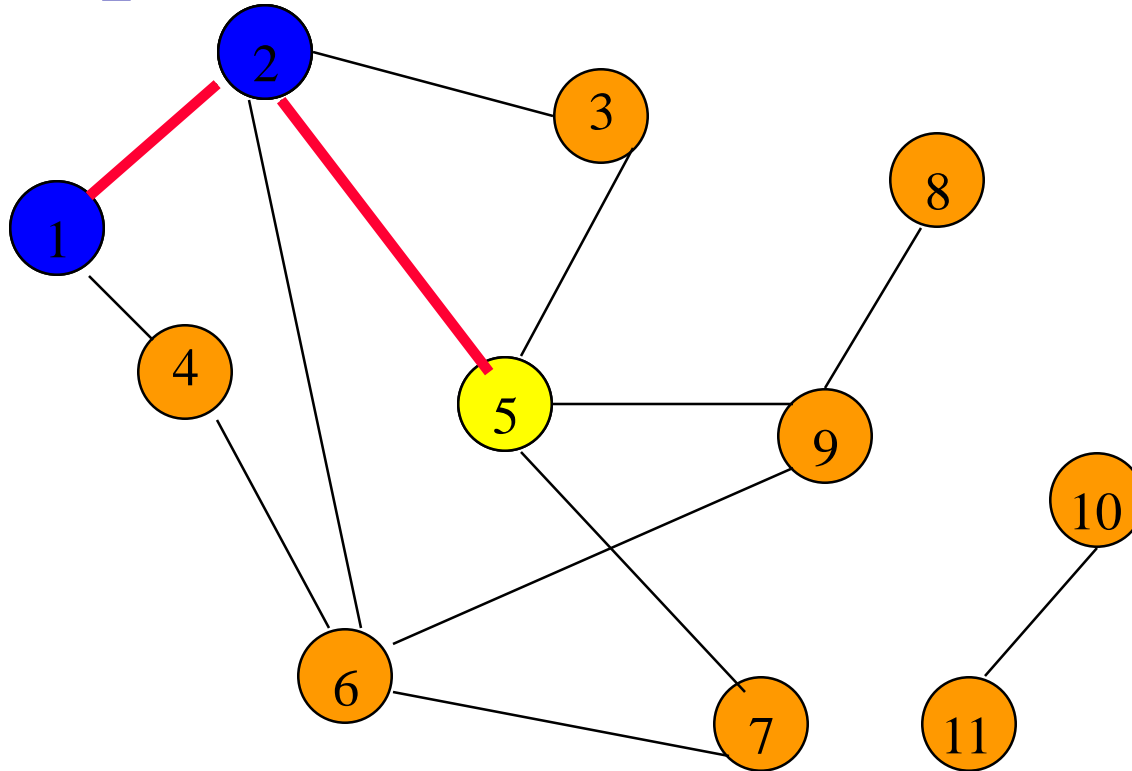
}

Depth-First Search Example



Start search at vertex **1**.
Label vertex **1** and do a depth first search
from either **2** or **4**.
Suppose that vertex **2** is selected.

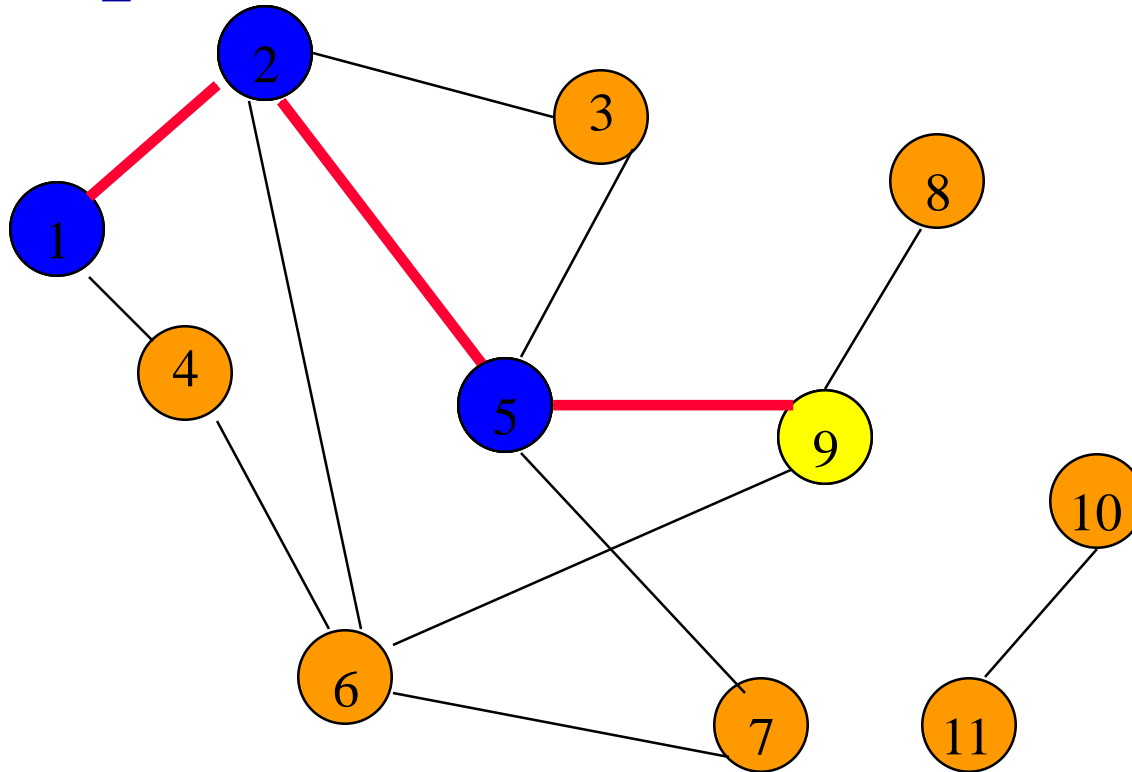
Depth-First Search Example



Label vertex **2** and do a depth first search from either **3**, **5**, or **6**.

Suppose that vertex **5** is selected.

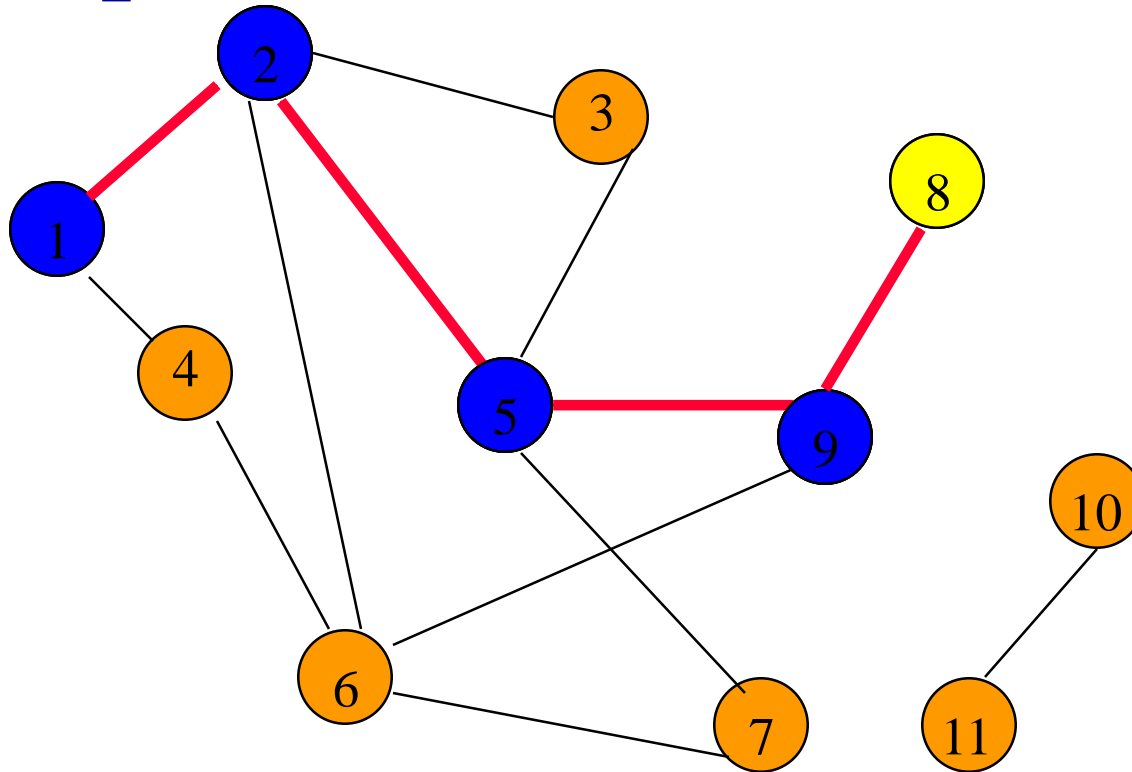
Depth-First Search Example



Label vertex **5** and do a depth first search from either **3**, **7**, or **9**.

Suppose that vertex **9** is selected.

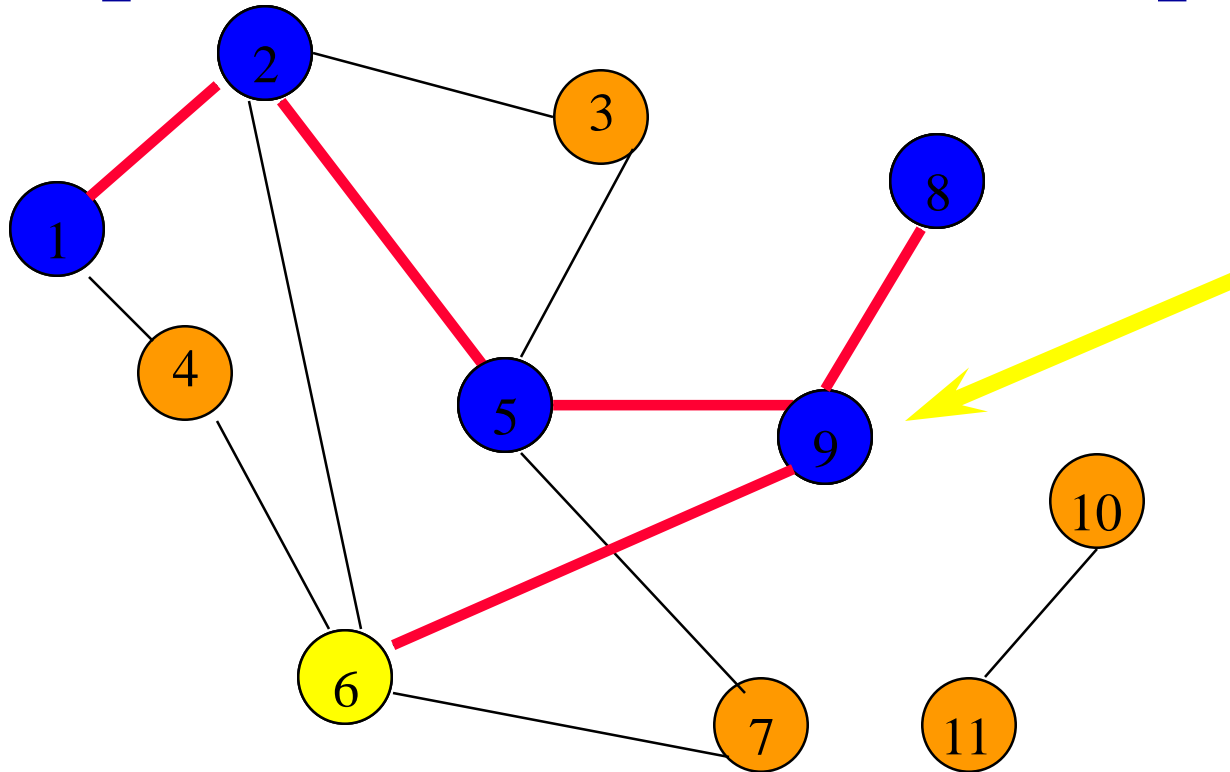
Depth-First Search Example



Label vertex 9 and do a depth first search from either 6 or 8.

Suppose that vertex 8 is selected.

