

Data Mining 2025

Naive Bayes and Logistic Regression

Exercise 1: Multinomial Naive Bayes for Text Classification

(a) The vocabulary consists of:

1. spaghetti
2. tomato
3. minced
4. meat
5. gorgonzola
6. eggplant
7. zucchini
8. olive
9. oil
10. garlic
11. feta
12. yogurt
13. cucumber

Hence, $|V| = 13$. The total number of words in Italian recipes is 12, and in Greek recipes 7. Hence, we get:

$$\begin{aligned}\hat{P}(\text{spaghetti} \mid \text{Italian}) &= \frac{3 + 1}{12 + 13} = \frac{4}{25} \\ \hat{P}(\text{spaghetti} \mid \text{Greek}) &= \frac{0 + 1}{7 + 13} = \frac{1}{20} \\ \hat{P}(\text{yogurt} \mid \text{Italian}) &= \frac{0 + 1}{12 + 13} = \frac{1}{25} \\ \hat{P}(\text{yogurt} \mid \text{Greek}) &= \frac{1 + 1}{7 + 13} = \frac{2}{20}\end{aligned}$$

(b)

$$\hat{P}(\text{Greek}) = \frac{2}{5} \quad \hat{P}(\text{Italian}) = \frac{3}{5}$$

(c)

$$\begin{aligned} \hat{P}(\text{Italian} \mid \text{spaghetti yogurt}) &\propto \hat{P}(\text{spaghetti} \mid \text{Italian}) \hat{P}(\text{yogurt} \mid \text{Italian}) \hat{P}(\text{Italian}) \\ &= \left(\frac{4}{25}\right) \left(\frac{1}{25}\right) \left(\frac{3}{5}\right) = \frac{12}{3125} \end{aligned}$$

$$\begin{aligned} \hat{P}(\text{Greek} \mid \text{spaghetti yogurt}) &\propto \hat{P}(\text{spaghetti} \mid \text{Greek}) \hat{P}(\text{yogurt} \mid \text{Greek}) \hat{P}(\text{Greek}) \\ &= \left(\frac{1}{20}\right) \left(\frac{2}{20}\right) \left(\frac{2}{5}\right) = \frac{4}{2000} \end{aligned}$$

$$\hat{P}(\text{Italian} \mid \text{spaghetti yogurt}) = \frac{12/3125}{12/3125 + 4/2000} \approx 0.66$$

Exercise 2: Naive Bayes for Text Classification

(a) We want to calculate

$$\begin{aligned} &\hat{P}(\text{Funk} \mid \text{s5}) \\ &= \frac{\hat{P}(\text{Funk}, \text{s5})}{\hat{P}(\text{s5})} && \text{(definition of conditional probability)} \\ &= \frac{\hat{P}(\text{Funk}) \hat{P}(\text{s5} \mid \text{Funk})}{\hat{P}(\text{s5})} && (\hat{P}(A, B) = \hat{P}(B) \hat{P}(B|A)) \\ &= \frac{\text{prior} \times \text{likelihood}}{\text{(marginal) probability of evidence}} \\ &\propto \hat{P}(\text{Funk}) \hat{P}(\text{s5} \mid \text{Funk}) && \text{(We don't need to compute } \hat{P}(\text{s5}) \text{ (yet).)} \\ &= \hat{P}(\text{Funk}) \prod_{w \in \text{s5}} \hat{P}(w \mid \text{Funk}) && \text{(Naive Bayes assumption)} \\ &= \hat{P}(\text{Funk}) \prod_{w_i \in \text{s5} \cap V} \frac{\text{count}(w_i, \text{Funk}) + 1}{\sum_{w_j \in V} \text{count}(w_j, \text{Funk}) + |V|} && \text{(smoothed probabilities)} \\ &= \hat{P}(\text{Funk}) \prod_{w_i \in \text{s5} \cap V} \frac{\text{count of } w_i \text{ in class Funk} + 1}{\text{total count of all words in class Funk} + |V|} \\ &= \hat{P}(\text{Funk}) \hat{P}(\text{hell} \mid \text{Funk}) \hat{P}(\text{ya} \mid \text{Funk}) && (\text{burn} \notin V, \text{ so ignore it}) \\ &= \frac{1}{2} \times \frac{0+1}{9+9} \times \frac{3+1}{9+9} = \frac{1}{162} \\ &= \hat{P}(\text{Funk}, \text{s5}) && \text{(Reminder: not done yet! This isn't } \hat{P}(\text{Funk} \mid \text{s5}).) \end{aligned}$$

The proportionality above holds because $\hat{P}(s5)$ does not depend on the class. In general it is the case that a conditional probability is proportional to the corresponding joint probability: $P(A | B) = P(A, B) \times \frac{1}{P(B)} \propto P(A, B)$.