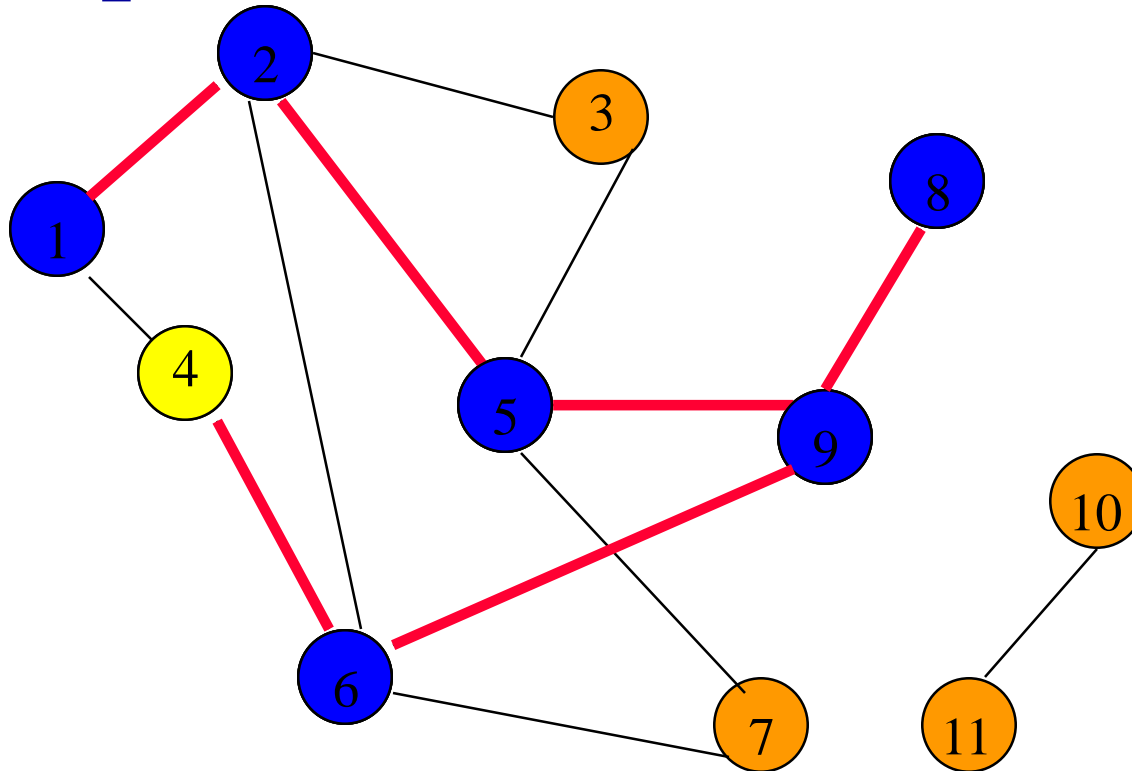
Depth-First Search Example



Label vertex **8** and return to vertex **9**.

From vertex **9** do a **DFS(6)**.

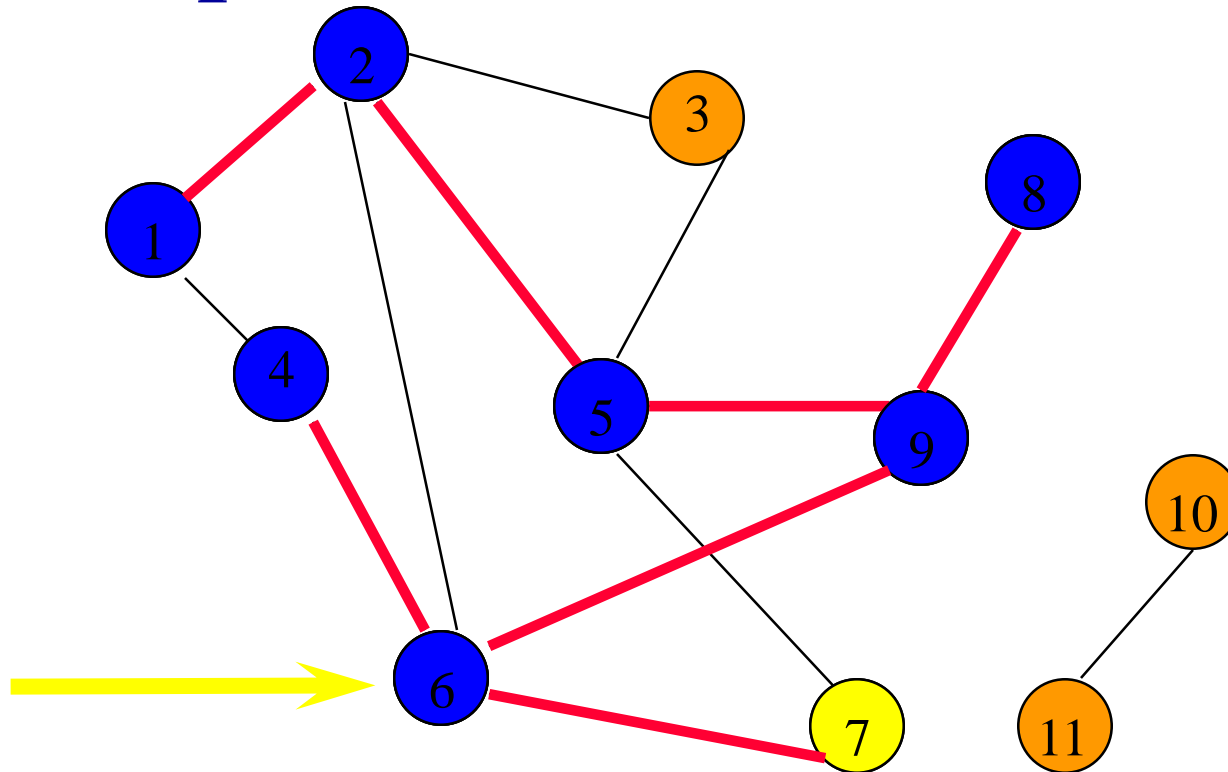
Depth-First Search Example



Label vertex **6** and do a depth first search from either **4** or **7**.

Suppose that vertex **4** is selected.

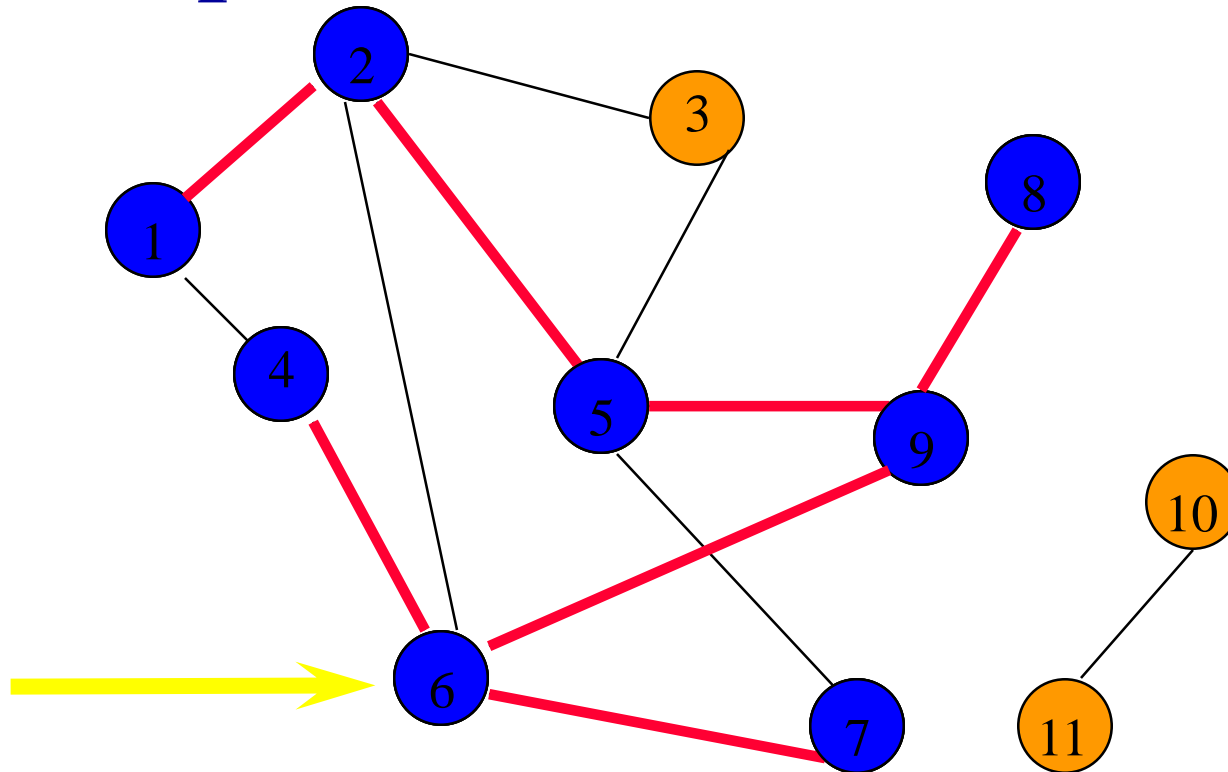
Depth-First Search Example



Label vertex 4 and return to 6.

From vertex 6 do a DFS(7).

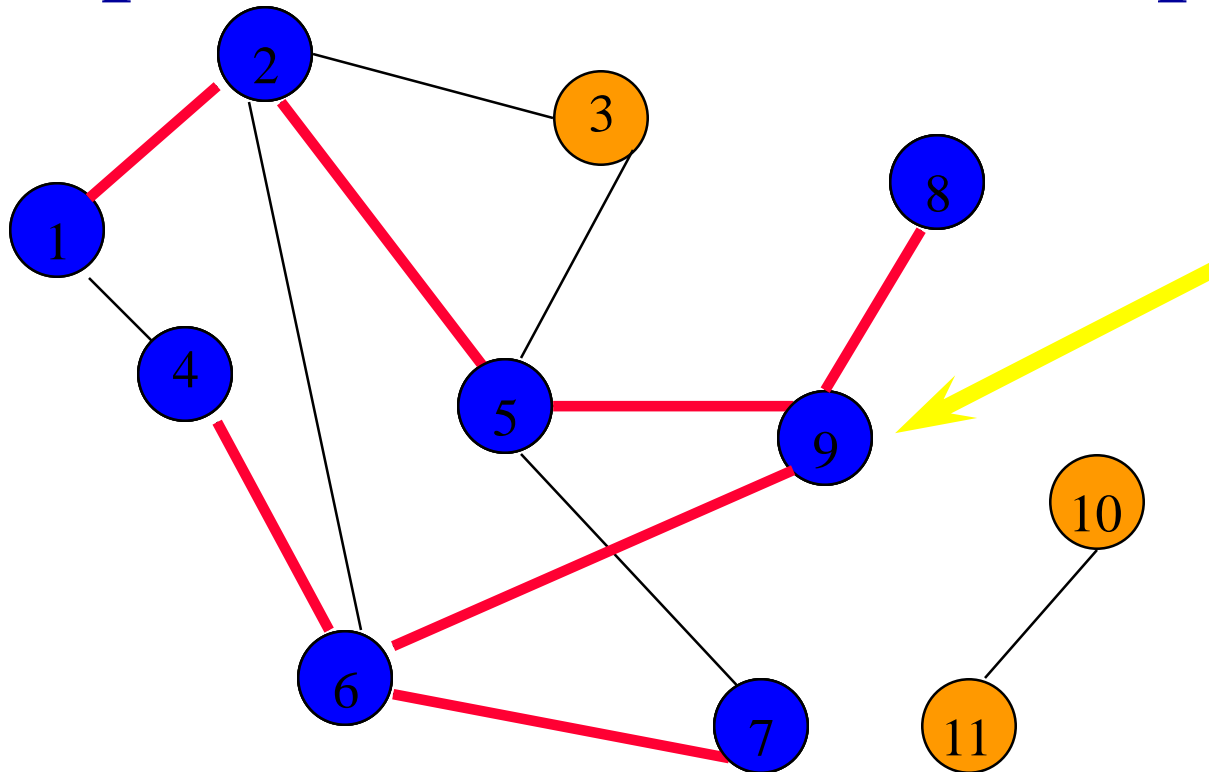
Depth-First Search Example



Label vertex **7** and return to **6**.

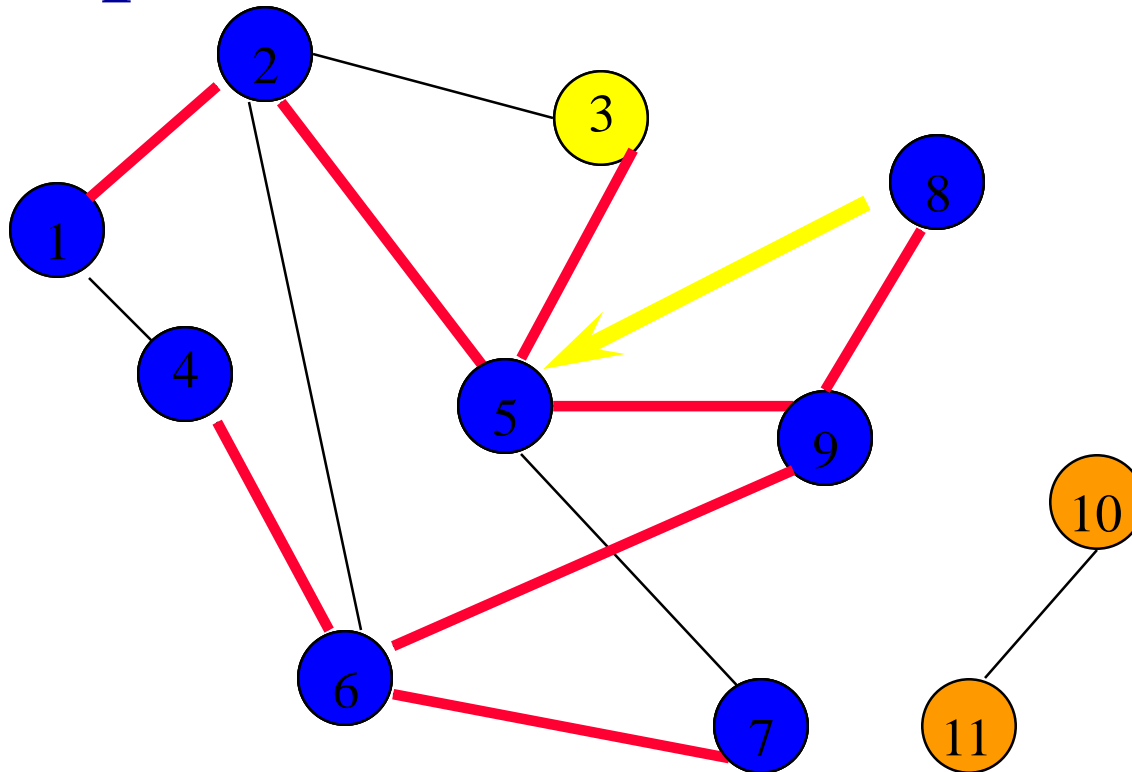
Return to **9**.

Depth-First Search Example



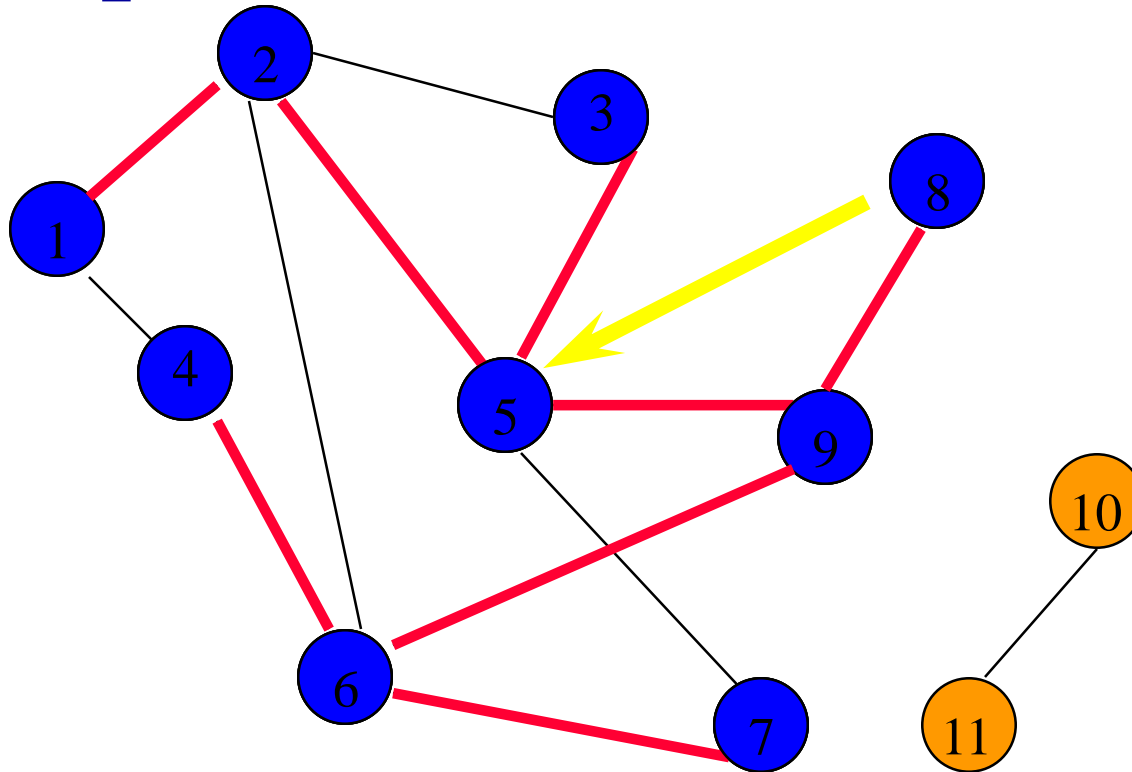
Return to 5.

Depth-First Search Example



Do a **DFS(3)**.

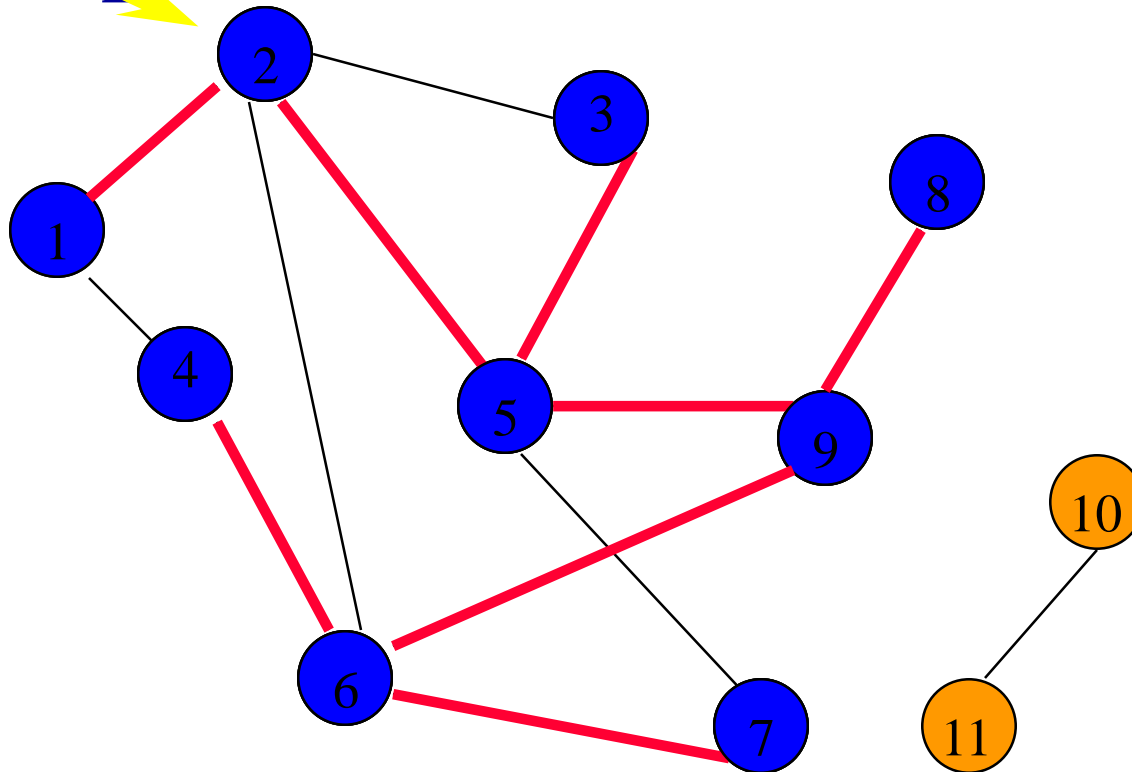
Depth-First Search Example



Label **3** and return to **5**.

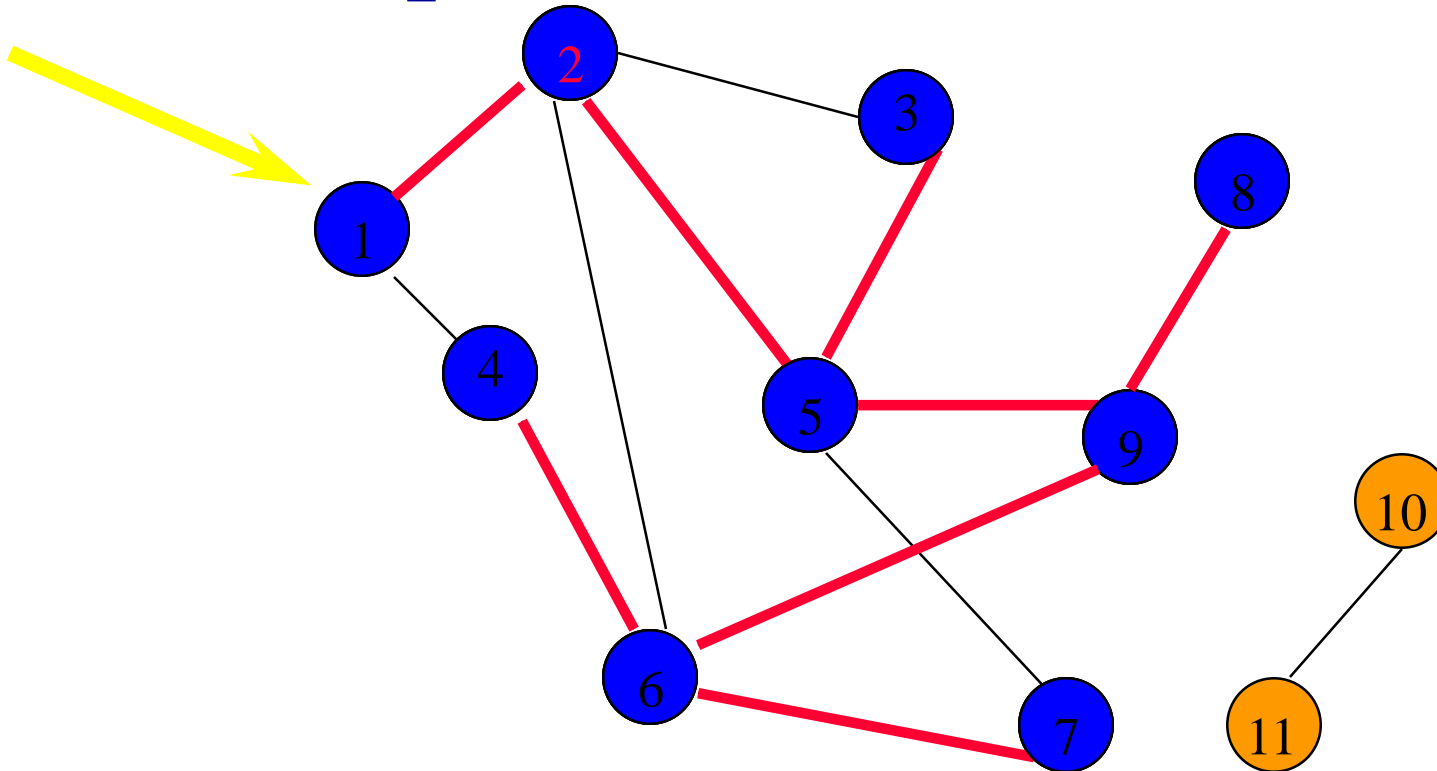
Return to **2**.

Depth-First Search Example



Return to 1.

Depth-First Search Example



Return to invoking method.

Depth-First Search Property

- All vertices reachable from the start vertex (including the start vertex) are visited.