Now we know the joint probability of class “Funk” and document s5. Let’s calculate the same for class “Metal”:

$$\begin{aligned}\hat{P}(\text{Metal}, s5) &= \hat{P}(\text{Metal})\hat{P}(\text{hell} | \text{Metal})\hat{P}(\text{ya} | \text{Metal}) && \text{(again ignore burn)} \\ &= \frac{1}{2} \times \frac{3+1}{7+9} \times \frac{0+1}{7+9} = \frac{1}{128}.\end{aligned}$$

Now we only need to calculate the marginal probability of the evidence (of observing s5): $\hat{P}(s5)$. We get this by marginalising over all (two) of the classes:

$$\hat{P}(s5) = \hat{P}(\text{Funk}, s5) + \hat{P}(\text{Metal}, s5) = \frac{1}{162} + \frac{1}{128} = \frac{145}{10368}.$$

Then, we can calculate our final answer:

$$\begin{aligned}\hat{P}(\text{Funk} | s5) &= \frac{\hat{P}(\text{Funk}, s5)}{\hat{P}(s5)} \\ &= \frac{1}{162} \div \frac{145}{10368} \\ &= \frac{1}{162} \times \frac{10368}{145} \\ &= \frac{64}{145} \approx 0.44.\end{aligned}$$

Similarly,

$$\begin{aligned}\hat{P}(\text{Metal} | s5) &= \frac{\hat{P}(\text{Metal}, s5)}{\hat{P}(s5)} \\ &= \frac{1}{128} \div \frac{145}{10368} \\ &= \frac{1}{128} \times \frac{10368}{145} \\ &= \frac{81}{145} \approx 0.56,\end{aligned}$$

though of course we could have calculated this also as

$$\begin{aligned}\hat{P}(\text{Metal} \mid s5) &= 1 - \hat{P}(\text{Funk} \mid s5) \\ &= 1 - \frac{64}{145} \\ &= \frac{81}{145} \approx 0.56.\end{aligned}$$

If all we care about is prediction of the class with maximum probability, we don't actually even need to calculate $\hat{P}(s5)$, since, as mentioned before, the joint probabilities are already proportional to the conditional ones (i.e. the class with highest joint probability is also the class with the highest conditional probability).

- (b) $\hat{P}(\text{ya} \mid \text{Metal}) = \frac{0}{7} = 0$ and $\hat{P}(\text{hell} \mid \text{Funk}) = \frac{0}{9} = 0$. Hence both $\hat{P}(\text{hell} \mid \text{Metal})\hat{P}(\text{ya} \mid \text{Metal})$ and $\hat{P}(\text{hell} \mid \text{Funk})\hat{P}(\text{ya} \mid \text{Funk})$ are zero, which leads to division by zero when we try to compute $\hat{P}(\text{Funk} \mid s5)$ and $\hat{P}(\text{Metal} \mid s5)$.

Exercise 3: Logistic Regression

- (a) If the players have the same average and checkout percentage, then the probability that the player who begins wins is:

$$\frac{e^{0.12}}{1 + e^{0.12}} = 0.53.$$

Hence the advantage is 6 percentage points (53% against 47%).

- (b) Yes. For example, β_1 is positive which means that the bigger the difference in average in a 's favor, the more likely it is that a will win the game. This is in accordance with common sense.
- (c) The difference in average is $102.7 - 92.6 = 10.1$, and the difference in checkout percentage is $46.2 - 40.4 = 5.8$. Hence the probability that van Gerwen wins is:

$$\frac{\exp(0.12 + 0.135 \times 10.1 + 0.025 \times 5.8)}{1 + \exp(0.12 + 0.135 \times 10.1 + 0.025 \times 5.8)} \approx 0.84$$

So approximately 84%.

- (d) The probability that van de Voort wins is:

$$\frac{\exp(0.12 + 0.135 \times -10.1 + 0.025 \times -5.8)}{1 + \exp(0.12 + 0.135 \times -10.1 + 0.025 \times -5.8)} \approx 0.20$$

So the probability that van Gerwen wins is approximately 80%.

- (e) If

$$0.135 \times (\text{Av}_a - \text{Av}_b) + 0.025 \times (\text{Check}_a - \text{Check}_b) > -0.12$$

then player a wins, otherwise player b wins.