Path From Vertex v To Vertex u

- Start a depth-first search at vertex v .
- Terminate when vertex u is visited or when **DFS** ends (whichever occurs first).
- Time
 - $O(n^2)$ when adjacency matrix used
 - $O(n+e)$ when adjacency lists used (e is number of edges)

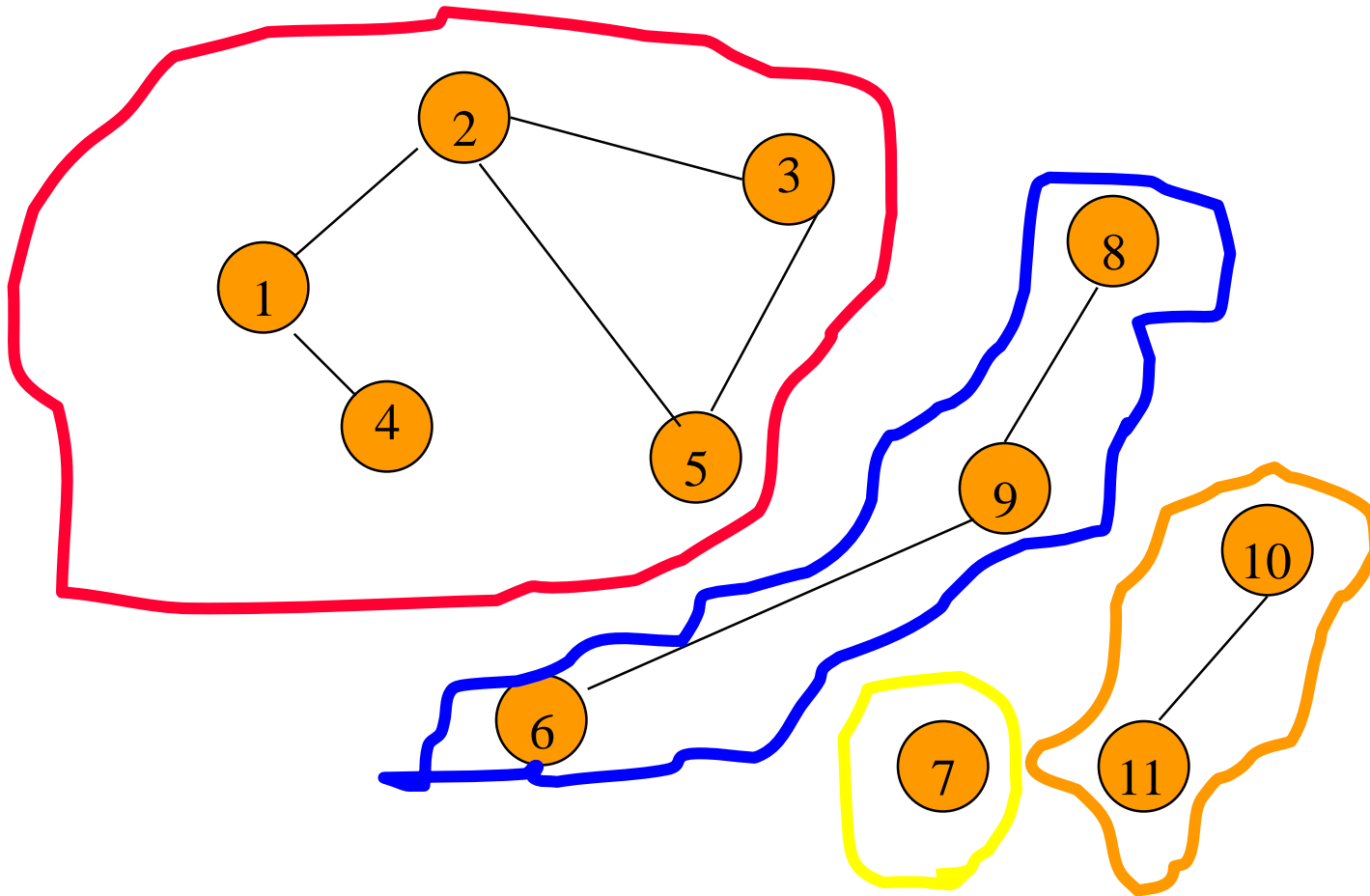
Is The Graph Connected?

- Start a depth-first search at any vertex of the graph.
- Graph is connected iff all n vertices get visited.
- Time
 - $O(n^2)$ when adjacency matrix used
 - $O(n+e)$ when adjacency lists used (e is number of edges)

Connected Components

- Start a depth-first search at any as yet unvisited vertex of the graph.
- Newly visited vertices (plus edges between them) define a component.
- Repeat until all vertices are visited.

Connected Components

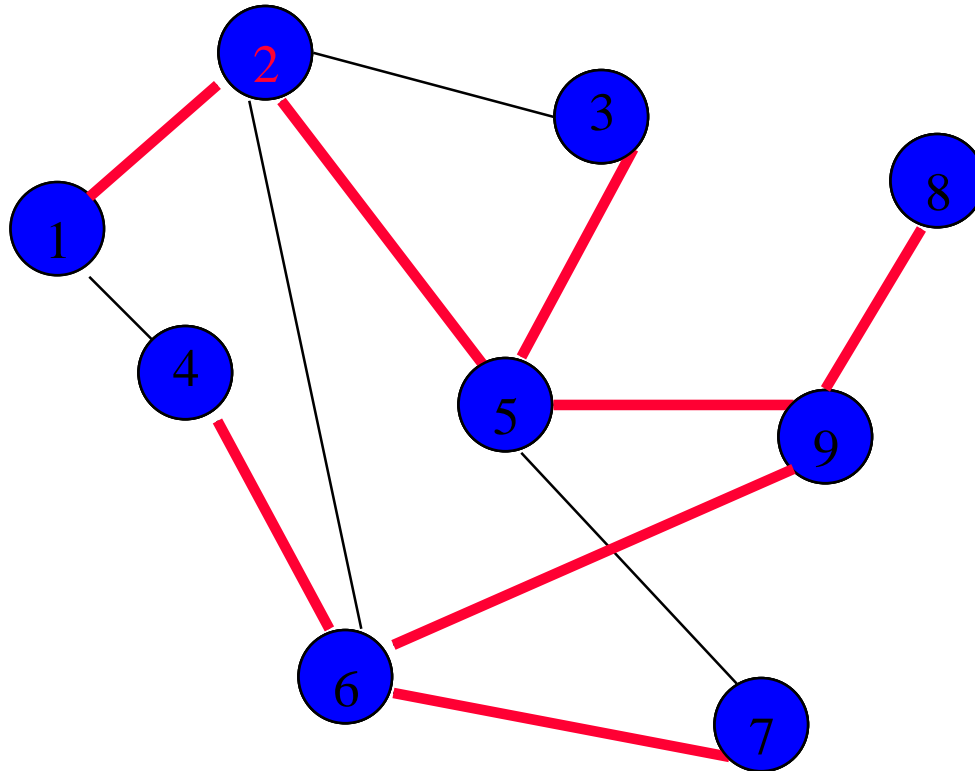


Time Complexity



- $O(n^2)$ when adjacency matrix used
- $O(n+e)$ when adjacency lists used (e is number of edges)

Spanning Tree



Depth-first search from vertex **1**.

Depth-first spanning tree.

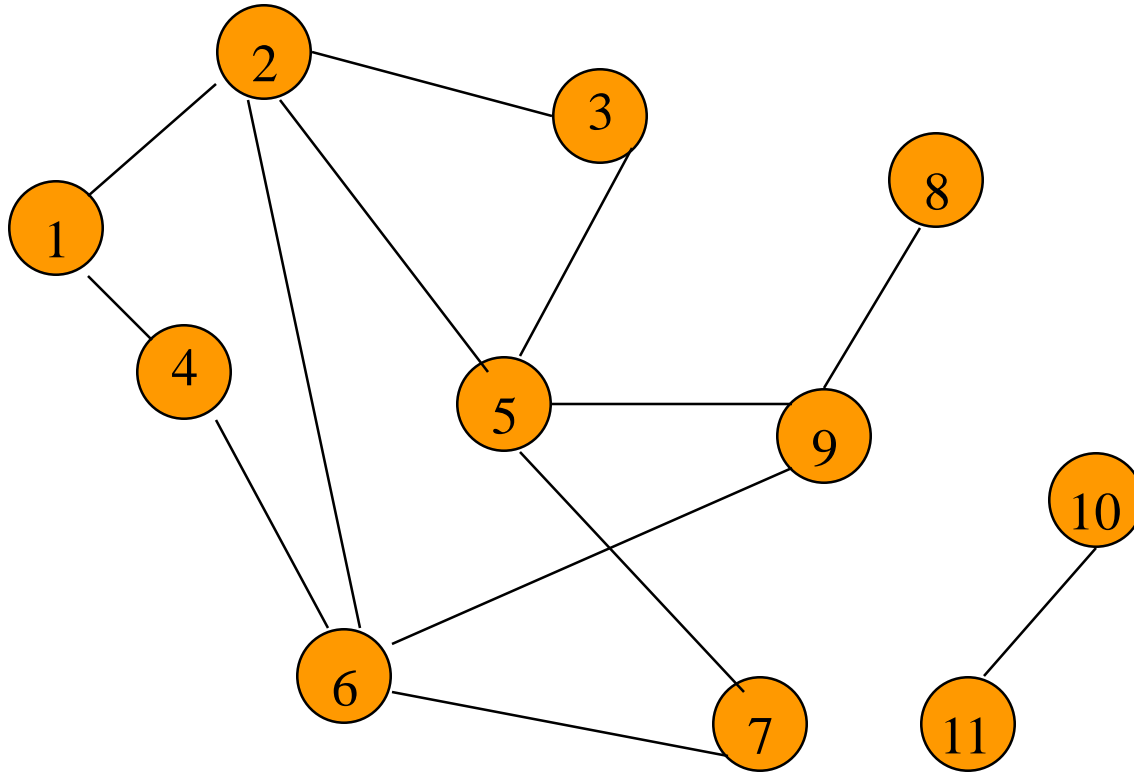
Spanning Tree

- Start a depth-first search at any vertex of the graph.
- If graph is connected, the $n-1$ edges used to get to unvisited vertices define a spanning tree (depth-first spanning tree).
- Time
 - $O(n^2)$ when adjacency matrix used
 - $O(n+e)$ when adjacency lists used (e is number of edges)

Breadth-First Search

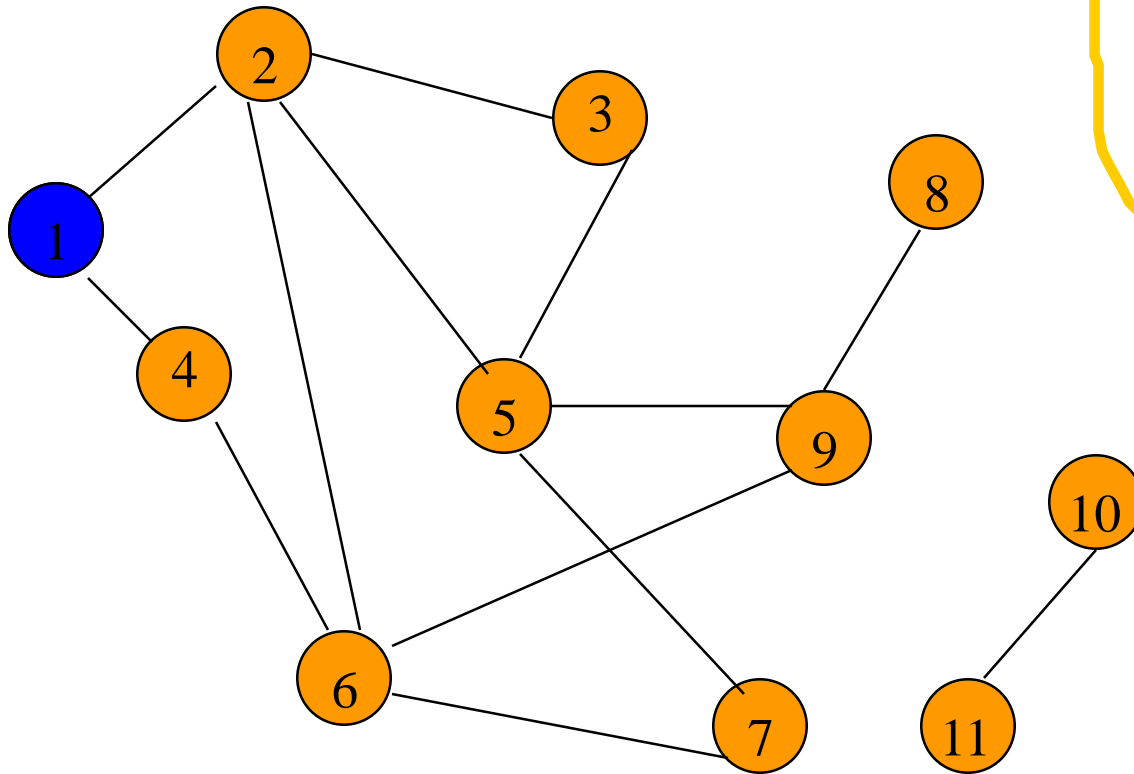
- Visit start vertex and put into a FIFO queue.
- Repeatedly remove a vertex from the queue, visit its unvisited adjacent vertices, put newly visited vertices into the queue.

Breadth-First Search Example



Start search at vertex **1**.

Breadth-First Search Example

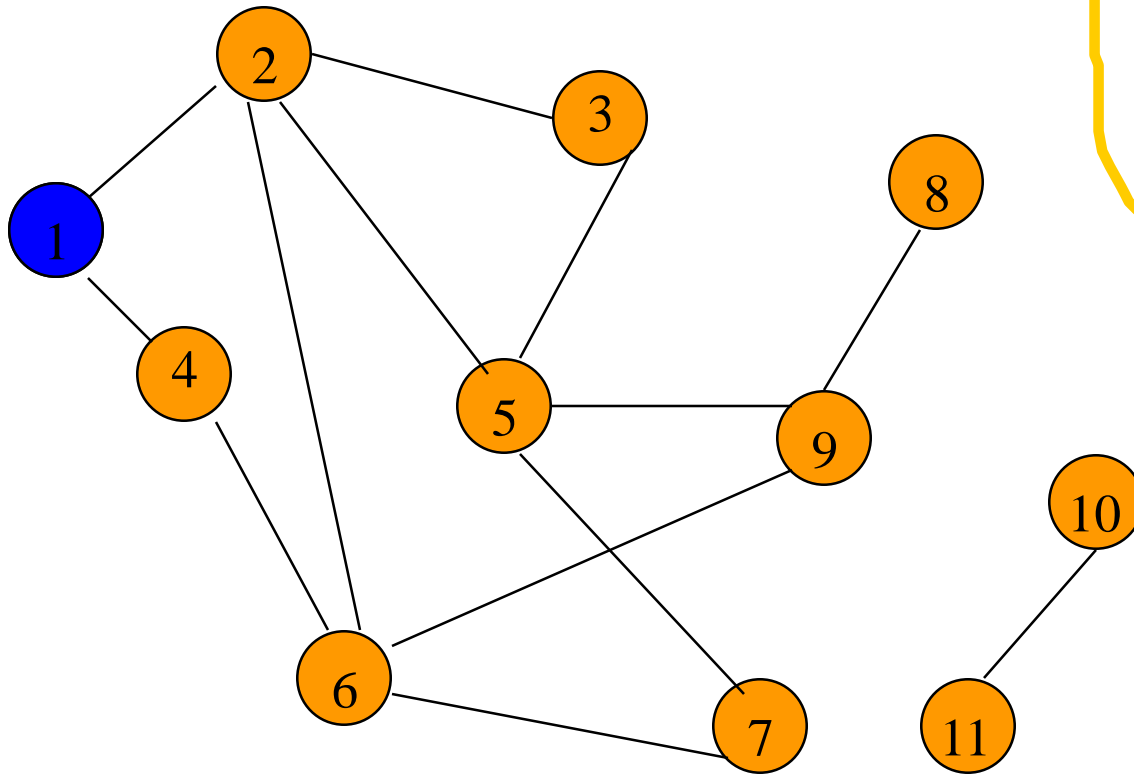


FIFO Queue

1

Visit/mark/label start vertex and put in a FIFO queue.

Breadth-First Search Example

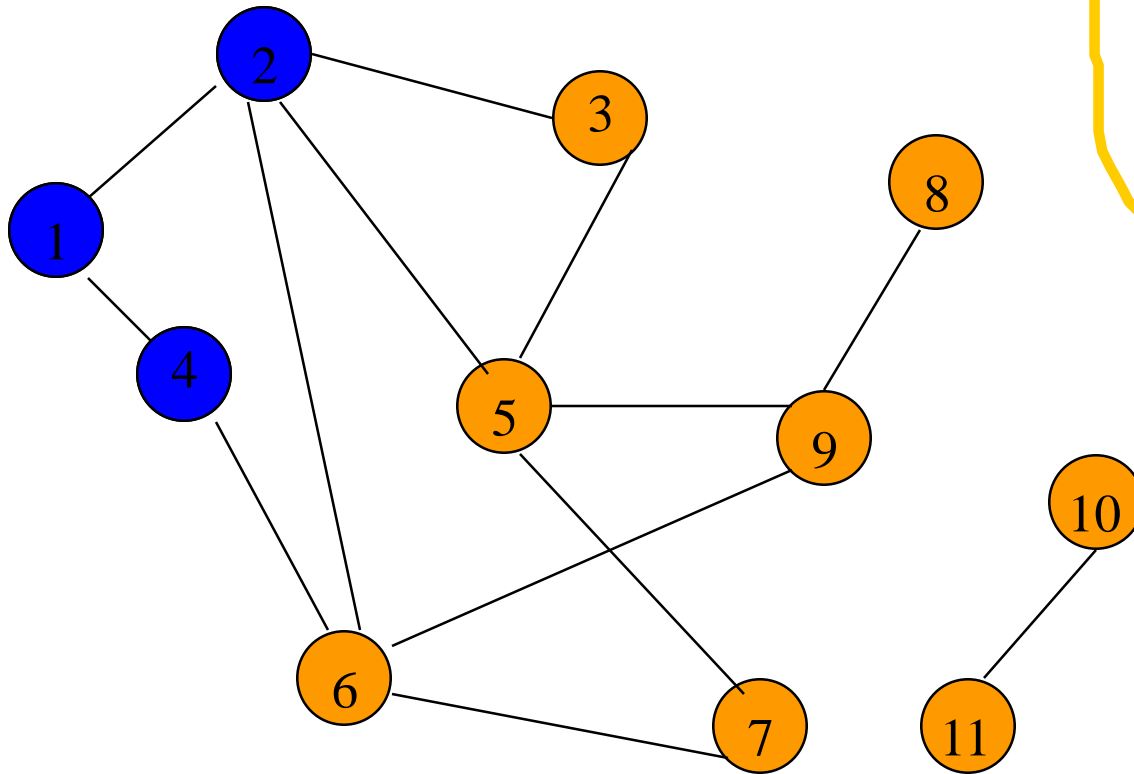


FIFO Queue

1

Remove 1 from Q; visit adjacent unvisited vertices;
put in Q.

Breadth-First Search Example

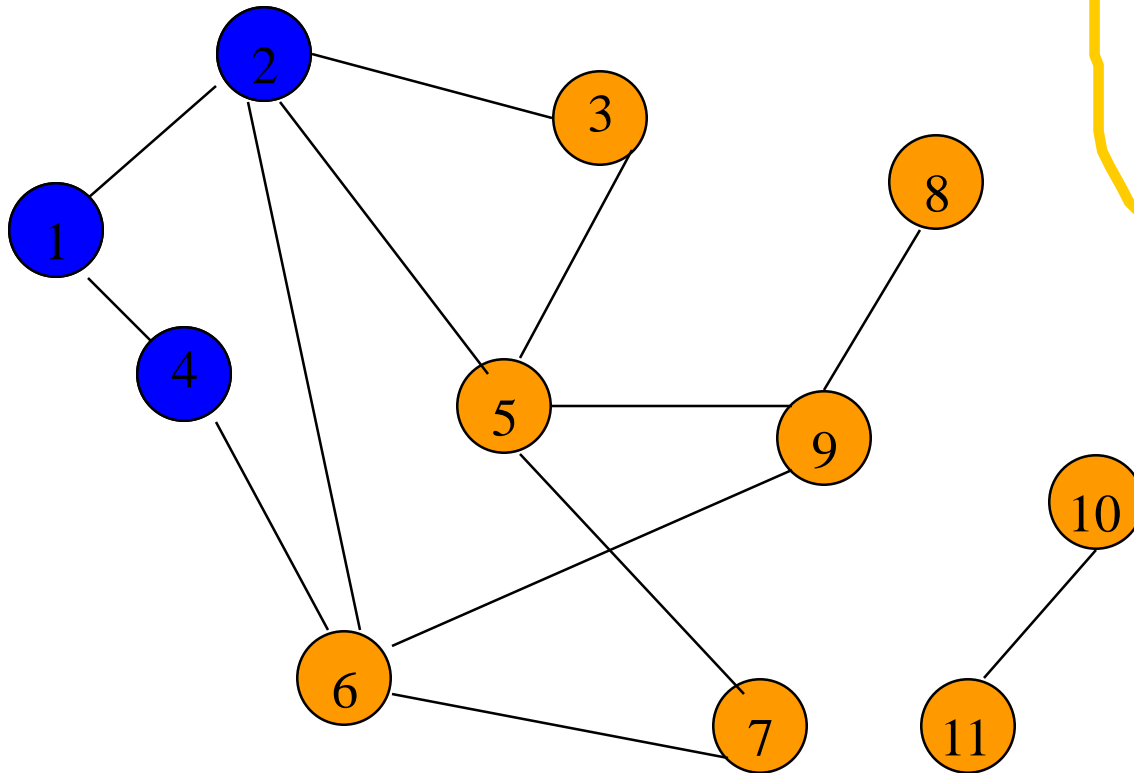


FIFO Queue

2 4

Remove 1 from Q; visit adjacent unvisited vertices;
put in Q.

Breadth-First Search Example

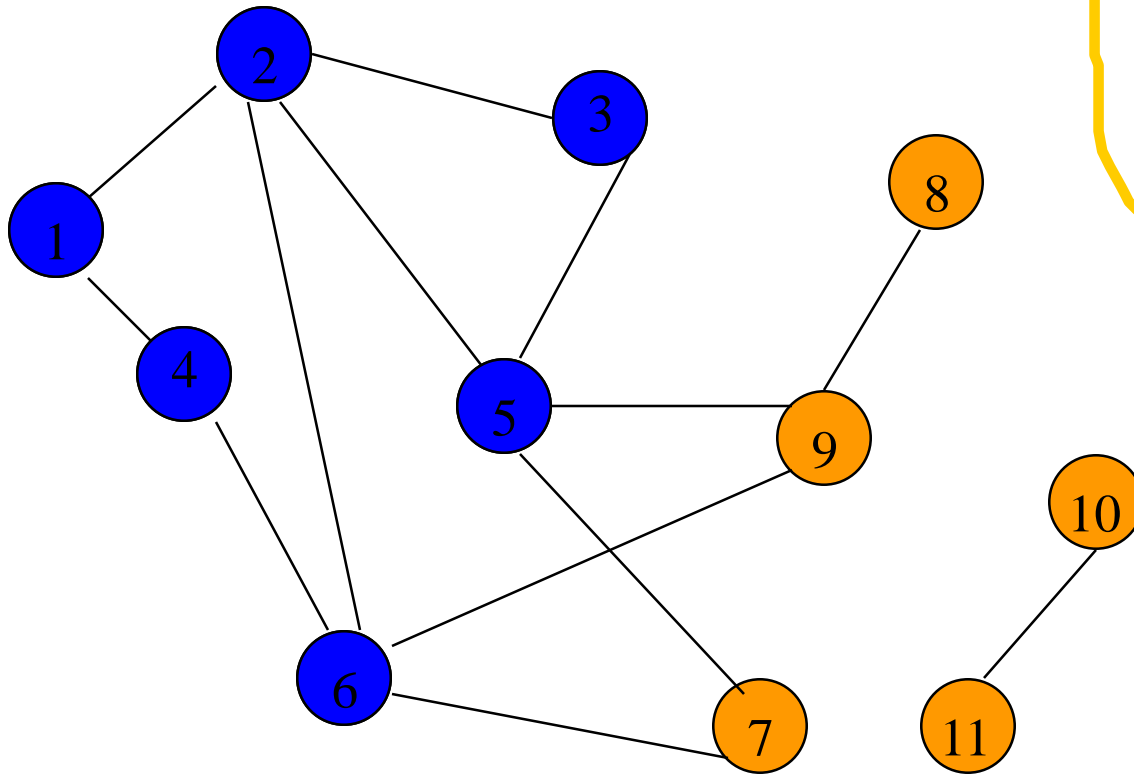


FIFO Queue

2 4

Remove 2 from Q; visit adjacent unvisited vertices;
put in Q.

Breadth-First Search Example

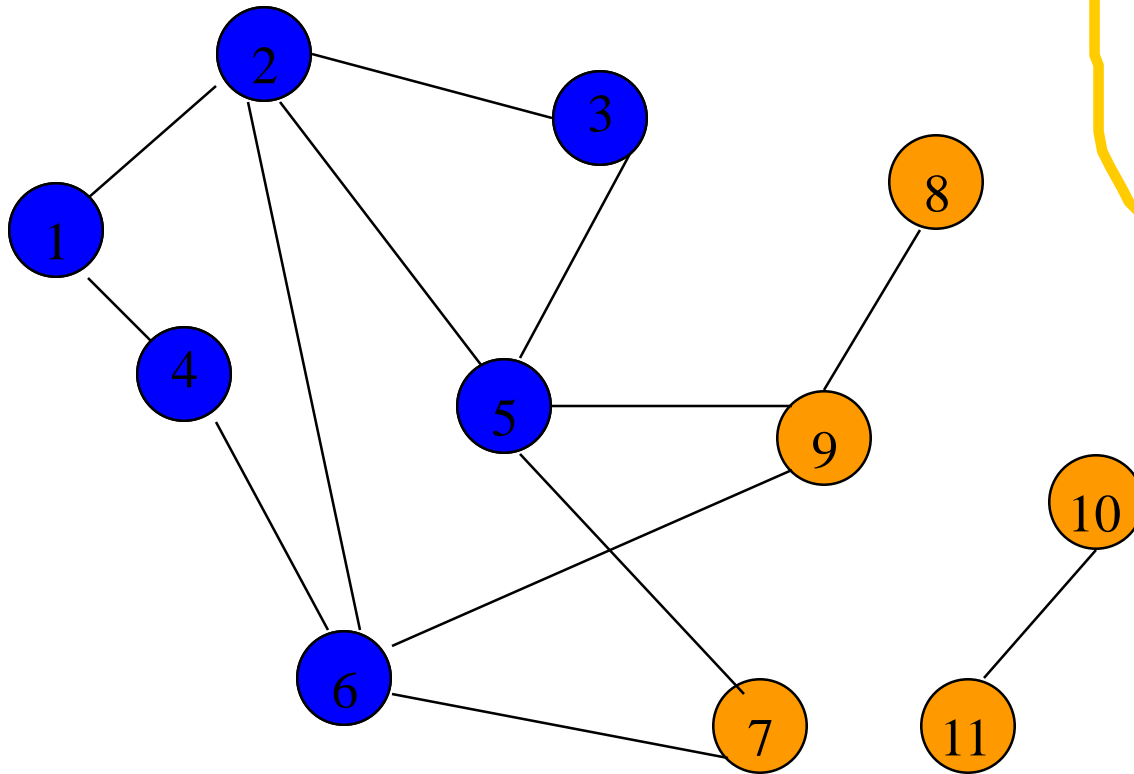


FIFO Queue

4 5 3 6

Remove 2 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

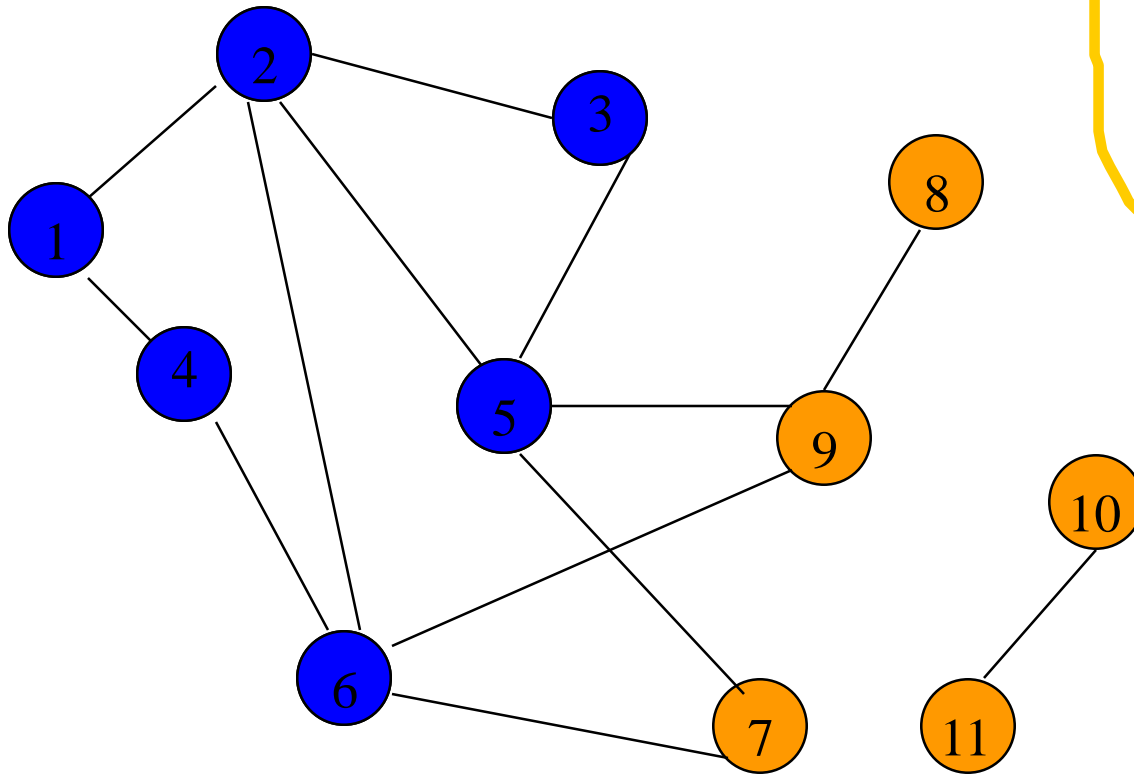


FIFO Queue

4 5 3 6

Remove 4 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

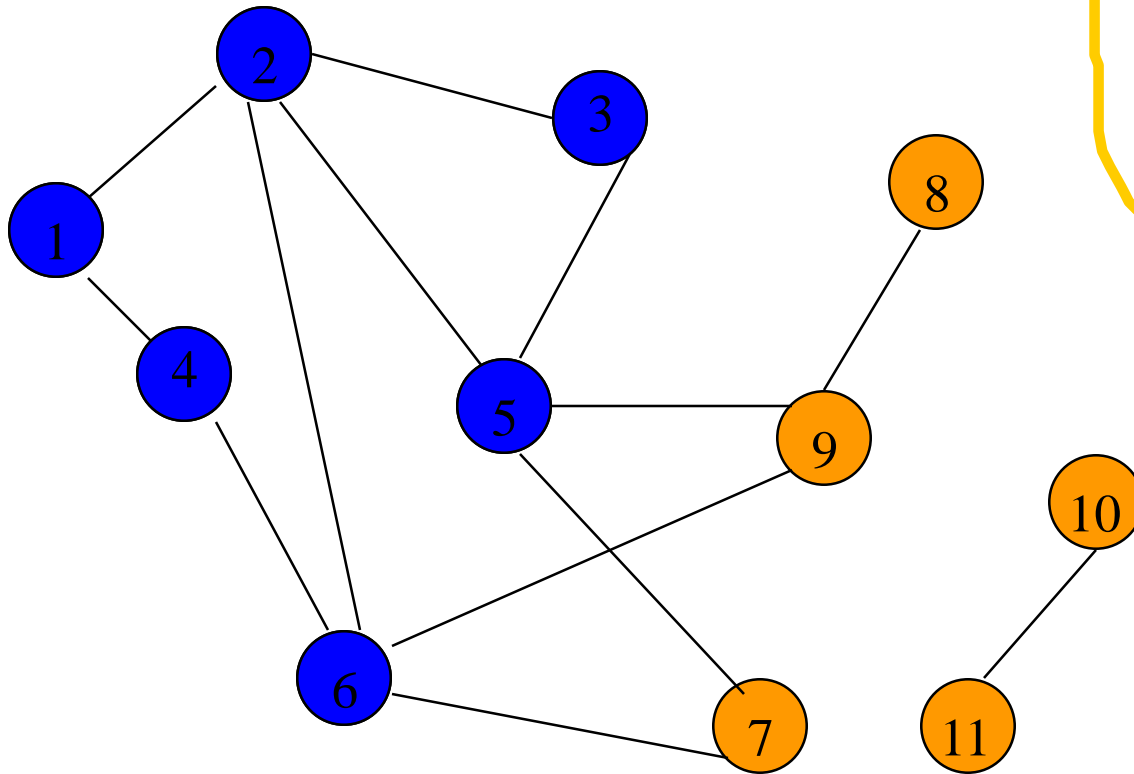


FIFO Queue

5 3 6

Remove 4 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

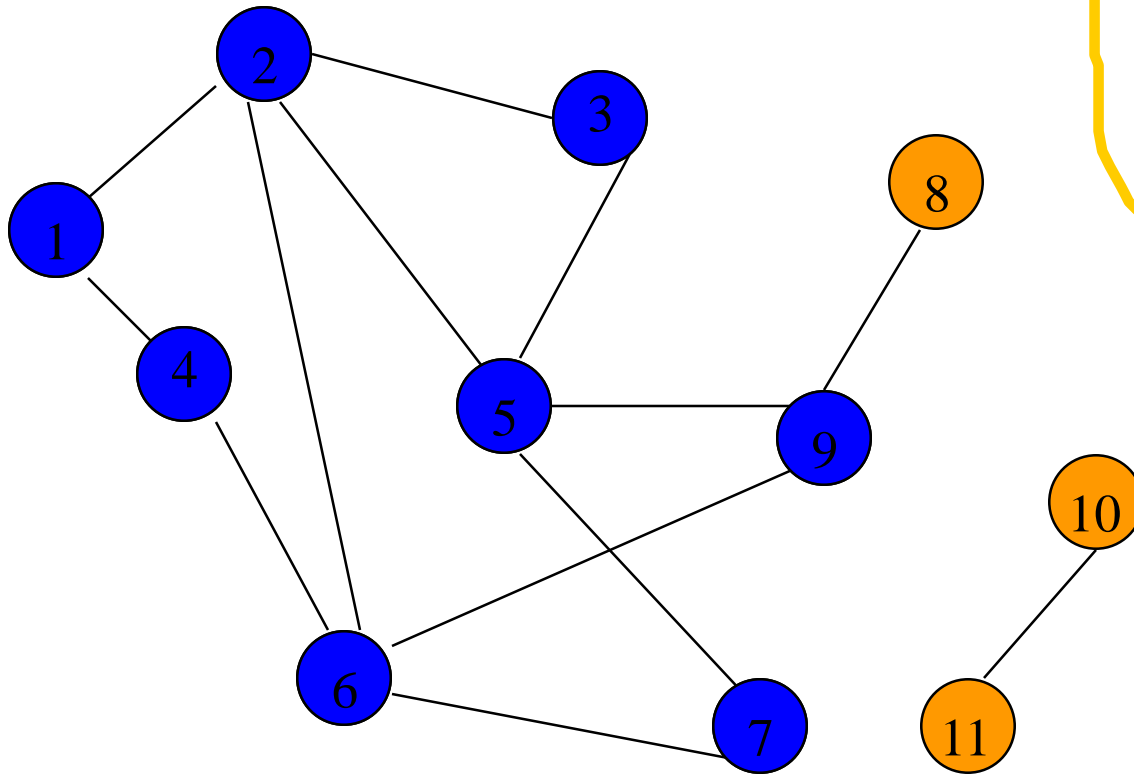


FIFO Queue

5 3 6

Remove 5 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

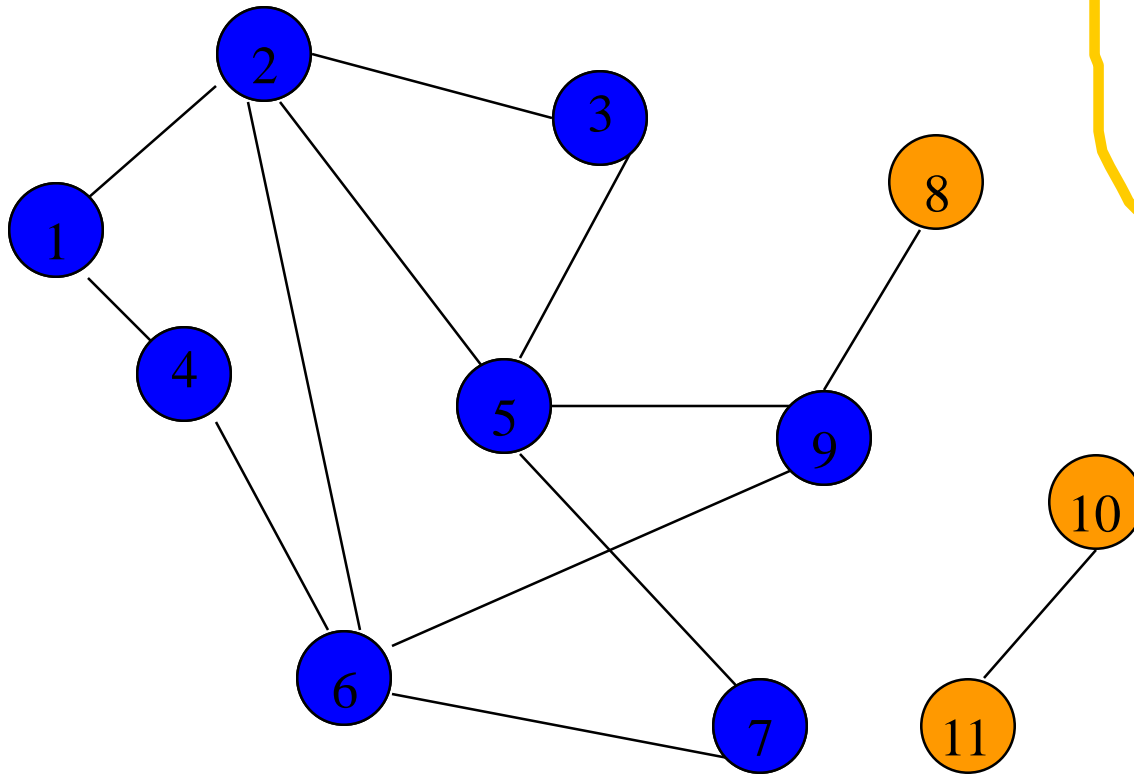


FIFO Queue

3 6 9 7

Remove 5 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

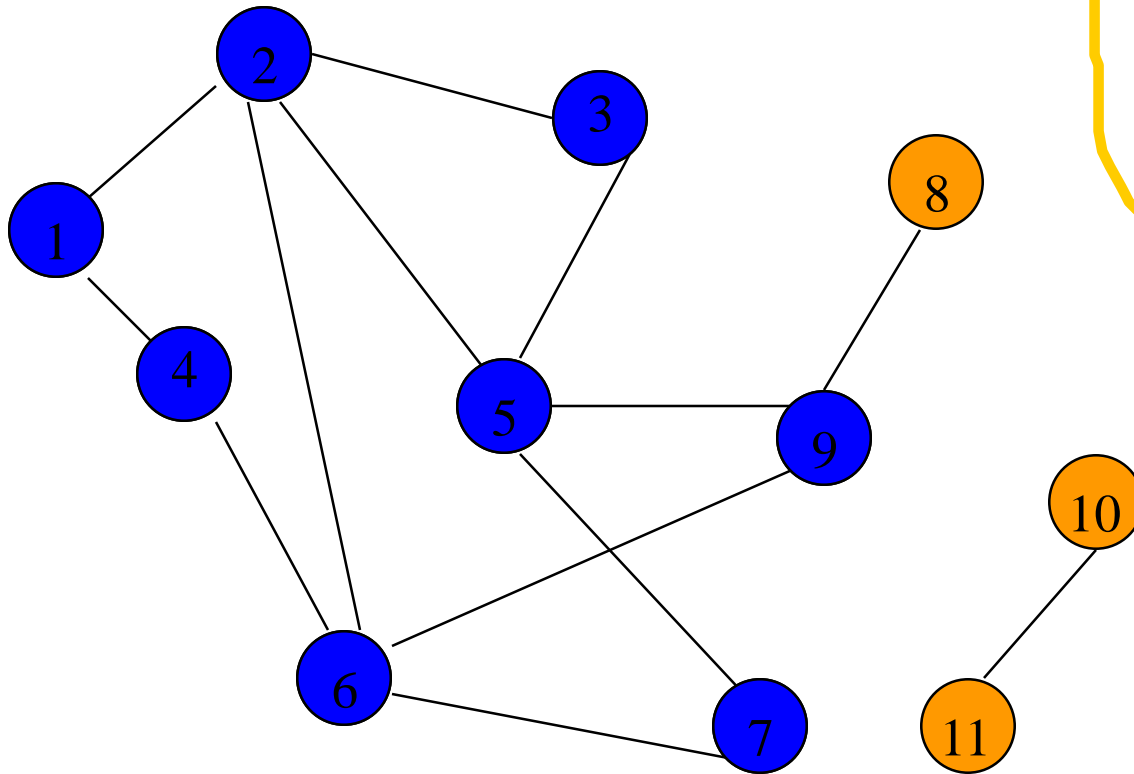


FIFO Queue

3 6 9 7

Remove 3 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

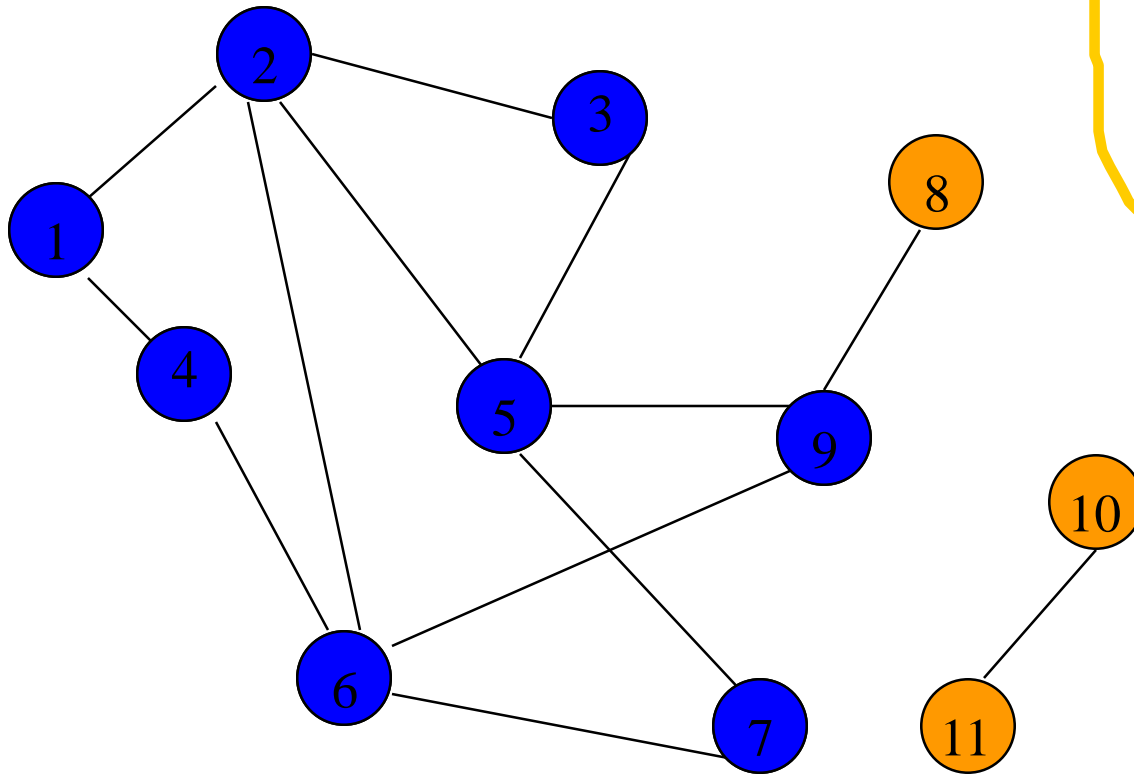


FIFO Queue

6 9 7

Remove 3 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

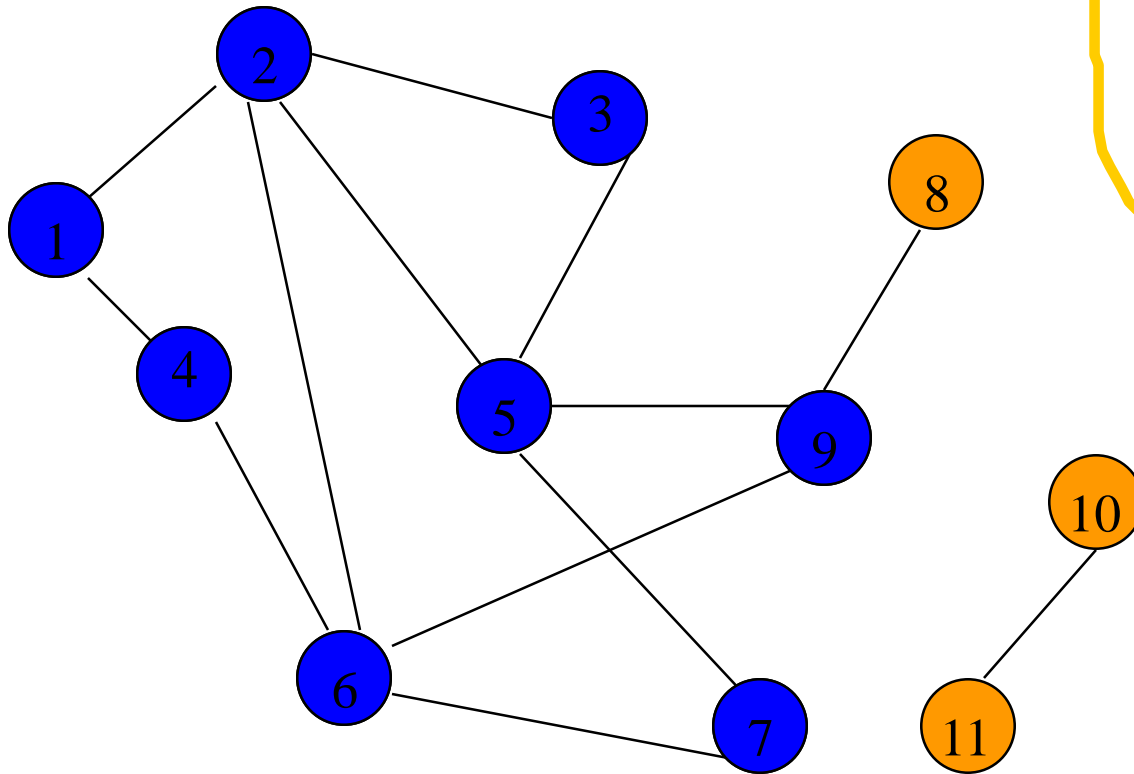


FIFO Queue

6 9 7

Remove 6 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

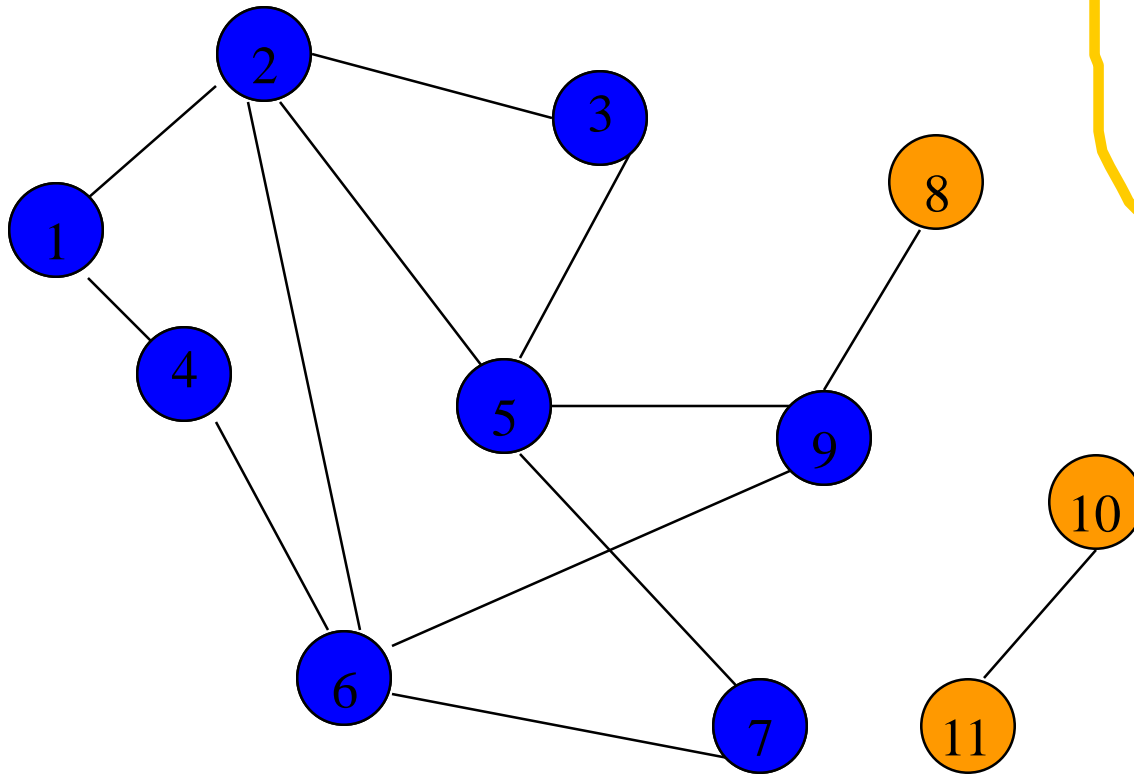


FIFO Queue

9 7

Remove 6 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

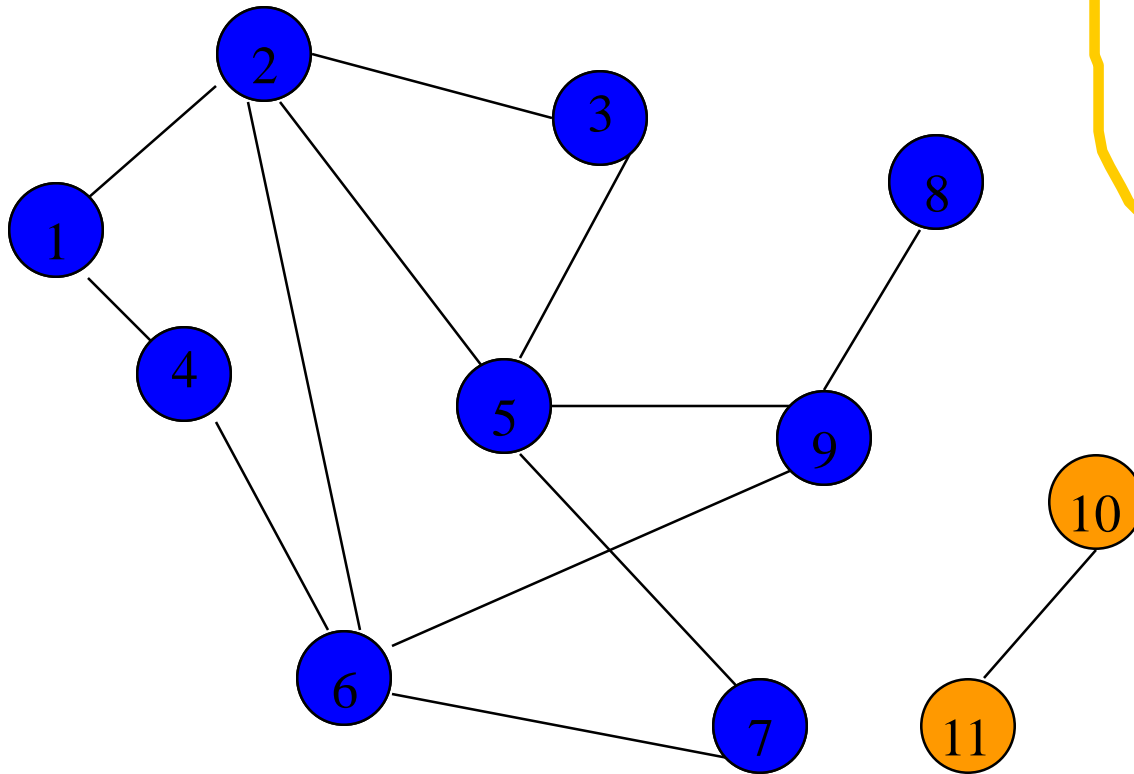


FIFO Queue

9 7

Remove 9 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

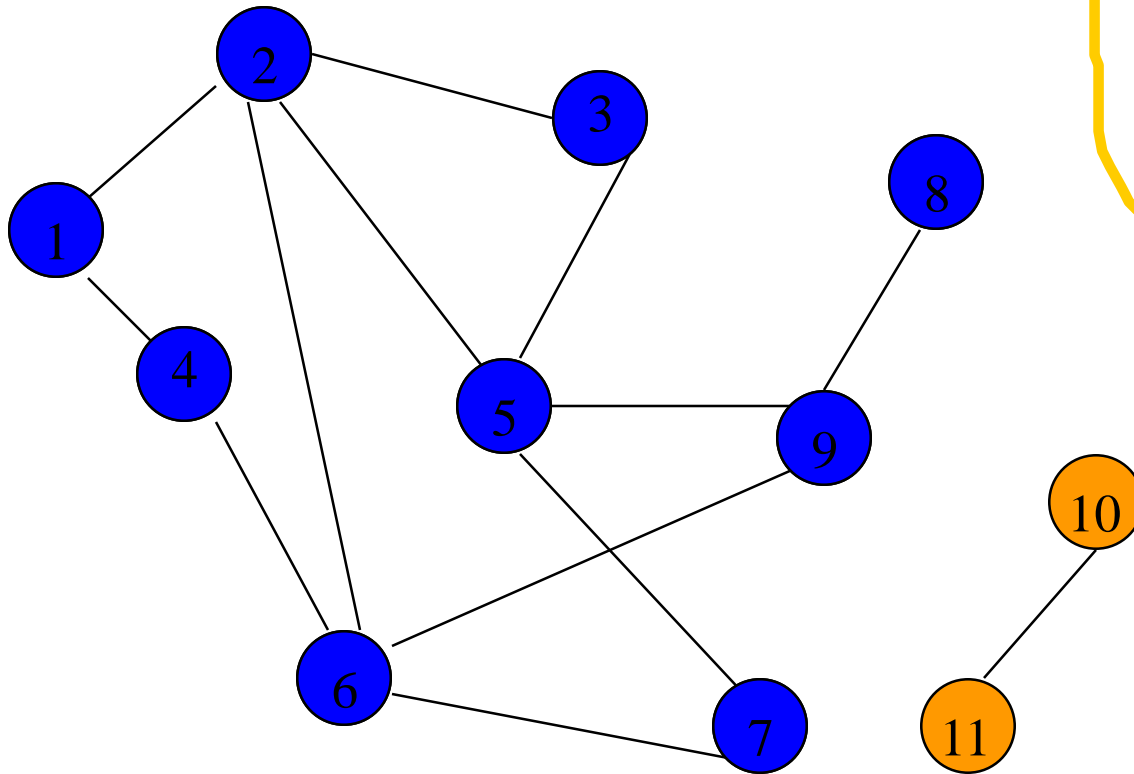


FIFO Queue

7 8

Remove 9 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

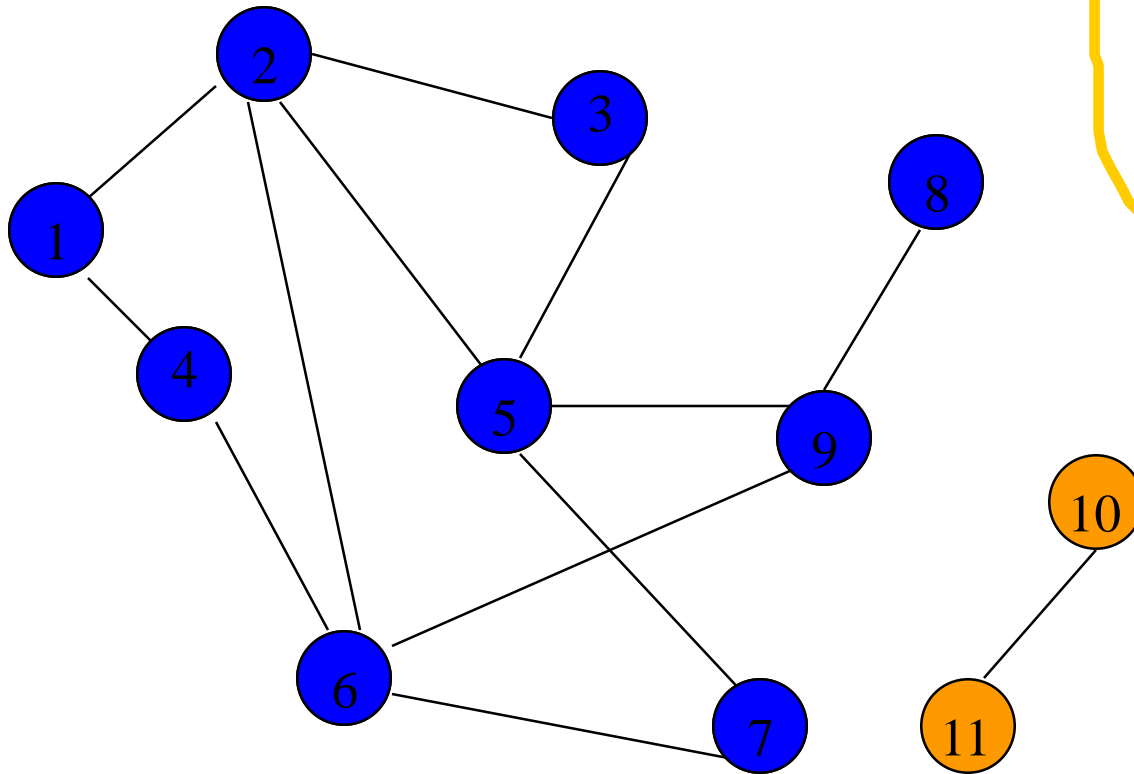


FIFO Queue

7 8

Remove 7 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

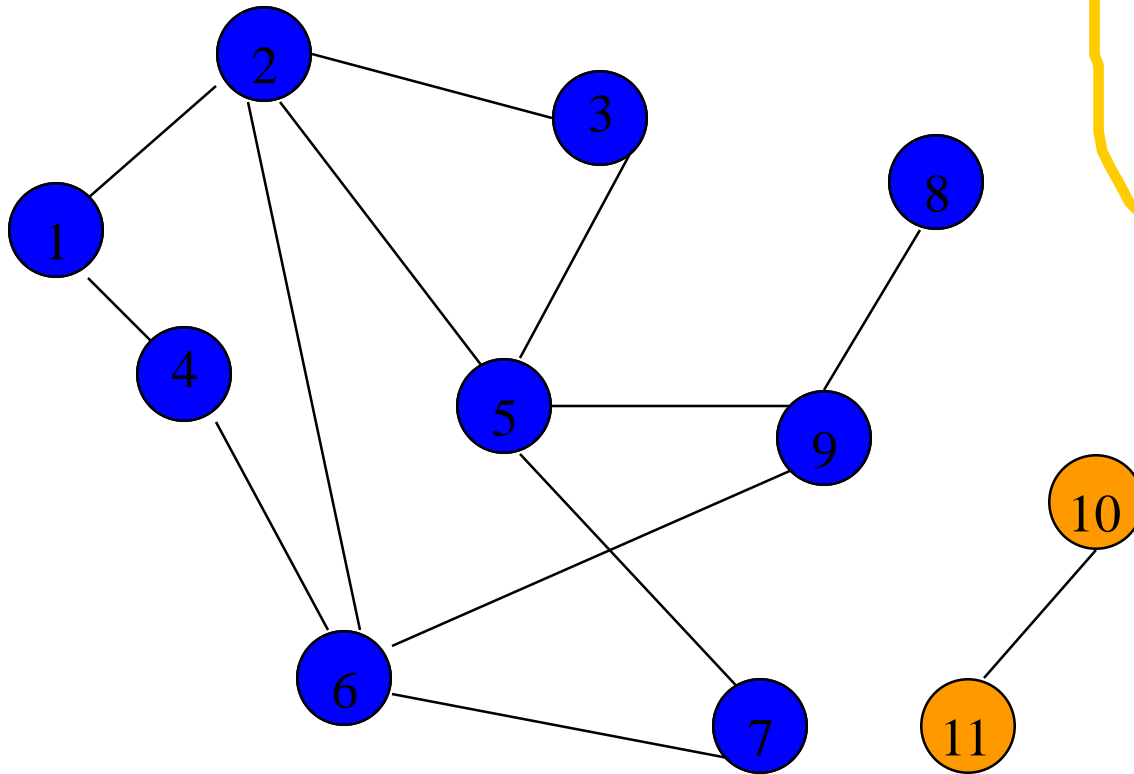


FIFO Queue

8

Remove 7 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example

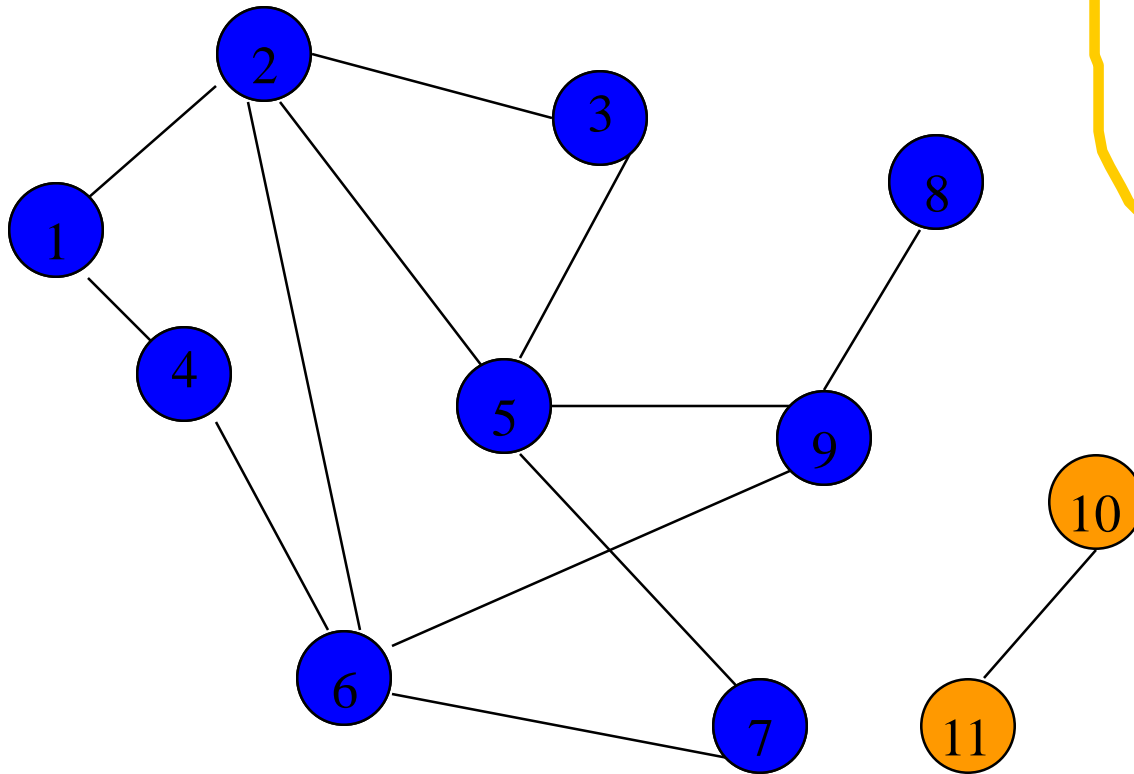


FIFO Queue

8

Remove 8 from Q ; visit adjacent unvisited vertices;
put in Q .

Breadth-First Search Example



FIFO Queue

Queue is empty. Search terminates.

Time Complexity



- Each visited vertex is put on (and so removed from) the queue exactly once.
- When a vertex is removed from the queue, we examine its adjacent vertices.
 - $O(n)$ if adjacency matrix used
 - $O(\text{vertex degree})$ if adjacency lists used
- Total time
 - $O(mn)$, where m is number of vertices in the component that is searched (adjacency matrix)

Time Complexity



- $O(n + \text{sum of component vertex degrees})$ (adj. lists)
 $= O(n + \text{number of edges in component})$

Breadth-First Search Properties

- Same complexity as DFS.
- Same properties with respect to path finding, connected components, and spanning trees.
- Edges used to reach unlabeled vertices define a depth-first spanning tree when the graph is connected.
- There are problems for which bfs is better than dfs and vice versa.